Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.1-Sine/79-4.1.7-d-trig-^m-a+b-c-sin-^n-^p

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [594]. This is test number [79].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.66 (592)	0.34 (2)
Mathematica	97.64 (580)	2.36 (14)
Maple	88.05 (523)	11.95 (71)
Fricas	80.98 (481)	19.02 (113)
Mupad	56.23 (334)	43.77 (260)
Maxima	55.89 (332)	44.11 (262)
Giac	52.36 (311)	47.64 (283)
Sympy	13.13 (78)	86.87 (516)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

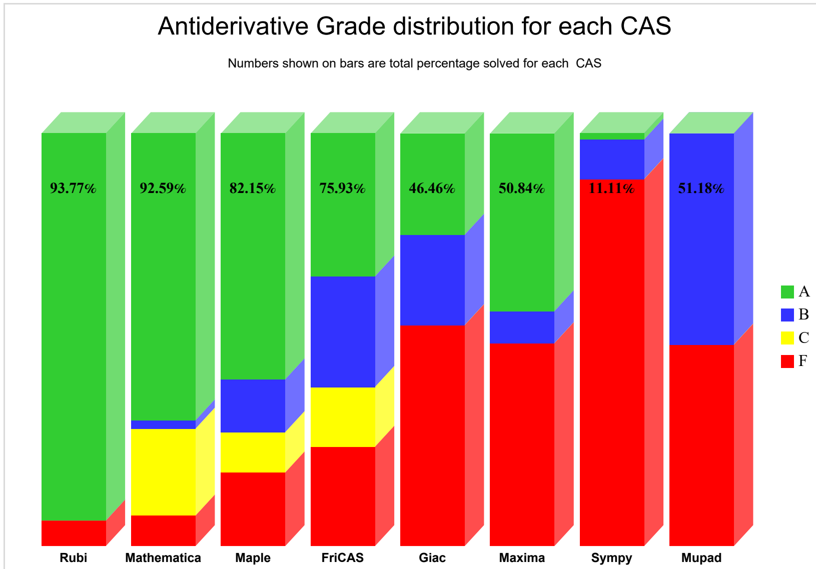
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

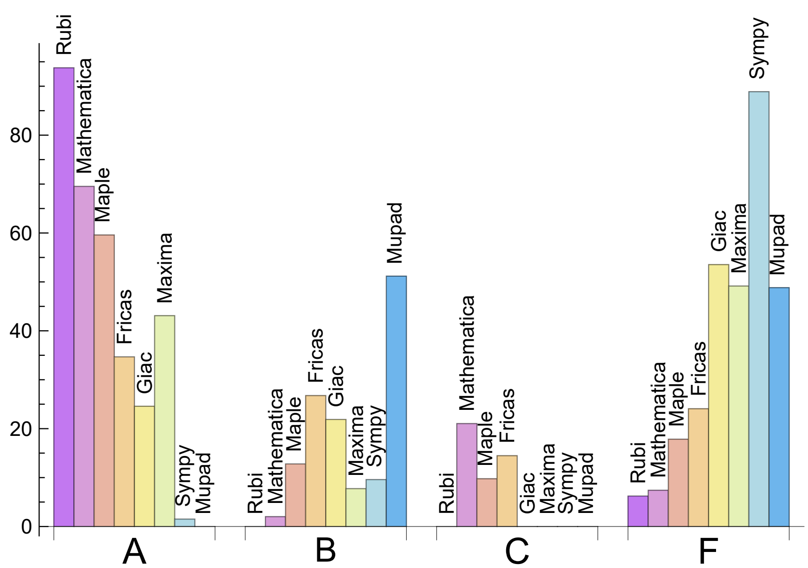
System	% A grade	% B grade	% C grade	% F grade
Rubi	93.771	0.000	0.000	6.229
Mathematica	69.529	2.020	21.044	7.407
Maple	59.596	12.795	9.764	17.845
Maxima	43.098	7.744	0.000	49.158
Fricas	34.680	26.768	14.478	24.074
Giac	24.579	21.886	0.000	53.535
Sympy	1.515	9.596	0.000	88.889
Mupad	0.000	51.178	0.000	48.822

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	2	100.00	0.00	0.00
Mathematica	14	78.57	21.43	0.00
Maple	71	100.00	0.00	0.00
Fricas	113	91.15	0.88	7.96
Mupad	260	0.00	100.00	0.00
Maxima	262	95.42	1.53	3.05
Giac	283	85.16	9.54	5.30
Sympy	516	56.78	43.22	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.37
Maxima	0.68
Fricas	1.26
Giac	1.50
Maple	1.71
Mathematica	2.19
Mupad	13.72
Sympy	18.95

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	129.88	1.00	92.50	1.00
Maxima	132.85	1.68	69.00	1.09
Mathematica	170.90	1.45	97.00	1.00
Giac	575.75	6.59	83.00	1.44
Maple	798.57	9.32	104.00	1.08
Mupad	2015.04	8.18	99.50	1.15
Sympy	5135.24	138.32	143.50	3.05
Fricas	5313.19	55.75	305.00	3.53

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

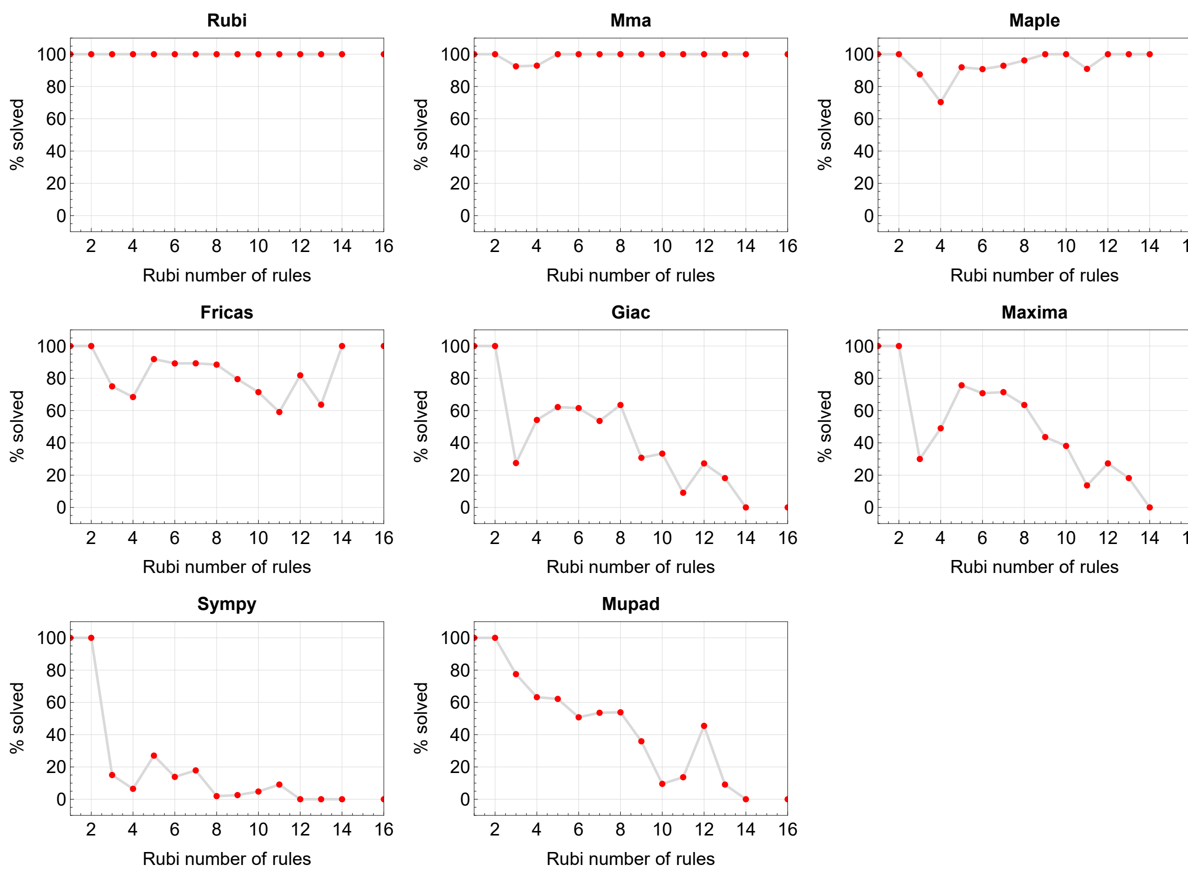


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

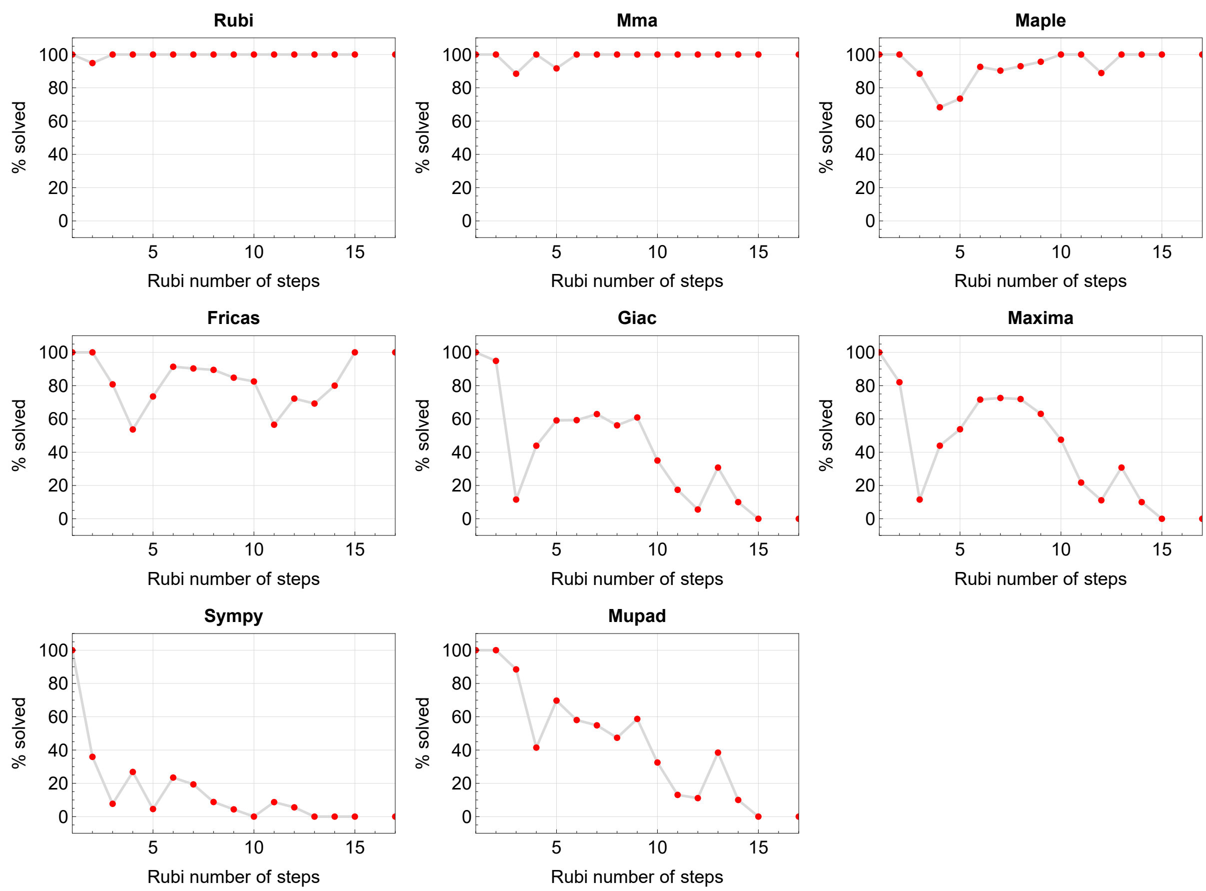


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

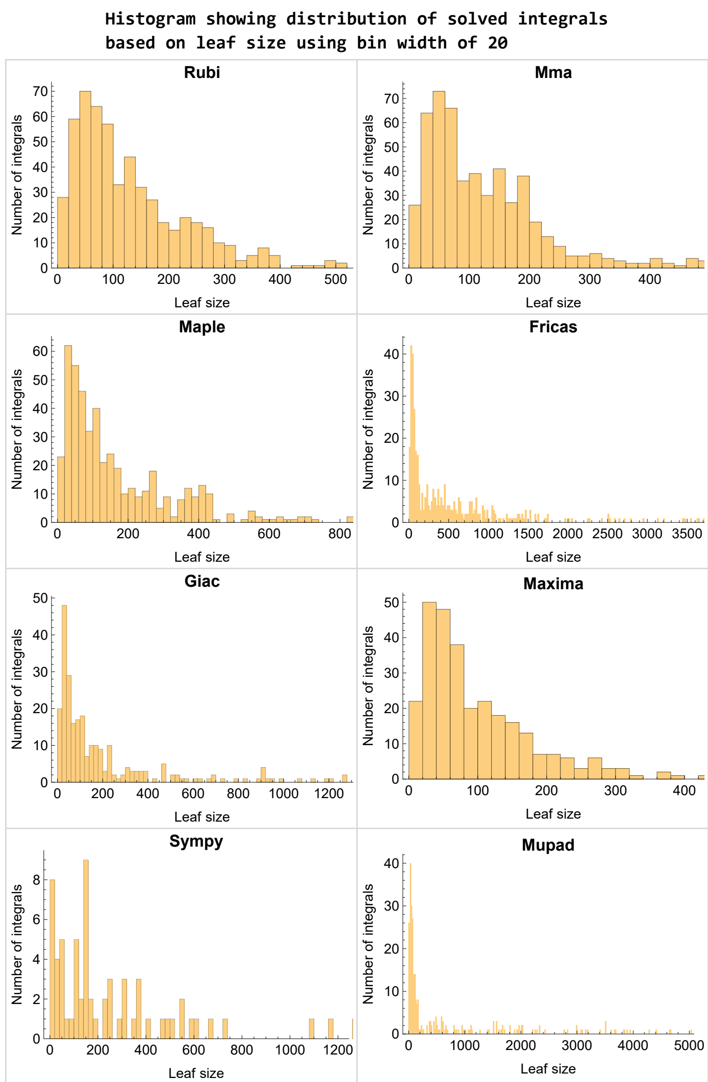


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

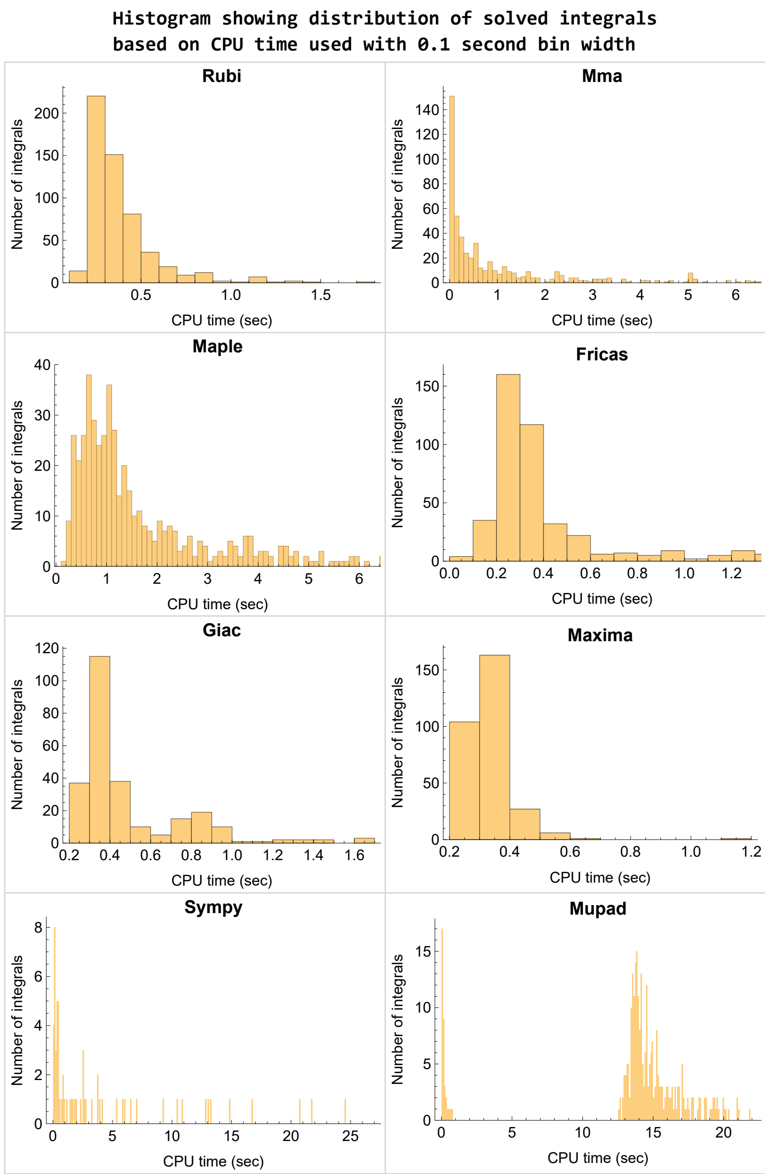


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

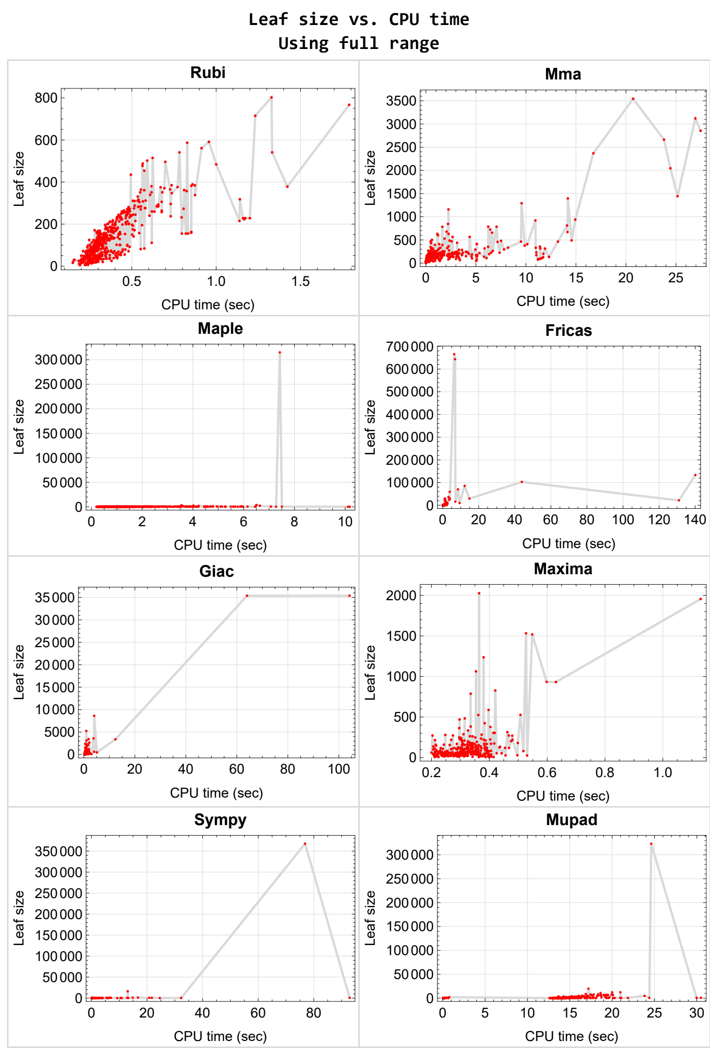


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{399, 400, 401, 402, 403, 419, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 564, 569, 570, 571, 572, 573, 577, 578, 581, 582, 583, 586, 587, 588, 589, 590}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {399, 400, 401, 402, 403}

Maple {}

Maxima {}

Fricas {399, 400, 401, 402, 403}

Sympy {}

Giac {}

Mupad {399, 400, 401, 402, 403}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {240, 244, 245, 591}

Mathematica {212, 213, 214, 215, 216, 217, 224, 225, 226, 227, 228, 229, 239, 240, 241, 242, 245, 249, 251, 252, 254, 328, 339, 348, 356, 357, 366, 372, 373, 378, 379, 392, 402, 403, 565, 566, 574, 575, 576, 594}

Maple {119, 533, 593, 594}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

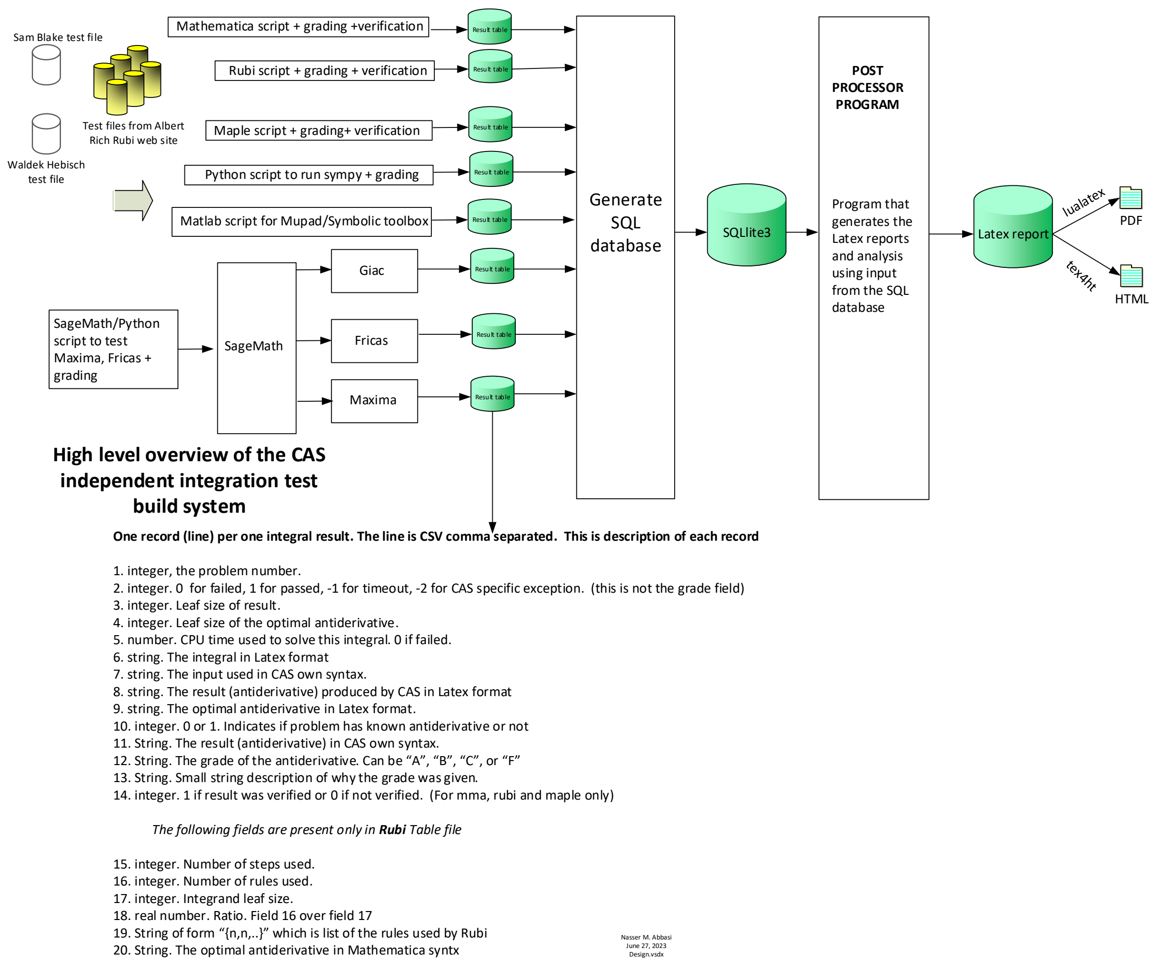
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 393, 394, 395, 396, 397, 398, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 420, 421, 422, 423, 424, 431, 432, 433, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 566, 567, 568, 574, 575, 576, 579, 580, 584, 585, 591, 592, 593, 594 }

B grade { }

C grade { }

F normal fail { 391, 392 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 77, 85, 86, 87, 88, 89, 90, 91, 92, 93, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 174, 178, 179, 203, 204, 205, 206, 207, 208, 209, 210, 211, 218, 219, 220, 221, 222, 223, 230, 231, 232, 233, 234, 235, 236, 256, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 337, 340, 341, 342, 343, 344, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 358, 359, 360, 361, 362, 363, 364, 365, 367, 368, 369, 370, 371, 372, 374, 375, 380, 384, 385, 395, 396, 407, 411, 412, 413, 414, 415, 416, 417, 418, 420, 421, 422, 431, 432, 433, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 482, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 527, 528, 529, 530, 531, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 567, 568, 579, 580, 584, 585, 592 }

B grade { 40, 54, 66, 67, 308, 310, 312, 378, 379, 565, 566, 593 }

C grade { 76, 78, 79, 80, 81, 82, 83, 84, 94, 95, 96, 97, 98, 99, 113, 114, 115, 172, 173, 182, 183, 185, 186, 187, 188, 189, 190, 191, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 212, 213, 214, 215, 216, 217, 224, 225, 226, 227, 228, 229, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 294, 328, 338, 339, 348, 356, 357, 366, 373, 383, 386, 387, 388, 389, 391, 392, 393, 394, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 408, 409, 410, 481, 484, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 536, 537, 561, 562, 563, 574, 575, 576, 591, 594 }

F normal fail { 175, 176, 177, 180, 181, 376, 377, 381, 382, 423, 424 }

F(-1) timedout fail { 184, 192, 390 }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 6, 13, 14, 15, 16, 17, 18, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 116, 117, 118, 120, 121, 127, 128, 129, 130, 131, 139, 140, 141, 142, 143, 148, 149, 151, 152, 154, 157, 158, 159, 160, 161, 162, 163, 164, 168, 170, 195, 196, 197, 198, 199, 200, 201, 202, 203, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 246, 256, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 334, 335, 340, 341, 342, 344, 345, 346, 349, 350, 352, 353, 354, 355, 358, 359, 360, 361, 362, 363, 365, 368, 383, 386, 387, 393, 394, 397, 398, 404, 405, 409, 410, 411, 412, 413, 414, 416, 417, 418, 441, 442, 443, 444, 445, 446, 447, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 491, 492, 493, 494, 495, 496, 497, 498, 502, 503, 504, 505, 506, 507, 508, 509, 513, 514, 515, 516, 519, 520, 524, 525, 526, 527, 528, 529, 530, 531, 541, 542, 555, 557, 558, 579, 580 }

B grade { 4, 5, 113, 115, 122, 123, 124, 125, 126, 132, 133, 134, 135, 136, 137, 138, 144, 145, 146, 147, 153, 155, 156, 165, 166, 167, 169, 247, 260, 298, 326, 327, 328, 336, 337, 338, 339, 343, 347, 348, 356, 357, 364, 366, 367, 369, 370, 371, 406, 408, 448, 484, 488, 489, 490, 499, 500, 501, 510, 511, 512, 517, 521, 522, 523, 532, 533, 534, 535, 536, 537, 538, 539, 540, 562, 591 }

C grade { 7, 8, 9, 10, 11, 12, 119, 150, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 204, 205, 206, 207, 208, 237, 238, 239, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 351, 384, 385, 388, 389, 390, 391, 392, 395, 396, 407, 415, 518, 552, 593, 594 }

F normal fail { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 240, 244, 245, 248, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 420, 421, 422, 423, 424, 431, 432, 433, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 556, 559, 560, 561, 563, 565, 566, 567, 568, 574, 575, 576, 584, 585, 592 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 5, 6, 13, 14, 15, 17, 18, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 97, 100, 116, 117, 118, 120, 121, 122, 125, 126, 132, 133, 136, 137, 147, 154, 163, 256, 261, 262, 263, 264, 266, 267, 268, 269, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 312, 317, 319, 324, 328, 334, 338, 339, 355, 364, 365, 385, 395, 441, 442, 443, 444, 445, 446, 451, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 499, 500, 501, 502, 503, 504, 510, 511, 512, 513, 514, 515, 521, 555, 558, 559, 560, 579, 580 }

B grade { 4, 16, 67, 84, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 119, 123, 124, 134, 135, 144, 145, 146, 153, 155, 156, 162, 164, 165, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 251, 254, 255, 257, 258, 260, 265, 270, 276, 277, 309, 311, 313, 314, 315, 316, 318, 320, 321, 322, 323, 325, 326, 327, 335, 336, 337, 345, 346, 347, 348, 354, 356, 357, 363, 366, 396, 404, 405, 406, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 447, 448, 449, 450, 452, 453, 454, 455, 456, 522, 523, 524, 525, 526, 532, 533, 534, 535, 536, 537, 556, 557 }

C grade { 7, 8, 9, 10, 11, 12, 130, 131, 142, 150, 151, 152, 159, 160, 161, 167, 168, 169, 170, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 238, 250, 253, 259, 332, 333, 344, 351, 352, 353, 360, 361, 362, 368, 369, 370, 371, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 399, 400, 401, 402, 403, 407, 516, 517, 518, 519, 520, 527, 528, 529, 530, 531, 538, 539, 540, 541, 542, 552, 591, 594 }

F normal fail { 25, 26, 27, 28, 30, 127, 128, 129, 138, 139, 140, 141, 143, 148, 149, 157, 158, 166, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 329, 330, 331, 340, 341, 342, 343, 349, 350, 358, 359, 367, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 420, 421, 422, 423, 424, 431, 432, 433, 494, 495, 496, 497, 498, 505, 506, 507, 508, 509, 543, 544, 545, 546, 547, 548, 549, 550, 551, 561, 562, 563, 565, 566, 567, 568, 574, 575, 576, 584, 585, 592, 593 }

F(-1) timedout fail { 553 }

F(-2) exception fail { 19, 20, 21, 22, 23, 24, 249, 252, 554 }

2.1.5 Maxima

A grade { 3, 4, 13, 14, 15, 16, 17, 18, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 118, 122, 123, 124, 132, 133, 134, 144, 145, 153, 154, 163, 164, 236, 256, 261, 262, 263, 264, 266, 267, 268, 269, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 334, 335, 336, 345, 346, 354, 355, 363, 364, 365, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 398, 404, 405, 406, 407, 408, 409, 410, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 464, 465, 466, 467, 468, 469, 472, 478, 479, 481, 482, 488, 489, 491, 492, 493, 499, 500, 502, 503, 504, 511, 513, 514, 515, 522, 524, 525, 526, 533, 535, 536, 537, 552, 553, 554, 555, 558, 559, 560, 579, 580 }

B grade { 5, 6, 34, 112, 119, 120, 121, 146, 155, 162, 165, 265, 270, 276, 277, 310, 312, 322, 323, 347, 356, 366, 441, 462, 463, 470, 471, 473, 474, 475, 476, 477, 480, 483, 484, 485, 486, 487, 490, 501, 510, 512, 521, 523, 532, 534 }

C grade { }

F normal fail { 1, 2, 7, 8, 9, 10, 11, 12, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 125, 126, 127, 128, 129, 130, 131, 135, 136, 137, 138, 139, 140, 141, 142, 143, 147, 148, 149, 150, 151, 152, 156, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 327, 328, 329, 330, 331, 332, 333, 337, 338, 339, 340, 341, 342, 343, 344, 348, 349, 350, 351, 352, 353, 357, 358, 359, 360, 361, 362, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 388, 389, 390, 391, 411, 412, 413, 414, 415, 416, 417, 418, 420, 421, 422, 423, 424, 431, 432, 433, 494, 495, 496, 497, 498, 505, 506, 507, 508, 509, 516, 517, 518, 519, 520, 528, 529, 530, 539, 540, 541, 543, 544, 545, 546, 547, 548, 549, 550, 551, 556, 557, 561, 562, 563, 565, 566, 567, 568, 574, 575, 576, 584, 585, 591, 592, 593, 594 }

F(-1) timedout fail { 527, 531, 538, 542 }

F(-2) exception fail { 188, 194, 392, 399, 400, 401, 402, 403 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 13, 14, 15, 16, 17, 18, 31, 32, 33, 34, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 72, 73, 74, 75, 76, 77, 80, 81, 85, 86, 87, 90, 91, 92, 93, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 114, 120, 121, 237, 238, 261, 262, 263, 264, 266, 267, 268, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 314, 315, 316, 317, 318, 319, 320, 321, 383, 384, 385, 386, 387, 445, 446, 452, 453, 454, 458, 468, 469, 478, 483, 552, 553, 554, 579, 580 }

B grade { 5, 6, 29, 35, 36, 39, 40, 41, 50, 53, 54, 66, 67, 71, 78, 79, 82, 83, 84, 88, 89, 94, 95, 98, 99, 112, 113, 115, 116, 117, 118, 122, 123, 125, 126, 132, 133, 136, 137, 198, 199, 203, 204, 205, 206, 207, 208, 209, 210, 211, 215, 216, 218, 219, 220, 221, 222, 223, 227, 228, 230, 231, 232, 233, 234, 235, 236, 256, 259, 260, 265, 269, 270, 276, 277, 310, 311, 312, 313, 322, 323, 355, 364, 365, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 441, 442, 443, 444, 447, 448, 449, 450, 451, 455, 456, 457, 459, 460, 461, 462, 463, 464, 465, 466, 467, 470, 473, 474, 479, 480, 488, 489, 490, 499, 501 }

C grade { }

F normal fail { 7, 8, 9, 10, 11, 12, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 119, 127, 128, 129, 130, 131, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 200, 201, 202, 212, 213, 214, 217, 224, 225, 226, 229, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 342, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 420, 421, 422, 423, 424, 431, 432, 433, 491, 494, 495, 496, 505, 506, 507, 510, 511, 512, 513, 515, 516, 517, 518, 519, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 556, 557, 561, 562, 565, 566, 567, 568, 574, 575, 576, 584, 585, 591, 592, 593, 594 }

F(-1) timedout fail { 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 492, 493, 497, 498, 502, 503, 504, 508, 509, 514, 520, 537, 555, 558, 559, 560, 563 }

F(-2) exception fail { 124, 134, 135, 471, 472, 475, 476, 477, 481, 482, 484, 485, 486, 487, 500 }

2.1.7 Mupad

A grade { }

B grade { 3, 16, 17, 18, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 118, 154, 163, 164, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 335, 346, 355, 364, 365, 375, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 422, 433, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 465, 466, 467, 468, 469, 470, 475, 476, 477, 478, 479, 480, 485, 486, 487, 552 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 324, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 366, 367, 368, 369, 370, 371, 372, 373, 374, 376, 377, 378, 379, 380, 381, 382, 420, 421, 423, 424, 431, 432, 460, 461, 462, 463, 464, 471, 472, 473, 474, 481, 482, 483, 484, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 566, 567, 568, 574, 575, 576, 579, 580, 584, 585, 591, 592, 593, 594 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 3, 31, 37, 70, 74, 269, 281, 288, 385 }

B grade { 29, 32, 33, 34, 35, 36, 38, 42, 43, 44, 45, 49, 50, 51, 52, 55, 56, 57, 58, 61, 62, 63, 64, 65, 68, 69, 75, 76, 77, 81, 89, 236, 261, 262, 263, 264, 265, 267, 268, 273, 274, 275, 276, 277, 279, 280, 282, 285, 286, 287, 293, 294, 295, 307, 319, 396, 407 }

C grade { }

F normal fail { 1, 2, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 39, 40, 41, 46, 47, 48, 53, 54, 59, 60, 66, 67, 71, 72, 73, 82, 83, 84, 90, 91, 92, 98, 99, 104, 105, 113, 114, 115, 117, 118, 119, 120, 121, 123, 124, 125, 126, 128, 129, 130, 131, 134, 140, 145, 146, 147, 149, 150, 151, 152, 154, 155, 156, 160, 161, 165, 169, 170, 175, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 200, 201, 202, 208, 209, 210, 239, 240, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 266, 270, 271, 272, 278, 283, 284, 289, 290, 296, 308, 309, 310, 311, 312, 313, 320, 321, 322, 323, 325, 326, 327, 328, 330, 331, 332, 333, 336, 342, 346, 347, 348, 350, 351, 352, 353, 355, 356, 357, 361, 362, 366, 370, 371, 386, 387, 388, 389, 390, 391, 392, 408, 409, 410, 416, 417, 433, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 501, 502, 503, 506, 507, 508, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 545, 546, 550, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 574, 575, 576, 579, 580, 584, 593, 594 }

F(-1) timedout fail { 7, 19, 78, 79, 80, 85, 86, 87, 88, 93, 94, 95, 96, 97, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 116, 122, 127, 132, 133, 135, 136, 137, 138, 139, 141, 142, 143, 144, 148, 153, 157, 158, 159, 162, 163, 164, 166, 167, 168, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 195, 196, 197, 198, 199, 203, 204, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 241, 242, 243, 246, 256, 259, 260, 291, 292, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 314, 315, 316, 317, 318, 324, 329, 334, 335, 337, 338, 339, 340, 341, 343, 344, 345, 349, 354, 358, 359, 360, 363, 364, 365, 367, 368, 369, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 393, 394, 395, 397, 398, 402, 403, 404, 405, 406, 411, 412, 413, 414, 415, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 431, 432, 435, 436, 439, 440, 487, 499, 500, 504, 505, 509, 543, 544, 547, 548, 549, 551, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 581, 582, 585, 586, 587, 590, 591, 592 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	56	36	32	0	43	0	45	0
N.S.	1	1.06	0.68	0.60	0.00	0.81	0.00	0.85	0.00
time (sec)	N/A	0.333	0.036	1.010	0.000	0.272	0.000	0.329	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	0	29	0	24	0
N.S.	1	1.00	0.76	0.71	0.00	0.85	0.00	0.71	0.00
time (sec)	N/A	0.267	0.039	0.476	0.000	0.269	0.000	0.336	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	16	13	19	17	17	40
N.S.	1	1.00	1.00	1.14	0.93	1.36	1.21	1.21	2.86
time (sec)	N/A	0.194	0.013	0.493	0.399	0.260	0.113	0.339	13.751

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	30	49	26	70	0	15	0
N.S.	1	1.00	1.76	2.88	1.53	4.12	0.00	0.88	0.00
time (sec)	N/A	0.199	0.023	0.722	0.456	0.272	0.000	0.362	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	55	70	314	58	0	81	0
N.S.	1	1.00	1.31	1.67	7.48	1.38	0.00	1.93	0.00
time (sec)	N/A	0.280	0.057	0.841	0.463	0.253	0.000	0.359	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	69	77	89	931	78	0	118	0
N.S.	1	1.13	1.26	1.46	15.26	1.28	0.00	1.93	0.00
time (sec)	N/A	0.368	0.147	0.832	0.630	0.277	0.000	0.371	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	101	65	192	0	110	0	0	0
N.S.	1	0.82	0.53	1.56	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.461	0.127	4.465	0.000	0.105	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	65	54	302	0	80	0	0	0
N.S.	1	0.89	0.74	4.14	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.330	0.078	1.448	0.000	0.095	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	47	41	163	0	69	0	0	0
N.S.	1	0.94	0.82	3.26	0.00	1.38	0.00	0.00	0.00
time (sec)	N/A	0.263	0.040	1.411	0.000	0.091	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	43	37	274	0	102	0	0	0
N.S.	1	0.90	0.77	5.71	0.00	2.12	0.00	0.00	0.00
time (sec)	N/A	0.266	0.036	1.169	0.000	0.100	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	67	48	239	0	139	0	0	0
N.S.	1	0.87	0.62	3.10	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	0.335	0.068	1.472	0.000	0.085	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	97	60	314	0	209	0	0	0
N.S.	1	0.79	0.49	2.55	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.460	0.162	1.368	0.000	0.099	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	92	53	61	85	82	0	57	0
N.S.	1	0.70	0.40	0.46	0.64	0.62	0.00	0.43	0.00
time (sec)	N/A	0.469	0.127	10.176	0.311	0.279	0.000	0.319	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	60	38	43	55	56	0	27	0
N.S.	1	0.77	0.49	0.55	0.71	0.72	0.00	0.35	0.00
time (sec)	N/A	0.327	0.080	0.577	0.320	0.269	0.000	0.308	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	29	25	24	22	36	0	15	0
N.S.	1	0.81	0.69	0.67	0.61	1.00	0.00	0.42	0.00
time (sec)	N/A	0.213	0.024	0.605	0.317	0.272	0.000	0.411	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	18	9	36	0	9	7
N.S.	1	1.00	1.00	1.12	0.56	2.25	0.00	0.56	0.44
time (sec)	N/A	0.217	0.018	0.559	0.296	0.272	0.000	0.413	13.215

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	38	34	37	23	74	0	23	44
N.S.	1	0.56	0.50	0.54	0.34	1.09	0.00	0.34	0.65
time (sec)	N/A	0.242	0.036	0.528	0.299	0.261	0.000	0.420	13.792

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	54	47	49	35	104	0	35	117
N.S.	1	0.46	0.40	0.42	0.30	0.88	0.00	0.30	0.99
time (sec)	N/A	0.254	0.054	0.641	0.324	0.279	0.000	0.405	15.910

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	74	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.279	0.122	0.000	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	72	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.270	0.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	88	68	0	0	0	0	0	0
N.S.	1	1.19	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.262	0.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	72	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.269	0.052	0.000	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	71	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.276	0.075	0.000	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	73	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.276	0.075	0.000	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	71	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.043	0.000	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	61	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	0.050	0.000	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	67	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.270	0.054	0.000	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	65	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.273	0.061	0.000	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	0	16	58	384	36
N.S.	1	1.00	1.00	1.16	0.00	0.64	2.32	15.36	1.44
time (sec)	N/A	0.232	0.035	1.102	0.000	0.264	0.356	0.738	14.438

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	73	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.280	0.036	0.000	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	17	12	15	17	11
N.S.	1	1.00	1.00	0.81	1.06	0.75	0.94	1.06	0.69
time (sec)	N/A	0.144	0.007	0.194	0.287	0.274	0.020	0.326	13.574

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	26	20	40	28	110	25	33
N.S.	1	1.00	0.79	0.61	1.21	0.85	3.33	0.76	1.00
time (sec)	N/A	0.255	0.002	0.651	0.279	0.278	0.129	0.302	13.533

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	48	34	26	69	37	233	34	42
N.S.	1	1.04	0.74	0.57	1.50	0.80	5.07	0.74	0.91
time (sec)	N/A	0.309	0.002	1.046	0.298	0.272	0.259	0.307	14.012

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	63	42	34	104	46	376	43	51
N.S.	1	1.07	0.71	0.58	1.76	0.78	6.37	0.73	0.86
time (sec)	N/A	0.373	0.006	1.359	0.286	0.271	0.489	0.310	13.919

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	45	58	45	50	46	314	149	54
N.S.	1	0.73	0.94	0.73	0.81	0.74	5.06	2.40	0.87
time (sec)	N/A	0.307	0.139	0.719	0.264	0.269	10.455	0.309	0.094

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	37	43	35	40	36	143	105	38
N.S.	1	0.80	0.93	0.76	0.87	0.78	3.11	2.28	0.83
time (sec)	N/A	0.304	0.081	0.506	0.275	0.258	4.102	0.302	0.061

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	25	25	23	27	25	36	29	25
N.S.	1	0.93	0.93	0.85	1.00	0.93	1.33	1.07	0.93
time (sec)	N/A	0.287	0.055	0.400	0.283	0.264	1.515	0.308	13.946

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	16	15	15	34	15	15
N.S.	1	1.00	1.00	1.23	1.15	1.15	2.62	1.15	1.15
time (sec)	N/A	0.243	0.010	0.526	0.290	0.249	0.592	0.300	13.810

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	23	46	39	46	55	0	62	31
N.S.	1	0.79	1.59	1.34	1.59	1.90	0.00	2.14	1.07
time (sec)	N/A	0.267	0.090	0.550	0.291	0.252	0.000	0.305	0.085

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	54	146	63	70	98	0	149	55
N.S.	1	0.93	2.52	1.09	1.21	1.69	0.00	2.57	0.95
time (sec)	N/A	0.303	0.265	0.639	0.315	0.266	0.000	0.329	14.181

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	85	132	87	90	135	0	181	74
N.S.	1	1.04	1.61	1.06	1.10	1.65	0.00	2.21	0.90
time (sec)	N/A	0.310	2.744	0.815	0.303	0.274	0.000	0.333	0.106

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	81	44	57	72	56	1161	63	68
N.S.	1	1.11	0.60	0.78	0.99	0.77	15.90	0.86	0.93
time (sec)	N/A	0.309	0.207	0.627	0.392	0.276	6.573	0.301	14.065

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	52	34	44	49	45	502	50	45
N.S.	1	1.06	0.69	0.90	1.00	0.92	10.24	1.02	0.92
time (sec)	N/A	0.299	0.152	0.463	0.383	0.273	2.564	0.305	13.719

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	27	24	26	34	100	26	20
N.S.	1	1.00	1.35	1.20	1.30	1.70	5.00	1.30	1.00
time (sec)	N/A	0.322	0.010	0.398	0.363	0.256	0.924	0.304	13.749

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	21	41	13	13
N.S.	1	1.00	1.00	1.08	1.00	1.62	3.15	1.00	1.00
time (sec)	N/A	0.224	0.005	0.336	0.295	0.238	0.467	0.296	13.109

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	22	16	25	28	36	0	19	17
N.S.	1	0.79	0.57	0.89	1.00	1.29	0.00	0.68	0.61
time (sec)	N/A	0.288	0.044	0.515	0.266	0.262	0.000	0.318	14.112

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	34	49	35	42	56	0	42	38
N.S.	1	0.74	1.07	0.76	0.91	1.22	0.00	0.91	0.83
time (sec)	N/A	0.298	0.111	0.596	0.341	0.259	0.000	0.335	13.850

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	44	70	45	52	77	0	52	50
N.S.	1	0.71	1.13	0.73	0.84	1.24	0.00	0.84	0.81
time (sec)	N/A	0.310	0.096	0.879	0.282	0.245	0.000	0.312	13.875

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	49	59	47	52	46	156	57	48
N.S.	1	0.75	0.91	0.72	0.80	0.71	2.40	0.88	0.74
time (sec)	N/A	0.311	0.110	0.722	0.266	0.254	24.600	0.337	13.236

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	35	42	37	41	38	156	106	36
N.S.	1	0.74	0.89	0.79	0.87	0.81	3.32	2.26	0.77
time (sec)	N/A	0.286	0.067	0.634	0.283	0.256	10.897	0.339	0.056

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	28	31	29	28	28	156	28	26
N.S.	1	0.85	0.94	0.88	0.85	0.85	4.73	0.85	0.79
time (sec)	N/A	0.283	0.068	0.543	0.284	0.260	3.916	0.326	0.053

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	156	16	16
N.S.	1	1.00	1.00	0.94	0.89	0.89	8.67	0.89	0.89
time (sec)	N/A	0.258	0.009	0.549	0.277	0.263	1.920	0.314	12.979

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	35	61	49	59	70	0	107	41
N.S.	1	0.74	1.30	1.04	1.26	1.49	0.00	2.28	0.87
time (sec)	N/A	0.287	0.075	0.756	0.250	0.256	0.000	0.296	0.111

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	66	208	75	86	118	0	175	70
N.S.	1	0.85	2.67	0.96	1.10	1.51	0.00	2.24	0.90
time (sec)	N/A	0.313	0.380	0.889	0.230	0.253	0.000	0.330	12.938

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	64	46	56	64	59	1275	68	66
N.S.	1	0.93	0.67	0.81	0.93	0.86	18.48	0.99	0.96
time (sec)	N/A	0.314	0.206	0.743	0.357	0.275	16.725	0.293	13.086

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	32	42	34	37	49	551	44	31
N.S.	1	0.84	1.11	0.89	0.97	1.29	14.50	1.16	0.82
time (sec)	N/A	0.318	0.011	0.668	0.355	0.273	7.023	0.304	13.098

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	32	94	16	16
N.S.	1	1.00	1.00	0.94	0.89	1.78	5.22	0.89	0.89
time (sec)	N/A	0.272	0.012	0.516	0.258	0.267	2.790	0.306	12.596

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	29	26	25	25	34	238	25	24
N.S.	1	0.91	0.81	0.78	0.78	1.06	7.44	0.78	0.75
time (sec)	N/A	0.245	0.029	0.454	0.309	0.246	1.187	0.293	12.694

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	36	50	37	40	46	0	48	36
N.S.	1	0.77	1.06	0.79	0.85	0.98	0.00	1.02	0.77
time (sec)	N/A	0.294	0.101	0.698	0.306	0.244	0.000	0.309	12.699

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	48	46	47	52	72	0	34	48
N.S.	1	0.74	0.71	0.72	0.80	1.11	0.00	0.52	0.74
time (sec)	N/A	0.308	0.041	0.939	0.314	0.266	0.000	0.306	13.135

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	26	23	20	22	25	362	22	21
N.S.	1	0.90	0.79	0.69	0.76	0.86	12.48	0.76	0.72
time (sec)	N/A	0.239	0.005	0.579	0.311	0.264	1.810	0.285	12.891

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	32	27	24	28	31	675	28	33
N.S.	1	0.86	0.73	0.65	0.76	0.84	18.24	0.76	0.89
time (sec)	N/A	0.244	0.006	0.926	0.497	0.272	5.396	0.288	12.853

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	42	39	32	34	37	1083	34	43
N.S.	1	0.82	0.76	0.63	0.67	0.73	21.24	0.67	0.84
time (sec)	N/A	0.250	0.007	1.460	0.313	0.248	14.843	0.280	13.138

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	46	77	54	43	43	107	67	44
N.S.	1	0.90	1.51	1.06	0.84	0.84	2.10	1.31	0.86
time (sec)	N/A	0.233	0.075	1.370	0.383	0.247	0.221	0.308	13.143

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	29	54	34	34	27	58	40	27
N.S.	1	0.94	1.74	1.10	1.10	0.87	1.87	1.29	0.87
time (sec)	N/A	0.205	0.046	0.810	0.303	0.267	0.111	0.295	0.049

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	63	28	38	42	0	58	23
N.S.	1	1.00	2.42	1.08	1.46	1.62	0.00	2.23	0.88
time (sec)	N/A	0.241	0.049	0.524	0.293	0.267	0.000	0.294	12.974

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	118	52	58	95	0	121	42
N.S.	1	1.00	2.95	1.30	1.45	2.38	0.00	3.02	1.05
time (sec)	N/A	0.262	0.070	0.883	0.285	0.251	0.000	0.301	0.098

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	85	70	62	104	69	258	68	92
N.S.	1	0.96	0.79	0.70	1.17	0.78	2.90	0.76	1.03
time (sec)	N/A	0.344	0.124	1.265	0.362	0.256	0.329	0.298	14.219

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	59	45	44	74	50	158	43	68
N.S.	1	0.97	0.74	0.72	1.21	0.82	2.59	0.70	1.11
time (sec)	N/A	0.270	0.109	0.959	0.361	0.261	0.159	0.279	13.832

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	33	24	29	29	51	25	27
N.S.	1	1.00	1.10	0.80	0.97	0.97	1.70	0.83	0.90
time (sec)	N/A	0.155	0.042	0.387	0.297	0.267	0.081	0.279	13.514

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	22	23	32	0	39	16
N.S.	1	1.00	1.00	1.38	1.44	2.00	0.00	2.44	1.00
time (sec)	N/A	0.184	0.034	0.881	0.401	0.266	0.000	0.412	13.617

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	49	35	28	54	0	37	29
N.S.	1	1.00	1.14	0.81	0.65	1.26	0.00	0.86	0.67
time (sec)	N/A	0.272	0.065	1.090	0.280	0.250	0.000	0.405	13.875

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	56	95	56	45	81	0	61	49
N.S.	1	0.86	1.46	0.86	0.69	1.25	0.00	0.94	0.75
time (sec)	N/A	0.290	0.068	1.111	0.290	0.255	0.000	0.432	14.089

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	17	16	15	17	15
N.S.	1	1.00	1.00	0.84	0.89	0.84	0.79	0.89	0.79
time (sec)	N/A	0.146	0.021	0.219	0.248	0.257	0.021	0.376	13.518

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	43	41	39	47	110	42	44
N.S.	1	1.00	0.86	0.82	0.78	0.94	2.20	0.84	0.88
time (sec)	N/A	0.193	0.046	0.671	0.242	0.271	0.132	0.379	13.890

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	92	80	70	71	81	246	76	118
N.S.	1	1.06	0.92	0.80	0.82	0.93	2.83	0.87	1.36
time (sec)	N/A	0.319	0.070	1.008	0.236	0.269	0.274	0.384	13.981

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	150	113	110	108	123	410	118	147
N.S.	1	1.07	0.81	0.79	0.77	0.88	2.93	0.84	1.05
time (sec)	N/A	0.496	0.110	1.439	0.272	0.258	0.496	0.403	13.570

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	99	180	110	116	272	0	332	112
N.S.	1	0.93	1.70	1.04	1.09	2.57	0.00	3.13	1.06
time (sec)	N/A	0.296	1.273	1.322	0.373	0.297	0.000	0.429	0.172

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	73	150	70	88	218	0	173	72
N.S.	1	0.95	1.95	0.91	1.14	2.83	0.00	2.25	0.94
time (sec)	N/A	0.273	0.483	0.970	0.389	0.293	0.000	0.418	13.421

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	51	125	45	67	165	0	57	44
N.S.	1	0.98	2.40	0.87	1.29	3.17	0.00	1.10	0.85
time (sec)	N/A	0.238	0.265	0.556	0.319	0.284	0.000	0.390	0.099

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	97	29	50	117	367693	37	29
N.S.	1	1.00	2.62	0.78	1.35	3.16	9937.65	1.00	0.78
time (sec)	N/A	0.214	0.864	0.391	0.396	0.280	76.845	0.383	0.095

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	54	143	62	83	161	0	100	457
N.S.	1	0.98	2.60	1.13	1.51	2.93	0.00	1.82	8.31
time (sec)	N/A	0.241	0.535	0.628	0.406	0.281	0.000	0.398	14.661

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	94	224	107	120	327	0	196	592
N.S.	1	1.11	2.64	1.26	1.41	3.85	0.00	2.31	6.96
time (sec)	N/A	0.286	1.815	0.848	0.454	0.284	0.000	0.413	13.808

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	146	657	168	181	612	0	334	1105
N.S.	1	1.17	5.26	1.34	1.45	4.90	0.00	2.67	8.84
time (sec)	N/A	0.344	6.632	0.993	0.334	0.339	0.000	0.440	13.558

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	202	133	168	192	453	0	233	2244
N.S.	1	1.24	0.82	1.03	1.18	2.78	0.00	1.43	13.77
time (sec)	N/A	0.476	1.175	1.628	0.329	0.327	0.000	0.448	15.307

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	144	95	118	128	372	0	157	1892
N.S.	1	1.23	0.81	1.01	1.09	3.18	0.00	1.34	16.17
time (sec)	N/A	0.386	0.512	1.049	0.361	0.303	0.000	0.433	14.189

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	94	69	81	78	305	0	114	481
N.S.	1	1.22	0.90	1.05	1.01	3.96	0.00	1.48	6.25
time (sec)	N/A	0.288	0.349	0.709	0.345	0.284	0.000	0.431	14.138

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	48	46	260	0	81	104
N.S.	1	1.00	1.00	1.04	1.00	5.65	0.00	1.76	2.26
time (sec)	N/A	0.291	1.407	0.409	0.396	0.291	0.000	0.397	13.852

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	30	29	236	16298	64	33
N.S.	1	1.00	1.00	0.83	0.81	6.56	452.72	1.78	0.92
time (sec)	N/A	0.199	0.733	0.386	0.324	0.282	13.057	0.438	14.950

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	51	53	50	48	313	0	83	45
N.S.	1	0.96	1.00	0.94	0.91	5.91	0.00	1.57	0.85
time (sec)	N/A	0.241	0.551	0.630	0.301	0.293	0.000	0.494	14.008

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	72	119	69	69	451	0	111	68
N.S.	1	0.94	1.55	0.90	0.90	5.86	0.00	1.44	0.88
time (sec)	N/A	0.287	0.804	0.844	0.313	0.271	0.000	0.460	13.885

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	101	147	96	98	595	0	155	95
N.S.	1	0.93	1.35	0.88	0.90	5.46	0.00	1.42	0.87
time (sec)	N/A	0.310	2.311	1.129	0.356	0.289	0.000	0.472	13.775

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	129	137	130	137	789	0	215	130
N.S.	1	0.92	0.98	0.93	0.98	5.64	0.00	1.54	0.93
time (sec)	N/A	0.340	5.818	1.531	0.370	0.310	0.000	0.481	15.183

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	122	194	116	154	529	0	322	123
N.S.	1	0.95	1.52	0.91	1.20	4.13	0.00	2.52	0.96
time (sec)	N/A	0.341	1.307	1.603	0.363	0.313	0.000	0.476	0.222

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	97	172	90	131	427	0	342	95
N.S.	1	0.95	1.69	0.88	1.28	4.19	0.00	3.35	0.93
time (sec)	N/A	0.307	0.899	1.105	0.323	0.283	0.000	0.362	0.187

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	82	160	77	111	327	0	93	71
N.S.	1	0.99	1.93	0.93	1.34	3.94	0.00	1.12	0.86
time (sec)	N/A	0.258	0.527	0.716	0.328	0.282	0.000	0.429	13.527

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	73	149	65	98	282	0	79	62
N.S.	1	0.99	2.01	0.88	1.32	3.81	0.00	1.07	0.84
time (sec)	N/A	0.227	0.259	0.585	0.392	0.276	0.000	0.469	0.110

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	113	194	107	149	455	0	246	2039
N.S.	1	1.10	1.88	1.04	1.45	4.42	0.00	2.39	19.80
time (sec)	N/A	0.288	1.149	0.916	0.354	0.315	0.000	0.425	14.481

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	166	390	152	223	838	0	512	2338
N.S.	1	1.08	2.55	0.99	1.46	5.48	0.00	3.35	15.28
time (sec)	N/A	0.372	1.650	1.137	0.403	0.350	0.000	0.421	15.462

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	169	106	134	181	623	0	223	2295
N.S.	1	1.14	0.72	0.91	1.22	4.21	0.00	1.51	15.51
time (sec)	N/A	0.398	1.191	1.245	0.332	0.318	0.000	0.397	16.770

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	111	93	101	109	492	0	140	1959
N.S.	1	1.19	1.00	1.09	1.17	5.29	0.00	1.51	21.06
time (sec)	N/A	0.297	11.371	0.704	0.336	0.327	0.000	0.392	15.210

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	74	75	74	419	0	109	72
N.S.	1	1.00	0.95	0.96	0.95	5.37	0.00	1.40	0.92
time (sec)	N/A	0.330	11.210	0.621	0.337	0.280	0.000	0.390	13.134

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	84	87	89	463	0	113	79
N.S.	1	1.00	0.97	1.00	1.02	5.32	0.00	1.30	0.91
time (sec)	N/A	0.322	11.201	0.602	0.388	0.293	0.000	0.365	13.450

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	131	155	103	133	588	0	179	132
N.S.	1	1.03	1.22	0.81	1.05	4.63	0.00	1.41	1.04
time (sec)	N/A	0.307	1.256	0.906	0.437	0.302	0.000	0.381	13.735

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	158	202	122	172	843	0	174	164
N.S.	1	0.98	1.25	0.75	1.06	5.20	0.00	1.07	1.01
time (sec)	N/A	0.374	2.326	1.249	0.382	0.300	0.000	0.364	14.868

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	178	134	158	234	950	0	224	3189
N.S.	1	1.20	0.91	1.07	1.58	6.42	0.00	1.51	21.55
time (sec)	N/A	0.410	12.307	1.197	0.371	0.310	0.000	0.403	17.722

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	115	97	109	158	683	0	152	149
N.S.	1	1.05	0.88	0.99	1.44	6.21	0.00	1.38	1.35
time (sec)	N/A	0.271	11.501	0.999	0.333	0.328	0.000	0.390	14.528

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	141	112	131	191	771	0	191	159
N.S.	1	1.08	0.85	1.00	1.46	5.89	0.00	1.46	1.21
time (sec)	N/A	0.512	11.641	1.053	0.327	0.322	0.000	0.383	14.107

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	157	125	148	211	843	0	211	176
N.S.	1	1.09	0.87	1.03	1.47	5.85	0.00	1.47	1.22
time (sec)	N/A	0.532	11.721	1.085	0.387	0.297	0.000	0.327	14.120

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	209	214	160	270	1003	0	232	251
N.S.	1	1.07	1.09	0.82	1.38	5.12	0.00	1.18	1.28
time (sec)	N/A	0.399	2.824	1.665	0.479	0.362	0.000	0.398	15.804

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	232	201	230	378	1361	0	344	339
N.S.	1	1.13	0.98	1.12	1.83	6.61	0.00	1.67	1.65
time (sec)	N/A	0.788	11.746	1.874	0.404	0.339	0.000	0.344	15.292

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	318	312	336	588	2017	0	524	450
N.S.	1	1.14	1.12	1.20	2.11	7.23	0.00	1.88	1.61
time (sec)	N/A	1.128	11.903	3.546	0.397	0.383	0.000	0.380	16.120

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	29	33	10	57	0	25	18
N.S.	1	1.00	2.64	3.00	0.91	5.18	0.00	2.27	1.64
time (sec)	N/A	0.187	0.050	0.682	0.342	0.270	0.000	0.328	0.053

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	53	51	25	71	0	25	0
N.S.	1	1.00	1.77	1.70	0.83	2.37	0.00	0.83	0.00
time (sec)	N/A	0.197	0.045	0.768	0.369	0.287	0.000	0.326	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	39	57	11	94	0	48	0
N.S.	1	1.00	2.60	3.80	0.73	6.27	0.00	3.20	0.00
time (sec)	N/A	0.193	0.149	0.704	0.406	0.288	0.000	0.528	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	56	33	32	31	40	0	84	0
N.S.	1	1.06	0.62	0.60	0.58	0.75	0.00	1.58	0.00
time (sec)	N/A	0.408	0.016	1.152	0.379	0.247	0.000	0.378	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	23	24	17	26	0	57	0
N.S.	1	1.00	0.68	0.71	0.50	0.76	0.00	1.68	0.00
time (sec)	N/A	0.318	0.008	0.589	0.371	0.258	0.000	0.342	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	6	15	0	27	46
N.S.	1	1.00	1.00	1.15	0.46	1.15	0.00	2.08	3.54
time (sec)	N/A	0.254	0.016	0.615	0.415	0.251	0.000	0.306	0.230

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	16	16	16	22	38	65	0	0	0
N.S.	1	1.00	1.00	1.38	2.38	4.06	0.00	0.00	0.00
time (sec)	N/A	0.265	0.006	0.309	0.399	0.268	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	26	41	304	40	0	47	0
N.S.	1	1.00	0.62	0.98	7.24	0.95	0.00	1.12	0.00
time (sec)	N/A	0.334	0.012	0.720	0.419	0.276	0.000	0.354	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	69	36	63	933	49	0	63	0
N.S.	1	1.13	0.59	1.03	15.30	0.80	0.00	1.03	0.00
time (sec)	N/A	0.439	0.026	0.834	0.598	0.272	0.000	0.335	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	121	119	311	176	501	0	2404	0
N.S.	1	0.97	0.95	2.49	1.41	4.01	0.00	19.23	0.00
time (sec)	N/A	0.275	0.483	1.134	0.340	0.523	0.000	0.496	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	77	93	184	70	433	0	723	0
N.S.	1	0.99	1.19	2.36	0.90	5.55	0.00	9.27	0.00
time (sec)	N/A	0.236	0.220	0.781	0.345	0.375	0.000	0.412	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	80	99	174	132	1158	0	0	0
N.S.	1	0.96	1.19	2.10	1.59	13.95	0.00	0.00	0.00
time (sec)	N/A	0.278	0.200	1.009	0.326	0.477	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	91	100	227	0	338	0	434	0
N.S.	1	1.08	1.19	2.70	0.00	4.02	0.00	5.17	0.00
time (sec)	N/A	0.267	0.365	0.928	0.000	0.320	0.000	0.488	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	151	127	379	0	520	0	914	0
N.S.	1	1.06	0.89	2.65	0.00	3.64	0.00	6.39	0.00
time (sec)	N/A	0.304	0.522	1.095	0.000	0.449	0.000	0.567	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	262	199	413	0	0	0	0	0
N.S.	1	1.01	0.77	1.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.475	1.080	3.372	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	159	266	0	0	0	0	0
N.S.	1	1.00	1.00	1.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.841	0.674	2.299	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	61	71	0	0	0	0	0
N.S.	1	1.00	1.20	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.276	0.096	1.444	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	176	137	156	0	626	0	0	0
N.S.	1	1.01	0.79	0.90	0.00	3.60	0.00	0.00	0.00
time (sec)	N/A	0.373	0.578	1.805	0.000	0.128	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	243	188	342	0	947	0	0	0
N.S.	1	1.04	0.80	1.46	0.00	4.05	0.00	0.00	0.00
time (sec)	N/A	0.454	2.213	2.456	0.000	0.140	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	157	152	446	248	579	0	5251	0
N.S.	1	0.93	0.90	2.64	1.47	3.43	0.00	31.07	0.00
time (sec)	N/A	0.307	0.700	1.081	0.352	1.216	0.000	0.934	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	113	113	309	106	495	0	2403	0
N.S.	1	0.99	0.99	2.71	0.93	4.34	0.00	21.08	0.00
time (sec)	N/A	0.261	0.312	1.122	0.314	0.544	0.000	0.629	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	120	141	255	179	1282	0	0	0
N.S.	1	0.98	1.16	2.09	1.47	10.51	0.00	0.00	0.00
time (sec)	N/A	0.334	0.677	1.130	0.498	0.576	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	134	147	287	0	1449	0	0	0
N.S.	1	1.05	1.15	2.24	0.00	11.32	0.00	0.00	0.00
time (sec)	N/A	0.331	0.894	1.165	0.000	0.588	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	141	114	376	0	484	0	912	0
N.S.	1	1.10	0.89	2.94	0.00	3.78	0.00	7.12	0.00
time (sec)	N/A	0.309	0.635	1.104	0.000	0.431	0.000	0.722	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	201	161	565	0	752	0	1623	0
N.S.	1	1.02	0.82	2.87	0.00	3.82	0.00	8.24	0.00
time (sec)	N/A	0.341	1.597	1.390	0.000	1.131	0.000	0.939	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	339	249	602	0	0	0	0	0
N.S.	1	1.04	0.77	1.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.618	1.865	4.042	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	223	201	429	0	0	0	0	0
N.S.	1	1.02	0.92	1.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.166	1.042	3.539	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	155	156	266	0	0	0	0	0
N.S.	1	1.01	1.01	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.781	0.559	2.388	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	183	141	174	0	0	0	0	0
N.S.	1	1.01	0.78	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.394	1.088	2.276	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	243	201	408	0	969	0	0	0
N.S.	1	1.03	0.85	1.73	0.00	4.11	0.00	0.00	0.00
time (sec)	N/A	0.487	3.064	2.604	0.000	0.138	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	215	194	437	0	0	0	0	0
N.S.	1	1.02	0.92	2.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.145	0.991	4.219	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	82	105	148	75	438	0	0	0
N.S.	1	0.99	1.27	1.78	0.90	5.28	0.00	0.00	0.00
time (sec)	N/A	0.268	0.369	0.908	0.399	0.362	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	53	99	24	370	0	0	0
N.S.	1	1.00	1.29	2.41	0.59	9.02	0.00	0.00	0.00
time (sec)	N/A	0.220	0.141	0.647	0.322	0.354	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	48	112	109	219	0	0	0
N.S.	1	1.00	1.17	2.73	2.66	5.34	0.00	0.00	0.00
time (sec)	N/A	0.242	0.221	0.877	0.325	0.305	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	96	102	165	0	347	0	0	0
N.S.	1	1.08	1.15	1.85	0.00	3.90	0.00	0.00	0.00
time (sec)	N/A	0.291	0.401	1.084	0.000	0.353	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	196	163	268	0	0	0	0	0
N.S.	1	0.95	0.79	1.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.396	0.730	1.514	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	78	93	0	0	0	0	0
N.S.	1	1.00	0.70	0.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.605	0.271	1.047	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	60	52	0	305	0	0	0
N.S.	1	1.00	1.18	1.02	0.00	5.98	0.00	0.00	0.00
time (sec)	N/A	0.282	0.099	0.442	0.000	0.108	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	181	138	140	0	643	0	0	0
N.S.	1	1.02	0.78	0.79	0.00	3.63	0.00	0.00	0.00
time (sec)	N/A	0.389	0.519	1.150	0.000	0.128	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	252	195	354	0	955	0	0	0
N.S.	1	1.03	0.80	1.45	0.00	3.91	0.00	0.00	0.00
time (sec)	N/A	0.466	2.677	1.670	0.000	0.141	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	78	96	156	81	564	0	0	0
N.S.	1	0.99	1.22	1.97	1.03	7.14	0.00	0.00	0.00
time (sec)	N/A	0.272	0.473	1.087	0.304	0.403	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	41	31	32	57	0	0	119
N.S.	1	1.00	1.21	0.91	0.94	1.68	0.00	0.00	3.50
time (sec)	N/A	0.219	0.156	0.634	0.221	0.302	0.000	0.000	14.897

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	78	93	165	165	422	0	0	0
N.S.	1	0.99	1.18	2.09	2.09	5.34	0.00	0.00	0.00
time (sec)	N/A	0.273	0.348	1.017	0.339	0.383	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	140	134	274	0	634	0	0	0
N.S.	1	1.04	1.00	2.04	0.00	4.73	0.00	0.00	0.00
time (sec)	N/A	0.340	0.610	1.249	0.000	0.523	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	275	197	405	0	0	0	0	0
N.S.	1	1.00	0.72	1.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.506	1.082	3.338	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	199	136	241	0	0	0	0	0
N.S.	1	0.99	0.67	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.398	0.611	2.532	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	163	138	191	0	780	0	0	0
N.S.	1	1.07	0.90	1.25	0.00	5.10	0.00	0.00	0.00
time (sec)	N/A	0.847	0.439	1.712	0.000	0.136	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	90	103	0	938	0	0	0
N.S.	1	1.00	0.89	1.02	0.00	9.29	0.00	0.00	0.00
time (sec)	N/A	0.422	0.143	1.292	0.000	0.155	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	252	170	199	0	1075	0	0	0
N.S.	1	1.07	0.72	0.85	0.00	4.57	0.00	0.00	0.00
time (sec)	N/A	0.490	1.029	2.275	0.000	0.170	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	151	133	243	283	885	0	0	0
N.S.	1	1.10	0.97	1.77	2.07	6.46	0.00	0.00	0.00
time (sec)	N/A	0.332	0.874	1.453	0.374	0.753	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	84	64	64	121	137	0	0	176
N.S.	1	1.04	0.79	0.79	1.49	1.69	0.00	0.00	2.17
time (sec)	N/A	0.266	0.402	0.618	0.298	0.412	0.000	0.000	20.107

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	72	60	55	63	134	0	0	159
N.S.	1	0.99	0.82	0.75	0.86	1.84	0.00	0.00	2.18
time (sec)	N/A	0.228	0.221	0.662	0.290	0.380	0.000	0.000	20.302

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	139	127	249	306	752	0	0	0
N.S.	1	1.08	0.98	1.93	2.37	5.83	0.00	0.00	0.00
time (sec)	N/A	0.324	0.538	1.357	0.414	0.558	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	288	192	698	0	0	0	0	0
N.S.	1	1.01	0.67	2.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.521	1.656	3.976	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	267	182	623	0	1432	0	0	0
N.S.	1	0.99	0.68	2.32	0.00	5.32	0.00	0.00	0.00
time (sec)	N/A	0.473	1.295	2.805	0.000	0.186	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	228	174	483	0	1400	0	0	0
N.S.	1	1.03	0.79	2.19	0.00	6.33	0.00	0.00	0.00
time (sec)	N/A	1.188	1.168	2.735	0.000	0.203	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	228	172	547	0	1531	0	0	0
N.S.	1	1.02	0.77	2.45	0.00	6.87	0.00	0.00	0.00
time (sec)	N/A	1.159	0.928	1.815	0.000	0.189	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	351	214	527	0	1719	0	0	0
N.S.	1	1.09	0.66	1.64	0.00	5.34	0.00	0.00	0.00
time (sec)	N/A	0.617	1.772	3.700	0.000	0.225	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	113	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	0.646	0.000	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	220	218	98	0	0	0	0	0	0
N.S.	1	0.99	0.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	0.585	0.000	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	130	98	0	0	0	0	0	0
N.S.	1	0.99	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.288	0.411	0.000	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	74	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	0.215	0.000	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.249	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.258	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.252	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	0.566	0.000	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	102	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.328	0.323	0.000	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.274	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.274	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	338	219	221	0	21338	0	0	1978
N.S.	1	1.01	0.65	0.66	0.00	63.70	0.00	0.00	5.90
time (sec)	N/A	0.851	0.563	1.924	0.000	1.426	0.000	0.000	15.449

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	255	141	0	29175	0	0	1962
N.S.	1	1.00	0.93	0.52	0.00	106.87	0.00	0.00	7.19
time (sec)	N/A	0.794	0.331	1.300	0.000	1.284	0.000	0.000	14.511

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	C	F	C	F(-1)	F	B
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	0	104	0	29221	0	0	1672
N.S.	1	1.00	0.00	0.40	0.00	112.82	0.00	0.00	6.46
time (sec)	N/A	0.628	0.000	0.775	0.000	1.249	0.000	0.000	14.442

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	270	172	78	0	18879	0	0	652
N.S.	1	1.01	0.64	0.29	0.00	70.71	0.00	0.00	2.44
time (sec)	N/A	0.538	11.047	0.797	0.000	1.321	0.000	0.000	15.875

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	264	96	0	29139	0	0	1439
N.S.	1	1.00	1.00	0.36	0.00	110.38	0.00	0.00	5.45
time (sec)	N/A	0.689	0.502	1.039	0.000	3.768	0.000	0.000	15.200

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	181	136	0	29431	0	0	1573
N.S.	1	1.00	0.63	0.47	0.00	102.55	0.00	0.00	5.48
time (sec)	N/A	0.620	0.401	1.488	0.000	14.834	0.000	0.000	15.061

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	347	290	196	0	21564	0	0	1560
N.S.	1	1.01	0.84	0.57	0.00	62.69	0.00	0.00	4.53
time (sec)	N/A	0.732	1.229	2.043	0.000	131.012	0.000	0.000	15.221

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	164	143	0	29350	0	0	1800
N.S.	1	1.00	0.56	0.49	0.00	100.17	0.00	0.00	6.14
time (sec)	N/A	0.623	0.384	1.508	0.000	1.231	0.000	0.000	15.211

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	284	186	104	0	21185	0	0	665
N.S.	1	1.01	0.66	0.37	0.00	75.39	0.00	0.00	2.37
time (sec)	N/A	0.638	0.241	1.056	0.000	1.328	0.000	0.000	15.324

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	231	76	0	25253	0	0	590
N.S.	1	1.00	0.96	0.32	0.00	105.22	0.00	0.00	2.46
time (sec)	N/A	0.594	11.045	0.809	0.000	1.175	0.000	0.000	14.792

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	C	F	C	F	F	B
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	0	83	0	25429	0	0	609
N.S.	1	1.00	0.00	0.34	0.00	103.79	0.00	0.00	2.49
time (sec)	N/A	0.506	0.000	0.699	0.000	1.216	0.000	0.000	15.552

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	284	196	114	0	21243	0	0	697
N.S.	1	1.01	0.70	0.41	0.00	75.60	0.00	0.00	2.48
time (sec)	N/A	0.634	0.370	1.153	0.000	1.336	0.000	0.000	14.461

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	333	162	0	29423	0	0	1503
N.S.	1	1.00	1.12	0.55	0.00	99.40	0.00	0.00	5.08
time (sec)	N/A	0.715	1.455	1.480	0.000	4.046	0.000	0.000	16.275

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	168	228	137	0	872	0	0	1067
N.S.	1	0.95	1.29	0.77	0.00	4.93	0.00	0.00	6.03
time (sec)	N/A	0.406	2.668	2.972	0.000	0.345	0.000	0.000	14.425

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	142	310	110	0	849	0	0	1119
N.S.	1	0.96	2.09	0.74	0.00	5.74	0.00	0.00	7.56
time (sec)	N/A	0.342	1.298	1.717	0.000	0.335	0.000	0.000	13.796

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	135	198	99	0	815	0	0	1001
N.S.	1	0.98	1.43	0.72	0.00	5.91	0.00	0.00	7.25
time (sec)	N/A	0.345	0.823	1.130	0.000	0.359	0.000	0.000	13.498

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	114	285	83	0	703	0	166	976
N.S.	1	0.99	2.48	0.72	0.00	6.11	0.00	1.44	8.49
time (sec)	N/A	0.290	3.613	0.789	0.000	0.325	0.000	0.754	0.523

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	124	183	87	0	703	0	183	361
N.S.	1	0.99	1.46	0.70	0.00	5.62	0.00	1.46	2.89
time (sec)	N/A	0.268	2.287	0.687	0.000	0.336	0.000	0.678	15.333

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	131	318	119	0	773	0	0	2031
N.S.	1	0.96	2.34	0.88	0.00	5.68	0.00	0.00	14.93
time (sec)	N/A	0.335	0.691	0.942	0.000	0.368	0.000	0.000	15.637

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	174	242	152	0	924	0	0	2779
N.S.	1	0.95	1.32	0.83	0.00	5.02	0.00	0.00	15.10
time (sec)	N/A	0.369	0.863	1.280	0.000	0.406	0.000	0.000	15.155

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	213	409	193	0	1089	0	0	3692
N.S.	1	0.93	1.79	0.84	0.00	4.76	0.00	0.00	16.12
time (sec)	N/A	0.412	1.330	1.933	0.000	0.427	0.000	0.000	15.245

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	198	172	209	0	1311	0	461	5022
N.S.	1	1.08	0.93	1.14	0.00	7.12	0.00	2.51	27.29
time (sec)	N/A	0.443	1.423	2.230	0.000	0.431	0.000	0.739	17.019

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	162	157	170	0	1275	0	695	1273
N.S.	1	1.05	1.01	1.10	0.00	8.23	0.00	4.48	8.21
time (sec)	N/A	0.370	1.311	1.307	0.000	0.447	0.000	0.780	16.086

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	131	143	128	0	1125	0	912	2991
N.S.	1	1.03	1.13	1.01	0.00	8.86	0.00	7.18	23.55
time (sec)	N/A	0.358	2.226	0.716	0.000	0.397	0.000	0.724	16.322

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	168	137	114	0	1087	0	397	443
N.S.	1	1.34	1.10	0.91	0.00	8.70	0.00	3.18	3.54
time (sec)	N/A	0.314	1.538	0.688	0.000	0.405	0.000	0.791	16.778

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	168	128	128	0	1079	0	361	671
N.S.	1	1.46	1.11	1.11	0.00	9.38	0.00	3.14	5.83
time (sec)	N/A	0.290	0.966	0.574	0.000	0.410	0.000	0.449	15.423

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	134	143	155	0	1229	0	672	371
N.S.	1	0.96	1.03	1.12	0.00	8.84	0.00	4.83	2.67
time (sec)	N/A	0.355	1.210	1.036	0.000	0.412	0.000	0.752	14.809

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	141	165	177	0	1365	0	937	1670
N.S.	1	0.95	1.11	1.19	0.00	9.16	0.00	6.29	11.21
time (sec)	N/A	0.373	1.644	1.443	0.000	0.409	0.000	0.700	15.907

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	167	174	195	0	1477	0	471	416
N.S.	1	0.94	0.98	1.10	0.00	8.30	0.00	2.65	2.34
time (sec)	N/A	0.385	3.617	2.016	0.000	0.462	0.000	0.740	14.362

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	183	193	213	0	1585	0	467	1704
N.S.	1	0.93	0.98	1.08	0.00	8.05	0.00	2.37	8.65
time (sec)	N/A	0.412	6.332	2.954	0.000	0.438	0.000	0.809	16.823

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	236	260	486	209	0	2649	0	0	3941
N.S.	1	1.10	2.06	0.89	0.00	11.22	0.00	0.00	16.70
time (sec)	N/A	0.622	2.369	4.122	0.000	0.595	0.000	0.000	15.996

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	210	228	565	214	0	2507	0	0	3612
N.S.	1	1.09	2.69	1.02	0.00	11.94	0.00	0.00	17.20
time (sec)	N/A	0.455	2.674	2.152	0.000	0.541	0.000	0.000	16.130

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	217	228	469	188	0	2507	0	0	3839
N.S.	1	1.05	2.16	0.87	0.00	11.55	0.00	0.00	17.69
time (sec)	N/A	0.415	2.584	2.084	0.000	0.601	0.000	0.000	16.688

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	186	211	345	204	0	2049	0	577	3060
N.S.	1	1.13	1.85	1.10	0.00	11.02	0.00	3.10	16.45
time (sec)	N/A	0.381	0.479	1.877	0.000	0.424	0.000	0.999	15.735

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	221	232	469	241	0	2269	0	693	3507
N.S.	1	1.05	2.12	1.09	0.00	10.27	0.00	3.14	15.87
time (sec)	N/A	0.408	0.481	2.372	0.000	0.561	0.000	0.867	16.516

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	325	311	600	243	0	2711	0	0	7491
N.S.	1	0.96	1.85	0.75	0.00	8.34	0.00	0.00	23.05
time (sec)	N/A	0.538	1.218	2.690	0.000	0.862	0.000	0.000	17.439

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	376	262	266	0	3544	0	1563	7640
N.S.	1	1.18	0.82	0.83	0.00	11.08	0.00	4.88	23.88
time (sec)	N/A	0.770	6.686	1.971	0.000	0.943	0.000	0.935	17.819

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	271	238	262	0	3135	0	1481	3400
N.S.	1	1.16	1.02	1.12	0.00	13.45	0.00	6.36	14.59
time (sec)	N/A	0.475	2.951	2.166	0.000	0.864	0.000	0.815	16.336

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	238	225	218	0	2796	0	1264	2980
N.S.	1	1.22	1.15	1.12	0.00	14.34	0.00	6.48	15.28
time (sec)	N/A	0.427	6.528	1.780	0.000	0.623	0.000	0.816	16.004

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	257	255	250	0	3445	0	1407	3842
N.S.	1	1.17	1.16	1.14	0.00	15.73	0.00	6.42	17.54
time (sec)	N/A	0.450	1.671	1.941	0.000	0.923	0.000	0.812	17.168

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	261	230	260	0	3477	0	1506	3675
N.S.	1	1.24	1.10	1.24	0.00	16.56	0.00	7.17	17.50
time (sec)	N/A	0.476	4.162	2.019	0.000	0.934	0.000	0.509	17.091

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	237	274	269	0	3648	0	1545	4411
N.S.	1	1.00	1.16	1.14	0.00	15.46	0.00	6.55	18.69
time (sec)	N/A	0.709	2.321	2.625	0.000	1.105	0.000	0.893	18.227

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	315	371	785	342	0	4640	0	0	6675
N.S.	1	1.18	2.49	1.09	0.00	14.73	0.00	0.00	21.19
time (sec)	N/A	0.682	7.067	5.587	0.000	1.069	0.000	0.000	19.407

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	290	305	630	303	0	4185	0	0	5824
N.S.	1	1.05	2.17	1.04	0.00	14.43	0.00	0.00	20.08
time (sec)	N/A	0.534	2.267	5.260	0.000	0.745	0.000	0.000	18.678

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	313	361	786	347	0	4524	0	0	6362
N.S.	1	1.15	2.51	1.11	0.00	14.45	0.00	0.00	20.33
time (sec)	N/A	0.566	6.238	5.971	0.000	0.975	0.000	0.000	19.510

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	288	316	631	403	0	4050	0	1076	5566
N.S.	1	1.10	2.19	1.40	0.00	14.06	0.00	3.74	19.33
time (sec)	N/A	0.537	1.142	5.865	0.000	0.752	0.000	1.646	18.396

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	313	364	784	430	0	4160	0	793	5753
N.S.	1	1.16	2.50	1.37	0.00	13.29	0.00	2.53	18.38
time (sec)	N/A	0.538	1.648	6.150	0.000	0.890	0.000	1.960	18.284

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	617	591	920	384	0	5020	0	0	12247
N.S.	1	0.96	1.49	0.62	0.00	8.14	0.00	0.00	19.85
time (sec)	N/A	0.940	10.945	7.505	0.000	2.204	0.000	0.000	20.969

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	351	331	374	0	5219	0	1987	5508
N.S.	1	1.10	1.04	1.17	0.00	16.36	0.00	6.23	17.27
time (sec)	N/A	0.678	10.985	4.523	0.000	1.327	0.000	1.317	19.354

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	382	350	432	0	5961	0	2231	6391
N.S.	1	1.11	1.02	1.26	0.00	17.38	0.00	6.50	18.63
time (sec)	N/A	0.832	6.226	4.787	0.000	2.031	0.000	1.398	20.090

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	357	316	372	0	5510	0	1986	5892
N.S.	1	1.14	1.01	1.19	0.00	17.60	0.00	6.35	18.82
time (sec)	N/A	0.804	11.440	4.438	0.000	1.437	0.000	1.405	18.728

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	389	457	422	0	6215	0	1184	6646
N.S.	1	1.12	1.32	1.22	0.00	17.91	0.00	3.41	19.15
time (sec)	N/A	0.855	7.167	5.493	0.000	2.366	0.000	1.981	19.029

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	362	333	416	0	6152	0	1131	6267
N.S.	1	1.13	1.04	1.30	0.00	19.29	0.00	3.55	19.65
time (sec)	N/A	0.810	8.211	5.664	0.000	2.512	0.000	0.710	19.239

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	378	357	437	0	6323	0	2203	7364
N.S.	1	1.06	1.00	1.22	0.00	17.71	0.00	6.17	20.63
time (sec)	N/A	1.381	11.400	6.472	0.000	2.654	0.000	1.670	19.944

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	18	17	43	724	51	17
N.S.	1	1.00	0.96	0.72	0.68	1.72	28.96	2.04	0.68
time (sec)	N/A	0.177	0.331	1.141	0.395	0.270	21.730	0.364	13.765

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	487	541	148	101	0	823	0	318	407
N.S.	1	1.11	0.30	0.21	0.00	1.69	0.00	0.65	0.84
time (sec)	N/A	1.320	4.015	0.621	0.000	0.384	0.000	0.839	14.971

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	200	45	126	0	117	0	170	236
N.S.	1	0.65	0.15	0.41	0.00	0.38	0.00	0.55	0.76
time (sec)	N/A	0.465	5.036	1.533	0.000	0.302	0.000	0.578	14.512

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	477	477	3120	439	0	0	0	0	0
N.S.	1	1.00	6.54	0.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.549	26.937	3.265	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	521	587	2045	0	0	0	0	0	0
N.S.	1	1.13	3.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.835	24.443	0.000	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	484	488	2854	837	0	0	0	0	0
N.S.	1	1.01	5.90	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.560	27.465	3.870	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	431	435	2663	398	0	0	0	0	0
N.S.	1	1.01	6.18	0.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.499	23.803	2.854	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	294	163	0	0	0	0	0
N.S.	1	1.00	1.72	0.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.277	7.810	2.047	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	469	496	489	0	0	0	0	0	0
N.S.	1	1.06	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.681	14.570	0.000	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	776	802	1442	0	0	0	0	0	0
N.S.	1	1.03	1.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.295	25.158	0.000	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	499	541	287	881	0	0	0	0	0
N.S.	1	1.08	0.58	1.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.801	3.316	4.699	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	195	396	0	0	0	0	0
N.S.	1	1.00	1.20	2.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.287	1.360	2.881	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	493	515	464	0	0	0	0	0	0
N.S.	1	1.04	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.624	13.203	0.000	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-2)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	384	384	149	109	0	0	0	0	1515
N.S.	1	1.00	0.39	0.28	0.00	0.00	0.00	0.00	3.95
time (sec)	N/A	0.891	5.125	1.057	0.000	0.000	0.000	0.000	19.689

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	148	68	0	15501	0	0	513
N.S.	1	1.00	0.87	0.40	0.00	90.65	0.00	0.00	3.00
time (sec)	N/A	0.525	5.076	2.319	0.000	1.713	0.000	0.000	15.278

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	245	257	174	85	0	665483	0	0	816
N.S.	1	1.05	0.71	0.35	0.00	2716.26	0.00	0.00	3.33
time (sec)	N/A	0.675	5.123	1.689	0.000	6.398	0.000	0.000	16.281

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-2)	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	379	385	149	109	0	0	0	0	1515
N.S.	1	1.02	0.39	0.29	0.00	0.00	0.00	0.00	4.00
time (sec)	N/A	0.732	5.117	1.006	0.000	0.000	0.000	0.000	18.730

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	148	71	0	16697	0	0	513
N.S.	1	1.00	0.85	0.41	0.00	95.41	0.00	0.00	2.93
time (sec)	N/A	0.505	5.069	2.142	0.000	1.760	0.000	0.000	14.755

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	174	88	0	643307	0	0	818
N.S.	1	1.00	0.82	0.41	0.00	3020.22	0.00	0.00	3.84
time (sec)	N/A	0.485	5.020	1.734	0.000	6.886	0.000	0.000	16.360

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	411	87	0	848	0	0	3513
N.S.	1	1.00	2.11	0.45	0.00	4.35	0.00	0.00	18.02
time (sec)	N/A	0.568	5.093	0.685	0.000	0.407	0.000	0.000	14.267

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	83	79	72	71	138	0	185	98
N.S.	1	0.81	0.77	0.70	0.69	1.34	0.00	1.80	0.95
time (sec)	N/A	0.334	5.094	2.174	0.332	0.286	0.000	0.297	13.958

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	129	141	71	0	893	0	0	945
N.S.	1	0.59	0.65	0.33	0.00	4.10	0.00	0.00	4.33
time (sec)	N/A	0.426	5.052	4.451	0.000	0.427	0.000	0.000	14.956

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	197	413	87	0	858	0	0	3513
N.S.	1	1.05	2.21	0.47	0.00	4.59	0.00	0.00	18.79
time (sec)	N/A	0.505	5.093	0.648	0.000	0.387	0.000	0.000	13.667

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	117	51	0	239	0	197	99
N.S.	1	1.00	1.65	0.72	0.00	3.37	0.00	2.77	1.39
time (sec)	N/A	0.498	1.584	0.721	0.000	0.322	0.000	0.349	13.561

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	69	64	176	0	166	0	220	141
N.S.	1	0.78	0.72	1.98	0.00	1.87	0.00	2.47	1.58
time (sec)	N/A	0.422	3.400	5.766	0.000	0.321	0.000	0.532	13.792

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	30	29	27	28	27	580	28	34
N.S.	1	0.79	0.76	0.71	0.74	0.71	15.26	0.74	0.89
time (sec)	N/A	0.271	0.014	0.367	0.240	0.260	12.862	0.301	0.106

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	26	23	20	22	21	311	22	19
N.S.	1	0.90	0.79	0.69	0.76	0.72	10.72	0.76	0.66
time (sec)	N/A	0.270	0.010	0.381	0.269	0.259	5.828	0.320	0.060

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	15	14	14	13	124	14	16
N.S.	1	1.00	0.83	0.78	0.78	0.72	6.89	0.78	0.89
time (sec)	N/A	0.254	0.006	0.316	0.244	0.278	2.588	0.310	0.043

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	15	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	2.50	1.00	1.00
time (sec)	N/A	0.214	0.008	0.234	0.211	0.254	0.811	0.326	13.750

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	21	20	19	23	7
N.S.	1	1.00	1.00	1.14	3.00	2.86	2.71	3.29	1.00
time (sec)	N/A	0.212	0.004	0.279	0.240	0.284	0.106	0.307	13.737

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	43	51	46	0	47	31
N.S.	1	1.00	0.86	1.23	1.46	1.31	0.00	1.34	0.89
time (sec)	N/A	0.353	0.008	0.675	0.232	0.288	0.000	0.305	13.605

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	26	20	37	23	473	36	25
N.S.	1	1.00	0.79	0.61	1.12	0.70	14.33	1.09	0.76
time (sec)	N/A	0.292	0.004	0.355	0.325	0.258	3.751	0.308	13.575

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	18	18	14	21	12	153	24	13
N.S.	1	0.90	0.90	0.70	1.05	0.60	7.65	1.20	0.65
time (sec)	N/A	0.247	0.003	0.243	0.326	0.264	1.448	0.302	13.335

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	2	14	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	0.40	2.80	1.00
time (sec)	N/A	0.191	0.001	0.258	0.400	0.275	0.458	0.299	12.919

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	20	20	32	37	37	0	38	25
N.S.	1	0.91	0.91	1.45	1.68	1.68	0.00	1.73	1.14
time (sec)	N/A	0.284	0.006	0.435	0.297	0.277	0.000	0.320	13.877

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	15	14	14	19	0	14	13
N.S.	1	1.00	0.83	0.78	0.78	1.06	0.00	0.78	0.72
time (sec)	N/A	0.256	0.006	0.388	0.313	0.266	0.000	0.308	13.680

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	26	23	20	22	25	0	22	21
N.S.	1	0.90	0.79	0.69	0.76	0.86	0.00	0.76	0.72
time (sec)	N/A	0.269	0.002	0.654	0.222	0.251	0.000	0.301	13.211

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	26	23	20	22	21	362	22	19
N.S.	1	0.90	0.79	0.69	0.76	0.72	12.48	0.76	0.66
time (sec)	N/A	0.257	0.003	0.328	0.257	0.268	32.285	0.301	13.051

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	15	14	14	13	144	14	16
N.S.	1	1.00	0.83	0.78	0.78	0.72	8.00	0.78	0.89
time (sec)	N/A	0.256	0.001	0.359	0.218	0.282	13.230	0.300	0.039

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	19	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	3.17	1.00	1.00
time (sec)	N/A	0.224	0.002	0.304	0.224	0.265	6.088	0.290	0.026

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	21	20	22	23	7
N.S.	1	1.00	1.00	1.14	3.00	2.86	3.14	3.29	1.00
time (sec)	N/A	0.235	0.001	0.320	0.236	0.252	2.323	0.295	0.058

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	20	20	32	41	37	117	38	30
N.S.	1	0.91	0.91	1.45	1.86	1.68	5.32	1.73	1.36
time (sec)	N/A	0.278	0.001	0.433	0.232	0.278	0.326	0.316	0.077

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	43	57	46	0	47	31
N.S.	1	1.00	0.86	1.23	1.63	1.31	0.00	1.34	0.89
time (sec)	N/A	0.350	0.001	0.544	0.267	0.277	0.000	0.302	13.475

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	26	20	43	23	549	31	29
N.S.	1	1.00	0.79	0.61	1.30	0.70	16.64	0.94	0.88
time (sec)	N/A	0.288	0.003	0.286	0.383	0.286	20.776	0.284	13.328

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	18	18	14	25	12	178	22	13
N.S.	1	0.90	0.90	0.70	1.25	0.60	8.90	1.10	0.65
time (sec)	N/A	0.233	0.003	0.225	0.531	0.270	9.268	0.291	13.588

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	3	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	0.60	1.00	1.00
time (sec)	N/A	0.195	0.001	0.328	0.387	0.254	3.776	0.297	13.955

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	10	20	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.67	3.33	1.00	1.00
time (sec)	N/A	0.238	0.005	0.400	0.364	0.290	1.616	0.286	13.893

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	26	23	20	22	25	0	22	21
N.S.	1	0.90	0.79	0.69	0.76	0.86	0.00	0.76	0.72
time (sec)	N/A	0.266	0.001	0.588	0.265	0.291	0.000	0.317	13.661

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	32	27	24	28	31	0	28	33
N.S.	1	0.86	0.73	0.65	0.76	0.84	0.00	0.76	0.89
time (sec)	N/A	0.265	0.001	0.894	0.298	0.287	0.000	0.306	13.663

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	125	87	78	122	78	354	83	119
N.S.	1	1.15	0.80	0.72	1.12	0.72	3.25	0.76	1.09
time (sec)	N/A	0.240	0.335	1.423	0.391	0.310	0.831	0.333	15.601

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	98	64	62	97	63	250	64	91
N.S.	1	1.18	0.77	0.75	1.17	0.76	3.01	0.77	1.10
time (sec)	N/A	0.239	0.186	1.306	0.327	0.314	0.402	0.314	14.563

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	71	46	40	69	47	150	39	67
N.S.	1	1.25	0.81	0.70	1.21	0.82	2.63	0.68	1.18
time (sec)	N/A	0.222	0.118	0.603	0.345	0.270	0.199	0.308	13.822

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	33	24	29	29	51	25	27
N.S.	1	1.00	1.10	0.80	0.97	0.97	1.70	0.83	0.90
time (sec)	N/A	0.160	0.068	0.371	0.249	0.285	0.094	0.312	13.567

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	25	36	30	30	35	0	31	26
N.S.	1	1.39	2.00	1.67	1.67	1.94	0.00	1.72	1.44
time (sec)	N/A	0.206	0.019	0.916	0.369	0.323	0.000	0.316	13.299

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	28	41	46	27	38	0	35	31
N.S.	1	0.93	1.37	1.53	0.90	1.27	0.00	1.17	1.03
time (sec)	N/A	0.217	0.085	1.041	0.304	0.274	0.000	0.328	13.206

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	45	64	76	43	59	0	59	45
N.S.	1	0.90	1.28	1.52	0.86	1.18	0.00	1.18	0.90
time (sec)	N/A	0.236	0.388	1.139	0.348	0.273	0.000	0.340	13.478

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	64	86	104	60	77	0	81	59
N.S.	1	0.89	1.19	1.44	0.83	1.07	0.00	1.12	0.82
time (sec)	N/A	0.254	0.489	1.140	0.264	0.302	0.000	0.366	13.651

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	157	96	96	169	114	481	104	160
N.S.	1	1.01	0.62	0.62	1.08	0.73	3.08	0.67	1.03
time (sec)	N/A	0.298	2.359	1.635	0.349	0.296	0.729	0.374	14.903

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	128	79	78	127	85	314	81	120
N.S.	1	1.10	0.68	0.67	1.09	0.73	2.71	0.70	1.03
time (sec)	N/A	0.303	2.242	0.838	0.337	0.293	0.380	0.349	14.507

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	58	56	68	63	168	58	77
N.S.	1	1.00	0.81	0.78	0.94	0.88	2.33	0.81	1.07
time (sec)	N/A	0.214	0.257	0.926	0.248	0.277	0.188	0.293	14.113

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	61	48	65	74	68	0	77	74
N.S.	1	1.20	0.94	1.27	1.45	1.33	0.00	1.51	1.45
time (sec)	N/A	0.275	0.913	1.332	0.332	0.287	0.000	0.314	13.844

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	49	59	76	53	70	0	74	46
N.S.	1	1.09	1.31	1.69	1.18	1.56	0.00	1.64	1.02
time (sec)	N/A	0.256	0.248	1.060	0.443	0.288	0.000	0.343	13.771

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	48	67	101	55	83	0	80	44
N.S.	1	0.91	1.26	1.91	1.04	1.57	0.00	1.51	0.83
time (sec)	N/A	0.249	1.331	1.011	0.259	0.269	0.000	0.356	13.783

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	72	92	149	81	108	0	118	72
N.S.	1	0.90	1.15	1.86	1.01	1.35	0.00	1.48	0.90
time (sec)	N/A	0.279	3.009	1.231	0.254	0.269	0.000	0.368	14.141

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	95	107	195	103	128	0	156	94
N.S.	1	0.90	1.01	1.84	0.97	1.21	0.00	1.47	0.89
time (sec)	N/A	0.299	6.059	1.686	0.268	0.268	0.000	0.373	14.001

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	109	96	86	233	0	98	99
N.S.	1	1.00	1.40	1.23	1.10	2.99	0.00	1.26	1.27
time (sec)	N/A	0.283	0.305	1.312	0.370	0.317	0.000	0.294	0.113

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	114	79	96	114	312	0	131	1804
N.S.	1	1.31	0.91	1.10	1.31	3.59	0.00	1.51	20.74
time (sec)	N/A	0.343	0.234	0.970	0.344	0.324	0.000	0.297	14.525

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	84	54	52	159	0	58	65
N.S.	1	1.00	1.56	1.00	0.96	2.94	0.00	1.07	1.20
time (sec)	N/A	0.259	0.200	0.797	0.369	0.319	0.000	0.292	0.103

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	74	55	63	64	239	0	92	119
N.S.	1	1.25	0.93	1.07	1.08	4.05	0.00	1.56	2.02
time (sec)	N/A	0.278	0.188	0.720	0.354	0.335	0.000	0.292	13.654

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	31	30	101	0	30	28
N.S.	1	1.00	1.00	0.86	0.83	2.81	0.00	0.83	0.78
time (sec)	N/A	0.225	0.026	0.640	0.364	0.317	0.000	0.304	0.096

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	41	39	38	35	206	0	62	272
N.S.	1	1.05	1.00	0.97	0.90	5.28	0.00	1.59	6.97
time (sec)	N/A	0.236	0.087	0.354	0.356	0.289	0.000	0.312	14.015

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	17	16	78	66	16	17
N.S.	1	1.00	1.00	0.68	0.64	3.12	2.64	0.64	0.68
time (sec)	N/A	0.199	0.009	0.366	0.377	0.293	0.372	0.306	14.319

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	96	55	47	116	0	49	856
N.S.	1	1.00	2.40	1.38	1.18	2.90	0.00	1.22	21.40
time (sec)	N/A	0.223	0.150	0.459	0.362	0.334	0.000	0.304	15.202

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	38	37	255	0	45	39
N.S.	1	1.00	1.00	0.97	0.95	6.54	0.00	1.15	1.00
time (sec)	N/A	0.229	0.128	0.604	0.366	0.318	0.000	0.289	14.102

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	77	147	96	104	203	0	102	1139
N.S.	1	1.26	2.41	1.57	1.70	3.33	0.00	1.67	18.67
time (sec)	N/A	0.267	0.348	0.784	0.341	0.335	0.000	0.287	13.898

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	62	72	343	0	134	77
N.S.	1	1.00	1.00	1.05	1.22	5.81	0.00	2.27	1.31
time (sec)	N/A	0.276	0.257	0.859	0.362	0.320	0.000	0.298	14.239

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	125	214	154	199	327	0	177	832
N.S.	1	1.34	2.30	1.66	2.14	3.52	0.00	1.90	8.95
time (sec)	N/A	0.318	1.325	1.159	0.326	0.396	0.000	0.294	17.833

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	90	109	126	459	0	254	121
N.S.	1	1.00	1.03	1.25	1.45	5.28	0.00	2.92	1.39
time (sec)	N/A	0.306	0.435	1.355	0.352	0.330	0.000	0.297	14.129

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	131	90	100	150	491	0	175	463
N.S.	1	1.16	0.80	0.88	1.33	4.35	0.00	1.55	4.10
time (sec)	N/A	0.364	0.353	1.139	0.383	0.344	0.000	0.301	14.918

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	118	77	79	296	0	82	96
N.S.	1	1.00	1.64	1.07	1.10	4.11	0.00	1.14	1.33
time (sec)	N/A	0.285	0.374	0.977	0.398	0.323	0.000	0.286	14.740

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	87	79	75	80	367	0	109	533
N.S.	1	1.16	1.05	1.00	1.07	4.89	0.00	1.45	7.11
time (sec)	N/A	0.286	0.374	0.735	0.518	0.335	0.000	0.297	14.613

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	53	53	206	0	56	47
N.S.	1	1.00	1.00	0.90	0.90	3.49	0.00	0.95	0.80
time (sec)	N/A	0.239	0.208	0.594	0.425	0.313	0.000	0.293	14.578

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	59	51	49	313	0	77	50
N.S.	1	1.00	1.09	0.94	0.91	5.80	0.00	1.43	0.93
time (sec)	N/A	0.233	0.184	0.493	0.388	0.342	0.000	0.307	13.866

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	39	38	165	289	38	36
N.S.	1	1.00	1.00	0.81	0.79	3.44	6.02	0.79	0.75
time (sec)	N/A	0.212	0.042	0.430	0.407	0.301	3.259	0.304	13.062

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	86	130	79	115	354	0	109	2213
N.S.	1	1.18	1.78	1.08	1.58	4.85	0.00	1.49	30.32
time (sec)	N/A	0.277	0.558	0.837	0.383	0.351	0.000	0.299	14.915

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	81	119	505	0	113	123
N.S.	1	1.00	1.00	1.07	1.57	6.64	0.00	1.49	1.62
time (sec)	N/A	0.303	0.554	1.035	0.468	0.319	0.000	0.315	13.911

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	130	183	119	220	560	0	194	2009
N.S.	1	1.19	1.68	1.09	2.02	5.14	0.00	1.78	18.43
time (sec)	N/A	0.338	1.005	1.485	0.480	0.409	0.000	0.299	14.644

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	97	110	170	653	0	270	176
N.S.	1	1.00	1.01	1.15	1.77	6.80	0.00	2.81	1.83
time (sec)	N/A	0.323	0.976	1.646	0.405	0.340	0.000	0.313	14.091

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	112	125	146	119	511	0	0	0
N.S.	1	0.96	1.07	1.25	1.02	4.37	0.00	0.00	0.00
time (sec)	N/A	0.284	0.470	0.579	0.279	0.593	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	70	96	60	46	453	0	0	61
N.S.	1	0.97	1.33	0.83	0.64	6.29	0.00	0.00	0.85
time (sec)	N/A	0.241	0.257	0.319	0.350	0.370	0.000	0.000	13.428

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	80	129	150	126	1246	0	0	0
N.S.	1	0.98	1.57	1.83	1.54	15.20	0.00	0.00	0.00
time (sec)	N/A	0.285	0.287	1.116	0.435	0.517	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	88	164	290	0	337	0	0	0
N.S.	1	1.07	2.00	3.54	0.00	4.11	0.00	0.00	0.00
time (sec)	N/A	0.274	2.234	1.344	0.000	0.333	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	143	150	669	570	0	443	0	0	0
N.S.	1	1.05	4.68	3.99	0.00	3.10	0.00	0.00	0.00
time (sec)	N/A	0.315	14.156	1.590	0.000	0.551	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	260	199	432	0	0	0	0	0
N.S.	1	1.18	0.90	1.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.454	1.790	3.780	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	186	158	265	0	0	0	0	0
N.S.	1	1.17	0.99	1.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.378	1.121	3.141	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	61	71	0	0	0	0	0
N.S.	1	1.00	1.20	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	0.066	1.385	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	174	134	222	0	640	0	0	0
N.S.	1	1.33	1.02	1.69	0.00	4.89	0.00	0.00	0.00
time (sec)	N/A	0.360	0.803	2.665	0.000	0.135	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	244	187	368	0	789	0	0	0
N.S.	1	1.24	0.95	1.88	0.00	4.03	0.00	0.00	0.00
time (sec)	N/A	0.433	2.193	3.694	0.000	0.160	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	144	149	215	174	577	0	0	0
N.S.	1	0.92	0.95	1.37	1.11	3.68	0.00	0.00	0.00
time (sec)	N/A	0.307	0.879	0.497	0.417	1.378	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	102	93	90	73	503	0	0	60
N.S.	1	0.98	0.89	0.87	0.70	4.84	0.00	0.00	0.58
time (sec)	N/A	0.252	0.514	0.354	0.333	0.510	0.000	0.000	14.240

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	120	233	269	168	1381	0	0	0
N.S.	1	0.99	1.93	2.22	1.39	11.41	0.00	0.00	0.00
time (sec)	N/A	0.338	0.720	1.311	0.391	0.692	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	133	210	402	0	1471	0	0	0
N.S.	1	1.05	1.65	3.17	0.00	11.58	0.00	0.00	0.00
time (sec)	N/A	0.341	0.962	1.485	0.000	0.713	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	134	63	406	0	413	0	0	0
N.S.	1	1.10	0.52	3.33	0.00	3.39	0.00	0.00	0.00
time (sec)	N/A	0.300	0.121	1.389	0.000	0.673	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	195	196	938	693	0	545	0	0	0
N.S.	1	1.01	4.81	3.55	0.00	2.79	0.00	0.00	0.00
time (sec)	N/A	0.347	14.936	2.079	0.000	2.276	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	331	247	590	0	0	0	0	0
N.S.	1	1.03	0.77	1.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.543	3.143	4.569	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	256	200	429	0	0	0	0	0
N.S.	1	0.99	0.77	1.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.462	1.668	5.092	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	155	156	266	0	0	0	0	0
N.S.	1	1.01	1.01	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.812	0.835	2.399	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	185	144	365	0	0	0	0	0
N.S.	1	1.02	0.79	2.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.390	1.615	3.867	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	253	190	375	0	784	0	0	0
N.S.	1	1.07	0.81	1.59	0.00	3.32	0.00	0.00	0.00
time (sec)	N/A	0.466	2.370	3.726	0.000	0.192	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	77	79	93	69	461	0	0	0
N.S.	1	0.97	1.00	1.18	0.87	5.84	0.00	0.00	0.00
time (sec)	N/A	0.262	0.106	0.444	0.246	0.394	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	34	21	394	0	0	33
N.S.	1	1.00	1.00	0.89	0.55	10.37	0.00	0.00	0.87
time (sec)	N/A	0.229	0.017	0.314	0.248	0.364	0.000	0.000	13.620

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	105	105	240	0	0	0
N.S.	1	1.00	1.00	2.50	2.50	5.71	0.00	0.00	0.00
time (sec)	N/A	0.247	0.036	1.059	0.354	0.357	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	91	97	408	360	0	361	0	0	0
N.S.	1	1.07	4.48	3.96	0.00	3.97	0.00	0.00	0.00
time (sec)	N/A	0.283	10.143	1.053	0.000	0.430	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	198	170	316	0	0	0	0	0
N.S.	1	1.18	1.01	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.396	1.185	2.431	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	133	83	111	0	0	0	0	0
N.S.	1	1.17	0.73	0.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.334	0.447	1.846	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	60	52	0	305	0	0	0
N.S.	1	1.00	1.18	1.02	0.00	5.98	0.00	0.00	0.00
time (sec)	N/A	0.281	0.057	0.469	0.000	0.112	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	185	141	278	0	632	0	0	0
N.S.	1	1.32	1.01	1.99	0.00	4.51	0.00	0.00	0.00
time (sec)	N/A	0.372	0.974	2.364	0.000	0.128	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	260	205	405	0	808	0	0	0
N.S.	1	1.23	0.97	1.91	0.00	3.81	0.00	0.00	0.00
time (sec)	N/A	0.445	2.572	3.893	0.000	0.160	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	73	88	85	74	559	0	0	0
N.S.	1	0.97	1.17	1.13	0.99	7.45	0.00	0.00	0.00
time (sec)	N/A	0.266	0.185	0.362	0.221	0.422	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	27	49	0	3392	117
N.S.	1	1.00	1.00	0.97	0.93	1.69	0.00	116.97	4.03
time (sec)	N/A	0.218	0.033	0.266	0.214	0.319	0.000	12.378	14.349

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	78	76	480	397	152	453	0	0	0
N.S.	1	0.97	6.15	5.09	1.95	5.81	0.00	0.00	0.00
time (sec)	N/A	0.270	7.517	1.144	0.334	0.390	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	134	142	224	3219	0	625	0	0	0
N.S.	1	1.06	1.67	24.02	0.00	4.66	0.00	0.00	0.00
time (sec)	N/A	0.339	7.367	3.545	0.000	0.597	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	273	184	415	0	0	0	0	0
N.S.	1	1.00	0.67	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.484	1.606	3.981	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	197	139	286	0	0	0	0	0
N.S.	1	0.98	0.69	1.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.405	0.874	2.530	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	178	133	145	0	777	0	0	0
N.S.	1	0.95	0.71	0.77	0.00	4.13	0.00	0.00	0.00
time (sec)	N/A	0.370	0.539	2.066	0.000	0.139	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	90	103	0	938	0	0	0
N.S.	1	1.00	0.89	1.02	0.00	9.29	0.00	0.00	0.00
time (sec)	N/A	0.430	0.132	1.061	0.000	0.158	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	254	167	360	0	1079	0	0	0
N.S.	1	1.06	0.70	1.50	0.00	4.50	0.00	0.00	0.00
time (sec)	N/A	0.441	1.573	2.978	0.000	0.165	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	137	128	208	207	799	0	0	0
N.S.	1	1.05	0.98	1.60	1.59	6.15	0.00	0.00	0.00
time (sec)	N/A	0.322	0.872	0.409	0.306	1.109	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	75	51	138	107	107	0	35378	183
N.S.	1	1.03	0.70	1.89	1.47	1.47	0.00	484.63	2.51
time (sec)	N/A	0.264	0.121	0.346	0.239	0.458	0.000	104.200	21.073

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	63	47	56	55	104	0	35375	164
N.S.	1	0.97	0.72	0.86	0.85	1.60	0.00	544.23	2.52
time (sec)	N/A	0.236	0.051	0.272	0.241	0.413	0.000	63.997	19.358

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	126	134	1291	899	272	775	0	0	0
N.S.	1	1.06	10.25	7.13	2.16	6.15	0.00	0.00	0.00
time (sec)	N/A	0.333	9.569	1.630	0.467	0.594	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	281	194	712	0	0	0	0	0
N.S.	1	1.16	0.80	2.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.500	2.847	4.943	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	271	171	485	0	1343	0	0	0
N.S.	1	1.22	0.77	2.17	0.00	6.02	0.00	0.00	0.00
time (sec)	N/A	0.475	1.807	2.794	0.000	0.225	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	254	175	549	0	1394	0	0	0
N.S.	1	1.17	0.81	2.53	0.00	6.42	0.00	0.00	0.00
time (sec)	N/A	0.436	1.794	3.102	0.000	0.195	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	228	172	547	0	1531	0	0	0
N.S.	1	1.02	0.77	2.45	0.00	6.87	0.00	0.00	0.00
time (sec)	N/A	1.182	1.282	1.620	0.000	0.201	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	351	245	1082	0	1746	0	0	0
N.S.	1	1.22	0.85	3.76	0.00	6.06	0.00	0.00	0.00
time (sec)	N/A	0.554	4.012	4.787	0.000	0.239	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	228	0	0	0	0	0	0
N.S.	1	1.00	1.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.316	1.432	0.000	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	214	211	191	0	0	0	0	0	0
N.S.	1	0.99	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.379	0.440	0.000	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	117	120	0	0	0	0	0	0
N.S.	1	0.94	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.286	0.208	0.000	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	0	0	0	64
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.96
time (sec)	N/A	0.224	0.030	0.000	0.000	0.000	0.000	0.000	15.621

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.253	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	199	0	0	0	0	0	0
N.S.	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	0.853	0.000	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	195	0	0	0	0	0	0
N.S.	1	1.00	2.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.260	0.752	0.000	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	145	0	0	0	0	0	0
N.S.	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.556	0.000	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.260	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.257	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	208	203	264	210	3216	0	221	229
N.S.	1	0.95	0.93	1.21	0.96	14.68	0.00	1.01	1.05
time (sec)	N/A	0.466	0.261	1.069	0.314	1.020	0.000	0.389	14.545

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	155	139	106	159	1049	0	156	153
N.S.	1	0.93	0.83	0.63	0.95	6.28	0.00	0.93	0.92
time (sec)	N/A	0.436	0.162	0.545	0.339	0.975	0.000	0.368	13.991

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	135	116	48	121	401	184	137	123
N.S.	1	0.94	0.81	0.33	0.84	2.78	1.28	0.95	0.85
time (sec)	N/A	0.340	0.059	0.421	0.303	0.350	2.619	0.338	14.505

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	276	268	300	288	4396	0	309	600
N.S.	1	0.95	0.92	1.03	0.99	15.16	0.00	1.07	2.07
time (sec)	N/A	0.576	0.234	0.901	0.311	1.224	0.000	0.387	0.260

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	365	362	372	470	10135	0	510	898
N.S.	1	0.95	0.94	0.97	1.22	26.32	0.00	1.32	2.33
time (sec)	N/A	0.725	2.263	1.889	0.297	2.131	0.000	0.370	14.961

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	764	767	300	121	0	23437	0	0	2338
N.S.	1	1.00	0.39	0.16	0.00	30.68	0.00	0.00	3.06
time (sec)	N/A	1.791	0.537	1.741	0.000	3.707	0.000	0.000	17.289

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	484	484	231	83	0	8236	0	0	951
N.S.	1	1.00	0.48	0.17	0.00	17.02	0.00	0.00	1.96
time (sec)	N/A	1.006	0.355	0.892	0.000	1.632	0.000	0.000	15.548

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	C	F	C	F	F	B
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	0	83	0	25429	0	0	609
N.S.	1	1.00	0.00	0.34	0.00	103.79	0.00	0.00	2.49
time (sec)	N/A	0.523	0.000	0.459	0.000	1.245	0.000	0.000	14.887

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	C	F	C	F	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	0	432	162	0	59362	0	0	19737
N.S.	1	0.00	1.44	0.54	0.00	198.54	0.00	0.00	66.01
time (sec)	N/A	0.000	0.519	2.230	0.000	3.891	0.000	0.000	17.197

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	C	F(-2)	C	F	F	B
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1093	0	679	291	0	85064	0	0	323390
N.S.	1	0.00	0.62	0.27	0.00	77.83	0.00	0.00	295.87
time (sec)	N/A	0.000	2.064	4.090	0.000	12.200	0.000	0.000	24.617

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	284	439	311	263	3624	0	0	384
N.S.	1	0.99	1.52	1.08	0.91	12.58	0.00	0.00	1.33
time (sec)	N/A	0.552	3.127	2.441	0.333	1.554	0.000	0.000	0.410

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	239	258	284	213	2255	0	0	203
N.S.	1	1.00	1.08	1.19	0.89	9.47	0.00	0.00	0.85
time (sec)	N/A	0.499	0.963	1.060	0.303	1.211	0.000	0.000	13.698

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	177	184	153	163	665	0	0	172
N.S.	1	0.97	1.01	0.84	0.89	3.63	0.00	0.00	0.94
time (sec)	N/A	0.412	0.735	1.016	0.323	0.359	0.000	0.000	0.368

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	170	152	134	155	655	617	0	165
N.S.	1	0.97	0.86	0.76	0.88	3.72	3.51	0.00	0.94
time (sec)	N/A	0.358	0.482	0.904	0.306	0.357	92.897	0.000	14.162

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	587	561	564	389	483	10855	0	0	980
N.S.	1	0.96	0.96	0.66	0.82	18.49	0.00	0.00	1.67
time (sec)	N/A	0.907	4.380	2.214	0.315	2.597	0.000	0.000	14.815

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	747	715	724	483	788	15989	0	0	1605
N.S.	1	0.96	0.97	0.65	1.05	21.40	0.00	0.00	2.15
time (sec)	N/A	1.196	6.433	4.779	0.336	7.045	0.000	0.000	14.274

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	C	N/A	F(-2)	C	N/A	N/A	B
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	394	241	0	9984	22	3	2431
N.S.	1	1.00	17.13	10.48	0.00	434.09	0.96	0.13	105.70
time (sec)	N/A	0.222	0.895	1.744	0.000	9.399	148.602	6.109	15.051

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	C	N/A	F(-2)	C	N/A	N/A	B
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	273	175	0	36403	22	3	1648
N.S.	1	1.00	11.87	7.61	0.00	1582.74	0.96	0.13	71.65
time (sec)	N/A	0.225	0.765	2.080	0.000	3.241	125.450	4.408	15.244

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	C	N/A	F(-2)	C	N/A	N/A	B
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	502	349	0	70185	15	3	1567
N.S.	1	1.00	35.86	24.93	0.00	5013.21	1.07	0.21	111.93
time (sec)	N/A	0.188	0.609	2.882	0.000	8.471	99.859	4.337	17.023

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	C	N/A	F(-2)	C	F(-1)	N/A	B
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	845	398	0	102913	0	3	3148
N.S.	1	1.00	36.74	17.30	0.00	4474.48	0.00	0.13	136.87
time (sec)	N/A	0.223	2.214	6.545	0.000	43.884	0.000	4.343	19.147

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	C	N/A	F(-2)	C	F(-1)	N/A	B
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	1158	525	0	133123	0	3	4657
N.S.	1	1.00	50.35	22.83	0.00	5787.96	0.00	0.13	202.48
time (sec)	N/A	0.222	2.255	10.454	0.000	140.116	0.000	4.525	23.827

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	123	207	178	177	1429	0	360	1931
N.S.	1	0.94	1.58	1.36	1.35	10.91	0.00	2.75	14.74
time (sec)	N/A	0.368	0.285	1.972	0.342	0.597	0.000	0.720	0.762

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	108	189	152	158	1041	0	311	1097
N.S.	1	0.96	1.67	1.35	1.40	9.21	0.00	2.75	9.71
time (sec)	N/A	0.346	0.222	1.243	0.311	0.438	0.000	0.638	14.401

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	96	160	136	121	631	0	280	489
N.S.	1	1.01	1.68	1.43	1.27	6.64	0.00	2.95	5.15
time (sec)	N/A	0.290	0.085	0.725	0.295	0.368	0.000	0.859	14.790

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	69	54	48	100	161	129	224	40
N.S.	1	0.97	0.76	0.68	1.41	2.27	1.82	3.15	0.56
time (sec)	N/A	0.251	0.026	0.588	0.290	0.329	2.566	0.847	0.104

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	112	184	185	167	1329	0	370	3891
N.S.	1	0.96	1.57	1.58	1.43	11.36	0.00	3.16	33.26
time (sec)	N/A	0.336	0.196	1.220	0.309	0.423	0.000	0.846	16.540

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	164	255	241	244	2529	0	475	7758
N.S.	1	0.94	1.46	1.38	1.39	14.45	0.00	2.71	44.33
time (sec)	N/A	0.402	1.098	2.086	0.300	0.830	0.000	0.824	18.342

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	232	317	322	363	3703	0	630	12217
N.S.	1	0.93	1.27	1.29	1.46	14.87	0.00	2.53	49.06
time (sec)	N/A	0.506	5.867	3.402	0.296	1.653	0.000	1.278	19.999

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	276	233	258	0	2948	0	896	10319
N.S.	1	1.10	0.92	1.02	0.00	11.70	0.00	3.56	40.95
time (sec)	N/A	0.584	1.607	4.143	0.000	1.633	0.000	1.015	18.627

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	200	200	224	0	2433	0	836	8773
N.S.	1	1.08	1.08	1.20	0.00	13.08	0.00	4.49	47.17
time (sec)	N/A	0.471	1.181	2.680	0.000	0.974	0.000	0.950	16.879

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	162	194	179	0	1751	0	995	3088
N.S.	1	1.05	1.25	1.15	0.00	11.30	0.00	6.42	19.92
time (sec)	N/A	0.437	0.904	1.579	0.000	0.741	0.000	0.857	16.547

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	131	171	148	0	1197	0	906	4299
N.S.	1	1.03	1.35	1.17	0.00	9.43	0.00	7.13	33.85
time (sec)	N/A	0.395	0.543	0.904	0.000	0.557	0.000	0.797	16.035

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	161	158	73	0	541	0	559	1409
N.S.	1	1.29	1.26	0.58	0.00	4.33	0.00	4.47	11.27
time (sec)	N/A	0.310	0.558	0.720	0.000	0.364	0.000	0.802	14.752

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	137	175	166	0	2589	0	1211	2832
N.S.	1	0.96	1.23	1.17	0.00	18.23	0.00	8.53	19.94
time (sec)	N/A	0.395	0.819	1.474	0.000	0.671	0.000	0.843	15.535

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	153	205	230	0	4113	0	2183	4664
N.S.	1	0.95	1.27	1.43	0.00	25.55	0.00	13.56	28.97
time (sec)	N/A	0.478	1.308	2.290	0.000	0.960	0.000	0.927	16.484

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	193	253	311	0	5587	0	3105	6534
N.S.	1	0.95	1.24	1.52	0.00	27.39	0.00	15.22	32.03
time (sec)	N/A	0.531	1.770	3.424	0.000	1.590	0.000	0.997	17.543

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	37	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.61	0.00	1.09	1.09
time (sec)	N/A	0.215	6.681	1.306	2.112	0.631	0.000	0.938	18.605

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	197	199	141	0	0	0	0	0	0
N.S.	1	1.01	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.410	0.161	0.000	0.000	0.000	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	138	106	0	0	0	0	0	0
N.S.	1	0.99	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.311	0.057	0.000	0.000	0.000	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	0	0	0	64
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.96
time (sec)	N/A	0.230	0.020	0.000	0.000	0.000	0.000	0.000	13.766

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	158	156	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.358	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	239	234	0	0	0	0	0	0	0
N.S.	1	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.410	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	37	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.61	0.00	1.09	1.09
time (sec)	N/A	0.216	3.462	1.936	3.537	0.309	0.000	0.895	15.247

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	37	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.61	0.00	1.09	1.09
time (sec)	N/A	0.216	4.644	1.106	2.112	0.329	0.000	0.908	14.412

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	28	0	16	16
N.S.	1	1.00	1.14	1.00	1.14	2.00	0.00	1.14	1.14
time (sec)	N/A	0.176	1.158	0.646	1.058	0.283	0.000	0.591	14.142

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	37	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.61	0.00	1.09	1.09
time (sec)	N/A	0.210	4.482	1.263	2.146	0.339	0.000	1.167	16.499

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	37	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.61	0.00	1.09	1.09
time (sec)	N/A	0.214	7.294	1.493	3.323	0.365	0.000	0.888	17.281

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	22	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.96	1.09	1.09
time (sec)	N/A	0.217	5.131	0.684	3.692	0.301	129.181	4.643	14.783

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	226	221	155	0	0	0	0	0	0
N.S.	1	0.98	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.410	0.230	0.000	0.000	0.000	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	146	114	0	0	0	0	0	0
N.S.	1	0.99	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.346	0.100	0.000	0.000	0.000	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	69	0	0	0	0	0	70
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.01
time (sec)	N/A	0.238	0.026	0.000	0.000	0.000	0.000	0.000	14.550

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	20	23	25
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.19
time (sec)	N/A	0.207	3.853	0.594	2.458	0.315	132.941	0.429	13.648

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.00	1.09	1.09
time (sec)	N/A	0.219	7.648	0.594	2.732	0.288	0.000	0.516	13.285

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.00	1.09	1.09
time (sec)	N/A	0.217	10.215	0.949	7.261	0.308	0.000	10.630	13.494

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	22	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.96	1.09	1.09
time (sec)	N/A	0.217	8.552	0.596	4.698	0.296	43.029	6.960	14.008

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	14	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	1.00	1.14	1.14
time (sec)	N/A	0.181	2.036	0.224	2.729	0.307	4.166	3.088	14.016

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.00	1.09	1.09
time (sec)	N/A	0.221	4.770	0.665	2.483	0.288	0.000	3.500	13.735

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.00	1.09	1.09
time (sec)	N/A	0.215	7.894	0.943	3.086	0.286	0.000	3.724	13.749

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	139	113	114	273	179	0	603	115
N.S.	1	1.09	0.88	0.89	2.13	1.40	0.00	4.71	0.90
time (sec)	N/A	0.342	0.309	4.582	0.274	0.561	0.000	4.014	13.911

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	102	78	83	159	118	0	393	90
N.S.	1	1.09	0.83	0.88	1.69	1.26	0.00	4.18	0.96
time (sec)	N/A	0.311	0.282	2.354	0.243	0.392	0.000	2.263	14.172

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	69	52	58	82	78	0	234	52
N.S.	1	1.08	0.81	0.91	1.28	1.22	0.00	3.66	0.81
time (sec)	N/A	0.278	0.105	1.306	0.255	0.352	0.000	0.870	13.585

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	47	37	42	43	37	0	110	28
N.S.	1	1.09	0.86	0.98	1.00	0.86	0.00	2.56	0.65
time (sec)	N/A	0.224	0.039	0.926	0.213	0.317	0.000	0.481	13.444

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	39	38	54	37	35	0	38	41
N.S.	1	1.03	1.00	1.42	0.97	0.92	0.00	1.00	1.08
time (sec)	N/A	0.221	0.026	0.958	0.220	0.309	0.000	0.661	13.596

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	58	50	101	56	91	0	108	69
N.S.	1	0.92	0.79	1.60	0.89	1.44	0.00	1.71	1.10
time (sec)	N/A	0.268	0.150	1.853	0.224	0.322	0.000	0.406	14.128

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	81	72	161	92	198	0	205	103
N.S.	1	0.91	0.81	1.81	1.03	2.22	0.00	2.30	1.16
time (sec)	N/A	0.291	0.561	3.896	0.217	0.353	0.000	0.425	13.448

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	110	100	253	137	371	0	353	138
N.S.	1	0.91	0.83	2.09	1.13	3.07	0.00	2.92	1.14
time (sec)	N/A	0.320	0.281	7.279	0.206	0.398	0.000	0.448	13.949

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	109	147	120	180	602	0	472	141
N.S.	1	0.91	1.22	1.00	1.50	5.02	0.00	3.93	1.18
time (sec)	N/A	0.319	4.555	6.447	0.307	0.368	0.000	5.089	14.826

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	89	111	121	130	472	0	296	112
N.S.	1	0.92	1.14	1.25	1.34	4.87	0.00	3.05	1.15
time (sec)	N/A	0.293	1.164	3.475	0.326	0.345	0.000	2.208	14.076

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	69	75	78	85	366	0	164	83
N.S.	1	0.93	1.01	1.05	1.15	4.95	0.00	2.22	1.12
time (sec)	N/A	0.289	0.550	1.744	0.351	0.330	0.000	0.913	13.946

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	51	53	51	51	300	0	86	53
N.S.	1	0.96	1.00	0.96	0.96	5.66	0.00	1.62	1.00
time (sec)	N/A	0.243	0.352	1.083	0.338	0.319	0.000	0.496	13.440

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	50	52	55	50	290	0	85	44
N.S.	1	0.96	1.00	1.06	0.96	5.58	0.00	1.63	0.85
time (sec)	N/A	0.244	0.517	1.491	0.353	0.311	0.000	0.466	14.540

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	74	72	79	76	402	0	120	64
N.S.	1	1.04	1.01	1.11	1.07	5.66	0.00	1.69	0.90
time (sec)	N/A	0.257	0.611	2.651	0.328	0.311	0.000	0.526	13.455

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	98	101	115	111	576	0	171	82
N.S.	1	1.02	1.05	1.20	1.16	6.00	0.00	1.78	0.85
time (sec)	N/A	0.280	1.294	5.266	0.305	0.349	0.000	0.535	13.963

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	122	135	155	154	834	0	238	100
N.S.	1	1.04	1.15	1.32	1.32	7.13	0.00	2.03	0.85
time (sec)	N/A	0.298	3.247	10.114	0.331	0.337	0.000	0.588	16.784

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	70	42	49	69	47	0	128	326
N.S.	1	1.09	0.66	0.77	1.08	0.73	0.00	2.00	5.09
time (sec)	N/A	0.363	0.217	0.839	0.247	0.299	0.000	2.141	17.391

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	49	29	28	46	34	0	37	69
N.S.	1	1.29	0.76	0.74	1.21	0.89	0.00	0.97	1.82
time (sec)	N/A	0.344	0.131	0.689	0.257	0.319	0.000	0.963	0.671

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	21	20	17	0	37	20
N.S.	1	1.00	1.00	1.11	1.05	0.89	0.00	1.95	1.05
time (sec)	N/A	0.291	0.039	0.407	0.384	0.294	0.000	0.369	14.503

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	52	47	54	70	57	0	524	0
N.S.	1	1.04	0.94	1.08	1.40	1.14	0.00	10.48	0.00
time (sec)	N/A	0.317	0.083	0.833	0.323	0.300	0.000	0.557	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	90	97	102	99	88	0	1996	0
N.S.	1	1.03	1.11	1.17	1.14	1.01	0.00	22.94	0.00
time (sec)	N/A	0.348	1.520	0.957	0.306	0.314	0.000	1.187	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	102	75	157	1955	87	0	234	0
N.S.	1	0.85	0.62	1.31	16.29	0.72	0.00	1.95	0.00
time (sec)	N/A	0.420	0.271	1.053	1.131	0.319	0.000	2.315	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	71	55	131	827	77	0	197	0
N.S.	1	0.78	0.60	1.44	9.09	0.85	0.00	2.16	0.00
time (sec)	N/A	0.402	0.217	1.042	0.421	0.299	0.000	1.217	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	40	40	96	73	55	0	127	0
N.S.	1	0.70	0.70	1.68	1.28	0.96	0.00	2.23	0.00
time (sec)	N/A	0.383	0.054	0.976	0.352	0.322	0.000	0.583	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	38	35	42	42	42	0	1491	88
N.S.	1	0.67	0.61	0.74	0.74	0.74	0.00	26.16	1.54
time (sec)	N/A	0.418	0.075	0.714	0.336	0.289	0.000	0.879	16.809

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	54	47	75	57	66	0	3443	364
N.S.	1	0.59	0.52	0.82	0.63	0.73	0.00	37.84	4.00
time (sec)	N/A	0.408	0.083	0.794	0.326	0.278	0.000	1.820	17.074

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	62	67	85	68	86	0	8626	555
N.S.	1	0.50	0.54	0.69	0.55	0.69	0.00	69.56	4.48
time (sec)	N/A	0.405	0.206	0.704	0.316	0.279	0.000	4.025	24.394

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	72	43	48	69	50	0	67	486
N.S.	1	1.11	0.66	0.74	1.06	0.77	0.00	1.03	7.48
time (sec)	N/A	0.357	0.180	0.644	0.213	0.293	0.000	1.661	21.860

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	51	31	38	46	40	0	54	100
N.S.	1	1.21	0.74	0.90	1.10	0.95	0.00	1.29	2.38
time (sec)	N/A	0.345	0.137	0.593	0.239	0.298	0.000	0.711	17.455

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	20	65	27	0	39	61
N.S.	1	1.00	1.00	1.11	3.61	1.50	0.00	2.17	3.39
time (sec)	N/A	0.286	0.029	0.393	0.308	0.269	0.000	0.419	0.388

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	31	51	84	0	0	0
N.S.	1	1.00	1.00	1.00	1.65	2.71	0.00	0.00	0.00
time (sec)	N/A	0.303	0.036	0.596	0.212	0.282	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	68	58	83	81	79	0	0	0
N.S.	1	1.03	0.88	1.26	1.23	1.20	0.00	0.00	0.00
time (sec)	N/A	0.340	0.200	0.883	0.332	0.323	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	81	66	132	1518	80	0	188	0
N.S.	1	0.89	0.73	1.45	16.68	0.88	0.00	2.07	0.00
time (sec)	N/A	0.549	0.137	1.157	0.548	0.332	0.000	1.457	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	55	43	106	527	67	0	158	0
N.S.	1	0.89	0.69	1.71	8.50	1.08	0.00	2.55	0.00
time (sec)	N/A	0.448	0.048	0.937	0.508	0.299	0.000	0.870	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	90	36	0	0	37
N.S.	1	1.00	1.00	1.28	3.60	1.44	0.00	0.00	1.48
time (sec)	N/A	0.387	0.033	0.496	0.386	0.289	0.000	0.000	13.608

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	46	37	64	525	58	0	0	118
N.S.	1	0.77	0.62	1.07	8.75	0.97	0.00	0.00	1.97
time (sec)	N/A	0.413	0.067	0.781	0.362	0.280	0.000	0.000	18.113

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	56	49	74	1236	79	0	0	491
N.S.	1	0.58	0.51	0.77	12.88	0.82	0.00	0.00	5.11
time (sec)	N/A	0.417	0.076	0.668	0.380	0.296	0.000	0.000	20.956

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	74	51	51	69	50	0	93	583
N.S.	1	1.09	0.75	0.75	1.01	0.74	0.00	1.37	8.57
time (sec)	N/A	0.372	0.229	1.299	0.213	0.299	0.000	2.967	29.967

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	53	34	41	48	40	0	80	389
N.S.	1	1.20	0.77	0.93	1.09	0.91	0.00	1.82	8.84
time (sec)	N/A	0.362	0.174	1.094	0.237	0.288	0.000	0.973	19.297

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	21	95	28	0	54	72
N.S.	1	1.00	1.00	1.00	4.52	1.33	0.00	2.57	3.43
time (sec)	N/A	0.305	0.032	0.362	0.302	0.285	0.000	0.626	17.005

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	55	37	76	73	58	0	0	0
N.S.	1	1.04	0.70	1.43	1.38	1.09	0.00	0.00	0.00
time (sec)	N/A	0.326	0.091	1.092	0.212	0.304	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	70	59	67	100	83	0	0	0
N.S.	1	1.06	0.89	1.02	1.52	1.26	0.00	0.00	0.00
time (sec)	N/A	0.363	0.127	0.934	0.206	0.292	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	84	59	104	1532	77	0	99	0
N.S.	1	0.79	0.56	0.98	14.45	0.73	0.00	0.93	0.00
time (sec)	N/A	0.579	0.101	0.823	0.527	0.290	0.000	0.831	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	44	44	123	220	66	0	0	0
N.S.	1	0.70	0.70	1.95	3.49	1.05	0.00	0.00	0.00
time (sec)	N/A	0.409	0.079	1.014	0.380	0.302	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	29	35	382	50	0	0	88
N.S.	1	1.00	0.76	0.92	10.05	1.32	0.00	0.00	2.32
time (sec)	N/A	0.398	0.042	0.628	0.336	0.283	0.000	0.000	17.716

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	53	41	47	1063	75	0	0	393
N.S.	1	0.69	0.53	0.61	13.81	0.97	0.00	0.00	5.10
time (sec)	N/A	0.425	0.105	0.662	0.354	0.291	0.000	0.000	19.597

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	65	51	57	2026	100	0	0	589
N.S.	1	0.57	0.44	0.50	17.62	0.87	0.00	0.00	5.12
time (sec)	N/A	0.433	0.138	0.662	0.365	0.295	0.000	0.000	30.488

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	143	721	230	354	0	2646	0
N.S.	1	1.00	0.81	4.07	1.30	2.00	0.00	14.95	0.00
time (sec)	N/A	0.348	0.611	2.150	0.297	0.836	0.000	2.132	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	113	84	403	127	234	0	959	0
N.S.	1	0.96	0.71	3.42	1.08	1.98	0.00	8.13	0.00
time (sec)	N/A	0.282	0.425	1.591	0.388	0.558	0.000	0.840	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	60	60	129	122	145	0	313	0
N.S.	1	1.03	1.03	2.22	2.10	2.50	0.00	5.40	0.00
time (sec)	N/A	0.230	0.067	1.205	0.305	0.438	0.000	0.432	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	56	51	58	43	135	0	0	0
N.S.	1	1.04	0.94	1.07	0.80	2.50	0.00	0.00	0.00
time (sec)	N/A	0.248	0.046	0.823	0.206	0.732	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	100	77	122	113	239	0	0	0
N.S.	1	0.91	0.70	1.11	1.03	2.17	0.00	0.00	0.00
time (sec)	N/A	0.288	0.212	1.050	0.211	0.942	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	152	103	212	215	415	0	0	0
N.S.	1	0.92	0.62	1.28	1.30	2.52	0.00	0.00	0.00
time (sec)	N/A	0.324	0.597	1.333	0.212	1.294	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	243	198	380	0	0	0	0	0
N.S.	1	1.04	0.85	1.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.474	2.445	4.566	0.000	0.000	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	164	140	222	0	0	0	0	0
N.S.	1	0.96	0.82	1.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.363	0.773	3.757	0.000	0.000	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	61	71	0	0	0	0	0
N.S.	1	1.00	1.20	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	0.068	1.567	0.000	0.000	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	167	143	156	0	0	0	0	0
N.S.	1	0.96	0.82	0.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	0.936	2.925	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	238	197	351	0	0	0	0	0
N.S.	1	1.03	0.85	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.469	3.618	3.455	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	202	160	711	238	385	0	3585	0
N.S.	1	0.92	0.73	3.23	1.08	1.75	0.00	16.30	0.00
time (sec)	N/A	0.356	2.119	1.708	0.303	0.693	0.000	3.773	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	138	116	426	169	265	0	0	0
N.S.	1	0.93	0.78	2.88	1.14	1.79	0.00	0.00	0.00
time (sec)	N/A	0.305	0.535	1.780	0.288	0.526	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	85	79	211	157	186	0	1270	0
N.S.	1	1.01	0.94	2.51	1.87	2.21	0.00	15.12	0.00
time (sec)	N/A	0.254	0.165	1.273	0.315	0.489	0.000	0.707	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	79	69	86	61	175	0	0	0
N.S.	1	1.01	0.88	1.10	0.78	2.24	0.00	0.00	0.00
time (sec)	N/A	0.265	0.116	0.934	0.200	0.721	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	123	90	165	148	282	0	0	0
N.S.	1	0.88	0.64	1.18	1.06	2.01	0.00	0.00	0.00
time (sec)	N/A	0.303	0.474	1.145	0.216	0.970	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	177	123	257	272	442	0	0	0
N.S.	1	0.85	0.59	1.24	1.31	2.12	0.00	0.00	0.00
time (sec)	N/A	0.340	0.842	1.280	0.204	1.355	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	283	211	419	0	0	0	0	0
N.S.	1	1.03	0.77	1.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.545	3.337	4.976	0.000	0.000	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	223	174	389	0	0	0	0	0
N.S.	1	1.00	0.78	1.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.423	3.168	5.137	0.000	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	155	156	266	0	0	0	0	0
N.S.	1	1.01	1.01	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.838	0.756	2.523	0.000	0.000	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	224	173	204	0	0	0	0	0
N.S.	1	1.00	0.78	0.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.445	2.616	4.186	0.000	0.000	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	282	218	419	0	0	0	0	0
N.S.	1	1.02	0.79	1.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.538	5.360	3.812	0.000	0.000	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	149	108	644	248	328	0	0	0
N.S.	1	1.11	0.81	4.81	1.85	2.45	0.00	0.00	0.00
time (sec)	N/A	0.334	0.497	1.582	0.296	0.394	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	83	77	353	124	220	0	0	0
N.S.	1	1.02	0.95	4.36	1.53	2.72	0.00	0.00	0.00
time (sec)	N/A	0.273	0.229	1.309	0.292	0.364	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	103	106	112	0	0	0
N.S.	1	1.00	1.06	2.86	2.94	3.11	0.00	0.00	0.00
time (sec)	N/A	0.231	0.038	1.373	0.319	0.332	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	42	25	100	0	0	0
N.S.	1	1.00	1.00	1.27	0.76	3.03	0.00	0.00	0.00
time (sec)	N/A	0.238	0.036	0.791	0.209	0.336	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	71	71	109	77	220	0	0	0
N.S.	1	0.95	0.95	1.45	1.03	2.93	0.00	0.00	0.00
time (sec)	N/A	0.269	0.165	0.946	0.229	0.366	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	127	101	205	158	388	0	0	0
N.S.	1	1.01	0.80	1.63	1.25	3.08	0.00	0.00	0.00
time (sec)	N/A	0.313	0.382	1.117	0.215	0.394	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	249	188	377	0	845	0	0	0
N.S.	1	1.01	0.76	1.53	0.00	3.43	0.00	0.00	0.00
time (sec)	N/A	0.455	2.563	3.796	0.000	0.172	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	123	100	222	0	730	0	0	0
N.S.	1	1.13	0.92	2.04	0.00	6.70	0.00	0.00	0.00
time (sec)	N/A	0.305	0.681	2.827	0.000	0.153	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	60	52	0	305	0	0	0
N.S.	1	1.00	1.18	1.02	0.00	5.98	0.00	0.00	0.00
time (sec)	N/A	0.276	0.054	0.546	0.000	0.116	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	120	101	120	0	723	0	0	0
N.S.	1	1.13	0.95	1.13	0.00	6.82	0.00	0.00	0.00
time (sec)	N/A	0.308	0.718	2.043	0.000	0.142	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	243	186	351	0	1045	0	0	0
N.S.	1	1.01	0.78	1.46	0.00	4.35	0.00	0.00	0.00
time (sec)	N/A	0.465	4.617	2.864	0.000	0.192	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	182	107	3767	334	593	0	0	0
N.S.	1	1.03	0.60	21.28	1.89	3.35	0.00	0.00	0.00
time (sec)	N/A	0.354	0.508	6.502	0.326	0.507	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	75	2537	193	445	0	0	0
N.S.	1	1.00	0.64	21.50	1.64	3.77	0.00	0.00	0.00
time (sec)	N/A	0.303	0.116	4.020	0.357	0.454	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	65	54	1317	143	281	0	0	0
N.S.	1	1.03	0.86	20.90	2.27	4.46	0.00	0.00	0.00
time (sec)	N/A	0.249	0.073	3.615	0.343	0.350	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	59	46	62	46	225	0	0	0
N.S.	1	1.04	0.81	1.09	0.81	3.95	0.00	0.00	0.00
time (sec)	N/A	0.255	0.055	0.882	0.251	0.380	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	103	70	148	117	406	0	0	0
N.S.	1	0.94	0.64	1.35	1.06	3.69	0.00	0.00	0.00
time (sec)	N/A	0.292	0.108	1.146	0.232	0.379	0.000	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	158	94	265	219	652	0	0	0
N.S.	1	0.95	0.56	1.59	1.31	3.90	0.00	0.00	0.00
time (sec)	N/A	0.331	0.332	1.296	0.269	0.414	0.000	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	311	197	368	0	1228	0	0	0
N.S.	1	1.07	0.67	1.26	0.00	4.21	0.00	0.00	0.00
time (sec)	N/A	0.506	2.783	4.669	0.000	0.211	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	233	145	278	0	1048	0	0	0
N.S.	1	1.04	0.65	1.24	0.00	4.68	0.00	0.00	0.00
time (sec)	N/A	0.432	1.196	3.883	0.000	0.184	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	90	103	0	938	0	0	0
N.S.	1	1.00	0.89	1.02	0.00	9.29	0.00	0.00	0.00
time (sec)	N/A	0.422	0.134	1.550	0.000	0.156	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	222	142	141	0	996	0	0	0
N.S.	1	1.06	0.68	0.67	0.00	4.77	0.00	0.00	0.00
time (sec)	N/A	0.441	1.114	3.211	0.000	0.178	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	312	199	353	0	1465	0	0	0
N.S.	1	1.05	0.67	1.19	0.00	4.93	0.00	0.00	0.00
time (sec)	N/A	0.582	4.304	3.732	0.000	0.187	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	212	107	1909	424	995	0	0	0
N.S.	1	0.97	0.49	8.76	1.94	4.56	0.00	0.00	0.00
time (sec)	N/A	0.384	0.504	4.226	0.384	0.580	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	153	150	76	834	262	769	0	0	0
N.S.	1	0.98	0.50	5.45	1.71	5.03	0.00	0.00	0.00
time (sec)	N/A	0.318	0.119	3.283	0.342	0.438	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	97	56	963	203	521	0	0	0
N.S.	1	1.07	0.62	10.58	2.23	5.73	0.00	0.00	0.00
time (sec)	N/A	0.267	0.085	2.342	0.476	0.373	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	87	49	260	66	382	0	0	0
N.S.	1	1.05	0.59	3.13	0.80	4.60	0.00	0.00	0.00
time (sec)	N/A	0.273	0.054	1.642	0.273	0.353	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	131	69	1038	156	666	0	0	0
N.S.	1	0.92	0.48	7.26	1.09	4.66	0.00	0.00	0.00
time (sec)	N/A	0.309	0.270	2.248	0.263	0.384	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	186	117	988	280	984	0	0	0
N.S.	1	0.89	0.56	4.75	1.35	4.73	0.00	0.00	0.00
time (sec)	N/A	0.352	0.879	3.512	0.248	0.433	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	380	235	667	0	1730	0	0	0
N.S.	1	1.09	0.68	1.92	0.00	4.97	0.00	0.00	0.00
time (sec)	N/A	0.623	4.171	5.885	0.000	0.334	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	310	199	851	0	1632	0	0	0
N.S.	1	1.06	0.68	2.91	0.00	5.59	0.00	0.00	0.00
time (sec)	N/A	0.524	3.221	5.924	0.000	0.257	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	228	172	547	0	1531	0	0	0
N.S.	1	1.02	0.77	2.45	0.00	6.87	0.00	0.00	0.00
time (sec)	N/A	1.201	1.341	2.112	0.000	0.197	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	308	209	411	0	1595	0	0	0
N.S.	1	1.07	0.73	1.43	0.00	5.56	0.00	0.00	0.00
time (sec)	N/A	0.560	3.241	5.289	0.000	0.221	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	375	226	633	0	1971	0	0	0
N.S.	1	1.08	0.65	1.82	0.00	5.66	0.00	0.00	0.00
time (sec)	N/A	0.678	3.708	4.472	0.000	0.285	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	147	121	0	0	0	0	0	0
N.S.	1	1.22	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.384	1.142	0.000	0.000	0.000	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	104	83	0	0	0	0	0	0
N.S.	1	1.02	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.277	0.249	0.000	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.228	0.067	0.000	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	0.063	0.000	0.000	0.000	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	91	73	0	0	0	0	0	0
N.S.	1	0.96	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.262	0.539	0.000	0.000	0.000	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.295	4.617	0.000	0.000	0.000	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.280	0.641	0.000	0.000	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	98	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.282	0.555	0.000	0.000	0.000	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.287	3.310	0.000	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	143	110	152	874	0	144	2003
N.S.	1	1.00	0.93	0.72	0.99	5.71	0.00	0.94	13.09
time (sec)	N/A	0.434	0.330	3.436	0.306	1.106	0.000	0.310	15.390

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	52	0	0	38	0
N.S.	1	1.00	1.00	0.00	1.16	0.00	0.00	0.84	0.00
time (sec)	N/A	0.244	0.030	0.000	0.291	0.000	0.000	0.296	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	39	0	0	24	0
N.S.	1	1.00	1.00	0.00	1.39	0.00	0.00	0.86	0.00
time (sec)	N/A	0.241	0.017	0.000	0.283	0.000	0.000	0.299	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	56	54	60	68	195	0	0	0
N.S.	1	0.95	0.92	1.02	1.15	3.31	0.00	0.00	0.00
time (sec)	N/A	0.261	0.057	0.621	0.301	0.388	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	93	89	0	0	361	0	0	0
N.S.	1	1.04	1.00	0.00	0.00	4.06	0.00	0.00	0.00
time (sec)	N/A	0.298	0.187	0.000	0.000	0.414	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	54	65	72	0	240	0	0	0
N.S.	1	1.06	1.27	1.41	0.00	4.71	0.00	0.00	0.00
time (sec)	N/A	0.236	0.100	2.186	0.000	0.393	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	42	50	140	0	0	0
N.S.	1	1.00	1.00	1.20	1.43	4.00	0.00	0.00	0.00
time (sec)	N/A	0.250	0.030	0.618	0.308	0.347	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	66	66	0	79	247	0	0	0
N.S.	1	0.94	0.94	0.00	1.13	3.53	0.00	0.00	0.00
time (sec)	N/A	0.281	0.096	0.000	0.303	0.355	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	109	141	0	166	371	0	0	0
N.S.	1	1.01	1.31	0.00	1.54	3.44	0.00	0.00	0.00
time (sec)	N/A	0.328	3.060	0.000	0.340	0.394	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	411	454	291	0	0	0	0	0	0
N.S.	1	1.10	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.581	4.949	0.000	0.000	0.000	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	195	396	0	0	0	0	0
N.S.	1	1.00	1.20	2.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	0.464	3.054	0.000	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	477	502	378	0	0	0	0	0	0
N.S.	1	1.05	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.593	9.877	0.000	0.000	0.000	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	37	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.61	0.00	1.09	1.09
time (sec)	N/A	0.211	5.573	1.908	2.423	1.674	0.000	2.871	17.103

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	279	265	810	0	0	0	0	0	0
N.S.	1	0.95	2.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.447	14.108	0.000	0.000	0.000	0.000	0.000	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	141	139	463	0	0	0	0	0	0
N.S.	1	0.99	3.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.301	9.505	0.000	0.000	0.000	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.236	0.095	0.000	0.000	0.000	0.000	0.000	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	126	119	0	0	0	0	0	0
N.S.	1	0.99	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	0.574	0.000	0.000	0.000	0.000	0.000	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	37	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.61	0.00	1.09	1.09
time (sec)	N/A	0.217	21.007	1.556	14.075	0.361	0.000	2.999	18.601

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	37	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.61	0.00	1.09	1.09
time (sec)	N/A	0.216	2.145	1.593	5.311	0.309	0.000	1.577	17.634

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	28	0	16	16
N.S.	1	1.00	1.14	1.00	1.14	2.00	0.00	1.14	1.14
time (sec)	N/A	0.180	0.205	0.944	1.086	0.277	0.000	0.847	14.779

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	37	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.61	0.00	1.09	1.09
time (sec)	N/A	0.218	1.912	1.464	5.354	0.299	0.000	1.559	16.690

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	37	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.61	0.00	1.09	1.09
time (sec)	N/A	0.211	14.591	1.630	14.883	0.353	0.000	1.638	17.767

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	306	306	3544	0	0	0	0	0	0
N.S.	1	1.00	11.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.681	20.717	0.000	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	215	215	2368	0	0	0	0	0	0
N.S.	1	1.00	11.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.527	16.740	0.000	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	124	124	1395	0	0	0	0	0	0
N.S.	1	1.00	11.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.365	14.193	0.000	0.000	0.000	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	20	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.87	1.09	1.09
time (sec)	N/A	0.225	5.255	1.089	1.489	0.283	5.172	1.376	13.931

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	43	22	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.87	0.96	1.09	1.09
time (sec)	N/A	0.225	47.942	1.204	3.247	0.289	107.532	1.804	15.282

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	45	45	38	57	97	0	46	0
N.S.	1	0.96	0.96	0.81	1.21	2.06	0.00	0.98	0.00
time (sec)	N/A	0.255	0.033	2.559	0.315	0.293	0.000	0.362	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	41	74	0	27	0
N.S.	1	1.00	1.00	0.83	1.41	2.55	0.00	0.93	0.00
time (sec)	N/A	0.256	0.018	0.747	0.312	0.318	0.000	0.369	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.00	1.09	1.09
time (sec)	N/A	0.221	5.774	1.214	4.441	0.289	0.000	6.229	15.319

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.00	1.09	1.09
time (sec)	N/A	0.221	25.095	0.911	5.112	0.263	0.000	8.010	15.446

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	20	23	23
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.10
time (sec)	N/A	0.204	1.508	0.823	3.130	0.268	109.766	6.171	14.697

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.251	0.061	0.000	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	134	129	0	0	0	0	0	0
N.S.	1	0.99	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.397	1.171	0.000	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.00	1.09	1.09
time (sec)	N/A	0.222	25.689	0.903	6.046	0.261	0.000	8.382	15.010

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.00	1.09	1.09
time (sec)	N/A	0.225	3.453	0.866	4.415	0.267	0.000	6.726	14.401

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	14	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	1.00	1.14	1.14
time (sec)	N/A	0.185	0.692	0.391	2.442	0.251	4.079	2.994	14.974

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	22	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.96	1.09	1.09
time (sec)	N/A	0.226	2.671	0.755	4.446	0.267	113.302	3.389	14.568

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	0	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.00	1.09	1.09
time (sec)	N/A	0.226	32.475	0.856	6.239	0.262	0.000	4.024	14.870

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	148	102	387	0	125	0	0	0
N.S.	1	1.38	0.95	3.62	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.420	0.201	7.072	0.000	0.103	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	135	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.403	2.523	0.000	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	79	79	325	315013	0	0	0	0	0
N.S.	1	1.00	4.11	3987.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.441	2.144	7.423	0.000	0.000	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	79	79	529	2174	0	1623	0	0	0
N.S.	1	1.00	6.70	27.52	0.00	20.54	0.00	0.00	0.00
time (sec)	N/A	0.456	1.861	6.638	0.000	0.140	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [13] had the largest ratio of [1.3000000000000004]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	8	1.06	10	0.800
2	A	6	6	1.00	10	0.600
3	A	4	4	1.00	10	0.400
4	A	4	4	1.00	10	0.400
5	A	6	6	1.00	10	0.600
6	A	8	8	1.13	10	0.800
7	A	12	12	0.82	10	1.200
8	A	8	8	0.89	10	0.800
9	A	6	6	0.94	10	0.600
10	A	6	6	0.90	10	0.600
11	A	8	8	0.87	10	0.800
12	A	12	12	0.79	10	1.200
13	A	13	13	0.70	10	1.300
14	A	9	9	0.77	10	0.900
15	A	5	5	0.81	10	0.500
16	A	6	5	1.00	10	0.500
17	A	6	5	0.56	10	0.500
18	A	6	5	0.46	10	0.500
19	A	4	4	1.00	14	0.286
20	A	4	4	1.00	14	0.286
21	A	4	4	1.19	14	0.286
22	A	4	4	1.00	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	4	4	1.00	14	0.286
24	A	4	4	1.00	14	0.286
25	A	4	4	1.00	12	0.333
26	A	4	4	1.00	12	0.333
27	A	4	4	1.00	12	0.333
28	A	4	4	1.00	12	0.333
29	A	4	4	1.00	14	0.286
30	A	4	4	1.00	14	0.286
31	A	1	1	1.00	9	0.111
32	A	7	7	1.00	11	0.636
33	A	9	9	1.04	11	0.818
34	A	11	11	1.07	11	1.000
35	A	7	6	0.73	24	0.250
36	A	7	6	0.80	24	0.250
37	A	7	6	0.93	24	0.250
38	A	6	5	1.00	22	0.227
39	A	8	7	0.79	22	0.318
40	A	8	7	0.93	24	0.292
41	A	10	9	1.04	24	0.375
42	A	9	8	1.11	24	0.333
43	A	8	7	1.06	24	0.292
44	A	8	7	1.00	24	0.292
45	A	6	5	1.00	15	0.333
46	A	7	6	0.79	24	0.250
47	A	7	6	0.74	24	0.250
48	A	7	6	0.71	24	0.250
49	A	7	6	0.75	24	0.250
50	A	7	6	0.74	24	0.250
51	A	6	5	0.85	24	0.208
52	A	6	5	1.00	22	0.227
53	A	8	7	0.74	22	0.318
54	A	8	7	0.85	24	0.292
55	A	8	7	0.93	24	0.292
56	A	7	7	0.84	24	0.292

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	6	5	1.00	24	0.208
58	A	6	5	0.91	15	0.333
59	A	7	6	0.77	24	0.250
60	A	7	6	0.74	24	0.250
61	A	6	5	0.90	11	0.455
62	A	6	5	0.86	11	0.455
63	A	6	5	0.82	11	0.455
64	A	5	4	0.90	21	0.190
65	A	4	3	0.94	19	0.158
66	A	4	4	1.00	19	0.211
67	A	4	4	1.00	21	0.190
68	A	7	7	0.96	21	0.333
69	A	5	5	0.97	21	0.238
70	A	1	1	1.00	12	0.083
71	A	3	3	1.00	21	0.143
72	A	6	5	1.00	21	0.238
73	A	6	5	0.86	21	0.238
74	A	1	1	1.00	8	0.125
75	A	2	2	1.00	10	0.200
76	A	4	4	1.06	10	0.400
77	A	6	6	1.07	10	0.600
78	A	5	4	0.93	23	0.174
79	A	5	4	0.95	23	0.174
80	A	5	4	0.98	23	0.174
81	A	4	3	1.00	21	0.143
82	A	6	5	0.98	21	0.238
83	A	7	6	1.11	23	0.261
84	A	8	7	1.17	23	0.304
85	A	10	9	1.24	23	0.391
86	A	8	7	1.23	23	0.304
87	A	7	6	1.22	23	0.261
88	A	6	5	1.00	23	0.217
89	A	4	3	1.00	14	0.214
90	A	5	4	0.96	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	5	4	0.94	23	0.174
92	A	5	4	0.93	23	0.174
93	A	5	4	0.92	23	0.174
94	A	5	4	0.95	23	0.174
95	A	5	4	0.95	23	0.174
96	A	5	4	0.99	23	0.174
97	A	5	4	0.99	21	0.190
98	A	8	7	1.10	21	0.333
99	A	9	8	1.08	23	0.348
100	A	9	8	1.14	23	0.348
101	A	7	6	1.19	23	0.261
102	A	7	6	1.00	23	0.261
103	A	8	7	1.00	14	0.500
104	A	7	6	1.03	23	0.261
105	A	7	6	0.98	23	0.261
106	A	8	7	1.20	23	0.304
107	A	6	5	1.05	23	0.217
108	A	9	8	1.08	23	0.348
109	A	10	9	1.09	14	0.643
110	A	9	8	1.07	23	0.348
111	A	12	11	1.13	14	0.786
112	A	14	13	1.14	14	0.929
113	A	4	3	1.00	13	0.231
114	A	5	4	1.00	13	0.308
115	A	4	3	1.00	21	0.143
116	A	10	10	1.06	13	0.769
117	A	8	8	1.00	13	0.615
118	A	6	6	1.00	13	0.462
119	A	6	6	1.00	13	0.462
120	A	8	8	1.00	13	0.615
121	A	10	10	1.13	13	0.769
122	A	7	6	0.97	25	0.240
123	A	6	5	0.99	23	0.217
124	A	8	7	0.96	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	6	5	1.08	25	0.200
126	A	7	6	1.06	25	0.240
127	A	10	9	1.01	25	0.360
128	A	11	11	1.00	25	0.440
129	A	4	4	1.00	16	0.250
130	A	10	9	1.01	25	0.360
131	A	12	11	1.04	25	0.440
132	A	8	7	0.93	25	0.280
133	A	7	6	0.99	23	0.261
134	A	10	9	0.98	23	0.391
135	A	10	9	1.05	25	0.360
136	A	7	6	1.10	25	0.240
137	A	8	7	1.02	25	0.280
138	A	13	12	1.04	25	0.480
139	A	13	13	1.02	25	0.520
140	A	11	11	1.01	16	0.688
141	A	10	9	1.01	25	0.360
142	A	12	11	1.03	25	0.440
143	A	13	13	1.02	16	0.812
144	A	6	5	0.99	25	0.200
145	A	5	4	1.00	23	0.174
146	A	5	4	1.00	23	0.174
147	A	6	5	1.08	25	0.200
148	A	9	8	0.95	25	0.320
149	A	9	9	1.00	25	0.360
150	A	4	4	1.00	16	0.250
151	A	11	10	1.02	25	0.400
152	A	12	11	1.03	25	0.440
153	A	6	5	0.99	25	0.200
154	A	4	3	1.00	23	0.130
155	A	6	5	0.99	23	0.217
156	A	9	8	1.04	25	0.320
157	A	11	10	1.00	25	0.400
158	A	9	8	0.99	25	0.320

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	11	11	1.07	25	0.440
160	A	7	7	1.00	16	0.438
161	A	12	11	1.07	25	0.440
162	A	8	7	1.10	25	0.280
163	A	5	4	1.04	25	0.160
164	A	5	4	0.99	23	0.174
165	A	9	8	1.08	23	0.348
166	A	10	9	1.01	25	0.360
167	A	11	10	0.99	25	0.400
168	A	13	13	1.03	25	0.520
169	A	14	14	1.02	16	0.875
170	A	14	13	1.09	25	0.520
171	A	5	4	1.00	25	0.160
172	A	7	6	0.99	23	0.261
173	A	6	5	0.99	23	0.217
174	A	5	4	1.00	21	0.190
175	A	5	4	1.00	21	0.190
176	A	5	4	1.00	23	0.174
177	A	5	4	1.00	23	0.174
178	A	5	4	1.00	23	0.174
179	A	5	4	1.00	23	0.174
180	A	5	4	1.00	23	0.174
181	A	5	4	1.00	23	0.174
182	A	3	3	1.01	23	0.130
183	A	3	3	1.00	23	0.130
184	A	3	3	1.00	23	0.130
185	A	3	3	1.01	21	0.143
186	A	3	3	1.00	21	0.143
187	A	3	3	1.00	23	0.130
188	A	3	3	1.01	23	0.130
189	A	3	3	1.00	23	0.130
190	A	3	3	1.01	23	0.130
191	A	3	3	1.00	23	0.130
192	A	3	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	3	3	1.01	23	0.130
194	A	3	3	1.00	23	0.130
195	A	5	4	0.95	24	0.167
196	A	5	4	0.96	24	0.167
197	A	5	4	0.98	24	0.167
198	A	6	5	0.99	24	0.208
199	A	6	5	0.99	22	0.227
200	A	5	4	0.96	22	0.182
201	A	5	4	0.95	24	0.167
202	A	5	4	0.93	24	0.167
203	A	5	4	1.08	24	0.167
204	A	5	4	1.05	24	0.167
205	A	5	4	1.03	24	0.167
206	A	5	4	1.34	24	0.167
207	A	5	4	1.46	15	0.267
208	A	5	4	0.96	24	0.167
209	A	5	4	0.95	24	0.167
210	A	5	4	0.94	24	0.167
211	A	5	4	0.93	24	0.167
212	A	7	6	1.10	24	0.250
213	A	8	7	1.09	24	0.292
214	A	8	7	1.05	24	0.292
215	A	8	7	1.13	24	0.292
216	A	8	7	1.05	22	0.318
217	A	5	4	0.96	22	0.182
218	A	13	12	1.18	24	0.500
219	A	8	7	1.16	24	0.292
220	A	9	8	1.22	24	0.333
221	A	7	6	1.17	24	0.250
222	A	7	6	1.24	15	0.400
223	A	7	6	1.00	24	0.250
224	A	10	9	1.18	24	0.375
225	A	10	9	1.05	24	0.375
226	A	10	9	1.15	24	0.375

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
227	A	10	9	1.10	24	0.375
228	A	10	9	1.16	22	0.409
229	A	5	4	0.96	22	0.182
230	A	13	12	1.10	24	0.500
231	A	9	8	1.11	24	0.333
232	A	9	8	1.14	24	0.333
233	A	9	8	1.12	24	0.333
234	A	9	8	1.13	15	0.533
235	A	9	8	1.06	24	0.333
236	A	5	4	1.00	10	0.400
237	A	10	9	1.11	10	0.900
238	A	9	8	0.65	8	1.000
239	A	8	7	1.00	23	0.304
240	A	8	7	1.13	23	0.304
241	A	8	7	1.01	25	0.280
242	A	6	5	1.01	25	0.200
243	A	4	3	1.00	23	0.130
244	A	6	5	1.06	23	0.217
245	A	12	11	1.03	25	0.440
246	A	7	6	1.08	25	0.240
247	A	4	3	1.00	16	0.188
248	A	9	8	1.04	25	0.320
249	A	3	3	1.00	10	0.300
250	A	6	5	1.00	10	0.500
251	A	6	5	1.05	10	0.500
252	A	3	3	1.02	11	0.273
253	A	6	5	1.00	11	0.455
254	A	6	5	1.00	11	0.455
255	A	3	3	1.00	8	0.375
256	A	6	5	0.81	8	0.625
257	A	6	5	0.59	8	0.625
258	A	3	3	1.05	10	0.300
259	A	10	9	1.00	10	0.900
260	A	10	9	0.78	10	0.900

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
261	A	6	5	0.79	16	0.312
262	A	6	5	0.90	16	0.312
263	A	6	5	1.00	16	0.312
264	A	4	4	1.00	16	0.250
265	A	4	4	1.00	14	0.286
266	A	8	8	1.00	16	0.500
267	A	7	7	1.00	16	0.438
268	A	5	5	0.90	16	0.312
269	A	3	3	1.00	16	0.188
270	A	6	6	0.91	14	0.429
271	A	6	5	1.00	16	0.312
272	A	6	5	0.90	16	0.312
273	A	6	5	0.90	16	0.312
274	A	6	5	1.00	16	0.312
275	A	4	4	1.00	16	0.250
276	A	4	4	1.00	16	0.250
277	A	6	6	0.91	14	0.429
278	A	8	8	1.00	14	0.571
279	A	7	7	1.00	16	0.438
280	A	5	5	0.90	16	0.312
281	A	3	3	1.00	16	0.188
282	A	6	5	1.00	16	0.312
283	A	6	5	0.90	16	0.312
284	A	6	5	0.86	16	0.312
285	A	8	7	1.15	21	0.333
286	A	7	6	1.18	21	0.286
287	A	6	5	1.25	21	0.238
288	A	1	1	1.00	12	0.083
289	A	5	4	1.39	21	0.190
290	A	4	3	0.93	21	0.143
291	A	5	4	0.90	21	0.190
292	A	5	4	0.89	21	0.190
293	A	8	7	1.01	23	0.304
294	A	7	6	1.10	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
295	A	2	2	1.00	14	0.143
296	A	5	4	1.20	23	0.174
297	A	5	4	1.09	23	0.174
298	A	5	4	0.91	23	0.174
299	A	5	4	0.90	23	0.174
300	A	5	4	0.90	23	0.174
301	A	5	4	1.00	15	0.267
302	A	8	7	1.31	15	0.467
303	A	5	4	1.00	15	0.267
304	A	7	6	1.25	15	0.400
305	A	5	4	1.00	15	0.267
306	A	6	5	1.05	15	0.333
307	A	4	3	1.00	13	0.231
308	A	6	5	1.00	13	0.385
309	A	5	4	1.00	15	0.267
310	A	7	6	1.26	15	0.400
311	A	5	4	1.00	15	0.267
312	A	8	7	1.34	15	0.467
313	A	5	4	1.00	15	0.267
314	A	9	8	1.16	15	0.533
315	A	5	4	1.00	15	0.267
316	A	7	6	1.16	15	0.400
317	A	5	4	1.00	15	0.267
318	A	5	4	1.00	15	0.267
319	A	5	4	1.00	13	0.308
320	A	7	6	1.18	13	0.462
321	A	5	4	1.00	15	0.267
322	A	9	8	1.19	15	0.533
323	A	5	4	1.00	15	0.267
324	A	7	6	0.96	25	0.240
325	A	6	5	0.97	23	0.217
326	A	8	7	0.98	23	0.304
327	A	6	5	1.07	25	0.200
328	A	7	6	1.05	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	11	10	1.18	25	0.400
330	A	10	9	1.17	25	0.360
331	A	4	4	1.00	16	0.250
332	A	10	9	1.33	25	0.360
333	A	12	11	1.24	25	0.440
334	A	8	7	0.92	25	0.280
335	A	7	6	0.98	23	0.261
336	A	10	9	0.99	23	0.391
337	A	10	9	1.05	25	0.360
338	A	7	6	1.10	25	0.240
339	A	8	7	1.01	25	0.280
340	A	13	12	1.03	25	0.480
341	A	12	11	0.99	25	0.440
342	A	11	11	1.01	16	0.688
343	A	10	9	1.02	25	0.360
344	A	12	11	1.07	25	0.440
345	A	6	5	0.97	25	0.200
346	A	5	4	1.00	23	0.174
347	A	5	4	1.00	23	0.174
348	A	6	5	1.07	25	0.200
349	A	9	8	1.18	25	0.320
350	A	8	7	1.17	25	0.280
351	A	4	4	1.00	16	0.250
352	A	10	9	1.32	25	0.360
353	A	11	10	1.23	25	0.400
354	A	6	5	0.97	25	0.200
355	A	4	3	1.00	23	0.130
356	A	6	5	0.97	23	0.217
357	A	9	8	1.06	25	0.320
358	A	11	10	1.00	25	0.400
359	A	10	9	0.98	25	0.360
360	A	10	9	0.95	25	0.360
361	A	7	7	1.00	16	0.438
362	A	12	11	1.06	25	0.440

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
363	A	8	7	1.05	25	0.280
364	A	5	4	1.03	25	0.160
365	A	5	4	0.97	23	0.174
366	A	8	7	1.06	23	0.304
367	A	11	10	1.16	25	0.400
368	A	11	10	1.22	25	0.400
369	A	12	11	1.17	25	0.440
370	A	14	14	1.02	16	0.875
371	A	14	13	1.22	25	0.520
372	A	5	4	1.00	25	0.160
373	A	7	6	0.99	23	0.261
374	A	6	5	0.94	23	0.217
375	A	5	4	1.00	21	0.190
376	A	5	4	1.00	21	0.190
377	A	5	4	1.00	23	0.174
378	A	5	4	1.00	23	0.174
379	A	5	4	1.00	23	0.174
380	A	5	4	1.00	14	0.286
381	A	5	4	1.00	23	0.174
382	A	5	4	1.00	23	0.174
383	A	5	4	0.95	23	0.174
384	A	13	12	0.93	23	0.522
385	A	11	10	0.94	21	0.476
386	A	5	4	0.95	21	0.190
387	A	5	4	0.95	23	0.174
388	A	3	3	1.00	23	0.130
389	A	3	3	1.00	23	0.130
390	A	3	3	1.00	14	0.214
391	F	0	0	N/A	0.000	N/A
392	F	0	0	N/A	0.000	N/A
393	A	7	6	0.99	23	0.261
394	A	13	12	1.00	23	0.522
395	A	13	12	0.97	23	0.522

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
396	A	12	11	0.97	21	0.524
397	A	5	4	0.96	21	0.190
398	A	5	4	0.96	23	0.174
399	N/A	2	0	1.00	23	0.000
400	N/A	2	0	1.00	23	0.000
401	N/A	2	0	1.00	14	0.000
402	N/A	2	0	1.00	23	0.000
403	N/A	2	0	1.00	23	0.000
404	A	5	4	0.94	24	0.167
405	A	5	4	0.96	24	0.167
406	A	6	5	1.01	24	0.208
407	A	6	5	0.97	22	0.227
408	A	5	4	0.96	22	0.182
409	A	5	4	0.94	24	0.167
410	A	5	4	0.93	24	0.167
411	A	5	4	1.10	24	0.167
412	A	5	4	1.08	24	0.167
413	A	5	4	1.05	24	0.167
414	A	5	4	1.03	24	0.167
415	A	5	4	1.29	24	0.167
416	A	5	4	0.96	24	0.167
417	A	5	4	0.95	24	0.167
418	A	5	4	0.95	24	0.167
419	N/A	2	0	1.00	23	0.000
420	A	7	6	1.01	23	0.261
421	A	5	4	0.99	23	0.174
422	A	5	4	1.00	21	0.190
423	A	5	4	0.99	21	0.190
424	A	5	4	0.98	23	0.174
425	N/A	2	0	1.00	23	0.000
426	N/A	2	0	1.00	23	0.000
427	N/A	2	0	1.00	14	0.000

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
428	N/A	2	0	1.00	23	0.000
429	N/A	2	0	1.00	23	0.000
430	N/A	2	0	1.00	23	0.000
431	A	5	4	0.98	23	0.174
432	A	5	4	0.99	23	0.174
433	A	5	4	1.00	21	0.190
434	N/A	2	0	1.00	21	0.000
435	N/A	2	0	1.00	23	0.000
436	N/A	2	0	1.00	23	0.000
437	N/A	2	0	1.00	23	0.000
438	N/A	2	0	1.00	14	0.000
439	N/A	2	0	1.00	23	0.000
440	N/A	2	0	1.00	23	0.000
441	A	5	4	1.09	23	0.174
442	A	5	4	1.09	23	0.174
443	A	5	4	1.08	23	0.174
444	A	5	4	1.09	21	0.190
445	A	6	5	1.03	21	0.238
446	A	5	4	0.92	23	0.174
447	A	5	4	0.91	23	0.174
448	A	5	4	0.91	23	0.174
449	A	5	4	0.91	23	0.174
450	A	5	4	0.92	23	0.174
451	A	5	4	0.93	23	0.174
452	A	5	4	0.96	23	0.174
453	A	5	4	0.96	23	0.174
454	A	6	5	1.04	23	0.217
455	A	7	6	1.02	23	0.261
456	A	8	7	1.04	23	0.304
457	A	9	8	1.09	26	0.308
458	A	9	8	1.29	26	0.308
459	A	8	7	1.00	24	0.292

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
460	A	9	8	1.04	24	0.333
461	A	11	10	1.03	26	0.385
462	A	11	10	0.85	26	0.385
463	A	10	9	0.78	26	0.346
464	A	9	8	0.70	26	0.308
465	A	9	8	0.67	26	0.308
466	A	9	8	0.59	26	0.308
467	A	9	8	0.50	26	0.308
468	A	9	8	1.11	26	0.308
469	A	9	8	1.21	26	0.308
470	A	8	7	1.00	24	0.292
471	A	8	7	1.00	24	0.292
472	A	10	9	1.03	26	0.346
473	A	10	10	0.89	26	0.385
474	A	8	8	0.89	26	0.308
475	A	9	8	1.00	26	0.308
476	A	9	8	0.77	26	0.308
477	A	10	9	0.58	26	0.346
478	A	9	8	1.09	26	0.308
479	A	9	8	1.20	26	0.308
480	A	8	7	1.00	24	0.292
481	A	9	8	1.04	24	0.333
482	A	10	9	1.06	26	0.346
483	A	10	10	0.79	26	0.385
484	A	10	9	0.70	26	0.346
485	A	9	8	1.00	26	0.308
486	A	11	10	0.69	26	0.385
487	A	10	9	0.57	26	0.346
488	A	9	8	1.00	25	0.320
489	A	7	6	0.96	25	0.240
490	A	6	5	1.03	23	0.217
491	A	6	5	1.04	23	0.217
492	A	7	6	0.91	25	0.240
493	A	9	8	0.92	25	0.320

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
494	A	11	10	1.04	25	0.400
495	A	9	8	0.96	25	0.320
496	A	4	4	1.00	16	0.250
497	A	10	9	0.96	25	0.360
498	A	12	11	1.03	25	0.440
499	A	10	9	0.92	25	0.360
500	A	8	7	0.93	25	0.280
501	A	7	6	1.01	23	0.261
502	A	7	6	1.01	23	0.261
503	A	8	7	0.88	25	0.280
504	A	10	9	0.85	25	0.360
505	A	14	13	1.03	25	0.520
506	A	11	10	1.00	25	0.400
507	A	11	11	1.01	16	0.688
508	A	12	11	1.00	25	0.440
509	A	14	13	1.02	25	0.520
510	A	8	7	1.11	25	0.280
511	A	6	5	1.02	25	0.200
512	A	5	4	1.00	23	0.174
513	A	5	4	1.00	23	0.174
514	A	6	5	0.95	25	0.200
515	A	8	7	1.01	25	0.280
516	A	11	10	1.01	25	0.400
517	A	6	5	1.13	25	0.200
518	A	4	4	1.00	16	0.250
519	A	7	6	1.13	25	0.240
520	A	11	10	1.01	25	0.400
521	A	9	8	1.03	25	0.320
522	A	7	6	1.00	25	0.240
523	A	6	5	1.03	23	0.217
524	A	6	5	1.04	23	0.217
525	A	7	6	0.94	25	0.240
526	A	9	8	0.95	25	0.320
527	A	14	13	1.07	25	0.520

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
528	A	12	11	1.04	25	0.440
529	A	7	7	1.00	16	0.438
530	A	11	10	1.06	25	0.400
531	A	13	12	1.05	25	0.480
532	A	10	9	0.97	25	0.360
533	A	8	7	0.98	25	0.280
534	A	7	6	1.07	23	0.261
535	A	7	6	1.05	23	0.261
536	A	8	7	0.92	25	0.280
537	A	10	9	0.89	25	0.360
538	A	17	16	1.09	25	0.640
539	A	14	13	1.06	25	0.520
540	A	14	14	1.02	16	0.875
541	A	13	12	1.07	25	0.480
542	A	15	14	1.08	25	0.560
543	A	6	5	1.22	25	0.200
544	A	5	4	1.02	23	0.174
545	A	4	3	1.00	21	0.143
546	A	4	3	1.00	21	0.143
547	A	5	4	0.96	23	0.174
548	A	5	4	1.00	23	0.174
549	A	5	4	1.00	23	0.174
550	A	5	4	1.00	23	0.174
551	A	5	4	1.00	23	0.174
552	A	5	4	1.00	15	0.267
553	A	7	6	1.00	15	0.400
554	A	6	5	1.00	15	0.333
555	A	7	6	0.95	23	0.261
556	A	6	5	1.04	25	0.200
557	A	5	4	1.06	23	0.174
558	A	6	5	1.00	23	0.217
559	A	7	6	0.94	25	0.240
560	A	8	7	1.01	25	0.280
561	A	7	6	1.10	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
562	A	4	3	1.00	16	0.188
563	A	9	8	1.05	25	0.320
564	N/A	2	0	1.00	23	0.000
565	A	12	11	0.95	23	0.478
566	A	8	7	0.99	21	0.333
567	A	5	4	1.00	21	0.190
568	A	8	7	0.99	23	0.304
569	N/A	2	0	1.00	23	0.000
570	N/A	2	0	1.00	23	0.000
571	N/A	2	0	1.00	14	0.000
572	N/A	2	0	1.00	23	0.000
573	N/A	2	0	1.00	23	0.000
574	A	3	3	1.00	23	0.130
575	A	3	3	1.00	23	0.130
576	A	3	3	1.00	21	0.143
577	N/A	2	0	1.00	23	0.000
578	N/A	2	0	1.00	23	0.000
579	A	7	6	0.96	15	0.400
580	A	6	5	1.00	15	0.333
581	N/A	2	0	1.00	23	0.000
582	N/A	2	0	1.00	23	0.000
583	N/A	2	0	1.00	21	0.000
584	A	5	4	1.00	21	0.190
585	A	5	4	0.99	23	0.174
586	N/A	2	0	1.00	23	0.000
587	N/A	2	0	1.00	23	0.000
588	N/A	2	0	1.00	14	0.000
589	N/A	2	0	1.00	23	0.000
590	N/A	2	0	1.00	23	0.000
591	A	9	8	1.38	37	0.216
592	A	5	4	1.00	35	0.114
593	A	6	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
594	A	6	6	1.00	25	0.240

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int \left(a\sin^2(x)\right)^{5/2}\,dx$

3.1.1 Optimal result

Integrand size = 10, antiderivative size = 53

$$\int \left(a\sin^2(x)\right)^{5/2}\,dx = -\frac{8}{15}a^2\cot(x)\sqrt{a\sin^2(x)}-\frac{4}{15}a\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

output
```
-4/15*a*cot(x)*(a*sin(x)^2)^(3/2)-1/5*cot(x)*(a*sin(x)^2)^(5/2)-8/15*a^2*c
ot(x)*(a*sin(x)^2)^(1/2)
```

3.1.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int \left(a\sin^2(x)\right)^{5/2}\,dx = -\frac{1}{240}a^2(150\cos(x)-25\cos(3x)+3\cos(5x))\csc(x)\sqrt{a\sin^2(x)}$$

input
```
Integrate[(a*Sin[x]^2)^(5/2),x]
```

output
```
-1/240*(a^2*(150*Cos[x] - 25*Cos[3*x] + 3*Cos[5*x])*Csc[x]*Sqrt[a*Sin[x]^2
])
```

3.1.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3682, 3042, 3682, 3042, 3686, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sin^2(x)\right)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin(x)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{4}{5}a\int \left(a\sin^2(x)\right)^{3/2}dx-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\int \left(a\sin(x)^2\right)^{3/2}dx-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3682}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\int \sqrt{a\sin^2(x)}dx-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}\right)-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\int \sqrt{a\sin(x)^2}dx-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}\right)-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3686}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\csc(x)\sqrt{a\sin^2(x)}\int \sin(x)dx-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}\right)-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\csc(x)\sqrt{a\sin^2(x)}\int \sin(x)dx-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}\right)-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3118}$$

$$\frac{4}{5}a\left(-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{2}{3}a\cot(x)\sqrt{a\sin^2(x)}\right)-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

input
```
Int[(a*Sin[x]^2)^(5/2),x]
```

output
```
-1/5*(Cot[x]*(a*Sin[x]^2)^(5/2)) + (4*a*((-2*a*Cot[x]*Sqrt[a*Sin[x]^2])/3
- (Cot[x]*(a*Sin[x]^2)^(3/2))/3))/5
```

3.1.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.1.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result
default	$-\frac{a^{3} \sin \left(x \right) \cos \left(x \right) \left(3 \left(\sin^{4}\left(x \right)\right)+4 \left(\sin^{2}\left(x \right)\right)+8\right)}{15 \sqrt{a \left(\sin^{2}\left(x \right)\right)}}$
risch	$-\frac{i a^{2} {\mathrm e}^{6 i x} \sqrt{-a \left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}}{160 \left({\mathrm e}^{2 i x}-1\right)}-\frac{5 i a^{2} {\mathrm e}^{2 i x} \sqrt{-a \left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}}{16 \left({\mathrm e}^{2 i x}-1\right)}-\frac{5 i \sqrt{-a \left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}\, a^{2}}{16 \left({\mathrm e}^{2 i x}-1\right)}+\frac{5 i a^{2} {\mathrm e}^{-2 i x} \sqrt{-a \left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}}{96 \left({\mathrm e}^{2 i x}-1\right)}$

input
```
int((a*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*a^3*sin(x)*cos(x)*(3*sin(x)^4+4*sin(x)^2+8)/(a*sin(x)^2)^(1/2)
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \left(a\sin^2(x)\right)^{5/2}\,dx = -\frac{\left(3\,a^2\cos\left(x\right)^5-10\,a^2\cos\left(x\right)^3+15\,a^2\cos\left(x\right)\right)\sqrt{-a\cos\left(x\right)^2+a}}{15\,\sin\left(x\right)}$$

input
```
integrate((a*sin(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(3*a^2*cos(x)^5 - 10*a^2*cos(x)^3 + 15*a^2*cos(x))*sqrt(-a*cos(x)^2
+ a)/sin(x)
```

3.1.6 Sympy [F]

$$\int \left(a\sin^2(x)\right)^{5/2}\,dx = \int \left(a\sin^2\left(x\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*sin(x)**2)**(5/2),x)
```

output
```
Integral((a*sin(x)**2)**(5/2), x)
```

3.1.7 Maxima [F]

$$\int \left(a\sin^2(x)\right)^{5/2}\,dx = \int \left(a\sin\left(x\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*sin(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(x)^2)^(5/2), x)
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \left(a\sin^2(x)\right)^{5/2} dx = \frac{1}{15}\left(8\,a^2\mathrm{sgn}(\sin(x)) - \left(3\,a^2\cos(x)^5 - 10\,a^2\cos(x)^3 + 15\,a^2\cos(x)\right)\mathrm{sgn}(\sin(x))\right)\sqrt{a}$$

input
```
integrate((a*sin(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/15*(8*a^2*sgn(sin(x)) - (3*a^2*cos(x)^5 - 10*a^2*cos(x)^3 + 15*a^2*cos(x
))*sgn(sin(x)))*sqrt(a)
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\sin^2(x)\right)^{5/2} dx = \int \left(a\sin(x)^2\right)^{5/2} dx$$

input
```
int((a*sin(x)^2)^(5/2),x)
```

output
```
int((a*sin(x)^2)^(5/2), x)
```

3.2 $\int \left(a\sin^2(x)\right)^{3/2}\,dx$

3.2.1 Optimal result

Integrand size = 10, antiderivative size = 34

$$\int \left(a\sin^2(x)\right)^{3/2}\,dx = -\frac{2}{3}a\cot(x)\sqrt{a\sin^2(x)}-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}$$

output
```
-1/3*cot(x)*(a*sin(x)^2)^(3/2)-2/3*a*cot(x)*(a*sin(x)^2)^(1/2)
```

3.2.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \left(a\sin^2(x)\right)^{3/2}\,dx = \frac{1}{12}a(-9\cos(x)+\cos(3x))\csc(x)\sqrt{a\sin^2(x)}$$

input
```
Integrate[(a*Sin[x]^2)^(3/2),x]
```

output
```
(a*(-9*Cos[x] + Cos[3*x])*Csc[x]*Sqrt[a*Sin[x]^2])/12
```

3.2.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3682, 3042, 3686, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a \sin^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a \sin(x)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{2}{3} a \int \sqrt{a \sin^2(x)} dx - \frac{1}{3} \cot(x) \left(a \sin^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3} a \int \sqrt{a \sin(x)^2} dx - \frac{1}{3} \cot(x) \left(a \sin^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3686}$$

$$\frac{2}{3} a \csc(x) \sqrt{a \sin^2(x)} \int \sin(x) dx - \frac{1}{3} \cot(x) \left(a \sin^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3} a \csc(x) \sqrt{a \sin^2(x)} \int \sin(x) dx - \frac{1}{3} \cot(x) \left(a \sin^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{1}{3} \cot(x) \left(a \sin^2(x)\right)^{3/2} - \frac{2}{3} a \cot(x) \sqrt{a \sin^2(x)}$$

input
```
Int[(a*Sin[x]^2)^(3/2),x]
```

output
```
(-2*a*Cot[x]*Sqrt[a*Sin[x]^2])/3 - (Cot[x]*(a*Sin[x]^2)^(3/2))/3
```

3.2.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.2.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	siz
default	$-\frac{a^{2} \sin \left(x \right) \cos \left(x \right) \left(\sin^{2}\left(x \right)+2\right)}{3 \sqrt{a \left(\sin^{2}\left(x \right)\right)}}$	24
risch	$\frac{i a \,{\mathrm e}^{4 i x} \sqrt{-a \left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}}{24 \,{\mathrm e}^{2 i x}-24}-\frac{3 i a \,{\mathrm e}^{2 i x} \sqrt{-a \left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}}{8 \left({\mathrm e}^{2 i x}-1\right)}-\frac{3 i \sqrt{-a \left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}\, a}{8 \left({\mathrm e}^{2 i x}-1\right)}+\frac{i a \,{\mathrm e}^{-2 i x} \sqrt{-a \left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}}{24 \,{\mathrm e}^{2 i x}-24}$	14

input
```
int((a*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2*sin(x)*cos(x)*(sin(x)^2+2)/(a*sin(x)^2)^(1/2)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \left(a\sin^2(x)\right)^{3/2}\,dx = \frac{\left(a\cos\left(x\right)^3 - 3\,a\cos\left(x\right)\right)\sqrt{-a\cos\left(x\right)^2 + a}}{3\,\sin\left(x\right)}$$

input
```
integrate((a*sin(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(a*cos(x)^3 - 3*a*cos(x))*sqrt(-a*cos(x)^2 + a)/sin(x)
```

3.2.6 Sympy [F]

$$\int \left(a\sin^2(x)\right)^{3/2}\,dx = \int \left(a\sin^2\left(x\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sin(x)**2)**(3/2),x)
```

output
```
Integral((a*sin(x)**2)**(3/2), x)
```

3.2.7 Maxima [F]

$$\int \left(a\sin^2(x)\right)^{3/2}\,dx = \int \left(a\sin\left(x\right)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sin(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(x)^2)^(3/2), x)
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a\sin^2(x)\right)^{3/2} dx = \frac{1}{3}\left(\left(\cos\left(x\right)^3 - 3\cos\left(x\right)\right)\mathrm{sgn}(\sin\left(x\right)) + 2\,\mathrm{sgn}(\sin\left(x\right))\right)a^{\frac{3}{2}}$$

input
```
integrate((a*sin(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/3*((cos(x)^3 - 3*cos(x))*sgn(sin(x)) + 2*sgn(sin(x)))*a^(3/2)
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\sin^2(x)\right)^{3/2} dx = \int \left(a\sin\left(x\right)^2\right)^{3/2} dx$$

input
```
int((a*sin(x)^2)^(3/2),x)
```

output
```
int((a*sin(x)^2)^(3/2), x)
```

3.3 $\int \sqrt{a \sin^2(x)} \, dx$

3.3.1 Optimal result

Integrand size = 10, antiderivative size = 14

$$\int \sqrt{a \sin^2(x)} \, dx = -\cot(x) \sqrt{a \sin^2(x)}$$

output
```
-cot(x)*(a*sin(x)^2)^(1/2)
```

3.3.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{a \sin^2(x)} \, dx = -\cot(x) \sqrt{a \sin^2(x)}$$

input
```
Integrate[Sqrt[a*Sin[x]^2],x]
```

output
```
-(Cot[x]*Sqrt[a*Sin[x]^2])
```

3.3.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3686, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \sin^2(x)} \, dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{a \sin(x)^2} dx$$

$$\Big\downarrow \ 3686$$

$$\csc(x)\sqrt{a \sin^2(x)} \int \sin(x) dx$$

$$\Big\downarrow \ 3042$$

$$\csc(x)\sqrt{a \sin^2(x)} \int \sin(x) dx$$

$$\Big\downarrow \ 3118$$

$$-\cot(x)\sqrt{a \sin^2(x)}$$

input
```
Int[Sqrt[a*Sin[x]^2],x]
```

output
```
-(Cot[x]*Sqrt[a*Sin[x]^2])
```

3.3.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.3.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{a\cos(x)\sin(x)}{\sqrt{a(\sin^2(x))}}$	16
risch	$-\frac{i\sqrt{-a(\mathrm{e}^{2ix}-1)^2\mathrm{e}^{-2ix}}\,\mathrm{e}^{2ix}}{2(\mathrm{e}^{2ix}-1)}-\frac{i\sqrt{-a(\mathrm{e}^{2ix}-1)^2\mathrm{e}^{-2ix}}}{2(\mathrm{e}^{2ix}-1)}$	69

input
```
int((a*sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*sin(x)^2)^(1/2)*a*cos(x)*sin(x)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int\sqrt{a\sin^2(x)}\,dx=-\frac{\sqrt{-a\cos{(x)}^2+a}\cos{(x)}}{\sin{(x)}}$$

input
```
integrate((a*sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-a*cos(x)^2 + a)*cos(x)/sin(x)
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \sqrt{a\sin^2(x)}\,dx = -\frac{\sqrt{a\sin^2(x)}\cos(x)}{\sin(x)}$$

input
```
integrate((a*sin(x)**2)**(1/2),x)
```

output
```
-sqrt(a*sin(x)**2)*cos(x)/sin(x)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \sqrt{a\sin^2(x)}\,dx = -\frac{\sqrt{a}}{\sqrt{\tan(x)^2+1}}$$

input
```
integrate((a*sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(a)/sqrt(tan(x)^2 + 1)
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \sqrt{a\sin^2(x)}\,dx = -(\cos(x)\operatorname{sgn}(\sin(x)) - \operatorname{sgn}(\sin(x)))\sqrt{a}$$

input
```
integrate((a*sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-(cos(x)*sgn(sin(x)) - sgn(sin(x)))*sqrt(a)
```

3.3.9 Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.86

$$\int \sqrt{a\sin^2(x)}\,dx = -\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{2\sin(x)^2}\left(-\sin(x)^2+\frac{\sin(2\,x)\,1\mathrm{i}}{2}+1\right)}{\sin(x)^2\,2\mathrm{i}+\sin(2\,x)}$$

input
```
int((a*sin(x)^2)^(1/2),x)
```

output
```
-(2^(1/2)*a^(1/2)*(2*sin(x)^2)^(1/2)*((sin(2*x)*1i)/2 - sin(x)^2 + 1))/(si
n(2*x) + sin(x)^2*2i)
```

3.4 $\int \frac{1}{\sqrt{a \sin^2(x)}} \, dx$

3.4.1 Optimal result

Integrand size = 10, antiderivative size = 17

$$\int \frac{1}{\sqrt{a \sin^2(x)}} \, dx = -\frac{\operatorname{arctanh}(\cos(x)) \sin(x)}{\sqrt{a \sin^2(x)}}$$

output
```
-arctanh(cos(x))*sin(x)/(a*sin(x)^2)^(1/2)
```

3.4.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \frac{1}{\sqrt{a \sin^2(x)}} \, dx = \frac{\left(-\log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)\right) \sin(x)}{\sqrt{a \sin^2(x)}}$$

input
```
Integrate[1/Sqrt[a*Sin[x]^2],x]
```

output
```
((-Log[Cos[x/2]] + Log[Sin[x/2]])*Sin[x])/Sqrt[a*Sin[x]^2]
```

3.4.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sin^2(x)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{a\sin(x)^2}}dx$$
$$\Big\downarrow \text{3686}$$
$$\frac{\sin(x)\int \csc(x)dx}{\sqrt{a\sin^2(x)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sin(x)\int \csc(x)dx}{\sqrt{a\sin^2(x)}}$$
$$\Big\downarrow \text{4257}$$
$$-\frac{\sin(x)\text{arctanh}(\cos(x))}{\sqrt{a\sin^2(x)}}$$

input
```
Int[1/Sqrt[a*Sin[x]^2],x]
```

output
```
-((ArcTanh[Cos[x]]*Sin[x])/Sqrt[a*Sin[x]^2])
```

3.4.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.4.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(15) = 30.

Time = 0.72 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.88

method	result	size
default	$-\frac{\sin(x)\sqrt{a(\cos^2(x))}\,\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^2(x)\right)}+2a}{\sin(x)}\right)}{\sqrt{a}\,\cos(x)\sqrt{a(\sin^2(x))}}$	49
risch	$-\frac{2\ln(e^{ix}+1)\sin(x)}{\sqrt{-a(e^{2ix}-1)^2 e^{-2ix}}}+\frac{2\ln(e^{ix}-1)\sin(x)}{\sqrt{-a(e^{2ix}-1)^2 e^{-2ix}}}$	64

input
```
int(1/(a*sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-sin(x)*(a*cos(x)^2)^(1/2)/a^(1/2)*ln(2*(a^(1/2)*(a*cos(x)^2)^(1/2)+a)/sin
(x))/cos(x)/(a*sin(x)^2)^(1/2)
```

3.4.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(15) = 30.

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 4.12

$$\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx = \left[\frac{\sqrt{-a\cos(x)^2+a}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)}{2\,a\sin(x)}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a\cos(x)^2+a}\sqrt{-a}\cos(x)}{a\sin(x)}\right)}{a}\right]$$

input
```
integrate(1/(a*sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(-a*cos(x)^2 + a)*log(-(cos(x) - 1)/(cos(x) + 1))/(a*sin(x)), sqr
t(-a)*arctan(sqrt(-a*cos(x)^2 + a)*sqrt(-a)*cos(x)/(a*sin(x)))/a]
```

3.4.6 Sympy [F]

$$\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{a\sin^2(x)}}\,dx$$

input
```
integrate(1/(a*sin(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a*sin(x)**2), x)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx = \frac{\sqrt{-a}(\arctan(\sin(x), \cos(x)+1) - \arctan(\sin(x), \cos(x)-1))}{a}$$

input
```
integrate(1/(a*sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(-a)*(arctan2(sin(x), cos(x) + 1) - arctan2(sin(x), cos(x) - 1))/a
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx = \frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{\sqrt{a}\mathrm{sgn}\left(\sin\left(x\right)\right)}$$

input
```
integrate(1/(a*sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
log(abs(tan(1/2*x)))/(sqrt(a)*sgn(sin(x)))
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{a\sin\left(x\right)^2}}\,dx$$

input
```
int(1/(a*sin(x)^2)^(1/2),x)
```

output
```
int(1/(a*sin(x)^2)^(1/2), x)
```

3.5 $\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx$

3.5.1 Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx=-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}-\frac{\operatorname{arctanh}(\cos(x))\sin(x)}{2a\sqrt{a\sin^2(x)}}$$

output
```
-1/2*cot(x)/a/(a*sin(x)^2)^(1/2)-1/2*arctanh(cos(x))*sin(x)/a/(a*sin(x)^2)
^(1/2)
```

3.5.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx=-\frac{\left(\csc^2\left(\frac{x}{2}\right)+4\log\left(\cos\left(\frac{x}{2}\right)\right)-4\log\left(\sin\left(\frac{x}{2}\right)\right)-\sec^2\left(\frac{x}{2}\right)\right)\sin^3(x)}{8\left(a\sin^2(x)\right)^{3/2}}$$

input
```
Integrate[(a*Sin[x]^2)^(-3/2),x]
```

output
```
-1/8*((Csc[x/2]^2 + 4*Log[Cos[x/2]] - 4*Log[Sin[x/2]] - Sec[x/2]^2)*Sin[x]
^3)/(a*Sin[x]^2)^(3/2)
```

3.5.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a \sin^2(x)\right)^{3/2}} dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a \sin(x)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3683}$$

$$\frac{\int \frac{1}{\sqrt{a \sin^2(x)}}dx}{2a}-\frac{\cot(x)}{2a\sqrt{a \sin^2(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{1}{\sqrt{a \sin(x)^2}}dx}{2a}-\frac{\cot(x)}{2a\sqrt{a \sin^2(x)}}$$

$$\Big\downarrow \text{ 3686}$$

$$\frac{\sin(x)\int \csc(x)dx}{2a\sqrt{a \sin^2(x)}}-\frac{\cot(x)}{2a\sqrt{a \sin^2(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sin(x)\int \csc(x)dx}{2a\sqrt{a \sin^2(x)}}-\frac{\cot(x)}{2a\sqrt{a \sin^2(x)}}$$

$$\Big\downarrow \text{ 4257}$$

$$-\frac{\sin(x)\text{arctanh}(\cos(x))}{2a\sqrt{a \sin^2(x)}}-\frac{\cot(x)}{2a\sqrt{a \sin^2(x)}}$$

input
```
Int[(a*Sin[x]^2)^(-3/2),x]
```

output
```
-1/2*Cot[x]/(a*Sqrt[a*Sin[x]^2]) - (ArcTanh[Cos[x]]*Sin[x])/(2*a*Sqrt[a*Si
n[x]^2])
```

3.5.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.5.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.84 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.67

method	result	size
default	$-\frac{\sqrt{a(\cos^2(x))}\left(\ln\left(\frac{2\sqrt{a}\sqrt{a(\cos^2(x))}+2a}{\sin(x)}\right)a(\sin^2(x))+\sqrt{a}\sqrt{a(\cos^2(x))}\right)}{2a^{\frac{5}{2}}\sin(x)\cos(x)\sqrt{a(\sin^2(x))}}$	70
risch	$-\frac{i(e^{2ix}+1)}{a(e^{2ix}-1)\sqrt{-a(e^{2ix}-1)^2e^{-2ix}}}+\frac{\ln(e^{ix}-1)\sin(x)}{a\sqrt{-a(e^{2ix}-1)^2e^{-2ix}}}-\frac{\ln(e^{ix}+1)\sin(x)}{a\sqrt{-a(e^{2ix}-1)^2e^{-2ix}}}$	110

input
```
int(1/(a*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^(5/2)/sin(x)*(a*cos(x)^2)^(1/2)*(ln(2*(a^(1/2)*(a*cos(x)^2)^(1/2)+a
)/sin(x))*a*sin(x)^2+a^(1/2)*(a*cos(x)^2)^(1/2))/cos(x)/(a*sin(x)^2)^(1/2)
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx = \frac{\sqrt{-a\cos(x)^2+a}\left(\left(\cos(x)^2-1\right)\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)+2\cos(x)\right)}{4\left(a^2\cos(x)^2-a^2\right)\sin(x)}$$

input
```
integrate(1/(a*sin(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(-a*cos(x)^2 + a)*((cos(x)^2 - 1)*log(-(cos(x) - 1)/(cos(x) + 1))
+ 2*cos(x))/((a^2*cos(x)^2 - a^2)*sin(x))
```

3.5.6 Sympy [F]

$$\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\sin^2(x)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)**2)**(3/2),x)
```

output
```
Integral((a*sin(x)**2)**(-3/2), x)
```

3.5.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(34) = 68.

Time = 0.46 (sec) , antiderivative size = 314, normalized size of antiderivative = 7.48

$$\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx =$$
$$-\frac{\left(\left(2\left(2\cos(2x)-1\right)\cos(4x)-\cos(4x)^2-4\cos(2x)^2-\sin(4x)^2+4\sin(4x)\sin(2x)-4\sin(2x)^2+\right.\right.}{}$$

input
```
integrate(1/(a*sin(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*((2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^
2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x), c
os(x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - si
n(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(si
n(x), cos(x) - 1) + 2*(sin(3*x) + sin(x))*cos(4*x) - 2*(cos(3*x) + cos(x))
*sin(4*x) - 2*(2*cos(2*x) - 1)*sin(3*x) + 4*cos(3*x)*sin(2*x) + 4*cos(x)*s
in(2*x) - 4*cos(2*x)*sin(x) + 2*sin(x))*sqrt(-a)/(a^2*cos(4*x)^2 + 4*a^2*c
os(2*x)^2 + a^2*sin(4*x)^2 - 4*a^2*sin(4*x)*sin(2*x) + 4*a^2*sin(2*x)^2 -
4*a^2*cos(2*x) + a^2 - 2*(2*a^2*cos(2*x) - a^2)*cos(4*x))
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(34) = 68$.

Time = 0.36 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.93

$$\int \frac{1}{(a\sin^2(x))^{3/2}}\,dx = -\frac{\frac{\left(\frac{2(\cos(x)-1)}{\cos(x)+1}-1\right)(\cos(x)+1)}{\sqrt{a}(\cos(x)-1)\mathrm{sgn}(\sin(x))} - \frac{2\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)}{\sqrt{a}\mathrm{sgn}(\sin(x))} + \frac{\cos(x)-1}{\sqrt{a}(\cos(x)+1)\mathrm{sgn}(\sin(x))}}{8a}$$

input
```
integrate(1/(a*sin(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/8*((2*(cos(x) - 1)/(cos(x) + 1) - 1)*(cos(x) + 1)/(sqrt(a)*(cos(x) - 1)
*sgn(sin(x))) - 2*log(-(cos(x) - 1)/(cos(x) + 1))/(sqrt(a)*sgn(sin(x))) +
(cos(x) - 1)/(sqrt(a)*(cos(x) + 1)*sgn(sin(x))))/a
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin^2(x))^{3/2}}\,dx = \int \frac{1}{\left(a\sin(x)^2\right)^{3/2}}\,dx$$

input
```
int(1/(a*sin(x)^2)^(3/2),x)
```

output
```
int(1/(a*sin(x)^2)^(3/2), x)
```

3.6 $\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx$

3.6.1 Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx=-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}-\frac{3\cot(x)}{8a^2\sqrt{a\sin^2(x)}}-\frac{3\text{arctanh}(\cos(x))\sin(x)}{8a^2\sqrt{a\sin^2(x)}}$$

output
```
-1/4*cot(x)/a/(a*sin(x)^2)^(3/2)-3/8*cot(x)/a^2/(a*sin(x)^2)^(1/2)-3/8*arc
tanh(cos(x))*sin(x)/a^2/(a*sin(x)^2)^(1/2)
```

3.6.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx = -\frac{\csc(x)\left(6\csc^2\left(\frac{x}{2}\right)+\csc^4\left(\frac{x}{2}\right)+24\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)-6\sec^2\left(\frac{x}{2}\right)-\sec^4\left(\frac{x}{2}\right)\right)\sqrt{a\sin^2(x)}}{64a^3}$$

input
```
Integrate[(a*Sin[x]^2)^(-5/2),x]
```

output
```
-1/64*(Csc[x]*(6*Csc[x/2]^2 + Csc[x/2]^4 + 24*(Log[Cos[x/2]] - Log[Sin[x/2
]]) - 6*Sec[x/2]^2 - Sec[x/2]^4)*Sqrt[a*Sin[x]^2])/a^3
```

3.6.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3683, 3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin^2(x))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sin(x)^2)^{5/2}}dx$$

$$\Big\downarrow \text{3683}$$

$$\frac{3\int \frac{1}{(a\sin^2(x))^{3/2}}dx}{4a}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{(a\sin(x)^2)^{3/2}}dx}{4a}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3683}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{a\sin^2(x)}}dx}{2a}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}\right)}{4a}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{a\sin(x)^2}}dx}{2a}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}\right)}{4a}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3686}$$

$$\frac{3\left(\frac{\sin(x)\int \csc(x)dx}{2a\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}\right)}{4a}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\sin(x)\int \csc(x)dx}{2a\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}\right)}{4a}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{3\left(-\frac{\sin(x)\text{arctanh}(\cos(x))}{2a\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}\right)}{4a}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

input
```
Int[(a*Sin[x]^2)^(-5/2),x]
```

output
```
-1/4*Cot[x]/(a*(a*Sin[x]^2)^(3/2)) + (3*(-1/2*Cot[x]/(a*Sqrt[a*Sin[x]^2])
- (ArcTanh[Cos[x]]*Sin[x])/(2*a*Sqrt[a*Sin[x]^2])))/(4*a)
```

3.6.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.6.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.46

method	result	size
default	$-\frac{\sqrt{a\left(\cos^2(x)\right)}\left(3\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^2(x)\right)}+2a}{\sin(x)}\right)a\left(\sin^4(x)\right)+3\sqrt{a\left(\cos^2(x)\right)}\left(\sin^2(x)\right)\sqrt{a}+2\sqrt{a}\,\sqrt{a\left(\cos^2(x)\right)}\right)}{8a^{\frac{7}{2}}\sin(x)^3\cos(x)\sqrt{a\left(\sin^2(x)\right)}}$	89
risch	$-\frac{i\left(3\,\mathrm{e}^{6ix}-11\,\mathrm{e}^{4ix}-11\,\mathrm{e}^{2ix}+3\right)}{4a^2\left(\mathrm{e}^{2ix}-1\right)^3\sqrt{-a\left(\mathrm{e}^{2ix}-1\right)^2\mathrm{e}^{-2ix}}}+\frac{3\ln\left(\mathrm{e}^{ix}-1\right)\sin(x)}{4a^2\sqrt{-a\left(\mathrm{e}^{2ix}-1\right)^2\mathrm{e}^{-2ix}}}-\frac{3\ln\left(\mathrm{e}^{ix}+1\right)\sin(x)}{4a^2\sqrt{-a\left(\mathrm{e}^{2ix}-1\right)^2\mathrm{e}^{-2ix}}}$	127

input
```
int(1/(a*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^(7/2)/sin(x)^3*(a*cos(x)^2)^(1/2)*(3*ln(2*(a^(1/2)*(a*cos(x)^2)^(1/
2)+a)/sin(x))*a*sin(x)^4+3*(a*cos(x)^2)^(1/2)*sin(x)^2*a^(1/2)+2*a^(1/2)*(
a*cos(x)^2)^(1/2))/cos(x)/(a*sin(x)^2)^(1/2)
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx = \frac{\sqrt{-a\cos\left(x\right)^2+a}\left(6\,\cos\left(x\right)^3+3\left(\cos\left(x\right)^4-2\,\cos\left(x\right)^2+1\right)\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)-10\,\cos\right.}{16\left(a^3\cos\left(x\right)^4-2\,a^3\cos\left(x\right)^2+a^3\right)\sin\left(x\right)}$$

input
```
integrate(1/(a*sin(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/16*sqrt(-a*cos(x)^2 + a)*(6*cos(x)^3 + 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log
(-(cos(x) - 1)/(cos(x) + 1)) - 10*cos(x))/((a^3*cos(x)^4 - 2*a^3*cos(x)^2
+ a^3)*sin(x))
```

3.6.6 Sympy [F]

$$\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sin^2{(x)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)**2)**(5/2),x)
```

output
```
Integral((a*sin(x)**2)**(-5/2), x)
```

3.6.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 931 vs. 2(49) = 98.

Time = 0.63 (sec) , antiderivative size = 931, normalized size of antiderivative = 15.26

$$\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sin(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/8*(3*(2*(4*cos(6*x) - 6*cos(4*x) + 4*cos(2*x) - 1)*cos(8*x) - cos(8*x)^
2 + 8*(6*cos(4*x) - 4*cos(2*x) + 1)*cos(6*x) - 16*cos(6*x)^2 + 12*(4*cos(2
*x) - 1)*cos(4*x) - 36*cos(4*x)^2 - 16*cos(2*x)^2 + 4*(2*sin(6*x) - 3*sin(
4*x) + 2*sin(2*x))*sin(8*x) - sin(8*x)^2 + 16*(3*sin(4*x) - 2*sin(2*x))*si
n(6*x) - 16*sin(6*x)^2 - 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x) - 16*sin(2*x
)^2 + 8*cos(2*x) - 1)*arctan2(sin(x), cos(x) + 1) - 3*(2*(4*cos(6*x) - 6*c
os(4*x) + 4*cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 8*(6*cos(4*x) - 4*cos(2*
x) + 1)*cos(6*x) - 16*cos(6*x)^2 + 12*(4*cos(2*x) - 1)*cos(4*x) - 36*cos(4
*x)^2 - 16*cos(2*x)^2 + 4*(2*sin(6*x) - 3*sin(4*x) + 2*sin(2*x))*sin(8*x)
- sin(8*x)^2 + 16*(3*sin(4*x) - 2*sin(2*x))*sin(6*x) - 16*sin(6*x)^2 - 36*
sin(4*x)^2 + 48*sin(4*x)*sin(2*x) - 16*sin(2*x)^2 + 8*cos(2*x) - 1)*arctan
2(sin(x), cos(x) - 1) + 2*(3*sin(7*x) - 11*sin(5*x) - 11*sin(3*x) + 3*sin(
x))*cos(8*x) + 12*(2*sin(6*x) - 3*sin(4*x) + 2*sin(2*x))*cos(7*x) + 8*(11*
sin(5*x) + 11*sin(3*x) - 3*sin(x))*cos(6*x) + 44*(3*sin(4*x) - 2*sin(2*x))
*cos(5*x) - 12*(11*sin(3*x) - 3*sin(x))*cos(4*x) - 2*(3*cos(7*x) - 11*cos(
5*x) - 11*cos(3*x) + 3*cos(x))*sin(8*x) - 6*(4*cos(6*x) - 6*cos(4*x) + 4*c
os(2*x) - 1)*sin(7*x) - 8*(11*cos(5*x) + 11*cos(3*x) - 3*cos(x))*sin(6*x)
- 22*(6*cos(4*x) - 4*cos(2*x) + 1)*sin(5*x) + 12*(11*cos(3*x) - 3*cos(x))*
sin(4*x) + 22*(4*cos(2*x) - 1)*sin(3*x) - 88*cos(3*x)*sin(2*x) + 24*cos(x)
*sin(2*x) - 24*cos(2*x)*sin(x) + 6*sin(x))*sqrt(-a)/(a^3*cos(8*x)^2 + 1...
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(49) = 98$.

Time = 0.37 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.93

$$\int \frac{1}{(a\sin^2(x))^{5/2}}\,dx = \frac{\left(\frac{8(\cos(x)-1)}{\cos(x)+1} - \frac{18(\cos(x)-1)^2}{(\cos(x)+1)^2} - 1\right)(\cos(x)+1)^2}{64\,a^{\frac{5}{2}}(\cos(x)-1)^2\mathrm{sgn}(\sin(x))} + \frac{3\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)}{16\,a^{\frac{5}{2}}\mathrm{sgn}(\sin(x))} - \frac{\frac{8\,a^{\frac{5}{2}}(\cos(x)-1)\mathrm{sgn}(\sin(x))}{\cos(x)+1} - \frac{a^{\frac{5}{2}}(\cos(x)-1)^2\mathrm{sgn}(\sin(x))}{(\cos(x)+1)^2}}{64\,a^5}$$

input
```
integrate(1/(a*sin(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/64*(8*(cos(x) - 1)/(cos(x) + 1) - 18*(cos(x) - 1)^2/(cos(x) + 1)^2 - 1)*
(cos(x) + 1)^2/(a^(5/2)*(cos(x) - 1)^2*sgn(sin(x))) + 3/16*log(-(cos(x) -
1)/(cos(x) + 1))/(a^(5/2)*sgn(sin(x))) - 1/64*(8*a^(5/2)*(cos(x) - 1)*sgn(
sin(x))/(cos(x) + 1) - a^(5/2)*(cos(x) - 1)^2*sgn(sin(x))/(cos(x) + 1)^2)/
a^5
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin^2(x))^{5/2}}\,dx = \int \frac{1}{\left(a\sin(x)^2\right)^{5/2}}\,dx$$

input
```
int(1/(a*sin(x)^2)^(5/2),x)
```

output
```
int(1/(a*sin(x)^2)^(5/2), x)
```

3.7 $\int (a\sin^3(x))^{5/2}\,dx$

3.7.1 Optimal result

Integrand size = 10, antiderivative size = 123

$$\int (a\sin^3(x))^{5/2}\,dx = -\frac{26}{77}a^2\cot(x)\sqrt{a\sin^3(x)} - \frac{26a^2\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{x}{2},2\right)\sqrt{a\sin^3(x)}}{77\sin^{\frac{3}{2}}(x)} - \frac{78}{385}a^2\cos(x)\sin(x)\sqrt{a\sin^3(x)} - \frac{26}{165}a^2\cos(x)\sin^3(x)\sqrt{a\sin^3(x)} - \frac{2}{15}a^2\cos(x)\sin^5(x)\sqrt{a\sin^3(x)}$$

output
```
-26/77*a^2*cot(x)*(a*sin(x)^3)^(1/2)-26/77*a^2*(sin(1/4*Pi+1/2*x)^2)^(1/2)
/sin(1/4*Pi+1/2*x)*EllipticF(cos(1/4*Pi+1/2*x),2^(1/2))*(a*sin(x)^3)^(1/2)
/sin(x)^(3/2)-78/385*a^2*cos(x)*sin(x)*(a*sin(x)^3)^(1/2)-26/165*a^2*cos(x
)*sin(x)^3*(a*sin(x)^3)^(1/2)-2/15*a^2*cos(x)*sin(x)^5*(a*sin(x)^3)^(1/2)
```

3.7.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.53

$$\int (a\sin^3(x))^{5/2}\,dx = \frac{a\left(-12480\operatorname{EllipticF}\left(\frac{1}{4}(\pi-2x),2\right)+(-15465\cos(x)+3657\cos(3x)-749\cos(5x)+\right.}{36960\sin^{\frac{9}{2}}(x)}$$

input
```
Integrate[(a*Sin[x]^3)^(5/2),x]
```

output
```
(a*(-12480*EllipticF[(Pi - 2*x)/4, 2] + (-15465*Cos[x] + 3657*Cos[3*x] - 7
49*Cos[5*x] + 77*Cos[7*x])*Sqrt[Sin[x]])*(a*Sin[x]^3)^(3/2))/(36960*Sin[x]
^(9/2))
```

3.7.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.82, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a \sin^3(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a \sin(x)^3\right)^{5/2} dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{a^2\sqrt{a\sin^3(x)}\int \sin^{\frac{15}{2}}(x)dx}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\sin^3(x)}\int \sin(x)^{15/2}dx}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a^2\sqrt{a\sin^3(x)}\left(\frac{13}{15}\int \sin^{\frac{11}{2}}(x)dx-\frac{2}{15}\sin^{\frac{13}{2}}(x)\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\sin^3(x)}\left(\frac{13}{15}\int \sin(x)^{11/2}dx-\frac{2}{15}\sin^{\frac{13}{2}}(x)\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a^2\sqrt{a\sin^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\int \sin^{\frac{7}{2}}(x)dx-\frac{2}{11}\sin^{\frac{9}{2}}(x)\cos(x)\right)-\frac{2}{15}\sin^{\frac{13}{2}}(x)\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$\Big\downarrow$ 3042

$$\frac{a^2\sqrt{a\sin^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\int\sin(x)^{7/2}dx-\frac{2}{11}\sin^{\frac{9}{2}}(x)\cos(x)\right)-\frac{2}{15}\sin^{\frac{13}{2}}(x)\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$\Big\downarrow$ 3115

$$\frac{a^2\sqrt{a\sin^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\int\sin^{\frac{3}{2}}(x)dx-\frac{2}{7}\sin^{\frac{5}{2}}(x)\cos(x)\right)-\frac{2}{11}\sin^{\frac{9}{2}}(x)\cos(x)\right)-\frac{2}{15}\sin^{\frac{13}{2}}(x)\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$\Big\downarrow$ 3042

$$\frac{a^2\sqrt{a\sin^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\int\sin(x)^{3/2}dx-\frac{2}{7}\sin^{\frac{5}{2}}(x)\cos(x)\right)-\frac{2}{11}\sin^{\frac{9}{2}}(x)\cos(x)\right)-\frac{2}{15}\sin^{\frac{13}{2}}(x)\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$\Big\downarrow$ 3115

$$\frac{a^2\sqrt{a\sin^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin(x)}}dx-\frac{2}{3}\sqrt{\sin(x)}\cos(x)\right)-\frac{2}{7}\sin^{\frac{5}{2}}(x)\cos(x)\right)-\frac{2}{11}\sin^{\frac{9}{2}}(x)\cos(x)\right)-\frac{2}{15}\sin^{\frac{13}{2}}(x\right.}{\sin^{\frac{3}{2}}(x)}$$

$\Big\downarrow$ 3042

$$\frac{a^2\sqrt{a\sin^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin(x)}}dx-\frac{2}{3}\sqrt{\sin(x)}\cos(x)\right)-\frac{2}{7}\sin^{\frac{5}{2}}(x)\cos(x)\right)-\frac{2}{11}\sin^{\frac{9}{2}}(x)\cos(x)\right)-\frac{2}{15}\sin^{\frac{13}{2}}(x\right.}{\sin^{\frac{3}{2}}(x)}$$

$\Big\downarrow$ 3120

$$\frac{a^2\sqrt{a\sin^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(-\frac{2}{3}\text{EllipticF}\left(\frac{\pi}{4}-\frac{x}{2},2\right)-\frac{2}{3}\sqrt{\sin(x)}\cos(x)\right)-\frac{2}{7}\sin^{\frac{5}{2}}(x)\cos(x)\right)-\frac{2}{11}\sin^{\frac{9}{2}}(x)\cos(x)\right)-\right.}{\sin^{\frac{3}{2}}(x)}$$

input
```
Int[(a*Sin[x]^3)^(5/2),x]
```

output
```
(a^2*Sqrt[a*Sin[x]^3]*((-2*Cos[x]*Sin[x]^(13/2))/15 + (13*((-2*Cos[x]*Sin[
x]^(9/2))/11 + (9*((5*((-2*EllipticF[Pi/4 - x/2, 2])/3 - (2*Cos[x]*Sqrt[Si
n[x]])/3))/7 - (2*Cos[x]*Sin[x]^(5/2))/7))/11))/15))/Sin[x]^(3/2)
```

3.7. $\int\left(a\sin^3(x)\right)^{5/2}\,dx$

3.7.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.7.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.46 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.56

method	result
default	$\frac{\sqrt{a\left(\sin^{3}(x)\right)}\,\left(77\left(\cos^{6}(x)\right)\cot(x)\sqrt{2}-322\left(\cos^{4}(x)\right)\cot(x)\sqrt{2}+195i\cot(x)\csc(x)\sqrt{-i(i+\cot(x)-\csc(x))}\,\sqrt{i(\csc(x)-\cot(x))}\,F\left(\sqrt{-\right.}\right.}{}$

input
```
int((a*sin(x)^3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2310*(a*sin(x)^3)^(1/2)*(77*cos(x)^6*cot(x)*2^(1/2)-322*cos(x)^4*cot(x)*
2^(1/2)+195*I*cot(x)*csc(x)*(-I*(I+cot(x)-csc(x)))^(1/2)*(I*(csc(x)-cot(x)
))^(1/2)*EllipticF((-I*(I-cot(x)+csc(x)))^(1/2),1/2*2^(1/2))*(-I*(I-cot(x)
+csc(x)))^(1/2)+195*I*csc(x)^2*(-I*(I+cot(x)-csc(x)))^(1/2)*(I*(csc(x)-cot
(x)))^(1/2)*EllipticF((-I*(I-cot(x)+csc(x)))^(1/2),1/2*2^(1/2))*(-I*(I-cot
(x)+csc(x)))^(1/2)+530*cos(x)^2*cot(x)*2^(1/2)-480*cot(x)*2^(1/2))*a^2*8^(
1/2)
```

3.7.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \left(a\sin^3(x)\right)^{5/2} dx = \frac{195\sqrt{2}\sqrt{-i\,a}a^2\sin(x)\,\text{weierstrassPInverse}(4,0,\cos(x)+i\,\sin(x))+195\sqrt{2}\sqrt{i\,a}a^2\,\text{s}}{}$$

input
```
integrate((a*sin(x)^3)^(5/2),x, algorithm="fricas")
```

output
```
1/1155*(195*sqrt(2)*sqrt(-I*a)*a^2*sin(x)*weierstrassPInverse(4, 0, cos(x)
 + I*sin(x)) + 195*sqrt(2)*sqrt(I*a)*a^2*sin(x)*weierstrassPInverse(4, 0,
cos(x) - I*sin(x)) + 2*(77*a^2*cos(x)^7 - 322*a^2*cos(x)^5 + 530*a^2*cos(x
)^3 - 480*a^2*cos(x))*sqrt(-(a*cos(x)^2 - a)*sin(x)))/sin(x)
```

3.7.6 Sympy [F(-1)]

Timed out.

$$\int \left(a\sin^3(x)\right)^{5/2} dx = \text{Timed out}$$

input
```
integrate((a*sin(x)**3)**(5/2),x)
```

output
```
Timed out
```

3.7.7 Maxima [F]

$$\int \left(a\sin^3(x)\right)^{5/2}\,dx = \int \left(a\sin{(x)}^3\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*sin(x)^3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(x)^3)^(5/2), x)
```

3.7.8 Giac [F]

$$\int \left(a\sin^3(x)\right)^{5/2}\,dx = \int \left(a\sin{(x)}^3\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*sin(x)^3)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sin(x)^3)^(5/2), x)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\sin^3(x)\right)^{5/2}\,dx = \int \left(a\sin{(x)}^3\right)^{5/2}\,dx$$

input
```
int((a*sin(x)^3)^(5/2),x)
```

output
```
int((a*sin(x)^3)^(5/2), x)
```

3.8 $\int (a\sin^3(x))^{3/2}\,dx$

3.8.1 Optimal result

Integrand size = 10, antiderivative size = 73

$$\int (a\sin^3(x))^{3/2}\,dx = -\frac{14}{45}a\cos(x)\sqrt{a\sin^3(x)} - \frac{14aE\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{15\sin^{\frac{3}{2}}(x)} - \frac{2}{9}a\cos(x)\sin^2(x)\sqrt{a\sin^3(x)}$$

output
```
-14/45*a*cos(x)*(a*sin(x)^3)^(1/2)-14/15*a*(sin(1/4*Pi+1/2*x)^2)^(1/2)/sin
(1/4*Pi+1/2*x)*EllipticE(cos(1/4*Pi+1/2*x),2^(1/2))*(a*sin(x)^3)^(1/2)/sin
(x)^(3/2)-2/9*a*cos(x)*sin(x)^2*(a*sin(x)^3)^(1/2)
```

3.8.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int (a\sin^3(x))^{3/2}\,dx = \frac{(a\sin^3(x))^{3/2}\left(-168E\left(\frac{1}{4}(\pi-2x)\middle|2\right)+\sqrt{\sin(x)}(-38\sin(2x)+5\sin(4x))\right)}{180\sin^{\frac{9}{2}}(x)}$$

input
```
Integrate[(a*Sin[x]^3)^(3/2),x]
```

output
```
((a*Sin[x]^3)^(3/2)*(-168*EllipticE[(Pi - 2*x)/4, 2] + Sqrt[Sin[x]]*(-38*S
in[2*x] + 5*Sin[4*x])))/(180*Sin[x]^(9/2))
```

3.8.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sin^3(x)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a\sin(x)^3\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3686}$$

$$\frac{a\sqrt{a\sin^3(x)}\int \sin^{\frac{9}{2}}(x)dx}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\sqrt{a\sin^3(x)}\int \sin(x)^{9/2}dx}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{a\sqrt{a\sin^3(x)}\left(\frac{7}{9}\int \sin^{\frac{5}{2}}(x)dx-\frac{2}{9}\sin^{\frac{7}{2}}(x)\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\sqrt{a\sin^3(x)}\left(\frac{7}{9}\int \sin(x)^{5/2}dx-\frac{2}{9}\sin^{\frac{7}{2}}(x)\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{a\sqrt{a\sin^3(x)}\left(\frac{7}{9}\left(\frac{3}{5}\int \sqrt{\sin(x)}dx-\frac{2}{5}\sin^{\frac{3}{2}}(x)\cos(x)\right)-\frac{2}{9}\sin^{\frac{7}{2}}(x)\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\sqrt{a\sin^3(x)}\left(\frac{7}{9}\left(\frac{3}{5}\int \sqrt{\sin(x)}dx-\frac{2}{5}\sin^{\frac{3}{2}}(x)\cos(x)\right)-\frac{2}{9}\sin^{\frac{7}{2}}(x)\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{a\sqrt{a\sin^3(x)}\left(\frac{7}{9}\left(-\frac{6}{5}E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)-\frac{2}{5}\sin^{\frac{3}{2}}(x)\cos(x)\right)-\frac{2}{9}\sin^{\frac{7}{2}}(x)\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

input
```
Int[(a*Sin[x]^3)^(3/2),x]
```

output
```
(a*Sqrt[a*Sin[x]^3]*((-2*Cos[x]*Sin[x]^(7/2))/9 + (7*((-6*EllipticE[Pi/4 -
 x/2, 2])/5 - (2*Cos[x]*Sin[x]^(3/2))/5))/9))/Sin[x]^(3/2)
```

3.8.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.8.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.45 (sec) , antiderivative size = 302, normalized size of antiderivative = 4.14

method	result
default	$-\frac{(\csc^2(x))a\left(5(\cos^5(x))\sqrt{2}-21\sqrt{-i(i-\cot(x)+\csc(x))}\,\sqrt{-i(i+\cot(x)-\csc(x))}\,\sqrt{i(\csc(x)-\cot(x))}\,F\left(\sqrt{-i(i-\cot(x)+\csc(x))},\frac{\sqrt{2}}{2}\right.\right.}{}$

input
```
int((a*sin(x)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/90*csc(x)^2*a*(5*cos(x)^5*2^(1/2)-21*(-I*(I-cot(x)+csc(x)))^(1/2)*(-I*(
I+cot(x)-csc(x)))^(1/2)*(I*(csc(x)-cot(x)))^(1/2)*EllipticF((-I*(I-cot(x)+
csc(x)))^(1/2),1/2*2^(1/2))*cos(x)+42*(-I*(I-cot(x)+csc(x)))^(1/2)*(-I*(I+
cot(x)-csc(x)))^(1/2)*(I*(csc(x)-cot(x)))^(1/2)*EllipticE((-I*(I-cot(x)+cs
c(x)))^(1/2),1/2*2^(1/2))*cos(x)-17*cos(x)^3*2^(1/2)-21*(-I*(I-cot(x)+csc(
x)))^(1/2)*(-I*(I+cot(x)-csc(x)))^(1/2)*(I*(csc(x)-cot(x)))^(1/2)*Elliptic
F((-I*(I-cot(x)+csc(x)))^(1/2),1/2*2^(1/2))+42*(-I*(I-cot(x)+csc(x)))^(1/2
)*(-I*(I+cot(x)-csc(x)))^(1/2)*(I*(csc(x)-cot(x)))^(1/2)*EllipticE((-I*(I-
cot(x)+csc(x)))^(1/2),1/2*2^(1/2))+33*cos(x)*2^(1/2)-21*2^(1/2))*(a*sin(x)
^3)^(1/2)*8^(1/2)
```

3.8.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int \left(a\sin^3(x)\right)^{3/2}\,dx = \frac{7}{15}i\sqrt{2}\sqrt{-i\,a}a\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(x)+i\,\sin(x))) - \frac{7}{15}i\sqrt{2}\sqrt{i\,a}a\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(x)-i\,\sin(x))) + \frac{2}{45}\left(5\,a\cos(x)^3-12\,a\cos(x)\right)\sqrt{-\left(a\cos(x)^2-a\right)\sin(x)}$$

input
```
integrate((a*sin(x)^3)^(3/2),x, algorithm="fricas")
```

output
```
7/15*I*sqrt(2)*sqrt(-I*a)*a*weierstrassZeta(4, 0, weierstrassPInverse(4, 0
, cos(x) + I*sin(x))) - 7/15*I*sqrt(2)*sqrt(I*a)*a*weierstrassZeta(4, 0, w
eierstrassPInverse(4, 0, cos(x) - I*sin(x))) + 2/45*(5*a*cos(x)^3 - 12*a*c
os(x))*sqrt(-(a*cos(x)^2 - a)*sin(x))
```

3.8.6 Sympy [F]

$$\int \left(a\sin^3(x)\right)^{3/2}\,dx = \int \left(a\sin^3{\left(x\right)}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sin(x)**3)**(3/2),x)
```

output
```
Integral((a*sin(x)**3)**(3/2), x)
```

3.8.7 Maxima [F]

$$\int \left(a\sin^3(x)\right)^{3/2}\,dx = \int \left(a\sin\left(x\right)^{3}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sin(x)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(x)^3)^(3/2), x)
```

3.8.8 Giac [F]

$$\int \left(a\sin^3(x)\right)^{3/2}\,dx = \int \left(a\sin\left(x\right)^{3}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sin(x)^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sin(x)^3)^(3/2), x)
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\sin^3(x)\right)^{3/2} dx = \int \left(a\sin{(x)}^3\right)^{3/2} dx$$

input
```
int((a*sin(x)^3)^(3/2),x)
```

output
```
int((a*sin(x)^3)^(3/2), x)
```

3.9 $\int \sqrt{a \sin^3(x)}\, dx$

3.9.1 Optimal result

Integrand size = 10, antiderivative size = 50

$$\int \sqrt{a \sin^3(x)}\, dx = -\frac{2}{3}\cot(x)\sqrt{a \sin^3(x)} - \frac{2\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{x}{2},2\right)\sqrt{a \sin^3(x)}}{3 \sin^{\frac{3}{2}}(x)}$$

output
```
-2/3*cot(x)*(a*sin(x)^3)^(1/2)-2/3*(sin(1/4*Pi+1/2*x)^2)^(1/2)/sin(1/4*Pi+
1/2*x)*EllipticF(cos(1/4*Pi+1/2*x),2^(1/2))*(a*sin(x)^3)^(1/2)/sin(x)^(3/2
)
```

3.9.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \sqrt{a \sin^3(x)}\, dx = -\frac{2\left(\text{EllipticF}\left(\frac{1}{4}(\pi-2x),2\right)+\cos(x)\sqrt{\sin(x)}\right)\sqrt{a \sin^3(x)}}{3 \sin^{\frac{3}{2}}(x)}$$

input
```
Integrate[Sqrt[a*Sin[x]^3],x]
```

output
```
(-2*(EllipticF[(Pi - 2*x)/4, 2] + Cos[x]*Sqrt[Sin[x]])*Sqrt[a*Sin[x]^3])/(
3*Sin[x]^(3/2))
```

3.9.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3686, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \sin^3(x)}\, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a \sin(x)^3}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sqrt{a \sin^3(x)} \int \sin^{\frac{3}{2}}(x)dx}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a \sin^3(x)} \int \sin(x)^{3/2}dx}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{a \sin^3(x)} \left(\frac{1}{3} \int \frac{1}{\sqrt{\sin(x)}}dx - \frac{2}{3}\sqrt{\sin(x)}\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a \sin^3(x)} \left(\frac{1}{3} \int \frac{1}{\sqrt{\sin(x)}}dx - \frac{2}{3}\sqrt{\sin(x)}\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{a \sin^3(x)} \left(-\frac{2}{3} \text{EllipticF}\left(\frac{\pi}{4} - \frac{x}{2}, 2\right) - \frac{2}{3}\sqrt{\sin(x)}\cos(x)\right)}{\sin^{\frac{3}{2}}(x)}$$

input
```
Int[Sqrt[a*Sin[x]^3],x]
```

output
```
(((-2*EllipticF[Pi/4 - x/2, 2])/3 - (2*Cos[x]*Sqrt[Sin[x]])/3)*Sqrt[a*Sin[
x]^3])/Sin[x]^(3/2)
```

3.9.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.9.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.26

method	result
default	$\frac{\left(-i\sqrt{-i(i-\cot(x)+\csc(x))}\,\sqrt{-i(i+\cot(x)-\csc(x))}\,\sqrt{i(\csc(x)-\cot(x))}\,F\left(\sqrt{-i(i-\cot(x)+\csc(x))},\frac{\sqrt{2}}{2}\right)\cos(x)-i\sqrt{-i(i-\cot(x)+\mathrm{cs}}{6(\cos(x)-1)(\cos(x}$

input
```
int((a*sin(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-I*(-I*(I-cot(x)+csc(x)))^(1/2)*(-I*(I+cot(x)-csc(x)))^(1/2)*(I*(csc(
x)-cot(x)))^(1/2)*EllipticF((-I*(I-cot(x)+csc(x)))^(1/2),1/2*2^(1/2))*cos(
x)-I*(-I*(I-cot(x)+csc(x)))^(1/2)*(-I*(I+cot(x)-csc(x)))^(1/2)*(I*(csc(x)-
cot(x)))^(1/2)*EllipticF((-I*(I-cot(x)+csc(x)))^(1/2),1/2*2^(1/2))+cos(x)*
sin(x)*2^(1/2))*(a*sin(x)^3)^(1/2)/(cos(x)-1)/(cos(x)+1)*8^(1/2)
```

3.9.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.38

$$\int \sqrt{a\sin^3(x)}\,dx = \frac{\sqrt{2}\sqrt{-i\,a}\sin(x)\,\text{weierstrassPInverse}(4,0,\cos(x)+i\,\sin(x))+\sqrt{2}\sqrt{i\,a}\sin(x)\,\text{weierstrassPInverse}(4,0,\text{c}}{3\,\sin(x)}$$

input
```
integrate((a*sin(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*sqrt(-I*a)*sin(x)*weierstrassPInverse(4, 0, cos(x) + I*sin(x)
) + sqrt(2)*sqrt(I*a)*sin(x)*weierstrassPInverse(4, 0, cos(x) - I*sin(x))
- 2*sqrt(-(a*cos(x)^2 - a)*sin(x))*cos(x))/sin(x)
```

3.9.6 Sympy [F]

$$\int \sqrt{a\sin^3(x)}\,dx = \int \sqrt{a\sin^3(x)}\,dx$$

input
```
integrate((a*sin(x)**3)**(1/2),x)
```

output
```
Integral(sqrt(a*sin(x)**3), x)
```

3.9.7 Maxima [F]

$$\int \sqrt{a\sin^3(x)}\,dx = \int \sqrt{a\sin(x)^3}\,dx$$

input
```
integrate((a*sin(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(x)^3), x)
```

3.9.8 Giac [F]

$$\int \sqrt{a\sin^3(x)}\,dx = \int \sqrt{a\sin(x)^3}\,dx$$

input
```
integrate((a*sin(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sin(x)^3), x)
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\sin^3(x)}\,dx = \int \sqrt{a\sin(x)^3}\,dx$$

input
```
int((a*sin(x)^3)^(1/2),x)
```

output
```
int((a*sin(x)^3)^(1/2), x)
```

3.10 $\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx$

3.10.1 Optimal result

Integrand size = 10, antiderivative size = 48

$$\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx=-\frac{2\cos(x)\sin(x)}{\sqrt{a\sin^3(x)}}+\frac{2E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sin^{\frac{3}{2}}(x)}{\sqrt{a\sin^3(x)}}$$

output
```
-2*cos(x)*sin(x)/(a*sin(x)^3)^(1/2)+2*(sin(1/4*Pi+1/2*x)^2)^(1/2)/sin(1/4*
Pi+1/2*x)*EllipticE(cos(1/4*Pi+1/2*x),2^(1/2))*sin(x)^(3/2)/(a*sin(x)^3)^(
1/2)
```

3.10.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx=\frac{2E\left(\frac{1}{4}(\pi-2x)\middle|2\right)\sin^{\frac{3}{2}}(x)-\sin(2x)}{\sqrt{a\sin^3(x)}}$$

input
```
Integrate[1/Sqrt[a*Sin[x]^3],x]
```

output
```
(2*EllipticE[(Pi - 2*x)/4, 2]*Sin[x]^(3/2) - Sin[2*x])/Sqrt[a*Sin[x]^3]
```

3.10.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3686, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sin^3(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin(x)^3}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sin^{\frac{3}{2}}(x)\int \frac{1}{\sin^{\frac{3}{2}}(x)}dx}{\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(x)\int \frac{1}{\sin(x)^{3/2}}dx}{\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(-\int \sqrt{\sin(x)}dx-\frac{2\cos(x)}{\sqrt{\sin(x)}}\right)}{\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(-\int \sqrt{\sin(x)}dx-\frac{2\cos(x)}{\sqrt{\sin(x)}}\right)}{\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(2E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)-\frac{2\cos(x)}{\sqrt{\sin(x)}}\right)}{\sqrt{a\sin^3(x)}}$$

input
```
Int[1/Sqrt[a*Sin[x]^3],x]
```

output
```
((2*EllipticE[Pi/4 - x/2, 2] - (2*Cos[x])/Sqrt[Sin[x]])*Sin[x]^(3/2))/Sqrt
[a*Sin[x]^3]
```

3.10.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.10.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 274, normalized size of antiderivative = 5.71

method	result
default	$\frac{\left(2\sqrt{-i(i-\cot(x)+\csc(x))}\,\sqrt{-i(i+\cot(x)-\csc(x))}\,\sqrt{i(\csc(x)-\cot(x))}\,E\left(\sqrt{-i(i-\cot(x)+\csc(x))},\frac{\sqrt{2}}{2}\right)\cos(x)-\sqrt{-i(i-\cot(x)+\csc(}\right.}{}$

input
```
int(1/(a*sin(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*(-I*(I-cot(x)+csc(x)))^(1/2)*(-I*(I+cot(x)-csc(x)))^(1/2)*(I*(csc(x
)-cot(x)))^(1/2)*EllipticE((-I*(I-cot(x)+csc(x)))^(1/2),1/2*2^(1/2))*cos(x
)-(-I*(I-cot(x)+csc(x)))^(1/2)*(-I*(I+cot(x)-csc(x)))^(1/2)*(I*(csc(x)-cot
(x)))^(1/2)*EllipticF((-I*(I-cot(x)+csc(x)))^(1/2),1/2*2^(1/2))*cos(x)+2*(
-I*(I-cot(x)+csc(x)))^(1/2)*(-I*(I+cot(x)-csc(x)))^(1/2)*(I*(csc(x)-cot(x)
))^(1/2)*EllipticE((-I*(I-cot(x)+csc(x)))^(1/2),1/2*2^(1/2))-(-I*(I-cot(x)
+csc(x)))^(1/2)*(-I*(I+cot(x)-csc(x)))^(1/2)*(I*(csc(x)-cot(x)))^(1/2)*Ell
ipticF((-I*(I-cot(x)+csc(x)))^(1/2),1/2*2^(1/2))-2^(1/2))*sin(x)/(a*sin(x)
^3)^(1/2)*8^(1/2)
```

3.10.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.12

$$\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx$$
$$= \frac{\left(-i\sqrt{2}\cos\left(x\right)^2+i\sqrt{2}\right)\sqrt{-i\,a}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(x\right)+i\,\sin\left(x\right)))+\left(i\sqrt{2}\right.}{}$$

input
```
integrate(1/(a*sin(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
((-I*sqrt(2)*cos(x)^2 + I*sqrt(2))*sqrt(-I*a)*weierstrassZeta(4, 0, weiers
trassPInverse(4, 0, cos(x) + I*sin(x))) + (I*sqrt(2)*cos(x)^2 - I*sqrt(2))
*sqrt(I*a)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(x) - I*sin(
x))) + 2*sqrt(-(a*cos(x)^2 - a)*sin(x))*cos(x))/(a*cos(x)^2 - a)
```

3.10.6 Sympy [F]

$$\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx = \int \frac{1}{\sqrt{a\sin^3\left(x\right)}}\,dx$$

input
```
integrate(1/(a*sin(x)**3)**(1/2),x)
```

output
```
Integral(1/sqrt(a*sin(x)**3), x)
```

3.10.7 Maxima [F]

$$\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx = \int \frac{1}{\sqrt{a\sin(x)^3}}\,dx$$

input
```
integrate(1/(a*sin(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*sin(x)^3), x)
```

3.10.8 Giac [F]

$$\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx = \int \frac{1}{\sqrt{a\sin(x)^3}}\,dx$$

input
```
integrate(1/(a*sin(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*sin(x)^3), x)
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx = \int \frac{1}{\sqrt{a\sin(x)^3}}\,dx$$

input
```
int(1/(a*sin(x)^3)^(1/2),x)
```

output
```
int(1/(a*sin(x)^3)^(1/2), x)
```

3.11 $\int \frac{1}{\left(a\sin^3(x)\right)^{3/2}}\,dx$

3.11.1 Optimal result

Integrand size = 10, antiderivative size = 77

$$\int \frac{1}{\left(a\sin^3(x)\right)^{3/2}}\,dx = -\frac{10\cos(x)}{21a\sqrt{a\sin^3(x)}} - \frac{2\cot(x)\csc(x)}{7a\sqrt{a\sin^3(x)}} - \frac{10\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{x}{2},2\right)\sin^{\frac{3}{2}}(x)}{21a\sqrt{a\sin^3(x)}}$$

output
```
-10/21*cos(x)/a/(a*sin(x)^3)^(1/2)-2/7*cot(x)*csc(x)/a/(a*sin(x)^3)^(1/2)-
10/21*(sin(1/4*Pi+1/2*x)^2)^(1/2)/sin(1/4*Pi+1/2*x)*EllipticF(cos(1/4*Pi+1
/2*x),2^(1/2))*sin(x)^(3/2)/a/(a*sin(x)^3)^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(a\sin^3(x)\right)^{3/2}}\,dx = -\frac{2\sin^2(x)\left(3\cot(x)+5\cos(x)\sin(x)+5\,\text{EllipticF}\left(\frac{1}{4}(\pi-2x),2\right)\sin^{\frac{5}{2}}(x)\right)}{21\left(a\sin^3(x)\right)^{3/2}}$$

input
```
Integrate[(a*Sin[x]^3)^(-3/2),x]
```

output
```
(-2*Sin[x]^2*(3*Cot[x] + 5*Cos[x]*Sin[x] + 5*EllipticF[(Pi - 2*x)/4, 2]*Si
n[x]^(5/2)))/(21*(a*Sin[x]^3)^(3/2))
```

3.11.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3686, 3042, 3116, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\sin^3(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin(x)^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sin^{\frac{3}{2}}(x)\int \frac{1}{\sin^{\frac{9}{2}}(x)}dx}{a\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(x)\int \frac{1}{\sin(x)^{9/2}}dx}{a\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{5}{7}\int \frac{1}{\sin^{\frac{5}{2}}(x)}dx-\frac{2\cos(x)}{7\sin^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{5}{7}\int \frac{1}{\sin(x)^{5/2}}dx-\frac{2\cos(x)}{7\sin^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(x)}}dx-\frac{2\cos(x)}{3\sin^{\frac{3}{2}}(x)}\right)-\frac{2\cos(x)}{7\sin^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin(x)}}dx-\frac{2\cos(x)}{3\sin^{\frac{3}{2}}(x)}\right)-\frac{2\cos(x)}{7\sin^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{5}{7}\left(-\frac{2}{3}\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{x}{2},2\right)-\frac{2\cos(x)}{3\sin^{\frac{3}{2}}(x)}\right)-\frac{2\cos(x)}{7\sin^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\sin^3(x)}}$$

input
```
Int[(a*Sin[x]^3)^(-3/2),x]
```

output
```
(((5*((-2*EllipticF[Pi/4 - x/2, 2])/3 - (2*Cos[x])/(3*Sin[x]^(3/2))))/7 -
(2*Cos[x])/(7*Sin[x]^(7/2)))*Sin[x]^(3/2))/(a*Sqrt[a*Sin[x]^3])
```

3.11.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.11.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.10

method	result
default	$\frac{(1-\cos(x))^2 \left(40i(\csc^5(x))\sqrt{-i(i-\cot(x)+\csc(x))}\sqrt{2}\sqrt{-i(i+\cot(x)-\csc(x))}\sqrt{i(\csc(x)-\cot(x))}\,F\left(\sqrt{-i(i-\cot(x)+\csc(x))}, \frac{\sqrt{2}}{2}\right)(1}{336\left(\frac{(\csc^3(x))a(1-\cos(x))^3}{((\csc^2(x))(1-\cos(x))^2+1)^3}\right)^{\frac{3}{2}}\left((\csc^2(x))(1-\cos(x))^2+1\right)^4\sqrt{\csc(x)\left((\csc^2(x))(1-\cos\right.}}$

input
```
int(1/(a*sin(x)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/336/(csc(x)^3*a*(1-cos(x))^3/(csc(x)^2*(1-cos(x))^2+1)^3)^(3/2)*(1-cos(x
))^2/(csc(x)^2*(1-cos(x))^2+1)^4/(csc(x)*(csc(x)^2*(1-cos(x))^2+1)*(1-cos(
x)))^(1/2)/(csc(x)^3*(1-cos(x))^3+csc(x)-cot(x))^(1/2)*(40*I*csc(x)^5*(-I*
(I-cot(x)+csc(x)))^(1/2)*2^(1/2)*(-I*(I+cot(x)-csc(x)))^(1/2)*(I*(csc(x)-c
ot(x)))^(1/2)*EllipticF((-I*(I-cot(x)+csc(x)))^(1/2),1/2*2^(1/2))*(1-cos(x
))^3+3*csc(x)^10*(1-cos(x))^8+26*csc(x)^8*(1-cos(x))^6-26*csc(x)^4*(1-cos(
x))^2-3*csc(x)^2)*8^(1/2)
```

3.11.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.81

$$\int \frac{1}{(a\sin^3(x))^{3/2}}\,dx = \frac{5\left(\sqrt{2}\cos(x)^4 - 2\sqrt{2}\cos(x)^2 + \sqrt{2}\right)\sqrt{-i\,a}\sin(x)\,\text{weierstrassPInverse}(4, 0, \cos(x) +}{}$$

input
```
integrate(1/(a*sin(x)^3)^(3/2),x, algorithm="fricas")
```

output
```
1/21*(5*(sqrt(2)*cos(x)^4 - 2*sqrt(2)*cos(x)^2 + sqrt(2))*sqrt(-I*a)*sin(x
)*weierstrassPInverse(4, 0, cos(x) + I*sin(x)) + 5*(sqrt(2)*cos(x)^4 - 2*s
qrt(2)*cos(x)^2 + sqrt(2))*sqrt(I*a)*sin(x)*weierstrassPInverse(4, 0, cos(
x) - I*sin(x)) + 2*(5*cos(x)^3 - 8*cos(x))*sqrt(-(a*cos(x)^2 - a)*sin(x)))
/((a^2*cos(x)^4 - 2*a^2*cos(x)^2 + a^2)*sin(x))
```

3.11.6 Sympy [F]

$$\int \frac{1}{(a\sin^3(x))^{3/2}}\,dx = \int \frac{1}{(a\sin^3(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)**3)**(3/2),x)
```

output
```
Integral((a*sin(x)**3)**(-3/2), x)
```

3.11.7 Maxima [F]

$$\int \frac{1}{(a\sin^3(x))^{3/2}}\,dx = \int \frac{1}{(a\sin(x)^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(x)^3)^(-3/2), x)
```

3.11.8 Giac [F]

$$\int \frac{1}{(a\sin^3(x))^{3/2}}\,dx = \int \frac{1}{(a\sin(x)^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sin(x)^3)^(-3/2), x)
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sin^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\sin(x)^3\right)^{3/2}}\,dx$$

input
```
int(1/(a*sin(x)^3)^(3/2),x)
```

output
```
int(1/(a*sin(x)^3)^(3/2), x)
```

3.12 $\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}\,dx$

3.12.1 Optimal result

Integrand size = 10, antiderivative size = 123

$$\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}\,dx=-\frac{154\cot(x)}{585a^2\sqrt{a\sin^3(x)}}-\frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}}-\frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}}-\frac{154\cos(x)\sin(x)}{195a^2\sqrt{a\sin^3(x)}}+\frac{154E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sin^{\frac{3}{2}}(x)}{195a^2\sqrt{a\sin^3(x)}}$$

output
```
-154/585*cot(x)/a^2/(a*sin(x)^3)^(1/2)-22/117*cot(x)*csc(x)^2/a^2/(a*sin(x
)^3)^(1/2)-2/13*cot(x)*csc(x)^4/a^2/(a*sin(x)^3)^(1/2)-154/195*cos(x)*sin(
x)/a^2/(a*sin(x)^3)^(1/2)+154/195*(sin(1/4*Pi+1/2*x)^2)^(1/2)/sin(1/4*Pi+1
/2*x)*EllipticE(cos(1/4*Pi+1/2*x),2^(1/2))*sin(x)^(3/2)/a^2/(a*sin(x)^3)^(
1/2)
```

3.12.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.49

$$\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}\,dx=-\frac{2\left(\cot(x)\left(77+55\csc^2(x)+45\csc^4(x)\right)+231\cos(x)\sin(x)-231E\left(\frac{1}{4}(\pi-2x)\middle|2\right)\sin^{\frac{3}{2}}(x)\right)}{585a^2\sqrt{a\sin^3(x)}}$$

input
```
Integrate[(a*Sin[x]^3)^(-5/2),x]
```

output
```
(-2*(Cot[x]*(77 + 55*Csc[x]^2 + 45*Csc[x]^4) + 231*Cos[x]*Sin[x] - 231*Ell
ipticE[(Pi - 2*x)/4, 2]*Sin[x]^(3/2)))/(585*a^2*Sqrt[a*Sin[x]^3])
```

3.12.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.79, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {3042, 3686, 3042, 3116, 3042, 3116, 3042, 3116, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin(x)^3\right)^{5/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sin^{\frac{3}{2}}(x)\int \frac{1}{\sin^{\frac{15}{2}}(x)}dx}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(x)\int \frac{1}{\sin(x)^{15/2}}dx}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{11}{13}\int \frac{1}{\sin^{\frac{11}{2}}(x)}dx-\frac{2\cos(x)}{13\sin^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{11}{13}\int \frac{1}{\sin(x)^{11/2}}dx-\frac{2\cos(x)}{13\sin^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\int\frac{1}{\sin^{\frac{7}{2}}(x)}dx-\frac{2\cos(x)}{9\sin^{\frac{9}{2}}(x)}\right)-\frac{2\cos(x)}{13\sin^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\int\frac{1}{\sin(x)^{7/2}}dx-\frac{2\cos(x)}{9\sin^{\frac{9}{2}}(x)}\right)-\frac{2\cos(x)}{13\sin^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \ 3116$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\int\frac{1}{\sin^{\frac{3}{2}}(x)}dx-\frac{2\cos(x)}{5\sin^{\frac{5}{2}}(x)}\right)-\frac{2\cos(x)}{9\sin^{\frac{9}{2}}(x)}\right)-\frac{2\cos(x)}{13\sin^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\int\frac{1}{\sin(x)^{3/2}}dx-\frac{2\cos(x)}{5\sin^{\frac{5}{2}}(x)}\right)-\frac{2\cos(x)}{9\sin^{\frac{9}{2}}(x)}\right)-\frac{2\cos(x)}{13\sin^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \ 3116$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\left(-\int\sqrt{\sin(x)}dx-\frac{2\cos(x)}{\sqrt{\sin(x)}}\right)-\frac{2\cos(x)}{5\sin^{\frac{5}{2}}(x)}\right)-\frac{2\cos(x)}{9\sin^{\frac{9}{2}}(x)}\right)-\frac{2\cos(x)}{13\sin^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\left(-\int\sqrt{\sin(x)}dx-\frac{2\cos(x)}{\sqrt{\sin(x)}}\right)-\frac{2\cos(x)}{5\sin^{\frac{5}{2}}(x)}\right)-\frac{2\cos(x)}{9\sin^{\frac{9}{2}}(x)}\right)-\frac{2\cos(x)}{13\sin^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sin^3(x)}}$$

$$\Big\downarrow \ 3119$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\left(2E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)-\frac{2\cos(x)}{\sqrt{\sin(x)}}\right)-\frac{2\cos(x)}{5\sin^{\frac{5}{2}}(x)}\right)-\frac{2\cos(x)}{9\sin^{\frac{9}{2}}(x)}\right)-\frac{2\cos(x)}{13\sin^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sin^3(x)}}$$

input
```
Int[(a*Sin[x]^3)^(-5/2),x]
```

output
```
(((11*((7*((3*(2*EllipticE[Pi/4 - x/2, 2] - (2*Cos[x])/Sqrt[Sin[x]]))/5 -
(2*Cos[x])/(5*Sin[x]^(5/2))))/9 - (2*Cos[x])/(9*Sin[x]^(9/2))))/13 - (2*Co
s[x])/(13*Sin[x]^(13/2)))*Sin[x]^(3/2))/(a^2*Sqrt[a*Sin[x]^3])
```

3.12. $\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}\,dx$

3.12.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.12.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.55

method	result
default	$-\frac{\left(231 \sin(x) \cos(x) \sqrt{-i(i-\cot(x)+\csc(x))}\, \sqrt{-i(i+\cot(x)-\csc(x))}\, \sqrt{i(\csc(x)-\cot(x))}\, F\left(\sqrt{-i(i-\cot(x)+\csc(x))}, \frac{\sqrt{2}}{2}\right)-462 \sin(x\right.}{}$

input
```
int(1/(a*sin(x)^3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1170/(a*sin(x)^3)^(1/2)/a^2*(231*sin(x)*cos(x)*(-I*(I-cot(x)+csc(x)))^(
1/2)*(-I*(I+cot(x)-csc(x)))^(1/2)*(I*(csc(x)-cot(x)))^(1/2)*EllipticF((-I*
(I-cot(x)+csc(x)))^(1/2),1/2*2^(1/2))-462*sin(x)*(-I*(I-cot(x)+csc(x)))^(1
/2)*(-I*(I+cot(x)-csc(x)))^(1/2)*(I*(csc(x)-cot(x)))^(1/2)*EllipticE((-I*(
I-cot(x)+csc(x)))^(1/2),1/2*2^(1/2))*cos(x)+231*(-I*(I-cot(x)+csc(x)))^(1/
2)*(-I*(I+cot(x)-csc(x)))^(1/2)*(I*(csc(x)-cot(x)))^(1/2)*EllipticF((-I*(I
-cot(x)+csc(x)))^(1/2),1/2*2^(1/2))*sin(x)-462*(-I*(I-cot(x)+csc(x)))^(1/2
)*(-I*(I+cot(x)-csc(x)))^(1/2)*(I*(csc(x)-cot(x)))^(1/2)*EllipticE((-I*(I-
cot(x)+csc(x)))^(1/2),1/2*2^(1/2))*sin(x)+231*sin(x)*2^(1/2)+77*cot(x)*2^(
1/2)+55*csc(x)^2*cot(x)*2^(1/2)+45*cot(x)*csc(x)^4*2^(1/2))*8^(1/2)
```

3.12.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.70

$$\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}\,dx =$$

$$-\frac{231\left(i\sqrt{2}\cos(x)^8-4i\sqrt{2}\cos(x)^6+6i\sqrt{2}\cos(x)^4-4i\sqrt{2}\cos(x)^2+i\sqrt{2}\right)\sqrt{-i\,a}\,\mathrm{weierstrassZeta}(4,0,\mathrm{we}}{}$$

input
```
integrate(1/(a*sin(x)^3)^(5/2),x, algorithm="fricas")
```

output
```
-1/585*(231*(I*sqrt(2)*cos(x)^8 - 4*I*sqrt(2)*cos(x)^6 + 6*I*sqrt(2)*cos(x
)^4 - 4*I*sqrt(2)*cos(x)^2 + I*sqrt(2))*sqrt(-I*a)*weierstrassZeta(4, 0, w
eierstrassPInverse(4, 0, cos(x) + I*sin(x))) + 231*(-I*sqrt(2)*cos(x)^8 +
4*I*sqrt(2)*cos(x)^6 - 6*I*sqrt(2)*cos(x)^4 + 4*I*sqrt(2)*cos(x)^2 - I*sqr
t(2))*sqrt(I*a)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(x) - I
*sin(x))) - 2*(231*cos(x)^7 - 770*cos(x)^5 + 902*cos(x)^3 - 408*cos(x))*sq
rt(-(a*cos(x)^2 - a)*sin(x)))/(a^3*cos(x)^8 - 4*a^3*cos(x)^6 + 6*a^3*cos(x
)^4 - 4*a^3*cos(x)^2 + a^3)
```

3.12.6 Sympy [F]

$$\int \frac{1}{(a\sin^3(x))^{5/2}}\,dx = \int \frac{1}{(a\sin^3(x))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)**3)**(5/2),x)
```

output
```
Integral((a*sin(x)**3)**(-5/2), x)
```

3.12.7 Maxima [F]

$$\int \frac{1}{(a\sin^3(x))^{5/2}}\,dx = \int \frac{1}{(a\sin(x)^3)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)^3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(x)^3)^(-5/2), x)
```

3.12.8 Giac [F]

$$\int \frac{1}{(a\sin^3(x))^{5/2}}\,dx = \int \frac{1}{(a\sin(x)^3)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)^3)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sin(x)^3)^(-5/2), x)
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sin\left(x\right)^3\right)^{5/2}}\,dx$$

input
```
int(1/(a*sin(x)^3)^(5/2),x)
```

output
```
int(1/(a*sin(x)^3)^(5/2), x)
```

3.13 $\int \left(a\sin^4(x)\right)^{5/2}\,dx$

3.13.1 Optimal result

Integrand size = 10, antiderivative size = 132

$$\begin{aligned}\int \left(a\sin^4(x)\right)^{5/2}\,dx = &-\frac{63}{256}a^2\cot(x)\sqrt{a\sin^4(x)}+\frac{63}{256}a^2x\csc^2(x)\sqrt{a\sin^4(x)}\\ &-\frac{21}{128}a^2\cos(x)\sin(x)\sqrt{a\sin^4(x)}-\frac{21}{160}a^2\cos(x)\sin^3(x)\sqrt{a\sin^4(x)}\\ &-\frac{9}{80}a^2\cos(x)\sin^5(x)\sqrt{a\sin^4(x)}-\frac{1}{10}a^2\cos(x)\sin^7(x)\sqrt{a\sin^4(x)}\end{aligned}$$

output
```
-63/256*a^2*cot(x)*(a*sin(x)^4)^(1/2)+63/256*a^2*x*csc(x)^2*(a*sin(x)^4)^(
1/2)-21/128*a^2*cos(x)*sin(x)*(a*sin(x)^4)^(1/2)-21/160*a^2*cos(x)*sin(x)^
3*(a*sin(x)^4)^(1/2)-9/80*a^2*cos(x)*sin(x)^5*(a*sin(x)^4)^(1/2)-1/10*a^2*
cos(x)*sin(x)^7*(a*sin(x)^4)^(1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.40

$$\int \left(a\sin^4(x)\right)^{5/2}\,dx = \frac{a\csc^6(x)\left(a\sin^4(x)\right)^{3/2}(2520x-2100\sin(2x)+600\sin(4x)-150\sin(6x)+25\sin(8x}{10240}$$

input
```
Integrate[(a*Sin[x]^4)^(5/2),x]
```

output
```
(a*Csc[x]^6*(a*Sin[x]^4)^(3/2)*(2520*x - 2100*Sin[2*x] + 600*Sin[4*x] - 15
0*Sin[6*x] + 25*Sin[8*x] - 2*Sin[10*x]))/10240
```

3.13.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.70, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sin^4(x)\right)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin(x)^4\right)^{5/2}dx$$

$$\Big\downarrow \text{3686}$$

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\int \sin^{10}(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\int \sin(x)^{10}dx$$

$$\Big\downarrow \text{3115}$$

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{9}{10}\int \sin^8(x)dx-\frac{1}{10}\sin^9(x)\cos(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{9}{10}\int \sin(x)^8dx-\frac{1}{10}\sin^9(x)\cos(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\int \sin^6(x)dx-\frac{1}{8}\sin^7(x)\cos(x)\right)-\frac{1}{10}\sin^9(x)\cos(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\int \sin(x)^6dx-\frac{1}{8}\sin^7(x)\cos(x)\right)-\frac{1}{10}\sin^9(x)\cos(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int \sin^4(x)dx-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\cos(x)\right)-\frac{1}{10}\sin^9(x)\cos(x)\right)$$

$$\Big\downarrow \; 3042$$

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int\sin(x)^4dx-\frac{1}{6}\sin^5(x)\cos(x)\right)-\frac{1}{8}\sin^7(x)\cos(x)\right)-\frac{1}{10}\sin^9(x)\cos(x)\right)$$

$$\Big\downarrow \; 3115$$

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\Bigg(\frac{9}{10}\Bigg(\frac{7}{8}\Bigg(\frac{5}{6}\Bigg(\frac{3}{4}\int\sin^2(x)dx-\frac{1}{4}\sin^3(x)\cos(x)\Bigg)-\frac{1}{6}\sin^5(x)\cos(x)\Bigg)-\frac{1}{8}\sin^7(x)\cos(x)\Bigg)-$$

$$\Big\downarrow \; 3042$$

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\Bigg(\frac{9}{10}\Bigg(\frac{7}{8}\Bigg(\frac{5}{6}\Bigg(\frac{3}{4}\int\sin(x)^2dx-\frac{1}{4}\sin^3(x)\cos(x)\Bigg)-\frac{1}{6}\sin^5(x)\cos(x)\Bigg)-\frac{1}{8}\sin^7(x)\cos(x)\Bigg)-$$

$$\Big\downarrow \; 3115$$

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\Bigg(\frac{9}{10}\Bigg(\frac{7}{8}\Bigg(\frac{5}{6}\Bigg(\frac{3}{4}\Bigg(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\Bigg)-\frac{1}{4}\sin^3(x)\cos(x)\Bigg)-\frac{1}{6}\sin^5(x)\cos(x)\Bigg)-\frac{1}{8}\sin^7($$

$$\Big\downarrow \; 24$$

$$a^2\csc^2(x)\sqrt{a\sin^4(x)}\Bigg(\frac{9}{10}\Bigg(\frac{7}{8}\Bigg(\frac{5}{6}\Bigg(\frac{3}{4}\Bigg(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\Bigg)-\frac{1}{4}\sin^3(x)\cos(x)\Bigg)-\frac{1}{6}\sin^5(x)\cos(x)\Bigg)-\frac{1}{8}\sin^7(x)\,c$$

input
```
Int[(a*Sin[x]^4)^(5/2),x]
```

output
```
a^2*Csc[x]^2*Sqrt[a*Sin[x]^4]*(-1/10*(Cos[x]*Sin[x]^9) + (9*(-1/8*(Cos[x]*
Sin[x]^7) + (7*(-1/6*(Cos[x]*Sin[x]^5) + (5*(-1/4*(Cos[x]*Sin[x]^3) + (3*(
x/2 - (Cos[x]*Sin[x])/2))/4))/6))/8))/10)
```

3.13.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.13.4 Maple [A] (verified)

Time = 10.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.46

method	result
default	$-\frac{a^2\sqrt{a(\sin^4(x))}\,(128(\cos^8(x))\cot(x)-656(\cos^6(x))\cot(x)+1368(\cos^4(x))\cot(x)-1490(\cos^2(x))\cot(x)+965\cot(x)-315(\csc^2(x)}{5120}$
risch	$-\frac{63a^2e^{2ix}\sqrt{a(e^{2ix}-1)^4e^{-4ix}}\,x}{256(e^{2ix}-1)^2}-\frac{ia^2e^{12ix}\sqrt{a(e^{2ix}-1)^4e^{-4ix}}}{10240(e^{2ix}-1)^2}+\frac{5ia^2e^{10ix}\sqrt{a(e^{2ix}-1)^4e^{-4ix}}}{4096(e^{2ix}-1)^2}-\frac{105ia^2e^{4ix}\sqrt{a(e^{2ix}-1)^4e^{-4}}}{1024(e^{2ix}-1)^2}$

input
```
int((a*sin(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5120*a^2*(a*sin(x)^4)^(1/2)*(128*cos(x)^8*cot(x)-656*cos(x)^6*cot(x)+13
68*cos(x)^4*cot(x)-1490*cos(x)^2*cot(x)+965*cot(x)-315*csc(x)^2*x)*16^(1/2
)
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.62

$$\int \left(a\sin^4(x)\right)^{5/2}\,dx =$$

$$-\frac{\sqrt{a\cos(x)^4-2\,a\cos(x)^2+a}\,\big(315\,a^2x-\big(128\,a^2\cos(x)^9-656\,a^2\cos(x)^7+1368\,a^2\cos(x)^5-1490\,a^2\cos}{1280\,\left(\cos(x)^2-1\right)}$$

input
```
integrate((a*sin(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
-1/1280*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(315*a^2*x - (128*a^2*cos(x)^9
 - 656*a^2*cos(x)^7 + 1368*a^2*cos(x)^5 - 1490*a^2*cos(x)^3 + 965*a^2*cos(
x))*sin(x))/(cos(x)^2 - 1)
```

3.13.6 Sympy [F]

$$\int \left(a\sin^4(x)\right)^{5/2}\,dx=\int \left(a\sin^4\left(x\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*sin(x)**4)**(5/2),x)
```

output
```
Integral((a*sin(x)**4)**(5/2), x)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.64

$$\int \left(a\sin^4(x)\right)^{5/2}\,dx=\frac{63}{256}\,a^{\frac{5}{2}}x-\frac{965\,a^{\frac{5}{2}}\tan\left(x\right)^9+2370\,a^{\frac{5}{2}}\tan\left(x\right)^7+2688\,a^{\frac{5}{2}}\tan\left(x\right)^5+1470\,a^{\frac{5}{2}}\tan\left(x\right)^3+315\,a^{\frac{5}{2}}\tan\left(x\right)}{1280\,\left(\tan\left(x\right)^{10}+5\,\tan\left(x\right)^8+10\,\tan\left(x\right)^6+10\,\tan\left(x\right)^4+5\,\tan\left(x\right)^2+1\right)}$$

input
```
integrate((a*sin(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
63/256*a^(5/2)*x - 1/1280*(965*a^(5/2)*tan(x)^9 + 2370*a^(5/2)*tan(x)^7 +
2688*a^(5/2)*tan(x)^5 + 1470*a^(5/2)*tan(x)^3 + 315*a^(5/2)*tan(x))/(tan(x
)^10 + 5*tan(x)^8 + 10*tan(x)^6 + 10*tan(x)^4 + 5*tan(x)^2 + 1)
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.43

$$\int \left(a\sin^4(x)\right)^{5/2}\,dx = \frac{1}{10240}\left(2520\,a^2x - 2\,a^2\sin\left(10\,x\right) + 25\,a^2\sin\left(8\,x\right) - 150\,a^2\sin\left(6\,x\right) + 600\,a^2\sin\left(4\,x\right)\right.$$

input
```
integrate((a*sin(x)^4)^(5/2),x, algorithm="giac")
```

output
```
1/10240*(2520*a^2*x - 2*a^2*sin(10*x) + 25*a^2*sin(8*x) - 150*a^2*sin(6*x)
 + 600*a^2*sin(4*x) - 2100*a^2*sin(2*x))*sqrt(a)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\sin^4(x)\right)^{5/2}\,dx = \int \left(a\sin\left(x\right)^4\right)^{5/2}\,dx$$

input
```
int((a*sin(x)^4)^(5/2),x)
```

output
```
int((a*sin(x)^4)^(5/2), x)
```

3.14 $\int (a\sin^4(x))^{3/2}\,dx$

3.14.1 Optimal result

Integrand size = 10, antiderivative size = 78

$$\int \left(a\sin^4(x)\right)^{3/2}\,dx = -\frac{5}{16}a\cot(x)\sqrt{a\sin^4(x)} + \frac{5}{16}ax\csc^2(x)\sqrt{a\sin^4(x)} - \frac{5}{24}a\cos(x)\sin(x)\sqrt{a\sin^4(x)} - \frac{1}{6}a\cos(x)\sin^3(x)\sqrt{a\sin^4(x)}$$

output
```
-5/16*a*cot(x)*(a*sin(x)^4)^(1/2)+5/16*a*x*csc(x)^2*(a*sin(x)^4)^(1/2)-5/2
4*a*cos(x)*sin(x)*(a*sin(x)^4)^(1/2)-1/6*a*cos(x)*sin(x)^3*(a*sin(x)^4)^(1
/2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \left(a\sin^4(x)\right)^{3/2}\,dx = -\frac{1}{192}\csc^6(x)\left(a\sin^4(x)\right)^{3/2}(-60x + 45\sin(2x) - 9\sin(4x) + \sin(6x))$$

input
```
Integrate[(a*Sin[x]^4)^(3/2),x]
```

output
```
-1/192*(Csc[x]^6*(a*Sin[x]^4)^(3/2)*(-60*x + 45*Sin[2*x] - 9*Sin[4*x] + Si
n[6*x]))
```

3.14.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sin^4(x)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin(x)^4\right)^{3/2}dx$$

$$\Big\downarrow \text{3686}$$

$$a\csc^2(x)\sqrt{a\sin^4(x)}\int \sin^6(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a\csc^2(x)\sqrt{a\sin^4(x)}\int \sin(x)^6dx$$

$$\Big\downarrow \text{3115}$$

$$a\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{5}{6}\int \sin^4(x)dx-\frac{1}{6}\sin^5(x)\cos(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{5}{6}\int \sin(x)^4dx-\frac{1}{6}\sin^5(x)\cos(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin^2(x)dx-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin(x)^2dx-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)$$

$$\Big\downarrow \text{24}$$

$$a\csc^2(x)\sqrt{a\sin^4(x)}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin^3(x)\cos(x)\right)-\frac{1}{6}\sin^5(x)\cos(x)\right)$$

input
```
Int[(a*Sin[x]^4)^(3/2),x]
```

output
```
a*Csc[x]^2*Sqrt[a*Sin[x]^4]*(-1/6*(Cos[x]*Sin[x]^5) + (5*(-1/4*(Cos[x]*Sin
[x]^3) + (3*(x/2 - (Cos[x]*Sin[x])/2))/4))/6)
```

3.14.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.14.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.55

method	result
default	$-\frac{a\sqrt{a\left(\sin^4(x)\right)}\,\left(8\left(\cos^4(x)\right)\cot(x)-26\left(\cos^2(x)\right)\cot(x)+33\cot(x)-15\left(\csc^2(x)\right)x\right)\sqrt{16}}{192}$
risch	$-\frac{5a\,e^{2ix}\sqrt{a(e^{2ix}-1)^4e^{-4ix}}\,x}{16(e^{2ix}-1)^2}-\frac{ia\,e^{8ix}\sqrt{a(e^{2ix}-1)^4e^{-4ix}}}{384(e^{2ix}-1)^2}+\frac{3ia\,e^{6ix}\sqrt{a(e^{2ix}-1)^4e^{-4ix}}}{128(e^{2ix}-1)^2}-\frac{15ia\,e^{4ix}\sqrt{a(e^{2ix}-1)^4e^{-4ix}}}{128(e^{2ix}-1)^2}+$ 1

input
```
int((a*sin(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*a*(a*sin(x)^4)^(1/2)*(8*cos(x)^4*cot(x)-26*cos(x)^2*cot(x)+33*cot(x
)-15*csc(x)^2*x)*16^(1/2)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.72

$$\int \left(a\sin^4(x)\right)^{3/2} dx = -\frac{\sqrt{a\cos\left(x\right)^4-2\,a\cos\left(x\right)^2+a}\left(15\,ax-\left(8\,a\cos\left(x\right)^5-26\,a\cos\left(x\right)^3+33\,a\cos\left(x\right)\right)\sin\left(x\right)\right)}{48\left(\cos\left(x\right)^2-1\right)}$$

input
```
integrate((a*sin(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
-1/48*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(15*a*x - (8*a*cos(x)^5 - 26*a*c
os(x)^3 + 33*a*cos(x))*sin(x))/(cos(x)^2 - 1)
```

3.14.6 Sympy [F]

$$\int \left(a\sin^4(x)\right)^{3/2} dx = \int \left(a\sin^4\left(x\right)\right)^{\frac{3}{2}}\, dx$$

input
```
integrate((a*sin(x)**4)**(3/2),x)
```

output
```
Integral((a*sin(x)**4)**(3/2), x)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \left(a\sin^4(x)\right)^{3/2} dx = \frac{5}{16}a^{\frac{3}{2}}x - \frac{33\,a^{\frac{3}{2}}\tan(x)^5 + 40\,a^{\frac{3}{2}}\tan(x)^3 + 15\,a^{\frac{3}{2}}\tan(x)}{48\left(\tan(x)^6 + 3\tan(x)^4 + 3\tan(x)^2 + 1\right)}$$

input
```
integrate((a*sin(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
5/16*a^(3/2)*x - 1/48*(33*a^(3/2)*tan(x)^5 + 40*a^(3/2)*tan(x)^3 + 15*a^(3
/2)*tan(x))/(tan(x)^6 + 3*tan(x)^4 + 3*tan(x)^2 + 1)
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.35

$$\int \left(a\sin^4(x)\right)^{3/2} dx = \frac{1}{192}a^{\frac{3}{2}}(60\,x - \sin(6\,x) + 9\sin(4\,x) - 45\sin(2\,x))$$

input
```
integrate((a*sin(x)^4)^(3/2),x, algorithm="giac")
```

output
```
1/192*a^(3/2)*(60*x - sin(6*x) + 9*sin(4*x) - 45*sin(2*x))
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\sin^4(x)\right)^{3/2} dx = \int \left(a\sin(x)^4\right)^{3/2} dx$$

input
```
int((a*sin(x)^4)^(3/2),x)
```

output
```
int((a*sin(x)^4)^(3/2), x)
```

3.15 $\int \sqrt{a\sin^4(x)}\,dx$

3.15.1 Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \sqrt{a\sin^4(x)}\,dx=-\frac{1}{2}\cot(x)\sqrt{a\sin^4(x)}+\frac{1}{2}x\csc^2(x)\sqrt{a\sin^4(x)}$$

output
```
-1/2*cot(x)*(a*sin(x)^4)^(1/2)+1/2*x*csc(x)^2*(a*sin(x)^4)^(1/2)
```

3.15.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \sqrt{a\sin^4(x)}\,dx=\frac{1}{2}\csc(x)(-\cos(x)+x\csc(x))\sqrt{a\sin^4(x)}$$

input
```
Integrate[Sqrt[a*Sin[x]^4],x]
```

output
```
(Csc[x]*(-Cos[x] + x*Csc[x])*Sqrt[a*Sin[x]^4])/2
```

3.15.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3686, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \sin^4(x)} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a \sin(x)^4}dx$$

$$\Big\downarrow \text{3686}$$

$$\csc^2(x)\sqrt{a \sin^4(x)} \int \sin^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc^2(x)\sqrt{a \sin^4(x)} \int \sin(x)^2dx$$

$$\Big\downarrow \text{3115}$$

$$\csc^2(x)\sqrt{a \sin^4(x)}\left(\frac{\int 1dx}{2} - \frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{24}$$

$$\csc^2(x)\sqrt{a \sin^4(x)}\left(\frac{x}{2} - \frac{1}{2}\sin(x)\cos(x)\right)$$

input
```
Int[Sqrt[a*Sin[x]^4],x]
```

output
```
Csc[x]^2*Sqrt[a*Sin[x]^4]*(x/2 - (Cos[x]*Sin[x])/2)
```

3.15.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.15.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

method	result	size
default	$-\frac{\sqrt{a\left(\sin^{4}\left(x\right)\right)}\,\left(\cot\left(x\right)-\left(\csc^{2}\left(x\right)\right)x\right)\sqrt{16}}{8}$	24
risch	$-\frac{\sqrt{a\left({\mathrm e}^{2 i x}-1\right)^{4}{\mathrm e}^{-4 i x}}\,{\mathrm e}^{2 i x}x}{2\left({\mathrm e}^{2 i x}-1\right)^{2}}-\frac{i\sqrt{a\left({\mathrm e}^{2 i x}-1\right)^{4}{\mathrm e}^{-4 i x}}\,{\mathrm e}^{4 i x}}{8\left({\mathrm e}^{2 i x}-1\right)^{2}}+\frac{i\sqrt{a\left({\mathrm e}^{2 i x}-1\right)^{4}{\mathrm e}^{-4 i x}}}{8\left({\mathrm e}^{2 i x}-1\right)^{2}}$	102

input
```
int((a*sin(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(a*sin(x)^4)^(1/2)*(cot(x)-csc(x)^2*x)*16^(1/2)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \sqrt{a\sin^4(x)}\,dx = \frac{\sqrt{a\cos(x)^4 - 2\,a\cos(x)^2 + a}(\cos(x)\sin(x) - x)}{2\left(\cos(x)^2 - 1\right)}$$

input
```
integrate((a*sin(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(cos(x)*sin(x) - x)/(cos(x)^2 - 1)
```

3.15.6 Sympy [F]

$$\int \sqrt{a\sin^4(x)}\,dx = \int \sqrt{a\sin^4(x)}\,dx$$

input
```
integrate((a*sin(x)**4)**(1/2),x)
```

output
```
Integral(sqrt(a*sin(x)**4), x)
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \sqrt{a\sin^4(x)}\,dx = \frac{1}{2}\sqrt{a}x - \frac{\sqrt{a}\tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

input
```
integrate((a*sin(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*x - 1/2*sqrt(a)*tan(x)/(tan(x)^2 + 1)
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.42

$$\int \sqrt{a\sin^4(x)}\,dx = \frac{1}{4}\sqrt{a}(2x - \sin(2x))$$

input
```
integrate((a*sin(x)^4)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(a)*(2*x - sin(2*x))
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\sin^4(x)}\,dx = \int \sqrt{a\sin(x)^4}\,dx$$

input
```
int((a*sin(x)^4)^(1/2),x)
```

output
```
int((a*sin(x)^4)^(1/2), x)
```

3.16 $\int \frac{1}{\sqrt{a \sin^4(x)}}\, dx$

3.16.1 Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \frac{1}{\sqrt{a \sin^4(x)}}\, dx = -\frac{\cos(x)\sin(x)}{\sqrt{a \sin^4(x)}}$$

output
```
-cos(x)*sin(x)/(a*sin(x)^4)^(1/2)
```

3.16.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a \sin^4(x)}}\, dx = -\frac{\cos(x)\sin(x)}{\sqrt{a \sin^4(x)}}$$

input
```
Integrate[1/Sqrt[a*Sin[x]^4],x]
```

output
```
-((Cos[x]*Sin[x])/Sqrt[a*Sin[x]^4])
```

3.16.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3686, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a \sin^4(x)}} \, dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\sqrt{a \sin(x)^4}} dx$$
$$\Big\downarrow \ 3686$$
$$\frac{\sin^2(x) \int \csc^2(x) dx}{\sqrt{a \sin^4(x)}}$$
$$\Big\downarrow \ 3042$$
$$\frac{\sin^2(x) \int \csc(x)^2 dx}{\sqrt{a \sin^4(x)}}$$
$$\Big\downarrow \ 4254$$
$$-\frac{\sin^2(x) \int 1 d \cot(x)}{\sqrt{a \sin^4(x)}}$$
$$\Big\downarrow \ 24$$
$$-\frac{\sin(x) \cos(x)}{\sqrt{a \sin^4(x)}}$$

input
```
Int[1/Sqrt[a*Sin[x]^4],x]
```

output
```
-((Cos[x]*Sin[x])/Sqrt[a*Sin[x]^4])
```

3.16.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.16.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

method	result	size
default	$-\frac{\cos(x)\sin(x)\sqrt{16}}{4\sqrt{a\left(\sin^4(x)\right)}}$	18
risch	$\frac{2i\left(1-\mathrm{e}^{-2ix}\right)}{\sqrt{a\left(\mathrm{e}^{2ix}-1\right)^4\mathrm{e}^{-4ix}}}$	31

input
```
int(1/(a*sin(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(x)*sin(x)/(a*sin(x)^4)^(1/2)*16^(1/2)
```

3.16.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(14) = 28.

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx = \frac{\sqrt{a\cos{(x)}^4 - 2\,a\cos{(x)}^2 + a}\cos{(x)}}{\left(a\cos{(x)}^2 - a\right)\sin{(x)}}$$

input
```
integrate(1/(a*sin(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*cos(x)/((a*cos(x)^2 - a)*sin(x))
```

3.16.6 Sympy [F]

$$\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx = \int \frac{1}{\sqrt{a\sin^4{(x)}}}\,dx$$

input
```
integrate(1/(a*sin(x)**4)**(1/2),x)
```

output
```
Integral(1/sqrt(a*sin(x)**4), x)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx = -\frac{1}{\sqrt{a}\tan{(x)}}$$

input
```
integrate(1/(a*sin(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/(sqrt(a)*tan(x))
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx = -\frac{1}{\sqrt{a}\tan(x)}$$

input
```
integrate(1/(a*sin(x)^4)^(1/2),x, algorithm="giac")
```

output
```
-1/(sqrt(a)*tan(x))
```

3.16.9 Mupad [B] (verification not implemented)

Time = 13.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx = -\frac{\cot(x)}{\sqrt{a}}$$

input
```
int(1/(a*sin(x)^4)^(1/2),x)
```

output
```
-cot(x)/a^(1/2)
```

3.17 $\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx$

3.17.1 Optimal result

Integrand size = 10, antiderivative size = 68

$$\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx = -\frac{2\cos^2(x)\cot(x)}{3a\sqrt{a\sin^4(x)}} - \frac{\cos^2(x)\cot^3(x)}{5a\sqrt{a\sin^4(x)}} - \frac{\cos(x)\sin(x)}{a\sqrt{a\sin^4(x)}}$$

output
```
-2/3*cos(x)^2*cot(x)/a/(a*sin(x)^4)^(1/2)-1/5*cos(x)^2*cot(x)^3/a/(a*sin(x
)^4)^(1/2)-cos(x)*sin(x)/a/(a*sin(x)^4)^(1/2)
```

3.17.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.50

$$\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx = -\frac{\cos(x)\left(8+4\csc^2(x)+3\csc^4(x)\right)\sin^5(x)}{15\left(a\sin^4(x)\right)^{3/2}}$$

input
```
Integrate[(a*Sin[x]^4)^(-3/2),x]
```

output
```
-1/15*(Cos[x]*(8 + 4*Csc[x]^2 + 3*Csc[x]^4)*Sin[x]^5)/(a*Sin[x]^4)^(3/2)
```

3.17.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3686, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin(x)^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sin^2(x)\int \csc^6(x)dx}{a\sqrt{a\sin^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^2(x)\int \csc(x)^6dx}{a\sqrt{a\sin^4(x)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\sin^2(x)\int \left(\cot^4(x)+2\cot^2(x)+1\right)d\cot(x)}{a\sqrt{a\sin^4(x)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin^2(x)\left(\frac{\cot^5(x)}{5}+\frac{2\cot^3(x)}{3}+\cot(x)\right)}{a\sqrt{a\sin^4(x)}}$$

input
```
Int[(a*Sin[x]^4)^(-3/2),x]
```

output
```
-(((Cot[x] + (2*Cot[x]^3)/3 + Cot[x]^5/5)*Sin[x]^2)/(a*Sqrt[a*Sin[x]^4]))
```

3.17.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.17.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{\left(\csc^2(x)\right)\cot(x)\left(8\left(\cos^4(x)\right)-20\left(\cos^2(x)\right)+15\right)\sqrt{16}}{60\sqrt{a\left(\sin^4(x)\right)}\,a}$	37
risch	$\frac{16i(-5+11\cos(2x)+9i\sin(2x))}{15a(\mathrm{e}^{2ix}-1)^3\sqrt{a(\mathrm{e}^{2ix}-1)^4\mathrm{e}^{-4ix}}}$	49

input
```
int(1/(a*sin(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*csc(x)^2*cot(x)*(8*cos(x)^4-20*cos(x)^2+15)/(a*sin(x)^4)^(1/2)/a*16^
(1/2)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx = \frac{\sqrt{a\cos(x)^4 - 2\,a\cos(x)^2 + a}\left(8\,\cos(x)^5 - 20\,\cos(x)^3 + 15\,\cos(x)\right)}{15\left(a^2\cos(x)^6 - 3\,a^2\cos(x)^4 + 3\,a^2\cos(x)^2 - a^2\right)\sin(x)}$$

input
```
integrate(1/(a*sin(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
1/15*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(8*cos(x)^5 - 20*cos(x)^3 + 15*co
s(x))/((a^2*cos(x)^6 - 3*a^2*cos(x)^4 + 3*a^2*cos(x)^2 - a^2)*sin(x))
```

3.17.6 Sympy [F]

$$\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\sin^4(x)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)**4)**(3/2),x)
```

output
```
Integral((a*sin(x)**4)**(-3/2), x)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.34

$$\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx = -\frac{15\,\tan(x)^4 + 10\,\tan(x)^2 + 3}{15\,a^{\frac{3}{2}}\tan(x)^5}$$

input
```
integrate(1/(a*sin(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
-1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/(a^(3/2)*tan(x)^5)
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.34

$$\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx = -\frac{15\,\tan\left(x\right)^4+10\,\tan\left(x\right)^2+3}{15\,a^{\frac{3}{2}}\,\tan\left(x\right)^5}$$

input
```
integrate(1/(a*sin(x)^4)^(3/2),x, algorithm="giac")
```

output
```
-1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/(a^(3/2)*tan(x)^5)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 13.79 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx = \frac{\frac{8\mathrm{i}}{15\,a^{3/2}}-\frac{4\,\left(2\sin(2\,x)^3-9\,\sin(2\,x)+3\,\sin(4\,x)+2\mathrm{i}\right)}{15\,a^{3/2}}}{\left(\cos\left(2\,x\right)-1\right)^3}$$

input
```
int(1/(a*sin(x)^4)^(3/2),x)
```

output
```
(8i/(15*a^(3/2)) - (4*(3*sin(4*x) - 9*sin(2*x) + 2*sin(2*x)^3 + 2i))/(15*a
^(3/2)))/(cos(2*x) - 1)^3
```

3.18 $\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx$

3.18.1 Optimal result

Integrand size = 10, antiderivative size = 118

$$\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx = -\frac{4\cos^2(x)\cot(x)}{3a^2\sqrt{a\sin^4(x)}} - \frac{6\cos^2(x)\cot^3(x)}{5a^2\sqrt{a\sin^4(x)}} - \frac{4\cos^2(x)\cot^5(x)}{7a^2\sqrt{a\sin^4(x)}} - \frac{\cos^2(x)\cot^7(x)}{9a^2\sqrt{a\sin^4(x)}} - \frac{\cos(x)\sin(x)}{a^2\sqrt{a\sin^4(x)}}$$

output
```
-4/3*cos(x)^2*cot(x)/a^2/(a*sin(x)^4)^(1/2)-6/5*cos(x)^2*cot(x)^3/a^2/(a*s
in(x)^4)^(1/2)-4/7*cos(x)^2*cot(x)^5/a^2/(a*sin(x)^4)^(1/2)-1/9*cos(x)^2*c
ot(x)^7/a^2/(a*sin(x)^4)^(1/2)-cos(x)*sin(x)/a^2/(a*sin(x)^4)^(1/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

$$\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx = -\frac{\cos(x)\left(128+64\csc^2(x)+48\csc^4(x)+40\csc^6(x)+35\csc^8(x)\right)\sin(x)}{315a^2\sqrt{a\sin^4(x)}}$$

input
```
Integrate[(a*Sin[x]^4)^(-5/2),x]
```

output
```
-1/315*(Cos[x]*(128 + 64*Csc[x]^2 + 48*Csc[x]^4 + 40*Csc[x]^6 + 35*Csc[x]^
8)*Sin[x])/(a^2*Sqrt[a*Sin[x]^4])
```

3.18.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.46, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3686, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin(x)^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sin^2(x)\int \csc^{10}(x)dx}{a^2\sqrt{a\sin^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^2(x)\int \csc(x)^{10}dx}{a^2\sqrt{a\sin^4(x)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\sin^2(x)\int \left(\cot^8(x)+4\cot^6(x)+6\cot^4(x)+4\cot^2(x)+1\right)d\cot(x)}{a^2\sqrt{a\sin^4(x)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin^2(x)\left(\frac{\cot^9(x)}{9}+\frac{4\cot^7(x)}{7}+\frac{6\cot^5(x)}{5}+\frac{4\cot^3(x)}{3}+\cot(x)\right)}{a^2\sqrt{a\sin^4(x)}}$$

input
```
Int[(a*Sin[x]^4)^(-5/2),x]
```

output
```
-(((Cot[x] + (4*Cot[x]^3)/3 + (6*Cot[x]^5)/5 + (4*Cot[x]^7)/7 + Cot[x]^9/9
)*Sin[x]^2)/(a^2*Sqrt[a*Sin[x]^4]))
```

3.18.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.18.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.42

method	result	size
default	$-\frac{\cot(x)\left(\csc^{6}(x)\right)\left(128\left(\cos^{8}(x)\right)-576\left(\cos^{6}(x)\right)+1008\left(\cos^{4}(x)\right)-840\left(\cos^{2}(x)\right)+315\right)\sqrt{16}}{1260\sqrt{a\left(\sin^{4}(x)\right)}\,a^{2}}$	49
risch	$\frac{256i\left(126\,{\rm e}^{6ix}-84\,{\rm e}^{4ix}-9+37\cos(2x)+35i\sin(2x)\right)}{315a^{2}\left({\rm e}^{2ix}-1\right)^{7}\sqrt{a\left({\rm e}^{2ix}-1\right)^{4}{\rm e}^{-4ix}}}$	63

input
```
int(1/(a*sin(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1260*cot(x)*csc(x)^6*(128*cos(x)^8-576*cos(x)^6+1008*cos(x)^4-840*cos(x
)^2+315)/(a*sin(x)^4)^(1/2)/a^2*16^(1/2)
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx = \frac{\left(128\,\cos\left(x\right)^9 - 576\,\cos\left(x\right)^7 + 1008\,\cos\left(x\right)^5 - 840\,\cos\left(x\right)^3 + 315\,\cos\left(x\right)\right)\sqrt{a\cos\left(x\right)}}{315\,\left(a^3\cos\left(x\right)^{10} - 5\,a^3\cos\left(x\right)^8 + 10\,a^3\cos\left(x\right)^6 - 10\,a^3\cos\left(x\right)^4 + 5\,a^3\cos\left(x\right)^2\right.}$$

input
```
integrate(1/(a*sin(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
1/315*(128*cos(x)^9 - 576*cos(x)^7 + 1008*cos(x)^5 - 840*cos(x)^3 + 315*co
s(x))*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)/((a^3*cos(x)^10 - 5*a^3*cos(x)^8
 + 10*a^3*cos(x)^6 - 10*a^3*cos(x)^4 + 5*a^3*cos(x)^2 - a^3)*sin(x))
```

3.18.6 Sympy [F]

$$\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sin^4\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sin(x)**4)**(5/2),x)
```

output
```
Integral((a*sin(x)**4)**(-5/2), x)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.30

$$\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx = -\frac{315\,\tan\left(x\right)^8 + 420\,\tan\left(x\right)^6 + 378\,\tan\left(x\right)^4 + 180\,\tan\left(x\right)^2 + 35}{315\,a^{\frac{5}{2}}\tan\left(x\right)^9}$$

input
```
integrate(1/(a*sin(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
-1/315*(315*tan(x)^8 + 420*tan(x)^6 + 378*tan(x)^4 + 180*tan(x)^2 + 35)/(a
^(5/2)*tan(x)^9)
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.30

$$\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx = -\frac{315\,\tan\left(x\right)^{8}+420\,\tan\left(x\right)^{6}+378\,\tan\left(x\right)^{4}+180\,\tan\left(x\right)^{2}+35}{315\,a^{\frac{5}{2}}\,\tan\left(x\right)^{9}}$$

input
```
integrate(1/(a*sin(x)^4)^(5/2),x, algorithm="giac")
```

output
```
-1/315*(315*tan(x)^8 + 420*tan(x)^6 + 378*tan(x)^4 + 180*tan(x)^2 + 35)/(a
^(5/2)*tan(x)^9)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 15.91 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx = \frac{256\left(\mathrm{e}^{x\,46\mathrm{i}}\,1\mathrm{i}-\mathrm{e}^{x\,48\mathrm{i}}\,9\mathrm{i}+\mathrm{e}^{x\,50\mathrm{i}}\,36\mathrm{i}-\mathrm{e}^{x\,52\mathrm{i}}\,84\mathrm{i}+\mathrm{e}^{x\,54\mathrm{i}}\,126\mathrm{i}\right)}{315\,a^{5/2}\,\left(\mathrm{e}^{x\,46\mathrm{i}}-9\,\mathrm{e}^{x\,48\mathrm{i}}+36\,\mathrm{e}^{x\,50\mathrm{i}}-84\,\mathrm{e}^{x\,52\mathrm{i}}+126\,\mathrm{e}^{x\,54\mathrm{i}}-126\,\mathrm{e}^{x\,56\mathrm{i}}+84\,\mathrm{e}^{x\,58\mathrm{i}}-36\,\mathrm{e}^{x}\cdots}$$

input
```
int(1/(a*sin(x)^4)^(5/2),x)
```

output
```
(256*(exp(x*46i)*1i - exp(x*48i)*9i + exp(x*50i)*36i - exp(x*52i)*84i + ex
p(x*54i)*126i))/(315*a^(5/2)*(exp(x*46i) - 9*exp(x*48i) + 36*exp(x*50i) -
84*exp(x*52i) + 126*exp(x*54i) - 126*exp(x*56i) + 84*exp(x*58i) - 36*exp(x
*60i) + 9*exp(x*62i) - exp(x*64i)))
```

3.19 $\int (c\sin^m(a+bx))^{5/2}\,dx$

3.19.1 Optimal result

Integrand size = 14, antiderivative size = 89

$$\int (c\sin^m(a+bx))^{5/2}\,dx = \frac{2c^2\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2+5m),\frac{1}{4}(6+5m),\sin^2(a+bx)\right)\sin^{1+2m}(a+bx)}{b(2+5m)\sqrt{\cos^2(a+bx)}}$$

output
```
2*c^2*cos(b*x+a)*hypergeom([1/2, 1/2+5/4*m],[3/2+5/4*m],sin(b*x+a)^2)*sin(
b*x+a)^(1+2*m)*(c*sin(b*x+a)^m)^(1/2)/b/(2+5*m)/(cos(b*x+a)^2)^(1/2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int (c\sin^m(a+bx))^{5/2}\,dx = \frac{2\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2+5m),\frac{1}{4}(6+5m),\sin^2(a+bx)\right)(c\sin^m(a+bx))}{b(2+5m)}$$

input
```
Integrate[(c*Sin[a + b*x]^m)^(5/2),x]
```

output
```
(2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 + 5*m)/4, (6 + 5*m)/4, S
in[a + b*x]^2]*(c*Sin[a + b*x]^m)^(5/2)*Tan[a + b*x])/(b*(2 + 5*m))
```

3.19.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin^m(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sin(a+bx)^m)^{5/2}\,dx$$

$$\Big\downarrow \text{3687}$$

$$c^2\sin^{-\frac{m}{2}}(a+bx)\sqrt{c\sin^m(a+bx)}\int \sin^{\frac{5m}{2}}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$c^2\sin^{-\frac{m}{2}}(a+bx)\sqrt{c\sin^m(a+bx)}\int \sin(a+bx)^{5m/2}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{2c^2\cos(a+bx)\sin^{2m+1}(a+bx)\sqrt{c\sin^m(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5m+2),\frac{1}{4}(5m+6),\sin^2(a+bx)\right)}{b(5m+2)\sqrt{\cos^2(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x]^m)^(5/2),x]
```

output
```
(2*c^2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 + 5*m)/4, (6 + 5*m)/4, Sin[a
 + b*x]^2]*Sin[a + b*x]^(1 + 2*m)*Sqrt[c*Sin[a + b*x]^m])/(b*(2 + 5*m)*Sqr
t[Cos[a + b*x]^2])
```

3.19.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.19.4 Maple [F]

$$\int (c(\sin^m(bx+a)))^{\frac{5}{2}}dx$$

input
```
int((c*sin(b*x+a)^m)^(5/2),x)
```

output
```
int((c*sin(b*x+a)^m)^(5/2),x)
```

3.19.5 Fricas [F(-2)]

Exception generated.

$$\int (c\sin^m(a+bx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*sin(b*x+a)^m)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.19.6 Sympy [F(-1)]

Timed out.

$$\int (c\sin^m(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a)**m)**(5/2),x)
```

output
```
Timed out
```

3.19.7 Maxima [F]

$$\int (c\sin^m(a+bx))^{5/2}\,dx = \int (c\sin(bx+a)^m)^{\frac{5}{2}}\,dx$$

input
```
integrate((c*sin(b*x+a)^m)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a)^m)^(5/2), x)
```

3.19.8 Giac [F]

$$\int (c\sin^m(a+bx))^{5/2}\,dx = \int (c\sin(bx+a)^m)^{\frac{5}{2}}\,dx$$

input
```
integrate((c*sin(b*x+a)^m)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^m)^(5/2), x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int (c\sin^m(a+bx))^{5/2}\,dx = \int (c\sin(a+b\,x)^m)^{5/2}\,dx$$

input
```
int((c*sin(a + b*x)^m)^(5/2),x)
```

output
```
int((c*sin(a + b*x)^m)^(5/2), x)
```

3.20 $\int (c\sin^m(a+bx))^{3/2}\,dx$

3.20.1 Optimal result

Integrand size = 14, antiderivative size = 83

$$\int (c\sin^m(a+bx))^{3/2}\,dx = \frac{2c\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2+3m),\frac{3(2+m)}{4},\sin^2(a+bx)\right)\sin^{1+m}(a+bx)\sqrt{c\sin^m(a+bx)}}{b(2+3m)\sqrt{\cos^2(a+bx)}}$$

output
```
2*c*cos(b*x+a)*hypergeom([1/2, 1/2+3/4*m],[3/2+3/4*m],sin(b*x+a)^2)*sin(b*
x+a)^(1+m)*(c*sin(b*x+a)^m)^(1/2)/b/(2+3*m)/(cos(b*x+a)^2)^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int (c\sin^m(a+bx))^{3/2}\,dx = \frac{2\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2+3m),\frac{3(2+m)}{4},\sin^2(a+bx)\right)(c\sin^m(a+bx))^{3/2}\tan(a+bx)}{b(2+3m)}$$

input
```
Integrate[(c*Sin[a + b*x]^m)^(3/2),x]
```

output
```
(2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 + 3*m)/4, (3*(2 + m))/4,
 Sin[a + b*x]^2]*(c*Sin[a + b*x]^m)^(3/2)*Tan[a + b*x])/(b*(2 + 3*m))
```

3.20.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin^m(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sin(a+bx)^m)^{3/2}\,dx$$

$$\Big\downarrow \text{3687}$$

$$c\sin^{-\frac{m}{2}}(a+bx)\sqrt{c\sin^m(a+bx)}\int \sin^{\frac{3m}{2}}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$c\sin^{-\frac{m}{2}}(a+bx)\sqrt{c\sin^m(a+bx)}\int \sin(a+bx)^{3m/2}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{2c\cos(a+bx)\sin^{m+1}(a+bx)\sqrt{c\sin^m(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3m+2),\frac{3(m+2)}{4},\sin^2(a+bx)\right)}{b(3m+2)\sqrt{\cos^2(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x]^m)^(3/2),x]
```

output
```
(2*c*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 + 3*m)/4, (3*(2 + m))/4, Sin[a
 + b*x]^2]*Sin[a + b*x]^(1 + m)*Sqrt[c*Sin[a + b*x]^m])/(b*(2 + 3*m)*Sqrt[
Cos[a + b*x]^2])
```

3.20.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.20.4 Maple [F]

$$\int (c(\sin^m (bx + a)))^{\frac{3}{2}} dx$$

input
```
int((c*sin(b*x+a)^m)^(3/2),x)
```

output
```
int((c*sin(b*x+a)^m)^(3/2),x)
```

3.20.5 Fricas [F(-2)]

Exception generated.

$$\int (c\sin^m(a + bx))^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*sin(b*x+a)^m)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.20.6 Sympy [F]

$$\int (c\sin^m(a+bx))^{3/2}\,dx = \int (c\sin^m(a+bx))^{\frac{3}{2}}\,dx$$

input
```
integrate((c*sin(b*x+a)**m)**(3/2),x)
```

output
```
Integral((c*sin(a + b*x)**m)**(3/2), x)
```

3.20.7 Maxima [F]

$$\int (c\sin^m(a+bx))^{3/2}\,dx = \int (c\sin{(bx+a)}^m)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*sin(b*x+a)^m)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a)^m)^(3/2), x)
```

3.20.8 Giac [F]

$$\int (c\sin^m(a+bx))^{3/2}\,dx = \int (c\sin{(bx+a)}^m)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*sin(b*x+a)^m)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^m)^(3/2), x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int (c\sin^m(a+bx))^{3/2}\,dx = \int (c\sin(a+b\,x)^m)^{3/2}\,dx$$

input
```
int((c*sin(a + b*x)^m)^(3/2),x)
```

output
```
int((c*sin(a + b*x)^m)^(3/2), x)
```

3.21 $\int \sqrt{c \sin^m(a+bx)}\,dx$

3.21.1 Optimal result

Integrand size = 14, antiderivative size = 74

$$\int \sqrt{c \sin^m(a+bx)}\,dx = \frac{2\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{4},\frac{6+m}{4},\sin^2(a+bx)\right)\sin(a+bx)\sqrt{c\sin^m(a+bx)}}{b(2+m)\sqrt{\cos^2(a+bx)}}$$

output
```
2*cos(b*x+a)*hypergeom([1/2, 1/2+1/4*m],[3/2+1/4*m],sin(b*x+a)^2)*sin(b*x+
a)*(c*sin(b*x+a)^m)^(1/2)/b/(2+m)/(cos(b*x+a)^2)^(1/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \sqrt{c \sin^m(a+bx)}\,dx = \frac{2\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{4},\frac{6+m}{4},\sin^2(a+bx)\right)\sqrt{c\sin^m(a+bx)}\tan(a+bx)}{b(2+m)}$$

input
```
Integrate[Sqrt[c*Sin[a + b*x]^m],x]
```

output
```
(2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 + m)/4, (6 + m)/4, Sin[a
 + b*x]^2]*Sqrt[c*Sin[a + b*x]^m]*Tan[a + b*x])/(b*(2 + m))
```

3.21.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c \sin^m(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c \sin(a+bx)^m}dx$$

$$\Big\downarrow \text{3687}$$

$$\sin^{-\frac{m}{2}}(a+bx)\sqrt{c \sin^m(a+bx)}\int \sin^{\frac{m}{2}}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-\frac{m}{2}}(a+bx)\sqrt{c \sin^m(a+bx)}\int \sin(a+bx)^{m/2}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{2\cos(a+bx)\sin^{\frac{m+2}{2}-\frac{m}{2}}(a+bx)\sqrt{c \sin^m(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{4},\frac{m+6}{4},\sin^2(a+bx)\right)}{b(m+2)\sqrt{\cos^2(a+bx)}}$$

input
```
Int[Sqrt[c*Sin[a + b*x]^m],x]
```

output
```
(2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 + m)/4, (6 + m)/4, Sin[a + b*x]^
2]*Sin[a + b*x]^(-1/2*m + (2 + m)/2)*Sqrt[c*Sin[a + b*x]^m])/(b*(2 + m)*Sq
rt[Cos[a + b*x]^2])
```

3.21.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.21.4 Maple [F]

$$\int \sqrt{c\left(\sin^{m}\left(bx+a\right)\right)}dx$$

input
```
int((c*sin(b*x+a)^m)^(1/2),x)
```

output
```
int((c*sin(b*x+a)^m)^(1/2),x)
```

3.21.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c\sin^{m}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*sin(b*x+a)^m)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.21.6 Sympy [F]

$$\int \sqrt{c \sin^m(a+bx)}\, dx = \int \sqrt{c \sin^m{(a+bx)}}\, dx$$

input
```
integrate((c*sin(b*x+a)**m)**(1/2),x)
```

output
```
Integral(sqrt(c*sin(a + b*x)**m), x)
```

3.21.7 Maxima [F]

$$\int \sqrt{c \sin^m(a+bx)}\, dx = \int \sqrt{c \sin{(bx+a)}^m}\, dx$$

input
```
integrate((c*sin(b*x+a)^m)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*sin(b*x + a)^m), x)
```

3.21.8 Giac [F]

$$\int \sqrt{c \sin^m(a+bx)}\, dx = \int \sqrt{c \sin{(bx+a)}^m}\, dx$$

input
```
integrate((c*sin(b*x+a)^m)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*sin(b*x + a)^m), x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c\sin^m(a+bx)}\,dx = \int \sqrt{c\sin\left(a+b\,x\right)^m}\,dx$$

input
```
int((c*sin(a + b*x)^m)^(1/2),x)
```

output
```
int((c*sin(a + b*x)^m)^(1/2), x)
```

3.22 $\int \frac{1}{\sqrt{c\sin^m(a+bx)}}\,dx$

3.22.1 Optimal result

Integrand size = 14, antiderivative size = 80

$$\int \frac{1}{\sqrt{c\sin^m(a+bx)}}\,dx = \frac{2\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{4},\frac{6-m}{4},\sin^2(a+bx)\right)\sin(a+bx)}{b(2-m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

output
```
2*cos(b*x+a)*hypergeom([1/2, 1/2-1/4*m],[3/2-1/4*m],sin(b*x+a)^2)*sin(b*x+
a)/b/(2-m)/(cos(b*x+a)^2)^(1/2)/(c*sin(b*x+a)^m)^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{c\sin^m(a+bx)}}\,dx = -\frac{2\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{4},\frac{6-m}{4},\sin^2(a+bx)\right)\tan(a+bx)}{b(-2+m)\sqrt{c\sin^m(a+bx)}}$$

input
```
Integrate[1/Sqrt[c*Sin[a + b*x]^m],x]
```

output
```
(-2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 - m)/4, (6 - m)/4, Sin[
a + b*x]^2]*Tan[a + b*x])/(b*(-2 + m)*Sqrt[c*Sin[a + b*x]^m])
```

3.22.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c\sin^m(a+bx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{c\sin(a+bx)^m}}dx$$

$$\Big\downarrow \text{ 3687}$$

$$\frac{\sin^{\frac{m}{2}}(a+bx)\int \sin^{-\frac{m}{2}}(a+bx)dx}{\sqrt{c\sin^m(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sin^{\frac{m}{2}}(a+bx)\int \sin(a+bx)^{-m/2}dx}{\sqrt{c\sin^m(a+bx)}}$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{2\sin(a+bx)\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{4},\frac{6-m}{4},\sin^2(a+bx)\right)}{b(2-m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

input
```
Int[1/Sqrt[c*Sin[a + b*x]^m],x]
```

output
```
(2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - m)/4, (6 - m)/4, Sin[a + b*x]^
2]*Sin[a + b*x])/(b*(2 - m)*Sqrt[Cos[a + b*x]^2]*Sqrt[c*Sin[a + b*x]^m])
```

3.22.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.22.4 Maple [F]

$$\int \frac{1}{\sqrt{c\left(\sin^{m}\left(bx+a\right)\right)}}dx$$

input
```
int(1/(c*sin(b*x+a)^m)^(1/2),x)
```

output
```
int(1/(c*sin(b*x+a)^m)^(1/2),x)
```

3.22.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c\sin^{m}(a+bx)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(c*sin(b*x+a)^m)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.22.6 Sympy [F]

$$\int \frac{1}{\sqrt{c\sin^m(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sin^m{(a+bx)}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a)**m)**(1/2),x)
```

output
```
Integral(1/sqrt(c*sin(a + b*x)**m), x)
```

3.22.7 Maxima [F]

$$\int \frac{1}{\sqrt{c\sin^m(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sin{(bx+a)}^m}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a)^m)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*sin(b*x + a)^m), x)
```

3.22.8 Giac [F]

$$\int \frac{1}{\sqrt{c\sin^m(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sin{(bx+a)}^m}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a)^m)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*sin(b*x + a)^m), x)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c\sin^m(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sin\left(a+b\,x\right)^m}}\,dx$$

input
```
int(1/(c*sin(a + b*x)^m)^(1/2),x)
```

output
```
int(1/(c*sin(a + b*x)^m)^(1/2), x)
```

3.23 $\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx$

3.23.1 Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx = \frac{2\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-3m),\frac{3(2-m)}{4},\sin^2(a+bx)\right)\sin^{1-m}(a}{bc(2-3m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

output
```
2*cos(b*x+a)*hypergeom([1/2, 1/2-3/4*m],[3/2-3/4*m],sin(b*x+a)^2)*sin(b*x+
a)^(1-m)/b/c/(2-3*m)/(cos(b*x+a)^2)^(1/2)/(c*sin(b*x+a)^m)^(1/2)
```

3.23.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

$$\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-3m),-\frac{3}{4}(-2+m),\sin^2(a+bx)\right)\,\text{t}}{\left(b-\frac{3bm}{2}\right)(c\sin^m(a+bx))^{3/2}}$$

input
```
Integrate[(c*Sin[a + b*x]^m)^(-3/2),x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 - 3*m)/4, (-3*(-2 + m))/4,
 Sin[a + b*x]^2]*Tan[a + b*x])/((b - (3*b*m)/2)*(c*Sin[a + b*x]^m)^(3/2))
```

3.23.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c\sin(a+bx)^m)^{3/2}}dx$$

$$\Big\downarrow \text{3687}$$

$$\frac{\sin^{\frac{m}{2}}(a+bx)\int \sin^{-\frac{3m}{2}}(a+bx)dx}{c\sqrt{c\sin^m(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{m}{2}}(a+bx)\int \sin(a+bx)^{-3m/2}dx}{c\sqrt{c\sin^m(a+bx)}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{2\cos(a+bx)\sin^{1-m}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-3m),\frac{3(2-m)}{4},\sin^2(a+bx)\right)}{bc(2-3m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x]^m)^(-3/2),x]
```

output
```
(2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - 3*m)/4, (3*(2 - m))/4, Sin[a +
 b*x]^2]*Sin[a + b*x]^(1 - m))/(b*c*(2 - 3*m)*Sqrt[Cos[a + b*x]^2]*Sqrt[c*
Sin[a + b*x]^m])
```

3.23.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.23.4 Maple [F]

$$\int \frac{1}{\left(c\left(\sin^{m}\left(bx+a\right)\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(c*sin(b*x+a)^m)^(3/2),x)
```

output
```
int(1/(c*sin(b*x+a)^m)^(3/2),x)
```

3.23.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c\sin^{m}(a+bx)\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(c*sin(b*x+a)^m)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.23.6 Sympy [F]

$$\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\sin^m{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a)**m)**(3/2),x)
```

output
```
Integral((c*sin(a + b*x)**m)**(-3/2), x)
```

3.23.7 Maxima [F]

$$\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\sin{(bx+a)}^m)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a)^m)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a)^m)^(-3/2), x)
```

3.23.8 Giac [F]

$$\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\sin{(bx+a)}^m)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a)^m)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^m)^(-3/2), x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx = \int \frac{1}{\left(c\sin\left(a+b\,x\right)^m\right)^{3/2}}\,dx$$

input
```
int(1/(c*sin(a + b*x)^m)^(3/2),x)
```

output
```
int(1/(c*sin(a + b*x)^m)^(3/2), x)
```

3.24 $\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx$

3.24.1 Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx = \frac{2\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-5m),\frac{1}{4}(6-5m),\sin^2(a+bx)\right)\sin^{1-}}{bc^2(2-5m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

output
```
2*cos(b*x+a)*hypergeom([1/2, 1/2-5/4*m],[3/2-5/4*m],sin(b*x+a)^2)*sin(b*x+
a)^(1-2*m)/b/c^2/(2-5*m)/(cos(b*x+a)^2)^(1/2)/(c*sin(b*x+a)^m)^(1/2)
```

3.24.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-5m),\frac{1}{4}(6-5m),\sin^2(a+bx)\right)\tan}{\left(b-\frac{5bm}{2}\right)(c\sin^m(a+bx))^{5/2}}$$

input
```
Integrate[(c*Sin[a + b*x]^m)^(-5/2),x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 - 5*m)/4, (6 - 5*m)/4, Sin
[a + b*x]^2]*Tan[a + b*x])/((b - (5*b*m)/2)*(c*Sin[a + b*x]^m)^(5/2))
```

3.24.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c\sin(a+bx)^m)^{5/2}}dx$$

$$\Big\downarrow \text{3687}$$

$$\frac{\sin^{\frac{m}{2}}(a+bx)\int \sin^{-\frac{5m}{2}}(a+bx)dx}{c^2\sqrt{c\sin^m(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{m}{2}}(a+bx)\int \sin(a+bx)^{-5m/2}dx}{c^2\sqrt{c\sin^m(a+bx)}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{2\cos(a+bx)\sin^{1-2m}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-5m),\frac{1}{4}(6-5m),\sin^2(a+bx)\right)}{bc^2(2-5m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x]^m)^(-5/2),x]
```

output
```
(2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - 5*m)/4, (6 - 5*m)/4, Sin[a + b
*x]^2]*Sin[a + b*x]^(1 - 2*m))/(b*c^2*(2 - 5*m)*Sqrt[Cos[a + b*x]^2]*Sqrt[
c*Sin[a + b*x]^m])
```

3.24.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.24.4 Maple [F]

$$\int \frac{1}{\left(c \left(\sin^{m}\left(b x +a \right)\right)\right)^{\frac{5}{2}}}d x$$

input
```
int(1/(c*sin(b*x+a)^m)^(5/2),x)
```

output
```
int(1/(c*sin(b*x+a)^m)^(5/2),x)
```

3.24.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*sin(b*x+a)^m)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.24.6 Sympy [F]

$$\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\sin^m{(a+bx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a)**m)**(5/2),x)
```

output
```
Integral((c*sin(a + b*x)**m)**(-5/2), x)
```

3.24.7 Maxima [F]

$$\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\sin{(bx+a)}^m)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a)^m)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a)^m)^(-5/2), x)
```

3.24.8 Giac [F]

$$\int \frac{1}{(c\sin^m(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\sin{(bx+a)}^m)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a)^m)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^m)^(-5/2), x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(c\sin^m(a+bx)\right)^{5/2}}\,dx = \int \frac{1}{\left(c\sin\left(a+b\,x\right)^m\right)^{5/2}}\,dx$$

input
```
int(1/(c*sin(a + b*x)^m)^(5/2),x)
```

output
```
int(1/(c*sin(a + b*x)^m)^(5/2), x)
```

3.25 $\int (b\sin^n(c+dx))^p \, dx$

3.25.1 Optimal result

Integrand size = 12, antiderivative size = 77

$$\int (b\sin^n(c+dx))^p \, dx = \frac{\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+np),\frac{1}{2}(3+np),\sin^2(c+dx)\right)\sin(c+dx)\,(b\sin^n(c+dx))^p}{d(1+np)\sqrt{\cos^2(c+dx)}}$$

output
```
cos(d*x+c)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin(d*x+c)^2)*sin(d*
x+c)*(b*sin(d*x+c)^n)^p/d/(n*p+1)/(cos(d*x+c)^2)^(1/2)
```

3.25.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int (b\sin^n(c+dx))^p \, dx = \frac{\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+np),\frac{1}{2}(3+np),\sin^2(c+dx)\right)(b\sin^n(c+dx))^p\tan(c+dx)}{d(1+np)}$$

input
```
Integrate[(b*Sin[c + d*x]^n)^p,x]
```

output
```
(Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin
[c + d*x]^2]*(b*Sin[c + d*x]^n)^p*Tan[c + d*x])/(d*(1 + n*p))
```

3.25.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sin^n(c+dx))^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (b\sin(c+dx)^n)^p\,dx$$

$$\Big\downarrow \text{ 3687}$$

$$\sin^{-np}(c+dx)\,(b\sin^n(c+dx))^p\int \sin^{np}(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sin^{-np}(c+dx)\,(b\sin^n(c+dx))^p\int \sin(c+dx)^{np}dx$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{\sin(c+dx)\cos(c+dx)\,(b\sin^n(c+dx))^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(np+1),\frac{1}{2}(np+3),\sin^2(c+dx)\right)}{d(np+1)\sqrt{\cos^2(c+dx)}}$$

input
```
Int[(b*Sin[c + d*x]^n)^p,x]
```

output
```
(Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[c + d*x
]^2]*Sin[c + d*x]*(b*Sin[c + d*x]^n)^p)/(d*(1 + n*p)*Sqrt[Cos[c + d*x]^2])
```

3.25.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.25.4 Maple [F]

$$\int (b(\sin^n(dx+c)))^p dx$$

input
```
int((b*sin(d*x+c)^n)^p,x)
```

output
```
int((b*sin(d*x+c)^n)^p,x)
```

3.25.5 Fricas [F]

$$\int (b\sin^n(c+dx))^p \, dx = \int (b\sin(dx+c)^n)^p \, dx$$

input
```
integrate((b*sin(d*x+c)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c)^n)^p, x)
```

3.25.6 Sympy [F]

$$\int (b\sin^n(c+dx))^p\,dx = \int (b\sin^n(c+dx))^p\,dx$$

input
```
integrate((b*sin(d*x+c)**n)**p,x)
```

output
```
Integral((b*sin(c + d*x)**n)**p, x)
```

3.25.7 Maxima [F]

$$\int (b\sin^n(c+dx))^p\,dx = \int (b\sin(dx+c)^n)^p\,dx$$

input
```
integrate((b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^n)^p, x)
```

3.25.8 Giac [F]

$$\int (b\sin^n(c+dx))^p\,dx = \int (b\sin(dx+c)^n)^p\,dx$$

input
```
integrate((b*sin(d*x+c)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^n)^p, x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int (b\sin^n(c+dx))^p\,dx = \int (b\sin(c+d\,x)^n)^p\,dx$$

input
```
int((b*sin(c + d*x)^n)^p,x)
```

output
```
int((b*sin(c + d*x)^n)^p, x)
```

3.26 $\int (c \sin^2(a+bx))^p \, dx$

3.26.1 Optimal result

Integrand size = 12, antiderivative size = 77

$$\int (c \sin^2(a+bx))^p \, dx = \frac{\cos(a+bx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(1+2p), \frac{1}{2}(3+2p), \sin^2(a+bx)\right) \sin(a+bx) (c \sin^2(a+bx))^p}{b(1+2p)\sqrt{\cos^2(a+bx)}}$$

output
```
cos(b*x+a)*hypergeom([1/2, 1/2+p],[3/2+p],sin(b*x+a)^2)*sin(b*x+a)*(c*sin(
b*x+a)^2)^p/b/(1+2*p)/(cos(b*x+a)^2)^(1/2)
```

3.26.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int (c \sin^2(a+bx))^p \, dx = \frac{\sqrt{\cos^2(a+bx)} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}+p, \frac{3}{2}+p, \sin^2(a+bx)\right) (c \sin^2(a+bx))^p \tan(a+bx)}{b+2bp}$$

input
```
Integrate[(c*Sin[a + b*x]^2)^p,x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 1/2 + p, 3/2 + p, Sin[a + b*x
]^2]*(c*Sin[a + b*x]^2)^p*Tan[a + b*x])/(b + 2*b*p)
```

3.26.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3686, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c\sin^2(a+bx)\right)^p \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(c\sin(a+bx)^2\right)^p dx$$

$$\Big\downarrow \text{ 3686}$$

$$\sin^{-2p}(a+bx)\left(c\sin^2(a+bx)\right)^p \int \sin^{2p}(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sin^{-2p}(a+bx)\left(c\sin^2(a+bx)\right)^p \int \sin(a+bx)^{2p}dx$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^2(a+bx)\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2p+1),\frac{1}{2}(2p+3),\sin^2(a+bx)\right)}{b(2p+1)\sqrt{\cos^2(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x]^2)^p,x]
```

output
```
(Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + 2*p)/2, (3 + 2*p)/2, Sin[a + b*x
]^2]*Sin[a + b*x]*(c*Sin[a + b*x]^2)^p)/(b*(1 + 2*p)*Sqrt[Cos[a + b*x]^2])
```

3.26.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.26.4 Maple [F]

$$\int \left(c\left(\sin^2\left(bx+a\right)\right)\right)^p dx$$

input
```
int((c*sin(b*x+a)^2)^p,x)
```

output
```
int((c*sin(b*x+a)^2)^p,x)
```

3.26.5 Fricas [F]

$$\int \left(c\sin^2(a+bx)\right)^p\,dx=\int \left(c\sin\left(bx+a\right)^2\right)^p\,dx$$

input
```
integrate((c*sin(b*x+a)^2)^p,x, algorithm="fricas")
```

output
```
integral((-c*cos(b*x + a)^2 + c)^p, x)
```

3.26.6 Sympy [F]

$$\int \left(c \sin^2(a+bx)\right)^p \, dx = \int \left(c \sin^2{(a+bx)}\right)^p \, dx$$

input
```
integrate((c*sin(b*x+a)**2)**p,x)
```

output
```
Integral((c*sin(a + b*x)**2)**p, x)
```

3.26.7 Maxima [F]

$$\int \left(c \sin^2(a+bx)\right)^p \, dx = \int \left(c \sin{(bx+a)}^2\right)^p \, dx$$

input
```
integrate((c*sin(b*x+a)^2)^p,x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a)^2)^p, x)
```

3.26.8 Giac [F]

$$\int \left(c \sin^2(a+bx)\right)^p \, dx = \int \left(c \sin{(bx+a)}^2\right)^p \, dx$$

input
```
integrate((c*sin(b*x+a)^2)^p,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^2)^p, x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int \left(c\sin^2(a+bx)\right)^p\,dx = \int \left(c\sin\left(a+b\,x\right)^2\right)^p dx$$

input
```
int((c*sin(a + b*x)^2)^p,x)
```

output
```
int((c*sin(a + b*x)^2)^p, x)
```

3.27 $\int (c\sin^3(a+bx))^p\,dx$

3.27.1 Optimal result

Integrand size = 12, antiderivative size = 75

$$\int (c\sin^3(a+bx))^p\,dx = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+3p),\frac{3(1+p)}{2},\sin^2(a+bx)\right)\sin(a+bx)\,(c\sin^3(a+bx))^p}{b(1+3p)\sqrt{\cos^2(a+bx)}}$$

output
```
cos(b*x+a)*hypergeom([1/2, 1/2+3/2*p],[3/2+3/2*p],sin(b*x+a)^2)*sin(b*x+a)
*(c*sin(b*x+a)^3)^p/b/(1+3*p)/(cos(b*x+a)^2)^(1/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int (c\sin^3(a+bx))^p\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+3p),\frac{3(1+p)}{2},\sin^2(a+bx)\right)(c\sin^3(a+bx))^p\tan(a+bx)}{b+3bp}$$

input
```
Integrate[(c*Sin[a + b*x]^3)^p,x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (1 + 3*p)/2, (3*(1 + p))/2, S
in[a + b*x]^2]*(c*Sin[a + b*x]^3)^p*Tan[a + b*x])/(b + 3*b*p)
```

3.27.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3686, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c\sin^3(a+bx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\sin(a+bx)^3\right)^p dx$$

$$\Big\downarrow \text{3686}$$

$$\sin^{-3p}(a+bx)\left(c\sin^3(a+bx)\right)^p\int \sin^{3p}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-3p}(a+bx)\left(c\sin^3(a+bx)\right)^p\int \sin(a+bx)^{3p}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^3(a+bx)\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(3p+1),\frac{3(p+1)}{2},\sin^2(a+bx)\right)}{b(3p+1)\sqrt{\cos^2(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x]^3)^p,x]
```

output
```
(Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + 3*p)/2, (3*(1 + p))/2, Sin[a + b
*x]^2]*Sin[a + b*x]*(c*Sin[a + b*x]^3)^p)/(b*(1 + 3*p)*Sqrt[Cos[a + b*x]^2
])
```

3.27.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.27.4 Maple [F]

$$\int \left(c\left(\sin^{3}\left(bx+a\right)\right)\right)^{p}dx$$

input
```
int((c*sin(b*x+a)^3)^p,x)
```

output
```
int((c*sin(b*x+a)^3)^p,x)
```

3.27.5 Fricas [F]

$$\int \left(c\sin^{3}(a+bx)\right)^{p}\,dx=\int \left(c\sin\left(bx+a\right)^{3}\right)^{p}\,dx$$

input
```
integrate((c*sin(b*x+a)^3)^p,x, algorithm="fricas")
```

output
```
integral((-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^p, x)
```

3.27.6 Sympy [F]

$$\int (c\sin^3(a+bx))^p \, dx = \int \left(c\sin^3{(a+bx)}\right)^p \, dx$$

input
```
integrate((c*sin(b*x+a)**3)**p,x)
```

output
```
Integral((c*sin(a + b*x)**3)**p, x)
```

3.27.7 Maxima [F]

$$\int (c\sin^3(a+bx))^p \, dx = \int \left(c\sin{(bx+a)}^3\right)^p \, dx$$

input
```
integrate((c*sin(b*x+a)^3)^p,x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a)^3)^p, x)
```

3.27.8 Giac [F]

$$\int (c\sin^3(a+bx))^p \, dx = \int \left(c\sin{(bx+a)}^3\right)^p \, dx$$

input
```
integrate((c*sin(b*x+a)^3)^p,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^3)^p, x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \left(c\sin^3(a+bx)\right)^p\,dx = \int \left(c\sin\left(a+b\,x\right)^3\right)^p\,dx$$

input
```
int((c*sin(a + b*x)^3)^p,x)
```

output
```
int((c*sin(a + b*x)^3)^p, x)
```

3.28 $\int (c\sin^4(a+bx))^p\,dx$

3.28.1 Optimal result

Integrand size = 12, antiderivative size = 77

$$\int (c\sin^4(a+bx))^p\,dx = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+4p),\frac{1}{2}(3+4p),\sin^2(a+bx)\right)\sin(a+bx)\,(c\sin^4(a+bx))^p}{b(1+4p)\sqrt{\cos^2(a+bx)}}$$

output
```
cos(b*x+a)*hypergeom([1/2, 1/2+2*p],[3/2+2*p],sin(b*x+a)^2)*sin(b*x+a)*(c*
sin(b*x+a)^4)^p/b/(1+4*p)/(cos(b*x+a)^2)^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int (c\sin^4(a+bx))^p\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}+2p,\frac{3}{2}+2p,\sin^2(a+bx)\right)(c\sin^4(a+bx))^p\tan(a+bx)}{b+4bp}$$

input
```
Integrate[(c*Sin[a + b*x]^4)^p,x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 1/2 + 2*p, 3/2 + 2*p, Sin[a +
 b*x]^2]*(c*Sin[a + b*x]^4)^p*Tan[a + b*x])/(b + 4*b*p)
```

3.28.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3686, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c\sin^4(a+bx)\right)^p\, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\sin(a+bx)^4\right)^p dx$$

$$\Big\downarrow \text{3686}$$

$$\sin^{-4p}(a+bx)\left(c\sin^4(a+bx)\right)^p\int \sin^{4p}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-4p}(a+bx)\left(c\sin^4(a+bx)\right)^p\int \sin(a+bx)^{4p}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^4(a+bx)\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(4p+1),\frac{1}{2}(4p+3),\sin^2(a+bx)\right)}{b(4p+1)\sqrt{\cos^2(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x]^4)^p,x]
```

output
```
(Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + 4*p)/2, (3 + 4*p)/2, Sin[a + b*x
]^2]*Sin[a + b*x]*(c*Sin[a + b*x]^4)^p)/(b*(1 + 4*p)*Sqrt[Cos[a + b*x]^2])
```

3.28.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.28.4 Maple [F]

$$\int \left(c\left(\sin^4\left(bx+a\right)\right)\right)^p dx$$

input
```
int((c*sin(b*x+a)^4)^p,x)
```

output
```
int((c*sin(b*x+a)^4)^p,x)
```

3.28.5 Fricas [F]

$$\int \left(c\sin^4(a+bx)\right)^p\, dx = \int \left(c\sin\left(bx+a\right)^4\right)^p\, dx$$

input
```
integrate((c*sin(b*x+a)^4)^p,x, algorithm="fricas")
```

output
```
integral((c*cos(b*x + a)^4 - 2*c*cos(b*x + a)^2 + c)^p, x)
```

3.28.6 Sympy [F]

$$\int \left(c\sin^4(a+bx)\right)^p\,dx = \int \left(c\sin^4{(a+bx)}\right)^p\,dx$$

input
```
integrate((c*sin(b*x+a)**4)**p,x)
```

output
```
Integral((c*sin(a + b*x)**4)**p, x)
```

3.28.7 Maxima [F]

$$\int \left(c\sin^4(a+bx)\right)^p\,dx = \int \left(c\sin{(bx+a)}^4\right)^p\,dx$$

input
```
integrate((c*sin(b*x+a)^4)^p,x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a)^4)^p, x)
```

3.28.8 Giac [F]

$$\int \left(c\sin^4(a+bx)\right)^p\,dx = \int \left(c\sin{(bx+a)}^4\right)^p\,dx$$

input
```
integrate((c*sin(b*x+a)^4)^p,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a)^4)^p, x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \left(c\sin^4(a+bx)\right)^p \, dx = \int \left(c\sin\left(a+b\,x\right)^4\right)^p \, dx$$

input
```
int((c*sin(a + b*x)^4)^p,x)
```

output
```
int((c*sin(a + b*x)^4)^p, x)
```

3.29 $\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx$

3.29.1 Optimal result

Integrand size = 14, antiderivative size = 25

$$\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx = -\frac{\cot(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}}{b}$$

output
```
-cot(b*x+a)*(c*sin(b*x+a)^n)^(1/n)/b
```

3.29.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx = -\frac{\cot(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}}{b}$$

input
```
Integrate[(c*Sin[a + b*x]^n)^n^(-1),x]
```

output
```
-((Cot[a + b*x]*(c*Sin[a + b*x]^n)^n^(-1))/b)
```

3.29.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c\sin(a+bx)^n)^{\frac{1}{n}}\,dx$$

$$\Big\downarrow \text{ 3687}$$

$$\csc(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}\int \sin(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\csc(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}\int \sin(a+bx)dx$$

$$\Big\downarrow \text{ 3118}$$

$$-\frac{\cot(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}}{b}$$

input
```
Int[(c*Sin[a + b*x]^n)^n^(-1),x]
```

output
```
-((Cot[a + b*x]*(c*Sin[a + b*x]^n)^n^(-1))/b)
```

3.29.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.29.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
parallelrisch	$-\frac{(c(\sin^n(bx+a)))^{\frac{1}{n}}\cot\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}$	29

input
```
int((c*sin(b*x+a)^n)^(1/n),x,method=_RETURNVERBOSE)
```

output
```
-1/b*(c*sin(b*x+a)^n)^(1/n)*cot(1/2*b*x+1/2*a)
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx = -\frac{c^{\left(\frac{1}{n}\right)}\cos(bx+a)}{b}$$

input
```
integrate((c*sin(b*x+a)^n)^(1/n),x, algorithm="fricas")
```

output
```
-c^(1/n)*cos(b*x + a)/b
```

3.29.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(22) = 44$.

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx = \begin{cases} x(c\sin^n(a))^{\frac{1}{n}} & \text{for } b=0 \\ x(0^n c)^{\frac{1}{n}} & \text{for } a=-bx \vee a=-bx+\pi \\ -\frac{(c\sin^n(a+bx))^{\frac{1}{n}}\cos(a+bx)}{b\sin(a+bx)} & \text{otherwise} \end{cases}$$

input
```
integrate((c*sin(b*x+a)**n)**(1/n),x)
```

output
```
Piecewise((x*(c*sin(a)**n)**(1/n), Eq(b, 0)), (x*(0**n*c)**(1/n), Eq(a, -b
*x) | Eq(a, -b*x + pi)), (-(c*sin(a + b*x)**n)**(1/n)*cos(a + b*x)/(b*sin(
a + b*x)), True))
```

3.29.7 Maxima [F]

$$\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx = \int (c\sin(bx+a)^n)^{\left(\frac{1}{n}\right)}\,dx$$

input
```
integrate((c*sin(b*x+a)^n)^(1/n),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a)^n)^(1/n), x)
```

3.29.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(25) = 50$.

Time = 0.74 (sec) , antiderivative size = 384, normalized size of antiderivative = 15.36

$$\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx = \frac{|c|^{\left(\frac{1}{n}\right)}\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{\pi\mathrm{sgn}(c)}{4\,n}-\frac{\pi}{4\,n}\right)^2\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^4-2\,|c|^{\left(\frac{1}{n}\right)}\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{\pi\mathrm{sgn}(c)}{4\,n}-\frac{\pi}{4\,n}\right)^2\tan\left(\frac{1}{2}\right.}{b\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{\pi\mathrm{sgn}(c)}{4\,n}-\frac{\pi}{4\,n}\right)^2}$$

input
```
integrate((c*sin(b*x+a)^n)^(1/n),x, algorithm="giac")
```

output
```
(abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)^2*tan(1/2*
b*x + 1/2*a)^4 - 2*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/
4*pi/n)^2*tan(1/2*b*x + 1/2*a)^2 + 4*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/
4*pi*sgn(c)/n - 1/4*pi/n)*tan(1/2*b*x + 1/2*a)^3 - abs(c)^(1/n)*tan(1/2*b*
x + 1/2*a)^4 + abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi
/n)^2 - 4*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)*t
an(1/2*b*x + 1/2*a) + 2*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a)^2 - abs(c)^(1/n)
)/(b*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)^2*tan(1/2*b*x + 1/2
*a)^4 + 2*b*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)^2*tan(1/2*b*
x + 1/2*a)^2 + b*tan(1/2*b*x + 1/2*a)^4 + b*tan(1/2*b*x + 1/2*a + 1/4*pi*s
gn(c)/n - 1/4*pi/n)^2 + 2*b*tan(1/2*b*x + 1/2*a)^2 + b)
```

3.29.9 Mupad [B] (verification not implemented)

Time = 14.44 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx = -\frac{\sin(2\,a+2\,b\,x)\,\left(c\sin(a+b\,x)^n\right)^{1/n}}{2\,b\sin(a+b\,x)^2}$$

input
```
int((c*sin(a + b*x)^n)^(1/n),x)
```

output
```
-(sin(2*a + 2*b*x)*(c*sin(a + b*x)^n)^(1/n))/(2*b*sin(a + b*x)^2)
```

3.30 $\int (a(b\sin(c+dx))^p)^n \, dx$

3.30.1 Optimal result

Integrand size = 14, antiderivative size = 79

$$\int (a(b\sin(c+dx))^p)^n \, dx = \frac{\cos(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(1+np), \frac{1}{2}(3+np), \sin^2(c+dx)\right) \sin(c+dx) (a(b\sin(c+dx))^p)^n}{d(1+np)\sqrt{\cos^2(c+dx)}}$$

output
```
cos(d*x+c)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin(d*x+c)^2)*sin(d*
x+c)*(a*(b*sin(d*x+c))^p)^n/d/(n*p+1)/(cos(d*x+c)^2)^(1/2)
```

3.30.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int (a(b\sin(c+dx))^p)^n \, dx = \frac{\sqrt{\cos^2(c+dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(1+np), \frac{1}{2}(3+np), \sin^2(c+dx)\right) (a(b\sin(c+dx))^p)^n \tan(c+d}{d(1+np)}$$

input
```
Integrate[(a*(b*Sin[c + d*x])^p)^n,x]
```

output
```
(Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin
[c + d*x]^2]*(a*(b*Sin[c + d*x])^p)^n*Tan[c + d*x])/(d*(1 + n*p))
```

3.30.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a(b\sin(c+dx))^p)^n \, dx$$

$$\Big\downarrow \; 3042$$

$$\int (a(b\sin(c+dx))^p)^n dx$$

$$\Big\downarrow \; 3687$$

$$(b\sin(c+dx))^{-np} (a(b\sin(c+dx))^p)^n \int (b\sin(c+dx))^{np}dx$$

$$\Big\downarrow \; 3042$$

$$(b\sin(c+dx))^{-np} (a(b\sin(c+dx))^p)^n \int (b\sin(c+dx))^{np}dx$$

$$\Big\downarrow \; 3122$$

$$\frac{\sin(c+dx)\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(np+1),\frac{1}{2}(np+3),\sin^2(c+dx)\right)(a(b\sin(c+dx))^p)^n}{d(np+1)\sqrt{\cos^2(c+dx)}}$$

input
```
Int[(a*(b*Sin[c + d*x])^p)^n,x]
```

output
```
(Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[c + d*x
]^2]*Sin[c + d*x]*(a*(b*Sin[c + d*x])^p)^n)/(d*(1 + n*p)*Sqrt[Cos[c + d*x]
^2])
```

3.30.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.30.4 Maple [F]

$$\int (a(b\sin (dx+c))^p)^n \, dx$$

input
```
int((a*(b*sin(d*x+c))^p)^n,x)
```

output
```
int((a*(b*sin(d*x+c))^p)^n,x)
```

3.30.5 Fricas [F]

$$\int (a(b\sin(c+dx))^p)^n \, dx = \int ((b\sin (dx+c))^p a)^n \, dx$$

input
```
integrate((a*(b*sin(d*x+c))^p)^n,x, algorithm="fricas")
```

output
```
integral(((b*sin(d*x + c))^p*a)^n, x)
```

3.30.6 Sympy [F]

$$\int (a(b\sin(c+dx))^p)^n \, dx = \int (a(b\sin{(c+dx)})^p)^n \, dx$$

input
```
integrate((a*(b*sin(d*x+c))**p)**n,x)
```

output
```
Integral((a*(b*sin(c + d*x))**p)**n, x)
```

3.30.7 Maxima [F]

$$\int (a(b\sin(c+dx))^p)^n \, dx = \int ((b\sin{(dx+c)})^p a)^n \, dx$$

input
```
integrate((a*(b*sin(d*x+c))^p)^n,x, algorithm="maxima")
```

output
```
integrate(((b*sin(d*x + c))^p*a)^n, x)
```

3.30.8 Giac [F]

$$\int (a(b\sin(c+dx))^p)^n \, dx = \int ((b\sin{(dx+c)})^p a)^n \, dx$$

input
```
integrate((a*(b*sin(d*x+c))^p)^n,x, algorithm="giac")
```

output
```
integrate(((b*sin(d*x + c))^p*a)^n, x)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int (a(b\sin(c+dx))^p)^n \, dx = \int (a\,(b\,\sin(c+d\,x))^p)^n \, dx$$

input
```
int((a*(b*sin(c + d*x))^p)^n,x)
```

output
```
int((a*(b*sin(c + d*x))^p)^n, x)
```

3.31 $\int (a - a\sin^2(x))\,dx$

3.31.1 Optimal result

Integrand size = 9, antiderivative size = 16

$$\int (a - a\sin^2(x))\,dx = \frac{ax}{2} + \frac{1}{2}a\cos(x)\sin(x)$$

output
```
1/2*a*x+1/2*a*cos(x)*sin(x)
```

3.31.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a - a\sin^2(x))\,dx = a\left(\frac{x}{2} + \frac{1}{4}\sin(2x)\right)$$

input
```
Integrate[a - a*Sin[x]^2,x]
```

output
```
a*(x/2 + Sin[2*x]/4)
```

3.31.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sin^2(x)\right)\,dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{ax}{2}+\frac{1}{2}a\sin(x)\cos(x)$$

input
```
Int[a - a*Sin[x]^2,x]
```

output
```
(a*x)/2 + (a*Cos[x]*Sin[x])/2
```

3.31.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.31.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{ax}{2}+\frac{a\sin(2x)}{4}$	13
default	$ax-a\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)$	18
parallelrisch	$-a\left(\frac{x}{2}-\frac{\sin(2x)}{4}\right)+ax$	18
parts	$ax-a\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)$	18
norman	$\frac{a\tan\left(\frac{x}{2}\right)+ax\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{ax}{2}-a\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{ax\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	51

input
```
int(a-a*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x+1/4*a*sin(2*x)
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \left(a - a\sin^2(x)\right)\,dx = \frac{1}{2}\,a\cos\left(x\right)\sin\left(x\right) + \frac{1}{2}\,ax$$

input
```
integrate(a-a*sin(x)^2,x, algorithm="fricas")
```

output
```
1/2*a*cos(x)*sin(x) + 1/2*a*x
```

3.31.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \left(a - a\sin^2(x)\right)\,dx = ax - a\left(\frac{x}{2} - \frac{\sin\left(x\right)\cos\left(x\right)}{2}\right)$$

input
```
integrate(a-a*sin(x)**2,x)
```

output
```
a*x - a*(x/2 - sin(x)*cos(x)/2)
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \left(a - a\sin^2(x)\right)\,dx = -\frac{1}{4}\,a(2\,x - \sin\left(2\,x\right)) + ax$$

input
```
integrate(a-a*sin(x)^2,x, algorithm="maxima")
```

output
```
-1/4*a*(2*x - sin(2*x)) + a*x
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \left(a - a\sin^2(x)\right) dx = -\frac{1}{4}a(2x - \sin(2x)) + ax$$

input
```
integrate(a-a*sin(x)^2,x, algorithm="giac")
```

output
```
-1/4*a*(2*x - sin(2*x)) + a*x
```

3.31.9 Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \left(a - a\sin^2(x)\right) dx = \frac{a(2x + \sin(2x))}{4}$$

input
```
int(a - a*sin(x)^2,x)
```

output
```
(a*(2*x + sin(2*x)))/4
```

3.32 $\int (a - a\sin^2(x))^2 \, dx$

3.32.1 Optimal result

Integrand size = 11, antiderivative size = 33

$$\int (a - a\sin^2(x))^2 \, dx = \frac{3a^2x}{8} + \frac{3}{8}a^2\cos(x)\sin(x) + \frac{1}{4}a^2\cos^3(x)\sin(x)$$

output
```
3/8*a^2*x+3/8*a^2*cos(x)*sin(x)+1/4*a^2*cos(x)^3*sin(x)
```

3.32.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int (a - a\sin^2(x))^2 \, dx = a^2\left(\frac{3x}{8} + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x)\right)$$

input
```
Integrate[(a - a*Sin[x]^2)^2,x]
```

output
```
a^2*((3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32)
```

3.32.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3042, 3654, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sin^2(x)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin(x)^2\right)^2 dx$$

$$\Big\downarrow \text{3654}$$

$$a^2\int \cos^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2\int \sin\left(x+\frac{\pi}{2}\right)^4 dx$$

$$\Big\downarrow \text{3115}$$

$$a^2\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2 dx+\frac{1}{4}\sin(x)\cos^3(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a^2\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)$$

$$\Big\downarrow \text{24}$$

$$a^2\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)$$

input
```
Int[(a - a*Sin[x]^2)^2,x]
```

output
```
a^2*((Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[x]*Sin[x])/2))/4)
```

3.32.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.32.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

method	result
parallelrisch	$\frac{a^2(12x+8\sin(2x)+\sin(4x))}{32}$
risch	$\frac{3a^2x}{8}+\frac{a^2\sin(4x)}{32}+\frac{a^2\sin(2x)}{4}$
default	$a^2\left(-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)-2a^2\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+a^2x$
parts	$a^2\left(-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)-2a^2\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+a^2x$
norman	$\frac{\frac{3a^2x}{8}+\frac{5a^2\tan\left(\frac{x}{2}\right)}{4}-\frac{3a^2\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}+\frac{3a^2\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}-\frac{5a^2\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{3a^2x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{9a^2x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{3a^2x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{3a^2}{}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$

input
```
int((a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*a^2*(12*x+8*sin(2*x)+sin(4*x))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \left(a-a\sin^2(x)\right)^2\,dx=\frac{3}{8}\,a^2x+\frac{1}{8}\left(2\,a^2\cos\left(x\right)^3+3\,a^2\cos\left(x\right)\right)\sin\left(x\right)$$

input
```
integrate((a-a*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
3/8*a^2*x + 1/8*(2*a^2*cos(x)^3 + 3*a^2*cos(x))*sin(x)
```

3.32.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(34) = 68.

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.33

$$\int \left(a-a\sin^2(x)\right)^2\,dx=\frac{3a^2x\sin^4\left(x\right)}{8}+\frac{3a^2x\sin^2\left(x\right)\cos^2\left(x\right)}{4}-a^2x\sin^2\left(x\right)+\frac{3a^2x\cos^4\left(x\right)}{8}-a^2x\cos^2\left(x\right)+a^2x-\frac{5a^2\sin^3\left(x\right)\cos\left(x\right)}{8}-\frac{3a^2\sin\left(x\right)\cos^3\left(x\right)}{8}+a^2\sin\left(x\right)\cos\left(x\right)$$

input
```
integrate((a-a*sin(x)**2)**2,x)
```

output
```
3*a**2*x*sin(x)**4/8 + 3*a**2*x*sin(x)**2*cos(x)**2/4 - a**2*x*sin(x)**2 +
 3*a**2*x*cos(x)**4/8 - a**2*x*cos(x)**2 + a**2*x - 5*a**2*sin(x)**3*cos(x
)/8 - 3*a**2*sin(x)*cos(x)**3/8 + a**2*sin(x)*cos(x)
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int \left(a-a\sin^2(x)\right)^2\,dx=\frac{1}{32}\,a^2(12\,x+\sin\left(4\,x\right)-8\,\sin\left(2\,x\right))-\frac{1}{2}\,a^2(2\,x-\sin\left(2\,x\right))+a^2x$$

input
```
integrate((a-a*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/32*a^2*(12*x + sin(4*x) - 8*sin(2*x)) - 1/2*a^2*(2*x - sin(2*x)) + a^2*x
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \left(a-a\sin^2(x)\right)^2\,dx=\frac{3}{8}\,a^2x+\frac{1}{32}\,a^2\sin\left(4\,x\right)+\frac{1}{4}\,a^2\sin\left(2\,x\right)$$

input
```
integrate((a-a*sin(x)^2)^2,x, algorithm="giac")
```

output
```
3/8*a^2*x + 1/32*a^2*sin(4*x) + 1/4*a^2*sin(2*x)
```

3.32.9 Mupad [B] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \left(a-a\sin^2(x)\right)^2\,dx=\frac{\frac{3\,a^2\tan(x)^3}{8}+\frac{5\,a^2\tan(x)}{8}}{\left(\tan\left(x\right)^2+1\right)^2}+\frac{3\,a^2\,x}{8}$$

input
```
int((a - a*sin(x)^2)^2,x)
```

output
```
((5*a^2*tan(x))/8 + (3*a^2*tan(x)^3)/8)/(tan(x)^2 + 1)^2 + (3*a^2*x)/8
```

3.33 $\int (a - a\sin^2(x))^3 \, dx$

3.33.1 Optimal result

Integrand size = 11, antiderivative size = 46

$$\int (a - a\sin^2(x))^3 \, dx = \frac{5a^3x}{16} + \frac{5}{16}a^3\cos(x)\sin(x) + \frac{5}{24}a^3\cos^3(x)\sin(x) + \frac{1}{6}a^3\cos^5(x)\sin(x)$$

output
```
5/16*a^3*x+5/16*a^3*cos(x)*sin(x)+5/24*a^3*cos(x)^3*sin(x)+1/6*a^3*cos(x)^
5*sin(x)
```

3.33.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int (a - a\sin^2(x))^3 \, dx = a^3\left(\frac{5x}{16} + \frac{15}{64}\sin(2x) + \frac{3}{64}\sin(4x) + \frac{1}{192}\sin(6x)\right)$$

input
```
Integrate[(a - a*Sin[x]^2)^3,x]
```

output
```
a^3*((5*x)/16 + (15*Sin[2*x])/64 + (3*Sin[4*x])/64 + Sin[6*x]/192)
```

3.33.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {3042, 3654, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sin^2(x)\right)^3 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin(x)^2\right)^3 dx$$

$$\Big\downarrow \text{3654}$$

$$a^3\int \cos^6(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a^3\int \sin\left(x+\frac{\pi}{2}\right)^6 dx$$

$$\Big\downarrow \text{3115}$$

$$a^3\left(\frac{5}{6}\int \cos^4(x)dx+\frac{1}{6}\sin(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3\left(\frac{5}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4 dx+\frac{1}{6}\sin(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a^3\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2 dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{24}$$

$$a^3\left(\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)\right)$$

input
```
Int[(a - a*Sin[x]^2)^3,x]
```

output
```
a^3*((Cos[x]^5*Sin[x])/6 + (5*((Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[x]*Sin
[x])/2))/4))/6)
```

3.33.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.33.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

method	result
parallelrisch	$\frac{a^3(60x+45\sin(2x)+\sin(6x)+9\sin(4x))}{192}$
risch	$\frac{5a^3x}{16}+\frac{a^3\sin(6x)}{192}+\frac{3a^3\sin(4x)}{64}+\frac{15a^3\sin(2x)}{64}$
default	$-a^3\left(-\frac{\left(\sin^5(x)+\frac{5(\sin^3(x))}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\frac{5x}{16}\right)+3a^3\left(-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)-3a^3\left(-\right.$
parts	$-a^3\left(-\frac{\left(\sin^5(x)+\frac{5(\sin^3(x))}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\frac{5x}{16}\right)+3a^3\left(-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)-3a^3\left(-\right.$
norman	$\frac{\frac{5a^3x}{16}+\frac{11a^3\tan\left(\frac{x}{2}\right)}{8}-\frac{5a^3\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}+\frac{15a^3\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}-\frac{15a^3\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{5a^3\left(\tan^9\left(\frac{x}{2}\right)\right)}{24}-\frac{11a^3\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{8}+\frac{15a^3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$

input
```
int((a-a*sin(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/192*a^3*(60*x+45*sin(2*x)+sin(6*x)+9*sin(4*x))
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \left(a-a\sin^2(x)\right)^3\,dx=\frac{5}{16}\,a^3x+\frac{1}{48}\left(8\,a^3\cos\left(x\right)^5+10\,a^3\cos\left(x\right)^3+15\,a^3\cos\left(x\right)\right)\sin\left(x\right)$$

input
```
integrate((a-a*sin(x)^2)^3,x, algorithm="fricas")
```

output
```
5/16*a^3*x + 1/48*(8*a^3*cos(x)^5 + 10*a^3*cos(x)^3 + 15*a^3*cos(x))*sin(x
)
```

3.33.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 233, normalized size of antiderivative = 5.07

$$\int \left(a-a\sin^2(x)\right)^3\,dx = -\frac{5a^3x\sin^6(x)}{16}-\frac{15a^3x\sin^4(x)\cos^2(x)}{16}+\frac{9a^3x\sin^4(x)}{8}-\frac{15a^3x\sin^2(x)\cos^4(x)}{16}+\frac{9a^3x\sin^2(x)\cos^2(x)}{4}-\frac{3a^3x\sin^2(x)}{2}-\frac{5a^3x\cos^6(x)}{16}+\frac{9a^3x\cos^4(x)}{8}-\frac{3a^3x\cos^2(x)}{2}+a^3x+\frac{11a^3\sin^5(x)\cos(x)}{16}+\frac{5a^3\sin^3(x)\cos^3(x)}{6}-\frac{15a^3\sin^3(x)\cos(x)}{8}+\frac{5a^3\sin(x)\cos^5(x)}{16}-\frac{9a^3\sin(x)\cos^3(x)}{8}+\frac{3a^3\sin(x)\cos(x)}{2}$$

input
```
integrate((a-a*sin(x)**2)**3,x)
```

output
```
-5*a**3*x*sin(x)**6/16 - 15*a**3*x*sin(x)**4*cos(x)**2/16 + 9*a**3*x*sin(x
)**4/8 - 15*a**3*x*sin(x)**2*cos(x)**4/16 + 9*a**3*x*sin(x)**2*cos(x)**2/4
 - 3*a**3*x*sin(x)**2/2 - 5*a**3*x*cos(x)**6/16 + 9*a**3*x*cos(x)**4/8 - 3
*a**3*x*cos(x)**2/2 + a**3*x + 11*a**3*sin(x)**5*cos(x)/16 + 5*a**3*sin(x)
**3*cos(x)**3/6 - 15*a**3*sin(x)**3*cos(x)/8 + 5*a**3*sin(x)*cos(x)**5/16
- 9*a**3*sin(x)*cos(x)**3/8 + 3*a**3*sin(x)*cos(x)/2
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

$$\int \left(a-a\sin^2(x)\right)^3\,dx = -\frac{1}{192}\left(4\sin(2x)^3+60x+9\sin(4x)-48\sin(2x)\right)a^3+\frac{3}{32}a^3(12x+\sin(4x)-8\sin(2x))-\frac{3}{4}a^3(2x-\sin(2x))+a^3x$$

input
```
integrate((a-a*sin(x)^2)^3,x, algorithm="maxima")
```

output
```
-1/192*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*a^3 + 3/32*a^3*(12
*x + sin(4*x) - 8*sin(2*x)) - 3/4*a^3*(2*x - sin(2*x)) + a^3*x
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \left(a-a\sin^2(x)\right)^3\,dx=\frac{5}{16}\,a^3x+\frac{1}{192}\,a^3\sin\left(6\,x\right)+\frac{3}{64}\,a^3\sin\left(4\,x\right)+\frac{15}{64}\,a^3\sin\left(2\,x\right)$$

input
```
integrate((a-a*sin(x)^2)^3,x, algorithm="giac")
```

output
```
5/16*a^3*x + 1/192*a^3*sin(6*x) + 3/64*a^3*sin(4*x) + 15/64*a^3*sin(2*x)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 14.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(a-a\sin^2(x)\right)^3\,dx=\frac{11\,a^3\cos\left(x\right)^5\,\sin\left(x\right)}{16}+\frac{5\,a^3\cos\left(x\right)^3\sin\left(x\right)^3}{6}+\frac{5\,a^3\,\cos\left(x\right)\,\sin\left(x\right)^5}{16}+\frac{5\,x\,a^3}{16}$$

input
```
int((a - a*sin(x)^2)^3,x)
```

output
```
(5*a^3*x)/16 + (5*a^3*cos(x)*sin(x)^5)/16 + (11*a^3*cos(x)^5*sin(x))/16 +
(5*a^3*cos(x)^3*sin(x)^3)/6
```

3.34 $\int (a-a\sin^2(x))^4\,dx$

3.34.1 Optimal result

Integrand size = 11, antiderivative size = 59

$$\int (a-a\sin^2(x))^4\,dx = \frac{35a^4x}{128}+\frac{35}{128}a^4\cos(x)\sin(x)+\frac{35}{192}a^4\cos^3(x)\sin(x)+\frac{7}{48}a^4\cos^5(x)\sin(x)+\frac{1}{8}a^4\cos^7(x)\sin(x)$$

output
```
35/128*a^4*x+35/128*a^4*cos(x)*sin(x)+35/192*a^4*cos(x)^3*sin(x)+7/48*a^4*
cos(x)^5*sin(x)+1/8*a^4*cos(x)^7*sin(x)
```

3.34.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int (a-a\sin^2(x))^4\,dx = a^4\left(\frac{35x}{128}+\frac{7}{32}\sin(2x)+\frac{7}{128}\sin(4x)+\frac{1}{96}\sin(6x)+\frac{\sin(8x)}{1024}\right)$$

input
```
Integrate[(a - a*Sin[x]^2)^4,x]
```

output
```
a^4*((35*x)/128 + (7*Sin[2*x])/32 + (7*Sin[4*x])/128 + Sin[6*x]/96 + Sin[8
*x]/1024)
```

3.34.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3654, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sin^2(x)\right)^4 dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a-a\sin(x)^2\right)^4 dx$$

$$\Big\downarrow \ 3654$$

$$a^4\int \cos^8(x)dx$$

$$\Big\downarrow \ 3042$$

$$a^4\int \sin\left(x+\frac{\pi}{2}\right)^8 dx$$

$$\Big\downarrow \ 3115$$

$$a^4\left(\frac{7}{8}\int \cos^6(x)dx+\frac{1}{8}\sin(x)\cos^7(x)\right)$$

$$\Big\downarrow \ 3042$$

$$a^4\left(\frac{7}{8}\int \sin\left(x+\frac{\pi}{2}\right)^6 dx+\frac{1}{8}\sin(x)\cos^7(x)\right)$$

$$\Big\downarrow \ 3115$$

$$a^4\left(\frac{7}{8}\left(\frac{5}{6}\int \cos^4(x)dx+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)$$

$$\Big\downarrow \ 3042$$

$$a^4\left(\frac{7}{8}\left(\frac{5}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4 dx+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)$$

$$\Big\downarrow \ 3115$$

$$a^4\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)$$

$$\Big\downarrow \ 3042$$

$$a^4\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)$$

$$\Big\downarrow \text{ 3115}$$

$$a^4\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)$$

$$\Big\downarrow \text{ 24}$$

$$a^4\left(\frac{1}{8}\sin(x)\cos^7(x)+\frac{7}{8}\left(\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)\right)\right)$$

input
```
Int[(a - a*Sin[x]^2)^4,x]
```

output
```
a^4*((Cos[x]^7*Sin[x])/8 + (7*((Cos[x]^5*Sin[x])/6 + (5*((Cos[x]^3*Sin[x])
/4 + (3*(x/2 + (Cos[x]*Sin[x])/2))/4))/6))/8)
```

3.34.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.34.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.58

method	result
parallelrisch	$\frac{a^4(840x+672\sin(2x)+3\sin(8x)+32\sin(6x)+168\sin(4x))}{3072}$
risch	$\frac{35a^4x}{128}+\frac{a^4\sin(8x)}{1024}+\frac{a^4\sin(6x)}{96}+\frac{7a^4\sin(4x)}{128}+\frac{7a^4\sin(2x)}{32}$
default	$a^4\left(-\frac{\left(\sin^7(x)+\frac{7(\sin^5(x))}{6}+\frac{35(\sin^3(x))}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8}+\frac{35x}{128}\right)-4a^4\left(-\frac{\left(\sin^5(x)+\frac{5(\sin^3(x))}{4}+\frac{15\sin(x)}{8}\right)\cos(}{6}\right.$
parts	$a^4\left(-\frac{\left(\sin^7(x)+\frac{7(\sin^5(x))}{6}+\frac{35(\sin^3(x))}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8}+\frac{35x}{128}\right)-4a^4\left(-\frac{\left(\sin^5(x)+\frac{5(\sin^3(x))}{4}+\frac{15\sin(x)}{8}\right)\cos(}{6}\right.$
norman	$\frac{\frac{35a^4x}{128}+\frac{93a^4\tan\left(\frac{x}{2}\right)}{64}+\frac{91a^4\left(\tan^3\left(\frac{x}{2}\right)\right)}{192}+\frac{1799a^4\left(\tan^5\left(\frac{x}{2}\right)\right)}{192}-\frac{1085a^4\left(\tan^7\left(\frac{x}{2}\right)\right)}{192}+\frac{1085a^4\left(\tan^9\left(\frac{x}{2}\right)\right)}{192}-\frac{1799a^4\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{192}-\frac{91a^4(\mathrm{ta}}{1}}{}$

input
```
int((a-a*sin(x)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3072*a^4*(840*x+672*sin(2*x)+3*sin(8*x)+32*sin(6*x)+168*sin(4*x))
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int \left(a-a\sin^2(x)\right)^4 dx$$
$$=\frac{35}{128}a^4x+\frac{1}{384}\left(48a^4\cos(x)^7+56a^4\cos(x)^5+70a^4\cos(x)^3+105a^4\cos(x)\right)\sin(x)$$

input
```
integrate((a-a*sin(x)^2)^4,x, algorithm="fricas")
```

output
```
35/128*a^4*x + 1/384*(48*a^4*cos(x)^7 + 56*a^4*cos(x)^5 + 70*a^4*cos(x)^3
+ 105*a^4*cos(x))*sin(x)
```

3.34.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(65) = 130$.

Time = 0.49 (sec) , antiderivative size = 376, normalized size of antiderivative = 6.37

$$\int \left(a-a\sin^2(x)\right)^4\,dx = \frac{35a^4x\sin^8(x)}{128} + \frac{35a^4x\sin^6(x)\cos^2(x)}{32} - \frac{5a^4x\sin^6(x)}{4} + \frac{105a^4x\sin^4(x)\cos^4(x)}{64} - \frac{15a^4x\sin^4(x)\cos^2(x)}{4} + \frac{9a^4x\sin^4(x)}{4} + \frac{35a^4x\sin^2(x)\cos^6(x)}{32} - \frac{15a^4x\sin^2(x)\cos^4(x)}{4} + \frac{9a^4x\sin^2(x)\cos^2(x)}{2} - 2a^4x\sin^2(x) + \frac{35a^4x\cos^8(x)}{128} - \frac{5a^4x\cos^6(x)}{4} + \frac{9a^4x\cos^4(x)}{4} - 2a^4x\cos^2(x) + a^4x - \frac{93a^4\sin^7(x)\cos(x)}{128} - \frac{511a^4\sin^5(x)\cos^3(x)}{384} + \frac{11a^4\sin^5(x)\cos(x)}{4} - \frac{385a^4\sin^3(x)\cos^5(x)}{384} + \frac{10a^4\sin^3(x)\cos^3(x)}{3} - \frac{15a^4\sin^3(x)\cos(x)}{4} - \frac{35a^4\sin(x)\cos^7(x)}{128} + \frac{5a^4\sin(x)\cos^5(x)}{4} - \frac{9a^4\sin(x)\cos^3(x)}{4} + 2a^4\sin(x)\cos(x)$$

input
```
integrate((a-a*sin(x)**2)**4,x)
```

output
```
35*a**4*x*sin(x)**8/128 + 35*a**4*x*sin(x)**6*cos(x)**2/32 - 5*a**4*x*sin(
x)**6/4 + 105*a**4*x*sin(x)**4*cos(x)**4/64 - 15*a**4*x*sin(x)**4*cos(x)**
2/4 + 9*a**4*x*sin(x)**4/4 + 35*a**4*x*sin(x)**2*cos(x)**6/32 - 15*a**4*x*
sin(x)**2*cos(x)**4/4 + 9*a**4*x*sin(x)**2*cos(x)**2/2 - 2*a**4*x*sin(x)**
2 + 35*a**4*x*cos(x)**8/128 - 5*a**4*x*cos(x)**6/4 + 9*a**4*x*cos(x)**4/4
- 2*a**4*x*cos(x)**2 + a**4*x - 93*a**4*sin(x)**7*cos(x)/128 - 511*a**4*si
n(x)**5*cos(x)**3/384 + 11*a**4*sin(x)**5*cos(x)/4 - 385*a**4*sin(x)**3*co
s(x)**5/384 + 10*a**4*sin(x)**3*cos(x)**3/3 - 15*a**4*sin(x)**3*cos(x)/4 -
 35*a**4*sin(x)*cos(x)**7/128 + 5*a**4*sin(x)*cos(x)**5/4 - 9*a**4*sin(x)*
cos(x)**3/4 + 2*a**4*sin(x)*cos(x)
```

3.34.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(49) = 98.

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.76

$$\int \left(a-a\sin^2(x)\right)^4\,dx$$
$$=\frac{1}{3072}\left(128\sin(2x)^3+840x+3\sin(8x)+168\sin(4x)-768\sin(2x)\right)a^4$$
$$-\frac{1}{48}\left(4\sin(2x)^3+60x+9\sin(4x)-48\sin(2x)\right)a^4$$
$$+\frac{3}{16}a^4(12x+\sin(4x)-8\sin(2x))-a^4(2x-\sin(2x))+a^4x$$

input
```
integrate((a-a*sin(x)^2)^4,x, algorithm="maxima")
```

output
```
1/3072*(128*sin(2*x)^3 + 840*x + 3*sin(8*x) + 168*sin(4*x) - 768*sin(2*x))
*a^4 - 1/48*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*a^4 + 3/16*a^
4*(12*x + sin(4*x) - 8*sin(2*x)) - a^4*(2*x - sin(2*x)) + a^4*x
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \left(a-a\sin^2(x)\right)^4\,dx=\frac{35}{128}a^4x+\frac{1}{1024}a^4\sin(8x)+\frac{1}{96}a^4\sin(6x)$$
$$+\frac{7}{128}a^4\sin(4x)+\frac{7}{32}a^4\sin(2x)$$

input
```
integrate((a-a*sin(x)^2)^4,x, algorithm="giac")
```

output
```
35/128*a^4*x + 1/1024*a^4*sin(8*x) + 1/96*a^4*sin(6*x) + 7/128*a^4*sin(4*x
) + 7/32*a^4*sin(2*x)
```

3.34.9 Mupad [B] (verification not implemented)

Time = 13.92 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \left(a - a\sin^2(x)\right)^4 \, dx = \frac{\frac{35\,a^4\tan(x)^7}{128} + \frac{385\,a^4\tan(x)^5}{384} + \frac{511\,a^4\tan(x)^3}{384} + \frac{93\,a^4\tan(x)}{128}}{\left(\tan\left(x\right)^2 + 1\right)^4} + \frac{35\,a^4\,x}{128}$$

input
```
int((a - a*sin(x)^2)^4,x)
```

output
```
((93*a^4*tan(x))/128 + (511*a^4*tan(x)^3)/384 + (385*a^4*tan(x)^5)/384 + (
35*a^4*tan(x)^7)/128)/(tan(x)^2 + 1)^4 + (35*a^4*x)/128
```

3.35 $\int \frac{\sin^7(c+dx)}{a-a\sin^2(c+dx)}\,dx$

3.35.1 Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{\sin^7(c+dx)}{a-a\sin^2(c+dx)}\,dx=\frac{3\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{ad}+\frac{\cos^5(c+dx)}{5ad}+\frac{\sec(c+dx)}{ad}$$

output
```
3*cos(d*x+c)/a/d-cos(d*x+c)^3/a/d+1/5*cos(d*x+c)^5/a/d+sec(d*x+c)/a/d
```

3.35.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{\sin^7(c+dx)}{a-a\sin^2(c+dx)}\,dx=\frac{\frac{19\cos(c+dx)}{8d}-\frac{3\cos(3(c+dx))}{16d}+\frac{\cos(5(c+dx))}{80d}+\frac{\sec(c+dx)}{d}}{a}$$

input
```
Integrate[Sin[c + d*x]^7/(a - a*Sin[c + d*x]^2),x]
```

output
```
((19*Cos[c + d*x])/(8*d) - (3*Cos[3*(c + d*x)])/(16*d) + Cos[5*(c + d*x)]/
(80*d) + Sec[c + d*x]/d)/a
```

3.35.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3654, 3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(c+dx)}{a-a\sin^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^7}{a-a\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sin^5(c+dx)\tan^2(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin(c+dx)^5\tan(c+dx)^2dx}{a}$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int \left(1-\cos^2(c+dx)\right)^3\sec^2(c+dx)d\cos(c+dx)}{ad}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(-\cos^4(c+dx)+3\cos^2(c+dx)+\sec^2(c+dx)-3\right)d\cos(c+dx)}{ad}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{5}\cos^5(c+dx)+\cos^3(c+dx)-3\cos(c+dx)-\sec(c+dx)}{ad}$$

input
```
Int[Sin[c + d*x]^7/(a - a*Sin[c + d*x]^2),x]
```

output
```
-((-3*Cos[c + d*x] + Cos[c + d*x]^3 - Cos[c + d*x]^5/5 - Sec[c + d*x])/(a*
d))
```

3.35.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.35.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\frac{\left(\cos^5(dx+c)\right)}{5}-\left(\cos^3(dx+c)\right)+3\cos(dx+c)+\frac{1}{\cos(dx+c)}}{da}$	45
default	$\frac{\frac{\left(\cos^5(dx+c)\right)}{5}-\left(\cos^3(dx+c)\right)+3\cos(dx+c)+\frac{1}{\cos(dx+c)}}{da}$	45
parallelrisch	$\frac{350+175\cos(2dx+2c)-14\cos(4dx+4c)+512\cos(dx+c)+\cos(6dx+6c)}{160ad\cos(dx+c)}$	58
risch	$\frac{19\,e^{i(dx+c)}}{16ad}+\frac{19\,e^{-i(dx+c)}}{16ad}+\frac{2\,e^{i(dx+c)}}{da\left(e^{2i(dx+c)}+1\right)}+\frac{\cos(5dx+5c)}{80ad}-\frac{3\cos(3dx+3c)}{16ad}$	100
norman	$\frac{-\frac{32\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{32}{5ad}-\frac{192\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5da}-\frac{448\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5da}-\frac{448\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	117

input
```
int(sin(d*x+c)^7/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(1/5*cos(d*x+c)^5-cos(d*x+c)^3+3*cos(d*x+c)+1/cos(d*x+c))
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int \frac{\sin^7(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\cos(dx+c)^6 - 5\cos(dx+c)^4 + 15\cos(dx+c)^2 + 5}{5\,ad\cos(dx+c)}$$

input
```
integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/5*(cos(d*x + c)^6 - 5*cos(d*x + c)^4 + 15*cos(d*x + c)^2 + 5)/(a*d*cos(d
*x + c))
```

3.35.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(46) = 92.

Time = 10.46 (sec) , antiderivative size = 314, normalized size of antiderivative = 5.06

$$\int \frac{\sin^7(c+dx)}{a-a\sin^2(c+dx)}\,dx$$
$$= \begin{cases} -\frac{160\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{5ad\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)+20ad\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+25ad\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)-25ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-20ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-5ad} - \frac{}{5ad\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)+20a \cdots} \\ \frac{x\sin^7(c)}{-a\sin^2(c)+a} \end{cases}$$

input
```
integrate(sin(d*x+c)**7/(a-a*sin(d*x+c)**2),x)
```

output
```
Piecewise((-160*tan(c/2 + d*x/2)**4/(5*a*d*tan(c/2 + d*x/2)**12 + 20*a*d*t
an(c/2 + d*x/2)**10 + 25*a*d*tan(c/2 + d*x/2)**8 - 25*a*d*tan(c/2 + d*x/2)
**4 - 20*a*d*tan(c/2 + d*x/2)**2 - 5*a*d) - 128*tan(c/2 + d*x/2)**2/(5*a*d
*tan(c/2 + d*x/2)**12 + 20*a*d*tan(c/2 + d*x/2)**10 + 25*a*d*tan(c/2 + d*x
/2)**8 - 25*a*d*tan(c/2 + d*x/2)**4 - 20*a*d*tan(c/2 + d*x/2)**2 - 5*a*d)
- 32/(5*a*d*tan(c/2 + d*x/2)**12 + 20*a*d*tan(c/2 + d*x/2)**10 + 25*a*d*ta
n(c/2 + d*x/2)**8 - 25*a*d*tan(c/2 + d*x/2)**4 - 20*a*d*tan(c/2 + d*x/2)**
2 - 5*a*d), Ne(d, 0)), (x*sin(c)**7/(-a*sin(c)**2 + a), True))
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{\sin^7(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\frac{\cos(dx+c)^5-5\cos(dx+c)^3+15\cos(dx+c)}{a}+\frac{5}{a\cos(dx+c)}}{5\,d}$$

input
```
integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/5*((cos(d*x + c)^5 - 5*cos(d*x + c)^3 + 15*cos(d*x + c))/a + 5/(a*cos(d*
x + c)))/d
```

3.35.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(60) = 120.

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.40

$$\int \frac{\sin^7(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{2\left(\frac{5}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}+\frac{\frac{50(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{80(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{30(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{5(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-11}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}\right)}{5\,d}$$

input
```
integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
2/5*(5/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + (50*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) - 80*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
30*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 5*(cos(d*x + c) - 1)^4/(cos
(d*x + c) + 1)^4 - 11)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5))/
d
```

3.35.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{\sin^7(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\frac{3\cos(c+dx)}{a}+\frac{1}{a\cos(c+dx)}-\frac{\cos(c+dx)^3}{a}+\frac{\cos(c+dx)^5}{5a}}{d}$$

input
```
int(sin(c + d*x)^7/(a - a*sin(c + d*x)^2),x)
```

output
```
((3*cos(c + d*x))/a + 1/(a*cos(c + d*x)) - cos(c + d*x)^3/a + cos(c + d*x)
^5/(5*a))/d
```

3.36 $\int \frac{\sin^5(c+dx)}{a-a\sin^2(c+dx)}\,dx$

3.36.1 Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{\sin^5(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{2\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}$$

output
```
2*cos(d*x+c)/a/d-1/3*cos(d*x+c)^3/a/d+sec(d*x+c)/a/d
```

3.36.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{\sin^5(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\frac{7\cos(c+dx)}{4d}-\frac{\cos(3(c+dx))}{12d}+\frac{\sec(c+dx)}{d}}{a}$$

input
```
Integrate[Sin[c + d*x]^5/(a - a*Sin[c + d*x]^2),x]
```

output
```
((7*Cos[c + d*x])/(4*d) - Cos[3*(c + d*x)]/(12*d) + Sec[c + d*x]/d)/a
```

3.36.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3654, 3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(c+dx)}{a-a\sin^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^5}{a-a\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sin^3(c+dx)\tan^2(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin(c+dx)^3\tan(c+dx)^2dx}{a}$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int \left(1-\cos^2(c+dx)\right)^2\sec^2(c+dx)d\cos(c+dx)}{ad}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\cos^2(c+dx)+\sec^2(c+dx)-2\right)d\cos(c+dx)}{ad}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{3}\cos^3(c+dx)-2\cos(c+dx)-\sec(c+dx)}{ad}$$

input
```
Int[Sin[c + d*x]^5/(a - a*Sin[c + d*x]^2),x]
```

output
```
-((-2*Cos[c + d*x] + Cos[c + d*x]^3/3 - Sec[c + d*x])/(a*d))
```

3.36.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.36.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{-\frac{\left(\cos^3(dx+c)\right)}{3}+2\cos(dx+c)+\frac{1}{\cos(dx+c)}}{da}$	35
default	$\frac{-\frac{\left(\cos^3(dx+c)\right)}{3}+2\cos(dx+c)+\frac{1}{\cos(dx+c)}}{da}$	35
parallelrisch	$\frac{45+20\cos(2dx+2c)-\cos(4dx+4c)+64\cos(dx+c)}{24ad\cos(dx+c)}$	49
risch	$\frac{7\,\mathrm{e}^{i(dx+c)}}{8ad}+\frac{7\,\mathrm{e}^{-i(dx+c)}}{8ad}+\frac{2\,\mathrm{e}^{i(dx+c)}}{da\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{\cos(3dx+3c)}{12ad}$	83
norman	$\frac{-\frac{32\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{16}{3ad}-\frac{64\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{80\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	98

input
```
int(sin(d*x+c)^5/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(-1/3*cos(d*x+c)^3+2*cos(d*x+c)+1/cos(d*x+c))
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{\sin^5(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{\cos(dx+c)^4-6\,\cos(dx+c)^2-3}{3\,ad\cos(dx+c)}$$

input
```
integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/3*(cos(d*x + c)^4 - 6*cos(d*x + c)^2 - 3)/(a*d*cos(d*x + c))
```

3.36.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(34) = 68.

Time = 4.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.11

$$\int \frac{\sin^5(c+dx)}{a-a\sin^2(c+dx)}\,dx$$
$$= \begin{cases} -\frac{32\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-6ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3ad} - \frac{16}{3ad\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-6ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3ad} & \text{fo} \\ \frac{x\sin^5(c)}{-a\sin^2(c)+a} & \text{otl} \end{cases}$$

input
```
integrate(sin(d*x+c)**5/(a-a*sin(d*x+c)**2),x)
```

output
```
Piecewise((-32*tan(c/2 + d*x/2)**2/(3*a*d*tan(c/2 + d*x/2)**8 + 6*a*d*tan(
c/2 + d*x/2)**6 - 6*a*d*tan(c/2 + d*x/2)**2 - 3*a*d) - 16/(3*a*d*tan(c/2 +
 d*x/2)**8 + 6*a*d*tan(c/2 + d*x/2)**6 - 6*a*d*tan(c/2 + d*x/2)**2 - 3*a*d
), Ne(d, 0)), (x*sin(c)**5/(-a*sin(c)**2 + a), True))
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{\sin^5(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{\frac{\cos(dx+c)^3-6\cos(dx+c)}{a}-\frac{3}{a\cos(dx+c)}}{3\,d}$$

input
```
integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/3*((cos(d*x + c)^3 - 6*cos(d*x + c))/a - 3/(a*cos(d*x + c)))/d
```

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(44) = 88.

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.28

$$\int \frac{\sin^5(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{2\left(\frac{3}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}+\frac{\frac{12\,(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-5}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^3}\right)}{3\,d}$$

input
```
integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
2/3*(3/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + (12*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) - 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 5
)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^3))/d
```

3.36.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{\sin^5(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{-\cos\left(c+d\,x\right)^4+6\cos\left(c+d\,x\right)^2+3}{3\,a\,d\,\cos\left(c+d\,x\right)}$$

input
```
int(sin(c + d*x)^5/(a - a*sin(c + d*x)^2),x)
```

output
```
(6*cos(c + d*x)^2 - cos(c + d*x)^4 + 3)/(3*a*d*cos(c + d*x))
```

3.37 $\int \frac{\sin^3(c+dx)}{a-a\sin^2(c+dx)}\,dx$

3.37.1 Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sin^3(c+dx)}{a-a\sin^2(c+dx)}\,dx=\frac{\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

output
```
cos(d*x+c)/a/d+sec(d*x+c)/a/d
```

3.37.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sin^3(c+dx)}{a-a\sin^2(c+dx)}\,dx=\frac{\frac{\cos(c+dx)}{d}+\frac{\sec(c+dx)}{d}}{a}$$

input
```
Integrate[Sin[c + d*x]^3/(a - a*Sin[c + d*x]^2),x]
```

output
```
(Cos[c + d*x]/d + Sec[c + d*x]/d)/a
```

3.37.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3654, 3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{a-a\sin^2(c+dx)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(c+dx)^3}{a-a\sin(c+dx)^2}dx$$
$$\Big\downarrow \text{ 3654}$$
$$\frac{\int \sin(c+dx)\tan^2(c+dx)dx}{a}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \sin(c+dx)\tan(c+dx)^2dx}{a}$$
$$\Big\downarrow \text{ 3070}$$
$$-\frac{\int \left(1-\cos^2(c+dx)\right)\sec^2(c+dx)d\cos(c+dx)}{ad}$$
$$\Big\downarrow \text{ 244}$$
$$-\frac{\int \left(\sec^2(c+dx)-1\right)d\cos(c+dx)}{ad}$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{-\cos(c+dx)-\sec(c+dx)}{ad}$$

input
```
Int[Sin[c + d*x]^3/(a - a*Sin[c + d*x]^2),x]
```

output
```
-((-Cos[c + d*x] - Sec[c + d*x])/(a*d))
```

3.37.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.37.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\cos(dx+c)+\frac{1}{\cos(dx+c)}}{da}$	23
default	$\frac{\cos(dx+c)+\frac{1}{\cos(dx+c)}}{da}$	23
parallelrisch	$\frac{3+\cos(2dx+2c)+4\cos(dx+c)}{2ad\cos(dx+c)}$	36
risch	$\frac{e^{3i(dx+c)}+7\cos(dx+c)+5i\sin(dx+c)}{2da\left(e^{2i(dx+c)}+1\right)}$	49
norman	$\frac{-\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{4}{ad}-\frac{8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	79

input
```
int(sin(d*x+c)^3/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(cos(d*x+c)+1/cos(d*x+c))
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sin^3(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\cos{(dx+c)}^2+1}{ad\cos{(dx+c)}}$$

input
```
integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
(cos(d*x + c)^2 + 1)/(a*d*cos(d*x + c))
```

3.37.6 Sympy [A] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \frac{\sin^3(c+dx)}{a-a\sin^2(c+dx)}\,dx = \begin{cases} -\frac{4}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-ad} & \text{for } d\neq 0 \\ \frac{x\sin^3{(c)}}{-a\sin^2{(c)}+a} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(d*x+c)**3/(a-a*sin(d*x+c)**2),x)
```

output
```
Piecewise((-4/(a*d*tan(c/2 + d*x/2)**4 - a*d), Ne(d, 0)), (x*sin(c)**3/(-a
*sin(c)**2 + a), True))
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\frac{\cos(dx+c)}{a}+\frac{1}{a\cos(dx+c)}}{d}$$

input
```
integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
(cos(d*x + c)/a + 1/(a*cos(d*x + c)))/d
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\sin^3(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\cos(dx+c)}{ad} + \frac{1}{ad\cos(dx+c)}$$

input
```
integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
cos(d*x + c)/(a*d) + 1/(a*d*cos(d*x + c))
```

3.37.9 Mupad [B] (verification not implemented)

Time = 13.95 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\sin^3(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\cos(c+d\,x)^2+1}{a\,d\,\cos(c+d\,x)}$$

input
```
int(sin(c + d*x)^3/(a - a*sin(c + d*x)^2),x)
```

output
```
(cos(c + d*x)^2 + 1)/(a*d*cos(c + d*x))
```

3.38 $\int \frac{\sin(c+dx)}{a-a\sin^2(c+dx)}\,dx$

3.38.1 Optimal result

Integrand size = 22, antiderivative size = 13

$$\int \frac{\sin(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\sec(c+dx)}{ad}$$

output
```
sec(d*x+c)/a/d
```

3.38.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\sin(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\sec(c+dx)}{ad}$$

input
```
Integrate[Sin[c + d*x]/(a - a*Sin[c + d*x]^2),x]
```

output
```
Sec[c + d*x]/(a*d)
```

3.38.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 3654, 3042, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{a-a\sin^2(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(c+dx)}{a-a\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{ 3654}$$

$$\frac{\int \sec(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sec(c+dx)\tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3086}$$

$$\frac{\int 1d\sec(c+dx)}{ad}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sec(c+dx)}{ad}$$

input
```
Int[Sin[c + d*x]/(a - a*Sin[c + d*x]^2),x]
```

output
```
Sec[c + d*x]/(a*d)
```

3.38.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.38.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

method	result	size
derivativedivides	$\frac{1}{da\cos(dx+c)}$	16
default	$\frac{1}{da\cos(dx+c)}$	16
parallelrisch	$-\frac{2}{da\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	24
risch	$\frac{2\,\mathrm{e}^{i(dx+c)}}{da\left(\mathrm{e}^{2i(dx+c)}+1\right)}$	31
norman	$\frac{-\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{2}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	60

input
```
int(sin(d*x+c)/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a/cos(d*x+c)
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\sin(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{1}{ad\cos(dx+c)}$$

input
```
integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/(a*d*cos(d*x + c))
```

3.38.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(8) = 16.

Time = 0.59 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.62

$$\int \frac{\sin(c+dx)}{a-a\sin^2(c+dx)}\,dx = \begin{cases} -\frac{2}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad} & \text{for } d\neq 0 \\ \frac{x\sin(c)}{-a\sin^2(c)+a} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(d*x+c)/(a-a*sin(d*x+c)**2),x)
```

output
```
Piecewise((-2/(a*d*tan(c/2 + d*x/2)**2 - a*d), Ne(d, 0)), (x*sin(c)/(-a*si
n(c)**2 + a), True))
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\sin(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{1}{ad\cos(dx+c)}$$

input
```
integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/(a*d*cos(d*x + c))
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\sin(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{1}{ad\cos(dx+c)}$$

input
```
integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/(a*d*cos(d*x + c))
```

3.38.9 Mupad [B] (verification not implemented)

Time = 13.81 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\sin(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{1}{a\,d\,\cos(c+d\,x)}$$

input
```
int(sin(c + d*x)/(a - a*sin(c + d*x)^2),x)
```

output
```
1/(a*d*cos(c + d*x))
```

3.39 $\int \frac{\csc(c+dx)}{a-a\sin^2(c+dx)}\,dx$

3.39.1 Optimal result

Integrand size = 22, antiderivative size = 29

$$\int \frac{\csc(c+dx)}{a-a\sin^2(c+dx)}\,dx=-\frac{\operatorname{arctanh}(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}$$

output
```
-arctanh(cos(d*x+c))/a/d+sec(d*x+c)/a/d
```

3.39.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.59

$$\int \frac{\csc(c+dx)}{a-a\sin^2(c+dx)}\,dx=\frac{-\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{\sec(c+dx)}{d}}{a}$$

input
```
Integrate[Csc[c + d*x]/(a - a*Sin[c + d*x]^2),x]
```

output
```
(-(Log[Cos[(c + d*x)/2]]/d) + Log[Sin[(c + d*x)/2]]/d + Sec[c + d*x]/d)/a
```

3.39.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3042, 3654, 3042, 3102, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{a-a\sin^2(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(c+dx)\left(a-a\sin(c+dx)^2\right)}dx$$

$$\Big\downarrow \text{ 3654}$$

$$\frac{\int \csc(c+dx)\sec^2(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc(c+dx)\sec(c+dx)^2dx}{a}$$

$$\Big\downarrow \text{ 3102}$$

$$\frac{\int -\frac{\sec^2(c+dx)}{1-\sec^2(c+dx)}d\sec(c+dx)}{ad}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\sec^2(c+dx)}{1-\sec^2(c+dx)}d\sec(c+dx)}{ad}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{\sec(c+dx)-\int \frac{1}{1-\sec^2(c+dx)}d\sec(c+dx)}{ad}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sec(c+dx)-\text{arctanh}(\sec(c+dx))}{ad}$$

input
```
Int[Csc[c + d*x]/(a - a*Sin[c + d*x]^2),x]
```

output
```
(-ArcTanh[Sec[c + d*x]] + Sec[c + d*x])/(a*d)
```

3.39.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.39.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

method	result	size
derivativedivides	$\frac{-\frac{\ln(1+\cos(dx+c))}{2}+\frac{1}{\cos(dx+c)}+\frac{\ln(\cos(dx+c)-1)}{2}}{da}$	39
default	$\frac{-\frac{\ln(1+\cos(dx+c))}{2}+\frac{1}{\cos(dx+c)}+\frac{\ln(\cos(dx+c)-1)}{2}}{da}$	39
norman	$-\frac{2}{da\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$	42
parallelrisch	$\frac{-2+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	59
risch	$\frac{2\,\mathrm{e}^{i(dx+c)}}{da\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{da}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{da}$	71

input
```
int(csc(d*x+c)/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(-1/2*ln(1+cos(d*x+c))+1/cos(d*x+c)+1/2*ln(cos(d*x+c)-1))
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.90

$$\int \frac{\csc(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-\cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2}{2\,ad\cos(dx+c)}$$

input
```
integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - cos(d*x + c)*log(-1/2*cos
(d*x + c) + 1/2) - 2)/(a*d*cos(d*x + c))
```

3.39.6 Sympy [F]

$$\int \frac{\csc(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{\int \frac{\csc(c+dx)}{\sin^2(c+dx)-1}\,dx}{a}$$

input
```
integrate(csc(d*x+c)/(a-a*sin(d*x+c)**2),x)
```

output
```
-Integral(csc(c + d*x)/(sin(c + d*x)**2 - 1), x)/a
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.59

$$\int \frac{\csc(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{\frac{\log(\cos(dx+c)+1)}{a} - \frac{\log(\cos(dx+c)-1)}{a} - \frac{2}{a\cos(dx+c)}}{2\,d}$$

input
```
integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/2*(log(cos(d*x + c) + 1)/a - log(cos(d*x + c) - 1)/a - 2/(a*cos(d*x + c
)))/d
```

3.39.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(29) = 58.

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.14

$$\int \frac{\csc(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a} + \frac{4}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{2\,d}$$

input
```
integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + 4/(a*((cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

3.39.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\csc(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{1}{a\,d\,\cos\left(c+d\,x\right)}-\frac{\operatorname{atanh}(\cos\left(c+d\,x\right))}{a\,d}$$

input
```
int(1/(sin(c + d*x)*(a - a*sin(c + d*x)^2)),x)
```

output
```
1/(a*d*cos(c + d*x)) - atanh(cos(c + d*x))/(a*d)
```

3.40 $\int \frac{\csc^3(c+dx)}{a-a\sin^2(c+dx)}\,dx$

3.40.1 Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{\csc^3(c+dx)}{a-a\sin^2(c+dx)}\,dx=-\frac{3\text{arctanh}(\cos(c+dx))}{2ad}+\frac{3\sec(c+dx)}{2ad}-\frac{\csc^2(c+dx)\sec(c+dx)}{2ad}$$

output
```
-3/2*arctanh(cos(d*x+c))/a/d+3/2*sec(d*x+c)/a/d-1/2*csc(d*x+c)^2*sec(d*x+c
)/a/d
```

3.40.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 146 vs. 2(58) = 116.

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.52

$$\int \frac{\csc^3(c+dx)}{a-a\sin^2(c+dx)}\,dx$$
$$=\frac{\csc^4(c+dx)\left(2-6\cos(2(c+dx))+2\cos(3(c+dx))+3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-3\cos(3(c\right.}{2ad\left(\csc^2\left(\frac{1}{2}(c+dx)\right)-\right.}$$

input
```
Integrate[Csc[c + d*x]^3/(a - a*Sin[c + d*x]^2),x]
```

output
```
(Csc[c + d*x]^4*(2 - 6*Cos[2*(c + d*x)] + 2*Cos[3*(c + d*x)] + 3*Cos[3*(c
+ d*x)]*Log[Cos[(c + d*x)/2]] - 3*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] +
 Cos[c + d*x]*(-2 - 3*Log[Cos[(c + d*x)/2]] + 3*Log[Sin[(c + d*x)/2]])))/(
2*a*d*(Csc[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2))
```

3.40.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 3654, 3042, 3102, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)}{a-a\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^3\left(a-a\sin(c+dx)^2\right)}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \csc^3(c+dx)\sec^2(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(c+dx)^3\sec(c+dx)^2dx}{a}$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int \frac{\sec^4(c+dx)}{\left(1-\sec^2(c+dx)\right)^2}d\sec(c+dx)}{ad}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\sec^3(c+dx)}{2\left(1-\sec^2(c+dx)\right)}-\frac{3}{2}\int \frac{\sec^2(c+dx)}{1-\sec^2(c+dx)}d\sec(c+dx)}{ad}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{\sec^3(c+dx)}{2\left(1-\sec^2(c+dx)\right)}-\frac{3}{2}\left(\int \frac{1}{1-\sec^2(c+dx)}d\sec(c+dx)-\sec(c+dx)\right)}{ad}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\sec^3(c+dx)}{2\left(1-\sec^2(c+dx)\right)}-\frac{3}{2}\left(\text{arctanh}(\sec(c+dx))-\sec(c+dx)\right)}{ad}$$

input
```
Int[Csc[c + d*x]^3/(a - a*Sin[c + d*x]^2),x]
```

output
```
((-3*(ArcTanh[Sec[c + d*x]] - Sec[c + d*x]))/2 + Sec[c + d*x]^3/(2*(1 - Se
c[c + d*x]^2)))/(a*d)
```

3.40.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.40.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\frac{1}{4+4\cos(dx+c)}-\frac{3\ln(1+\cos(dx+c))}{4}+\frac{1}{\cos(dx+c)}+\frac{1}{4\cos(dx+c)-4}+\frac{3\ln(\cos(dx+c)-1)}{4}}{da}$	63
default	$\frac{\frac{1}{4+4\cos(dx+c)}-\frac{3\ln(1+\cos(dx+c))}{4}+\frac{1}{\cos(dx+c)}+\frac{1}{4\cos(dx+c)-4}+\frac{3\ln(\cos(dx+c)-1)}{4}}{da}$	63
parallelrisch	$\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)+12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\cot^2\left(\frac{dx}{2}+\frac{c}{2}\right)-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-18}{8da\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	83
norman	$\frac{\frac{1}{8ad}+\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}-\frac{9\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}$	94
risch	$\frac{3\,e^{5i(dx+c)}-2\,e^{3i(dx+c)}+3\,e^{i(dx+c)}}{da\left(e^{2i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)}+\frac{3\ln\left(e^{i(dx+c)}-1\right)}{2da}-\frac{3\ln\left(e^{i(dx+c)}+1\right)}{2da}$	109

input
```
int(csc(d*x+c)^3/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(1/4/(1+cos(d*x+c))-3/4*ln(1+cos(d*x+c))+1/cos(d*x+c)+1/4/(cos(d*x+c
)-1)+3/4*ln(cos(d*x+c)-1))
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.69

$$\int \frac{\csc^3(c+dx)}{a-a\sin^2(c+dx)}\,dx$$
$$= \frac{6\,\cos(dx+c)^2-3\left(\cos(dx+c)^3-\cos(dx+c)\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)+3\left(\cos(dx+c)^3-\cos(dx+\right.}{4\left(ad\cos(dx+c)^3-ad\cos(dx+c)\right)}$$

input
```
integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/4*(6*cos(d*x + c)^2 - 3*(cos(d*x + c)^3 - cos(d*x + c))*log(1/2*cos(d*x
+ c) + 1/2) + 3*(cos(d*x + c)^3 - cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/
2) - 4)/(a*d*cos(d*x + c)^3 - a*d*cos(d*x + c))
```

3.40.6 Sympy [F]

$$\int \frac{\csc^3(c+dx)}{a-a\sin^2(c+dx)}\,dx=-\frac{\int \frac{\csc^3(c+dx)}{\sin^2(c+dx)-1}\,dx}{a}$$

input
```
integrate(csc(d*x+c)**3/(a-a*sin(d*x+c)**2),x)
```

output
```
-Integral(csc(c + d*x)**3/(sin(c + d*x)**2 - 1), x)/a
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{\csc^3(c+dx)}{a-a\sin^2(c+dx)}\,dx=\frac{\frac{2\left(3\cos(dx+c)^2-2\right)}{a\cos(dx+c)^3-a\cos(dx+c)}-\frac{3\log(\cos(dx+c)+1)}{a}+\frac{3\log(\cos(dx+c)-1)}{a}}{4d}$$

input
```
integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/4*(2*(3*cos(d*x + c)^2 - 2)/(a*cos(d*x + c)^3 - a*cos(d*x + c)) - 3*log(
cos(d*x + c) + 1)/a + 3*log(cos(d*x + c) - 1)/a)/d
```

3.40.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(52) = 104$.

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.57

$$\int \frac{\csc^3(c+dx)}{a-a\sin^2(c+dx)}\,dx=\frac{\frac{6\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}+\frac{\frac{14(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}-\frac{\cos(dx+c)-1}{a(\cos(dx+c)+1)}}{8d}$$

input
```
integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/8*(6*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + (14*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
 + 1)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + (cos(d*x + c) - 1)^2/(co
s(d*x + c) + 1)^2)) - (cos(d*x + c) - 1)/(a*(cos(d*x + c) + 1)))/d
```

3.40.9 Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\csc^3(c+dx)}{a-a\sin^2(c+dx)}\,dx=-\frac{\frac{3\cos(c+d\,x)^2}{2}-1}{d\,\left(a\,\cos\left(c+d\,x\right)-a\cos\left(c+d\,x\right)^3\right)}-\frac{3\,\mathrm{atanh}(\cos\left(c+d\,x\right))}{2\,a\,d}$$

input
```
int(1/(sin(c + d*x)^3*(a - a*sin(c + d*x)^2)),x)
```

output
```
- ((3*cos(c + d*x)^2)/2 - 1)/(d*(a*cos(c + d*x) - a*cos(c + d*x)^3)) - (3*
atanh(cos(c + d*x)))/(2*a*d)
```

3.41 $\int \frac{\csc^5(c+dx)}{a-a\sin^2(c+dx)}\,dx$

3.41.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{\csc^5(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{15\text{arctanh}(\cos(c+dx))}{8ad}+\frac{15\sec(c+dx)}{8ad} -\frac{5\csc^2(c+dx)\sec(c+dx)}{8ad}-\frac{\csc^4(c+dx)\sec(c+dx)}{4ad}$$

output
```
-15/8*arctanh(cos(d*x+c))/a/d+15/8*sec(d*x+c)/a/d-5/8*csc(d*x+c)^2*sec(d*x
+c)/a/d-1/4*csc(d*x+c)^4*sec(d*x+c)/a/d
```

3.41.2 Mathematica [A] (verified)

Time = 2.74 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.61

$$\int \frac{\csc^5(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{14\csc^2\left(\frac{1}{2}(c+dx)\right)+\csc^4\left(\frac{1}{2}(c+dx)\right)+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left(78+\cos(c+dx)\left(-8\left(8+15\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-15\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.\right.}{-1+\tan^2\left(\frac{1}{2}(c+dx)\right)}}{64ad}$$

input
```
Integrate[Csc[c + d*x]^5/(a - a*Sin[c + d*x]^2),x]
```

output
```
-1/64*(14*Csc[(c + d*x)/2]^2 + Csc[(c + d*x)/2]^4 + (Sec[(c + d*x)/2]^2*(7
8 + Cos[c + d*x]*(-8*(8 + 15*Log[Cos[(c + d*x)/2]] - 15*Log[Sin[(c + d*x)/
2]]) + Sec[(c + d*x)/2]^4) - 14*Tan[(c + d*x)/2]^2))/(-1 + Tan[(c + d*x)/2
]^2))/(a*d)
```

3.41.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3654, 3042, 3102, 25, 252, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(c+dx)}{a-a\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^5\left(a-a\sin(c+dx)^2\right)}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \csc^5(c+dx)\sec^2(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(c+dx)^5\sec(c+dx)^2dx}{a}$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\sec^6(c+dx)}{(1-\sec^2(c+dx))^3}d\sec(c+dx)}{ad}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^6(c+dx)}{(1-\sec^2(c+dx))^3}d\sec(c+dx)}{ad}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{5}{4}\int \frac{\sec^4(c+dx)}{(1-\sec^2(c+dx))^2}d\sec(c+dx)-\frac{\sec^5(c+dx)}{4(1-\sec^2(c+dx))^2}}{ad}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{5}{4}\left(\frac{\sec^3(c+dx)}{2(1-\sec^2(c+dx))}-\frac{3}{2}\int\frac{\sec^2(c+dx)}{1-\sec^2(c+dx)}d\sec(c+dx)\right)-\frac{\sec^5(c+dx)}{4(1-\sec^2(c+dx))^2}}{ad}$$

$\Big\downarrow$ 262

$$\frac{\frac{5}{4}\left(\frac{\sec^3(c+dx)}{2(1-\sec^2(c+dx))}-\frac{3}{2}\left(\int\frac{1}{1-\sec^2(c+dx)}d\sec(c+dx)-\sec(c+dx)\right)\right)-\frac{\sec^5(c+dx)}{4(1-\sec^2(c+dx))^2}}{ad}$$

$\Big\downarrow$ 219

$$\frac{\frac{5}{4}\left(\frac{\sec^3(c+dx)}{2(1-\sec^2(c+dx))}-\frac{3}{2}(\text{arctanh}(\sec(c+dx))-\sec(c+dx))\right)-\frac{\sec^5(c+dx)}{4(1-\sec^2(c+dx))^2}}{ad}$$

input
```
Int[Csc[c + d*x]^5/(a - a*Sin[c + d*x]^2),x]
```

output
```
(-1/4*Sec[c + d*x]^5/(1 - Sec[c + d*x]^2)^2 + (5*((-3*(ArcTanh[Sec[c + d*x
]] - Sec[c + d*x]))/2 + Sec[c + d*x]^3/(2*(1 - Sec[c + d*x]^2))))/4)/(a*d)
```

3.41.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.41.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\frac{1}{16(1+\cos(dx+c))^2}+\frac{7}{16(1+\cos(dx+c))}-\frac{15\ln(1+\cos(dx+c))}{16}-\frac{1}{16(\cos(dx+c)-1)^2}+\frac{7}{16(\cos(dx+c)-1)}+\frac{15\ln(\cos(dx+c)-1)}{16}+\frac{1}{\cos(dx}}{da}$
default	$\frac{\frac{1}{16(1+\cos(dx+c))^2}+\frac{7}{16(1+\cos(dx+c))}-\frac{15\ln(1+\cos(dx+c))}{16}-\frac{1}{16(\cos(dx+c)-1)^2}+\frac{7}{16(\cos(dx+c)-1)}+\frac{15\ln(\cos(dx+c)-1)}{16}+\frac{1}{\cos(dx}}{da}$
parallelrisch	$\frac{-160+120\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)+\cot^4\left(\frac{dx}{2}+\frac{c}{2}\right)+15\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15\left(\cot^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{64da\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$
norman	$\frac{\frac{1}{64ad}+\frac{15\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64da}+\frac{15\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64da}+\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{64da}-\frac{5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{15\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}$
risch	$\frac{15\,e^{9i(dx+c)}-40\,e^{7i(dx+c)}+18\,e^{5i(dx+c)}-40\,e^{3i(dx+c)}+15\,e^{i(dx+c)}}{4da\left(e^{2i(dx+c)}-1\right)^4\left(e^{2i(dx+c)}+1\right)}-\frac{15\ln\left(e^{i(dx+c)}+1\right)}{8da}+\frac{15\ln\left(e^{i(dx+c)}-1\right)}{8da}$

input
```
int(csc(d*x+c)^5/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(1/16/(1+cos(d*x+c))^2+7/16/(1+cos(d*x+c))-15/16*ln(1+cos(d*x+c))-1/
16/(cos(d*x+c)-1)^2+7/16/(cos(d*x+c)-1)+15/16*ln(cos(d*x+c)-1)+1/cos(d*x+c
))
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.65

$$\int \frac{\csc^5(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{30\cos(dx+c)^4 - 50\cos(dx+c)^2 - 15\left(\cos(dx+c)^5 - 2\cos(dx+c)^3 + \cos(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+\right.}{16\left(ad\cos(dx+c)^5 - 2ad\cos(dx+\right.}$$

input
```
integrate(csc(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/16*(30*cos(d*x + c)^4 - 50*cos(d*x + c)^2 - 15*(cos(d*x + c)^5 - 2*cos(d
*x + c)^3 + cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^5
 - 2*cos(d*x + c)^3 + cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 16)/(a*
d*cos(d*x + c)^5 - 2*a*d*cos(d*x + c)^3 + a*d*cos(d*x + c))
```

3.41.6 Sympy [F]

$$\int \frac{\csc^5(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{\int \frac{\csc^5(c+dx)}{\sin^2(c+dx)-1}\,dx}{a}$$

input
```
integrate(csc(d*x+c)**5/(a-a*sin(d*x+c)**2),x)
```

output
```
-Integral(csc(c + d*x)**5/(sin(c + d*x)**2 - 1), x)/a
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

$$\int \frac{\csc^5(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\frac{2\left(15\cos(dx+c)^4-25\cos(dx+c)^2+8\right)}{a\cos(dx+c)^5-2a\cos(dx+c)^3+a\cos(dx+c)} - \frac{15\log(\cos(dx+c)+1)}{a} + \frac{15\log(\cos(dx+c)-1)}{a}}{16\,d}$$

input
```
integrate(csc(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/16*(2*(15*cos(d*x + c)^4 - 25*cos(d*x + c)^2 + 8)/(a*cos(d*x + c)^5 - 2*
a*cos(d*x + c)^3 + a*cos(d*x + c)) - 15*log(cos(d*x + c) + 1)/a + 15*log(c
os(d*x + c) - 1)/a)/d
```

3.41.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(74) = 148$.

Time = 0.33 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.21

$$\int \frac{\csc^5(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\frac{\left(\frac{16\,(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{90\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^2}{a(\cos(dx+c)-1)^2}+\frac{60\,\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}-\frac{\frac{16\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^2}+\frac{128}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{64\,d}$$

input
```
integrate(csc(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/64*((16*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 90*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a*(cos(d*x + c) - 1)^2) +
60*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a - (16*a*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/
a^2 + 128/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

3.41.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90

$$\int \frac{\csc^5(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\frac{15\cos(c+d\,x)^4}{8}-\frac{25\cos(c+d\,x)^2}{8}+1}{d\,\left(a\cos\left(c+d\,x\right)^5-2\,a\cos\left(c+d\,x\right)^3+a\,\cos\left(c+d\,x\right)\right)} - \frac{15\,\mathrm{atanh}(\cos\left(c+d\,x\right))}{8\,a\,d}$$

input
```
int(1/(sin(c + d*x)^5*(a - a*sin(c + d*x)^2)),x)
```

output
```
((15*cos(c + d*x)^4)/8 - (25*cos(c + d*x)^2)/8 + 1)/(d*(a*cos(c + d*x) - 2
*a*cos(c + d*x)^3 + a*cos(c + d*x)^5)) - (15*atanh(cos(c + d*x)))/(8*a*d)
```

3.41. $\int \frac{\csc^5(c+dx)}{a-a\sin^2(c+dx)}\,dx$

3.42 $\int \frac{\sin^6(c+dx)}{a-a\sin^2(c+dx)}\,dx$

3.42.1 Optimal result

Integrand size = 24, antiderivative size = 73

$$\int \frac{\sin^6(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{15x}{8a}+\frac{15\tan(c+dx)}{8ad}-\frac{5\sin^2(c+dx)\tan(c+dx)}{8ad} -\frac{\sin^4(c+dx)\tan(c+dx)}{4ad}$$

output
```
-15/8*x/a+15/8*tan(d*x+c)/a/d-5/8*sin(d*x+c)^2*tan(d*x+c)/a/d-1/4*sin(d*x+
c)^4*tan(d*x+c)/a/d
```

3.42.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int \frac{\sin^6(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{60c+60dx-16\sin(2(c+dx))+\sin(4(c+dx))-32\tan(c+dx)}{32ad}$$

input
```
Integrate[Sin[c + d*x]^6/(a - a*Sin[c + d*x]^2),x]
```

output
```
-1/32*(60*c + 60*d*x - 16*Sin[2*(c + d*x)] + Sin[4*(c + d*x)] - 32*Tan[c +
 d*x])/(a*d)
```

3.42.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3654, 3042, 3071, 252, 252, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(c+dx)}{a-a\sin^2(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(c+dx)^6}{a-a\sin(c+dx)^2}dx$$

$$\Big\downarrow \; 3654$$

$$\frac{\int \sin^4(c+dx)\tan^2(c+dx)dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \sin(c+dx)^4\tan(c+dx)^2dx}{a}$$

$$\Big\downarrow \; 3071$$

$$\frac{\int \frac{\tan^6(c+dx)}{(\tan^2(c+dx)+1)^3}d\tan(c+dx)}{ad}$$

$$\Big\downarrow \; 252$$

$$\frac{\frac{5}{4}\int \frac{\tan^4(c+dx)}{(\tan^2(c+dx)+1)^2}d\tan(c+dx)-\frac{\tan^5(c+dx)}{4(\tan^2(c+dx)+1)^2}}{ad}$$

$$\Big\downarrow \; 252$$

$$\frac{\frac{5}{4}\left(\frac{3}{2}\int \frac{\tan^2(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)-\frac{\tan^3(c+dx)}{2(\tan^2(c+dx)+1)}\right)-\frac{\tan^5(c+dx)}{4(\tan^2(c+dx)+1)^2}}{ad}$$

$$\Big\downarrow \; 262$$

$$\frac{\frac{5}{4}\left(\frac{3}{2}\left(\tan(c+dx)-\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)\right)-\frac{\tan^3(c+dx)}{2(\tan^2(c+dx)+1)}\right)-\frac{\tan^5(c+dx)}{4(\tan^2(c+dx)+1)^2}}{ad}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{5}{4}\left(\frac{3}{2}(\tan(c+dx)-\arctan(\tan(c+dx)))-\frac{\tan^3(c+dx)}{2(\tan^2(c+dx)+1)}\right)-\frac{\tan^5(c+dx)}{4(\tan^2(c+dx)+1)^2}}{ad}$$

---

input
```
Int[Sin[c + d*x]^6/(a - a*Sin[c + d*x]^2),x]
```

output
```
(-1/4*Tan[c + d*x]^5/(1 + Tan[c + d*x]^2)^2 + (5*((3*(-ArcTan[Tan[c + d*x]
] + Tan[c + d*x]))/2 - Tan[c + d*x]^3/(2*(1 + Tan[c + d*x]^2))))/4)/(a*d)
```

3.42.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.42.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\tan(dx+c)-\frac{-\frac{9\left(\tan^3(dx+c)\right)}{8}-\frac{7\tan(dx+c)}{8}}{\left(1+\tan^2(dx+c)\right)^2}-\frac{15\arctan(\tan(dx+c))}{8}}{da}$
default	$\frac{\tan(dx+c)-\frac{-\frac{9\left(\tan^3(dx+c)\right)}{8}-\frac{7\tan(dx+c)}{8}}{\left(1+\tan^2(dx+c)\right)^2}-\frac{15\arctan(\tan(dx+c))}{8}}{da}$
parallelrisch	$\frac{-120dx\cos(dx+c)+80\sin(dx+c)-\sin(5dx+5c)+15\sin(3dx+3c)}{64ad\cos(dx+c)}$
risch	$-\frac{15x}{8a}-\frac{i\mathrm{e}^{2i(dx+c)}}{4da}+\frac{i\mathrm{e}^{-2i(dx+c)}}{4da}+\frac{2i}{da\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{\sin(4dx+4c)}{32ad}$
norman	$\frac{15x}{8a}-\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}-\frac{35\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}-\frac{113\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}-\frac{29\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{113\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}-\frac{35\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}$

input
```
int(sin(d*x+c)^6/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(tan(d*x+c)-(-9/8*tan(d*x+c)^3-7/8*tan(d*x+c))/(1+tan(d*x+c)^2)^2-15
/8*arctan(tan(d*x+c)))
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{\sin^6(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{15\,dx\cos(dx+c)+\left(2\,\cos(dx+c)^4-9\,\cos(dx+c)^2-8\right)\sin(dx+c)}{8\,ad\cos(dx+c)}$$

input
```
integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/8*(15*d*x*cos(d*x + c) + (2*cos(d*x + c)^4 - 9*cos(d*x + c)^2 - 8)*sin(
d*x + c))/(a*d*cos(d*x + c))
```

3.42.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. $2(61) = 122$.

Time = 6.57 (sec) , antiderivative size = 1161, normalized size of antiderivative = 15.90

$$\int \frac{\sin^6(c+dx)}{a-a\sin^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)**6/(a-a*sin(d*x+c)**2),x)
```

output
```
Piecewise((-15*d*x*tan(c/2 + d*x/2)**10/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a
*d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x
/2)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) - 45*d*x*tan(c/2 + d*x/2)**8/
(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2
+ d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*
a*d) - 30*d*x*tan(c/2 + d*x/2)**6/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan
(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4
 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) + 30*d*x*tan(c/2 + d*x/2)**4/(8*a*d
*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/
2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) +
 45*d*x*tan(c/2 + d*x/2)**2/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 +
 d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*
a*d*tan(c/2 + d*x/2)**2 - 8*a*d) + 15*d*x/(8*a*d*tan(c/2 + d*x/2)**10 + 24
*a*d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d
*x/2)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) - 30*tan(c/2 + d*x/2)**9/(8
*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 +
d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*
d) - 80*tan(c/2 + d*x/2)**7/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 +
 d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*
a*d*tan(c/2 + d*x/2)**2 - 8*a*d) - 36*tan(c/2 + d*x/2)**5/(8*a*d*tan(c/...
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \frac{\sin^6(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\frac{9\tan(dx+c)^3+7\tan(dx+c)}{a\tan(dx+c)^4+2a\tan(dx+c)^2+a} - \frac{15(dx+c)}{a} + \frac{8\tan(dx+c)}{a}}{8d}$$

input
```
integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/8*((9*tan(d*x + c)^3 + 7*tan(d*x + c))/(a*tan(d*x + c)^4 + 2*a*tan(d*x +
 c)^2 + a) - 15*(d*x + c)/a + 8*tan(d*x + c)/a)/d
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{\sin^6(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{\frac{15\,(dx+c)}{a} - \frac{8\,\tan(dx+c)}{a} - \frac{9\,\tan(dx+c)^3+7\,\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)^2 a}}{8\,d}$$

input
```
integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/8*(15*(d*x + c)/a - 8*tan(d*x + c)/a - (9*tan(d*x + c)^3 + 7*tan(d*x +
c))/((tan(d*x + c)^2 + 1)^2*a))/d
```

3.42.9 Mupad [B] (verification not implemented)

Time = 14.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{\sin^6(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\tan(c+d\,x)}{a\,d} - \frac{15\,x}{8\,a} + \frac{\frac{9\tan(c+d\,x)^3}{8} + \frac{7\tan(c+d\,x)}{8}}{d\,\left(a\tan\left(c+d\,x\right)^4 + 2\,a\tan\left(c+d\,x\right)^2 + a\right)}$$

input
```
int(sin(c + d*x)^6/(a - a*sin(c + d*x)^2),x)
```

output
```
tan(c + d*x)/(a*d) - (15*x)/(8*a) + ((7*tan(c + d*x))/8 + (9*tan(c + d*x)^
3)/8)/(d*(a + 2*a*tan(c + d*x)^2 + a*tan(c + d*x)^4))
```

3.43 $\int \frac{\sin^4(c+dx)}{a-a\sin^2(c+dx)}\,dx$

3.43.1 Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \frac{\sin^4(c+dx)}{a-a\sin^2(c+dx)}\,dx=-\frac{3x}{2a}+\frac{3\tan(c+dx)}{2ad}-\frac{\sin^2(c+dx)\tan(c+dx)}{2ad}$$

output
```
-3/2*x/a+3/2*tan(d*x+c)/a/d-1/2*sin(d*x+c)^2*tan(d*x+c)/a/d
```

3.43.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int \frac{\sin^4(c+dx)}{a-a\sin^2(c+dx)}\,dx=\frac{-6(c+dx)+\sin(2(c+dx))+4\tan(c+dx)}{4ad}$$

input
```
Integrate[Sin[c + d*x]^4/(a - a*Sin[c + d*x]^2),x]
```

output
```
(-6*(c + d*x) + Sin[2*(c + d*x)] + 4*Tan[c + d*x])/(4*a*d)
```

3.43.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 3654, 3042, 3071, 252, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{a-a\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(c+dx)^4}{a-a\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{ 3654}$$

$$\frac{\int \sin^2(c+dx)\tan^2(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sin(c+dx)^2\tan(c+dx)^2dx}{a}$$

$$\Big\downarrow \text{ 3071}$$

$$\frac{\int \frac{\tan^4(c+dx)}{(\tan^2(c+dx)+1)^2}d\tan(c+dx)}{ad}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\frac{3}{2}\int \frac{\tan^2(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)-\frac{\tan^3(c+dx)}{2(\tan^2(c+dx)+1)}}{ad}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{\frac{3}{2}\left(\tan(c+dx)-\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)\right)-\frac{\tan^3(c+dx)}{2(\tan^2(c+dx)+1)}}{ad}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{3}{2}(\tan(c+dx)-\arctan(\tan(c+dx)))-\frac{\tan^3(c+dx)}{2(\tan^2(c+dx)+1)}}{ad}$$

input
```
Int[Sin[c + d*x]^4/(a - a*Sin[c + d*x]^2),x]
```

output
```
((3*(-ArcTan[Tan[c + d*x]] + Tan[c + d*x]))/2 - Tan[c + d*x]^3/(2*(1 + Tan
[c + d*x]^2)))/(a*d)
```

3.43.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.43. $\int \frac{\sin^4(c+dx)}{a-a\sin^2(c+dx)}\,dx$

3.43.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\tan(dx+c)+\frac{\tan(dx+c)}{2+2\left(\tan^2(dx+c)\right)}-\frac{3\arctan(\tan(dx+c))}{2}}{da}$
default	$\frac{\tan(dx+c)+\frac{\tan(dx+c)}{2+2\left(\tan^2(dx+c)\right)}-\frac{3\arctan(\tan(dx+c))}{2}}{da}$
parallelrisch	$\frac{-12dx\cos(dx+c)+9\sin(dx+c)+\sin(3dx+3c)}{8ad\cos(dx+c)}$
risch	$-\frac{3x}{2a}-\frac{i\mathrm{e}^{2i(dx+c)}}{8da}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8da}+\frac{2i}{da\left(\mathrm{e}^{2i(dx+c)}+1\right)}$
norman	$\frac{\frac{3x}{2a}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}-\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{10\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{8\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{9x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{3x(\mathrm{ta}}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

input
```
int(sin(d*x+c)^4/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(tan(d*x+c)+1/2*tan(d*x+c)/(1+tan(d*x+c)^2)-3/2*arctan(tan(d*x+c)))
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{\sin^4(c+dx)}{a-a\sin^2(c+dx)}\,dx=-\frac{3\,dx\cos\left(dx+c\right)-\left(\cos\left(dx+c\right)^2+2\right)\sin\left(dx+c\right)}{2\,ad\cos\left(dx+c\right)}$$

input
```
integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(3*d*x*cos(d*x + c) - (cos(d*x + c)^2 + 2)*sin(d*x + c))/(a*d*cos(d*x
 + c))
```

3.43.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(39) = 78.

Time = 2.56 (sec) , antiderivative size = 502, normalized size of antiderivative = 10.24

$$\int \frac{\sin^4(c+dx)}{a-a\sin^2(c+dx)}\,dx$$
$$= \begin{cases} -\frac{3dx\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad} - \frac{3dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad} + \frac{}{2a} \\ \frac{x\sin^4(c)}{-a\sin^2(c)+a} \end{cases}$$

input
```
integrate(sin(d*x+c)**4/(a-a*sin(d*x+c)**2),x)
```

output
```
Piecewise((-3*d*x*tan(c/2 + d*x/2)**6/(2*a*d*tan(c/2 + d*x/2)**6 + 2*a*d*t
an(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 + d*x/2)**2 - 2*a*d) - 3*d*x*tan(c/2 +
d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**6 + 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*t
an(c/2 + d*x/2)**2 - 2*a*d) + 3*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 + d
*x/2)**6 + 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 + d*x/2)**2 - 2*a*d)
+ 3*d*x/(2*a*d*tan(c/2 + d*x/2)**6 + 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan
(c/2 + d*x/2)**2 - 2*a*d) - 6*tan(c/2 + d*x/2)**5/(2*a*d*tan(c/2 + d*x/2)*
*6 + 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 + d*x/2)**2 - 2*a*d) - 4*ta
n(c/2 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**6 + 2*a*d*tan(c/2 + d*x/2)**4 -
 2*a*d*tan(c/2 + d*x/2)**2 - 2*a*d) - 6*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 +
d*x/2)**6 + 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 + d*x/2)**2 - 2*a*d)
, Ne(d, 0)), (x*sin(c)**4/(-a*sin(c)**2 + a), True))
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{\sin^4(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{\frac{3\,(dx+c)}{a} - \frac{\tan(dx+c)}{a\tan(dx+c)^2+a} - \frac{2\,\tan(dx+c)}{a}}{2\,d}$$

input
```
integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/2*(3*(d*x + c)/a - tan(d*x + c)/(a*tan(d*x + c)^2 + a) - 2*tan(d*x + c)
/a)/d
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{\sin^4(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{\frac{3\,(dx+c)}{a}-\frac{2\,\tan(dx+c)}{a}-\frac{\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)a}}{2\,d}$$

input
```
integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/2*(3*(d*x + c)/a - 2*tan(d*x + c)/a - tan(d*x + c)/((tan(d*x + c)^2 + 1
)*a))/d
```

3.43.9 Mupad [B] (verification not implemented)

Time = 13.72 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{\sin^4(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\tan(c+d\,x)}{2\,d\,\left(a\tan\left(c+d\,x\right)^2+a\right)}-\frac{3\,x}{2\,a}+\frac{\tan(c+d\,x)}{a\,d}$$

input
```
int(sin(c + d*x)^4/(a - a*sin(c + d*x)^2),x)
```

output
```
tan(c + d*x)/(2*d*(a + a*tan(c + d*x)^2)) - (3*x)/(2*a) + tan(c + d*x)/(a*
d)
```

3.44 $\int \frac{\sin^2(c+dx)}{a-a\sin^2(c+dx)}\,dx$

3.44.1 Optimal result

Integrand size = 24, antiderivative size = 20

$$\int \frac{\sin^2(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{x}{a}+\frac{\tan(c+dx)}{ad}$$

output
```
-x/a+tan(d*x+c)/a/d
```

3.44.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{\sin^2(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{-\frac{\arctan(\tan(c+dx))}{d}+\frac{\tan(c+dx)}{d}}{a}$$

input
```
Integrate[Sin[c + d*x]^2/(a - a*Sin[c + d*x]^2),x]
```

output
```
(-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d)/a
```

3.44.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 3650, 3042, 3654, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{a-a\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^2}{a-a\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3650}$$

$$\int \frac{1}{a-a\sin^2(c+dx)}dx-\frac{x}{a}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-a\sin(c+dx)^2}dx-\frac{x}{a}$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sec^2(c+dx)dx}{a}-\frac{x}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a}-\frac{x}{a}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d(-\tan(c+dx))}{ad}-\frac{x}{a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan(c+dx)}{ad}-\frac{x}{a}$$

input
```
Int[Sin[c + d*x]^2/(a - a*Sin[c + d*x]^2),x]
```

output
```
-(x/a) + Tan[c + d*x]/(a*d)
```

3.44.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3650
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Simp[(A*b - a*B)/b   Int[1/(a
+ b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.44.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{\tan(dx+c)-\arctan(\tan(dx+c))}{da}$	24
default	$\frac{\tan(dx+c)-\arctan(\tan(dx+c))}{da}$	24
risch	$-\frac{x}{a}+\frac{2i}{da\left(e^{2i(dx+c)}+1\right)}$	30
parallelrisch	$\frac{-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)xd+dx-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	53
norman	$\frac{\frac{x}{a}+\frac{x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	143

input
```
int(sin(d*x+c)^2/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(tan(d*x+c)-arctan(tan(d*x+c)))
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{\sin^2(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{dx\cos(dx+c)-\sin(dx+c)}{ad\cos(dx+c)}$$

input
```
integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
-(d*x*cos(d*x + c) - sin(d*x + c))/(a*d*cos(d*x + c))
```

3.44.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(12) = 24.

Time = 0.92 (sec) , antiderivative size = 100, normalized size of antiderivative = 5.00

$$\int \frac{\sin^2(c+dx)}{a-a\sin^2(c+dx)}\,dx = \begin{cases} -\frac{dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad}+\frac{dx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad} & \text{for } d\neq 0 \\ \frac{x\sin^2(c)}{-a\sin^2(c)+a} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(d*x+c)**2/(a-a*sin(d*x+c)**2),x)
```

output
```
Piecewise((-d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 - a*d) + d*x/
(a*d*tan(c/2 + d*x/2)**2 - a*d) - 2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)
**2 - a*d), Ne(d, 0)), (x*sin(c)**2/(-a*sin(c)**2 + a), True))
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\sin^2(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{\frac{dx+c}{a}-\frac{\tan(dx+c)}{a}}{d}$$

input
```
integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-((d*x + c)/a - tan(d*x + c)/a)/d
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\sin^2(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{\frac{dx+c}{a}-\frac{\tan(dx+c)}{a}}{d}$$

input
```
integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-((d*x + c)/a - tan(d*x + c)/a)/d
```

3.44.9 Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\tan(c+d\,x)}{a\,d}-\frac{x}{a}$$

input
```
int(sin(c + d*x)^2/(a - a*sin(c + d*x)^2),x)
```

output
```
tan(c + d*x)/(a*d) - x/a
```

3.45 $\int \frac{1}{a-a\sin^2(c+dx)}\,dx$

3.45.1 Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{1}{a-a\sin^2(c+dx)}\,dx=\frac{\tan(c+dx)}{ad}$$

output
```
tan(d*x+c)/a/d
```

3.45.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{a-a\sin^2(c+dx)}\,dx=\frac{\tan(c+dx)}{ad}$$

input
```
Integrate[(a - a*Sin[c + d*x]^2)^(-1),x]
```

output
```
Tan[c + d*x]/(a*d)
```

3.45.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3654, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-a\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-a\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sec^2(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d(-\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan(c+dx)}{ad}$$

input
```
Int[(a - a*Sin[c + d*x]^2)^(-1),x]
```

output
```
Tan[c + d*x]/(a*d)
```

3.45.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.45.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\tan(dx+c)}{ad}$	14
default	$\frac{\tan(dx+c)}{ad}$	14
risch	$\frac{2i}{da\left(e^{2i(dx+c)}+1\right)}$	23
norman	$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	33
parallelrisch	$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	33

input
```
int(1/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
tan(d*x+c)/a/d
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{a - a\sin^2(c + dx)}\,dx = \frac{\sin(dx + c)}{ad\cos(dx + c)}$$

input
```
integrate(1/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
sin(d*x + c)/(a*d*cos(d*x + c))
```

3.45.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(8) = 16.

Time = 0.47 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.15

$$\int \frac{1}{a - a\sin^2(c + dx)}\,dx = \begin{cases} -\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad} & \text{for } d \neq 0 \\ \frac{x}{-a\sin^2(c)+a} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a-a*sin(d*x+c)**2),x)
```

output
```
Piecewise((-2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 - a*d), Ne(d, 0)),
 (x/(-a*sin(c)**2 + a), True))
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{a - a\sin^2(c + dx)}\,dx = \frac{\tan(dx + c)}{ad}$$

input
```
integrate(1/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
tan(d*x + c)/(a*d)
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{a - a\sin^2(c+dx)}\,dx = \frac{\tan(dx+c)}{ad}$$

input
```
integrate(1/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
tan(d*x + c)/(a*d)
```

3.45.9 Mupad [B] (verification not implemented)

Time = 13.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{a - a\sin^2(c+dx)}\,dx = \frac{\tan(c+d\,x)}{a\,d}$$

input
```
int(1/(a - a*sin(c + d*x)^2),x)
```

output
```
tan(c + d*x)/(a*d)
```

3.46 $\int \frac{\csc^2(c+dx)}{a-a\sin^2(c+dx)}\,dx$

3.46.1 Optimal result

Integrand size = 24, antiderivative size = 28

$$\int \frac{\csc^2(c+dx)}{a-a\sin^2(c+dx)}\,dx=-\frac{\cot(c+dx)}{ad}+\frac{\tan(c+dx)}{ad}$$

output
```
-cot(d*x+c)/a/d+tan(d*x+c)/a/d
```

3.46.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int \frac{\csc^2(c+dx)}{a-a\sin^2(c+dx)}\,dx=-\frac{2\cot(2(c+dx))}{ad}$$

input
```
Integrate[Csc[c + d*x]^2/(a - a*Sin[c + d*x]^2),x]
```

output
```
(-2*Cot[2*(c + d*x)])/(a*d)
```

3.46.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3654, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{a-a\sin^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^2\,(a-a\sin(c+dx)^2)}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \csc^2(c+dx)\sec^2(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(c+dx)^2\sec(c+dx)^2dx}{a}$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^2(c+dx)\left(\tan^2(c+dx)+1\right)d\tan(c+dx)}{ad}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^2(c+dx)+1\right)d\tan(c+dx)}{ad}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tan(c+dx)-\cot(c+dx)}{ad}$$

input
```
Int[Csc[c + d*x]^2/(a - a*Sin[c + d*x]^2),x]
```

output
```
(-Cot[c + d*x] + Tan[c + d*x])/(a*d)
```

3.46.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.46.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\tan(dx+c)-\frac{1}{\tan(dx+c)}}{da}$	25
default	$\frac{\tan(dx+c)-\frac{1}{\tan(dx+c)}}{da}$	25
risch	$-\frac{4i}{da\left(e^{2i(dx+c)}+1\right)\left(e^{2i(dx+c)}-1\right)}$	36
parallelrisch	$\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	56
norman	$\frac{\frac{1}{2ad}-\frac{3\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	75

input
```
int(csc(d*x+c)^2/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(tan(d*x+c)-1/tan(d*x+c))
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\csc^2(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{2\,\cos(dx+c)^2-1}{ad\cos(dx+c)\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
-(2*cos(d*x + c)^2 - 1)/(a*d*cos(d*x + c)*sin(d*x + c))
```

3.46.6 Sympy [F]

$$\int \frac{\csc^2(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{\int \frac{\csc^2(c+dx)}{\sin^2(c+dx)-1}\,dx}{a}$$

input
```
integrate(csc(d*x+c)**2/(a-a*sin(d*x+c)**2),x)
```

output
```
-Integral(csc(c + d*x)**2/(sin(c + d*x)**2 - 1), x)/a
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\frac{\tan(dx+c)}{a}-\frac{1}{a\tan(dx+c)}}{d}$$

input
```
integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
(tan(d*x + c)/a - 1/(a*tan(d*x + c)))/d
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{\csc^2(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{2}{ad\tan(2\,dx+2\,c)}$$

input
```
integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-2/(a*d*tan(2*d*x + 2*c))
```

3.46.9 Mupad [B] (verification not implemented)

Time = 14.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int \frac{\csc^2(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{2\cot(2\,c+2\,d\,x)}{a\,d}$$

input
```
int(1/(sin(c + d*x)^2*(a - a*sin(c + d*x)^2)),x)
```

output
```
-(2*cot(2*c + 2*d*x))/(a*d)
```

3.47 $\int \frac{\csc^4(c+dx)}{a-a\sin^2(c+dx)}\,dx$

3.47.1 Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{\csc^4(c+dx)}{a-a\sin^2(c+dx)}\,dx=-\frac{2\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}$$

output
```
-2*cot(d*x+c)/a/d-1/3*cot(d*x+c)^3/a/d+tan(d*x+c)/a/d
```

3.47.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{\csc^4(c+dx)}{a-a\sin^2(c+dx)}\,dx=\frac{-\frac{5\cot(c+dx)}{3d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3d}+\frac{\tan(c+dx)}{d}}{a}$$

input
```
Integrate[Csc[c + d*x]^4/(a - a*Sin[c + d*x]^2),x]
```

output
```
((-5*Cot[c + d*x])/(3*d) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) + Tan[c + d
*x]/d)/a
```

3.47.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3654, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(c+dx)}{a-a\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(c+dx)^4\left(a-a\sin(c+dx)^2\right)}dx$$

$$\Big\downarrow \text{ 3654}$$

$$\frac{\int \csc^4(c+dx)\sec^2(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc(c+dx)^4\sec(c+dx)^2dx}{a}$$

$$\Big\downarrow \text{ 3100}$$

$$\frac{\int \cot^4(c+dx)\left(\tan^2(c+dx)+1\right)^2d\tan(c+dx)}{ad}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\int \left(\cot^4(c+dx)+2\cot^2(c+dx)+1\right)d\tan(c+dx)}{ad}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\tan(c+dx)-\frac{1}{3}\cot^3(c+dx)-2\cot(c+dx)}{ad}$$

input
```
Int[Csc[c + d*x]^4/(a - a*Sin[c + d*x]^2),x]
```

output
```
(-2*Cot[c + d*x] - Cot[c + d*x]^3/3 + Tan[c + d*x])/(a*d)
```

3.47.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.47.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{\tan(dx+c)-\frac{1}{3\tan(dx+c)^3}-\frac{2}{\tan(dx+c)}}{da}$	35
default	$\frac{\tan(dx+c)-\frac{1}{3\tan(dx+c)^3}-\frac{2}{\tan(dx+c)}}{da}$	35
risch	$\frac{16i\left(2\,e^{2i(dx+c)}-1\right)}{3da\left(e^{2i(dx+c)}-1\right)^3\left(e^{2i(dx+c)}+1\right)}$	49
parallelrisch	$\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)+20\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\cot^3\left(\frac{dx}{2}+\frac{c}{2}\right)-90\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+20\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{24da\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	82
norman	$\frac{\frac{1}{24ad}+\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}-\frac{15\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}+\frac{5\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}+\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{24da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	113

input
```
int(csc(d*x+c)^4/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(tan(d*x+c)-1/3/tan(d*x+c)^3-2/tan(d*x+c))
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{\csc^4(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{8\,\cos\left(dx+c\right)^4-12\,\cos\left(dx+c\right)^2+3}{3\left(ad\cos\left(dx+c\right)^3-ad\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}$$

input
```
integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/3*(8*cos(d*x + c)^4 - 12*cos(d*x + c)^2 + 3)/((a*d*cos(d*x + c)^3 - a*d
*cos(d*x + c))*sin(d*x + c))
```

3.47.6 Sympy [F]

$$\int \frac{\csc^4(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{\int \frac{\csc^4(c+dx)}{\sin^2(c+dx)-1}\,dx}{a}$$

input
```
integrate(csc(d*x+c)**4/(a-a*sin(d*x+c)**2),x)
```

output
```
-Integral(csc(c + d*x)**4/(sin(c + d*x)**2 - 1), x)/a
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{\csc^4(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\frac{3\,\tan(dx+c)}{a}-\frac{6\,\tan(dx+c)^2+1}{a\tan(dx+c)^3}}{3\,d}$$

input
```
integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/3*(3*tan(d*x + c)/a - (6*tan(d*x + c)^2 + 1)/(a*tan(d*x + c)^3))/d
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{\csc^4(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\frac{3\tan(dx+c)}{a} - \frac{6\tan(dx+c)^2+1}{a\tan(dx+c)^3}}{3\,d}$$

input
```
integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(3*tan(d*x + c)/a - (6*tan(d*x + c)^2 + 1)/(a*tan(d*x + c)^3))/d
```

3.47.9 Mupad [B] (verification not implemented)

Time = 13.85 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{\csc^4(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{-\tan(c+d\,x)^4+2\tan(c+d\,x)^2+\frac{1}{3}}{a\,d\tan\left(c+d\,x\right)^3}$$

input
```
int(1/(sin(c + d*x)^4*(a - a*sin(c + d*x)^2)),x)
```

output
```
-(2*tan(c + d*x)^2 - tan(c + d*x)^4 + 1/3)/(a*d*tan(c + d*x)^3)
```

3.48 $\int \frac{\csc^6(c+dx)}{a-a\sin^2(c+dx)}\,dx$

3.48.1 Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{\csc^6(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{3\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{ad}-\frac{\cot^5(c+dx)}{5ad}+\frac{\tan(c+dx)}{ad}$$

output
```
-3*cot(d*x+c)/a/d-cot(d*x+c)^3/a/d-1/5*cot(d*x+c)^5/a/d+tan(d*x+c)/a/d
```

3.48.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{\csc^6(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{-\frac{11\cot(c+dx)}{5d}-\frac{3\cot(c+dx)\csc^2(c+dx)}{5d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5d}+\frac{\tan(c+dx)}{d}}{a}$$

input
```
Integrate[Csc[c + d*x]^6/(a - a*Sin[c + d*x]^2),x]
```

output
```
((-11*Cot[c + d*x])/(5*d) - (3*Cot[c + d*x]*Csc[c + d*x]^2)/(5*d) - (Cot[c
 + d*x]*Csc[c + d*x]^4)/(5*d) + Tan[c + d*x]/d)/a
```

3.48.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.71, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3654, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(c+dx)}{a-a\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^6\left(a-a\sin(c+dx)^2\right)}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \csc^6(c+dx)\sec^2(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(c+dx)^6\sec(c+dx)^2dx}{a}$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^6(c+dx)\left(\tan^2(c+dx)+1\right)^3d\tan(c+dx)}{ad}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^6(c+dx)+3\cot^4(c+dx)+3\cot^2(c+dx)+1\right)d\tan(c+dx)}{ad}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tan(c+dx)-\frac{1}{5}\cot^5(c+dx)-\cot^3(c+dx)-3\cot(c+dx)}{ad}$$

input
```
Int[Csc[c + d*x]^6/(a - a*Sin[c + d*x]^2),x]
```

output
```
(-3*Cot[c + d*x] - Cot[c + d*x]^3 - Cot[c + d*x]^5/5 + Tan[c + d*x])/(a*d)
```

3.48.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.48.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{\tan(dx+c)-\frac{1}{5\tan(dx+c)^5}-\frac{3}{\tan(dx+c)}-\frac{1}{\tan(dx+c)^3}}{da}$
default	$\frac{\tan(dx+c)-\frac{1}{5\tan(dx+c)^5}-\frac{3}{\tan(dx+c)}-\frac{1}{\tan(dx+c)^3}}{da}$
risch	$-\frac{32i\left(5\,\mathrm{e}^{4i(dx+c)}-4\,\mathrm{e}^{2i(dx+c)}+1\right)}{5da\left(\mathrm{e}^{2i(dx+c)}-1\right)^5\left(\mathrm{e}^{2i(dx+c)}+1\right)}$
parallelrisch	$\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)+14\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\cot^5\left(\frac{dx}{2}+\frac{c}{2}\right)+175\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+14\left(\cot^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-700\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+175\,\mathrm{co}}{160da\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$
norman	$\frac{\frac{1}{160ad}+\frac{7\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80da}+\frac{35\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32da}-\frac{35\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da}+\frac{35\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32da}+\frac{7\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80da}+\frac{\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)}{160da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

input
```
int(csc(d*x+c)^6/(a-a*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(tan(d*x+c)-1/5/tan(d*x+c)^5-3/tan(d*x+c)-1/tan(d*x+c)^3)
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.24

$$\int \frac{\csc^6(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{16\cos(dx+c)^6-40\cos(dx+c)^4+30\cos(dx+c)^2-5}{5\left(ad\cos(dx+c)^5-2\,ad\cos(dx+c)^3+ad\cos(dx+c)\right)\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/5*(16*cos(d*x + c)^6 - 40*cos(d*x + c)^4 + 30*cos(d*x + c)^2 - 5)/((a*d
*cos(d*x + c)^5 - 2*a*d*cos(d*x + c)^3 + a*d*cos(d*x + c))*sin(d*x + c))
```

3.48.6 Sympy [F]

$$\int \frac{\csc^6(c+dx)}{a-a\sin^2(c+dx)}\,dx = -\frac{\int \frac{\csc^6(c+dx)}{\sin^2(c+dx)-1}\,dx}{a}$$

input
```
integrate(csc(d*x+c)**6/(a-a*sin(d*x+c)**2),x)
```

output
```
-Integral(csc(c + d*x)**6/(sin(c + d*x)**2 - 1), x)/a
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{\csc^6(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\frac{5\tan(dx+c)}{a}-\frac{15\tan(dx+c)^4+5\tan(dx+c)^2+1}{a\tan(dx+c)^5}}{5\,d}$$

input
```
integrate(csc(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/5*(5*tan(d*x + c)/a - (15*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)/(a*tan(
d*x + c)^5))/d
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{\csc^6(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\frac{5\tan(dx+c)}{a} - \frac{15\tan(dx+c)^4+5\tan(dx+c)^2+1}{a\tan(dx+c)^5}}{5\,d}$$

input
```
integrate(csc(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/5*(5*tan(d*x + c)/a - (15*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)/(a*tan(
d*x + c)^5))/d
```

3.48.9 Mupad [B] (verification not implemented)

Time = 13.88 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{\csc^6(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\tan(c+d\,x)}{a\,d} - \frac{3\tan(c+d\,x)^4+\tan(c+d\,x)^2+\frac{1}{5}}{a\,d\tan\left(c+d\,x\right)^5}$$

input
```
int(1/(sin(c + d*x)^6*(a - a*sin(c + d*x)^2)),x)
```

output
```
tan(c + d*x)/(a*d) - (tan(c + d*x)^2 + 3*tan(c + d*x)^4 + 1/5)/(a*d*tan(c
+ d*x)^5)
```

3.49 $\int \frac{\sin^7(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx$

3.49.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{\sin^7(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = -\frac{3\cos(c+dx)}{a^2d}+\frac{\cos^3(c+dx)}{3a^2d}-\frac{3\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}$$

output
```
-3*cos(d*x+c)/a^2/d+1/3*cos(d*x+c)^3/a^2/d-3*sec(d*x+c)/a^2/d+1/3*sec(d*x+
c)^3/a^2/d
```

3.49.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{\sin^7(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{-\frac{11\cos(c+dx)}{4d}+\frac{\cos(3(c+dx))}{12d}-\frac{3\sec(c+dx)}{d}+\frac{\sec^3(c+dx)}{3d}}{a^2}$$

input
```
Integrate[Sin[c + d*x]^7/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
((-11*Cos[c + d*x])/(4*d) + Cos[3*(c + d*x)]/(12*d) - (3*Sec[c + d*x])/d +
 Sec[c + d*x]^3/(3*d))/a^2
```

3.49.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3654, 3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(c+dx)^7}{\left(a-a\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{ 3654}$$

$$\frac{\int \sin^3(c+dx)\tan^4(c+dx)dx}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sin(c+dx)^3\tan(c+dx)^4dx}{a^2}$$

$$\Big\downarrow \text{ 3070}$$

$$-\frac{\int \left(1-\cos^2(c+dx)\right)^3\sec^4(c+dx)d\cos(c+dx)}{a^2d}$$

$$\Big\downarrow \text{ 244}$$

$$-\frac{\int \left(\sec^4(c+dx)-3\sec^2(c+dx)-\cos^2(c+dx)+3\right)d\cos(c+dx)}{a^2d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-\frac{1}{3}\cos^3(c+dx)+3\cos(c+dx)-\frac{1}{3}\sec^3(c+dx)+3\sec(c+dx)}{a^2d}$$

input
```
Int[Sin[c + d*x]^7/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
-((3*Cos[c + d*x] - Cos[c + d*x]^3/3 + 3*Sec[c + d*x] - Sec[c + d*x]^3/3)/
(a^2*d))
```

3.49.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.49.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{\frac{\left(\cos^3(dx+c)\right)}{3}-3\cos(dx+c)+\frac{1}{3\cos(dx+c)^3}-\frac{3}{\cos(dx+c)}}{d\,a^2}$	47
default	$\frac{\frac{\left(\cos^3(dx+c)\right)}{3}-3\cos(dx+c)+\frac{1}{3\cos(dx+c)^3}-\frac{3}{\cos(dx+c)}}{d\,a^2}$	47
parallelrisch	$\frac{-210-273\cos(2dx+2c)-30\cos(4dx+4c)-128\cos(3dx+3c)-384\cos(dx+c)+\cos(6dx+6c)}{24a^2d(\cos(3dx+3c)+3\cos(dx+c))}$	81
risch	$-\frac{30\,\mathrm{e}^{7i(dx+c)}+273\,\mathrm{e}^{5i(dx+c)}-\mathrm{e}^{9i(dx+c)}+303\cos(dx+c)+243i\sin(dx+c)+419\cos(3dx+3c)+421i\sin(3dx+3c)}{24d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	96

input
```
int(sin(d*x+c)^7/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(1/3*cos(d*x+c)^3-3*cos(d*x+c)+1/3/cos(d*x+c)^3-3/cos(d*x+c))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \frac{\sin^7(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx=\frac{\cos\left(dx+c\right)^6-9\,\cos\left(dx+c\right)^4-9\,\cos\left(dx+c\right)^2+1}{3\,a^2d\cos\left(dx+c\right)^3}$$

input
```
integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(cos(d*x + c)^6 - 9*cos(d*x + c)^4 - 9*cos(d*x + c)^2 + 1)/(a^2*d*cos(
d*x + c)^3)
```

3.49.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(56) = 112.

Time = 24.60 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.40

$$\int \frac{\sin^7(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$$
$$=\begin{cases}-\frac{96\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d}+\frac{32}{3a^2d\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-3} \\ \frac{x\sin^7(c)}{\left(-a\sin^2(c)+a\right)^2}\end{cases}$$

input
```
integrate(sin(d*x+c)**7/(a-a*sin(d*x+c)**2)**2,x)
```

output
```
Piecewise((-96*tan(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**12 - 9*a**2
*d*tan(c/2 + d*x/2)**8 + 9*a**2*d*tan(c/2 + d*x/2)**4 - 3*a**2*d) + 32/(3*
a**2*d*tan(c/2 + d*x/2)**12 - 9*a**2*d*tan(c/2 + d*x/2)**8 + 9*a**2*d*tan(
c/2 + d*x/2)**4 - 3*a**2*d), Ne(d, 0)), (x*sin(c)**7/(-a*sin(c)**2 + a)**2
, True))
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int \frac{\sin^7(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\frac{\cos(dx+c)^3-9\cos(dx+c)}{a^2} - \frac{9\cos(dx+c)^2-1}{a^2\cos(dx+c)^3}}{3d}$$

input
```
integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*((cos(d*x + c)^3 - 9*cos(d*x + c))/a^2 - (9*cos(d*x + c)^2 - 1)/(a^2*c
os(d*x + c)^3))/d
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{\sin^7(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = -\frac{32\left(\frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)}{3a^2d\left(\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)^3}$$

input
```
integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-32/3*(3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1)/(a^2*d*((cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2 - 1)^3)
```

3.49.9 Mupad [B] (verification not implemented)

Time = 13.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{\sin^7(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = -\frac{-\cos(c+d\,x)^6+9\cos(c+d\,x)^4+9\cos(c+d\,x)^2-1}{3\,a^2\,d\cos(c+d\,x)^3}$$

input
```
int(sin(c + d*x)^7/(a - a*sin(c + d*x)^2)^2,x)
```

output
```
-(9*cos(c + d*x)^2 + 9*cos(c + d*x)^4 - cos(c + d*x)^6 - 1)/(3*a^2*d*cos(c
 + d*x)^3)
```

3.50 $\int \frac{\sin^5(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

3.50.1 Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{\sin^5(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx=-\frac{\cos(c+dx)}{a^2d}-\frac{2\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}$$

output
```
-cos(d*x+c)/a^2/d-2*sec(d*x+c)/a^2/d+1/3*sec(d*x+c)^3/a^2/d
```

3.50.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\sin^5(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx=\frac{-\frac{\cos(c+dx)}{d}-\frac{2\sec(c+dx)}{d}+\frac{\sec^3(c+dx)}{3d}}{a^2}$$

input
```
Integrate[Sin[c + d*x]^5/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
(-(Cos[c + d*x]/d) - (2*Sec[c + d*x])/d + Sec[c + d*x]^3/(3*d))/a^2
```

3.50.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3654, 3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(c+dx)^5}{\left(a-a\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \; 3654$$

$$\frac{\int \sin(c+dx)\tan^4(c+dx)dx}{a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \sin(c+dx)\tan(c+dx)^4dx}{a^2}$$

$$\Big\downarrow \; 3070$$

$$-\frac{\int \left(1-\cos^2(c+dx)\right)^2\sec^4(c+dx)d\cos(c+dx)}{a^2d}$$

$$\Big\downarrow \; 244$$

$$-\frac{\int \left(\sec^4(c+dx)-2\sec^2(c+dx)+1\right)d\cos(c+dx)}{a^2d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\cos(c+dx)-\frac{1}{3}\sec^3(c+dx)+2\sec(c+dx)}{a^2d}$$

input
```
Int[Sin[c + d*x]^5/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
-((Cos[c + d*x] + 2*Sec[c + d*x] - Sec[c + d*x]^3/3)/(a^2*d))
```

3.50.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.50.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{-\cos(dx+c)-\frac{2}{\cos(dx+c)}+\frac{1}{3\cos(dx+c)^3}}{d\,a^2}$	37
default	$\frac{-\cos(dx+c)-\frac{2}{\cos(dx+c)}+\frac{1}{3\cos(dx+c)^3}}{d\,a^2}$	37
parallelrisch	$\frac{-25-36\cos(2dx+2c)-3\cos(4dx+4c)-48\cos(dx+c)-16\cos(3dx+3c)}{6a^2d(\cos(3dx+3c)+3\cos(dx+c))}$	72
risch	$-\frac{3\,\mathrm{e}^{7i(dx+c)}+36\,\mathrm{e}^{5i(dx+c)}+50\,\mathrm{e}^{3i(dx+c)}+39\cos(dx+c)+33i\sin(dx+c)}{6d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	73
norman	$\frac{-\frac{112\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{16}{3ad}+\frac{32\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{32\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{128\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{32\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$	139

input
```
int(sin(d*x+c)^5/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(-cos(d*x+c)-2/cos(d*x+c)+1/3/cos(d*x+c)^3)
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{\sin^5(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = -\frac{3\,\cos(dx+c)^4+6\,\cos(dx+c)^2-1}{3\,a^2d\cos(dx+c)^3}$$

input
```
integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(3*cos(d*x + c)^4 + 6*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3)
```

3.50.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(39) = 78.

Time = 10.90 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.32

$$\int \frac{\sin^5(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx$$
$$= \begin{cases} -\frac{32\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)-6a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d}+\frac{16}{3a^2d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)-6a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a} \\ \frac{x\sin^5(c)}{(-a\sin^2(c)+a)^2} \end{cases}$$

input
```
integrate(sin(d*x+c)**5/(a-a*sin(d*x+c)**2)**2,x)
```

output
```
Piecewise((-32*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**8 - 6*a**2*
d*tan(c/2 + d*x/2)**6 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 16/(3*a
**2*d*tan(c/2 + d*x/2)**8 - 6*a**2*d*tan(c/2 + d*x/2)**6 + 6*a**2*d*tan(c/
2 + d*x/2)**2 - 3*a**2*d), Ne(d, 0)), (x*sin(c)**5/(-a*sin(c)**2 + a)**2,
True))
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{\sin^5(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = -\frac{\frac{3\cos(dx+c)}{a^2} + \frac{6\cos(dx+c)^2-1}{a^2\cos(dx+c)^3}}{3\,d}$$

input
```
integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/3*(3*cos(d*x + c)/a^2 + (6*cos(d*x + c)^2 - 1)/(a^2*cos(d*x + c)^3))/d
```

3.50.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(45) = 90$.

Time = 0.34 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.26

$$\int \frac{\sin^5(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{2\left(\frac{3}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)} - \frac{\frac{12(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+5}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}\right)}{3\,d}$$

input
```
integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
2/3*(3/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (12*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
 5)/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3))/d
```

3.50.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{\sin^5(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = -\frac{\cos(c+d\,x)^4 + 2\cos(c+d\,x)^2 - \frac{1}{3}}{a^2\,d\cos(c+d\,x)^3}$$

input
```
int(sin(c + d*x)^5/(a - a*sin(c + d*x)^2)^2,x)
```

output
```
-(2*cos(c + d*x)^2 + cos(c + d*x)^4 - 1/3)/(a^2*d*cos(c + d*x)^3)
```

3.51 $\int \frac{\sin^3(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx$

3.51.1 Optimal result

Integrand size = 24, antiderivative size = 33

$$\int \frac{\sin^3(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = -\frac{\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}$$

output
```
-sec(d*x+c)/a^2/d+1/3*sec(d*x+c)^3/a^2/d
```

3.51.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{\sin^3(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{-\frac{\sec(c+dx)}{d}+\frac{\sec^3(c+dx)}{3d}}{a^2}$$

input
```
Integrate[Sin[c + d*x]^3/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
(-(Sec[c + d*x]/d) + Sec[c + d*x]^3/(3*d))/a^2
```

3.51.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3042, 3654, 3042, 3086, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^3}{\left(a-a\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sec(c+dx)\tan^3(c+dx)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sec(c+dx)\tan(c+dx)^3dx}{a^2}$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int \left(\sec^2(c+dx)-1\right)d\sec(c+dx)}{a^2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}\sec^3(c+dx)-\sec(c+dx)}{a^2d}$$

input
```
Int[Sin[c + d*x]^3/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
(-Sec[c + d*x] + Sec[c + d*x]^3/3)/(a^2*d)
```

3.51.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.51.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\frac{1}{3\cos(dx+c)^3}-\frac{1}{\cos(dx+c)}}{d\,a^2}$	29
default	$\frac{\frac{1}{3\cos(dx+c)^3}-\frac{1}{\cos(dx+c)}}{d\,a^2}$	29
risch	$-\frac{2\left(3\,e^{5i(dx+c)}+2\,e^{3i(dx+c)}+3\,e^{i(dx+c)}\right)}{3d\,a^2\left(e^{2i(dx+c)}+1\right)^3}$	56
parallelrisch	$\frac{-2-6\cos(2dx+2c)-6\cos(dx+c)-2\cos(3dx+3c)}{3a^2d(\cos(3dx+3c)+3\cos(dx+c))}$	61
norman	$\frac{-\frac{8\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{32\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{4}{3ad}-\frac{4\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$	101

input
```
int(sin(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(1/3/cos(d*x+c)^3-1/cos(d*x+c))
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{\sin^3(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = -\frac{3\cos(dx+c)^2-1}{3\,a^2d\cos(dx+c)^3}$$

input
```
integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(3*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3)
```

3.51.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(26) = 52.

Time = 3.92 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.73

$$\int \frac{\sin^3(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \begin{cases} -\frac{12\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} + \frac{4}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a} \\ \frac{x\sin^3(c)}{(-a\sin^2(c)+a)^2} \end{cases}$$

input
```
integrate(sin(d*x+c)**3/(a-a*sin(d*x+c)**2)**2,x)
```

output
```
Piecewise((-12*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*
d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 4/(3*a*
*2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2
 + d*x/2)**2 - 3*a**2*d), Ne(d, 0)), (x*sin(c)**3/(-a*sin(c)**2 + a)**2, T
rue))
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{\sin^3(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx=-\frac{3\,\cos{(dx+c)}^2-1}{3\,a^2d\cos{(dx+c)}^3}$$

input
```
integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/3*(3*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3)
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{\sin^3(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx=-\frac{3\,\cos{(dx+c)}^2-1}{3\,a^2d\cos{(dx+c)}^3}$$

input
```
integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/3*(3*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3)
```

3.51.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{\sin^3(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx=-\frac{\cos{(c+d\,x)}^2-\frac{1}{3}}{a^2\,d\cos{(c+d\,x)}^3}$$

input
```
int(sin(c + d*x)^3/(a - a*sin(c + d*x)^2)^2,x)
```

output
```
-(cos(c + d*x)^2 - 1/3)/(a^2*d*cos(c + d*x)^3)
```

3.52 $\int \frac{\sin(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx$

3.52.1 Optimal result

Integrand size = 22, antiderivative size = 18

$$\int \frac{\sin(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\sec^3(c+dx)}{3a^2d}$$

output
```
1/3*sec(d*x+c)^3/a^2/d
```

3.52.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\sec^3(c+dx)}{3a^2d}$$

input
```
Integrate[Sin[c + d*x]/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
Sec[c + d*x]^3/(3*a^2*d)
```

3.52.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 3654, 3042, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(c+dx)}{(a-a\sin(c+dx)^2)^2}dx$$

$$\Big\downarrow \ 3654$$

$$\frac{\int \sec^3(c+dx)\tan(c+dx)dx}{a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \sec(c+dx)^3\tan(c+dx)dx}{a^2}$$

$$\Big\downarrow \ 3086$$

$$\frac{\int \sec^2(c+dx)d\sec(c+dx)}{a^2d}$$

$$\Big\downarrow \ 15$$

$$\frac{\sec^3(c+dx)}{3a^2d}$$

input
```
Int[Sin[c + d*x]/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
Sec[c + d*x]^3/(3*a^2*d)
```

3.52.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.52.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{1}{3d\,a^2\cos(dx+c)^3}$	17
default	$\frac{1}{3d\,a^2\cos(dx+c)^3}$	17
risch	$\frac{8\,e^{3i(dx+c)}}{3d\,a^2\left(e^{2i(dx+c)}+1\right)^3}$	31
parallelrisch	$\frac{4+3\cos(dx+c)+\cos(3dx+3c)}{3a^2d(\cos(3dx+3c)+3\cos(dx+c))}$	48
norman	$\frac{-\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{2}{3ad}-\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$	101

input
```
int(sin(d*x+c)/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/d/a^2/cos(d*x+c)^3
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\sin(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx=\frac{1}{3\,a^2d\cos\left(dx+c\right)^3}$$

input
```
integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/3/(a^2*d*cos(d*x + c)^3)
```

3.52.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 1.92 (sec) , antiderivative size = 156, normalized size of antiderivative = 8.67

$$\int \frac{\sin(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$$
$$=\begin{cases} -\frac{6\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d}-\frac{2}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a} \\ \frac{x\sin\left(c\right)}{\left(-a\sin^2\left(c\right)+a\right)^2} \end{cases}$$

input
```
integrate(sin(d*x+c)/(a-a*sin(d*x+c)**2)**2,x)
```

output
```
Piecewise((-6*tan(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d
*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) - 2/(3*a**
2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2
+ d*x/2)**2 - 3*a**2*d), Ne(d, 0)), (x*sin(c)/(-a*sin(c)**2 + a)**2, True)
)
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\sin(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{1}{3\,a^2d\cos{(dx+c)}^3}$$

input
```
integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3/(a^2*d*cos(d*x + c)^3)
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\sin(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{1}{3\,a^2d\cos{(dx+c)}^3}$$

input
```
integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/3/(a^2*d*cos(d*x + c)^3)
```

3.52.9 Mupad [B] (verification not implemented)

Time = 12.98 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\sin(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{1}{3\,a^2\,d\cos{(c+d\,x)}^3}$$

input
```
int(sin(c + d*x)/(a - a*sin(c + d*x)^2)^2,x)
```

output
```
1/(3*a^2*d*cos(c + d*x)^3)
```

3.53 $\int \frac{\csc(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

3.53.1 Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{\csc(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx=-\frac{\operatorname{arctanh}(\cos(c+dx))}{a^2d}+\frac{\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}$$

output
```
-arctanh(cos(d*x+c))/a^2/d+sec(d*x+c)/a^2/d+1/3*sec(d*x+c)^3/a^2/d
```

3.53.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.30

$$\int \frac{\csc(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx=\frac{-\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{\sec(c+dx)}{d}+\frac{\sec^3(c+dx)}{3d}}{a^2}$$

input
```
Integrate[Csc[c + d*x]/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
(-(Log[Cos[(c + d*x)/2]]/d) + Log[Sin[(c + d*x)/2]]/d + Sec[c + d*x]/d + S
ec[c + d*x]^3/(3*d))/a^2
```

3.53.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3042, 3654, 3042, 3102, 25, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)\left(a-a\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \csc(c+dx)\sec^4(c+dx)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(c+dx)\sec(c+dx)^4dx}{a^2}$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\sec^4(c+dx)}{1-\sec^2(c+dx)}d\sec(c+dx)}{a^2d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^4(c+dx)}{1-\sec^2(c+dx)}d\sec(c+dx)}{a^2d}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\int \left(-\sec^2(c+dx)+\frac{1}{1-\sec^2(c+dx)}-1\right)d\sec(c+dx)}{a^2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\text{arctanh}(\sec(c+dx))+\frac{1}{3}\sec^3(c+dx)+\sec(c+dx)}{a^2d}$$

input
```
Int[Csc[c + d*x]/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
(-ArcTanh[Sec[c + d*x]] + Sec[c + d*x] + Sec[c + d*x]^3/3)/(a^2*d)
```

3.53.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.53.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}+\frac{\ln(\cos(dx+c)-1)}{2}-\frac{\ln(1+\cos(dx+c))}{2}}{d\,a^2}$	49
default	$\frac{\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}+\frac{\ln(\cos(dx+c)-1)}{2}-\frac{\ln(1+\cos(dx+c))}{2}}{d\,a^2}$	49
norman	$\frac{\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{8}{3ad}-\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$	85
parallelrisch	$\frac{(9\cos(dx+c)+3\cos(3dx+3c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+12\cos(dx+c)+6\cos(2dx+2c)+4\cos(3dx+3c)+10}{3a^2d(\cos(3dx+3c)+3\cos(dx+c))}$	92
risch	$\frac{2\,e^{5i(dx+c)}+\frac{20\,e^{3i(dx+c)}}{3}+2\,e^{i(dx+c)}}{d\,a^2\left(e^{2i(dx+c)}+1\right)^3}+\frac{\ln(e^{i(dx+c)}-1)}{d\,a^2}-\frac{\ln(e^{i(dx+c)}+1)}{d\,a^2}$	96

input
```
int(csc(d*x+c)/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(1/3/cos(d*x+c)^3+1/cos(d*x+c)+1/2*ln(cos(d*x+c)-1)-1/2*ln(1+cos(d
*x+c)))
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.49

$$\int \frac{\csc(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = -\frac{3\cos(dx+c)^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\cos(dx+c)^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-6\cos(dx+c)^2-}{6\,a^2d\cos(dx+c)^3}$$

input
```
integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/6*(3*cos(d*x + c)^3*log(1/2*cos(d*x + c) + 1/2) - 3*cos(d*x + c)^3*log(
-1/2*cos(d*x + c) + 1/2) - 6*cos(d*x + c)^2 - 2)/(a^2*d*cos(d*x + c)^3)
```

3.53.6 Sympy [F]

$$\int \frac{\csc(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\int \frac{\csc(c+dx)}{\sin^4(c+dx)-2\sin^2(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(csc(d*x+c)/(a-a*sin(d*x+c)**2)**2,x)
```

output
```
Integral(csc(c + d*x)/(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1), x)/a**2
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{\csc(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = -\frac{\frac{3\log(\cos(dx+c)+1)}{a^2} - \frac{3\log(\cos(dx+c)-1)}{a^2} - \frac{2\left(3\cos(dx+c)^2+1\right)}{a^2\cos(dx+c)^3}}{6\,d}$$

input
```
integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*log(cos(d*x + c) + 1)/a^2 - 3*log(cos(d*x + c) - 1)/a^2 - 2*(3*cos
(d*x + c)^2 + 1)/(a^2*cos(d*x + c)^3))/d
```

3.53.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(45) = 90.

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.28

$$\int \frac{\csc(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\frac{3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2} + \frac{8\left(\frac{3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + 2\right)}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6\,d}$$

input
```
integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/6*(3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + 8*(3*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 + 2)/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3))/d
```

3.53.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{\csc(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\cos(c+dx)^2+\frac{1}{3}}{a^2\,d\cos(c+dx)^3} - \frac{\operatorname{atanh}(\cos(c+dx))}{a^2\,d}$$

input
```
int(1/(sin(c + d*x)*(a - a*sin(c + d*x)^2)^2),x)
```

output
```
(cos(c + d*x)^2 + 1/3)/(a^2*d*cos(c + d*x)^3) - atanh(cos(c + d*x))/(a^2*d
)
```

3.54 $\int \frac{\csc^3(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

3.54.1 Optimal result

Integrand size = 24, antiderivative size = 78

$$\int \frac{\csc^3(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx = -\frac{5\text{arctanh}(\cos(c+dx))}{2a^2d}+\frac{5\sec(c+dx)}{2a^2d}+\frac{5\sec^3(c+dx)}{6a^2d}-\frac{\csc^2(c+dx)\sec^3(c+dx)}{2a^2d}$$

output
```
-5/2*arctanh(cos(d*x+c))/a^2/d+5/2*sec(d*x+c)/a^2/d+5/6*sec(d*x+c)^3/a^2/d
-1/2*csc(d*x+c)^2*sec(d*x+c)^3/a^2/d
```

3.54.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 208 vs. 2(78) = 156.

Time = 0.38 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.67

$$\int \frac{\csc^3(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx = \frac{2\csc^8(c+dx)\left(22-40\cos(2(c+dx))+13\cos(3(c+dx))-30\cos(4(c+dx))+13\cos(5(c+dx))+15\,\mathrm{c}\right.}{}$$

input
```
Integrate[Csc[c + d*x]^3/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
(2*Csc[c + d*x]^8*(22 - 40*Cos[2*(c + d*x)] + 13*Cos[3*(c + d*x)] - 30*Cos
[4*(c + d*x)] + 13*Cos[5*(c + d*x)] + 15*Cos[3*(c + d*x)]*Log[Cos[(c + d*x
)/2]] + 15*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 15*Cos[3*(c + d*x)]*Lo
g[Sin[(c + d*x)/2]] - 15*Cos[5*(c + d*x)]*Log[Sin[(c + d*x)/2]] + Cos[c +
d*x]*(-26 - 30*Log[Cos[(c + d*x)/2]] + 30*Log[Sin[(c + d*x)/2]])))/(3*a^2*
d*(Csc[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2)^3)
```

3.54.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 3654, 3042, 3102, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^3\left(a-a\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \csc^3(c+dx)\sec^4(c+dx)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(c+dx)^3\sec(c+dx)^4dx}{a^2}$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int \frac{\sec^6(c+dx)}{\left(1-\sec^2(c+dx)\right)^2}d\sec(c+dx)}{a^2d}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\sec^5(c+dx)}{2\left(1-\sec^2(c+dx)\right)}-\frac{5}{2}\int \frac{\sec^4(c+dx)}{1-\sec^2(c+dx)}d\sec(c+dx)}{a^2d}$$

$$\Big\downarrow \text{254}$$

$$\frac{\frac{\sec^5(c+dx)}{2(1-\sec^2(c+dx))}-\frac{5}{2}\int\left(-\sec^2(c+dx)+\frac{1}{1-\sec^2(c+dx)}-1\right)d\sec(c+dx)}{a^2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sec^5(c+dx)}{2(1-\sec^2(c+dx))}-\frac{5}{2}\left(\operatorname{arctanh}(\sec(c+dx))-\frac{1}{3}\sec^3(c+dx)-\sec(c+dx)\right)}{a^2d}$$

input
```
Int[Csc[c + d*x]^3/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
(Sec[c + d*x]^5/(2*(1 - Sec[c + d*x]^2)) - (5*(ArcTanh[Sec[c + d*x]] - Sec
[c + d*x] - Sec[c + d*x]^3/3))/2)/(a^2*d)
```

3.54.3.1 Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.54.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{1}{4+4\cos(dx+c)}-\frac{5\ln(1+\cos(dx+c))}{4}+\frac{1}{3\cos(dx+c)^3}+\frac{2}{\cos(dx+c)}+\frac{1}{4\cos(dx+c)-4}+\frac{5\ln(\cos(dx+c)-1)}{4}}{d\,a^2}$
default	$\frac{\frac{1}{4+4\cos(dx+c)}-\frac{5\ln(1+\cos(dx+c))}{4}+\frac{1}{3\cos(dx+c)^3}+\frac{2}{\cos(dx+c)}+\frac{1}{4\cos(dx+c)-4}+\frac{5\ln(\cos(dx+c)-1)}{4}}{d\,a^2}$
parallelrisch	$\frac{60\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-165\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\cot^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{24a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$
risch	$\frac{15\,e^{9i(dx+c)}+20\,e^{7i(dx+c)}-22\,e^{5i(dx+c)}+20\,e^{3i(dx+c)}+15\,e^{i(dx+c)}}{3d\,a^2\left(e^{2i(dx+c)}+1\right)^3\left(e^{2i(dx+c)}-1\right)^2}-\frac{5\ln\left(e^{i(dx+c)}+1\right)}{2d\,a^2}+\frac{5\ln\left(e^{i(dx+c)}-1\right)}{2d\,a^2}$
norman	$\frac{\frac{1}{8ad}+\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{75\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da}-\frac{65\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12da}-\frac{55\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da}}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2d}$

input
```
int(csc(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(1/4/(1+cos(d*x+c))-5/4*ln(1+cos(d*x+c))+1/3/cos(d*x+c)^3+2/cos(d*
x+c)+1/4/(cos(d*x+c)-1)+5/4*ln(cos(d*x+c)-1))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.51

$$\int \frac{\csc^3(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx$$
$$=\frac{30\,\cos(dx+c)^4-20\,\cos(dx+c)^2-15\left(\cos(dx+c)^5-\cos(dx+c)^3\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)+15\,(\cos}{12\left(a^2d\cos(dx+c)^5-a^2d\cos(dx+c)^3\right)}$$

input
```
integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/12*(30*cos(d*x + c)^4 - 20*cos(d*x + c)^2 - 15*(cos(d*x + c)^5 - cos(d*x
 + c)^3)*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^5 - cos(d*x + c)^3
)*log(-1/2*cos(d*x + c) + 1/2) - 4)/(a^2*d*cos(d*x + c)^5 - a^2*d*cos(d*x
+ c)^3)
```

3.54.6 Sympy [F]

$$\int \frac{\csc^3(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\int \frac{\csc^3(c+dx)}{\sin^4(c+dx)-2\sin^2(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(csc(d*x+c)**3/(a-a*sin(d*x+c)**2)**2,x)
```

output
```
Integral(csc(c + d*x)**3/(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1), x)/a**
2
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\frac{2\left(15\cos(dx+c)^4-10\cos(dx+c)^2-2\right)}{a^2\cos(dx+c)^5-a^2\cos(dx+c)^3} - \frac{15\log(\cos(dx+c)+1)}{a^2} + \frac{15\log(\cos(dx+c)-1)}{a^2}}{12\,d}$$

input
```
integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/12*(2*(15*cos(d*x + c)^4 - 10*cos(d*x + c)^2 - 2)/(a^2*cos(d*x + c)^5 -
a^2*cos(d*x + c)^3) - 15*log(cos(d*x + c) + 1)/a^2 + 15*log(cos(d*x + c) -
 1)/a^2)/d
```

3.54.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(70) = 140$.

Time = 0.33 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.24

$$\int \frac{\csc^3(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = -\frac{\frac{3\left(\frac{10(\cos(dx+c)-1)}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)}{a^2(\cos(dx+c)-1)} - \frac{30\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2} + \frac{3(\cos(dx+c)-1)}{a^2(\cos(dx+c)+1)} - \frac{16\left(\frac{12(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+7\right)}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{24\,d}$$

input
```
integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/24*(3*(10*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)*(cos(d*x + c) + 1)
/(a^2*(cos(d*x + c) - 1)) - 30*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c)
 + 1))/a^2 + 3*(cos(d*x + c) - 1)/(a^2*(cos(d*x + c) + 1)) - 16*(12*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 9*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 + 7)/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3))/d
```

3.54.9 Mupad [B] (verification not implemented)

Time = 12.94 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \frac{\csc^3(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{-\frac{5\cos(c+d\,x)^4}{2}+\frac{5\cos(c+d\,x)^2}{3}+\frac{1}{3}}{d\,\left(a^2\cos\left(c+d\,x\right)^3-a^2\cos\left(c+d\,x\right)^5\right)} - \frac{5\,\mathrm{atanh}(\cos\left(c+d\,x\right))}{2\,a^2\,d}$$

input
```
int(1/(sin(c + d*x)^3*(a - a*sin(c + d*x)^2)^2),x)
```

output
```
((5*cos(c + d*x)^2)/3 - (5*cos(c + d*x)^4)/2 + 1/3)/(d*(a^2*cos(c + d*x)^3
 - a^2*cos(c + d*x)^5)) - (5*atanh(cos(c + d*x)))/(2*a^2*d)
```

3.55 $\int \frac{\sin^6(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx$

3.55.1 Optimal result

Integrand size = 24, antiderivative size = 69

$$\int \frac{\sin^6(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{5x}{2a^2}-\frac{5\tan(c+dx)}{2a^2d}+\frac{5\tan^3(c+dx)}{6a^2d}-\frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d}$$

output
```
5/2*x/a^2-5/2*tan(d*x+c)/a^2/d+5/6*tan(d*x+c)^3/a^2/d-1/2*sin(d*x+c)^2*tan
(d*x+c)^3/a^2/d
```

3.55.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int \frac{\sin^6(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{30(c+dx)-3\sin(2(c+dx))+4(-7+\sec^2(c+dx))\tan(c+dx)}{12a^2d}$$

input
```
Integrate[Sin[c + d*x]^6/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
(30*(c + d*x) - 3*Sin[2*(c + d*x)] + 4*(-7 + Sec[c + d*x]^2)*Tan[c + d*x])
/(12*a^2*d)
```

3.55.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 3654, 3042, 3071, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^6}{\left(a-a\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sin^2(c+dx)\tan^4(c+dx)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin(c+dx)^2\tan(c+dx)^4dx}{a^2}$$

$$\Big\downarrow \text{3071}$$

$$\frac{\int \frac{\tan^6(c+dx)}{\left(\tan^2(c+dx)+1\right)^2}d\tan(c+dx)}{a^2d}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{5}{2}\int \frac{\tan^4(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)-\frac{\tan^5(c+dx)}{2\left(\tan^2(c+dx)+1\right)}}{a^2d}$$

$$\Big\downarrow \text{254}$$

$$\frac{\frac{5}{2}\int \left(\tan^2(c+dx)+\frac{1}{\tan^2(c+dx)+1}-1\right)d\tan(c+dx)-\frac{\tan^5(c+dx)}{2\left(\tan^2(c+dx)+1\right)}}{a^2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{5}{2}\left(\arctan(\tan(c+dx))+\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)-\frac{\tan^5(c+dx)}{2\left(\tan^2(c+dx)+1\right)}}{a^2d}$$

input
```
Int[Sin[c + d*x]^6/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
(-1/2*Tan[c + d*x]^5/(1 + Tan[c + d*x]^2) + (5*(ArcTan[Tan[c + d*x]] - Tan
[c + d*x] + Tan[c + d*x]^3/3))/2)/(a^2*d)
```

3.55.3.1 Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.55. $\int \frac{\sin^6(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx$

3.55.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}-2\tan(dx+c)-\frac{\tan(dx+c)}{2\left(1+\tan^2(dx+c)\right)}+\frac{5\arctan(\tan(dx+c))}{2}}{d\,a^2}$	56
default	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}-2\tan(dx+c)-\frac{\tan(dx+c)}{2\left(1+\tan^2(dx+c)\right)}+\frac{5\arctan(\tan(dx+c))}{2}}{d\,a^2}$	56
parallelrisch	$\frac{180dx\cos(dx+c)+60dx\cos(3dx+3c)-65\sin(3dx+3c)-30\sin(dx+c)-3\sin(5dx+5c)}{24a^2d(\cos(3dx+3c)+3\cos(dx+c))}$	83
risch	$\frac{5x}{2a^2}+\frac{i\mathrm{e}^{2i(dx+c)}}{8d\,a^2}-\frac{i\mathrm{e}^{-2i(dx+c)}}{8d\,a^2}-\frac{2i\left(9\,\mathrm{e}^{4i(dx+c)}+12\,\mathrm{e}^{2i(dx+c)}+7\right)}{3d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	90

input
```
int(sin(d*x+c)^6/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(1/3*tan(d*x+c)^3-2*tan(d*x+c)-1/2*tan(d*x+c)/(1+tan(d*x+c)^2)+5/2
*arctan(tan(d*x+c)))
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{\sin^6(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{15\,dx\cos\left(dx+c\right)^3-\left(3\,\cos\left(dx+c\right)^4+14\,\cos\left(dx+c\right)^2-2\right)\sin\left(dx+c\right)}{6\,a^2d\cos\left(dx+c\right)^3}$$

input
```
integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/6*(15*d*x*cos(d*x + c)^3 - (3*cos(d*x + c)^4 + 14*cos(d*x + c)^2 - 2)*si
n(d*x + c))/(a^2*d*cos(d*x + c)^3)
```

3.55.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1275 vs. $2(63) = 126$.

Time = 16.73 (sec) , antiderivative size = 1275, normalized size of antiderivative = 18.48

$$\int \frac{\sin^6(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)**6/(a-a*sin(d*x+c)**2)**2,x)
```

output
```
Piecewise((15*d*x*tan(c/2 + d*x/2)**10/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*
a**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan
(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) - 15*d*x*tan(c
/2 + d*x/2)**8/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)*
*8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**
2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) - 30*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d
*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 +
 d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2
- 6*a**2*d) + 30*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**10 -
6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*t
an(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) + 15*d*x*tan
(c/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2
)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a
**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) - 15*d*x/(6*a**2*d*tan(c/2 + d*x/2)*
*10 - 6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a*
*2*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) + 30*t
an(c/2 + d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x
/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6
*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) - 40*tan(c/2 + d*x/2)**7/(6*a**2*d
*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/...
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int \frac{\sin^6(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = -\frac{\frac{3\tan(dx+c)}{a^2\tan(dx+c)^2+a^2} - \frac{2\left(\tan(dx+c)^3-6\tan(dx+c)\right)}{a^2} - \frac{15\,(dx+c)}{a^2}}{6\,d}$$

input
```
integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*tan(d*x + c)/(a^2*tan(d*x + c)^2 + a^2) - 2*(tan(d*x + c)^3 - 6*ta
n(d*x + c))/a^2 - 15*(d*x + c)/a^2)/d
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{\sin^6(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\frac{15\,(dx+c)}{a^2} - \frac{3\,\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)a^2} + \frac{2\left(a^4\tan(dx+c)^3-6\,a^4\tan(dx+c)\right)}{a^6}}{6\,d}$$

input
```
integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/6*(15*(d*x + c)/a^2 - 3*tan(d*x + c)/((tan(d*x + c)^2 + 1)*a^2) + 2*(a^4
*tan(d*x + c)^3 - 6*a^4*tan(d*x + c))/a^6)/d
```

3.55.9 Mupad [B] (verification not implemented)

Time = 13.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{\sin^6(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{5\,x}{2\,a^2} - \frac{\tan(c+d\,x)}{2\,d\,\left(a^2\tan\left(c+d\,x\right)^2+a^2\right)} - \frac{2\tan(c+d\,x)}{a^2\,d} + \frac{\tan(c+d\,x)^3}{3\,a^2\,d}$$

input
```
int(sin(c + d*x)^6/(a - a*sin(c + d*x)^2)^2,x)
```

output
```
(5*x)/(2*a^2) - tan(c + d*x)/(2*d*(a^2 + a^2*tan(c + d*x)^2)) - (2*tan(c +
 d*x))/(a^2*d) + tan(c + d*x)^3/(3*a^2*d)
```

3.56 $\int \frac{\sin^4(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

3.56.1 Optimal result

Integrand size = 24, antiderivative size = 38

$$\int \frac{\sin^4(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx=\frac{x}{a^2}-\frac{\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}$$

output
```
x/a^2-tan(d*x+c)/a^2/d+1/3*tan(d*x+c)^3/a^2/d
```

3.56.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{\sin^4(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx=\frac{\frac{\arctan(\tan(c+dx))}{d}-\frac{\tan(c+dx)}{d}+\frac{\tan^3(c+dx)}{3d}}{a^2}$$

input
```
Integrate[Sin[c + d*x]^4/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
(ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d))/a^2
```

3.56.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 3654, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^4}{\left(a-a\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \tan^4(c+dx)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \tan(c+dx)^4dx}{a^2}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\frac{\tan^3(c+dx)}{3d}-\int \tan^2(c+dx)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\tan^3(c+dx)}{3d}-\int \tan(c+dx)^2dx}{a^2}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\int 1dx+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}}{a^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}+x}{a^2}$$

input
```
Int[Sin[c + d*x]^4/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
(x - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d))/a^2
```

3.56.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.56.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{\left(\tan^{3}(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d\,a^{2}}$
default	$\frac{\frac{\left(\tan^{3}(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d\,a^{2}}$
risch	$\frac{x}{a^{2}}-\frac{4i\left(3\,e^{4i(dx+c)}+3\,e^{2i(dx+c)}+2\right)}{3d\,a^{2}\left(e^{2i(dx+c)}+1\right)^{3}}$
parallelrisch	$\frac{3\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)xd-9\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)xd+6\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+9\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)xd-20\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3dx+6\tan(}{3d\,a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}$
norman	$\frac{\frac{x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{x}{a}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{4\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{38\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{24\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{38(t}{}}{(1+ta}$

input
```
int(sin(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+arctan(tan(d*x+c)))
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \frac{\sin^4(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{3\,dx\cos(dx+c)^3-\left(4\,\cos(dx+c)^2-1\right)\sin(dx+c)}{3\,a^2d\cos(dx+c)^3}$$

input
```
integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*d*x*cos(d*x + c)^3 - (4*cos(d*x + c)^2 - 1)*sin(d*x + c))/(a^2*d*co
s(d*x + c)^3)
```

3.56.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. 2(31) = 62.

Time = 7.02 (sec) , antiderivative size = 551, normalized size of antiderivative = 14.50

$$\int \frac{\sin^4(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx$$
$$= \begin{cases} \frac{3dx\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} - \frac{9dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} \\ \frac{x\sin^4(c)}{(-a\sin^2(c)+a)^2} \end{cases}$$

input
```
integrate(sin(d*x+c)**4/(a-a*sin(d*x+c)**2)**2,x)
```

output
```
Piecewise((3*d*x*tan(c/2 + d*x/2)**6/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**
2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) - 9*d*x
*tan(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*
x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 9*d*x*tan(c/2 + d*x/2
)**2/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2
*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) - 3*d*x/(3*a**2*d*tan(c/2 + d*x/2)**6 -
 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) +
 6*tan(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 +
d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) - 20*tan(c/2 + d*x/2)
**3/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*
d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 6*tan(c/2 + d*x/2)/(3*a**2*d*tan(c/2 +
 d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 -
 3*a**2*d), Ne(d, 0)), (x*sin(c)**4/(-a*sin(c)**2 + a)**2, True))
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{\sin^4(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\frac{\tan(dx+c)^3-3\tan(dx+c)}{a^2}+\frac{3(dx+c)}{a^2}}{3d}$$

input
```
integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*((tan(d*x + c)^3 - 3*tan(d*x + c))/a^2 + 3*(d*x + c)/a^2)/d
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{\sin^4(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\frac{3(dx+c)}{a^2}+\frac{a^4\tan(dx+c)^3-3a^4\tan(dx+c)}{a^6}}{3d}$$

input
```
integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)/a^2 + (a^4*tan(d*x + c)^3 - 3*a^4*tan(d*x + c))/a^6)/d
```

3.56.9 Mupad [B] (verification not implemented)

Time = 13.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{\sin^4(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx = \frac{x}{a^2} - \frac{\tan(c+d\,x) - \frac{\tan(c+d\,x)^3}{3}}{a^2\,d}$$

input
```
int(sin(c + d*x)^4/(a - a*sin(c + d*x)^2)^2,x)
```

output
```
x/a^2 - (tan(c + d*x) - tan(c + d*x)^3/3)/(a^2*d)
```

3.57 $\int \frac{\sin^2(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx$

3.57.1 Optimal result

Integrand size = 24, antiderivative size = 18

$$\int \frac{\sin^2(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\tan^3(c+dx)}{3a^2d}$$

output
```
1/3*tan(d*x+c)^3/a^2/d
```

3.57.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\tan^3(c+dx)}{3a^2d}$$

input
```
Integrate[Sin[c + d*x]^2/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
Tan[c + d*x]^3/(3*a^2*d)
```

3.57.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3042, 3654, 3042, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{(a-a\sin^2(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(c+dx)^2}{(a-a\sin(c+dx)^2)^2}dx$$
$$\Big\downarrow \text{3654}$$
$$\frac{\int \sec^2(c+dx)\tan^2(c+dx)dx}{a^2}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \sec(c+dx)^2\tan(c+dx)^2dx}{a^2}$$
$$\Big\downarrow \text{3087}$$
$$\frac{\int \tan^2(c+dx)d\tan(c+dx)}{a^2d}$$
$$\Big\downarrow \text{15}$$
$$\frac{\tan^3(c+dx)}{3a^2d}$$

input
```
Int[Sin[c + d*x]^2/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
Tan[c + d*x]^3/(3*a^2*d)
```

3.57.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.57.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\tan^3(dx+c)}{3a^2d}$	17
default	$\frac{\tan^3(dx+c)}{3a^2d}$	17
risch	$-\frac{2i\left(3\,\mathrm{e}^{4i(dx+c)}+1\right)}{3d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	36
parallelrisch	$\frac{-\sin(3dx+3c)+3\sin(dx+c)}{3a^2d(\cos(3dx+3c)+3\cos(dx+c))}$	49
norman	$\frac{-\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{16\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{8\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}}{a\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$	93

input
```
int(sin(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*tan(d*x+c)^3/a^2/d
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \frac{\sin^2(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = -\frac{\left(\cos(dx+c)^2-1\right)\sin(dx+c)}{3\,a^2d\cos(dx+c)^3}$$

input
```
integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(cos(d*x + c)^2 - 1)*sin(d*x + c)/(a^2*d*cos(d*x + c)^3)
```

3.57.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(14) = 28.

Time = 2.79 (sec) , antiderivative size = 94, normalized size of antiderivative = 5.22

$$\int \frac{\sin^2(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \begin{cases} -\frac{8\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} & \text{for } d\neq 0 \\ \frac{x\sin^2(c)}{(-a\sin^2(c)+a)^2} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(d*x+c)**2/(a-a*sin(d*x+c)**2)**2,x)
```

output
```
Piecewise((-8*tan(c/2 + d*x/2)**3/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d
*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d), Ne(d, 0))
, (x*sin(c)**2/(-a*sin(c)**2 + a)**2, True))
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\sin^2(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\tan(dx+c)^3}{3\,a^2d}$$

input
```
integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*tan(d*x + c)^3/(a^2*d)
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\sin^2(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\tan{(dx+c)}^3}{3\,a^2d}$$

input
```
integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/3*tan(d*x + c)^3/(a^2*d)
```

3.57.9 Mupad [B] (verification not implemented)

Time = 12.60 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\sin^2(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\tan(c+d\,x)^3}{3\,a^2\,d}$$

input
```
int(sin(c + d*x)^2/(a - a*sin(c + d*x)^2)^2,x)
```

output
```
tan(c + d*x)^3/(3*a^2*d)
```

3.58 $\int \frac{1}{(a-a\sin^2(c+dx))^2}\,dx$

3.58.1 Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\tan(c+dx)}{a^2d} + \frac{\tan^3(c+dx)}{3a^2d}$$

output
```
tan(d*x+c)/a^2/d+1/3*tan(d*x+c)^3/a^2/d
```

3.58.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)}{a^2d}$$

input
```
Integrate[(a - a*Sin[c + d*x]^2)^(-2),x]
```

output
```
(Tan[c + d*x] + Tan[c + d*x]^3/3)/(a^2*d)
```

3.58.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-a\sin^2(c+dx)\right)^2}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(a-a\sin(c+dx)^2)^2}dx$$
$$\Big\downarrow \text{ 3654}$$
$$\frac{\int \sec^4(c+dx)dx}{a^2}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{a^2}$$
$$\Big\downarrow \text{ 4254}$$
$$-\frac{\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{a^2d}$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)}{a^2d}$$

input
```
Int[(a - a*Sin[c + d*x]^2)^(-2),x]
```

output
```
-((-Tan[c + d*x] - Tan[c + d*x]^3/3)/(a^2*d))
```

3.58.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.58.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)}{d\,a^2}$	25
default	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)}{d\,a^2}$	25
risch	$\frac{4i\left(3\,\mathrm{e}^{2i(dx+c)}+1\right)}{3d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	36
parallelrisch	$\frac{2\sin(3dx+3c)+6\sin(dx+c)}{3a^2d(\cos(3dx+3c)+3\cos(dx+c))}$	49
norman	$\frac{-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$	76

input
```
int(1/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(1/3*tan(d*x+c)^3+tan(d*x+c))
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a - a\sin^2(c+dx))^2}\,dx = \frac{(2\cos(dx+c)^2 + 1)\sin(dx+c)}{3\,a^2 d\cos(dx+c)^3}$$

input
```
integrate(1/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(2*cos(d*x + c)^2 + 1)*sin(d*x + c)/(a^2*d*cos(d*x + c)^3)
```

3.58.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(26) = 52.

Time = 1.19 (sec) , antiderivative size = 238, normalized size of antiderivative = 7.44

$$\int \frac{1}{(a - a\sin^2(c+dx))^2}\,dx$$
$$= \begin{cases} -\frac{6\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} + \frac{4\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a} \\ \frac{x}{(-a\sin^2(c)+a)^2} \end{cases}$$

input
```
integrate(1/(a-a*sin(d*x+c)**2)**2,x)
```

output
```
Piecewise((-6*tan(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d
*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 4*tan(c/
2 + d*x/2)**3/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4
 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) - 6*tan(c/2 + d*x/2)/(3*a**2*d
*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d
*x/2)**2 - 3*a**2*d), Ne(d, 0)), (x/(-a*sin(c)**2 + a)**2, True))
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a-a\sin^2(c+dx)\right)^2}\,dx=\frac{\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)}{3\,a^2d}$$

input
```
integrate(1/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))/(a^2*d)
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a-a\sin^2(c+dx)\right)^2}\,dx=\frac{\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)}{3\,a^2d}$$

input
```
integrate(1/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))/(a^2*d)
```

3.58.9 Mupad [B] (verification not implemented)

Time = 12.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(a-a\sin^2(c+dx)\right)^2}\,dx=\frac{\tan(c+d\,x)\,\left({\tan(c+d\,x)}^2+3\right)}{3\,a^2\,d}$$

input
```
int(1/(a - a*sin(c + d*x)^2)^2,x)
```

output
```
(tan(c + d*x)*(tan(c + d*x)^2 + 3))/(3*a^2*d)
```

3.59 $\int \frac{\csc^2(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

3.59.1 Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{\csc^2(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx=-\frac{\cot(c+dx)}{a^2d}+\frac{2\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}$$

output
```
-cot(d*x+c)/a^2/d+2*tan(d*x+c)/a^2/d+1/3*tan(d*x+c)^3/a^2/d
```

3.59.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{\csc^2(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx=\frac{-\frac{\cot(c+dx)}{d}+\frac{5\tan(c+dx)}{3d}+\frac{\sec^2(c+dx)\tan(c+dx)}{3d}}{a^2}$$

input
```
Integrate[Csc[c + d*x]^2/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
(-(Cot[c + d*x]/d) + (5*Tan[c + d*x])/(3*d) + (Sec[c + d*x]^2*Tan[c + d*x]
)/(3*d))/a^2
```

3.59.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3654, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{(a-a\sin^2(c+dx))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(c+dx)^2\,(a-a\sin(c+dx)^2)^2}dx$$

$$\Big\downarrow \; 3654$$

$$\frac{\int \csc^2(c+dx)\sec^4(c+dx)dx}{a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \csc(c+dx)^2\sec(c+dx)^4dx}{a^2}$$

$$\Big\downarrow \; 3100$$

$$\frac{\int \cot^2(c+dx)\left(\tan^2(c+dx)+1\right)^2d\tan(c+dx)}{a^2d}$$

$$\Big\downarrow \; 244$$

$$\frac{\int \left(\cot^2(c+dx)+\tan^2(c+dx)+2\right)d\tan(c+dx)}{a^2d}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{3}\tan^3(c+dx)+2\tan(c+dx)-\cot(c+dx)}{a^2d}$$

input
```
Int[Csc[c + d*x]^2/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
(-Cot[c + d*x] + 2*Tan[c + d*x] + Tan[c + d*x]^3/3)/(a^2*d)
```

3.59.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.59.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}+2\tan(dx+c)-\frac{1}{\tan(dx+c)}}{d\,a^2}$	37
default	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}+2\tan(dx+c)-\frac{1}{\tan(dx+c)}}{d\,a^2}$	37
risch	$-\frac{16i\left(2\,\mathrm{e}^{2i(dx+c)}+1\right)}{3d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^3\left(\mathrm{e}^{2i(dx+c)}-1\right)}$	49
parallelrisch	$-\frac{2\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\csc\left(\frac{dx}{2}+\frac{c}{2}\right)(\cos(4dx+4c)+2\cos(2dx+2c))}{3a^2d(\cos(3dx+3c)+3\cos(dx+c))}$	68
norman	$\frac{\frac{1}{2ad}-\frac{6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{25\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{6\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$	116

input
```
int(csc(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(1/3*tan(d*x+c)^3+2*tan(d*x+c)-1/tan(d*x+c))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{\csc^2(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = -\frac{8\,\cos(dx+c)^4-4\,\cos(dx+c)^2-1}{3\,a^2d\cos(dx+c)^3\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(8*cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3*sin(d
*x + c))
```

3.59.6 Sympy [F]

$$\int \frac{\csc^2(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\int \frac{\csc^2(c+dx)}{\sin^4(c+dx)-2\sin^2(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(csc(d*x+c)**2/(a-a*sin(d*x+c)**2)**2,x)
```

output
```
Integral(csc(c + d*x)**2/(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1), x)/a**
2
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{\csc^2(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\frac{\tan(dx+c)^3+6\,\tan(dx+c)}{a^2}-\frac{3}{a^2\tan(dx+c)}}{3\,d}$$

input
```
integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*((tan(d*x + c)^3 + 6*tan(d*x + c))/a^2 - 3/(a^2*tan(d*x + c)))/d
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{\csc^2(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = -\frac{\frac{3}{a^2\tan(dx+c)} - \frac{a^4\tan(dx+c)^3+6a^4\tan(dx+c)}{a^6}}{3d}$$

input
```
integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/3*(3/(a^2*tan(d*x + c)) - (a^4*tan(d*x + c)^3 + 6*a^4*tan(d*x + c))/a^6
)/d
```

3.59.9 Mupad [B] (verification not implemented)

Time = 12.70 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{\csc^2(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\tan(c+d\,x)^4+6\tan(c+d\,x)^2-3}{3\,a^2\,d\tan(c+d\,x)}$$

input
```
int(1/(sin(c + d*x)^2*(a - a*sin(c + d*x)^2)^2),x)
```

output
```
(6*tan(c + d*x)^2 + tan(c + d*x)^4 - 3)/(3*a^2*d*tan(c + d*x))
```

3.60 $\int \frac{\csc^4(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx$

3.60.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{\csc^4(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx=-\frac{3\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{3\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}$$

output
```
-3*cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d+3*tan(d*x+c)/a^2/d+1/3*tan(d*x+
c)^3/a^2/d
```

3.60.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \frac{\csc^4(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx=\frac{16\left(-\frac{\cot(2(c+dx))}{3d}-\frac{\cot(2(c+dx))\csc^2(2(c+dx))}{6d}\right)}{a^2}$$

input
```
Integrate[Csc[c + d*x]^4/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
(16*(-1/3*Cot[2*(c + d*x)]/d - (Cot[2*(c + d*x)]*Csc[2*(c + d*x)]^2)/(6*d)
))/a^2
```

3.60.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3654, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^4\left(a-a\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \csc^4(c+dx)\sec^4(c+dx)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(c+dx)^4\sec(c+dx)^4dx}{a^2}$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^4(c+dx)\left(\tan^2(c+dx)+1\right)^3d\tan(c+dx)}{a^2d}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^4(c+dx)+3\cot^2(c+dx)+\tan^2(c+dx)+3\right)d\tan(c+dx)}{a^2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}\tan^3(c+dx)+3\tan(c+dx)-\frac{1}{3}\cot^3(c+dx)-3\cot(c+dx)}{a^2d}$$

input
```
Int[Csc[c + d*x]^4/(a - a*Sin[c + d*x]^2)^2,x]
```

output
```
(-3*Cot[c + d*x] - Cot[c + d*x]^3/3 + 3*Tan[c + d*x] + Tan[c + d*x]^3/3)/(
a^2*d)
```

3.60.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.60.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{-\frac{1}{3\tan(dx+c)^3}-\frac{3}{\tan(dx+c)}+\frac{\left(\tan^3(dx+c)\right)}{3}+3\tan(dx+c)}{d\,a^2}$
default	$\frac{-\frac{1}{3\tan(dx+c)^3}-\frac{3}{\tan(dx+c)}+\frac{\left(\tan^3(dx+c)\right)}{3}+3\tan(dx+c)}{d\,a^2}$
risch	$\frac{32i\left(3\,\mathrm{e}^{4i(dx+c)}-1\right)}{3d\,a^2\left(\mathrm{e}^{2i(dx+c)}-1\right)^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$
parallelrisch	$\frac{(-3\cos(2dx+2c)+\cos(6dx+6c))\left(\sec^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\csc^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12a^2d(\cos(3dx+3c)+3\cos(dx+c))}$
norman	$\frac{\frac{1}{24ad}+\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}-\frac{91\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da}+\frac{35\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}-\frac{91\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da}+\frac{5\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}+\frac{\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24da}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$

input
```
int(csc(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(-1/3/tan(d*x+c)^3-3/tan(d*x+c)+1/3*tan(d*x+c)^3+3*tan(d*x+c))
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \frac{\csc^4(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = -\frac{16\,\cos(dx+c)^6-24\,\cos(dx+c)^4+6\,\cos(dx+c)^2+1}{3\,(a^2d\cos(dx+c)^5-a^2d\cos(dx+c)^3)\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(16*cos(d*x + c)^6 - 24*cos(d*x + c)^4 + 6*cos(d*x + c)^2 + 1)/((a^2*
d*cos(d*x + c)^5 - a^2*d*cos(d*x + c)^3)*sin(d*x + c))
```

3.60.6 Sympy [F]

$$\int \frac{\csc^4(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\int \frac{\csc^4(c+dx)}{\sin^4(c+dx)-2\sin^2(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(csc(d*x+c)**4/(a-a*sin(d*x+c)**2)**2,x)
```

output
```
Integral(csc(c + d*x)**4/(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1), x)/a**
2
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int \frac{\csc^4(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = \frac{\frac{\tan(dx+c)^3+9\,\tan(dx+c)}{a^2}-\frac{9\,\tan(dx+c)^2+1}{a^2\tan(dx+c)^3}}{3\,d}$$

input
```
integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*((tan(d*x + c)^3 + 9*tan(d*x + c))/a^2 - (9*tan(d*x + c)^2 + 1)/(a^2*t
an(d*x + c)^3))/d
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \frac{\csc^4(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = -\frac{8\left(3\tan\left(2\,dx+2\,c\right)^2+1\right)}{3\,a^2d\tan\left(2\,dx+2\,c\right)^3}$$

input
```
integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-8/3*(3*tan(2*d*x + 2*c)^2 + 1)/(a^2*d*tan(2*d*x + 2*c)^3)
```

3.60.9 Mupad [B] (verification not implemented)

Time = 13.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{\csc^4(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx = -\frac{-\tan(c+d\,x)^6-9\tan(c+d\,x)^4+9\tan(c+d\,x)^2+1}{3\,a^2\,d\tan\left(c+d\,x\right)^3}$$

input
```
int(1/(sin(c + d*x)^4*(a - a*sin(c + d*x)^2)^2),x)
```

output
```
-(9*tan(c + d*x)^2 - 9*tan(c + d*x)^4 - tan(c + d*x)^6 + 1)/(3*a^2*d*tan(c
 + d*x)^3)
```

3.61 $\int \frac{1}{(a-a\sin^2(x))^3}\,dx$

3.61.1 Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \frac{1}{(a-a\sin^2(x))^3}\,dx = \frac{\tan(x)}{a^3} + \frac{2\tan^3(x)}{3a^3} + \frac{\tan^5(x)}{5a^3}$$

output
```
tan(x)/a^3+2/3*tan(x)^3/a^3+1/5*tan(x)^5/a^3
```

3.61.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a-a\sin^2(x))^3}\,dx = \frac{\tan(x) + \frac{2\tan^3(x)}{3} + \frac{\tan^5(x)}{5}}{a^3}$$

input
```
Integrate[(a - a*Sin[x]^2)^(-3),x]
```

output
```
(Tan[x] + (2*Tan[x]^3)/3 + Tan[x]^5/5)/a^3
```

3.61.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-a\sin(x)^2\right)^3}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sec^6(x)dx}{a^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^6dx}{a^3}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int \left(\tan^4(x)+2\tan^2(x)+1\right)d(-\tan(x))}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{5}\tan^5(x)-\frac{2\tan^3(x)}{3}-\tan(x)}{a^3}$$

input
```
Int[(a - a*Sin[x]^2)^(-3),x]
```

output
```
-((-Tan[x] - (2*Tan[x]^3)/3 - Tan[x]^5/5)/a^3)
```

3.61.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.61.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\frac{\left(\tan^5(x)\right)}{5}+\frac{2\left(\tan^3(x)\right)}{3}+\tan(x)}{a^3}$	20
parallelrisch	$\frac{\tan(x)\left(3\left(\sec^4(x)\right)+4\left(\sec^2(x)\right)+8\right)}{15a^3}$	22
risch	$\frac{16i\left(10\,e^{4ix}+5\,e^{2ix}+1\right)}{15\left(e^{2ix}+1\right)^5a^3}$	32
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{8\left(\tan^3\left(\frac{x}{2}\right)\right)}{3a}-\frac{116\left(\tan^5\left(\frac{x}{2}\right)\right)}{15a}+\frac{8\left(\tan^7\left(\frac{x}{2}\right)\right)}{3a}-\frac{2\left(\tan^9\left(\frac{x}{2}\right)\right)}{a}}{a^2\left(\tan^2\left(\frac{x}{2}\right)-1\right)^5}$	69

input
```
int(1/(a-a*sin(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/5*tan(x)^5+2/3*tan(x)^3+tan(x))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a - a\sin^2(x)\right)^3}\,dx = \frac{\left(8\,\cos\left(x\right)^4 + 4\,\cos\left(x\right)^2 + 3\right)\sin\left(x\right)}{15\,a^3\cos\left(x\right)^5}$$

input
```
integrate(1/(a-a*sin(x)^2)^3,x, algorithm="fricas")
```

output
```
1/15*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/(a^3*cos(x)^5)
```

3.61.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(27) = 54.

Time = 1.81 (sec) , antiderivative size = 362, normalized size of antiderivative = 12.48

$$\begin{aligned}\int \frac{1}{\left(a - a\sin^2(x)\right)^3}\,dx =& -\frac{30\tan^9\left(\frac{x}{2}\right)}{15a^3\tan^{10}\left(\frac{x}{2}\right) - 75a^3\tan^8\left(\frac{x}{2}\right) + 150a^3\tan^6\left(\frac{x}{2}\right) - 150a^3\tan^4\left(\frac{x}{2}\right) + 75a^3\tan^2\left(\frac{x}{2}\right) - 15a^3} \\ &+\frac{40\tan^7\left(\frac{x}{2}\right)}{15a^3\tan^{10}\left(\frac{x}{2}\right) - 75a^3\tan^8\left(\frac{x}{2}\right) + 150a^3\tan^6\left(\frac{x}{2}\right) - 150a^3\tan^4\left(\frac{x}{2}\right) + 75a^3\tan^2\left(\frac{x}{2}\right) - 15a^3} \\ &-\frac{116\tan^5\left(\frac{x}{2}\right)}{15a^3\tan^{10}\left(\frac{x}{2}\right) - 75a^3\tan^8\left(\frac{x}{2}\right) + 150a^3\tan^6\left(\frac{x}{2}\right) - 150a^3\tan^4\left(\frac{x}{2}\right) + 75a^3\tan^2\left(\frac{x}{2}\right) - 15a^3} \\ &+\frac{40\tan^3\left(\frac{x}{2}\right)}{15a^3\tan^{10}\left(\frac{x}{2}\right) - 75a^3\tan^8\left(\frac{x}{2}\right) + 150a^3\tan^6\left(\frac{x}{2}\right) - 150a^3\tan^4\left(\frac{x}{2}\right) + 75a^3\tan^2\left(\frac{x}{2}\right) - 15a^3} \\ &-\frac{30\tan\left(\frac{x}{2}\right)}{15a^3\tan^{10}\left(\frac{x}{2}\right) - 75a^3\tan^8\left(\frac{x}{2}\right) + 150a^3\tan^6\left(\frac{x}{2}\right) - 150a^3\tan^4\left(\frac{x}{2}\right) + 75a^3\tan^2\left(\frac{x}{2}\right) - 15a^3}\end{aligned}$$

input
```
integrate(1/(a-a*sin(x)**2)**3,x)
```

output
```
-30*tan(x/2)**9/(15*a**3*tan(x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan
(x/2)**6 - 150*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**2 - 15*a**3) + 40*tan(
x/2)**7/(15*a**3*tan(x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6
 - 150*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**2 - 15*a**3) - 116*tan(x/2)**5
/(15*a**3*tan(x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6 - 150*
a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**2 - 15*a**3) + 40*tan(x/2)**3/(15*a**
3*tan(x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6 - 150*a**3*tan
(x/2)**4 + 75*a**3*tan(x/2)**2 - 15*a**3) - 30*tan(x/2)/(15*a**3*tan(x/2)*
*10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6 - 150*a**3*tan(x/2)**4 +
75*a**3*tan(x/2)**2 - 15*a**3)
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a - a\sin^2(x)\right)^3}\,dx = \frac{3\,\tan\left(x\right)^5 + 10\,\tan\left(x\right)^3 + 15\,\tan\left(x\right)}{15\,a^3}$$

input
```
integrate(1/(a-a*sin(x)^2)^3,x, algorithm="maxima")
```

output
```
1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^3
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a - a\sin^2(x)\right)^3}\,dx = \frac{3\,\tan\left(x\right)^5 + 10\,\tan\left(x\right)^3 + 15\,\tan\left(x\right)}{15\,a^3}$$

input
```
integrate(1/(a-a*sin(x)^2)^3,x, algorithm="giac")
```

output
```
1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^3
```

3.61.9 Mupad [B] (verification not implemented)

Time = 12.89 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^3}\,dx=\frac{\tan(x)\ \left(3\tan(x)^4+10\tan(x)^2+15\right)}{15\,a^3}$$

input
```
int(1/(a - a*sin(x)^2)^3,x)
```

output
```
(tan(x)*(10*tan(x)^2 + 3*tan(x)^4 + 15))/(15*a^3)
```

3.62 $\int \frac{1}{(a-a\sin^2(x))^4} dx$

3.62.1 Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{1}{(a-a\sin^2(x))^4} dx = \frac{\tan(x)}{a^4} + \frac{\tan^3(x)}{a^4} + \frac{3\tan^5(x)}{5a^4} + \frac{\tan^7(x)}{7a^4}$$

output
```
tan(x)/a^4+tan(x)^3/a^4+3/5*tan(x)^5/a^4+1/7*tan(x)^7/a^4
```

3.62.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a-a\sin^2(x))^4} dx = \frac{\tan(x) + \tan^3(x) + \frac{3\tan^5(x)}{5} + \frac{\tan^7(x)}{7}}{a^4}$$

input
```
Integrate[(a - a*Sin[x]^2)^(-4),x]
```

output
```
(Tan[x] + Tan[x]^3 + (3*Tan[x]^5)/5 + Tan[x]^7/7)/a^4
```

3.62.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-a\sin(x)^2)^4}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sec^8(x)dx}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^8dx}{a^4}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int \left(\tan^6(x)+3\tan^4(x)+3\tan^2(x)+1\right)d(-\tan(x))}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{7}\tan^7(x)-\frac{3\tan^5(x)}{5}-\tan^3(x)-\tan(x)}{a^4}$$

input
```
Int[(a - a*Sin[x]^2)^(-4),x]
```

output
```
-((-Tan[x] - Tan[x]^3 - (3*Tan[x]^5)/5 - Tan[x]^7/7)/a^4)
```

3.62.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.62.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\frac{\left(\tan^{7}\left(x\right)\right)}{7}+\frac{3\left(\tan^{5}\left(x\right)\right)}{5}+\tan^{3}\left(x\right)+\tan\left(x\right)}{a^{4}}$	24
parallelrisch	$\frac{\tan\left(x\right)\left(\sec^{6}\left(x\right)\right)\left(32+\cos\left(6x\right)+8\cos\left(4x\right)+29\cos\left(2x\right)\right)}{70a^{4}}$	30
risch	$\frac{32i\left(35\,{\mathrm e}^{6ix}+21\,{\mathrm e}^{4ix}+7\,{\mathrm e}^{2ix}+1\right)}{35\left({\mathrm e}^{2ix}+1\right)^{7}a^{4}}$	39
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{4\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{a}-\frac{86\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{5a}+\frac{424\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{35a}-\frac{86\left(\tan^{9}\left(\frac{x}{2}\right)\right)}{5a}+\frac{4\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{a}-\frac{2\left(\tan^{13}\left(\frac{x}{2}\right)\right)}{a}}{a^{3}\left(\tan^{2}\left(\frac{x}{2}\right)-1\right)^{7}}$	91

input
```
int(1/(a-a*sin(x)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/7*tan(x)^7+3/5*tan(x)^5+tan(x)^3+tan(x))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^4}\,dx = \frac{\left(16\cos(x)^6+8\cos(x)^4+6\cos(x)^2+5\right)\sin(x)}{35\,a^4\cos(x)^7}$$

input
```
integrate(1/(a-a*sin(x)^2)^4,x, algorithm="fricas")
```

output
```
1/35*(16*cos(x)^6 + 8*cos(x)^4 + 6*cos(x)^2 + 5)*sin(x)/(a^4*cos(x)^7)
```

3.62.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. 2(36) = 72.

Time = 5.40 (sec) , antiderivative size = 675, normalized size of antiderivative = 18.24

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^4}\,dx =$$

$$-\frac{70\tan^{13}\left(\frac{x}{2}\right)}{35a^4\tan^{14}\left(\frac{x}{2}\right)-245a^4\tan^{12}\left(\frac{x}{2}\right)+735a^4\tan^{10}\left(\frac{x}{2}\right)-1225a^4\tan^8\left(\frac{x}{2}\right)+1225a^4\tan^6\left(\frac{x}{2}\right)-735a^4\tan^4}$$
$$+\frac{140\tan^{11}\left(\frac{x}{2}\right)}{35a^4\tan^{14}\left(\frac{x}{2}\right)-245a^4\tan^{12}\left(\frac{x}{2}\right)+735a^4\tan^{10}\left(\frac{x}{2}\right)-1225a^4\tan^8\left(\frac{x}{2}\right)+1225a^4\tan^6\left(\frac{x}{2}\right)-735a^4\tan^4}$$
$$-\frac{602\tan^{9}\left(\frac{x}{2}\right)}{35a^4\tan^{14}\left(\frac{x}{2}\right)-245a^4\tan^{12}\left(\frac{x}{2}\right)+735a^4\tan^{10}\left(\frac{x}{2}\right)-1225a^4\tan^8\left(\frac{x}{2}\right)+1225a^4\tan^6\left(\frac{x}{2}\right)-735a^4\tan^4}$$
$$+\frac{424\tan^{7}\left(\frac{x}{2}\right)}{35a^4\tan^{14}\left(\frac{x}{2}\right)-245a^4\tan^{12}\left(\frac{x}{2}\right)+735a^4\tan^{10}\left(\frac{x}{2}\right)-1225a^4\tan^8\left(\frac{x}{2}\right)+1225a^4\tan^6\left(\frac{x}{2}\right)-735a^4\tan^4}$$
$$-\frac{602\tan^{5}\left(\frac{x}{2}\right)}{35a^4\tan^{14}\left(\frac{x}{2}\right)-245a^4\tan^{12}\left(\frac{x}{2}\right)+735a^4\tan^{10}\left(\frac{x}{2}\right)-1225a^4\tan^8\left(\frac{x}{2}\right)+1225a^4\tan^6\left(\frac{x}{2}\right)-735a^4\tan^4}$$
$$+\frac{140\tan^{3}\left(\frac{x}{2}\right)}{35a^4\tan^{14}\left(\frac{x}{2}\right)-245a^4\tan^{12}\left(\frac{x}{2}\right)+735a^4\tan^{10}\left(\frac{x}{2}\right)-1225a^4\tan^8\left(\frac{x}{2}\right)+1225a^4\tan^6\left(\frac{x}{2}\right)-735a^4\tan^4}$$
$$-\frac{70\tan\left(\frac{x}{2}\right)}{35a^4\tan^{14}\left(\frac{x}{2}\right)-245a^4\tan^{12}\left(\frac{x}{2}\right)+735a^4\tan^{10}\left(\frac{x}{2}\right)-1225a^4\tan^8\left(\frac{x}{2}\right)+1225a^4\tan^6\left(\frac{x}{2}\right)-735a^4\tan^4}$$

input
```
integrate(1/(a-a*sin(x)**2)**4,x)
```

output
```
-70*tan(x/2)**13/(35*a**4*tan(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*
tan(x/2)**10 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*ta
n(x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**4) + 140*tan(x/2)**11/(35*a**4*ta
n(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**10 - 1225*a**4*tan
(x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 + 245*a**4*tan(x/2
)**2 - 35*a**4) - 602*tan(x/2)**9/(35*a**4*tan(x/2)**14 - 245*a**4*tan(x/2
)**12 + 735*a**4*tan(x/2)**10 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(x/2)
**6 - 735*a**4*tan(x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**4) + 424*tan(x/2
)**7/(35*a**4*tan(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**10
 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 +
245*a**4*tan(x/2)**2 - 35*a**4) - 602*tan(x/2)**5/(35*a**4*tan(x/2)**14 -
245*a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**10 - 1225*a**4*tan(x/2)**8 + 12
25*a**4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**
4) + 140*tan(x/2)**3/(35*a**4*tan(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a
**4*tan(x/2)**10 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**
4*tan(x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**4) - 70*tan(x/2)/(35*a**4*tan
(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**10 - 1225*a**4*tan(
x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 + 245*a**4*tan(x/2)
**2 - 35*a**4)
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a-a\sin^2(x))^4}\,dx = \frac{5\,\tan(x)^7+21\,\tan(x)^5+35\,\tan(x)^3+35\,\tan(x)}{35\,a^4}$$

input
```
integrate(1/(a-a*sin(x)^2)^4,x, algorithm="maxima")
```

output
```
1/35*(5*tan(x)^7 + 21*tan(x)^5 + 35*tan(x)^3 + 35*tan(x))/a^4
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a - a\sin^2(x))^4} \, dx = \frac{5\,\tan(x)^7 + 21\,\tan(x)^5 + 35\,\tan(x)^3 + 35\,\tan(x)}{35\,a^4}$$

input
```
integrate(1/(a-a*sin(x)^2)^4,x, algorithm="giac")
```

output
```
1/35*(5*tan(x)^7 + 21*tan(x)^5 + 35*tan(x)^3 + 35*tan(x))/a^4
```

3.62.9 Mupad [B] (verification not implemented)

Time = 12.85 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a - a\sin^2(x))^4} \, dx = \frac{\tan(x)}{a^4} + \frac{\tan(x)^3}{a^4} + \frac{3\tan(x)^5}{5\,a^4} + \frac{\tan(x)^7}{7\,a^4}$$

input
```
int(1/(a - a*sin(x)^2)^4,x)
```

output
```
tan(x)/a^4 + tan(x)^3/a^4 + (3*tan(x)^5)/(5*a^4) + tan(x)^7/(7*a^4)
```

3.63 $\int \frac{1}{\left(a-a\sin^2(x)\right)^5}\,dx$

3.63.1 Optimal result

Integrand size = 11, antiderivative size = 51

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^5}\,dx=\frac{\tan(x)}{a^5}+\frac{4\tan^3(x)}{3a^5}+\frac{6\tan^5(x)}{5a^5}+\frac{4\tan^7(x)}{7a^5}+\frac{\tan^9(x)}{9a^5}$$

output
```
tan(x)/a^5+4/3*tan(x)^3/a^5+6/5*tan(x)^5/a^5+4/7*tan(x)^7/a^5+1/9*tan(x)^9
/a^5
```

3.63.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^5}\,dx=\frac{\tan(x)+\frac{4\tan^3(x)}{3}+\frac{6\tan^5(x)}{5}+\frac{4\tan^7(x)}{7}+\frac{\tan^9(x)}{9}}{a^5}$$

input
```
Integrate[(a - a*Sin[x]^2)^(-5),x]
```

output
```
(Tan[x] + (4*Tan[x]^3)/3 + (6*Tan[x]^5)/5 + (4*Tan[x]^7)/7 + Tan[x]^9/9)/a
^5
```

3.63.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-a\sin(x)^2)^5}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sec^{10}(x)dx}{a^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^{10}dx}{a^5}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int \left(\tan^8(x)+4\tan^6(x)+6\tan^4(x)+4\tan^2(x)+1\right)d(-\tan(x))}{a^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{9}\tan^9(x)-\frac{4\tan^7(x)}{7}-\frac{6\tan^5(x)}{5}-\frac{4\tan^3(x)}{3}-\tan(x)}{a^5}$$

input
```
Int[(a - a*Sin[x]^2)^(-5),x]
```

output
```
-((-Tan[x] - (4*Tan[x]^3)/3 - (6*Tan[x]^5)/5 - (4*Tan[x]^7)/7 - Tan[x]^9/9
)/a^5)
```

3.63.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.63.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{\frac{\left(\tan^{9}(x)\right)}{9}+\frac{4\left(\tan^{7}(x)\right)}{7}+\frac{6\left(\tan^{5}(x)\right)}{5}+\frac{4\left(\tan^{3}(x)\right)}{3}+\tan(x)}{a^{5}}$	32
parallelrisch	$\frac{\tan(x)\left(\sec^{8}(x)\right)\left(128+\cos(8x)+10\cos(6x)+46\cos(4x)+130\cos(2x)\right)}{315a^{5}}$	36
risch	$\frac{256i\left(126\,\mathrm{e}^{8ix}+84\,\mathrm{e}^{6ix}+36\,\mathrm{e}^{4ix}+9\,\mathrm{e}^{2ix}+1\right)}{315\left(\mathrm{e}^{2ix}+1\right)^{9}a^{5}}$	46

input
```
int(1/(a-a*sin(x)^2)^5,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/9*tan(x)^9+4/7*tan(x)^7+6/5*tan(x)^5+4/3*tan(x)^3+tan(x))
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^5}\,dx=\frac{\left(128\,\cos\left(x\right)^{8}+64\,\cos\left(x\right)^{6}+48\,\cos\left(x\right)^{4}+40\,\cos\left(x\right)^{2}+35\right)\sin\left(x\right)}{315\,a^{5}\cos\left(x\right)^{9}}$$

input
```
integrate(1/(a-a*sin(x)^2)^5,x, algorithm="fricas")
```

output
```
1/315*(128*cos(x)^8 + 64*cos(x)^6 + 48*cos(x)^4 + 40*cos(x)^2 + 35)*sin(x)
/(a^5*cos(x)^9)
```

3.63.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1083 vs. 2(51) = 102.

Time = 14.84 (sec) , antiderivative size = 1083, normalized size of antiderivative = 21.24

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^5}\,dx=\text{Too large to display}$$

input
```
integrate(1/(a-a*sin(x)**2)**5,x)
```

output
```
-630*tan(x/2)**17/(315*a**5*tan(x/2)**18 - 2835*a**5*tan(x/2)**16 + 11340*
a**5*tan(x/2)**14 - 26460*a**5*tan(x/2)**12 + 39690*a**5*tan(x/2)**10 - 39
690*a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)**6 - 11340*a**5*tan(x/2)**4 + 2
835*a**5*tan(x/2)**2 - 315*a**5) + 1680*tan(x/2)**15/(315*a**5*tan(x/2)**1
8 - 2835*a**5*tan(x/2)**16 + 11340*a**5*tan(x/2)**14 - 26460*a**5*tan(x/2)
**12 + 39690*a**5*tan(x/2)**10 - 39690*a**5*tan(x/2)**8 + 26460*a**5*tan(x
/2)**6 - 11340*a**5*tan(x/2)**4 + 2835*a**5*tan(x/2)**2 - 315*a**5) - 9576
*tan(x/2)**13/(315*a**5*tan(x/2)**18 - 2835*a**5*tan(x/2)**16 + 11340*a**5
*tan(x/2)**14 - 26460*a**5*tan(x/2)**12 + 39690*a**5*tan(x/2)**10 - 39690*
a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)**6 - 11340*a**5*tan(x/2)**4 + 2835*
a**5*tan(x/2)**2 - 315*a**5) + 10224*tan(x/2)**11/(315*a**5*tan(x/2)**18 -
 2835*a**5*tan(x/2)**16 + 11340*a**5*tan(x/2)**14 - 26460*a**5*tan(x/2)**1
2 + 39690*a**5*tan(x/2)**10 - 39690*a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)
**6 - 11340*a**5*tan(x/2)**4 + 2835*a**5*tan(x/2)**2 - 315*a**5) - 21316*t
an(x/2)**9/(315*a**5*tan(x/2)**18 - 2835*a**5*tan(x/2)**16 + 11340*a**5*ta
n(x/2)**14 - 26460*a**5*tan(x/2)**12 + 39690*a**5*tan(x/2)**10 - 39690*a**
5*tan(x/2)**8 + 26460*a**5*tan(x/2)**6 - 11340*a**5*tan(x/2)**4 + 2835*a**
5*tan(x/2)**2 - 315*a**5) + 10224*tan(x/2)**7/(315*a**5*tan(x/2)**18 - 283
5*a**5*tan(x/2)**16 + 11340*a**5*tan(x/2)**14 - 26460*a**5*tan(x/2)**12 +
39690*a**5*tan(x/2)**10 - 39690*a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)*...
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^5}\,dx = \frac{35\tan(x)^9+180\tan(x)^7+378\tan(x)^5+420\tan(x)^3+315\tan(x)}{315\,a^5}$$

input
```
integrate(1/(a-a*sin(x)^2)^5,x, algorithm="maxima")
```

output
```
1/315*(35*tan(x)^9 + 180*tan(x)^7 + 378*tan(x)^5 + 420*tan(x)^3 + 315*tan(
x))/a^5
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a - a\sin^2(x))^5} \, dx = \frac{35 \tan(x)^9 + 180 \tan(x)^7 + 378 \tan(x)^5 + 420 \tan(x)^3 + 315 \tan(x)}{315 \, a^5}$$

input
```
integrate(1/(a-a*sin(x)^2)^5,x, algorithm="giac")
```

output
```
1/315*(35*tan(x)^9 + 180*tan(x)^7 + 378*tan(x)^5 + 420*tan(x)^3 + 315*tan(
x))/a^5
```

3.63.9 Mupad [B] (verification not implemented)

Time = 13.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a - a\sin^2(x))^5} \, dx = \frac{\tan(x)}{a^5} + \frac{4\tan(x)^3}{3\,a^5} + \frac{6\tan(x)^5}{5\,a^5} + \frac{4\tan(x)^7}{7\,a^5} + \frac{\tan(x)^9}{9\,a^5}$$

input
```
int(1/(a - a*sin(x)^2)^5,x)
```

output
```
tan(x)/a^5 + (4*tan(x)^3)/(3*a^5) + (6*tan(x)^5)/(5*a^5) + (4*tan(x)^7)/(7
*a^5) + tan(x)^9/(9*a^5)
```

3.64 $\int \sin^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

3.64.1 Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \sin^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=-\frac{(a+b)\cos(c+dx)}{d}+\frac{(a+2b)\cos^3(c+dx)}{3d}-\frac{b\cos^5(c+dx)}{5d}$$

output
```
-(a+b)*cos(d*x+c)/d+1/3*(a+2*b)*cos(d*x+c)^3/d-1/5*b*cos(d*x+c)^5/d
```

3.64.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.51

$$\int \sin^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=-\frac{3a\cos(c+dx)}{4d}-\frac{5b\cos(c+dx)}{8d}+\frac{a\cos(3(c+dx))}{12d}+\frac{5b\cos(3(c+dx))}{48d}-\frac{b\cos(5(c+dx))}{80d}$$

input
```
Integrate[Sin[c + d*x]^3*(a + b*Sin[c + d*x]^2),x]
```

output
```
(-3*a*Cos[c + d*x])/(4*d) - (5*b*Cos[c + d*x])/(8*d) + (a*Cos[3*(c + d*x)]
)/(12*d) + (5*b*Cos[3*(c + d*x)])/(48*d) - (b*Cos[5*(c + d*x)])/(80*d)
```

3.64.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3492, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(c+dx)\left(a+b\sin^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)^3\left(a+b\sin(c+dx)^2\right)dx$$

$$\Big\downarrow \text{3492}$$

$$-\frac{\int \left(1-\cos^2(c+dx)\right)\left(-b\cos^2(c+dx)+a+b\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$-\frac{\int \left(b\cos^4(c+dx)-(a+2b)\cos^2(c+dx)+a\left(\frac{b}{a}+1\right)\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{3}(a+2b)\cos^3(c+dx)+(a+b)\cos(c+dx)+\frac{1}{5}b\cos^5(c+dx)}{d}$$

input
```
Int[Sin[c + d*x]^3*(a + b*Sin[c + d*x]^2),x]
```

output
```
-(((a + b)*Cos[c + d*x] - ((a + 2*b)*Cos[c + d*x]^3)/3 + (b*Cos[c + d*x]^5
)/5)/d)
```

3.64.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

3.64.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{-\frac{b\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}-\frac{a\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}}{d}$	54
default	$\frac{-\frac{b\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}-\frac{a\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}}{d}$	54
parallelrisch	$\frac{(20a+25b)\cos(3dx+3c)-3b\cos(5dx+5c)+(-180a-150b)\cos(dx+c)-160a-128b}{240d}$	56
parts	$-\frac{a(2+\sin^2(dx+c))\cos(dx+c)}{3d}-\frac{b\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5d}$	56
risch	$-\frac{3a\cos(dx+c)}{4d}-\frac{5b\cos(dx+c)}{8d}-\frac{b\cos(5dx+5c)}{80d}+\frac{a\cos(3dx+3c)}{12d}+\frac{5\cos(3dx+3c)b}{48d}$	71
norman	$\frac{-\frac{20a+16b}{15d}-\frac{4a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2(14a+16b)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{(20a+16b)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$	93

input
```
int(sin(d*x+c)^3*(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/5*b*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)-1/3*a*(2+sin(d*
x+c)^2)*cos(d*x+c))
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \sin^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = -\frac{3\,b\cos\left(dx+c\right)^5-5\,(a+2\,b)\cos\left(dx+c\right)^3+15\,(a+b)\cos\left(dx+c\right)}{15\,d}$$

input
```
integrate(sin(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/15*(3*b*cos(d*x + c)^5 - 5*(a + 2*b)*cos(d*x + c)^3 + 15*(a + b)*cos(d*
x + c))/d
```

3.64.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(44) = 88.

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.10

$$\int \sin^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = \begin{cases} -\frac{a\sin^2(c+dx)\cos(c+dx)}{d}-\frac{2a\cos^3(c+dx)}{3d}-\frac{b\sin^4(c+dx)\cos(c+dx)}{d}-\frac{4b\sin^2(c+dx)\cos^3(c+dx)}{3d}-\frac{8b\cos^5(c+dx)}{15d} & \text{for } d\neq 0 \\ x\left(a+b\sin^2(c)\right)\sin^3(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(d*x+c)**3*(a+b*sin(d*x+c)**2),x)
```

output
```
Piecewise((-a*sin(c + d*x)**2*cos(c + d*x)/d - 2*a*cos(c + d*x)**3/(3*d) -
 b*sin(c + d*x)**4*cos(c + d*x)/d - 4*b*sin(c + d*x)**2*cos(c + d*x)**3/(3
*d) - 8*b*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*sin(c)**2)*sin(c)**
3, True))
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \sin^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = -\frac{3\,b\cos\left(dx+c\right)^5-5\,(a+2\,b)\cos\left(dx+c\right)^3+15\,(a+b)\cos\left(dx+c\right)}{15\,d}$$

input
```
integrate(sin(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/15*(3*b*cos(d*x + c)^5 - 5*(a + 2*b)*cos(d*x + c)^3 + 15*(a + b)*cos(d*
x + c))/d
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \sin^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = -\frac{b\cos\left(dx+c\right)^5}{5\,d}+\frac{a\cos\left(dx+c\right)^3}{3\,d}+\frac{2\,b\cos\left(dx+c\right)^3}{3\,d}-\frac{a\cos\left(dx+c\right)}{d}-\frac{b\cos\left(dx+c\right)}{d}$$

input
```
integrate(sin(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/5*b*cos(d*x + c)^5/d + 1/3*a*cos(d*x + c)^3/d + 2/3*b*cos(d*x + c)^3/d
- a*cos(d*x + c)/d - b*cos(d*x + c)/d
```

3.64.9 Mupad [B] (verification not implemented)

Time = 13.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \sin^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = -\frac{\frac{b\cos(c+d\,x)^5}{5}+\left(-\frac{a}{3}-\frac{2\,b}{3}\right)\,\cos\left(c+d\,x\right)^3+(a+b)\,\cos\left(c+d\,x\right)}{d}$$

input
```
int(sin(c + d*x)^3*(a + b*sin(c + d*x)^2),x)
```

output
```
-((b*cos(c + d*x)^5)/5 - cos(c + d*x)^3*(a/3 + (2*b)/3) + cos(c + d*x)*(a
+ b))/d
```

3.64. $\int \sin^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

3.65 $\int \sin(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

3.65.1 Optimal result

Integrand size = 19, antiderivative size = 31

$$\int \sin(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=-\frac{(a+b)\cos(c+dx)}{d}+\frac{b\cos^3(c+dx)}{3d}$$

output
```
-(a+b)*cos(d*x+c)/d+1/3*b*cos(d*x+c)^3/d
```

3.65.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \sin(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=-\frac{a\cos(c)\cos(dx)}{d}-\frac{3b\cos(c+dx)}{4d}+\frac{b\cos(3(c+dx))}{12d}+\frac{a\sin(c)\sin(dx)}{d}$$

input
```
Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x]^2),x]
```

output
```
-((a*Cos[c]*Cos[d*x])/d) - (3*b*Cos[c + d*x])/(4*d) + (b*Cos[3*(c + d*x)])
/(12*d) + (a*Sin[c]*Sin[d*x])/d
```

3.65.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx) \left(a+b\sin^2(c+dx)\right) \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(c+dx) \left(a+b\sin(c+dx)^2\right) dx$$

$$\Big\downarrow \text{ 3492}$$

$$-\frac{\int \left(-b\cos^2(c+dx)+a+b\right) d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(a+b)\cos(c+dx)-\frac{1}{3}b\cos^3(c+dx)}{d}$$

input
```
Int[Sin[c + d*x]*(a + b*Sin[c + d*x]^2),x]
```

output
```
-(((a + b)*Cos[c + d*x] - (b*Cos[c + d*x]^3)/3)/d)
```

3.65.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

3.65.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{-\frac{b\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-\cos(dx+c)a}{d}$	34
default	$\frac{-\frac{b\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-\cos(dx+c)a}{d}$	34
parts	$-\frac{a\cos(dx+c)}{d}-\frac{b(2+\sin^2(dx+c))\cos(dx+c)}{3d}$	36
parallelrisch	$\frac{\cos(3dx+3c)b+(-12a-9b)\cos(dx+c)-12a-8b}{12d}$	38
risch	$-\frac{a\cos(dx+c)}{d}-\frac{3b\cos(dx+c)}{4d}+\frac{\cos(3dx+3c)b}{12d}$	41
norman	$\frac{-\frac{6a+4b}{3d}-\frac{2a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{(4a+4b)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$	70

input
```
int(sin(d*x+c)*(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*b*(2+sin(d*x+c)^2)*cos(d*x+c)-cos(d*x+c)*a)
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \sin(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=\frac{b\cos\left(dx+c\right)^3-3\left(a+b\right)\cos\left(dx+c\right)}{3\,d}$$

input
```
integrate(sin(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/3*(b*cos(d*x + c)^3 - 3*(a + b)*cos(d*x + c))/d
```

3.65.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(26) = 52$.

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int \sin(c+dx) \left(a+b\sin^2(c+dx)\right) \, dx = \begin{cases} -\frac{a\cos{(c+dx)}}{d} - \frac{b\sin^2{(c+dx)}\cos{(c+dx)}}{d} - \frac{2b\cos^3{(c+dx)}}{3d} & \text{for } d \neq 0 \\ x(a+b\sin^2{(c)})\sin{(c)} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(d*x+c)*(a+b*sin(d*x+c)**2),x)
```

output
```
Piecewise((-a*cos(c + d*x)/d - b*sin(c + d*x)**2*cos(c + d*x)/d - 2*b*cos(
c + d*x)**3/(3*d), Ne(d, 0)), (x*(a + b*sin(c)**2)*sin(c), True))
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \sin(c+dx) \left(a+b\sin^2(c+dx)\right) \, dx = \frac{(\cos{(dx+c)}^3 - 3\cos{(dx+c)})b - 3a\cos{(dx+c)}}{3d}$$

input
```
integrate(sin(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/3*((cos(d*x + c)^3 - 3*cos(d*x + c))*b - 3*a*cos(d*x + c))/d
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \sin(c+dx) \left(a+b\sin^2(c+dx)\right) \, dx = \frac{1}{3}\left(\frac{\cos{(dx+c)}^3}{d} - \frac{3\cos{(dx+c)}}{d}\right)b - \frac{a\cos{(dx+c)}}{d}$$

input
```
integrate(sin(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(cos(d*x + c)^3/d - 3*cos(d*x + c)/d)*b - a*cos(d*x + c)/d
```

3.65.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \sin(c+dx) \left(a+b\sin^2(c+dx)\right) \, dx = \frac{\frac{b\cos(c+d\,x)^3}{3} - \cos\left(c+d\,x\right)\,\left(a+b\right)}{d}$$

input
```
int(sin(c + d*x)*(a + b*sin(c + d*x)^2),x)
```

output
```
((b*cos(c + d*x)^3)/3 - cos(c + d*x)*(a + b))/d
```

3.66 $\int \csc(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

3.66.1 Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \csc(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=-\frac{a\text{arctanh}(\cos(c+dx))}{d}-\frac{b\cos(c+dx)}{d}$$

output
```
-a*arctanh(cos(d*x+c))/d-b*cos(d*x+c)/d
```

3.66.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 63 vs. 2(26) = 52.

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.42

$$\int \csc(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=-\frac{b\cos(c)\cos(dx)}{d}-\frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{b\sin(c)\sin(dx)}{d}$$

input
```
Integrate[Csc[c + d*x]*(a + b*Sin[c + d*x]^2),x]
```

output
```
-((b*Cos[c]*Cos[d*x])/d) - (a*Log[Cos[c/2 + (d*x)/2]])/d + (a*Log[Sin[c/2
+ (d*x)/2]])/d + (b*Sin[c]*Sin[d*x])/d
```

3.66.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3493, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(c+dx)^2}{\sin(c+dx)}dx$$

$$\Big\downarrow \text{3493}$$

$$a\int \csc(c+dx)dx-\frac{b\cos(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc(c+dx)dx-\frac{b\cos(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{a\text{arctanh}(\cos(c+dx))}{d}-\frac{b\cos(c+dx)}{d}$$

input
```
Int[Csc[c + d*x]*(a + b*Sin[c + d*x]^2),x]
```

output
```
-((a*ArcTanh[Cos[c + d*x]])/d) - (b*Cos[c + d*x])/d
```

3.66.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.66.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

method	result	size
parallelrisch	$\frac{-\cos(dx+c)b+a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b}{d}$	28
derivativedivides	$\frac{a\ln(\csc(dx+c)-\cot(dx+c))-\cos(dx+c)b}{d}$	33
default	$\frac{a\ln(\csc(dx+c)-\cot(dx+c))-\cos(dx+c)b}{d}$	33
risch	$-\frac{b\,e^{i(dx+c)}}{2d}-\frac{b\,e^{-i(dx+c)}}{2d}+\frac{a\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{d}$	67
norman	$\frac{\frac{2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	68

input
```
int(csc(d*x+c)*(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-cos(d*x+c)*b+a*ln(tan(1/2*d*x+1/2*c))+b)
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \csc(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$$
$$=-\frac{2\,b\cos\left(dx+c\right)+a\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,d}$$

input
```
integrate(csc(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*b*cos(d*x + c) + a*log(1/2*cos(d*x + c) + 1/2) - a*log(-1/2*cos(d*
x + c) + 1/2))/d
```

3.66.6 Sympy [F]

$$\int \csc(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = \int \left(a+b\sin^2\left(c+dx\right)\right)\csc\left(c+dx\right)dx$$

input
```
integrate(csc(d*x+c)*(a+b*sin(d*x+c)**2),x)
```

output
```
Integral((a + b*sin(c + d*x)**2)*csc(c + d*x), x)
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \csc(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = -\frac{2\,b\cos\left(dx+c\right)+a\log\left(\cos\left(dx+c\right)+1\right)-a\log\left(\cos\left(dx+c\right)-1\right)}{2\,d}$$

input
```
integrate(csc(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/2*(2*b*cos(d*x + c) + a*log(cos(d*x + c) + 1) - a*log(cos(d*x + c) - 1)
)/d
```

3.66.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(26) = 52.

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.23

$$\int \csc(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = \frac{a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)+\frac{4\,b}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1}}{2\,d}$$

input
```
integrate(csc(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + 4*b/((cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 1))/d
```

3.66.9 Mupad [B] (verification not implemented)

Time = 12.97 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \csc(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = -\frac{b\,\cos\left(c+d\,x\right)+a\,\mathrm{atanh}(\cos\left(c+d\,x\right))}{d}$$

input
```
int((a + b*sin(c + d*x)^2)/sin(c + d*x),x)
```

output
```
-(b*cos(c + d*x) + a*atanh(cos(c + d*x)))/d
```

3.67 $\int \csc^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

3.67.1 Optimal result

Integrand size = 21, antiderivative size = 40

$$\int \csc^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=-\frac{(a+2b)\text{arctanh}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

output
```
-1/2*(a+2*b)*arctanh(cos(d*x+c))/d-1/2*a*cot(d*x+c)*csc(d*x+c)/d
```

3.67.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 118 vs. 2(40) = 80.

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.95

$$\int \csc^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{b\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

input
```
Integrate[Csc[c + d*x]^3*(a + b*Sin[c + d*x]^2),x]
```

output
```
-1/8*(a*Csc[(c + d*x)/2]^2)/d - (b*Log[Cos[c/2 + (d*x)/2]])/d - (a*Log[Cos
[(c + d*x)/2]])/(2*d) + (b*Log[Sin[c/2 + (d*x)/2]])/d + (a*Log[Sin[(c + d*
x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

3.67.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3491, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(c+dx)^2}{\sin(c+dx)^3}dx$$

$$\Big\downarrow \text{3491}$$

$$\frac{1}{2}(a+2b)\int \csc(c+dx)dx-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(a+2b)\int \csc(c+dx)dx-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{(a+2b)\text{arctanh}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

input
```
Int[Csc[c + d*x]^3*(a + b*Sin[c + d*x]^2),x]
```

output
```
-1/2*((a + 2*b)*ArcTanh[Cos[c + d*x]])/d - (a*Cot[c + d*x]*Csc[c + d*x])/(
2*d)
```

3.67.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.67.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

method	result	size
parallelrisch	$\frac{(4a+8b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\cot^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{8d}$	52
derivativedivides	$\frac{a\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b\ln(\csc(dx+c)-\cot(dx+c))}{d}$	59
default	$\frac{a\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b\ln(\csc(dx+c)-\cot(dx+c))}{d}$	59
norman	$\frac{-\frac{a}{8d}+\frac{a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{(a+2b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}$	107
risch	$\frac{a\left(e^{3i(dx+c)}+e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}-1\right)^2}+\frac{a\ln\left(e^{i(dx+c)}-1\right)}{2d}+\frac{b\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{2d}-\frac{b\ln\left(e^{i(dx+c)}+1\right)}{d}$	110

input
```
int(csc(d*x+c)^3*(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/8*((4*a+8*b)*ln(tan(1/2*d*x+1/2*c))-cot(1/2*d*x+1/2*c)^2*a+tan(1/2*d*x+1
/2*c)^2*a)/d
```

3.67.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(36) = 72$.

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.38

$$\int \csc^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$$
$$= \frac{2\,a\cos\left(dx+c\right)-\left((a+2\,b)\cos\left(dx+c\right)^2-a-2\,b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left((a+2\,b)\cos\left(dx+c\right)^2-\right.}{4\left(d\cos\left(dx+c\right)^2-d\right)}$$

input
```
integrate(csc(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/4*(2*a*cos(d*x + c) - ((a + 2*b)*cos(d*x + c)^2 - a - 2*b)*log(1/2*cos(d
*x + c) + 1/2) + ((a + 2*b)*cos(d*x + c)^2 - a - 2*b)*log(-1/2*cos(d*x + c
) + 1/2))/(d*cos(d*x + c)^2 - d)
```

3.67.6 Sympy [F]

$$\int \csc^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = \int \left(a+b\sin^2\left(c+dx\right)\right)\csc^3\left(c+dx\right)dx$$

input
```
integrate(csc(d*x+c)**3*(a+b*sin(d*x+c)**2),x)
```

output
```
Integral((a + b*sin(c + d*x)**2)*csc(c + d*x)**3, x)
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \csc^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$$
$$= -\frac{(a+2\,b)\log\left(\cos\left(dx+c\right)+1\right)-(a+2\,b)\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\,a\cos(dx+c)}{\cos(dx+c)^2-1}}{4\,d}$$

input
```
integrate(csc(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/4*((a + 2*b)*log(cos(d*x + c) + 1) - (a + 2*b)*log(cos(d*x + c) - 1) -
2*a*cos(d*x + c)/(cos(d*x + c)^2 - 1))/d
```

3.67.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(36) = 72.

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.02

$$\int \csc^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = \frac{2\,(a+2\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)+\frac{\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{8\,d}$$

input
```
integrate(csc(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/8*(2*(a + 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + (a -
2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1) - a*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1))/d
```

3.67.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \csc^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = \frac{a\,\cos(c+d\,x)}{2\,d\,\left(\cos(c+d\,x)^2-1\right)}-\frac{\mathrm{atanh}(\cos(c+d\,x))\,\left(\frac{a}{2}+b\right)}{d}$$

input
```
int((a + b*sin(c + d*x)^2)/sin(c + d*x)^3,x)
```

output
```
(a*cos(c + d*x))/(2*d*(cos(c + d*x)^2 - 1)) - (atanh(cos(c + d*x))*(a/2 +
b))/d
```

3.68 $\int \sin^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

3.68.1 Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \sin^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=\frac{1}{16}(6a+5b)x-\frac{(6a+5b)\cos(c+dx)\sin(c+dx)}{16d}-\frac{(6a+5b)\cos(c+dx)\sin^3(c+dx)}{24d}-\frac{b\cos(c+dx)\sin^5(c+dx)}{6d}$$

output
```
1/16*(6*a+5*b)*x-1/16*(6*a+5*b)*cos(d*x+c)*sin(d*x+c)/d-1/24*(6*a+5*b)*cos
(d*x+c)*sin(d*x+c)^3/d-1/6*b*cos(d*x+c)*sin(d*x+c)^5/d
```

3.68.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.79

$$\int \sin^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=\frac{72ac+60bc+72adx+60bdx-3(16a+15b)\sin(2(c+dx))+(6a+9b)\sin(4(c+dx))-b\sin(6(c+dx))}{192d}$$

input
```
Integrate[Sin[c + d*x]^4*(a + b*Sin[c + d*x]^2),x]
```

output
```
(72*a*c + 60*b*c + 72*a*d*x + 60*b*d*x - 3*(16*a + 15*b)*Sin[2*(c + d*x)]
+ (6*a + 9*b)*Sin[4*(c + d*x)] - b*Sin[6*(c + d*x)])/(192*d)
```

3.68.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3493, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(c+dx)^4\left(a+b\sin(c+dx)^2\right)dx$$

$$\Big\downarrow \text{ 3493}$$

$$\frac{1}{6}(6a+5b)\int \sin^4(c+dx)dx-\frac{b\sin^5(c+dx)\cos(c+dx)}{6d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{6}(6a+5b)\int \sin(c+dx)^4dx-\frac{b\sin^5(c+dx)\cos(c+dx)}{6d}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{1}{6}(6a+5b)\left(\frac{3}{4}\int \sin^2(c+dx)dx-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)-\frac{b\sin^5(c+dx)\cos(c+dx)}{6d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{6}(6a+5b)\left(\frac{3}{4}\int \sin(c+dx)^2dx-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)-\frac{b\sin^5(c+dx)\cos(c+dx)}{6d}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{1}{6}(6a+5b)\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)-\frac{b\sin^5(c+dx)\cos(c+dx)}{6d}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{1}{6}(6a+5b)\left(\frac{3}{4}\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)-\frac{b\sin^5(c+dx)\cos(c+dx)}{6d}$$

input
```
Int[Sin[c + d*x]^4*(a + b*Sin[c + d*x]^2),x]
```

output
```
-1/6*(b*Cos[c + d*x]*Sin[c + d*x]^5)/d + ((6*a + 5*b)*(-1/4*(Cos[c + d*x]*
Sin[c + d*x]^3)/d + (3*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/6
```

3.68.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

3.68.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{(-48a-45b)\sin(2dx+2c)+(6a+9b)\sin(4dx+4c)-b\sin(6dx+6c)+72d\left(a+\frac{5b}{6}\right)x}{192d}$
risch	$\frac{3ax}{8}+\frac{5bx}{16}-\frac{b\sin(6dx+6c)}{192d}+\frac{a\sin(4dx+4c)}{32d}+\frac{3b\sin(4dx+4c)}{64d}-\frac{\sin(2dx+2c)a}{4d}-\frac{15\sin(2dx+2c)b}{64d}$
derivativedivides	$\frac{b\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+a\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\right.}{d}$
default	$\frac{b\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+a\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\right.}{d}$
parts	$\frac{a\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5d}{16}\right.}{d}$
norman	$\left(\frac{3a}{8}+\frac{5b}{16}\right)x+\left(\frac{3a}{8}+\frac{5b}{16}\right)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{9a}{4}+\frac{15b}{8}\right)x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{9a}{4}+\frac{15b}{8}\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{15a}{2}+\frac{25b}{4}\right)$

input
```
int(sin(d*x+c)^4*(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/192*((-48*a-45*b)*sin(2*d*x+2*c)+(6*a+9*b)*sin(4*d*x+4*c)-b*sin(6*d*x+6*
c)+72*d*(a+5/6*b)*x)/d
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \sin^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$$
$$=\frac{3\,(6\,a+5\,b)dx-\left(8\,b\cos\left(dx+c\right)^5-2\,(6\,a+13\,b)\cos\left(dx+c\right)^3+3\,(10\,a+11\,b)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d}$$

input
```
integrate(sin(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/48*(3*(6*a + 5*b)*d*x - (8*b*cos(d*x + c)^5 - 2*(6*a + 13*b)*cos(d*x + c
)^3 + 3*(10*a + 11*b)*cos(d*x + c))*sin(d*x + c))/d
```

3.68.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(82) = 164$.

Time = 0.33 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.90

$$\int \sin^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$$
$$= \begin{cases} \frac{3ax\sin^4(c+dx)}{8} + \frac{3ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3ax\cos^4(c+dx)}{8} - \frac{5a\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{3a\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{5bx\sin\cdots}{1\cdots} \\ x(a+b\sin^2(c))\sin^4(c) \end{cases}$$

input
```
integrate(sin(d*x+c)**4*(a+b*sin(d*x+c)**2),x)
```

output
```
Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2
/4 + 3*a*x*cos(c + d*x)**4/8 - 5*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*
a*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 5*b*x*sin(c + d*x)**6/16 + 15*b*x*s
in(c + d*x)**4*cos(c + d*x)**2/16 + 15*b*x*sin(c + d*x)**2*cos(c + d*x)**4
/16 + 5*b*x*cos(c + d*x)**6/16 - 11*b*sin(c + d*x)**5*cos(c + d*x)/(16*d)
- 5*b*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - 5*b*sin(c + d*x)*cos(c + d*x
)**5/(16*d), Ne(d, 0)), (x*(a + b*sin(c)**2)*sin(c)**4, True))
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

$$\int \sin^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$$
$$= \frac{3\,(dx+c)(6\,a+5\,b) - \frac{3\,(10\,a+11\,b)\tan(dx+c)^5+8\,(6\,a+5\,b)\tan(dx+c)^3+3\,(6\,a+5\,b)\tan(dx+c)}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{48\,d}$$

input
```
integrate(sin(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/48*(3*(d*x + c)*(6*a + 5*b) - (3*(10*a + 11*b)*tan(d*x + c)^5 + 8*(6*a +
 5*b)*tan(d*x + c)^3 + 3*(6*a + 5*b)*tan(d*x + c))/(tan(d*x + c)^6 + 3*tan
(d*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \sin^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = \frac{1}{16}\,(6\,a+5\,b)x - \frac{b\sin\left(6\,dx+6\,c\right)}{192\,d} + \frac{(2\,a+3\,b)\sin\left(4\,dx+4\,c\right)}{64\,d} - \frac{(16\,a+15\,b)\sin\left(2\,dx+2\,c\right)}{64\,d}$$

input
```
integrate(sin(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/16*(6*a + 5*b)*x - 1/192*b*sin(6*d*x + 6*c)/d + 1/64*(2*a + 3*b)*sin(4*d
*x + 4*c)/d - 1/64*(16*a + 15*b)*sin(2*d*x + 2*c)/d
```

3.68.9 Mupad [B] (verification not implemented)

Time = 14.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \sin^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = x\left(\frac{3\,a}{8}+\frac{5\,b}{16}\right) - \frac{\left(\frac{5\,a}{8}+\frac{11\,b}{16}\right)\tan(c+d\,x)^5+\left(a+\frac{5\,b}{6}\right)\tan(c+d\,x)^3+\left(\frac{3\,a}{8}+\frac{5\,b}{16}\right)\tan(c+d\,x)}{d\,\left(\tan\left(c+d\,x\right)^6+3\tan\left(c+d\,x\right)^4+3\tan\left(c+d\,x\right)^2+1\right)}$$

input
```
int(sin(c + d*x)^4*(a + b*sin(c + d*x)^2),x)
```

output
```
x*((3*a)/8 + (5*b)/16) - (tan(c + d*x)^5*((5*a)/8 + (11*b)/16) + tan(c + d
*x)*((3*a)/8 + (5*b)/16) + tan(c + d*x)^3*(a + (5*b)/6))/(d*(3*tan(c + d*x
)^2 + 3*tan(c + d*x)^4 + tan(c + d*x)^6 + 1))
```

3.69 $\int \sin^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

3.69.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \sin^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=\frac{1}{8}(4a+3b)x-\frac{(4a+3b)\cos(c+dx)\sin(c+dx)}{8d}-\frac{b\cos(c+dx)\sin^3(c+dx)}{4d}$$

output
```
1/8*(4*a+3*b)*x-1/8*(4*a+3*b)*cos(d*x+c)*sin(d*x+c)/d-1/4*b*cos(d*x+c)*sin
(d*x+c)^3/d
```

3.69.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \sin^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=\frac{4(4a+3b)(c+dx)-8(a+b)\sin(2(c+dx))+b\sin(4(c+dx))}{32d}$$

input
```
Integrate[Sin[c + d*x]^2*(a + b*Sin[c + d*x]^2),x]
```

output
```
(4*(4*a + 3*b)*(c + d*x) - 8*(a + b)*Sin[2*(c + d*x)] + b*Sin[4*(c + d*x)]
)/(32*d)
```

3.69.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3493, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(c+dx)\left(a+b\sin^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)^2\left(a+b\sin(c+dx)^2\right)dx$$

$$\Big\downarrow \text{3493}$$

$$\frac{1}{4}(4a+3b)\int \sin^2(c+dx)dx-\frac{b\sin^3(c+dx)\cos(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}(4a+3b)\int \sin(c+dx)^2dx-\frac{b\sin^3(c+dx)\cos(c+dx)}{4d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{4}(4a+3b)\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{b\sin^3(c+dx)\cos(c+dx)}{4d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{4}(4a+3b)\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{b\sin^3(c+dx)\cos(c+dx)}{4d}$$

input
```
Int[Sin[c + d*x]^2*(a + b*Sin[c + d*x]^2),x]
```

output
```
-1/4*(b*Cos[c + d*x]*Sin[c + d*x]^3)/d + ((4*a + 3*b)*(x/2 - (Cos[c + d*x]
*Sin[c + d*x])/(2*d)))/4
```

3.69.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

3.69.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{(-8a-8b)\sin(2dx+2c)+\sin(4dx+4c)b+16d\left(a+\frac{3b}{4}\right)x}{32d}$
risch	$\frac{ax}{2}+\frac{3bx}{8}+\frac{b\sin(4dx+4c)}{32d}-\frac{\sin(2dx+2c)a}{4d}-\frac{\sin(2dx+2c)b}{4d}$
derivativedivides	$\frac{b\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{b\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
parts	$\frac{a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
norman	$\frac{\left(\frac{a}{2}+\frac{3b}{8}\right)x+\left(2a+\frac{3b}{2}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2a+\frac{3b}{2}\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3a+\frac{9b}{4}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{a}{2}+\frac{3b}{8}\right)x\left(\tan^8\left(\ldots\right.\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\ldots\right.\right.}$

input
```
int(sin(d*x+c)^2*(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/32*((-8*a-8*b)*sin(2*d*x+2*c)+sin(4*d*x+4*c)*b+16*d*(a+3/4*b)*x)/d
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \sin^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = \frac{(4\,a+3\,b)dx+\left(2\,b\cos\left(dx+c\right)^3-(4\,a+5\,b)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

input
```
integrate(sin(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/8*((4*a + 3*b)*d*x + (2*b*cos(d*x + c)^3 - (4*a + 5*b)*cos(d*x + c))*sin
(d*x + c))/d
```

3.69.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(53) = 106.

Time = 0.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.59

$$\int \sin^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = \begin{cases} \frac{ax\sin^2(c+dx)}{2}+\frac{ax\cos^2(c+dx)}{2}-\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{3bx\sin^4(c+dx)}{8}+\frac{3bx\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3bx\cos^4(c+dx)}{8}-5b \\ x(a+b\sin^2(c))\sin^2(c) \end{cases}$$

input
```
integrate(sin(d*x+c)**2*(a+b*sin(d*x+c)**2),x)
```

output
```
Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 - a*sin(c + d*x)*
cos(c + d*x)/(2*d) + 3*b*x*sin(c + d*x)**4/8 + 3*b*x*sin(c + d*x)**2*cos(c
 + d*x)**2/4 + 3*b*x*cos(c + d*x)**4/8 - 5*b*sin(c + d*x)**3*cos(c + d*x)/
(8*d) - 3*b*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*sin(c
)**2)*sin(c)**2, True))
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.21

$$\int \sin^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = \frac{(dx+c)(4a+3b) - \frac{(4a+5b)\tan(dx+c)^3+(4a+3b)\tan(dx+c)}{\tan(dx+c)^4+2\tan(dx+c)^2+1}}{8d}$$

input
```
integrate(sin(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/8*((d*x + c)*(4*a + 3*b) - ((4*a + 5*b)*tan(d*x + c)^3 + (4*a + 3*b)*tan
(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \sin^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = \frac{1}{8}(4a+3b)x + \frac{b\sin(4dx+4c)}{32d} - \frac{(a+b)\sin(2dx+2c)}{4d}$$

input
```
integrate(sin(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/8*(4*a + 3*b)*x + 1/32*b*sin(4*d*x + 4*c)/d - 1/4*(a + b)*sin(2*d*x + 2*
c)/d
```

3.69.9 Mupad [B] (verification not implemented)

Time = 13.83 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \sin^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = x\left(\frac{a}{2}+\frac{3b}{8}\right) - \frac{\left(\frac{a}{2}+\frac{5b}{8}\right)\tan(c+dx)^3+\left(\frac{a}{2}+\frac{3b}{8}\right)\tan(c+dx)}{d\left(\tan(c+dx)^4+2\tan(c+dx)^2+1\right)}$$

input
```
int(sin(c + d*x)^2*(a + b*sin(c + d*x)^2),x)
```

output
```
x*(a/2 + (3*b)/8) - (tan(c + d*x)^3*(a/2 + (5*b)/8) + tan(c + d*x)*(a/2 +
(3*b)/8))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

3.70 $\int (a+b\sin^2(c+dx)) \, dx$

3.70.1 Optimal result

Integrand size = 12, antiderivative size = 30

$$\int (a+b\sin^2(c+dx)) \, dx = ax+\frac{bx}{2}-\frac{b\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
a*x+1/2*b*x-1/2*b*cos(d*x+c)*sin(d*x+c)/d
```

3.70.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

$$\int (a+b\sin^2(c+dx)) \, dx = ax+\frac{b(c+dx)}{2d}-\frac{b\sin(2(c+dx))}{4d}$$

input
```
Integrate[a + b*Sin[c + d*x]^2,x]
```

output
```
a*x + (b*(c + d*x))/(2*d) - (b*Sin[2*(c + d*x)])/(4*d)
```

3.70.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin^2(c+dx)\right)\,dx$$

$$\Big\downarrow \ 2009$$

$$ax-\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

input
```
Int[a + b*Sin[c + d*x]^2,x]
```

output
```
a*x + (b*x)/2 - (b*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

3.70.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.70.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
risch	$ax+\frac{bx}{2}-\frac{\sin(2dx+2c)b}{4d}$	24
parallelrisch	$\frac{b(2dx-\sin(2dx+2c))}{4d}+ax$	27
default	$ax+\frac{b\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	32
parts	$ax+\frac{b\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	32
derivativedivides	$\frac{(dx+c)a+b\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	37
norman	$\frac{\left(a+\frac{b}{2}\right)x+\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\left(a+\frac{b}{2}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(2a+b)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	92

input
```
int(a+b*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/2*b*x-1/4/d*sin(2*d*x+2*c)*b
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \left(a+b\sin^2(c+dx)\right)\,dx=\frac{(2\,a+b)dx-b\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate(a+b*sin(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*((2*a + b)*d*x - b*cos(d*x + c)*sin(d*x + c))/d
```

3.70.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.70

$$\int \left(a+b\sin^2(c+dx)\right)\,dx$$
$$=ax+b\left(\begin{cases}\frac{x\sin^2\left(c+dx\right)}{2}+\frac{x\cos^2\left(c+dx\right)}{2}-\frac{\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d} & \text{for}\ d\neq 0\\ x\sin^2\left(c\right) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*sin(d*x+c)**2,x)
```

output
```
a*x + b*Piecewise((x*sin(c + d*x)**2/2 + x*cos(c + d*x)**2/2 - sin(c + d*x
)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*sin(c)**2, True))
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \left(a + b\sin^2(c + dx)\right) \, dx = ax + \frac{(2\,dx + 2\,c - \sin\left(2\,dx + 2\,c\right))b}{4\,d}$$

input
```
integrate(a+b*sin(d*x+c)^2,x, algorithm="maxima")
```

output
```
a*x + 1/4*(2*d*x + 2*c - sin(2*d*x + 2*c))*b/d
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \left(a + b\sin^2(c + dx)\right) \, dx = \frac{1}{4}\,b\left(2\,x - \frac{\sin\left(2\,dx + 2\,c\right)}{d}\right) + ax$$

input
```
integrate(a+b*sin(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*b*(2*x - sin(2*d*x + 2*c)/d) + a*x
```

3.70.9 Mupad [B] (verification not implemented)

Time = 13.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \left(a + b\sin^2(c + dx)\right) \, dx = -\frac{\frac{b\,\sin(2\,c+2\,d\,x)}{4} - d\,x\,\left(a + \frac{b}{2}\right)}{d}$$

input
```
int(a + b*sin(c + d*x)^2,x)
```

output
```
-((b*sin(2*c + 2*d*x))/4 - d*x*(a + b/2))/d
```

3.71 $\int \csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

3.71.1 Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=bx-\frac{a\cot(c+dx)}{d}$$

output
```
b*x-a*cot(d*x+c)/d
```

3.71.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=bx-\frac{a\cot(c+dx)}{d}$$

input
```
Integrate[Csc[c + d*x]^2*(a + b*Sin[c + d*x]^2),x]
```

output
```
b*x - (a*Cot[c + d*x])/d
```

3.71.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3491, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a+b\sin(c+dx)^2}{\sin(c+dx)^2}dx$$

$$\Big\downarrow \ 3491$$

$$b\int 1dx-\frac{a\cot(c+dx)}{d}$$

$$\Big\downarrow \ 24$$

$$bx-\frac{a\cot(c+dx)}{d}$$

input
```
Int[Csc[c + d*x]^2*(a + b*Sin[c + d*x]^2),x]
```

output
```
b*x - (a*Cot[c + d*x])/d
```

3.71.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

3.71.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{-a\cot(dx+c)+b(dx+c)}{d}$
default	$\frac{-a\cot(dx+c)+b(dx+c)}{d}$
risch	$bx-\frac{2ia}{d\left(\mathrm{e}^{2i(dx+c)}-1\right)}$
parallelrisch	$\frac{2bxd+a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\cot\left(\frac{dx}{2}+\frac{c}{2}\right)a}{2d}$
norman	$\frac{bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+bx\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a}{2d}-\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+2bx\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

input
```
int(csc(d*x+c)^2*(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a*cot(d*x+c)+b*(d*x+c))
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=\frac{bdx\sin(dx+c)-a\cos(dx+c)}{d\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
(b*d*x*sin(d*x + c) - a*cos(d*x + c))/(d*sin(d*x + c))
```

3.71.6 Sympy [F]

$$\int \csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=\int \left(a+b\sin^2(c+dx)\right)\csc^2(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**2*(a+b*sin(d*x+c)**2),x)
```

output
```
Integral((a + b*sin(c + d*x)**2)*csc(c + d*x)**2, x)
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = \frac{(dx+c)b-\frac{a}{\tan(dx+c)}}{d}$$

input
```
integrate(csc(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
((d*x + c)*b - a/tan(d*x + c))/d
```

3.71.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(16) = 32.

Time = 0.41 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = \frac{2\,(dx+c)b+a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

input
```
integrate(csc(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)*b + a*tan(1/2*d*x + 1/2*c) - a/tan(1/2*d*x + 1/2*c))/d
```

3.71.9 Mupad [B] (verification not implemented)

Time = 13.62 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = b\,x-\frac{a\cot(c+d\,x)}{d}$$

input
```
int((a + b*sin(c + d*x)^2)/sin(c + d*x)^2,x)
```

output
```
b*x - (a*cot(c + d*x))/d
```

3.72 $\int \csc^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

3.72.1 Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \csc^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=-\frac{(2a+3b)\cot(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}$$

output
```
-1/3*(2*a+3*b)*cot(d*x+c)/d-1/3*a*cot(d*x+c)*csc(d*x+c)^2/d
```

3.72.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \csc^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=-\frac{2a\cot(c+dx)}{3d}-\frac{b\cot(c+dx)}{d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}$$

input
```
Integrate[Csc[c + d*x]^4*(a + b*Sin[c + d*x]^2),x]
```

output
```
(-2*a*Cot[c + d*x])/(3*d) - (b*Cot[c + d*x])/d - (a*Cot[c + d*x]*Csc[c + d
*x]^2)/(3*d)
```

3.72.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3491, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(c+dx)^2}{\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{3491}$$

$$\frac{1}{3}(2a+3b)\int \csc^2(c+dx)dx-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}(2a+3b)\int \csc(c+dx)^2dx-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{(2a+3b)\int 1d\cot(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}$$

$$\Big\downarrow \text{24}$$

$$-\frac{(2a+3b)\cot(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}$$

input
```
Int[Csc[c + d*x]^4*(a + b*Sin[c + d*x]^2),x]
```

output
```
-1/3*((2*a + 3*b)*Cot[c + d*x])/d - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d)
```

3.72.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.72.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{a\left(-\frac{2}{3}-\frac{\left(\csc^{2}(dx+c)\right)}{3}\right)\cot(dx+c)-b\cot(dx+c)}{d}$
default	$\frac{a\left(-\frac{2}{3}-\frac{\left(\csc^{2}(dx+c)\right)}{3}\right)\cot(dx+c)-b\cot(dx+c)}{d}$
risch	$-\frac{2i\left(3b\,\mathrm{e}^{4i(dx+c)}-6a\,\mathrm{e}^{2i(dx+c)}-6b\,\mathrm{e}^{2i(dx+c)}+2a+3b\right)}{3d\left(\mathrm{e}^{2i(dx+c)}-1\right)^{3}}$
parallelrisch	$-\frac{\left(-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\cot\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\left(\cot^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+\cot\left(\frac{dx}{2}+\frac{c}{2}\right)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+9a+12b\right)}{24d}$
norman	$\frac{-\frac{a}{24d}+\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{(5a+6b)\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{(5a+6b)\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{(11a+12b)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{(11a+12b)\left(\tan^{8}\left(\right.\right.}{24d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}$

input
```
int(csc(d*x+c)^4*(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)-b*cot(d*x+c))
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \csc^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = -\frac{(2\,a+3\,b)\cos{(dx+c)}^3-3\,(a+b)\cos{(dx+c)}}{3\left(d\cos{(dx+c)}^2-d\right)\sin{(dx+c)}}$$

input
```
integrate(csc(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/3*((2*a + 3*b)*cos(d*x + c)^3 - 3*(a + b)*cos(d*x + c))/((d*cos(d*x + c
)^2 - d)*sin(d*x + c))
```

3.72.6 Sympy [F]

$$\int \csc^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = \int \left(a+b\sin^2{(c+dx)}\right)\csc^4{(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**4*(a+b*sin(d*x+c)**2),x)
```

output
```
Integral((a + b*sin(c + d*x)**2)*csc(c + d*x)**4, x)
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \csc^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = -\frac{3\,(a+b)\tan{(dx+c)}^2+a}{3\,d\tan{(dx+c)}^3}$$

input
```
integrate(csc(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/3*(3*(a + b)*tan(d*x + c)^2 + a)/(d*tan(d*x + c)^3)
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \csc^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = -\frac{3\,a\tan\left(dx+c\right)^2+3\,b\tan\left(dx+c\right)^2+a}{3\,d\tan\left(dx+c\right)^3}$$

input
```
integrate(csc(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/3*(3*a*tan(d*x + c)^2 + 3*b*tan(d*x + c)^2 + a)/(d*tan(d*x + c)^3)
```

3.72.9 Mupad [B] (verification not implemented)

Time = 13.87 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \csc^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = -\frac{a\cot(c+d\,x)^3}{3\,d}-\frac{\cot(c+d\,x)\,(a+b)}{d}$$

input
```
int((a + b*sin(c + d*x)^2)/sin(c + d*x)^4,x)
```

output
```
- (a*cot(c + d*x)^3)/(3*d) - (cot(c + d*x)*(a + b))/d
```

3.73 $\int \csc^6(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

3.73.1 Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \csc^6(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=-\frac{(4a+5b)\cot(c+dx)}{5d}-\frac{(4a+5b)\cot^3(c+dx)}{15d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}$$

output
```
-1/5*(4*a+5*b)*cot(d*x+c)/d-1/15*(4*a+5*b)*cot(d*x+c)^3/d-1/5*a*cot(d*x+c)
*csc(d*x+c)^4/d
```

3.73.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.46

$$\int \csc^6(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx=-\frac{8a\cot(c+dx)}{15d}-\frac{2b\cot(c+dx)}{3d}-\frac{4a\cot(c+dx)\csc^2(c+dx)}{15d}-\frac{b\cot(c+dx)\csc^2(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}$$

input
```
Integrate[Csc[c + d*x]^6*(a + b*Sin[c + d*x]^2),x]
```

output
```
(-8*a*Cot[c + d*x])/(15*d) - (2*b*Cot[c + d*x])/(3*d) - (4*a*Cot[c + d*x]*
Csc[c + d*x]^2)/(15*d) - (b*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) - (a*Cot[c
+ d*x]*Csc[c + d*x]^4)/(5*d)
```

3.73.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3491, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(c+dx)^2}{\sin(c+dx)^6}dx$$

$$\Big\downarrow \text{3491}$$

$$\frac{1}{5}(4a+5b)\int \csc^4(c+dx)dx-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}(4a+5b)\int \csc(c+dx)^4dx-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{(4a+5b)\int \left(\cot^2(c+dx)+1\right)d\cot(c+dx)}{5d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(4a+5b)\left(\frac{1}{3}\cot^3(c+dx)+\cot(c+dx)\right)}{5d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}$$

input
```
Int[Csc[c + d*x]^6*(a + b*Sin[c + d*x]^2),x]
```

output
```
-1/5*((4*a + 5*b)*(Cot[c + d*x] + Cot[c + d*x]^3/3))/d - (a*Cot[c + d*x]*C
sc[c + d*x]^4)/(5*d)
```

3.73.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.73.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{a\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx+c)+b\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)}{d}$
default	$\frac{a\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx+c)+b\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)}{d}$
risch	$\frac{4i\left(15b\,e^{6i(dx+c)}-40a\,e^{4i(dx+c)}-35b\,e^{4i(dx+c)}+20a\,e^{2i(dx+c)}+25b\,e^{2i(dx+c)}-4a-5b\right)}{15d\left(e^{2i(dx+c)}-1\right)^5}$
parallelrisch	$\frac{-3\left(\cot^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+(-25a-20b)\left(\cot^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-150a-180b)\cot\left(\frac{dx}{2}+\frac{c}{2}\right)+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+\cdots\right)}{480d}$
norman	$\frac{-\frac{a}{160d}+\frac{a\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160d}-\frac{5(7a+8b)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}+\frac{5(7a+8b)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{(31a+20b)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{480d}+\frac{(31a+20b)\left(\tan\cdots\right)}{480\cdots}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

input
```
int(csc(d*x+c)^6*(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c)+b*(-2/3-1/3*c
sc(d*x+c)^2)*cot(d*x+c))
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int \csc^6(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = -\frac{2\,(4\,a+5\,b)\cos\left(dx+c\right)^5-5\,(4\,a+5\,b)\cos\left(dx+c\right)^3+15\,(a+b)\cos\left(dx+c\right)}{15\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

input
```
integrate(csc(d*x+c)^6*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/15*(2*(4*a + 5*b)*cos(d*x + c)^5 - 5*(4*a + 5*b)*cos(d*x + c)^3 + 15*(a
 + b)*cos(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x +
 c))
```

3.73.6 Sympy [F]

$$\int \csc^6(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = \int \left(a+b\sin^2\left(c+dx\right)\right)\csc^6\left(c+dx\right)dx$$

input
```
integrate(csc(d*x+c)**6*(a+b*sin(d*x+c)**2),x)
```

output
```
Integral((a + b*sin(c + d*x)**2)*csc(c + d*x)**6, x)
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \csc^6(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = -\frac{15\,(a+b)\tan\left(dx+c\right)^4+5\,(2\,a+b)\tan\left(dx+c\right)^2+3\,a}{15\,d\tan\left(dx+c\right)^5}$$

input
```
integrate(csc(d*x+c)^6*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/15*(15*(a + b)*tan(d*x + c)^4 + 5*(2*a + b)*tan(d*x + c)^2 + 3*a)/(d*ta
n(d*x + c)^5)
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \csc^6(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = -\frac{15\,a\tan\left(dx+c\right)^4+15\,b\tan\left(dx+c\right)^4+10\,a\tan\left(dx+c\right)^2+5\,b\tan\left(dx+c\right)^2+3\,a}{15\,d\tan\left(dx+c\right)^5}$$

input
```
integrate(csc(d*x+c)^6*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/15*(15*a*tan(d*x + c)^4 + 15*b*tan(d*x + c)^4 + 10*a*tan(d*x + c)^2 + 5
*b*tan(d*x + c)^2 + 3*a)/(d*tan(d*x + c)^5)
```

3.73.9 Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \csc^6(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx = -\frac{a\cot(c+d\,x)^5}{5\,d}-\frac{\cot(c+d\,x)\,\left(a+b\right)}{d}-\frac{\cot(c+d\,x)^3\,\left(\frac{2\,a}{3}+\frac{b}{3}\right)}{d}$$

input
```
int((a + b*sin(c + d*x)^2)/sin(c + d*x)^6,x)
```

output
```
- (a*cot(c + d*x)^5)/(5*d) - (cot(c + d*x)*(a + b))/d - (cot(c + d*x)^3*((
2*a)/3 + b/3))/d
```

3.74 $\int \left(a+b\sin^2(x)\right) \, dx$

3.74.1 Optimal result

Integrand size = 8, antiderivative size = 19

$$\int \left(a+b\sin^2(x)\right) \, dx = ax+\frac{bx}{2}-\frac{1}{2}b\cos(x)\sin(x)$$

output
```
a*x+1/2*b*x-1/2*b*cos(x)*sin(x)
```

3.74.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \left(a+b\sin^2(x)\right) \, dx = ax+\frac{bx}{2}-\frac{1}{4}b\sin(2x)$$

input
```
Integrate[a + b*Sin[x]^2,x]
```

output
```
a*x + (b*x)/2 - (b*Sin[2*x])/4
```

3.74.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin^2(x)\right)\,dx$$
$$\Big\downarrow \; 2009$$
$$ax+\frac{bx}{2}-\frac{1}{2}b\sin(x)\cos(x)$$

input
```
Int[a + b*Sin[x]^2,x]
```

output
```
a*x + (b*x)/2 - (b*Cos[x]*Sin[x])/2
```

3.74.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.74.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
risch	$ax+\frac{bx}{2}-\frac{b\sin(2x)}{4}$	16
default	$ax+b\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)$	17
parallelrisch	$b\left(\frac{x}{2}-\frac{\sin(2x)}{4}\right)+ax$	17
parts	$ax+b\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)$	17
norman	$\frac{b\left(\tan^3\left(\frac{x}{2}\right)\right)+\left(a+\frac{b}{2}\right)x+\left(a+\frac{b}{2}\right)x\left(\tan^4\left(\frac{x}{2}\right)\right)+(2a+b)x\left(\tan^2\left(\frac{x}{2}\right)\right)-b\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	61

input
```
int(a+b*sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/2*b*x-1/4*b*sin(2*x)
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \left(a+b\sin^2(x)\right)\,dx=-\frac{1}{2}\,b\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,(2\,a+b)x$$

input
```
integrate(a+b*sin(x)^2,x, algorithm="fricas")
```

output
```
-1/2*b*cos(x)*sin(x) + 1/2*(2*a + b)*x
```

3.74.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\sin^2(x)\right)\,dx=ax+b\left(\frac{x}{2}-\frac{\sin\left(x\right)\cos\left(x\right)}{2}\right)$$

input
```
integrate(a+b*sin(x)**2,x)
```

output
```
a*x + b*(x/2 - sin(x)*cos(x)/2)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(a+b\sin^2(x)\right)\,dx=\frac{1}{4}\,b(2\,x-\sin\left(2\,x\right))+ax$$

input
```
integrate(a+b*sin(x)^2,x, algorithm="maxima")
```

output
```
1/4*b*(2*x - sin(2*x)) + a*x
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(a+b\sin^2(x)\right)\,dx=\frac{1}{4}b(2x-\sin(2x))+ax$$

input
```
integrate(a+b*sin(x)^2,x, algorithm="giac")
```

output
```
1/4*b*(2*x - sin(2*x)) + a*x
```

3.74.9 Mupad [B] (verification not implemented)

Time = 13.52 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\sin^2(x)\right)\,dx=x\left(a+\frac{b}{2}\right)-\frac{b\sin(2x)}{4}$$

input
```
int(a + b*sin(x)^2,x)
```

output
```
x*(a + b/2) - (b*sin(2*x))/4
```

3.75 $\int \left(a+b\sin^2(x)\right)^2 \, dx$

3.75.1 Optimal result

Integrand size = 10, antiderivative size = 50

$$\int \left(a+b\sin^2(x)\right)^2 \, dx = \frac{1}{8}\left(8a^2+8ab+3b^2\right)x-\frac{1}{8}b(8a+3b)\cos(x)\sin(x)-\frac{1}{4}b^2\cos(x)\sin^3(x)$$

output
```
1/8*(8*a^2+8*a*b+3*b^2)*x-1/8*b*(8*a+3*b)*cos(x)*sin(x)-1/4*b^2*cos(x)*sin
(x)^3
```

3.75.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \left(a+b\sin^2(x)\right)^2 \, dx = \frac{1}{32}\left(4\left(8a^2+8ab+3b^2\right)x-8b(2a+b)\sin(2x)+b^2\sin(4x)\right)$$

input
```
Integrate[(a + b*Sin[x]^2)^2,x]
```

output
```
(4*(8*a^2 + 8*a*b + 3*b^2)*x - 8*b*(2*a + b)*Sin[2*x] + b^2*Sin[4*x])/32
```

3.75.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3658}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin^2(x)\right)^2 dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a+b\sin(x)^2\right)^2 dx$$

$$\Big\downarrow \; 3658$$

$$\frac{1}{8}x\left(8a^2+8ab+3b^2\right)-\frac{1}{8}b(8a+3b)\sin(x)\cos(x)-\frac{1}{4}b^2\sin^3(x)\cos(x)$$

input
```
Int[(a + b*Sin[x]^2)^2,x]
```

output
```
((8*a^2 + 8*a*b + 3*b^2)*x)/8 - (b*(8*a + 3*b)*Cos[x]*Sin[x])/8 - (b^2*Cos
[x]*Sin[x]^3)/4
```

3.75.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3658
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^2, x_Symbol] :> Simp[(8*a^2 +
8*a*b + 3*b^2)*(x/8), x] + (-Simp[b^2*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)),
x] - Simp[b*(8*a + 3*b)*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a,
 b, e, f}, x]
```

3.75.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

method	result
parallelrisch	$\frac{(-2ab-b^2)\sin(2x)}{4}+\frac{b^2\sin(4x)}{32}+\left(a^2+ab+\frac{3}{8}b^2\right)x$
default	$b^2\left(-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)+2ab\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+a^2x$
parts	$b^2\left(-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)+2ab\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+a^2x$
risch	$a^2x+abx+\frac{3b^2x}{8}+\frac{b^2\sin(4x)}{32}-\frac{\sin(2x)ab}{2}-\frac{\sin(2x)b^2}{4}$
norman	$\frac{\left(-2ab-\frac{11}{4}b^2\right)\left(\tan^3\left(\frac{x}{2}\right)\right)+\left(-2ab-\frac{3}{4}b^2\right)\tan\left(\frac{x}{2}\right)+\left(2ab+\frac{3}{4}b^2\right)\left(\tan^7\left(\frac{x}{2}\right)\right)+\left(2ab+\frac{11}{4}b^2\right)\left(\tan^5\left(\frac{x}{2}\right)\right)+\left(a^2+ab+\frac{3}{8}b^2\right)x+\left(a^2+ab+\frac{3}{8}\right.}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$

input
```
int((a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-2*a*b-b^2)*sin(2*x)+1/32*b^2*sin(4*x)+(a^2+a*b+3/8*b^2)*x
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \left(a+b\sin^2(x)\right)^2\,dx=\frac{1}{8}\left(8a^2+8ab+3b^2\right)x+\frac{1}{8}\left(2b^2\cos(x)^3-\left(8ab+5b^2\right)\cos(x)\right)\sin(x)$$

input
```
integrate((a+b*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
1/8*(8*a^2 + 8*a*b + 3*b^2)*x + 1/8*(2*b^2*cos(x)^3 - (8*a*b + 5*b^2)*cos(
x))*sin(x)
```

3.75.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(44) = 88$.

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.20

$$\int \left(a+b\sin^2(x)\right)^2\,dx = a^2x + abx\sin^2(x) + abx\cos^2(x) - ab\sin(x)\cos(x) + \frac{3b^2x\sin^4(x)}{8} + \frac{3b^2x\sin^2(x)\cos^2(x)}{4} + \frac{3b^2x\cos^4(x)}{8} - \frac{5b^2\sin^3(x)\cos(x)}{8} - \frac{3b^2\sin(x)\cos^3(x)}{8}$$

input
```
integrate((a+b*sin(x)**2)**2,x)
```

output
```
a**2*x + a*b*x*sin(x)**2 + a*b*x*cos(x)**2 - a*b*sin(x)*cos(x) + 3*b**2*x*
sin(x)**4/8 + 3*b**2*x*sin(x)**2*cos(x)**2/4 + 3*b**2*x*cos(x)**4/8 - 5*b*
*2*sin(x)**3*cos(x)/8 - 3*b**2*sin(x)*cos(x)**3/8
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \left(a+b\sin^2(x)\right)^2\,dx = \frac{1}{32}b^2(12\,x+\sin(4\,x)-8\,\sin(2\,x)) + \frac{1}{2}ab(2\,x-\sin(2\,x)) + a^2x$$

input
```
integrate((a+b*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/32*b^2*(12*x + sin(4*x) - 8*sin(2*x)) + 1/2*a*b*(2*x - sin(2*x)) + a^2*x
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \left(a+b\sin^2(x)\right)^2\,dx = \frac{1}{32}b^2\sin(4\,x) + \frac{1}{8}\left(8\,a^2+8\,ab+3\,b^2\right)x - \frac{1}{4}\left(2\,ab+b^2\right)\sin(2\,x)$$

input
```
integrate((a+b*sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/32*b^2*sin(4*x) + 1/8*(8*a^2 + 8*a*b + 3*b^2)*x - 1/4*(2*a*b + b^2)*sin(
2*x)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 13.89 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \left(a+b\sin^2(x)\right)^2\,dx = x\,a^2-\sin(x)\;a\,b\;\cos(x)+x\,a\,b+\frac{\sin(x)\;b^2\cos(x)^3}{4}-\frac{5\;\sin(x)\;b^2\;\cos(x)}{8}+\frac{3\,x\,b^2}{8}$$

input
```
int((a + b*sin(x)^2)^2,x)
```

output
```
a^2*x + (3*b^2*x)/8 + (b^2*cos(x)^3*sin(x))/4 + a*b*x - (5*b^2*cos(x)*sin(
x))/8 - a*b*cos(x)*sin(x)
```

3.76 $\int (a+b\sin^2(x))^3\,dx$

3.76.1 Optimal result

Integrand size = 10, antiderivative size = 87

$$\int (a+b\sin^2(x))^3\,dx=\frac{1}{16}(2a+b)\left(8a^2+8ab+5b^2\right)x-\frac{1}{48}b\left(64a^2+54ab+15b^2\right)\cos(x)\sin(x)-\frac{5}{24}b^2(2a+b)\cos(x)\sin^3(x)-\frac{1}{6}b\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^2$$

output
```
1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*x-1/48*b*(64*a^2+54*a*b+15*b^2)*cos(x)*si
n(x)-5/24*b^2*(2*a+b)*cos(x)*sin(x)^3-1/6*b*cos(x)*sin(x)*(a+b*sin(x)^2)^2
```

3.76.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int (a+b\sin^2(x))^3\,dx=\frac{1}{192}(12(2a+b)\left(8a^2+8ab+5b^2\right)x+9ib(4ia+(1+2i)b)(4a+(2+i)b)\sin(2x)+9b^2(2a+b)\sin(4x)-b^3\sin(6x))$$

input
```
Integrate[(a + b*Sin[x]^2)^3,x]
```

output
```
(12*(2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*x + (9*I)*b*((4*I)*a + (1 + 2*I)*b)*
(4*a + (2 + I)*b)*Sin[2*x] + 9*b^2*(2*a + b)*Sin[4*x] - b^3*Sin[6*x])/192
```

3.76.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3659, 3042, 3648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin^2(x)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin(x)^2\right)^3\,dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{6}\int \left(b\sin^2(x)+a\right)\left(5b(2a+b)\sin^2(x)+a(6a+b)\right)dx-\frac{1}{6}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \left(b\sin(x)^2+a\right)\left(5b(2a+b)\sin(x)^2+a(6a+b)\right)dx-\frac{1}{6}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^2$$

$$\Big\downarrow \text{3648}$$

$$\frac{1}{6}\left(\frac{3}{8}x(2a+b)\left(8a^2+8ab+5b^2\right)-\frac{1}{8}b\left(64a^2+54ab+15b^2\right)\sin(x)\cos(x)-\frac{5}{4}b^2(2a+b)\sin^3(x)\cos(x)\right)-\frac{1}{6}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^2$$

input
```
Int[(a + b*Sin[x]^2)^3,x]
```

output
```
-1/6*(b*Cos[x]*Sin[x]*(a + b*Sin[x]^2)^2) + ((3*(2*a + b)*(8*a^2 + 8*a*b +
 5*b^2)*x)/8 - (b*(64*a^2 + 54*a*b + 15*b^2)*Cos[x]*Sin[x])/8 - (5*b^2*(2*
a + b)*Cos[x]*Sin[x]^3)/4)/6
```

3.76.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3648
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(4*A*(2*a + b) + B*(4*a + 3*b))*(x/8), x] +
(-Simp[b*B*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)), x] - Simp[(4*A*b + B*(4*a +
 3*b))*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

3.76.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

method	result
parallelrisch	$\frac{3\left(-16a^2b-16a\,b^2-5b^3\right)\sin(2x)}{64}+\frac{3\left(2a\,b^2+b^3\right)\sin(4x)}{64}-\frac{b^3\sin(6x)}{192}+\left(a^2+ab+\frac{5}{8}b^2\right)\left(a+\frac{b}{2}\right)x$
default	$b^3\left(-\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\frac{5x}{16}\right)+3a\,b^2\left(-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)+3a^2b\Big(-$
parts	$b^3\left(-\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\frac{5x}{16}\right)+3a\,b^2\left(-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)+3a^2b\Big(-$
risch	$a^3x+\frac{3a^2bx}{2}+\frac{9a\,b^2x}{8}+\frac{5b^3x}{16}-\frac{b^3\sin(6x)}{192}+\frac{3\sin(4x)a\,b^2}{32}+\frac{3\sin(4x)b^3}{64}-\frac{3\sin(2x)a^2b}{4}-\frac{3\sin(2x)a\,b^2}{4}-$
norman	$\left(-9a^2b-\frac{51}{4}a\,b^2-\frac{85}{24}b^3\right)\left(\tan^3\left(\frac{x}{2}\right)\right)+\left(-6a^2b-\frac{21}{2}a\,b^2-\frac{33}{4}b^3\right)\left(\tan^5\left(\frac{x}{2}\right)\right)+\left(-3a^2b-\frac{9}{4}a\,b^2-\frac{5}{8}b^3\right)\tan\left(\frac{x}{2}\right)+\left(3a^2b+\frac{9}{4}a\,b^2+\frac{5}{8}b^3\right)$

input
```
int((a+b*sin(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
3/64*(-16*a^2*b-16*a*b^2-5*b^3)*sin(2*x)+3/64*(2*a*b^2+b^3)*sin(4*x)-1/192
*b^3*sin(6*x)+(a^2+a*b+5/8*b^2)*(a+1/2*b)*x
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \left(a+b\sin^2(x)\right)^3\,dx = \frac{1}{16}\left(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3\right)x - \frac{1}{48}\left(8\,b^3\cos(x)^5-2\left(18\,ab^2+13\,b^3\right)\cos(x)^3+3\left(24\,a^2b+30\,ab^2+11\,b^3\right)\cos(x)\right)\sin(x)$$

input
```
integrate((a+b*sin(x)^2)^3,x, algorithm="fricas")
```

output
```
1/16*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*x - 1/48*(8*b^3*cos(x)^5 - 2*(
18*a*b^2 + 13*b^3)*cos(x)^3 + 3*(24*a^2*b + 30*a*b^2 + 11*b^3)*cos(x))*sin
(x)
```

3.76.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(88) = 176.

Time = 0.27 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.83

$$\int \left(a+b\sin^2(x)\right)^3\,dx = a^3x+\frac{3a^2bx\sin^2(x)}{2}+\frac{3a^2bx\cos^2(x)}{2}-\frac{3a^2b\sin(x)\cos(x)}{2}+\frac{9ab^2x\sin^4(x)}{8}+\frac{9ab^2x\sin^2(x)\cos^2(x)}{4}+\frac{9ab^2x\cos^4(x)}{8}-\frac{15ab^2\sin^3(x)\cos(x)}{8}-\frac{9ab^2\sin(x)\cos^3(x)}{8}+\frac{5b^3x\sin^6(x)}{16}+\frac{15b^3x\sin^4(x)\cos^2(x)}{16}+\frac{15b^3x\sin^2(x)\cos^4(x)}{16}+\frac{5b^3x\cos^6(x)}{16}-\frac{11b^3\sin^5(x)\cos(x)}{16}-\frac{5b^3\sin^3(x)\cos^3(x)}{6}-\frac{5b^3\sin(x)\cos^5(x)}{16}$$

input
```
integrate((a+b*sin(x)**2)**3,x)
```

output
```
a**3*x + 3*a**2*b*x*sin(x)**2/2 + 3*a**2*b*x*cos(x)**2/2 - 3*a**2*b*sin(x)
*cos(x)/2 + 9*a*b**2*x*sin(x)**4/8 + 9*a*b**2*x*sin(x)**2*cos(x)**2/4 + 9*
a*b**2*x*cos(x)**4/8 - 15*a*b**2*sin(x)**3*cos(x)/8 - 9*a*b**2*sin(x)*cos(
x)**3/8 + 5*b**3*x*sin(x)**6/16 + 15*b**3*x*sin(x)**4*cos(x)**2/16 + 15*b*
*3*x*sin(x)**2*cos(x)**4/16 + 5*b**3*x*cos(x)**6/16 - 11*b**3*sin(x)**5*co
s(x)/16 - 5*b**3*sin(x)**3*cos(x)**3/6 - 5*b**3*sin(x)*cos(x)**5/16
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \left(a+b\sin^2(x)\right)^3 \, dx = \frac{1}{192}\left(4\,\sin\left(2\,x\right)^3+60\,x+9\,\sin\left(4\,x\right)-48\,\sin\left(2\,x\right)\right)b^3 + \frac{3}{32}\,ab^2(12\,x+\sin\left(4\,x\right)-8\,\sin\left(2\,x\right))+\frac{3}{4}\,a^2b(2\,x-\sin\left(2\,x\right))+a^3x$$

input
```
integrate((a+b*sin(x)^2)^3,x, algorithm="maxima")
```

output
```
1/192*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*b^3 + 3/32*a*b^2*(1
2*x + sin(4*x) - 8*sin(2*x)) + 3/4*a^2*b*(2*x - sin(2*x)) + a^3*x
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int \left(a+b\sin^2(x)\right)^3 \, dx = -\frac{1}{192}\,b^3\sin\left(6\,x\right)+\frac{1}{16}\left(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3\right)x + \frac{3}{64}\left(2\,ab^2+b^3\right)\sin\left(4\,x\right)-\frac{3}{64}\left(16\,a^2b+16\,ab^2+5\,b^3\right)\sin\left(2\,x\right)$$

input
```
integrate((a+b*sin(x)^2)^3,x, algorithm="giac")
```

output
```
-1/192*b^3*sin(6*x) + 1/16*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*x + 3/64
*(2*a*b^2 + b^3)*sin(4*x) - 3/64*(16*a^2*b + 16*a*b^2 + 5*b^3)*sin(2*x)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 13.98 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.36

$$\int \left(a+b\sin^2(x)\right)^3 \, dx = a^3\,x+\frac{5\,b^3\,x}{16} - \frac{\left(72\,a^2\,b+90\,a\,b^2+33\,b^3\right)\,\tan(x)^5+\left(144\,a^2\,b+144\,a\,b^2+40\,b^3\right)\,\tan(x)^3+\left(72\,a^2\,b+54\,a\,b^2+15\,b^3\right)\,\mathrm{ta}}{48\tan\left(x\right)^6+144\tan\left(x\right)^4+144\tan\left(x\right)^2+48} + \frac{9\,a\,b^2\,x}{8}+\frac{3\,a^2\,b\,x}{2}$$

input
```
int((a + b*sin(x)^2)^3,x)
```

output
```
a^3*x + (5*b^3*x)/16 - (tan(x)^5*(90*a*b^2 + 72*a^2*b + 33*b^3) + tan(x)^3
*(144*a*b^2 + 144*a^2*b + 40*b^3) + tan(x)*(54*a*b^2 + 72*a^2*b + 15*b^3))
/(144*tan(x)^2 + 144*tan(x)^4 + 48*tan(x)^6 + 48) + (9*a*b^2*x)/8 + (3*a^2
*b*x)/2
```

3.77 $\int (a+b\sin^2(x))^4 \, dx$

3.77.1 Optimal result

Integrand size = 10, antiderivative size = 140

$$\begin{aligned}\int (a+b\sin^2(x))^4 \, dx = &\frac{1}{128}\left(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4\right)x\\ &-\frac{1}{384}b\left(608a^3+808a^2b+480ab^2+105b^3\right)\cos(x)\sin(x)\\ &-\frac{1}{192}b^2\left(104a^2+104ab+35b^2\right)\cos(x)\sin^3(x)\\ &-\frac{7}{48}b(2a+b)\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^2\\ &-\frac{1}{8}b\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^3\end{aligned}$$

output
```
1/128*(128*a^4+256*a^3*b+288*a^2*b^2+160*a*b^3+35*b^4)*x-1/384*b*(608*a^3+
808*a^2*b+480*a*b^2+105*b^3)*cos(x)*sin(x)-1/192*b^2*(104*a^2+104*a*b+35*b
^2)*cos(x)*sin(x)^3-7/48*b*(2*a+b)*cos(x)*sin(x)*(a+b*sin(x)^2)^2-1/8*b*co
s(x)*sin(x)*(a+b*sin(x)^2)^3
```

3.77.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.81

$$\int \left(a+b\sin^2(x)\right)^4\,dx$$
$$=\frac{24(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4)\,x-96b(2a+b)\left(16a^2+16ab+7b^2\right)\sin(2x)+24b^2(24a^2}{3072}$$

input
```
Integrate[(a + b*Sin[x]^2)^4,x]
```

output
```
(24*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*x - 96*b*(2*a
 + b)*(16*a^2 + 16*a*b + 7*b^2)*Sin[2*x] + 24*b^2*(24*a^2 + 24*a*b + 7*b^2
)*Sin[4*x] - 32*b^3*(2*a + b)*Sin[6*x] + 3*b^4*Sin[8*x])/3072
```

3.77.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3659, 3042, 3649, 3042, 3648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin^2(x)\right)^4\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \left(a+b\sin(x)^2\right)^4dx$$
$$\Big\downarrow \; 3659$$
$$\frac{1}{8}\int \left(b\sin^2(x)+a\right)^2\left(7b(2a+b)\sin^2(x)+a(8a+b)\right)dx-\frac{1}{8}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^3$$
$$\Big\downarrow \; 3042$$
$$\frac{1}{8}\int \left(b\sin(x)^2+a\right)^2\left(7b(2a+b)\sin(x)^2+a(8a+b)\right)dx-\frac{1}{8}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^3$$
$$\Big\downarrow \; 3649$$

$$\frac{1}{8}\left(\frac{1}{6}\int \left(b\sin^2(x)+a\right)\left(b\left(104a^2+104ba+35b^2\right)\sin^2(x)+a\left(48a^2+20ba+7b^2\right)\right)dx-\frac{7}{6}b(2a+b)\sin(x)\cos(x)\left(\right.\right.$$
$$\frac{1}{8}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^3$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\int \left(b\sin(x)^2+a\right)\left(b\left(104a^2+104ba+35b^2\right)\sin(x)^2+a\left(48a^2+20ba+7b^2\right)\right)dx-\frac{7}{6}b(2a+b)\sin(x)\cos(x)\left(\right.\right.$$
$$\frac{1}{8}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^3$$

$\Big\downarrow$ 3648

$$\frac{1}{8}\left(\frac{1}{6}\left(-\frac{1}{4}b^2\left(104a^2+104ab+35b^2\right)\sin^3(x)\cos(x)-\frac{1}{8}b\left(608a^3+808a^2b+480ab^2+105b^3\right)\sin(x)\cos(x)+\frac{3}{8}x(12\right.\right.$$
$$\frac{1}{8}b\sin(x)\cos(x)\left(a+b\sin^2(x)\right)^3$$

input
```
Int[(a + b*Sin[x]^2)^4,x]
```

output
```
-1/8*(b*Cos[x]*Sin[x]*(a + b*Sin[x]^2)^3) + ((-7*b*(2*a + b)*Cos[x]*Sin[x]
*(a + b*Sin[x]^2)^2)/6 + ((3*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^
3 + 35*b^4)*x)/8 - (b*(608*a^3 + 808*a^2*b + 480*a*b^2 + 105*b^3)*Cos[x]*S
in[x])/8 - (b^2*(104*a^2 + 104*a*b + 35*b^2)*Cos[x]*Sin[x]^3)/4)/6)/8
```

3.77.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3648
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(4*A*(2*a + b) + B*(4*a + 3*b))*(x/8), x] +
(-Simp[b*B*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)), x] - Simp[(4*A*b + B*(4*a +
 3*b))*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

3.77.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

method	result
default	$b^4\left(-\frac{\left(\sin^7(x)+\frac{7\left(\sin^5(x)\right)}{6}+\frac{35\left(\sin^3(x)\right)}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8}+\frac{35x}{128}\right)+4a\,b^3\left(-\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\mathrm{co}}{6}\right.$
parts	$b^4\left(-\frac{\left(\sin^7(x)+\frac{7\left(\sin^5(x)\right)}{6}+\frac{35\left(\sin^3(x)\right)}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8}+\frac{35x}{128}\right)+4a\,b^3\left(-\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\mathrm{co}}{6}\right.$
parallelrisch	$\frac{\left(-32a^3b-48a^2b^2-30a\,b^3-7b^4\right)\sin(2x)}{32}+\frac{\left(24a^2b^2+24a\,b^3+7b^4\right)\sin(4x)}{128}+\frac{\left(-2a\,b^3-b^4\right)\sin(6x)}{96}+\frac{b^4\sin(8x)}{1024}+\left(a^4+\right.$
risch	$a^4x+2a^3bx+\frac{9a^2b^2x}{4}+\frac{5a\,b^3x}{4}+\frac{35b^4x}{128}+\frac{b^4\sin(8x)}{1024}-\frac{\sin(6x)a\,b^3}{48}-\frac{\sin(6x)b^4}{96}+\frac{3\sin(4x)a^2b^2}{16}+\frac{3\sin(4x)}{16}$
norman	$\frac{\left(-36a^3b-\frac{153}{2}a^2b^2-\frac{383}{6}a\,b^3-\frac{2681}{192}b^4\right)\left(\tan^5\left(\frac{x}{2}\right)\right)+\left(-20a^3b-\frac{93}{2}a^2b^2-\frac{283}{6}a\,b^3-\frac{5053}{192}b^4\right)\left(\tan^7\left(\frac{x}{2}\right)\right)+\left(-20a^3b-\frac{69}{2}a^2b^2-\frac{115}{6}a\,b\right.}{}$

input
```
int((a+b*sin(x)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
b^4*(-1/8*(sin(x)^7+7/6*sin(x)^5+35/24*sin(x)^3+35/16*sin(x))*cos(x)+35/12
8*x)+4*a*b^3*(-1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x)+6*a^
2*b^2*(-1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)+4*a^3*b*(-1/2*cos(x)*sin(x
)+1/2*x)+a^4*x
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int \left(a+b\sin^2(x)\right)^4\,dx=\frac{1}{128}\left(128\,a^4+256\,a^3b+288\,a^2b^2+160\,ab^3+35\,b^4\right)x$$
$$+\frac{1}{384}\left(48\,b^4\cos\left(x\right)^7-8\left(32\,ab^3+25\,b^4\right)\cos\left(x\right)^5+2\left(288\,a^2b^2+416\,ab^3+163\,b^4\right)\cos\left(x\right)^3-3\left(256\,a^3b+\right.\right.$$

input
```
integrate((a+b*sin(x)^2)^4,x, algorithm="fricas")
```

output
```
1/128*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*x + 1/384*(
48*b^4*cos(x)^7 - 8*(32*a*b^3 + 25*b^4)*cos(x)^5 + 2*(288*a^2*b^2 + 416*a*
b^3 + 163*b^4)*cos(x)^3 - 3*(256*a^3*b + 480*a^2*b^2 + 352*a*b^3 + 93*b^4)
*cos(x))*sin(x)
```

3.77.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(146) = 292$.

Time = 0.50 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.93

$$\begin{aligned}\int \left(a+b\sin^2(x)\right)^4\,dx = {} & a^4x+2a^3bx\sin^2\left(x\right)+2a^3bx\cos^2\left(x\right)-2a^3b\sin\left(x\right)\cos\left(x\right)\\
& +\frac{9a^2b^2x\sin^4\left(x\right)}{4}+\frac{9a^2b^2x\sin^2\left(x\right)\cos^2\left(x\right)}{2}+\frac{9a^2b^2x\cos^4\left(x\right)}{4}\\
& -\frac{15a^2b^2\sin^3\left(x\right)\cos\left(x\right)}{4}-\frac{9a^2b^2\sin\left(x\right)\cos^3\left(x\right)}{4}+\frac{5ab^3x\sin^6\left(x\right)}{4}\\
& +\frac{15ab^3x\sin^4\left(x\right)\cos^2\left(x\right)}{4}+\frac{15ab^3x\sin^2\left(x\right)\cos^4\left(x\right)}{4}\\
& +\frac{5ab^3x\cos^6\left(x\right)}{4}-\frac{11ab^3\sin^5\left(x\right)\cos\left(x\right)}{4}-\frac{10ab^3\sin^3\left(x\right)\cos^3\left(x\right)}{3}\\
& -\frac{5ab^3\sin\left(x\right)\cos^5\left(x\right)}{4}+\frac{35b^4x\sin^8\left(x\right)}{128}+\frac{35b^4x\sin^6\left(x\right)\cos^2\left(x\right)}{32}\\
& +\frac{105b^4x\sin^4\left(x\right)\cos^4\left(x\right)}{64}+\frac{35b^4x\sin^2\left(x\right)\cos^6\left(x\right)}{32}\\
& +\frac{35b^4x\cos^8\left(x\right)}{128}-\frac{93b^4\sin^7\left(x\right)\cos\left(x\right)}{128}-\frac{511b^4\sin^5\left(x\right)\cos^3\left(x\right)}{384}\\
& -\frac{385b^4\sin^3\left(x\right)\cos^5\left(x\right)}{384}-\frac{35b^4\sin\left(x\right)\cos^7\left(x\right)}{128}\end{aligned}$$

input
```
integrate((a+b*sin(x)**2)**4,x)
```

output
```
a**4*x + 2*a**3*b*x*sin(x)**2 + 2*a**3*b*x*cos(x)**2 - 2*a**3*b*sin(x)*cos
(x) + 9*a**2*b**2*x*sin(x)**4/4 + 9*a**2*b**2*x*sin(x)**2*cos(x)**2/2 + 9*
a**2*b**2*x*cos(x)**4/4 - 15*a**2*b**2*sin(x)**3*cos(x)/4 - 9*a**2*b**2*si
n(x)*cos(x)**3/4 + 5*a*b**3*x*sin(x)**6/4 + 15*a*b**3*x*sin(x)**4*cos(x)**
2/4 + 15*a*b**3*x*sin(x)**2*cos(x)**4/4 + 5*a*b**3*x*cos(x)**6/4 - 11*a*b*
*3*sin(x)**5*cos(x)/4 - 10*a*b**3*sin(x)**3*cos(x)**3/3 - 5*a*b**3*sin(x)*
cos(x)**5/4 + 35*b**4*x*sin(x)**8/128 + 35*b**4*x*sin(x)**6*cos(x)**2/32 +
 105*b**4*x*sin(x)**4*cos(x)**4/64 + 35*b**4*x*sin(x)**2*cos(x)**6/32 + 35
*b**4*x*cos(x)**8/128 - 93*b**4*sin(x)**7*cos(x)/128 - 511*b**4*sin(x)**5*
cos(x)**3/384 - 385*b**4*sin(x)**3*cos(x)**5/384 - 35*b**4*sin(x)*cos(x)**
7/128
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.77

$$\begin{aligned}\int \left(a+b\sin^2(x)\right)^4\,dx &= \frac{1}{48}\left(4\sin\left(2x\right)^3+60x+9\sin\left(4x\right)-48\sin\left(2x\right)\right)ab^3\\ &\quad+\frac{1}{3072}\left(128\sin\left(2x\right)^3+840x+3\sin\left(8x\right)+168\sin\left(4x\right)-768\sin\left(2x\right)\right)b^4\\ &\quad+\frac{3}{16}a^2b^2(12x+\sin\left(4x\right)-8\sin\left(2x\right))+a^3b(2x-\sin\left(2x\right))+a^4x\end{aligned}$$

input
```
integrate((a+b*sin(x)^2)^4,x, algorithm="maxima")
```

output
```
1/48*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*a*b^3 + 1/3072*(128*
sin(2*x)^3 + 840*x + 3*sin(8*x) + 168*sin(4*x) - 768*sin(2*x))*b^4 + 3/16*
a^2*b^2*(12*x + sin(4*x) - 8*sin(2*x)) + a^3*b*(2*x - sin(2*x)) + a^4*x
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

$$\int \left(a+b\sin^2(x)\right)^4\,dx = \frac{1}{1024}\,b^4\sin(8\,x) + \frac{1}{128}\left(128\,a^4+256\,a^3b+288\,a^2b^2+160\,ab^3+35\,b^4\right)x - \frac{1}{96}\left(2\,ab^3+b^4\right)\sin(6\,x) + \frac{1}{128}\left(24\,a^2b^2+24\,ab^3+7\,b^4\right)\sin(4\,x) - \frac{1}{32}\left(32\,a^3b+48\,a^2b^2+30\,ab^3+7\,b^4\right)\sin(2\,x)$$

input
```
integrate((a+b*sin(x)^2)^4,x, algorithm="giac")
```

output
```
1/1024*b^4*sin(8*x) + 1/128*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3
 + 35*b^4)*x - 1/96*(2*a*b^3 + b^4)*sin(6*x) + 1/128*(24*a^2*b^2 + 24*a*b^
3 + 7*b^4)*sin(4*x) - 1/32*(32*a^3*b + 48*a^2*b^2 + 30*a*b^3 + 7*b^4)*sin(
2*x)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

$$\int \left(a+b\sin^2(x)\right)^4\,dx = x\,a^4-2\,\sin(x)\,a^3\,b\,\cos(x)+2\,x\,a^3\,b+\frac{3\,\sin(x)\,a^2\,b^2\cos(x)^3}{2} - \frac{15\,\sin(x)\,a^2\,b^2\,\cos(x)}{4}+\frac{9\,x\,a^2\,b^2}{4}-\frac{2\,\sin(x)\,a\,b^3\cos(x)^5}{3} + \frac{13\,\sin(x)\,a\,b^3\cos(x)^3}{6}-\frac{11\,\sin(x)\,a\,b^3\,\cos(x)}{4} + \frac{5\,x\,a\,b^3}{4}+\frac{\sin(x)\,b^4\cos(x)^7}{8}-\frac{25\,\sin(x)\,b^4\cos(x)^5}{48} + \frac{163\,\sin(x)\,b^4\cos(x)^3}{192}-\frac{93\,\sin(x)\,b^4\,\cos(x)}{128}+\frac{35\,x\,b^4}{128}$$

input
```
int((a + b*sin(x)^2)^4,x)
```

output
```
a^4*x + (35*b^4*x)/128 + (163*b^4*cos(x)^3*sin(x))/192 - (25*b^4*cos(x)^5*
sin(x))/48 + (b^4*cos(x)^7*sin(x))/8 + (9*a^2*b^2*x)/4 - (93*b^4*cos(x)*si
n(x))/128 + (5*a*b^3*x)/4 + 2*a^3*b*x + (3*a^2*b^2*cos(x)^3*sin(x))/2 - (1
1*a*b^3*cos(x)*sin(x))/4 - 2*a^3*b*cos(x)*sin(x) - (15*a^2*b^2*cos(x)*sin(
x))/4 + (13*a*b^3*cos(x)^3*sin(x))/6 - (2*a*b^3*cos(x)^5*sin(x))/3
```

3.77. $\int \left(a + b\sin^2(x)\right)^4 \, dx$

3.78 $\int \frac{\sin^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.78.1 Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \frac{\sin^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{a^3\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}d} - \frac{(a^2-ab+b^2)\cos(c+dx)}{b^3d} - \frac{(a-2b)\cos^3(c+dx)}{3b^2d} - \frac{\cos^5(c+dx)}{5bd}$$

output
```
-(a^2-a*b+b^2)*cos(d*x+c)/b^3/d-1/3*(a-2*b)*cos(d*x+c)^3/b^2/d-1/5*cos(d*x
+c)^5/b/d+a^3*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/b^(7/2)/d/(a+b)^(1/2
)
```

3.78.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.70

$$\int \frac{\sin^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{-240a^3\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right) - 240a^3\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right) - 2\sqrt{-a-b}\sqrt{b}\cos(c+dx)\,(120}{240\sqrt{-a-b}b^{7/2}d}$$

input
```
Integrate[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-240*a^3*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] - 24
0*a^3*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] - 2*Sqrt
[-a - b]*Sqrt[b]*Cos[c + d*x]*(120*a^2 - 100*a*b + 89*b^2 + 4*(5*a - 7*b)*
b*Cos[2*(c + d*x)] + 3*b^2*Cos[4*(c + d*x)]))/(240*Sqrt[-a - b]*b^(7/2)*d)
```

3.78.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3665, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^7}{a+b\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{(1-\cos^2(c+dx))^3}{-b\cos^2(c+dx)+a+b}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$-\frac{\int \left(\frac{\cos^4(c+dx)}{b}+\frac{(a-2b)\cos^2(c+dx)}{b^2}+\frac{a^2-ba+b^2}{b^3}-\frac{a^3}{b^3(-b\cos^2(c+dx)+a+b)}\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{a^3\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}}+\frac{(a^2-ab+b^2)\cos(c+dx)}{b^3}+\frac{(a-2b)\cos^3(c+dx)}{3b^2}+\frac{\cos^5(c+dx)}{5b}}{d}$$

input
```
Int[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

output
```
-((-((a^3*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(b^(7/2)*Sqrt[a + b
])) + ((a^2 - a*b + b^2)*Cos[c + d*x])/b^3 + ((a - 2*b)*Cos[c + d*x]^3)/(3
*b^2) + Cos[c + d*x]^5/(5*b))/d)
```

3.78.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.78.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{-\frac{\frac{\left(\cos^{5}(dx+c)\right)b^{2}}{5}+\frac{ab\left(\cos^{3}(dx+c)\right)}{3}-\frac{2b^{2}\left(\cos^{3}(dx+c)\right)}{3}+\cos(dx+c)a^{2}-ab\cos(dx+c)+b^{2}\cos(dx+c)}{b^{3}}+\frac{a^{3}\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{b^{3}\sqrt{(a+b)b}}}{d}$
default	$\frac{-\frac{\frac{\left(\cos^{5}(dx+c)\right)b^{2}}{5}+\frac{ab\left(\cos^{3}(dx+c)\right)}{3}-\frac{2b^{2}\left(\cos^{3}(dx+c)\right)}{3}+\cos(dx+c)a^{2}-ab\cos(dx+c)+b^{2}\cos(dx+c)}{b^{3}}+\frac{a^{3}\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{b^{3}\sqrt{(a+b)b}}}{d}$
risch	$-\frac{e^{i(dx+c)}a^{2}}{2b^{3}d}+\frac{3a\,e^{i(dx+c)}}{8d\,b^{2}}-\frac{5\,e^{i(dx+c)}}{16bd}-\frac{e^{-i(dx+c)}a^{2}}{2b^{3}d}+\frac{3\,e^{-i(dx+c)}a}{8b^{2}d}-\frac{5\,e^{-i(dx+c)}}{16bd}-\frac{ia^{3}\ln\left(e^{2i(dx+c)}-\frac{2i}{}\right.}{2\sqrt{-ab-}}$

input
```
int(sin(d*x+c)^7/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^3*(1/5*cos(d*x+c)^5*b^2+1/3*a*b*cos(d*x+c)^3-2/3*b^2*cos(d*x+c)^
3+cos(d*x+c)*a^2-a*b*cos(d*x+c)+b^2*cos(d*x+c))+a^3/b^3/((a+b)*b)^(1/2)*ar
ctanh(b*cos(d*x+c)/((a+b)*b)^(1/2)))
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.57

$$\int \frac{\sin^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \left[-\frac{6\,(ab^3+b^4)\cos(dx+c)^5 - 15\,\sqrt{ab+b^2}a^3\log\left(\frac{b\cos(dx+c)^2+2\,\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right) + 10\,(a^2b^2-ab^3-2\,b^4)\cdots}{30\,(ab^4+b^5)d}\right.$$
$$-\frac{3\,(ab^3+b^4)\cos(dx+c)^5 + 15\,\sqrt{-ab-b^2}a^3\arctan\left(\frac{\sqrt{-ab-b^2}\cos(dx+c)}{a+b}\right) + 5\,(a^2b^2-ab^3-2\,b^4)\cos(dx+\cdots}{15\,(ab^4+b^5)d}$$

input
```
integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/30*(6*(a*b^3 + b^4)*cos(d*x + c)^5 - 15*sqrt(a*b + b^2)*a^3*log((b*cos
(d*x + c)^2 + 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 -
a - b)) + 10*(a^2*b^2 - a*b^3 - 2*b^4)*cos(d*x + c)^3 + 30*(a^3*b + b^4)*c
os(d*x + c))/((a*b^4 + b^5)*d), -1/15*(3*(a*b^3 + b^4)*cos(d*x + c)^5 + 15
*sqrt(-a*b - b^2)*a^3*arctan(sqrt(-a*b - b^2)*cos(d*x + c)/(a + b)) + 5*(a
^2*b^2 - a*b^3 - 2*b^4)*cos(d*x + c)^3 + 15*(a^3*b + b^4)*cos(d*x + c))/((
a*b^4 + b^5)*d)]
```

3.78.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**7/(a+b*sin(d*x+c)**2),x)
```

output
```
Timed out
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09

$$\int \frac{\sin^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= -\frac{\frac{15\,a^3\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}b^3}+\frac{2\left(3\,b^2\cos(dx+c)^5+5\,(ab-2\,b^2)\cos(dx+c)^3+15\,(a^2-ab+b^2)\cos(dx+c)\right)}{b^3}}{30\,d}$$

input
```
integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/30*(15*a^3*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqr
t((a + b)*b)))/(sqrt((a + b)*b)*b^3) + 2*(3*b^2*cos(d*x + c)^5 + 5*(a*b -
2*b^2)*cos(d*x + c)^3 + 15*(a^2 - a*b + b^2)*cos(d*x + c))/b^3)/d
```

3.78.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(94) = 188.

Time = 0.43 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.13

$$\int \frac{\sin^7(c+dx)}{a+b\sin^2(c+dx)}\,dx =$$
$$-\frac{\frac{15\,a^3\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}b^3}-\frac{2\left(15\,a^2-10\,ab+8\,b^2-\frac{60\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{50\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{40\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{90\,a^2(\cos(\cdots}{(\cos(\cdots}\cdots\right.}{\cdots}}{15\,d}$$

input
```
integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/15*(15*a^3*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x +
c) + sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*b^3) - 2*(15*a^2 - 10*a*b + 8*b^
2 - 60*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 50*a*b*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - 40*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 90*
a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 70*a*b*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 + 80*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
- 60*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 30*a*b*(cos(d*x + c)
- 1)^3/(cos(d*x + c) + 1)^3 + 15*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) +
1)^4)/(b^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5))/d
```

3.78.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06

$$\int \frac{\sin^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{a^3\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos(c+d\,x)}{\sqrt{a+b}}\right)}{b^{7/2}\,d\,\sqrt{a+b}} - \frac{\cos\left(c+d\,x\right)^5}{5\,b\,d} - \frac{\cos\left(c+d\,x\right)^3\,\left(\frac{a+b}{3\,b^2}-\frac{1}{b}\right)}{d} - \frac{\cos\left(c+d\,x\right)\,\left(\frac{3}{b}+\frac{(a+b)\,\left(\frac{a+b}{b^2}-\frac{3}{b}\right)}{b}\right)}{d}$$

input
```
int(sin(c + d*x)^7/(a + b*sin(c + d*x)^2),x)
```

output
```
(a^3*atanh((b^(1/2)*cos(c + d*x))/(a + b)^(1/2)))/(b^(7/2)*d*(a + b)^(1/2)
) - cos(c + d*x)^5/(5*b*d) - (cos(c + d*x)^3*((a + b)/(3*b^2) - 1/b))/d -
(cos(c + d*x)*(3/b + ((a + b)*((a + b)/b^2 - 3/b))/b))/d
```

3.79 $\int \frac{\sin^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.79.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\sin^5(c+dx)}{a+b\sin^2(c+dx)}\,dx=-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}d}+\frac{(a-b)\cos(c+dx)}{b^2d}+\frac{\cos^3(c+dx)}{3bd}$$

output
```
(a-b)*cos(d*x+c)/b^2/d+1/3*cos(d*x+c)^3/b/d-a^2*arctanh(cos(d*x+c)*b^(1/2)
/(a+b)^(1/2))/b^(5/2)/d/(a+b)^(1/2)
```

3.79.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.95

$$\int \frac{\sin^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$=\frac{6a^2\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)+6a^2\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)+\sqrt{-a-b}\sqrt{b}\cos(c+dx)(6a-5b+b}{6\sqrt{-a-b}b^{5/2}d}$$

input
```
Integrate[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

output
```
(6*a^2*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + 6*a^2
*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + Sqrt[-a - b
]*Sqrt[b]*Cos[c + d*x]*(6*a - 5*b + b*Cos[2*(c + d*x)]))/(6*Sqrt[-a - b]*b
^(5/2)*d)
```

3.79.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3665, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^5}{a+b\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{\left(1-\cos^2(c+dx)\right)^2}{-b\cos^2(c+dx)+a+b}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$-\frac{\int \left(\frac{a^2}{b^2\left(-b\cos^2(c+dx)+a+b\right)}-\frac{\cos^2(c+dx)}{b}-\frac{a-b}{b^2}\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(a-b)\cos(c+dx)}{b^2}-\frac{\cos^3(c+dx)}{3b}}{d}$$

input
```
Int[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

output
```
-(((a^2*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b])
 - ((a - b)*Cos[c + d*x])/b^2 - Cos[c + d*x]^3/(3*b))/d)
```

3.79.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.79.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{\frac{b\left(\cos^{3}(dx+c)\right)}{3}+\cos(dx+c)a-\cos(dx+c)b}{b^{2}}-\frac{a^{2}\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{b^{2}\sqrt{(a+b)b}}}{d}$
default	$\frac{\frac{\frac{b\left(\cos^{3}(dx+c)\right)}{3}+\cos(dx+c)a-\cos(dx+c)b}{b^{2}}-\frac{a^{2}\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{b^{2}\sqrt{(a+b)b}}}{d}$
risch	$\frac{a\,e^{i(dx+c)}}{2d\,b^{2}}-\frac{3\,e^{i(dx+c)}}{8bd}+\frac{e^{-i(dx+c)}a}{2b^{2}d}-\frac{3\,e^{-i(dx+c)}}{8bd}+\frac{i a^{2}\ln\left(e^{2i(dx+c)}-\frac{2i(a+b)e^{i(dx+c)}}{\sqrt{-ab-b^{2}}}+1\right)}{2\sqrt{-ab-b^{2}}\,d\,b^{2}}-\frac{i a^{2}\ln\left(e^{2i(dx+\right.}{2\sqrt{}}$

input
```
int(sin(d*x+c)^5/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^2*(1/3*b*cos(d*x+c)^3+cos(d*x+c)*a-cos(d*x+c)*b)-a^2/b^2/((a+b)*b
)^(1/2)*arctanh(b*cos(d*x+c)/((a+b)*b)^(1/2)))
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.83

$$\int \frac{\sin^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \left[\frac{2\,(ab^2+b^3)\cos{(dx+c)}^3+3\,\sqrt{ab+b^2}a^2\log\left(-\frac{b\cos(dx+c)^2-2\,\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)+6\,(a^2b-b^3)\cos{(dx+}}{6\,(ab^3+b^4)d}\right.$$

input
```
integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/6*(2*(a*b^2 + b^3)*cos(d*x + c)^3 + 3*sqrt(a*b + b^2)*a^2*log(-(b*cos(d
*x + c)^2 - 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a
- b)) + 6*(a^2*b - b^3)*cos(d*x + c))/((a*b^3 + b^4)*d), 1/3*((a*b^2 + b^3
)*cos(d*x + c)^3 + 3*sqrt(-a*b - b^2)*a^2*arctan(sqrt(-a*b - b^2)*cos(d*x
+ c)/(a + b)) + 3*(a^2*b - b^3)*cos(d*x + c))/((a*b^3 + b^4)*d)]
```

3.79.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**5/(a+b*sin(d*x+c)**2),x)
```

output
```
Timed out
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int \frac{\sin^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{3\,a^2\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}b^2}+\frac{2\left(b\cos(dx+c)^3+3\,(a-b)\cos(dx+c)\right)}{b^2}}{6\,d}$$

input
```
integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/6*(3*a^2*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt((
a + b)*b)))/(sqrt((a + b)*b)*b^2) + 2*(b*cos(d*x + c)^3 + 3*(a - b)*cos(d*
x + c))/b^2)/d
```

3.79.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(67) = 134$.

Time = 0.42 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.25

$$\int \frac{\sin^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{3\,a^2\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}b^2}-\frac{2\left(3\,a-2\,b-\frac{6\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{6\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{b^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^3}}{3\,d}$$

input
```
integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(3*a^2*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x + c)
+ sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*b^2) - 2*(3*a - 2*b - 6*a*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 3*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(b^2*((cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 1)^3))/d
```

3.79.9 Mupad [B] (verification not implemented)

Time = 13.42 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{\sin^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\cos\left(c+d\,x\right)\,\left(\frac{a+b}{b^2}-\frac{2}{b}\right)}{d}+\frac{{\cos\left(c+d\,x\right)}^3}{3\,b\,d}-\frac{a^2\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos\left(c+d\,x\right)}{\sqrt{a+b}}\right)}{b^{5/2}\,d\,\sqrt{a+b}}$$

input
```
int(sin(c + d*x)^5/(a + b*sin(c + d*x)^2),x)
```

output
```
(cos(c + d*x)*((a + b)/b^2 - 2/b))/d + cos(c + d*x)^3/(3*b*d) - (a^2*atanh
((b^(1/2)*cos(c + d*x))/(a + b)^(1/2)))/(b^(5/2)*d*(a + b)^(1/2))
```

3.80 $\int \frac{\sin^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.80.1 Optimal result

Integrand size = 23, antiderivative size = 52

$$\int \frac{\sin^3(c+dx)}{a+b\sin^2(c+dx)}\,dx=\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}d}-\frac{\cos(c+dx)}{bd}$$

output
```
-cos(d*x+c)/b/d+a*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/b^(3/2)/d/(a+b)^
(1/2)
```

3.80.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.40

$$\int \frac{\sin^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{a\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)+a\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)+\sqrt{-a-b}\sqrt{b}\cos(c+dx)}{\sqrt{-a-b}b^{3/2}d}$$

input
```
Integrate[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

output
```
-((a*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + a*ArcTa
n[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + Sqrt[-a - b]*Sqrt
[b]*Cos[c + d*x])/(Sqrt[-a - b]*b^(3/2)*d))
```

3.80.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3665, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(c+dx)^3}{a+b\sin(c+dx)^2}dx$$

$$\Big\downarrow \; 3665$$

$$-\frac{\int \frac{1-\cos^2(c+dx)}{-b\cos^2(c+dx)+a+b}d\cos(c+dx)}{d}$$

$$\Big\downarrow \; 299$$

$$-\frac{\frac{\cos(c+dx)}{b}-\frac{a\int \frac{1}{-b\cos^2(c+dx)+a+b}d\cos(c+dx)}{b}}{d}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{\cos(c+dx)}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}}{d}$$

input
```
Int[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

output
```
-((-((a*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b])
) + Cos[c + d*x]/b)/d)
```

3.80.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.80.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{-\frac{\cos(dx+c)}{b}+\frac{a \operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{b\sqrt{(a+b)b}}}{d}$	45
default	$\frac{-\frac{\cos(dx+c)}{b}+\frac{a \operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{b\sqrt{(a+b)b}}}{d}$	45
risch	$-\frac{e^{i(dx+c)}}{2bd}-\frac{e^{-i(dx+c)}}{2bd}-\frac{ia\ln\left(e^{2i(dx+c)}-\frac{2i(a+b)e^{i(dx+c)}}{\sqrt{-ab-b^2}}+1\right)}{2\sqrt{-ab-b^2}\,db}+\frac{ia\ln\left(e^{2i(dx+c)}+\frac{2i(a+b)e^{i(dx+c)}}{\sqrt{-ab-b^2}}+1\right)}{2\sqrt{-ab-b^2}\,db}$	158

input
```
int(sin(d*x+c)^3/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-cos(d*x+c)/b+1/b*a/((a+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/((a+b)*b)^(1
/2)))
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.17

$$\int \frac{\sin^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = \left[\frac{\sqrt{ab+b^2}a\log\left(\frac{b\cos(dx+c)^2+2\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)-2\left(ab+b^2\right)\cos\left(dx+c\right)}{2\left(ab^2+b^3\right)d}, -\frac{\sqrt{-ab-b^2}a\arctan\left(\frac{\sqrt{-ab-b^2}\cos(dx+c)}{a+b}\right)+\left(ab+b^2\right)\cos\left(dx+c\right)}{\left(ab^2+b^3\right)d}\right]$$

input
```
integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a*b + b^2)*a*log((b*cos(d*x + c)^2 + 2*sqrt(a*b + b^2)*cos(d*x
+ c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - 2*(a*b + b^2)*cos(d*x + c))/((
a*b^2 + b^3)*d), -(sqrt(-a*b - b^2)*a*arctan(sqrt(-a*b - b^2)*cos(d*x + c)
/(a + b)) + (a*b + b^2)*cos(d*x + c))/((a*b^2 + b^3)*d)]
```

3.80.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)**2),x)
```

output
```
Timed out
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.29

$$\int \frac{\sin^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{a\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}b}+\frac{2\cos(dx+c)}{b}}{2\,d}$$

input
```
integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/2*(a*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt((a +
 b)*b)))/(sqrt((a + b)*b)*b) + 2*cos(d*x + c)/b)/d
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.10

$$\int \frac{\sin^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{a\arctan\left(\frac{b\cos(dx+c)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}bd}-\frac{\cos(dx+c)}{bd}$$

input
```
integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-a*arctan(b*cos(d*x + c)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*b*d) - cos(d*
x + c)/(b*d)
```

3.80.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\sin^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos(c+d\,x)}{\sqrt{a+b}}\right)}{b^{3/2}\,d\,\sqrt{a+b}}-\frac{\cos(c+d\,x)}{b\,d}$$

input
```
int(sin(c + d*x)^3/(a + b*sin(c + d*x)^2),x)
```

output
```
(a*atanh((b^(1/2)*cos(c + d*x))/(a + b)^(1/2)))/(b^(3/2)*d*(a + b)^(1/2))
- cos(c + d*x)/(b*d)
```

3.81 $\int \frac{\sin(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.81.1 Optimal result

Integrand size = 21, antiderivative size = 37

$$\int \frac{\sin(c+dx)}{a+b\sin^2(c+dx)}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}d}$$

output
```
-arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/d/b^(1/2)/(a+b)^(1/2)
```

3.81.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.62

$$\int \frac{\sin(c+dx)}{a+b\sin^2(c+dx)}\,dx=\frac{\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)+\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}\sqrt{b}d}$$

input
```
Integrate[Sin[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

output
```
(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + ArcTan[(Sqr
t[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]])/(Sqrt[-a - b]*Sqrt[b]*d)
```

3.81.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3665, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin(c+dx)}{a+b\sin(c+dx)^2}dx$$
$$\Big\downarrow \; 3665$$
$$-\frac{\int \frac{1}{-b\cos^2(c+dx)+a+b}d\cos(c+dx)}{d}$$
$$\Big\downarrow \; 221$$
$$-\frac{\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}d\sqrt{a+b}}$$

input
```
Int[Sin[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

output
```
-(ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d))
```

3.81.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.81.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{d\sqrt{(a+b)b}}$	29
default	$-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{d\sqrt{(a+b)b}}$	29
risch	$\frac{i\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{2i(a+b)\mathrm{e}^{i(dx+c)}}{\sqrt{-ab-b^2}}+1\right)}{2\sqrt{-ab-b^2}\,d}-\frac{i\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2i(a+b)\mathrm{e}^{i(dx+c)}}{\sqrt{-ab-b^2}}+1\right)}{2\sqrt{-ab-b^2}\,d}$	116

input
```
int(sin(d*x+c)/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/((a+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/((a+b)*b)^(1/2))
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.16

$$\int \frac{\sin(c+dx)}{a+b\sin^2(c+dx)}\,dx = \left[\frac{\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)}{2\sqrt{ab+b^2}d}, \frac{\sqrt{-ab-b^2}\arctan\left(\frac{\sqrt{-ab-b^2}\cos(dx+c)}{a+b}\right)}{(ab+b^2)d}\right]$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*log(-(b*cos(d*x + c)^2 - 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*c
os(d*x + c)^2 - a - b))/(sqrt(a*b + b^2)*d), sqrt(-a*b - b^2)*arctan(sqrt(
-a*b - b^2)*cos(d*x + c)/(a + b))/((a*b + b^2)*d)]
```

3.81.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367693 vs. $2(34) = 68$.

Time = 76.85 (sec) , antiderivative size = 367693, normalized size of antiderivative = 9937.65

$$\int \frac{\sin(c+dx)}{a+b\sin^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(tan(c/2 + d
*x/2))/(b*d), Eq(a, 0)), (2/(b*d*tan(c/2 + d*x/2)**2 - b*d), Eq(a, -b)), (
-cos(c + d*x)/(a*d), Eq(b, 0)), (x*sin(c)/(a + b*sin(c)**2), Eq(d, 0)), (7
4*a**37*b*log(-sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2))
/(2*a**38*b*d + 5478*a**37*b**2*d - 148*a**37*b*d*sqrt(a*b + b**2) + 25025
32*a**36*b**3*d - 135124*a**36*b**2*d*sqrt(a*b + b**2) + 456961248*a**35*b
**4*d - 36983424*a**35*b**3*d*sqrt(a*b + b**2) + 44602414272*a**34*b**5*d
- 4809599808*a**34*b**4*d*sqrt(a*b + b**2) + 2698911348224*a**33*b**6*d -
363524561920*a**33*b**5*d*sqrt(a*b + b**2) + 110776036340736*a**32*b**7*d
- 17891931206656*a**32*b**6*d*sqrt(a*b + b**2) + 3275718126403584*a**31*b*
*8*d - 616808259780608*a**31*b**7*d*sqrt(a*b + b**2) + 72854727629602816*a
**30*b**9*d - 15666762815766528*a**30*b**8*d*sqrt(a*b + b**2) + 1258467596
957384704*a**29*b**10*d - 304230303833522176*a**29*b**9*d*sqrt(a*b + b**2)
 + 17306140891880620032*a**28*b**11*d - 4645206174395269120*a**28*b**10*d*
sqrt(a*b + b**2) + 193199008739227598848*a**27*b**12*d - 57001938802859573
248*a**27*b**11*d*sqrt(a*b + b**2) + 1778515685235870400512*a**26*b**13*d
- 572029907419376123904*a**26*b**12*d*sqrt(a*b + b**2) + 13673782930644613
988352*a**25*b**14*d - 4761020109769125396480*a**25*b**13*d*sqrt(a*b + b**
2) + 88722183139577965838336*a**24*b**15*d - 33244276082712682430464*a**24
*b**14*d*sqrt(a*b + b**2) + 490030319626953299066880*a**23*b**16*d - 19...
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \frac{\sin(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}d}$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt((a + b)*
b)))/(sqrt((a + b)*b)*d)
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\sin(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\arctan\left(\frac{b\cos(dx+c)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}d}$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
arctan(b*cos(d*x + c)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*d)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sin(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,d\,\sqrt{a+b}}$$

input
```
int(sin(c + d*x)/(a + b*sin(c + d*x)^2),x)
```

output
```
-atanh((b^(1/2)*cos(c + d*x))/(a + b)^(1/2))/(b^(1/2)*d*(a + b)^(1/2))
```

3.82 $\int \frac{\csc(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.82.1 Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\csc(c+dx)}{a+b\sin^2(c+dx)}\,dx=-\frac{\operatorname{arctanh}(\cos(c+dx))}{ad}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}d}$$

output
```
-arctanh(cos(d*x+c))/a/d+arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a
/d/(a+b)^(1/2)
```

3.82.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.60

$$\int \frac{\csc(c+dx)}{a+b\sin^2(c+dx)}\,dx=-\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

input
```
Integrate[Csc[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

output
```
-(((Sqrt[b]*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]])/S
qrt[-a - b] + (Sqrt[b]*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[
-a - b]])/Sqrt[-a - b] + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])/(a
*d))
```

3.82.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3665, 303, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(c+dx)\left(a+b\sin(c+dx)^2\right)}dx$$

$$\Big\downarrow \ 3665$$

$$-\frac{\int \frac{1}{(1-\cos^2(c+dx))(-b\cos^2(c+dx)+a+b)}d\cos(c+dx)}{d}$$

$$\Big\downarrow \ 303$$

$$-\frac{\frac{\int \frac{1}{1-\cos^2(c+dx)}d\cos(c+dx)}{a}-\frac{b\int \frac{1}{-b\cos^2(c+dx)+a+b}d\cos(c+dx)}{a}}{d}$$

$$\Big\downarrow \ 219$$

$$-\frac{\frac{\text{arctanh}(\cos(c+dx))}{a}-\frac{b\int \frac{1}{-b\cos^2(c+dx)+a+b}d\cos(c+dx)}{a}}{d}$$

$$\Big\downarrow \ 221$$

$$-\frac{\frac{\text{arctanh}(\cos(c+dx))}{a}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{d}$$

input
```
Int[Csc[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

output
```
-((ArcTanh[Cos[c + d*x]]/a - (Sqrt[b]*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[
a + b]])/(a*Sqrt[a + b]))/d)
```

3.82.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.82.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{b\,\mathrm{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}-\frac{\ln(1+\cos(dx+c))}{2a}+\frac{\ln(\cos(dx+c)-1)}{2a}}{d}$
default	$\frac{\frac{b\,\mathrm{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}-\frac{\ln(1+\cos(dx+c))}{2a}+\frac{\ln(\cos(dx+c)-1)}{2a}}{d}$
risch	$\frac{\ln(\mathrm{e}^{i(dx+c)}-1)}{da}-\frac{\ln(\mathrm{e}^{i(dx+c)}+1)}{da}+\frac{i\sqrt{-(a+b)b}\,\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2i\sqrt{-(a+b)b}\,\mathrm{e}^{i(dx+c)}}{b}+1\right)}{2(a+b)da}-\frac{i\sqrt{-(a+b)b}\,\ln\left(\mathrm{e}^{2i(d}\right.}{}$

input
```
int(csc(d*x+c)/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a*b/((a+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/((a+b)*b)^(1/2))-1/2/a*ln(
1+cos(d*x+c))+1/2/a*ln(cos(d*x+c)-1))
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.93

$$\int \frac{\csc(c+dx)}{a+b\sin^2(c+dx)}\,dx = \left[\frac{\sqrt{\frac{b}{a+b}}\log\left(\frac{b\cos(dx+c)^2+2\,(a+b)\sqrt{\frac{b}{a+b}}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)-\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,ad}, -\frac{2\,\sqrt{-\frac{b}{a+b}}\arctan\left(\sqrt{-\frac{b}{a+b}}\cos\left(dx+c\right)\right)+\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,ad}\right]$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b/(a + b))*log((b*cos(d*x + c)^2 + 2*(a + b)*sqrt(b/(a + b))*co
s(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - log(1/2*cos(d*x + c) + 1
/2) + log(-1/2*cos(d*x + c) + 1/2))/(a*d), -1/2*(2*sqrt(-b/(a + b))*arctan
(sqrt(-b/(a + b))*cos(d*x + c)) + log(1/2*cos(d*x + c) + 1/2) - log(-1/2*c
os(d*x + c) + 1/2))/(a*d)]
```

3.82.6 Sympy [F]

$$\int \frac{\csc(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\csc{(c+dx)}}{a+b\sin^2{(c+dx)}}\,dx$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(csc(c + d*x)/(a + b*sin(c + d*x)**2), x)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \frac{\csc(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{b\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a} + \frac{\log(\cos(dx+c)+1)}{a} - \frac{\log(\cos(dx+c)-1)}{a}}{2\,d}$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/2*(b*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt((a +
 b)*b)))/(sqrt((a + b)*b)*a) + log(cos(d*x + c) + 1)/a - log(cos(d*x + c)
- 1)/a)/d
```

3.82.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(47) = 94$.

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.82

$$\int \frac{\csc(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{2\,b\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}a} - \frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}}{2\,d}$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/2*(2*b*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x + c) +
 sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*a) - log(abs(-cos(d*x + c) + 1)/abs(
cos(d*x + c) + 1))/a)/d
```

3.82.9 Mupad [B] (verification not implemented)

Time = 14.66 (sec) , antiderivative size = 457, normalized size of antiderivative = 8.31

$$\int \frac{\csc(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\operatorname{atanh}(\cos(c+d\,x))}{a\,d} - \frac{\operatorname{atan}\left(\frac{\frac{\left(2\,b^3\,\cos(c+d\,x)+\frac{\left(2\,a^2\,b^2-\frac{\cos(c+d\,x)\,\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\,\sqrt{b\,(a+b)}}{4\,\left(a^2+b\,a\right)}\right)\,\sqrt{b\,(a+b)}}{2\,\left(a^2+b\,a\right)}\right)\,\sqrt{b\,(a+b)}\,1i}{a^2+b\,a}+\frac{\left(2\,b^3\,\cos(c+d\,x)-\frac{\left(2\,a^2\,b^2+\frac{\cos(c+d\,x)\,\left(8\,a^3\,b^2\cdots\right.}{4\,\left(a^2\cdots\right.}\right.}{2\,\left(a\cdots\right.}\right.}{a^2+\cdots}}{\frac{\left(2\,b^3\,\cos(c+d\,x)+\frac{\left(2\,a^2\,b^2-\frac{\cos(c+d\,x)\,\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\,\sqrt{b\,(a+b)}}{4\,\left(a^2+b\,a\right)}\right)\,\sqrt{b\,(a+b)}}{2\,\left(a^2+b\,a\right)}\right)\,\sqrt{b\,(a+b)}}{a^2+b\,a}-\frac{\left(2\,b^3\,\cos(c+d\,x)-\frac{\left(2\,a^2\,b^2+\frac{\cos(c+d\,x)\,\left(8\,a^3\,b^2\cdots\right.}{4\,\left(a^2\cdots\right.}\right.}{2\,\left(a\cdots\right.}\right.}{a^2+b\cdots}}\right.}{d\,\left(a^2+b\,a\right)}$$

input
```
int(1/(sin(c + d*x)*(a + b*sin(c + d*x)^2)),x)
```

output
```
- atanh(cos(c + d*x))/(a*d) - (atan((((2*b^3*cos(c + d*x) + ((2*a^2*b^2 -
(cos(c + d*x)*(16*a^2*b^3 + 8*a^3*b^2)*(b*(a + b))^(1/2))/(4*(a*b + a^2)))
*(b*(a + b))^(1/2))/(2*(a*b + a^2)))*(b*(a + b))^(1/2)*1i)/(a*b + a^2) + (
(2*b^3*cos(c + d*x) - ((2*a^2*b^2 + (cos(c + d*x)*(16*a^2*b^3 + 8*a^3*b^2)
*(b*(a + b))^(1/2))/(4*(a*b + a^2)))*(b*(a + b))^(1/2))/(2*(a*b + a^2)))*(
b*(a + b))^(1/2)*1i)/(a*b + a^2))/(((2*b^3*cos(c + d*x) + ((2*a^2*b^2 - (c
os(c + d*x)*(16*a^2*b^3 + 8*a^3*b^2)*(b*(a + b))^(1/2))/(4*(a*b + a^2)))*(
b*(a + b))^(1/2))/(2*(a*b + a^2)))*(b*(a + b))^(1/2))/(a*b + a^2) - ((2*b^
3*cos(c + d*x) - ((2*a^2*b^2 + (cos(c + d*x)*(16*a^2*b^3 + 8*a^3*b^2)*(b*(
a + b))^(1/2))/(4*(a*b + a^2)))*(b*(a + b))^(1/2))/(2*(a*b + a^2)))*(b*(a
+ b))^(1/2))/(a*b + a^2)))*(b*(a + b))^(1/2)*1i)/(d*(a*b + a^2))
```

3.82. $\int \frac{\csc(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.83 $\int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.83.1 Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{(a-2b)\text{arctanh}(\cos(c+dx))}{2a^2d} - \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

output
```
-1/2*(a-2*b)*arctanh(cos(d*x+c))/a^2/d-1/2*cot(d*x+c)*csc(d*x+c)/a/d-b^(3/
2)*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/a^2/d/(a+b)^(1/2)
```

3.83.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.82 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.64

$$\int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{(2a+b-b\cos(2(c+dx)))\csc^2(c+dx)\left(-8b^{3/2}\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)-8b^{3/2}\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tan(}{\sqrt{-a}}\right.\right.}{16a^2\sqrt{-}}$$

input
```
Integrate[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

output
```
-1/16*((2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*(-8*b^(3/2)*ArcTan[(S
qrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] - 8*b^(3/2)*ArcTan[(Sqr
t[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + Sqrt[-a - b]*(a*Csc[(c
+ d*x)/2]^2 + 4*(a - 2*b)*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])
- a*Sec[(c + d*x)/2]^2)))/(a^2*Sqrt[-a - b]*d*(b + a*Csc[c + d*x]^2))
```

3.83.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3665, 316, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(c+dx)^3\left(a+b\sin(c+dx)^2\right)}dx$$

$$\Big\downarrow \ 3665$$

$$-\frac{\int \frac{1}{\left(1-\cos^2(c+dx)\right)^2\left(-b\cos^2(c+dx)+a+b\right)}d\cos(c+dx)}{d}$$

$$\Big\downarrow \ 316$$

$$-\frac{\frac{\int \frac{-b\cos^2(c+dx)+a-b}{\left(1-\cos^2(c+dx)\right)\left(-b\cos^2(c+dx)+a+b\right)}d\cos(c+dx)}{2a}+\frac{\cos(c+dx)}{2a\left(1-\cos^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \ 397$$

$$-\frac{\frac{\frac{2b^2\int \frac{1}{-b\cos^2(c+dx)+a+b}d\cos(c+dx)}{a}+\frac{(a-2b)\int \frac{1}{1-\cos^2(c+dx)}d\cos(c+dx)}{a}}{2a}+\frac{\cos(c+dx)}{2a\left(1-\cos^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \ 219$$

$$-\frac{\frac{\frac{2b^2\int \frac{1}{-b\cos^2(c+dx)+a+b}d\cos(c+dx)}{a}+\frac{(a-2b)\text{arctanh}(\cos(c+dx))}{a}}{2a}+\frac{\cos(c+dx)}{2a\left(1-\cos^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \ 221$$

$$-\frac{\frac{\frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}+\frac{(a-2b)\text{arctanh}(\cos(c+dx))}{a}}{2a}+\frac{\cos(c+dx)}{2a(1-\cos^2(c+dx))}}{d}$$

input
```
Int[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

output
```
-(((((a - 2*b)*ArcTanh[Cos[c + d*x]])/a + (2*b^(3/2)*ArcTanh[(Sqrt[b]*Cos[
c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a) + Cos[c + d*x]/(2*a*(1 - Co
s[c + d*x]^2)))/d)
```

3.83.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.83.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{\frac{1}{4a(1+\cos(dx+c))}+\frac{(-a+2b)\ln(1+\cos(dx+c))}{4a^2}+\frac{1}{4a(\cos(dx+c)-1)}+\frac{(a-2b)\ln(\cos(dx+c)-1)}{4a^2}-\frac{b^2\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{a^2\sqrt{(a+b)b}}}{d}$
default	$\frac{\frac{1}{4a(1+\cos(dx+c))}+\frac{(-a+2b)\ln(1+\cos(dx+c))}{4a^2}+\frac{1}{4a(\cos(dx+c)-1)}+\frac{(a-2b)\ln(\cos(dx+c)-1)}{4a^2}-\frac{b^2\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{a^2\sqrt{(a+b)b}}}{d}$
risch	$\frac{e^{3i(dx+c)}+e^{i(dx+c)}}{da\left(e^{2i(dx+c)}-1\right)^2}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{2da}+\frac{b\ln\left(e^{i(dx+c)}+1\right)}{a^2d}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2da}-\frac{b\ln\left(e^{i(dx+c)}-1\right)}{a^2d}-\frac{i\sqrt{-(a+b)b}}{}$

input
```
int(csc(d*x+c)^3/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4/a/(1+cos(d*x+c))+1/4/a^2*(-a+2*b)*ln(1+cos(d*x+c))+1/4/a/(cos(d*x
+c)-1)+1/4*(a-2*b)/a^2*ln(cos(d*x+c)-1)-b^2/a^2/((a+b)*b)^(1/2)*arctanh(b*
cos(d*x+c)/((a+b)*b)^(1/2)))
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.85

$$\int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \left[\frac{2\left(b\cos\left(dx+c\right)^2-b\right)\sqrt{\frac{b}{a+b}}\log\left(-\frac{b\cos(dx+c)^2-2\,(a+b)\sqrt{\frac{b}{a+b}}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)+2\,a\cos\left(dx+c\right)-\left((a-2\,b)\,\mathrm{co}\right.}{4\left(a^2d\cos\right.}\right.$$

input
```
integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/4*(2*(b*cos(d*x + c)^2 - b)*sqrt(b/(a + b))*log(-(b*cos(d*x + c)^2 - 2*
(a + b)*sqrt(b/(a + b))*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b))
+ 2*a*cos(d*x + c) - ((a - 2*b)*cos(d*x + c)^2 - a + 2*b)*log(1/2*cos(d*x
+ c) + 1/2) + ((a - 2*b)*cos(d*x + c)^2 - a + 2*b)*log(-1/2*cos(d*x + c) +
 1/2))/(a^2*d*cos(d*x + c)^2 - a^2*d), 1/4*(4*(b*cos(d*x + c)^2 - b)*sqrt(
-b/(a + b))*arctan(sqrt(-b/(a + b))*cos(d*x + c)) + 2*a*cos(d*x + c) - ((a
 - 2*b)*cos(d*x + c)^2 - a + 2*b)*log(1/2*cos(d*x + c) + 1/2) + ((a - 2*b)
*cos(d*x + c)^2 - a + 2*b)*log(-1/2*cos(d*x + c) + 1/2))/(a^2*d*cos(d*x +
c)^2 - a^2*d)]
```

3.83.6 Sympy [F]

$$\int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**3/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(csc(c + d*x)**3/(a + b*sin(c + d*x)**2), x)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.41

$$\int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{2\,b^2\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^2} + \frac{2\,\cos(dx+c)}{a\cos(dx+c)^2-a} - \frac{(a-2\,b)\log(\cos(dx+c)+1)}{a^2} + \frac{(a-2\,b)\log(\cos(dx+c)-1)}{a^2}}{4\,d}$$

input
```
integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/4*(2*b^2*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt((
a + b)*b)))/(sqrt((a + b)*b)*a^2) + 2*cos(d*x + c)/(a*cos(d*x + c)^2 - a)
- (a - 2*b)*log(cos(d*x + c) + 1)/a^2 + (a - 2*b)*log(cos(d*x + c) - 1)/a^
2)/d
```

3.83.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(73) = 146$.

Time = 0.41 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.31

$$\int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{8\,b^2\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}a^2} + \frac{2\,(a-2\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2} + \frac{\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{a^2(\cos(dx+c)-1)} -}{8\,d}$$

input
```
integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/8*(8*b^2*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x + c)
+ sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*a^2) + 2*(a - 2*b)*log(abs(-cos(d*x
 + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + (a - 2*a*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) +
1)/(a^2*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/(a*(cos(d*x + c) + 1)))/d
```

3.83.9 Mupad [B] (verification not implemented)

Time = 13.81 (sec) , antiderivative size = 592, normalized size of antiderivative = 6.96

$$\int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{a\left(b\,\cos\left(c+d\,x\right)-b\,\mathrm{atanh}(\cos\left(c+d\,x\right))+b\cos\left(c+d\,x\right)^2\mathrm{atanh}(\cos\left(c+d\,x\right))\right)+a^2\left(\cos\left(c+d\,x\right)+a\right.}{}$$

input
```
int(1/(sin(c + d*x)^3*(a + b*sin(c + d*x)^2)),x)
```

output
```
-(a*(b*cos(c + d*x) - b*atanh(cos(c + d*x)) + b*cos(c + d*x)^2*atanh(cos(c
 + d*x))) + a^2*(cos(c + d*x) + atanh(cos(c + d*x)) - cos(c + d*x)^2*atanh
(cos(c + d*x))) - 2*b^2*atanh(cos(c + d*x)) + atan((b^5*cos(c + d*x)*(a*b^
3 + b^4)^(1/2)*8i - b*cos(c + d*x)*(a*b^3 + b^4)^(3/2)*8i - a*cos(c + d*x)
*(a*b^3 + b^4)^(3/2)*4i + a^2*b^3*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*1i - a^
3*b^2*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*2i + a*b^4*cos(c + d*x)*(a*b^3 + b^
4)^(1/2)*12i + a^4*b*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*1i)/(3*a^2*b^5 + 5*a
^3*b^4 + a^4*b^3 - a^5*b^2))*(a*b^3 + b^4)^(1/2)*2i + 2*b^2*cos(c + d*x)^2
*atanh(cos(c + d*x)) - cos(c + d*x)^2*atan((b^5*cos(c + d*x)*(a*b^3 + b^4)
^(1/2)*8i - b*cos(c + d*x)*(a*b^3 + b^4)^(3/2)*8i - a*cos(c + d*x)*(a*b^3
+ b^4)^(3/2)*4i + a^2*b^3*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*1i - a^3*b^2*co
s(c + d*x)*(a*b^3 + b^4)^(1/2)*2i + a*b^4*cos(c + d*x)*(a*b^3 + b^4)^(1/2)
*12i + a^4*b*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*1i)/(3*a^2*b^5 + 5*a^3*b^4 +
 a^4*b^3 - a^5*b^2))*(a*b^3 + b^4)^(1/2)*2i)/(d*(2*a^2*b + 2*a^3 - 2*a^3*c
os(c + d*x)^2 - 2*a^2*b*cos(c + d*x)^2))
```

3.83. $\int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.84 $\int \frac{\csc^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.84.1 Optimal result

Integrand size = 23, antiderivative size = 125

$$\int \frac{\csc^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{(3a^2-4ab+8b^2)\,\text{arctanh}(\cos(c+dx))}{8a^3d} + \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}d} - \frac{(3a-4b)\cot(c+dx)\csc(c+dx)}{8a^2d} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad}$$

output
```
-1/8*(3*a^2-4*a*b+8*b^2)*arctanh(cos(d*x+c))/a^3/d-1/8*(3*a-4*b)*cot(d*x+c
)*csc(d*x+c)/a^2/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d+b^(5/2)*arctanh(cos(d*x
+c)*b^(1/2)/(a+b)^(1/2))/a^3/d/(a+b)^(1/2)
```

3.84.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.63 (sec) , antiderivative size = 657, normalized size of antiderivative = 5.26

$$
\begin{aligned}
&\int \frac{\csc^5(c+dx)}{a+b\sin^2(c+dx)}\,dx \\
&= \frac{b^{5/2}\arctan\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{b}\cos\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-a-b}}\right)(-2a-b+b\cos(2(c+dx)))\csc^2(c+dx)}{2a^3\sqrt{-a-b}d\,(b+a\csc^2(c+dx))} \\
&\quad+\frac{b^{5/2}\arctan\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{b}\cos\left(\frac{1}{2}(c+dx)\right)+i\sqrt{a}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-a-b}}\right)(-2a-b+b\cos(2(c+dx)))\csc^2(c+dx)}{2a^3\sqrt{-a-b}d\,(b+a\csc^2(c+dx))} \\
&\quad+\frac{(3a-4b)(-2a-b+b\cos(2(c+dx)))\csc^2\left(\frac{1}{2}(c+dx)\right)\csc^2(c+dx)}{64a^2d\,(b+a\csc^2(c+dx))} \\
&\quad+\frac{(-2a-b+b\cos(2(c+dx)))\csc^4\left(\frac{1}{2}(c+dx)\right)\csc^2(c+dx)}{128ad\,(b+a\csc^2(c+dx))} \\
&\quad+\frac{(3a^2-4ab+8b^2)(-2a-b+b\cos(2(c+dx)))\csc^2(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16a^3d\,(b+a\csc^2(c+dx))} \\
&\quad+\frac{(-3a^2+4ab-8b^2)(-2a-b+b\cos(2(c+dx)))\csc^2(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16a^3d\,(b+a\csc^2(c+dx))} \\
&\quad+\frac{(-3a+4b)(-2a-b+b\cos(2(c+dx)))\csc^2(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}{64a^2d\,(b+a\csc^2(c+dx))} \\
&\quad-\frac{(-2a-b+b\cos(2(c+dx)))\csc^2(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)}{128ad\,(b+a\csc^2(c+dx))}
\end{aligned}
$$

input
```
Integrate[Csc[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

output
```
(b^(5/2)*ArcTan[(Sec[(c + d*x)/2]*(Sqrt[b]*Cos[(c + d*x)/2] - I*Sqrt[a]*Si
n[(c + d*x)/2]))/Sqrt[-a - b]]*(-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x
]^2)/(2*a^3*Sqrt[-a - b]*d*(b + a*Csc[c + d*x]^2)) + (b^(5/2)*ArcTan[(Sec[
(c + d*x)/2]*(Sqrt[b]*Cos[(c + d*x)/2] + I*Sqrt[a]*Sin[(c + d*x)/2]))/Sqrt
[-a - b]]*(-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^2)/(2*a^3*Sqrt[-a -
 b]*d*(b + a*Csc[c + d*x]^2)) + ((3*a - 4*b)*(-2*a - b + b*Cos[2*(c + d*x)
])*Csc[(c + d*x)/2]^2*Csc[c + d*x]^2)/(64*a^2*d*(b + a*Csc[c + d*x]^2)) +
((-2*a - b + b*Cos[2*(c + d*x)])*Csc[(c + d*x)/2]^4*Csc[c + d*x]^2)/(128*a
*d*(b + a*Csc[c + d*x]^2)) + ((3*a^2 - 4*a*b + 8*b^2)*(-2*a - b + b*Cos[2*
(c + d*x)])*Csc[c + d*x]^2*Log[Cos[(c + d*x)/2]])/(16*a^3*d*(b + a*Csc[c +
 d*x]^2)) + ((-3*a^2 + 4*a*b - 8*b^2)*(-2*a - b + b*Cos[2*(c + d*x)])*Csc[
c + d*x]^2*Log[Sin[(c + d*x)/2]])/(16*a^3*d*(b + a*Csc[c + d*x]^2)) + ((-3
*a + 4*b)*(-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*Sec[(c + d*x)/2]^
2)/(64*a^2*d*(b + a*Csc[c + d*x]^2)) - ((-2*a - b + b*Cos[2*(c + d*x)])*Cs
c[c + d*x]^2*Sec[(c + d*x)/2]^4)/(128*a*d*(b + a*Csc[c + d*x]^2))
```

3.84.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 3665, 316, 402, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(c+dx)^5\left(a+b\sin(c+dx)^2\right)}dx$$

$$\Big\downarrow \; 3665$$

$$-\frac{\int \frac{1}{(1-\cos^2(c+dx))^3(-b\cos^2(c+dx)+a+b)}d\cos(c+dx)}{d}$$

$$\Big\downarrow \; 316$$

$$-\frac{\frac{\int \frac{-3b\cos^2(c+dx)+3a-b}{\left(1-\cos^2(c+dx)\right)^2\left(-b\cos^2(c+dx)+a+b\right)}d\cos(c+dx)}{4a}+\frac{\cos(c+dx)}{4a(1-\cos^2(c+dx))^2}}{d}$$

$$\Big\downarrow \; 402$$

$$-\frac{\frac{\frac{\int \frac{3a^2-ba+4b^2-(3a-4b)b\cos^2(c+dx)}{(1-\cos^2(c+dx))(-b\cos^2(c+dx)+a+b)}d\cos(c+dx)}{2a}+\frac{(3a-4b)\cos(c+dx)}{2a(1-\cos^2(c+dx))}}{4a}+\frac{\cos(c+dx)}{4a(1-\cos^2(c+dx))^2}}{d}$$

$$\Big\downarrow \; 397$$

$$-\frac{\frac{\frac{\frac{(3a^2-4ab+8b^2)\int \frac{1}{1-\cos^2(c+dx)}d\cos(c+dx)}{a}-\frac{8b^3\int \frac{1}{-b\cos^2(c+dx)+a+b}d\cos(c+dx)}{a}}{2a}+\frac{(3a-4b)\cos(c+dx)}{2a(1-\cos^2(c+dx))}}{4a}+\frac{\cos(c+dx)}{4a(1-\cos^2(c+dx))^2}}{d}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{\frac{\frac{(3a^2-4ab+8b^2)\text{arctanh}(\cos(c+dx))}{a}-\frac{8b^3\int \frac{1}{-b\cos^2(c+dx)+a+b}d\cos(c+dx)}{a}}{2a}+\frac{(3a-4b)\cos(c+dx)}{2a(1-\cos^2(c+dx))}}{4a}+\frac{\cos(c+dx)}{4a(1-\cos^2(c+dx))^2}}{d}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{\frac{\frac{(3a^2-4ab+8b^2)\text{arctanh}(\cos(c+dx))}{a}-\frac{8b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a}+\frac{(3a-4b)\cos(c+dx)}{2a(1-\cos^2(c+dx))}}{4a}+\frac{\cos(c+dx)}{4a(1-\cos^2(c+dx))^2}}{d}$$

input
```
Int[Csc[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

output
```
-((Cos[c + d*x]/(4*a*(1 - Cos[c + d*x]^2)^2) + ((((3*a^2 - 4*a*b + 8*b^2)*
ArcTanh[Cos[c + d*x]])/a - (8*b^(5/2)*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[
a + b]])/(a*Sqrt[a + b]))/(2*a) + ((3*a - 4*b)*Cos[c + d*x])/(2*a*(1 - Cos
[c + d*x]^2)))/(4*a))/d)
```

3.84.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.84. $\int \frac{\csc^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.84.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{\frac{b^3 \operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{a^3\sqrt{(a+b)b}}+\frac{1}{16a(1+\cos(dx+c))^2}-\frac{-3a+4b}{16a^2(1+\cos(dx+c))}+\frac{(-3a^2+4ab-8b^2)\ln(1+\cos(dx+c))}{16a^3}-\frac{1}{16a(\cos(dx+c)-1)^2}}{d}$
default	$\frac{\frac{b^3 \operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{a^3\sqrt{(a+b)b}}+\frac{1}{16a(1+\cos(dx+c))^2}-\frac{-3a+4b}{16a^2(1+\cos(dx+c))}+\frac{(-3a^2+4ab-8b^2)\ln(1+\cos(dx+c))}{16a^3}-\frac{1}{16a(\cos(dx+c)-1)^2}}{d}$
risch	$\frac{3a\,e^{7i(dx+c)}-4b\,e^{7i(dx+c)}-11a\,e^{5i(dx+c)}+4b\,e^{5i(dx+c)}-11a\,e^{3i(dx+c)}+4b\,e^{3i(dx+c)}+3a\,e^{i(dx+c)}-4b\,e^{i(dx+c)}}{4d\,a^2\left(e^{2i(dx+c)}-1\right)^4}+\frac{3\ln(e}{}$

input
```
int(csc(d*x+c)^5/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3/a^3/((a+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/((a+b)*b)^(1/2))+1/16/a/
(1+cos(d*x+c))^2-1/16*(-3*a+4*b)/a^2/(1+cos(d*x+c))+1/16/a^3*(-3*a^2+4*a*b
-8*b^2)*ln(1+cos(d*x+c))-1/16/a/(cos(d*x+c)-1)^2-1/16*(-3*a+4*b)/a^2/(cos(
d*x+c)-1)+1/16*(3*a^2-4*a*b+8*b^2)/a^3*ln(cos(d*x+c)-1))
```

3.84.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(111) = 222$.

Time = 0.34 (sec) , antiderivative size = 612, normalized size of antiderivative = 4.90

$$\int \frac{\csc^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \left[\frac{2\,(3\,a^2-4\,ab)\cos(dx+c)^3+8\left(b^2\cos(dx+c)^4-2\,b^2\cos(dx+c)^2+b^2\right)\sqrt{\frac{b}{a+b}}\log\left(\frac{b\cos(dx+c)^2+2\,(a+b)\sqrt{}}{b\cos(dx+c)}\right.}{}\right.$$

input
```
integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/16*(2*(3*a^2 - 4*a*b)*cos(d*x + c)^3 + 8*(b^2*cos(d*x + c)^4 - 2*b^2*co
s(d*x + c)^2 + b^2)*sqrt(b/(a + b))*log((b*cos(d*x + c)^2 + 2*(a + b)*sqrt
(b/(a + b))*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - 2*(5*a^2 -
 4*a*b)*cos(d*x + c) - ((3*a^2 - 4*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a^2
- 4*a*b + 8*b^2)*cos(d*x + c)^2 + 3*a^2 - 4*a*b + 8*b^2)*log(1/2*cos(d*x +
 c) + 1/2) + ((3*a^2 - 4*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4*a*b +
8*b^2)*cos(d*x + c)^2 + 3*a^2 - 4*a*b + 8*b^2)*log(-1/2*cos(d*x + c) + 1/2
))/(a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d), 1/16*(2*(3*a^2
 - 4*a*b)*cos(d*x + c)^3 - 16*(b^2*cos(d*x + c)^4 - 2*b^2*cos(d*x + c)^2 +
 b^2)*sqrt(-b/(a + b))*arctan(sqrt(-b/(a + b))*cos(d*x + c)) - 2*(5*a^2 -
4*a*b)*cos(d*x + c) - ((3*a^2 - 4*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a^2 -
 4*a*b + 8*b^2)*cos(d*x + c)^2 + 3*a^2 - 4*a*b + 8*b^2)*log(1/2*cos(d*x +
c) + 1/2) + ((3*a^2 - 4*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4*a*b + 8
*b^2)*cos(d*x + c)^2 + 3*a^2 - 4*a*b + 8*b^2)*log(-1/2*cos(d*x + c) + 1/2)
)/(a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)]
```

3.84.6 Sympy [F]

$$\int \frac{\csc^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\csc^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**5/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(csc(c + d*x)**5/(a + b*sin(c + d*x)**2), x)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.45

$$\int \frac{\csc^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{8\,b^3\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^3} - \frac{2\left((3\,a-4\,b)\cos(dx+c)^3-(5\,a-4\,b)\cos(dx+c)\right)}{a^2\cos(dx+c)^4-2\,a^2\cos(dx+c)^2+a^2} + \frac{(3\,a^2-4\,ab+8\,b^2)\log(\cos(dx+c)+1)}{a^3} - \frac{(3\,a^2-4\,ab+\cdots}{\cdots}}{16\,d}$$

input
```
integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/16*(8*b^3*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt
((a + b)*b)))/(sqrt((a + b)*b)*a^3) - 2*((3*a - 4*b)*cos(d*x + c)^3 - (5*a
 - 4*b)*cos(d*x + c))/(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2) +
(3*a^2 - 4*a*b + 8*b^2)*log(cos(d*x + c) + 1)/a^3 - (3*a^2 - 4*a*b + 8*b^2
)*log(cos(d*x + c) - 1)/a^3)/d
```

3.84.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(111) = 222$.

Time = 0.44 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.67

$$\int \frac{\csc^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{64\,b^3\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^3} + \frac{\frac{8\,a(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{8\,b(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^2} - \frac{4\,(3\,a^2-4\,ab+8\,b^2)\log\left(\frac{|-\cos(dx+}{|\cos(dx+c}\right.}{a^3}}{64\,a}$$

input
```
integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/64*(64*b^3*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x +
c) + sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*a^3) + (8*a*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 8*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2)/a^2 - 4*(3*a^2 - 4*a*b + 8*b^2)*log(abs
(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + (a^2 - 8*a^2*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 8*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
18*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 24*a*b*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 + 48*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2)*(cos(d*x + c) + 1)^2/(a^3*(cos(d*x + c) - 1)^2))/d
```

3.84.9 Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 1105, normalized size of antiderivative = 8.84

$$\int \frac{\csc^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(c + d*x)^5*(a + b*sin(c + d*x)^2)),x)
```

output
```
- ((cos(c + d*x)*(5*a - 4*b))/(8*a^2) - (cos(c + d*x)^3*(3*a - 4*b))/(8*a^
2))/(d*(cos(c + d*x)^4 - cos(c + d*x)^2 + sin(c + d*x)^2)) - (atanh((63*b^
4*cos(c + d*x))/(64*((63*b^4)/64 - (81*a*b^3)/256 + (27*a^2*b^2)/256 - (35
*b^5)/(32*a) + (5*b^6)/(4*a^2))) - (81*b^3*cos(c + d*x))/(256*((27*a*b^2)/
256 - (81*b^3)/256 + (63*b^4)/(64*a) - (35*b^5)/(32*a^2) + (5*b^6)/(4*a^3)
)) - (35*b^5*cos(c + d*x))/(32*((63*a*b^4)/64 - (35*b^5)/32 - (81*a^2*b^3)
/256 + (27*a^3*b^2)/256 + (5*b^6)/(4*a))) + (5*b^6*cos(c + d*x))/(4*((5*b^
6)/4 - (35*a*b^5)/32 + (63*a^2*b^4)/64 - (81*a^3*b^3)/256 + (27*a^4*b^2)/2
56)) + (27*b^2*cos(c + d*x))/(256*((27*b^2)/256 - (81*b^3)/(256*a) + (63*b
^4)/(64*a^2) - (35*b^5)/(32*a^3) + (5*b^6)/(4*a^4))))*(3*a^2 - 4*a*b + 8*b
^2))/(8*a^3*d) - (atan((((b^5*(a + b))^(1/2)*((cos(c + d*x)*(128*b^7 - 64*
a*b^6 + 64*a^2*b^5 - 24*a^3*b^4 + 9*a^4*b^3))/(64*a^4) + ((b^5*(a + b))^(1
/2)*((2*a^6*b^4 - (a^7*b^3)/2 + (3*a^8*b^2)/2)/(2*a^6) - (cos(c + d*x)*(51
2*a^6*b^3 + 256*a^7*b^2)*(b^5*(a + b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2
*(a^3*b + a^4)))*1i)/(a^3*b + a^4) + ((b^5*(a + b))^(1/2)*((cos(c + d*x)*(
128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a^3*b^4 + 9*a^4*b^3))/(64*a^4) - ((b^
5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b^3)/2 + (3*a^8*b^2)/2)/(2*a^6) + (cos
(c + d*x)*(512*a^6*b^3 + 256*a^7*b^2)*(b^5*(a + b))^(1/2))/(128*a^4*(a^3*b
 + a^4))))/(2*(a^3*b + a^4)))*1i)/(a^3*b + a^4))/((((5*a*b^7)/4 - b^8 - (3*
a^2*b^6)/4 + (9*a^3*b^5)/32)/a^6 + ((b^5*(a + b))^(1/2)*((cos(c + d*x)*...
```

3.84. $\int \frac{\csc^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.85 $\int \frac{\sin^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.85.1 Optimal result

Integrand size = 23, antiderivative size = 163

$$\begin{aligned}\int \frac{\sin^8(c+dx)}{a+b\sin^2(c+dx)}\,dx &= -\frac{(16a^3-8a^2b+6ab^2-5b^3)\,x}{16b^4}+\frac{a^{7/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^4\sqrt{a+b}d}\\ &\quad-\frac{(8a^2-6ab+5b^2)\cos(c+dx)\sin(c+dx)}{16b^3d}\\ &\quad+\frac{(6a-5b)\cos(c+dx)\sin^3(c+dx)}{24b^2d}-\frac{\cos(c+dx)\sin^5(c+dx)}{6bd}\end{aligned}$$

output
```
-1/16*(16*a^3-8*a^2*b+6*a*b^2-5*b^3)*x/b^4-1/16*(8*a^2-6*a*b+5*b^2)*cos(d*
x+c)*sin(d*x+c)/b^3/d+1/24*(6*a-5*b)*cos(d*x+c)*sin(d*x+c)^3/b^2/d-1/6*cos
(d*x+c)*sin(d*x+c)^5/b/d+a^(7/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/b^
4/d/(a+b)^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

$$\int \frac{\sin^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{12(16a^3-8a^2b+6ab^2-5b^3)\,(c+dx)-\frac{192a^{7/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}}+3b(16a^2-16ab+15b^2)\sin(2(c+}{192b^4d}$$

input
```
Integrate[Sin[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

output
```
-1/192*(12*(16*a^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*(c + d*x) - (192*a^(7/2)*A
rcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a + b] + 3*b*(16*a^2 - 16*
a*b + 15*b^2)*Sin[2*(c + d*x)] + 3*(2*a - 3*b)*b^2*Sin[4*(c + d*x)] + b^3*
Sin[6*(c + d*x)])/(b^4*d)
```

3.85.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3666, 372, 440, 27, 440, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^8(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^8}{a+b\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3666}$$

$$\frac{\int \frac{\tan^8(c+dx)}{(\tan^2(c+dx)+1)^4((a+b)\tan^2(c+dx)+a)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\int \frac{\tan^4(c+dx)\left(5a-(a-5b)\tan^2(c+dx)\right)}{\left(\tan^2(c+dx)+1\right)^3\left((a+b)\tan^2(c+dx)+a\right)}d\tan(c+dx)}{6b}-\frac{\tan^5(c+dx)}{6b(\tan^2(c+dx)+1)^3}}{d}$$

$$\Big\downarrow \text{440}$$

$$\frac{\frac{\frac{(6a-5b)\tan^3(c+dx)}{4b\left(\tan^2(c+dx)+1\right)^2}-\frac{\int \frac{3\tan^2(c+dx)\left(a(6a-5b)-\left(2a^2-ba+5b^2\right)\tan^2(c+dx)\right)}{\left(\tan^2(c+dx)+1\right)^2\left((a+b)\tan^2(c+dx)+a\right)}d\tan(c+dx)}{4b}}{6b}-\frac{\tan^5(c+dx)}{6b(\tan^2(c+dx)+1)^3}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{(6a-5b)\tan^3(c+dx)}{4b\left(\tan^2(c+dx)+1\right)^2}-\frac{3\int\frac{\tan^2(c+dx)\left(a(6a-5b)-\left(2a^2-ba+5b^2\right)\tan^2(c+dx)\right)}{\left(\tan^2(c+dx)+1\right)^2\left((a+b)\tan^2(c+dx)+a\right)}d\tan(c+dx)}{4b}}{6b}-\frac{\tan^5(c+dx)}{6b(\tan^2(c+dx)+1)^3}}{d}$$

$\Big\downarrow$ 440

$$\frac{\frac{\frac{(6a-5b)\tan^3(c+dx)}{4b\left(\tan^2(c+dx)+1\right)^2}-\frac{3\left(\frac{\left(8a^2-6ab+5b^2\right)\tan(c+dx)}{2b\left(\tan^2(c+dx)+1\right)}-\frac{\int\frac{a\left(8a^2-6ba+5b^2\right)-\left(8a^3-2ba^2+b^2a-5b^3\right)\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left((a+b)\tan^2(c+dx)+a\right)}d\tan(c+dx)}{2b}\right)}{4b}}{6b}-\frac{\tan^5(c+dx)}{6b(\tan^2(c+dx)+1)^3}}{d}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{(6a-5b)\tan^3(c+dx)}{4b\left(\tan^2(c+dx)+1\right)^2}-\frac{3\left(\frac{\left(8a^2-6ab+5b^2\right)\tan(c+dx)}{2b\left(\tan^2(c+dx)+1\right)}-\frac{\frac{16a^4\int\frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{b}-\frac{\left(16a^3-8a^2b+6ab^2-5b^3\right)\int\frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)}{b}}{2b}\right)}{4b}}{6b}-}{d}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{(6a-5b)\tan^3(c+dx)}{4b\left(\tan^2(c+dx)+1\right)^2}-\frac{3\left(\frac{\left(8a^2-6ab+5b^2\right)\tan(c+dx)}{2b\left(\tan^2(c+dx)+1\right)}-\frac{\frac{16a^4\int\frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{b}-\frac{\left(16a^3-8a^2b+6ab^2-5b^3\right)\arctan(\tan(c+dx))}{b}}{2b}\right)}{4b}}{6b}-}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{(6a-5b)\tan^3(c+dx)}{4b\left(\tan^2(c+dx)+1\right)^2}-\frac{3\left(\frac{\left(8a^2-6ab+5b^2\right)\tan(c+dx)}{2b\left(\tan^2(c+dx)+1\right)}-\frac{\frac{16a^{7/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b\sqrt{a+b}}-\frac{\left(16a^3-8a^2b+6ab^2-5b^3\right)\arctan(\tan(c+dx))}{b}}{2b}\right)}{4b}}{6b}-\frac{\tan^5(c+dx}{6b(\tan^2(c+dx)}}{d}$$

input
```
Int[Sin[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

3.85. $\int\frac{\sin^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

output
```
(-1/6*Tan[c + d*x]^5/(b*(1 + Tan[c + d*x]^2)^3) + (((6*a - 5*b)*Tan[c + d*
x]^3)/(4*b*(1 + Tan[c + d*x]^2)^2) - (3*(-1/2*(-(((16*a^3 - 8*a^2*b + 6*a*
b^2 - 5*b^3)*ArcTan[Tan[c + d*x]])/b) + (16*a^(7/2)*ArcTan[(Sqrt[a + b]*Ta
n[c + d*x])/Sqrt[a]])/(b*Sqrt[a + b]))/b + ((8*a^2 - 6*a*b + 5*b^2)*Tan[c
+ d*x])/(2*b*(1 + Tan[c + d*x]^2))))/(4*b))/(6*b))/d
```

3.85.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.85.4 Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{\frac{a^4 \arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{b^4\sqrt{a(a+b)}}-\frac{\frac{\left(\frac{1}{2}a^2b-\frac{5}{8}a\,b^2+\frac{11}{16}b^3\right)\left(\tan^5(dx+c)\right)+\left(a^2b-a\,b^2+\frac{5}{6}b^3\right)\left(\tan^3(dx+c)\right)+\left(\frac{1}{2}a^2b-\frac{3}{8}a\,b^2+\frac{5}{16}b^3\right)\tan(}{\left(1+\tan^2(dx+c)\right)^3}}{b^4}}{d}$
default	$\frac{\frac{a^4 \arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{b^4\sqrt{a(a+b)}}-\frac{\frac{\left(\frac{1}{2}a^2b-\frac{5}{8}a\,b^2+\frac{11}{16}b^3\right)\left(\tan^5(dx+c)\right)+\left(a^2b-a\,b^2+\frac{5}{6}b^3\right)\left(\tan^3(dx+c)\right)+\left(\frac{1}{2}a^2b-\frac{3}{8}a\,b^2+\frac{5}{16}b^3\right)\tan(}{\left(1+\tan^2(dx+c)\right)^3}}{b^4}}{d}$
risch	$-\frac{x\,a^3}{b^4}+\frac{x\,a^2}{2b^3}-\frac{3ax}{8b^2}+\frac{5x}{16b}+\frac{i\mathrm{e}^{2i(dx+c)}a^2}{8b^3d}-\frac{i\mathrm{e}^{2i(dx+c)}a}{8b^2d}+\frac{15i\mathrm{e}^{2i(dx+c)}}{128bd}-\frac{i\mathrm{e}^{-2i(dx+c)}a^2}{8b^3d}+\frac{i\mathrm{e}^{-2i(dx+c)}a}{8b^2d}$

input
```
int(sin(d*x+c)^8/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^4*a^4/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/
b^4*(((1/2*a^2*b-5/8*a*b^2+11/16*b^3)*tan(d*x+c)^5+(a^2*b-a*b^2+5/6*b^3)*t
an(d*x+c)^3+(1/2*a^2*b-3/8*a*b^2+5/16*b^3)*tan(d*x+c))/(1+tan(d*x+c)^2)^3+
1/16*(16*a^3-8*a^2*b+6*a*b^2-5*b^3)*arctan(tan(d*x+c))))
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.78

$$\int \frac{\sin^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \left[\frac{12\,a^3\sqrt{-\frac{a}{a+b}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4-2\,(4\,a^2+5\,ab+b^2)\cos(dx+c)^2-4\,\left((2\,a^2+3\,ab+b^2)\cos(dx+c)^3-(a^2+2\,ab+b^2)\cos(dx+c\right.}{b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2\,ab+b^2}\right.}{} \right.$$
$$-\frac{24\,a^3\sqrt{\frac{a}{a+b}}\arctan\left(\frac{\left((2\,a+b)\cos(dx+c)^2-a-b\right)\sqrt{\frac{a}{a+b}}}{2\,a\cos(dx+c)\sin(dx+c)}\right)+3\,(16\,a^3-8\,a^2b+6\,ab^2-5\,b^3)dx+\left(8\,b^3\cos\left(dx+c\right.\right.}{48\,b^4d}$$

input
```
integrate(sin(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/48*(12*a^3*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 -
 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + 3*a*b + b^2)*cos(d*x
 + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c)
+ a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 +
a^2 + 2*a*b + b^2)) - 3*(16*a^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*d*x - (8*b^3*
cos(d*x + c)^5 + 2*(6*a*b^2 - 13*b^3)*cos(d*x + c)^3 + 3*(8*a^2*b - 10*a*b
^2 + 11*b^3)*cos(d*x + c))*sin(d*x + c))/(b^4*d), -1/48*(24*a^3*sqrt(a/(a
+ b))*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos
(d*x + c)*sin(d*x + c))) + 3*(16*a^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*d*x + (8
*b^3*cos(d*x + c)^5 + 2*(6*a*b^2 - 13*b^3)*cos(d*x + c)^3 + 3*(8*a^2*b - 1
0*a*b^2 + 11*b^3)*cos(d*x + c))*sin(d*x + c))/(b^4*d)]
```

3.85.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**8/(a+b*sin(d*x+c)**2),x)
```

output
```
Timed out
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.18

$$\int \frac{\sin^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \frac{\frac{48\,a^4\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}b^4} - \frac{3\,(8\,a^2-10\,ab+11\,b^2)\tan(dx+c)^5+8\,(6\,a^2-6\,ab+5\,b^2)\tan(dx+c)^3+3\,(8\,a^2-6\,ab+5\,b^2)\tan(dx+c)}{b^3\tan(dx+c)^6+3\,b^3\tan(dx+c)^4+3\,b^3\tan(dx+c)^2+b^3} - \frac{3\,(16\,a}{}}{48\,d}$$

input
```
integrate(sin(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/48*(48*a^4*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)
*b^4) - (3*(8*a^2 - 10*a*b + 11*b^2)*tan(d*x + c)^5 + 8*(6*a^2 - 6*a*b + 5
*b^2)*tan(d*x + c)^3 + 3*(8*a^2 - 6*a*b + 5*b^2)*tan(d*x + c))/(b^3*tan(d*
x + c)^6 + 3*b^3*tan(d*x + c)^4 + 3*b^3*tan(d*x + c)^2 + b^3) - 3*(16*a^3
- 8*a^2*b + 6*a*b^2 - 5*b^3)*(d*x + c)/b^4)/d
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.43

$$\int \frac{\sin^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^4}{\sqrt{a^2+ab}b^4} - \frac{3\,(16\,a^3-8\,a^2b+6\,ab^2-5\,b^3)(dx+c)}{b^4} - \frac{24\,a^2\tan(dx+c)^5-30\,ab\tan(d}{}}{48\,d}$$

input
```
integrate(sin(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/48*(48*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x
+ c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^4/(sqrt(a^2 + a*b)*b^4) - 3*(16
*a^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*(d*x + c)/b^4 - (24*a^2*tan(d*x + c)^5 -
 30*a*b*tan(d*x + c)^5 + 33*b^2*tan(d*x + c)^5 + 48*a^2*tan(d*x + c)^3 - 4
8*a*b*tan(d*x + c)^3 + 40*b^2*tan(d*x + c)^3 + 24*a^2*tan(d*x + c) - 18*a*
b*tan(d*x + c) + 15*b^2*tan(d*x + c))/((tan(d*x + c)^2 + 1)^3*b^3))/d
```

3.85.9 Mupad [B] (verification not implemented)

Time = 15.31 (sec) , antiderivative size = 2244, normalized size of antiderivative = 13.77

$$\int \frac{\sin^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^8/(a + b*sin(c + d*x)^2),x)
```

output
```
(atan(((((tan(c + d*x)*(15*a*b^8 + 768*a^8*b + 512*a^9 + 25*b^9 + 11*a^2*b
^7 - 63*a^3*b^6 - 224*a^4*b^5 - 140*a^5*b^4 + 256*a^7*b^2))/(128*b^6) - ((
((5*a*b^12)/4 + a^2*b^11 + (a^3*b^10)/4 + (5*a^4*b^9)/2 + 2*a^5*b^8)/b^9 -
 (tan(c + d*x)*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i)*(4096*a*b^10 + 102
4*b^11 + 5120*a^2*b^9 + 2048*a^3*b^8))/(4096*b^10))*(a*b^2*6i - a^2*b*8i +
 a^3*16i - b^3*5i))/(32*b^4))*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i)*1i)
/(32*b^4) + (((tan(c + d*x)*(15*a*b^8 + 768*a^8*b + 512*a^9 + 25*b^9 + 11*
a^2*b^7 - 63*a^3*b^6 - 224*a^4*b^5 - 140*a^5*b^4 + 256*a^7*b^2))/(128*b^6)
 + ((((5*a*b^12)/4 + a^2*b^11 + (a^3*b^10)/4 + (5*a^4*b^9)/2 + 2*a^5*b^8)/
b^9 + (tan(c + d*x)*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i)*(4096*a*b^10
+ 1024*b^11 + 5120*a^2*b^9 + 2048*a^3*b^8))/(4096*b^10))*(a*b^2*6i - a^2*b
*8i + a^3*16i - b^3*5i))/(32*b^4))*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i
)*1i)/(32*b^4))/(((a^10*b)/4 + a^11 + (25*a^4*b^7)/128 - (5*a^5*b^6)/64 +
(21*a^6*b^5)/128 - (21*a^7*b^4)/32 - (15*a^8*b^3)/32 - (a^9*b^2)/8)/b^9 -
(((tan(c + d*x)*(15*a*b^8 + 768*a^8*b + 512*a^9 + 25*b^9 + 11*a^2*b^7 - 63
*a^3*b^6 - 224*a^4*b^5 - 140*a^5*b^4 + 256*a^7*b^2))/(128*b^6) - ((((5*a*b
^12)/4 + a^2*b^11 + (a^3*b^10)/4 + (5*a^4*b^9)/2 + 2*a^5*b^8)/b^9 - (tan(c
 + d*x)*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i)*(4096*a*b^10 + 1024*b^11
+ 5120*a^2*b^9 + 2048*a^3*b^8))/(4096*b^10))*(a*b^2*6i - a^2*b*8i + a^3*16
i - b^3*5i))/(32*b^4))*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i))/(32*b^...
```

3.86 $\int \frac{\sin^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.86.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{\sin^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{(8a^2-4ab+3b^2)\,x}{8b^3} - \frac{a^{5/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^3\sqrt{a+b}d} + \frac{(4a-3b)\cos(c+dx)\sin(c+dx)}{8b^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{4bd}$$

output
```
1/8*(8*a^2-4*a*b+3*b^2)*x/b^3+1/8*(4*a-3*b)*cos(d*x+c)*sin(d*x+c)/b^2/d-1/
4*cos(d*x+c)*sin(d*x+c)^3/b/d-a^(5/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2
))/b^3/d/(a+b)^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

$$\int \frac{\sin^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{4(8a^2-4ab+3b^2)(c+dx) - \frac{32a^{5/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}} + 8(a-b)b\sin(2(c+dx)) + b^2\sin(4(c+dx))}{32b^3d}$$

input
```
Integrate[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

output
```
(4*(8*a^2 - 4*a*b + 3*b^2)*(c + d*x) - (32*a^(5/2)*ArcTan[(Sqrt[a + b]*Tan
[c + d*x])/Sqrt[a]])/Sqrt[a + b] + 8*(a - b)*b*Sin[2*(c + d*x)] + b^2*Sin[
4*(c + d*x)])/(32*b^3*d)
```

3.86.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 3666, 372, 440, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(c+dx)^6}{a+b\sin(c+dx)^2}dx$$

$$\Big\downarrow \; 3666$$

$$\frac{\int \frac{\tan^6(c+dx)}{(\tan^2(c+dx)+1)^3((a+b)\tan^2(c+dx)+a)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \; 372$$

$$\frac{\frac{\int \frac{\tan^2(c+dx)\left(3a-(a-3b)\tan^2(c+dx)\right)}{\left(\tan^2(c+dx)+1\right)^2\left((a+b)\tan^2(c+dx)+a\right)}d\tan(c+dx)}{4b}-\frac{\tan^3(c+dx)}{4b(\tan^2(c+dx)+1)^2}}{d}$$

$$\Big\downarrow \; 440$$

$$\frac{\frac{\frac{(4a-3b)\tan(c+dx)}{2b\left(\tan^2(c+dx)+1\right)}-\frac{\int \frac{a(4a-3b)-\left(4a^2-ba+3b^2\right)\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left((a+b)\tan^2(c+dx)+a\right)}d\tan(c+dx)}{2b}}{4b}-\frac{\tan^3(c+dx)}{4b(\tan^2(c+dx)+1)^2}}{d}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{(4a-3b)\tan(c+dx)}{2b\left(\tan^2(c+dx)+1\right)}-\frac{\frac{8a^3\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{b}-\frac{\left(8a^2-4ab+3b^2\right)\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)}{b}}{2b}}{4b}-\frac{\tan^3(c+dx)}{4b(\tan^2(c+dx)+1)^2}}{d}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{(4a-3b)\tan(c+dx)}{2b\left(\tan^2(c+dx)+1\right)}-\frac{\frac{8a^3\int\frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{b}-\frac{\left(8a^2-4ab+3b^2\right)\arctan(\tan(c+dx))}{b}}{2b}}{4b}-\frac{\tan^3(c+dx)}{4b(\tan^2(c+dx)+1)^2}}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{(4a-3b)\tan(c+dx)}{2b\left(\tan^2(c+dx)+1\right)}-\frac{\frac{8a^{5/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b\sqrt{a+b}}-\frac{\left(8a^2-4ab+3b^2\right)\arctan(\tan(c+dx))}{b}}{2b}}{4b}-\frac{\tan^3(c+dx)}{4b(\tan^2(c+dx)+1)^2}}{d}$$

input
```
Int[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-1/4*Tan[c + d*x]^3/(b*(1 + Tan[c + d*x]^2)^2) + (-1/2*(-(((8*a^2 - 4*a*b
 + 3*b^2)*ArcTan[Tan[c + d*x]])/b) + (8*a^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c
+ d*x])/Sqrt[a]])/(b*Sqrt[a + b]))/b + ((4*a - 3*b)*Tan[c + d*x])/(2*b*(1
+ Tan[c + d*x]^2)))/(4*b))/d
```

3.86.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.86.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{1}{2}ab-\frac{5}{8}b^2\right)\left(\tan^3(dx+c)\right)+\left(\frac{1}{2}ab-\frac{3}{8}b^2\right)\tan(dx+c)}{\left(1+\tan^2(dx+c)\right)^2}+\frac{\left(8a^2-4ab+3b^2\right)\arctan(\tan(dx+c))}{8}}{b^3}-\frac{a^3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{b^3\sqrt{a(a+b)}}}{d}$
default	$\frac{\frac{\frac{\left(\frac{1}{2}ab-\frac{5}{8}b^2\right)\left(\tan^3(dx+c)\right)+\left(\frac{1}{2}ab-\frac{3}{8}b^2\right)\tan(dx+c)}{\left(1+\tan^2(dx+c)\right)^2}+\frac{\left(8a^2-4ab+3b^2\right)\arctan(\tan(dx+c))}{8}}{b^3}-\frac{a^3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{b^3\sqrt{a(a+b)}}}{d}$
risch	$\frac{x\,a^2}{b^3}-\frac{ax}{2b^2}+\frac{3x}{8b}-\frac{i e^{2i(dx+c)}a}{8b^2d}+\frac{i e^{2i(dx+c)}}{8bd}+\frac{i e^{-2i(dx+c)}a}{8b^2d}-\frac{i e^{-2i(dx+c)}}{8bd}-\frac{\sqrt{-a(a+b)}\,a^2\ln\left(e^{2i(dx+c)}+\right.}{2(a+b)d\,b^3}$

input
```
int(sin(d*x+c)^6/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^3*(((1/2*a*b-5/8*b^2)*tan(d*x+c)^3+(1/2*a*b-3/8*b^2)*tan(d*x+c))/
(1+tan(d*x+c)^2)^2+1/8*(8*a^2-4*a*b+3*b^2)*arctan(tan(d*x+c)))-a^3/b^3/(a*
(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.18

$$\int \frac{\sin^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \left[\frac{2\,a^2\sqrt{-\frac{a}{a+b}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4-2\,(4\,a^2+5\,ab+b^2)\cos(dx+c)^2+4\,\left((2\,a^2+3\,ab+b^2)\cos(dx+c)^3-(a^2+2\,ab+b^2)\cos(dx+c)\right.}{b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2\,ab+b^2}\right.}{} \right.$$

input
```
integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/8*(2*a^2*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2
*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(d*x +
 c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) +
a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^
2 + 2*a*b + b^2)) + (8*a^2 - 4*a*b + 3*b^2)*d*x + (2*b^2*cos(d*x + c)^3 +
(4*a*b - 5*b^2)*cos(d*x + c))*sin(d*x + c))/(b^3*d), 1/8*(4*a^2*sqrt(a/(a
+ b))*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos
(d*x + c)*sin(d*x + c))) + (8*a^2 - 4*a*b + 3*b^2)*d*x + (2*b^2*cos(d*x +
c)^3 + (4*a*b - 5*b^2)*cos(d*x + c))*sin(d*x + c))/(b^3*d)]
```

3.86.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**6/(a+b*sin(d*x+c)**2),x)
```

output
```
Timed out
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09

$$\int \frac{\sin^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{8\,a^3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}b^3} - \frac{(4\,a-5\,b)\tan(dx+c)^3+(4\,a-3\,b)\tan(dx+c)}{b^2\tan(dx+c)^4+2\,b^2\tan(dx+c)^2+b^2} - \frac{(8\,a^2-4\,ab+3\,b^2)(dx+c)}{b^3}}{8\,d}$$

input
```
integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/8*(8*a^3*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*
b^3) - ((4*a - 5*b)*tan(d*x + c)^3 + (4*a - 3*b)*tan(d*x + c))/(b^2*tan(d*
x + c)^4 + 2*b^2*tan(d*x + c)^2 + b^2) - (8*a^2 - 4*a*b + 3*b^2)*(d*x + c)
/b^3)/d
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.34

$$\int \frac{\sin^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{8\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^3}{\sqrt{a^2+ab}b^3} - \frac{(8\,a^2-4\,ab+3\,b^2)(dx+c)}{b^3} - \frac{4\,a\tan(dx+c)^3-5\,b\tan(dx+c)^3+4\,a\tan\cdots}{\left(\tan(dx+c)^2+1\right)^2\cdots}}{8\,d}$$

input
```
integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/8*(8*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x +
 c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^3/(sqrt(a^2 + a*b)*b^3) - (8*a^2
 - 4*a*b + 3*b^2)*(d*x + c)/b^3 - (4*a*tan(d*x + c)^3 - 5*b*tan(d*x + c)^3
 + 4*a*tan(d*x + c) - 3*b*tan(d*x + c))/((tan(d*x + c)^2 + 1)^2*b^2))/d
```

3.86.9 Mupad [B] (verification not implemented)

Time = 14.19 (sec) , antiderivative size = 1892, normalized size of antiderivative = 16.17

$$\int \frac{\sin^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^6/(a + b*sin(c + d*x)^2),x)
```

output
```
((tan(c + d*x)*(4*a - 3*b))/(8*b^2) + (tan(c + d*x)^3*(4*a - 5*b))/(8*b^2)
)/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1)) - (atan(((((((((3*a*b^9)/2 +
a^2*b^8 - (5*a^3*b^7)/2 - 2*a^4*b^6)/(2*b^6) - (tan(c + d*x)*(-a^5*(a + b)
)^(1/2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(128*b^4*(a*b
^3 + b^4)))*(-a^5*(a + b))^(1/2))/(2*(a*b^3 + b^4)) - (tan(c + d*x)*(3*a*b
^6 + 192*a^6*b + 128*a^7 + 9*b^7 + 19*a^2*b^5 + 65*a^3*b^4 + 40*a^4*b^3 +
64*a^5*b^2))/(64*b^4))*(-a^5*(a + b))^(1/2)*1i)/(a*b^3 + b^4) - ((((((3*a*
b^9)/2 + a^2*b^8 - (5*a^3*b^7)/2 - 2*a^4*b^6)/(2*b^6) + (tan(c + d*x)*(-a^
5*(a + b))^(1/2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(128
*b^4*(a*b^3 + b^4)))*(-a^5*(a + b))^(1/2))/(2*(a*b^3 + b^4)) + (tan(c + d*
x)*(3*a*b^6 + 192*a^6*b + 128*a^7 + 9*b^7 + 19*a^2*b^5 + 65*a^3*b^4 + 40*a
^4*b^3 + 64*a^5*b^2))/(64*b^4))*(-a^5*(a + b))^(1/2)*1i)/(a*b^3 + b^4))/((
(((((3*a*b^9)/2 + a^2*b^8 - (5*a^3*b^7)/2 - 2*a^4*b^6)/(2*b^6) - (tan(c +
d*x)*(-a^5*(a + b))^(1/2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b
^6))/(128*b^4*(a*b^3 + b^4)))*(-a^5*(a + b))^(1/2))/(2*(a*b^3 + b^4)) - (t
an(c + d*x)*(3*a*b^6 + 192*a^6*b + 128*a^7 + 9*b^7 + 19*a^2*b^5 + 65*a^3*b
^4 + 40*a^4*b^3 + 64*a^5*b^2))/(64*b^4))*(-a^5*(a + b))^(1/2))/(a*b^3 + b^
4) - ((a^7*b)/4 + a^8 + (9*a^3*b^5)/32 - (3*a^4*b^4)/16 + (25*a^5*b^3)/32
+ (a^6*b^2)/2)/b^6 + ((((((3*a*b^9)/2 + a^2*b^8 - (5*a^3*b^7)/2 - 2*a^4*b^
6)/(2*b^6) + (tan(c + d*x)*(-a^5*(a + b))^(1/2)*(1024*a*b^8 + 256*b^9 +...
```

3.87 $\int \frac{\sin^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.87.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\sin^4(c+dx)}{a+b\sin^2(c+dx)}\,dx=-\frac{(2a-b)x}{2b^2}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^2\sqrt{a+b}d}-\frac{\cos(c+dx)\sin(c+dx)}{2bd}$$

output
```
-1/2*(2*a-b)*x/b^2-1/2*cos(d*x+c)*sin(d*x+c)/b/d+a^(3/2)*arctan((a+b)^(1/2
)*tan(d*x+c)/a^(1/2))/b^2/d/(a+b)^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \frac{\sin^4(c+dx)}{a+b\sin^2(c+dx)}\,dx=-\frac{2(2a-b)(c+dx)-\frac{4a^{3/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}}+b\sin(2(c+dx))}{4b^2d}$$

input
```
Integrate[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

output
```
-1/4*(2*(2*a - b)*(c + d*x) - (4*a^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])
/Sqrt[a]])/Sqrt[a + b] + b*Sin[2*(c + d*x)])/(b^2*d)
```

3.87.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3666, 372, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^4}{a+b\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3666}$$

$$\frac{\int \frac{\tan^4(c+dx)}{(\tan^2(c+dx)+1)^2((a+b)\tan^2(c+dx)+a)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\int \frac{a-(a-b)\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left((a+b)\tan^2(c+dx)+a\right)}d\tan(c+dx)}{2b}-\frac{\tan(c+dx)}{2b(\tan^2(c+dx)+1)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{2a^2\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{b}-\frac{(2a-b)\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)}{b}}{2b}-\frac{\tan(c+dx)}{2b(\tan^2(c+dx)+1)}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{2a^2\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{b}-\frac{(2a-b)\arctan(\tan(c+dx))}{b}}{2b}-\frac{\tan(c+dx)}{2b(\tan^2(c+dx)+1)}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b\sqrt{a+b}}-\frac{(2a-b)\arctan(\tan(c+dx))}{b}}{2b}-\frac{\tan(c+dx)}{2b(\tan^2(c+dx)+1)}}{d}$$

input
```
Int[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

output
```
((-(((2*a - b)*ArcTan[Tan[c + d*x]])/b) + (2*a^(3/2)*ArcTan[(Sqrt[a + b]*T
an[c + d*x])/Sqrt[a]])/(b*Sqrt[a + b]))/(2*b) - Tan[c + d*x]/(2*b*(1 + Tan
[c + d*x]^2)))/d
```

3.87.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.87.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{-\frac{\frac{\tan(dx+c)b}{2+2\left(\tan^2(dx+c)\right)}+\frac{(2a-b)\arctan(\tan(dx+c))}{2}}{b^2}+\frac{a^2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{b^2\sqrt{a(a+b)}}}{d}$
default	$\frac{-\frac{\frac{\tan(dx+c)b}{2+2\left(\tan^2(dx+c)\right)}+\frac{(2a-b)\arctan(\tan(dx+c))}{2}}{b^2}+\frac{a^2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{b^2\sqrt{a(a+b)}}}{d}$
risch	$-\frac{ax}{b^2}+\frac{x}{2b}+\frac{ie^{2i(dx+c)}}{8bd}-\frac{ie^{-2i(dx+c)}}{8bd}+\frac{\sqrt{-a(a+b)}\,a\ln\left(e^{2i(dx+c)}+\frac{2i\sqrt{-a(a+b)}-2a-b}{b}\right)}{2(a+b)d\,b^2}-\frac{\sqrt{-a(a+b)}\,a\ln\left(e\right.}{}$

input
```
int(sin(d*x+c)^4/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^2*(1/2*b*tan(d*x+c)/(1+tan(d*x+c)^2)+1/2*(2*a-b)*arctan(tan(d*x+
c)))+a^2/b^2/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.96

$$\int \frac{\sin^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \left[-\frac{2\,(2\,a-b)dx+2\,b\cos\left(dx+c\right)\sin\left(dx+c\right)-a\sqrt{-\frac{a}{a+b}}\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(dx+c)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(dx+c}{b^2\cos(}\right.}{4\,b^2d}\right.$$

$$\left.-\frac{(2\,a-b)dx+b\cos\left(dx+c\right)\sin\left(dx+c\right)+a\sqrt{\frac{a}{a+b}}\arctan\left(\frac{\left((2\,a+b)\cos(dx+c)^2-a-b\right)\sqrt{\frac{a}{a+b}}}{2\,a\cos(dx+c)\sin(dx+c)}\right)}{2\,b^2d}\right]$$

input
```
integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/4*(2*(2*a - b)*d*x + 2*b*cos(d*x + c)*sin(d*x + c) - a*sqrt(-a/(a + b)
)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(
d*x + c)^2 - 4*((2*a^2 + 3*a*b + b^2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)
*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos
(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)))/(b^2*d),
 -1/2*((2*a - b)*d*x + b*cos(d*x + c)*sin(d*x + c) + a*sqrt(a/(a + b))*arc
tan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)
*sin(d*x + c))))/(b^2*d)]
```

3.87.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**4/(a+b*sin(d*x+c)**2),x)
```

output
```
Timed out
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{\sin^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{2\,a^2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}b^2} - \frac{(dx+c)(2\,a-b)}{b^2} - \frac{\tan(dx+c)}{b\tan(dx+c)^2+b}}{2\,d}$$

input
```
integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*(2*a^2*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b
^2) - (d*x + c)*(2*a - b)/b^2 - tan(d*x + c)/(b*tan(d*x + c)^2 + b))/d
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.48

$$\int \frac{\sin^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{2\left(\pi\left\lfloor \frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^2}{\sqrt{a^2+ab}b^2} - \frac{(dx+c)(2\,a-b)}{b^2} - \frac{\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)b}}{2\,d}$$

input

```
integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output

```
1/2*(2*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x +
c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^2/(sqrt(a^2 + a*b)*b^2) - (d*x +
c)*(2*a - b)/b^2 - tan(d*x + c)/((tan(d*x + c)^2 + 1)*b))/d
```

3.87.9 Mupad [B] (verification not implemented)

Time = 14.14 (sec) , antiderivative size = 481, normalized size of antiderivative = 6.25

$$\int \frac{\sin^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{b^2\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)}{d\,\left(2\,b^3+2\,a\,b^2\right)} - \frac{2\,a^2\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)}{d\,\left(2\,b^3+2\,a\,b^2\right)} - \frac{b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d\,\left(2\,b^3+2\,a\,b^2\right)} - \frac{a\,b\,\sin\left(2\,c+2\,d\,x\right)}{2\,d\,\left(2\,b^3+2\,a\,b^2\right)} - \frac{a\,b\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)}{d\,\left(2\,b^3+2\,a\,b^2\right)} - \frac{\mathrm{atan}\left(\frac{a\,\sin(c+d\,x)\,\left(-a^4-b\,a^3\right)^{3/2}\,8\mathrm{i}+b\,\sin(c+d\,x)\,\left(-a^4-b\,a^3\right)^{3/2}\,4\mathrm{i}+a^5\,\sin(c+d\,x)\,\sqrt{-a^4-b\,a^3}\,8\mathrm{i}+b^5\,\sin(c+d\,x)\,\sqrt{-a^4-b\,a^3}\,1\mathrm{i}-a\,b^4\,\sin(\cdots}{3\,\cos(c+d\,x)\,a^5\,b^2+5\,\cos(c+d\,x)\,a^4\,b^3+\cos(\cdots}\right)\cdots}{d\,\left(2\,b^3\cdots\right.}$$

input

```
int(sin(c + d*x)^4/(a + b*sin(c + d*x)^2),x)
```

output
```
(b^2*atan(sin(c + d*x)/cos(c + d*x)))/(d*(2*a*b^2 + 2*b^3)) - (2*a^2*atan(
sin(c + d*x)/cos(c + d*x)))/(d*(2*a*b^2 + 2*b^3)) - (b^2*sin(2*c + 2*d*x))
/(2*d*(2*a*b^2 + 2*b^3)) - (atan((a*sin(c + d*x)*(- a^3*b - a^4)^(3/2)*8i
+ b*sin(c + d*x)*(- a^3*b - a^4)^(3/2)*4i + a^5*sin(c + d*x)*(- a^3*b - a^
4)^(1/2)*8i + b^5*sin(c + d*x)*(- a^3*b - a^4)^(1/2)*1i - a*b^4*sin(c + d*
x)*(- a^3*b - a^4)^(1/2)*1i + a^4*b*sin(c + d*x)*(- a^3*b - a^4)^(1/2)*12i
 - a^2*b^3*sin(c + d*x)*(- a^3*b - a^4)^(1/2)*5i + a^3*b^2*sin(c + d*x)*(-
 a^3*b - a^4)^(1/2)*1i)/(a^3*b^4*cos(c + d*x) - a^2*b^5*cos(c + d*x) + 5*a
^4*b^3*cos(c + d*x) + 3*a^5*b^2*cos(c + d*x)))*(- a^3*b - a^4)^(1/2)*2i)/(
d*(2*a*b^2 + 2*b^3)) - (a*b*sin(2*c + 2*d*x))/(2*d*(2*a*b^2 + 2*b^3)) - (a
*b*atan(sin(c + d*x)/cos(c + d*x)))/(d*(2*a*b^2 + 2*b^3))
```

3.87. $\int \frac{\sin^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.88 $\int \frac{\sin^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.88.1 Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{\sin^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{x}{b} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b\sqrt{a+b}d}$$

output
```
x/b-arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))*a^(1/2)/b/d/(a+b)^(1/2)
```

3.88.2 Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{c+dx-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}}}{bd}$$

input
```
Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

output
```
(c + d*x - (Sqrt[a]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a + b
])/(b*d)
```

3.88.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3650, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^2}{a+b\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3650}$$

$$\frac{x}{b}-\frac{a\int \frac{1}{b\sin^2(c+dx)+a}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{b}-\frac{a\int \frac{1}{b\sin(c+dx)^2+a}dx}{b}$$

$$\Big\downarrow \text{3660}$$

$$\frac{x}{b}-\frac{a\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{bd}$$

$$\Big\downarrow \text{218}$$

$$\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{bd\sqrt{a+b}}$$

input
```
Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

output
```
x/b - (Sqrt[a]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(b*Sqrt[a + b]*
d)
```

3.88.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3650
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Simp[(A*b - a*B)/b   Int[1/(a
+ b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.88.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\frac{\arctan(\tan(dx+c))}{b}-\frac{a\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{b\sqrt{a(a+b)}}}{d}$	48
default	$\frac{\frac{\arctan(\tan(dx+c))}{b}-\frac{a\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{b\sqrt{a(a+b)}}}{d}$	48
risch	$\frac{x}{b}-\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2i(dx+c)}+\frac{2i\sqrt{-a(a+b)}-2a-b}{b}\right)}{2(a+b)db}+\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2i(dx+c)}-\frac{2i\sqrt{-a(a+b)}+2a+b}{b}\right)}{2(a+b)db}$	114

input
```
int(sin(d*x+c)^2/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b*arctan(tan(d*x+c))-1/b*a/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/
(a*(a+b))^(1/2)))
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 260, normalized size of antiderivative = 5.65

$$\int \frac{\sin^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \left[\frac{4\,dx+\sqrt{-\frac{a}{a+b}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4-2\,(4\,a^2+5\,ab+b^2)\cos(dx+c)^2+4\left((2\,a^2+3\,ab+b^2)\cos(dx+c)^3-(a^2+2\,ab+b^2)\cos(dx\right.}{b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2\,ab+b^2}}{4\,bd}\right.$$

input
```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/4*(4*d*x + sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 -
 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(d*x
 + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c)
+ a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 +
a^2 + 2*a*b + b^2)))/(b*d), 1/2*(2*d*x + sqrt(a/(a + b))*arctan(1/2*((2*a
+ b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d*x + c))
))/(b*d)]
```

3.88.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**2),x)
```

output
```
Timed out
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{a\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}b} - \frac{dx+c}{b}}{d}$$

input
```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-(a*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b) - (d*
x + c)/b)/d
```

3.88.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(38) = 76.

Time = 0.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int \frac{\sin^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a}{\sqrt{a^2+ab}b} - \frac{dx+c}{b}}{d}$$

input
```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) +
b*tan(d*x + c))/sqrt(a^2 + a*b)))*a/(sqrt(a^2 + a*b)*b) - (d*x + c)/b)/d
```

3.88.9 Mupad [B] (verification not implemented)

Time = 13.85 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.26

$$\int \frac{\sin^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\mathrm{atan}\left(\frac{2ab^2\tan(c+dx)}{2a^2b+2ab^2}+\frac{2a^2b\tan(c+dx)}{2a^2b+2ab^2}\right)}{bd} + \frac{\mathrm{atanh}\left(\frac{\tan(c+dx)\sqrt{-a(a+b)}}{a}\right)\sqrt{-a(a+b)}}{d(b^2+ab)}$$

input
```
int(sin(c + d*x)^2/(a + b*sin(c + d*x)^2),x)
```

output
```
atan((2*a*b^2*tan(c + d*x))/(2*a*b^2 + 2*a^2*b) + (2*a^2*b*tan(c + d*x))/(
2*a*b^2 + 2*a^2*b))/(b*d) + (atanh((tan(c + d*x)*(-a*(a + b))^(1/2))/a)*(-
a*(a + b))^(1/2))/(d*(a*b + b^2))
```

3.88. $\int \frac{\sin^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.89 $\int \frac{1}{a+b\sin^2(c+dx)}\,dx$

3.89.1 Optimal result

Integrand size = 14, antiderivative size = 36

$$\int \frac{1}{a+b\sin^2(c+dx)}\,dx=\frac{\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}d}$$

output
```
arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/d/a^(1/2)/(a+b)^(1/2)
```

3.89.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sin^2(c+dx)}\,dx=\frac{\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}d}$$

input
```
Integrate[(a + b*Sin[c + d*x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

3.89.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3660}$$

$$\frac{\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a+b}}$$

input
```
Int[(a + b*Sin[c + d*x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

3.89.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.89.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\sqrt{a(a+b)}}$	30
default	$\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\sqrt{a(a+b)}}$	30
risch	$-\frac{\ln\left(e^{2i(dx+c)}-\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,d}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia^2+2iab-2a\sqrt{-a^2-ab}-b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,d}$	174

input
```
int(1/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))
```

3.89.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(28) = 56.

Time = 0.28 (sec) , antiderivative size = 236, normalized size of antiderivative = 6.56

$$\int \frac{1}{a+b\sin^2(c+dx)}\,dx = \left[-\frac{\sqrt{-a^2-ab}\log\left(\frac{(8a^2+8ab+b^2)\cos(dx+c)^4-2(4a^2+5ab+b^2)\cos(dx+c)^2+4\left((2a+b)\cos(dx+c)^3-(a+b)\cos(dx+c)\right)\sqrt{-a^2-ab}\sin}{b^2\cos(dx+c)^4-2(ab+b^2)\cos(dx+c)^2+a^2+2ab+b^2}\right.}{4(a^2+ab)d}\right.$$

$$\left.-\frac{\arctan\left(\frac{(2a+b)\cos(dx+c)^2-a-b}{2\sqrt{a^2+ab}\cos(dx+c)\sin(dx+c)}\right)}{2\sqrt{a^2+ab}d}\right]$$

input
```
integrate(1/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^
2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*co
s(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*
x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))/((a^2 + a*b)
*d), -1/2*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*c
os(d*x + c)*sin(d*x + c)))/(sqrt(a^2 + a*b)*d)]
```

3.89.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16298 vs. $2(32) = 64$.

Time = 13.06 (sec) , antiderivative size = 16298, normalized size of antiderivative = 452.72

$$\int \frac{1}{a + b\sin^2(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x/sin(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((tan(c/2 + d
*x/2)/(2*d) - 1/(2*d*tan(c/2 + d*x/2)))/b, Eq(a, 0)), (2*tan(c/2 + d*x/2)/
(b*d*tan(c/2 + d*x/2)**2 - b*d), Eq(a, -b)), (x/a, Eq(b, 0)), (x/(a + b*si
n(c)**2), Eq(d, 0)), (6*a**3*b*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*log
(-sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(10*a**4*b*d
*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b*
*2)/a) - 2*a**4*d*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)
*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + 50*a**3*b**2*d*sqrt(-1 - 2*b/a
- 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 26*a**3*
b*d*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b
/a + 2*sqrt(a*b + b**2)/a) + 72*a**2*b**3*d*sqrt(-1 - 2*b/a - 2*sqrt(a*b +
 b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 56*a**2*b**2*d*sqrt(a*
b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt
(a*b + b**2)/a) + 32*a*b**4*d*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt
(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 32*a*b**3*d*sqrt(a*b + b**2)*sqrt(-1
 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a))
- 6*a**3*b*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*log(sqrt(-1 - 2*b/a + 2
*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(10*a**4*b*d*sqrt(-1 - 2*b/a - 2*
sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) - 2*a**4*d*sqr
t(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*sqrt(-1 - 2*b/a +...
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{1}{a+b\sin^2(c+dx)}\,dx = \frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}d}$$

input
```
integrate(1/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*d)
```

3.89.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(28) = 56.

Time = 0.44 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.78

$$\int \frac{1}{a+b\sin^2(c+dx)}\,dx = \frac{\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}d}$$

input
```
integrate(1/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b*
tan(d*x + c))/sqrt(a^2 + a*b)))/(sqrt(a^2 + a*b)*d)
```

3.89.9 Mupad [B] (verification not implemented)

Time = 14.95 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{1}{a+b\sin^2(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{\tan(c+d\,x)\,(a+b)}{\sqrt{a^2+b\,a}}\right)}{d\,\sqrt{a^2+b\,a}}$$

input
```
int(1/(a + b*sin(c + d*x)^2),x)
```

output
```
atan((tan(c + d*x)*(a + b))/(a*b + a^2)^(1/2))/(d*(a*b + a^2)^(1/2))
```

3.90 $\int \frac{\csc^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.90.1 Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{\csc^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{b\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}d}-\frac{\cot(c+dx)}{ad}$$

output
```
-cot(d*x+c)/a/d-b*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/d/(a+b)^(
1/2)
```

3.90.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{-\frac{b\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}}-\sqrt{a}\cot(c+dx)}{a^{3/2}d}$$

input
```
Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-((b*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a + b]) - Sqrt[a]*C
ot[c + d*x])/(a^(3/2)*d)
```

3.90.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3666, 359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^2\left(a+b\sin(c+dx)^2\right)}dx$$

$$\Big\downarrow \text{3666}$$

$$\frac{\int \frac{\cot^2(c+dx)\left(\tan^2(c+dx)+1\right)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{359}$$

$$\frac{-\frac{b\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{a}-\frac{\cot(c+dx)}{a}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{b\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}}-\frac{\cot(c+dx)}{a}}{d}$$

input
```
Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-((b*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a + b])) -
 Cot[c + d*x]/a)/d
```

3.90.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.90.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\frac{1}{a\tan(dx+c)}-\frac{b\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a\sqrt{a(a+b)}}}{d}$
default	$\frac{-\frac{1}{a\tan(dx+c)}-\frac{b\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a\sqrt{a(a+b)}}}{d}$
risch	$-\frac{2i}{ad\left(e^{2i(dx+c)}-1\right)}+\frac{b\ln\left(e^{2i(dx+c)}-\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,da}-\frac{b\ln\left(e^{2i(dx+c)}+\frac{2ia^2+2iab-2a\sqrt{-a^2-}}{b\sqrt{-a^2-}}\right.}{2\sqrt{-a^2-ab}\,da}$

input
```
int(csc(d*x+c)^2/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a/tan(d*x+c)-1/a*b/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b
))^(1/2)))
```

3.90.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(45) = 90.

Time = 0.29 (sec) , antiderivative size = 313, normalized size of antiderivative = 5.91

$$\int \frac{\csc^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \left[-\frac{\sqrt{-a^2-ab}b\log\left(\frac{(8a^2+8ab+b^2)\cos(dx+c)^4-2(4a^2+5ab+b^2)\cos(dx+c)^2-4\left((2a+b)\cos(dx+c)^3-(a+b)\cos(dx+c)\right)\sqrt{-a^2-ab}\,\mathrm{si}}{b^2\cos(dx+c)^4-2(ab+b^2)\cos(dx+c)^2+a^2+2ab+b^2}\right.}{4(a^3+a^2b)d\sin(dx+c)}\right.$$

input
```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(-a^2 - a*b)*b*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4
*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a + b)*cos(d*x + c)^3 - (a + b)
*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos
(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x +
 c) + 4*(a^2 + a*b)*cos(d*x + c))/((a^3 + a^2*b)*d*sin(d*x + c)), 1/2*(sqr
t(a^2 + a*b)*b*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a
*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) - 2*(a^2 + a*b)*cos(d*x + c))
/((a^3 + a^2*b)*d*sin(d*x + c))]
```

3.90.6 Sympy [F]

$$\int \frac{\csc^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\csc^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(csc(c + d*x)**2/(a + b*sin(c + d*x)**2), x)
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{\csc^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{b\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a} + \frac{1}{a\tan(dx+c)}}{d}$$

input
```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-(b*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a) + 1/(
a*tan(d*x + c)))/d
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.57

$$\int \frac{\csc^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)b}{\sqrt{a^2+ab}a} + \frac{1}{a\tan(dx+c)}}{d}$$

input
```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) +
b*tan(d*x + c))/sqrt(a^2 + a*b)))*b/(sqrt(a^2 + a*b)*a) + 1/(a*tan(d*x + c
)))/d
```

3.90.9 Mupad [B] (verification not implemented)

Time = 14.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{\csc^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\cot(c+d\,x)}{a\,d} - \frac{b\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)}{a^{3/2}\,d\,\sqrt{a+b}}$$

input
```
int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x)^2)),x)
```

output
```
- cot(c + d*x)/(a*d) - (b*atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2)))/(a^(
3/2)*d*(a + b)^(1/2))
```

3.91 $\int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.91.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx=\frac{b^2\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}d}-\frac{(a-b)\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3ad}$$

output
```
-(a-b)*cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a/d+b^2*arctan((a+b)^(1/2)*tan(d*
x+c)/a^(1/2))/a^(5/2)/d/(a+b)^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.55

$$\int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx=-\frac{(2a+b-b\cos(2(c+dx)))\csc^2(c+dx)\left(-3b^2\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)+\sqrt{a}\sqrt{a+b}\cot(c+dx)\,(2a-3}{6a^{5/2}\sqrt{a+b}d\,(b+a\csc^2(c+dx))}$$

input
```
Integrate[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

output
```
-1/6*((2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*(-3*b^2*ArcTan[(Sqrt[a
 + b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*Sqrt[a + b]*Cot[c + d*x]*(2*a - 3*b
 + a*Csc[c + d*x]^2)))/(a^(5/2)*Sqrt[a + b]*d*(b + a*Csc[c + d*x]^2))
```

3.91.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3666, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^4\left(a+b\sin(c+dx)^2\right)}dx$$

$$\Big\downarrow \text{3666}$$

$$\frac{\int \frac{\cot^4(c+dx)\left(\tan^2(c+dx)+1\right)^2}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(\frac{\cot^4(c+dx)}{a}+\frac{(a-b)\cot^2(c+dx)}{a^2}+\frac{b^2}{a^2\left((a+b)\tan^2(c+dx)+a\right)}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}}-\frac{(a-b)\cot(c+dx)}{a^2}-\frac{\cot^3(c+dx)}{3a}}{d}$$

input
```
Int[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

output
```
((b^2*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a + b]) -
((a - b)*Cot[c + d*x])/a^2 - Cot[c + d*x]^3/(3*a))/d
```

3.91.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.91.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{1}{3a\tan(dx+c)^3}-\frac{a-b}{a^2\tan(dx+c)}+\frac{b^2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a^2\sqrt{a(a+b)}}}{d}$
default	$\frac{-\frac{1}{3a\tan(dx+c)^3}-\frac{a-b}{a^2\tan(dx+c)}+\frac{b^2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a^2\sqrt{a(a+b)}}}{d}$
risch	$\frac{2i\left(3b\,e^{4i(dx+c)}+6a\,e^{2i(dx+c)}-6b\,e^{2i(dx+c)}-2a+3b\right)}{3d\,a^2\left(e^{2i(dx+c)}-1\right)^3}-\frac{b^2\ln\left(e^{2i(dx+c)}-\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,d\,a^2}+\frac{b^2\ln}{}$

input
```
int(csc(d*x+c)^4/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3/a/tan(d*x+c)^3-(a-b)/a^2/tan(d*x+c)+b^2/a^2/(a*(a+b))^(1/2)*arct
an((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
```

3.91.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(67) = 134$.

Time = 0.27 (sec) , antiderivative size = 451, normalized size of antiderivative = 5.86

$$\int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \left[-\frac{4\,(2\,a^3 - a^2b - 3\,ab^2)\cos(dx+c)^3 + 3\,(b^2\cos(dx+c)^2 - b^2)\sqrt{-a^2-ab}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4 - 2\,(\ldots}{\ldots}\right.}{12\,((a^4+a^3b)d\cos\ldots}\right.$$
$$-\frac{2\,(2\,a^3 - a^2b - 3\,ab^2)\cos(dx+c)^3 + 3\,(b^2\cos(dx+c)^2 - b^2)\sqrt{a^2+ab}\arctan\left(\frac{(2\,a+b)\cos(dx+c)^2 - a - b}{2\,\sqrt{a^2+ab}\cos(dx+c)\sin(dx+c)}\ldots\right.}{6\,((a^4+a^3b)d\cos(dx+c)^2 - (a^4+a^3b)d)\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/12*(4*(2*a^3 - a^2*b - 3*a*b^2)*cos(d*x + c)^3 + 3*(b^2*cos(d*x + c)^2
 - b^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*
a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*
cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(
d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x +
c) - 12*(a^3 - a*b^2)*cos(d*x + c))/(((a^4 + a^3*b)*d*cos(d*x + c)^2 - (a^
4 + a^3*b)*d)*sin(d*x + c)), -1/6*(2*(2*a^3 - a^2*b - 3*a*b^2)*cos(d*x + c
)^3 + 3*(b^2*cos(d*x + c)^2 - b^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*c
os(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*
x + c) - 6*(a^3 - a*b^2)*cos(d*x + c))/(((a^4 + a^3*b)*d*cos(d*x + c)^2 -
(a^4 + a^3*b)*d)*sin(d*x + c))]
```

3.91.6 Sympy [F]

$$\int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**4/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(csc(c + d*x)**4/(a + b*sin(c + d*x)**2), x)
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{3\,b^2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a^2} - \frac{3\,(a-b)\tan(dx+c)^2+a}{a^2\tan(dx+c)^3}}{3\,d}$$

input
```
integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/3*(3*b^2*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a
^2) - (3*(a - b)*tan(d*x + c)^2 + a)/(a^2*tan(d*x + c)^3))/d
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.44

$$\int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)b^2}{\sqrt{a^2+ab}a^2} - \frac{3\,a\tan(dx+c)^2-3\,b\tan(dx+c)^2+a}{a^2\tan(dx+c)^3}}{3\,d}$$

input
```
integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x +
c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*b^2/(sqrt(a^2 + a*b)*a^2) - (3*a*ta
n(d*x + c)^2 - 3*b*tan(d*x + c)^2 + a)/(a^2*tan(d*x + c)^3))/d
```

3.91.9 Mupad [B] (verification not implemented)

Time = 13.89 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{b^2\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)}{a^{5/2}\,d\,\sqrt{a+b}} - \frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\,(a-b)}{a^2}}{d\tan\left(c+d\,x\right)^3}$$

input
```
int(1/(sin(c + d*x)^4*(a + b*sin(c + d*x)^2)),x)
```

output
```
(b^2*atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2)))/(a^(5/2)*d*(a + b)^(1/2))
 - (1/(3*a) + (tan(c + d*x)^2*(a - b))/a^2)/(d*tan(c + d*x)^3)
```

3.91. $\int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.92 $\int \frac{\csc^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.92.1 Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{\csc^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{b^3\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a+b}d} - \frac{(a^2-ab+b^2)\cot(c+dx)}{a^3d} - \frac{(2a-b)\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{5ad}$$

output
```
-(a^2-a*b+b^2)*cot(d*x+c)/a^3/d-1/3*(2*a-b)*cot(d*x+c)^3/a^2/d-1/5*cot(d*x
+c)^5/a/d-b^3*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(7/2)/d/(a+b)^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 2.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.35

$$\int \frac{\csc^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{(2a+b-b\cos(2(c+dx)))\csc^2(c+dx)\left(15b^3\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)+\sqrt{a}\sqrt{a+b}\cot(c+dx)\,(8a^2-1\ldots\right.}{30a^{7/2}\sqrt{a+b}d\,(b+a\csc^2(c+dx))}$$

input
```
Integrate[Csc[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

output
```
-1/30*((2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*(15*b^3*ArcTan[(Sqrt[
a + b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*Sqrt[a + b]*Cot[c + d*x]*(8*a^2 -
10*a*b + 15*b^2 + a*(4*a - 5*b)*Csc[c + d*x]^2 + 3*a^2*Csc[c + d*x]^4)))/(
a^(7/2)*Sqrt[a + b]*d*(b + a*Csc[c + d*x]^2))
```

3.92.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3666, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^6\left(a+b\sin(c+dx)^2\right)}dx$$

$$\Big\downarrow \text{3666}$$

$$\frac{\int \frac{\cot^6(c+dx)\left(\tan^2(c+dx)+1\right)^3}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(\frac{\cot^6(c+dx)}{a}+\frac{(2a-b)\cot^4(c+dx)}{a^2}+\frac{(a^2-ba+b^2)\cot^2(c+dx)}{a^3}+\frac{b^3}{a^3(-((a+b)\tan^2(c+dx))-a)}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^3\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a+b}}-\frac{(2a-b)\cot^3(c+dx)}{3a^2}-\frac{(a^2-ab+b^2)\cot(c+dx)}{a^3}-\frac{\cot^5(c+dx)}{5a}}{d}$$

input
```
Int[Csc[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-((b^3*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(7/2)*Sqrt[a + b]))
 - ((a^2 - a*b + b^2)*Cot[c + d*x])/a^3 - ((2*a - b)*Cot[c + d*x]^3)/(3*a^
2) - Cot[c + d*x]^5/(5*a))/d
```

3.92.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.92.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-\frac{b^3 \arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a^3\sqrt{a(a+b)}}-\frac{1}{5a\tan(dx+c)^5}-\frac{2a-b}{3a^2\tan(dx+c)^3}-\frac{a^2-ab+b^2}{a^3\tan(dx+c)}}{d}$
default	$\frac{-\frac{b^3 \arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a^3\sqrt{a(a+b)}}-\frac{1}{5a\tan(dx+c)^5}-\frac{2a-b}{3a^2\tan(dx+c)^3}-\frac{a^2-ab+b^2}{a^3\tan(dx+c)}}{d}$
risch	$-\frac{2i\left(15b^2 e^{8i(dx+c)}+30ab\, e^{6i(dx+c)}-60b^2 e^{6i(dx+c)}+80a^2 e^{4i(dx+c)}-70ab\, e^{4i(dx+c)}+90b^2 e^{4i(dx+c)}-40a^2 e^{2i(dx+c)}+50a\right.}{15d\, a^3\left(e^{2i(dx+c)}-1\right)^5}$

input
```
int(csc(d*x+c)^6/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b^3/a^3/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/5
/a/tan(d*x+c)^5-1/3*(2*a-b)/a^2/tan(d*x+c)^3-(a^2-a*b+b^2)/a^3/tan(d*x+c))
```

3.92.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(97) = 194$.

Time = 0.29 (sec) , antiderivative size = 595, normalized size of antiderivative = 5.46

$$\int \frac{\csc^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

$$= \Bigg[-\frac{4\,(8\,a^4-2\,a^3b+5\,a^2b^2+15\,ab^3)\cos{(dx+c)}^5-20\,(4\,a^4-a^3b+a^2b^2+6\,ab^3)\cos{(dx+c)}^3+15\,\big(b^3\,\mathrm{co}}{}$$

$$-\frac{2\,(8\,a^4-2\,a^3b+5\,a^2b^2+15\,ab^3)\cos{(dx+c)}^5-10\,(4\,a^4-a^3b+a^2b^2+6\,ab^3)\cos{(dx+c)}^3-15\,\big(b^3\,\mathrm{co}}{30\,\big((a^5+a^4b)d\cos{(dx+c)}^4-2\,(}$$

input
```
integrate(csc(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/60*(4*(8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3)*cos(d*x + c)^5 - 20*(4*
a^4 - a^3*b + a^2*b^2 + 6*a*b^3)*cos(d*x + c)^3 + 15*(b^3*cos(d*x + c)^4 -
 2*b^3*cos(d*x + c)^2 + b^3)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*c
os(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a + b)*cos(
d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2
*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a
*b + b^2))*sin(d*x + c) + 60*(a^4 + a*b^3)*cos(d*x + c))/(((a^5 + a^4*b)*d
*cos(d*x + c)^4 - 2*(a^5 + a^4*b)*d*cos(d*x + c)^2 + (a^5 + a^4*b)*d)*sin(
d*x + c)), -1/30*(2*(8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3)*cos(d*x + c)^
5 - 10*(4*a^4 - a^3*b + a^2*b^2 + 6*a*b^3)*cos(d*x + c)^3 - 15*(b^3*cos(d*
x + c)^4 - 2*b^3*cos(d*x + c)^2 + b^3)*sqrt(a^2 + a*b)*arctan(1/2*((2*a +
b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*si
n(d*x + c) + 30*(a^4 + a*b^3)*cos(d*x + c))/(((a^5 + a^4*b)*d*cos(d*x + c)
^4 - 2*(a^5 + a^4*b)*d*cos(d*x + c)^2 + (a^5 + a^4*b)*d)*sin(d*x + c))]
```

3.92.6 Sympy [F]

$$\int \frac{\csc^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\csc^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**6/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(csc(c + d*x)**6/(a + b*sin(c + d*x)**2), x)
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

$$\int \frac{\csc^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{15\,b^3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a^3} + \frac{15\,(a^2-ab+b^2)\tan(dx+c)^4+5\,(2\,a^2-ab)\tan(dx+c)^2+3\,a^2}{a^3\tan(dx+c)^5}}{15\,d}$$

input
```
integrate(csc(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/15*(15*b^3*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a
)*a^3) + (15*(a^2 - a*b + b^2)*tan(d*x + c)^4 + 5*(2*a^2 - a*b)*tan(d*x +
c)^2 + 3*a^2)/(a^3*tan(d*x + c)^5))/d
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.42

$$\int \frac{\csc^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{15\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)b^3}{\sqrt{a^2+ab}a^3} + \frac{15\,a^2\tan(dx+c)^4-15\,ab\tan(dx+c)^4+15\,b^2\tan(dx+c)^4+10\,a^2\tan(}{a^3\tan(dx+c)^5}}{15\,d}$$

input
```
integrate(csc(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/15*(15*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x
 + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*b^3/(sqrt(a^2 + a*b)*a^3) + (15*
a^2*tan(d*x + c)^4 - 15*a*b*tan(d*x + c)^4 + 15*b^2*tan(d*x + c)^4 + 10*a^
2*tan(d*x + c)^2 - 5*a*b*tan(d*x + c)^2 + 3*a^2)/(a^3*tan(d*x + c)^5))/d
```

3.92.9 Mupad [B] (verification not implemented)

Time = 13.77 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.87

$$\int \frac{\csc^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\tan(c+d\,x)^4\,(a^2-a\,b+b^2)+\frac{a^2}{5}-\tan(c+d\,x)^2\,\left(\frac{a\,b}{3}-\frac{2\,a^2}{3}\right)}{a^3\,d\tan\left(c+d\,x\right)^5} - \frac{b^3\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)}{a^{7/2}\,d\,\sqrt{a+b}}$$

input
```
int(1/(sin(c + d*x)^6*(a + b*sin(c + d*x)^2)),x)
```

output
```
- (tan(c + d*x)^4*(a^2 - a*b + b^2) + a^2/5 - tan(c + d*x)^2*((a*b)/3 - (2
*a^2)/3))/(a^3*d*tan(c + d*x)^5) - (b^3*atan((tan(c + d*x)*(a + b)^(1/2))/
a^(1/2)))/(a^(7/2)*d*(a + b)^(1/2))
```

3.93 $\int \frac{\csc^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.93.1 Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{\csc^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{b^4\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}\sqrt{a+b}d} - \frac{(a-b)\left(a^2+b^2\right)\cot(c+dx)}{a^4d} - \frac{\left(3a^2-2ab+b^2\right)\cot^3(c+dx)}{3a^3d} - \frac{(3a-b)\cot^5(c+dx)}{5a^2d} - \frac{\cot^7(c+dx)}{7ad}$$

output
```
-(a-b)*(a^2+b^2)*cot(d*x+c)/a^4/d-1/3*(3*a^2-2*a*b+b^2)*cot(d*x+c)^3/a^3/d
-1/5*(3*a-b)*cot(d*x+c)^5/a^2/d-1/7*cot(d*x+c)^7/a/d+b^4*arctan((a+b)^(1/2
)*tan(d*x+c)/a^(1/2))/a^(9/2)/d/(a+b)^(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 5.82 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.98

$$\int \frac{\csc^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{b^4\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}\sqrt{a+b}d} - \frac{\cot(c+dx)\left(48a^3-56a^2b+70ab^2-105b^3+a(24a^2-28ab+35b^2)\csc^2(c+dx)+3a^2(6a-7b)\csc^4(c\right.}{105a^4d}$$

input
```
Integrate[Csc[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

output
```
(b^4*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(9/2)*Sqrt[a + b]*d) -
 (Cot[c + d*x]*(48*a^3 - 56*a^2*b + 70*a*b^2 - 105*b^3 + a*(24*a^2 - 28*a*
b + 35*b^2)*Csc[c + d*x]^2 + 3*a^2*(6*a - 7*b)*Csc[c + d*x]^4 + 15*a^3*Csc
[c + d*x]^6))/(105*a^4*d)
```

3.93.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3666, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^8(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^8\left(a+b\sin(c+dx)^2\right)}dx$$

$$\Big\downarrow \text{3666}$$

$$\frac{\int \frac{\cot^8(c+dx)\left(\tan^2(c+dx)+1\right)^4}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(\frac{\cot^8(c+dx)}{a}+\frac{(3a-b)\cot^6(c+dx)}{a^2}+\frac{\left(3a^2-2ba+b^2\right)\cot^4(c+dx)}{a^3}+\frac{(a-b)\left(a^2+b^2\right)\cot^2(c+dx)}{a^4}+\frac{b^4}{a^4\left((a+b)\tan^2(c+dx)+a\right)}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^4\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}\sqrt{a+b}}-\frac{(3a-b)\cot^5(c+dx)}{5a^2}-\frac{(a-b)\left(a^2+b^2\right)\cot(c+dx)}{a^4}-\frac{\left(3a^2-2ab+b^2\right)\cot^3(c+dx)}{3a^3}-\frac{\cot^7(c+dx)}{7a}}{d}$$

input
```
Int[Csc[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

output
```
((b^4*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(9/2)*Sqrt[a + b]) -
((a - b)*(a^2 + b^2)*Cot[c + d*x])/a^4 - ((3*a^2 - 2*a*b + b^2)*Cot[c + d*
x]^3)/(3*a^3) - ((3*a - b)*Cot[c + d*x]^5)/(5*a^2) - Cot[c + d*x]^7/(7*a))
/d
```

3.93.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.93.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\frac{b^4 \arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a^4\sqrt{a(a+b)}}-\frac{1}{7a\tan(dx+c)^7}-\frac{3a-b}{5a^2\tan(dx+c)^5}-\frac{3a^2-2ab+b^2}{3a^3\tan(dx+c)^3}-\frac{a^3-a^2b+a\,b^2-b^3}{a^4\tan(dx+c)}}{d}$
default	$\frac{\frac{b^4 \arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a^4\sqrt{a(a+b)}}-\frac{1}{7a\tan(dx+c)^7}-\frac{3a-b}{5a^2\tan(dx+c)^5}-\frac{3a^2-2ab+b^2}{3a^3\tan(dx+c)^3}-\frac{a^3-a^2b+a\,b^2-b^3}{a^4\tan(dx+c)}}{d}$
risch	$2i\left(105b^3e^{12i(dx+c)}+210a\,b^2e^{10i(dx+c)}-630b^3e^{10i(dx+c)}+560a^2b\,e^{8i(dx+c)}-910a\,b^2e^{8i(dx+c)}+1575b^3e^{8i(dx+c)}+1680a$

input
```
int(csc(d*x+c)^8/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^4/a^4/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/7/
a/tan(d*x+c)^7-1/5*(3*a-b)/a^2/tan(d*x+c)^5-1/3*(3*a^2-2*a*b+b^2)/a^3/tan(
d*x+c)^3-(a^3-a^2*b+a*b^2-b^3)/a^4/tan(d*x+c))
```

3.93.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(126) = 252$.

Time = 0.31 (sec) , antiderivative size = 789, normalized size of antiderivative = 5.64

$$\int \frac{\csc^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

$$= \left[-\frac{4\left(48\,a^5-8\,a^4b+14\,a^3b^2-35\,a^2b^3-105\,ab^4\right)\cos\left(dx+c\right)^7-28\left(24\,a^5-4\,a^4b+7\,a^3b^2-10\,a^2b^3-45\right.}{} \right.$$

$$-\frac{2\left(48\,a^5-8\,a^4b+14\,a^3b^2-35\,a^2b^3-105\,ab^4\right)\cos\left(dx+c\right)^7-14\left(24\,a^5-4\,a^4b+7\,a^3b^2-10\,a^2b^3-45\right.}{}$$

input
```
integrate(csc(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/420*(4*(48*a^5 - 8*a^4*b + 14*a^3*b^2 - 35*a^2*b^3 - 105*a*b^4)*cos(d*
x + c)^7 - 28*(24*a^5 - 4*a^4*b + 7*a^3*b^2 - 10*a^2*b^3 - 45*a*b^4)*cos(d
*x + c)^5 + 140*(6*a^5 - a^4*b + a^3*b^2 - a^2*b^3 - 9*a*b^4)*cos(d*x + c)
^3 + 105*(b^4*cos(d*x + c)^6 - 3*b^4*cos(d*x + c)^4 + 3*b^4*cos(d*x + c)^2
 - b^4)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*
a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*
cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(
d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x +
c) - 420*(a^5 - a*b^4)*cos(d*x + c))/(((a^6 + a^5*b)*d*cos(d*x + c)^6 - 3*
(a^6 + a^5*b)*d*cos(d*x + c)^4 + 3*(a^6 + a^5*b)*d*cos(d*x + c)^2 - (a^6 +
 a^5*b)*d)*sin(d*x + c)), -1/210*(2*(48*a^5 - 8*a^4*b + 14*a^3*b^2 - 35*a^
2*b^3 - 105*a*b^4)*cos(d*x + c)^7 - 14*(24*a^5 - 4*a^4*b + 7*a^3*b^2 - 10*
a^2*b^3 - 45*a*b^4)*cos(d*x + c)^5 + 70*(6*a^5 - a^4*b + a^3*b^2 - a^2*b^3
 - 9*a*b^4)*cos(d*x + c)^3 + 105*(b^4*cos(d*x + c)^6 - 3*b^4*cos(d*x + c)^
4 + 3*b^4*cos(d*x + c)^2 - b^4)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(
d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x +
 c) - 210*(a^5 - a*b^4)*cos(d*x + c))/(((a^6 + a^5*b)*d*cos(d*x + c)^6 - 3
*(a^6 + a^5*b)*d*cos(d*x + c)^4 + 3*(a^6 + a^5*b)*d*cos(d*x + c)^2 - (a^6
+ a^5*b)*d)*sin(d*x + c))]
```

3.93.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**8/(a+b*sin(d*x+c)**2),x)
```

output
```
Timed out
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.98

$$\int \frac{\csc^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{105\,b^4\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a^4} - \frac{105\,(a^3-a^2b+ab^2-b^3)\tan(dx+c)^6+35\,(3\,a^3-2\,a^2b+ab^2)\tan(dx+c)^4+15\,a^3+21\,(3\,a^3-a^2b)\tan(dx+c)^2}{a^4\tan(dx+c)^7}}{105\,d}$$

input
```
integrate(csc(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/105*(105*b^4*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*
a)*a^4) - (105*(a^3 - a^2*b + a*b^2 - b^3)*tan(d*x + c)^6 + 35*(3*a^3 - 2*
a^2*b + a*b^2)*tan(d*x + c)^4 + 15*a^3 + 21*(3*a^3 - a^2*b)*tan(d*x + c)^2
)/(a^4*tan(d*x + c)^7))/d
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.54

$$\int \frac{\csc^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{105\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)b^4}{\sqrt{a^2+ab}a^4} - \frac{105a^3\tan(dx+c)^6-105a^2b\tan(dx+c)^6+105ab^2\tan(dx+c)^6-105b^3}{}}{105d}$$

input
```
integrate(csc(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/105*(105*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*
x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*b^4/(sqrt(a^2 + a*b)*a^4) - (10
5*a^3*tan(d*x + c)^6 - 105*a^2*b*tan(d*x + c)^6 + 105*a*b^2*tan(d*x + c)^6
 - 105*b^3*tan(d*x + c)^6 + 105*a^3*tan(d*x + c)^4 - 70*a^2*b*tan(d*x + c)
^4 + 35*a*b^2*tan(d*x + c)^4 + 63*a^3*tan(d*x + c)^2 - 21*a^2*b*tan(d*x +
c)^2 + 15*a^3)/(a^4*tan(d*x + c)^7))/d
```

3.93.9 Mupad [B] (verification not implemented)

Time = 15.18 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.93

$$\int \frac{\csc^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{b^4\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)}{a^{9/2}\,d\,\sqrt{a+b}} - \frac{\tan(c+d\,x)^4\left(a^3-\frac{2\,a^2\,b}{3}+\frac{a\,b^2}{3}\right)-\tan(c+d\,x)^2\left(\frac{a^2\,b}{5}-\frac{3\,a^3}{5}\right)+\tan(c+d\,x)^6\left(a^3-a^2\,b+a\,b^2-b^3\right)+}{a^4\,d\tan\left(c+d\,x\right)^7}$$

input
```
int(1/(sin(c + d*x)^8*(a + b*sin(c + d*x)^2)),x)
```

output
```
(b^4*atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2)))/(a^(9/2)*d*(a + b)^(1/2))
 - (tan(c + d*x)^4*((a*b^2)/3 - (2*a^2*b)/3 + a^3) - tan(c + d*x)^2*((a^2*
b)/5 - (3*a^3)/5) + tan(c + d*x)^6*(a*b^2 - a^2*b + a^3 - b^3) + a^3/7)/(a
^4*d*tan(c + d*x)^7)
```

3.93. $\int \frac{\csc^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.94 $\int \frac{\sin^7(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

3.94.1 Optimal result

Integrand size = 23, antiderivative size = 128

$$\int \frac{\sin^7(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{a^2(5a+6b)\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{7/2}(a+b)^{3/2}d} + \frac{(2a-b)\cos(c+dx)}{b^3d} + \frac{\cos^3(c+dx)}{3b^2d} + \frac{a^3\cos(c+dx)}{2b^3(a+b)d\left(a+b-b\cos^2(c+dx)\right)}$$

output
```
-1/2*a^2*(5*a+6*b)*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/b^(7/2)/(a+b)^(
3/2)/d+(2*a-b)*cos(d*x+c)/b^3/d+1/3*cos(d*x+c)^3/b^2/d+1/2*a^3*cos(d*x+c)/
b^3/(a+b)/d/(a+b-b*cos(d*x+c)^2)
```

3.94.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.52

$$\int \frac{\sin^7(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{-\frac{6a^2(5a+6b)\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}} - \frac{6a^2(5a+6b)\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}} + \sqrt{b}\Big(\cos(c+dx)\Big(24a-9b+\frac{}{(a-}}{12b^{7/2}d}$$

input
```
Integrate[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
((-6*a^2*(5*a + 6*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a
 - b]])/(-a - b)^(3/2) - (6*a^2*(5*a + 6*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Ta
n[(c + d*x)/2])/Sqrt[-a - b]])/(-a - b)^(3/2) + Sqrt[b]*(Cos[c + d*x]*(24*
a - 9*b + (12*a^3)/((a + b)*(2*a + b - b*Cos[2*(c + d*x)]))) + b*Cos[3*(c
+ d*x)]))/(12*b^(7/2)*d)
```

3.94.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3665, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^7}{\left(a+b\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{\left(1-\cos^2(c+dx)\right)^3}{\left(-b\cos^2(c+dx)+a+b\right)^2}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$-\frac{\int \left(-\frac{\cos^2(c+dx)}{b^2}+\frac{a^2(2a+3b)-3a^2b\cos^2(c+dx)}{b^3\left(-b\cos^2(c+dx)+a+b\right)^2}-\frac{2a-b}{b^3}\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{a^3\cos(c+dx)}{2b^3(a+b)\left(a-b\cos^2(c+dx)+b\right)}+\frac{a^2(5a+6b)\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{7/2}(a+b)^{3/2}}-\frac{(2a-b)\cos(c+dx)}{b^3}-\frac{\cos^3(c+dx)}{3b^2}}{d}$$

input
```
Int[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
-(((a^2*(5*a + 6*b)*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(2*b^(7/2
)*(a + b)^(3/2)) - ((2*a - b)*Cos[c + d*x])/b^3 - Cos[c + d*x]^3/(3*b^2) -
 (a^3*Cos[c + d*x])/(2*b^3*(a + b)*(a + b - b*Cos[c + d*x]^2)))/d)
```

3.94.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.94.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{\frac{b\left(\cos^{3}(dx+c)\right)}{3}+2\cos(dx+c)a-\cos(dx+c)b}{b^{3}}-\frac{a^{2}\left(-\frac{a\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^{2}(dx+c)\right)\right)}+\frac{(5a+6b)\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{b^{3}}}{d}$
default	$\frac{\frac{\frac{b\left(\cos^{3}(dx+c)\right)}{3}+2\cos(dx+c)a-\cos(dx+c)b}{b^{3}}-\frac{a^{2}\left(-\frac{a\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^{2}(dx+c)\right)\right)}+\frac{(5a+6b)\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{b^{3}}}{d}$
risch	$\frac{e^{3i(dx+c)}}{24b^{2}d}+\frac{a\,e^{i(dx+c)}}{b^{3}d}-\frac{3\,e^{i(dx+c)}}{8b^{2}d}+\frac{e^{-i(dx+c)}a}{b^{3}d}-\frac{3\,e^{-i(dx+c)}}{8b^{2}d}+\frac{e^{-3i(dx+c)}}{24b^{2}d}-\frac{a^{3}\left(e^{3i(dx+c)}+\right.}{b^{3}(a+b)d\left(b\,e^{4i(dx+c)}-4a\,e^{2i\right.}}$

input
```
int(sin(d*x+c)^7/(a+b*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^3*(1/3*b*cos(d*x+c)^3+2*cos(d*x+c)*a-cos(d*x+c)*b)-1/b^3*a^2*(-1/
2*a/(a+b)*cos(d*x+c)/(a+b-b*cos(d*x+c)^2)+1/2*(5*a+6*b)/(a+b)/((a+b)*b)^(1
/2)*arctanh(b*cos(d*x+c)/((a+b)*b)^(1/2))))
```

3.94.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(117) = 234$.

Time = 0.31 (sec) , antiderivative size = 529, normalized size of antiderivative = 4.13

$$\int \frac{\sin^7(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$$
$$= \left[\frac{4\,(a^2b^3+2\,ab^4+b^5)\cos{(dx+c)}^5+4\,(5\,a^3b^2+6\,a^2b^3-3\,ab^4-4\,b^5)\cos{(dx+c)}^3-3\,\left(5\,a^4+11\,a^3b+6\right.}{12\,\left((a^2b^5+2\,ab\right.}\right.$$

input
```
integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*(a^2*b^3 + 2*a*b^4 + b^5)*cos(d*x + c)^5 + 4*(5*a^3*b^2 + 6*a^2*b
^3 - 3*a*b^4 - 4*b^5)*cos(d*x + c)^3 - 3*(5*a^4 + 11*a^3*b + 6*a^2*b^2 - (
5*a^3*b + 6*a^2*b^2)*cos(d*x + c)^2)*sqrt(a*b + b^2)*log(-(b*cos(d*x + c)^
2 - 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) -
6*(5*a^4*b + 11*a^3*b^2 + 6*a^2*b^3 - 2*a*b^4 - 2*b^5)*cos(d*x + c))/((a^2
*b^5 + 2*a*b^6 + b^7)*d*cos(d*x + c)^2 - (a^3*b^4 + 3*a^2*b^5 + 3*a*b^6 +
b^7)*d), 1/6*(2*(a^2*b^3 + 2*a*b^4 + b^5)*cos(d*x + c)^5 + 2*(5*a^3*b^2 +
6*a^2*b^3 - 3*a*b^4 - 4*b^5)*cos(d*x + c)^3 - 3*(5*a^4 + 11*a^3*b + 6*a^2*
b^2 - (5*a^3*b + 6*a^2*b^2)*cos(d*x + c)^2)*sqrt(-a*b - b^2)*arctan(sqrt(-
a*b - b^2)*cos(d*x + c)/(a + b)) - 3*(5*a^4*b + 11*a^3*b^2 + 6*a^2*b^3 - 2
*a*b^4 - 2*b^5)*cos(d*x + c))/((a^2*b^5 + 2*a*b^6 + b^7)*d*cos(d*x + c)^2
- (a^3*b^4 + 3*a^2*b^5 + 3*a*b^6 + b^7)*d)]
```

3.94.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**7/(a+b*sin(d*x+c)**2)**2,x)
```

output
```
Timed out
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.20

$$\int \frac{\sin^7(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{\frac{6\,a^3\cos(dx+c)}{a^2b^3+2\,ab^4+b^5-(ab^4+b^5)\cos(dx+c)^2}+\frac{3\,(5\,a+6\,b)a^2\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{(ab^3+b^4)\sqrt{(a+b)b}}+\frac{4\left(b\cos(dx+c)^3+3\,(2\,a-b)\cos(dx+c)\right)}{b^3}}{12\,d}$$

input
```
integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/12*(6*a^3*cos(d*x + c)/(a^2*b^3 + 2*a*b^4 + b^5 - (a*b^4 + b^5)*cos(d*x
+ c)^2) + 3*(5*a + 6*b)*a^2*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(
d*x + c) + sqrt((a + b)*b)))/((a*b^3 + b^4)*sqrt((a + b)*b)) + 4*(b*cos(d*
x + c)^3 + 3*(2*a - b)*cos(d*x + c))/b^3)/d
```

3.94.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(117) = 234$.

Time = 0.48 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.52

$$\int \frac{\sin^7(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{\frac{3\,(5\,a^3+6\,a^2b)\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{(ab^3+b^4)\sqrt{-ab-b^2}}+\frac{6\left(a^3-\frac{a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{(ab^3+b^4)\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}-\frac{8\left(3\,a-b-\frac{6}{\cdots}\right.}{\cdots}}{6\,d}$$

input
```
integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/6*(3*(5*a^3 + 6*a^2*b)*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)
*cos(d*x + c) + sqrt(-a*b - b^2)))/((a*b^3 + b^4)*sqrt(-a*b - b^2)) + 6*(a
^3 - a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a^2*b*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1))/((a*b^3 + b^4)*(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2)) - 8*(3*a - b - 6*a*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) + 3*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a*(cos(d*x + c)
 - 1)^2/(cos(d*x + c) + 1)^2)/(b^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1)
- 1)^3))/d
```

3.94.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.96

$$\int \frac{\sin^7(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{\cos\left(c+d\,x\right)\,\left(\frac{2\,\left(a+b\right)}{b^3}-\frac{3}{b^2}\right)}{d}+\frac{{\cos\left(c+d\,x\right)}^3}{3\,b^2\,d} + \frac{a^3\,\cos\left(c+d\,x\right)}{2\,d\,\left(a+b\right)\,\left(-b^4\,{\cos\left(c+d\,x\right)}^2+b^4+a\,b^3\right)} - \frac{a^2\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos\left(c+d\,x\right)}{\sqrt{a+b}}\right)\,\left(5\,a+6\,b\right)}{2\,b^{7/2}\,d\,{\left(a+b\right)}^{3/2}}$$

input
```
int(sin(c + d*x)^7/(a + b*sin(c + d*x)^2)^2,x)
```

output
```
(cos(c + d*x)*((2*(a + b))/b^3 - 3/b^2))/d + cos(c + d*x)^3/(3*b^2*d) + (a
^3*cos(c + d*x))/(2*d*(a + b)*(a*b^3 + b^4 - b^4*cos(c + d*x)^2)) - (a^2*a
tanh((b^(1/2)*cos(c + d*x))/(a + b)^(1/2))*(5*a + 6*b))/(2*b^(7/2)*d*(a +
b)^(3/2))
```

3.95 $\int \frac{\sin^5(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$

3.95.1 Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{\sin^5(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \frac{a(3a+4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}d} - \frac{\cos(c+dx)}{b^2d} - \frac{a^2\cos(c+dx)}{2b^2(a+b)d\,(a+b-b\cos^2(c+dx))}$$

output
```
1/2*a*(3*a+4*b)*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/b^(5/2)/(a+b)^(3/2
)/d-cos(d*x+c)/b^2/d-1/2*a^2*cos(d*x+c)/b^2/(a+b)/d/(a+b-b*cos(d*x+c)^2)
```

3.95.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.69

$$\int \frac{\sin^5(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \frac{\frac{a(3a+4b)\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}} + \frac{a(3a+4b)\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}} + 2\sqrt{b}\cos(c+dx)\left(-1-\frac{a^2}{(a+b)(2a+b-b\cos}\right.}{2b^{5/2}d}$$

input
```
Integrate[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
((a*(3*a + 4*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]
])/(-a - b)^(3/2) + (a*(3*a + 4*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*
x)/2])/Sqrt[-a - b]])/(-a - b)^(3/2) + 2*Sqrt[b]*Cos[c + d*x]*(-1 - a^2/((
a + b)*(2*a + b - b*Cos[2*(c + d*x)]))))/(2*b^(5/2)*d)
```

3.95.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3665, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(c+dx)}{(a+b\sin^2(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^5}{(a+b\sin(c+dx)^2)^2}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{(1-\cos^2(c+dx))^2}{(-b\cos^2(c+dx)+a+b)^2}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$-\frac{\int \left(\frac{1}{b^2}-\frac{a(a+2b)-2ab\cos^2(c+dx)}{b^2(-b\cos^2(c+dx)+a+b)^2}\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{a^2\cos(c+dx)}{2b^2(a+b)(a-b\cos^2(c+dx)+b)}-\frac{a(3a+4b)\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}}+\frac{\cos(c+dx)}{b^2}}{d}$$

input
```
Int[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
-((-1/2*(a*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(b^(5/
2)*(a + b)^(3/2)) + Cos[c + d*x]/b^2 + (a^2*Cos[c + d*x])/(2*b^2*(a + b)*(
a + b - b*Cos[c + d*x]^2)))/d)
```

3.95.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.95.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-\frac{\cos(dx+c)}{b^2}+\frac{a\left(-\frac{a\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^2(dx+c)\right)\right)}+\frac{(3a+4b)\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{b^2}}{d}$
default	$\frac{-\frac{\cos(dx+c)}{b^2}+\frac{a\left(-\frac{a\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^2(dx+c)\right)\right)}+\frac{(3a+4b)\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{b^2}}{d}$
risch	$-\frac{e^{i(dx+c)}}{2b^2d}-\frac{e^{-i(dx+c)}}{2b^2d}+\frac{a^2\left(e^{3i(dx+c)}+e^{i(dx+c)}\right)}{b^2(a+b)d\left(b\,e^{4i(dx+c)}-4a\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}+b\right)}+\frac{3ia^2\ln\left(e^{2i(dx+c)}+\frac{2i(a+b)e^{i(dx}}{\sqrt{-ab-b^2}}\right.}{4\sqrt{-ab-b^2}\,(a+b)d\,b^2}$

input
```
int(sin(d*x+c)^5/(a+b*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-cos(d*x+c)/b^2+a/b^2*(-1/2*a/(a+b)*cos(d*x+c)/(a+b-b*cos(d*x+c)^2)+1
/2*(3*a+4*b)/(a+b)/((a+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/((a+b)*b)^(1/2))))
```

3.95.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(93) = 186$.

Time = 0.28 (sec) , antiderivative size = 427, normalized size of antiderivative = 4.19

$$\int \frac{\sin^5(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \left[-\frac{4\,(a^2b^2+2\,ab^3+b^4)\cos{(dx+c)}^3+\left(3\,a^3+7\,a^2b+4\,ab^2-(3\,a^2b+4\,ab^2)\cos{(dx+c)}^2\right)\sqrt{ab+b^2}\log}{4\,\left((a^2b^4+2\,ab^5+b^6)d\cos{(dx+c)}^2-(a^3b^3+\right.} \right.$$
$$-\frac{2\,(a^2b^2+2\,ab^3+b^4)\cos{(dx+c)}^3-\left(3\,a^3+7\,a^2b+4\,ab^2-(3\,a^2b+4\,ab^2)\cos{(dx+c)}^2\right)\sqrt{-ab-b^2}\,\mathrm{arc}}{2\,\left((a^2b^4+2\,ab^5+b^6)d\cos{(dx+c)}^2-(a^3b^3+3\,a^2b\right.}$$

input
```
integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*(a^2*b^2 + 2*a*b^3 + b^4)*cos(d*x + c)^3 + (3*a^3 + 7*a^2*b + 4*a
*b^2 - (3*a^2*b + 4*a*b^2)*cos(d*x + c)^2)*sqrt(a*b + b^2)*log((b*cos(d*x
+ c)^2 + 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b
)) - 2*(3*a^3*b + 7*a^2*b^2 + 6*a*b^3 + 2*b^4)*cos(d*x + c))/((a^2*b^4 + 2
*a*b^5 + b^6)*d*cos(d*x + c)^2 - (a^3*b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6)*d),
 -1/2*(2*(a^2*b^2 + 2*a*b^3 + b^4)*cos(d*x + c)^3 - (3*a^3 + 7*a^2*b + 4*a
*b^2 - (3*a^2*b + 4*a*b^2)*cos(d*x + c)^2)*sqrt(-a*b - b^2)*arctan(sqrt(-a
*b - b^2)*cos(d*x + c)/(a + b)) - (3*a^3*b + 7*a^2*b^2 + 6*a*b^3 + 2*b^4)*
cos(d*x + c))/((a^2*b^4 + 2*a*b^5 + b^6)*d*cos(d*x + c)^2 - (a^3*b^3 + 3*a
^2*b^4 + 3*a*b^5 + b^6)*d)]
```

3.95.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**5/(a+b*sin(d*x+c)**2)**2,x)
```

output
```
Timed out
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.28

$$\int \frac{\sin^5(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\frac{2a^2\cos(dx+c)}{a^2b^2+2ab^3+b^4-(ab^3+b^4)\cos(dx+c)^2} + \frac{(3a+4b)a\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{(ab^2+b^3)\sqrt{(a+b)b}} + \frac{4\cos(dx+c)}{b^2}}{4d}$$

input
```
integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*a^2*cos(d*x + c)/(a^2*b^2 + 2*a*b^3 + b^4 - (a*b^3 + b^4)*cos(d*x
+ c)^2) + (3*a + 4*b)*a*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x
+ c) + sqrt((a + b)*b)))/((a*b^2 + b^3)*sqrt((a + b)*b)) + 4*cos(d*x + c)/
b^2)/d
```

3.95.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(93) = 186.

Time = 0.36 (sec) , antiderivative size = 342, normalized size of antiderivative = 3.35

$$\int \frac{\sin^5(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\frac{(3a^2+4ab)\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{(ab^2+b^3)\sqrt{-ab-b^2}} + \frac{2\left(3a^2+2ab-\frac{6a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{14ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3a^2(\cos(dx\cdots}{(\cos(dx+\cdots}\cdots\right.}{(ab^2+b^3)\left(a-\frac{3a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{4b(\cos(dx+\cdots}{(\cos(dx+c\cdots}\cdots\right.}}{2d}$$

input
```
integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((3*a^2 + 4*a*b)*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*co
s(d*x + c) + sqrt(-a*b - b^2)))/((a*b^2 + b^3)*sqrt(-a*b - b^2)) + 2*(3*a^
2 + 2*a*b - 6*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 14*a*b*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 8*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 + 3*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4*a*b*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2)/((a*b^2 + b^3)*(a - 3*a*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) - 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 - a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)))/d
```

3.95.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.93

$$\int \frac{\sin^5(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \frac{a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos(c+d\,x)}{\sqrt{a+b}}\right)\,(3\,a+4\,b)}{2\,b^{5/2}\,d\,(a+b)^{3/2}} - \frac{a^2\,\cos(c+d\,x)}{2\,d\,(a+b)\,\left(-b^3\cos(c+d\,x)^2+b^3+a\,b^2\right)} - \frac{\cos(c+d\,x)}{b^2\,d}$$

input
```
int(sin(c + d*x)^5/(a + b*sin(c + d*x)^2)^2,x)
```

output
```
(a*atanh((b^(1/2)*cos(c + d*x))/(a + b)^(1/2))*(3*a + 4*b))/(2*b^(5/2)*d*(
a + b)^(3/2)) - (a^2*cos(c + d*x))/(2*d*(a + b)*(a*b^2 + b^3 - b^3*cos(c +
 d*x)^2)) - cos(c + d*x)/(b^2*d)
```

3.96 $\int \frac{\sin^3(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$

3.96.1 Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{\sin^3(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = -\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}d} + \frac{a\cos(c+dx)}{2b(a+b)d\,(a+b-b\cos^2(c+dx))}$$

output
```
-1/2*(a+2*b)*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/b^(3/2)/(a+b)^(3/2)/d
+1/2*a*cos(d*x+c)/b/(a+b)/d/(a+b-b*cos(d*x+c)^2)
```

3.96.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.93

$$\int \frac{\sin^3(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \frac{\frac{(a+2b)\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}} + \frac{(a+2b)\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}} + \frac{2a\sqrt{b}\cos(c+dx)}{2a+b-b\cos(2(c+dx))}}{2b^{3/2}(a+b)d}$$

input
```
Integrate[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
(((a + 2*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]])/S
qrt[-a - b] + ((a + 2*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqr
t[-a - b]])/Sqrt[-a - b] + (2*a*Sqrt[b]*Cos[c + d*x])/(2*a + b - b*Cos[2*(
c + d*x)]))/(2*b^(3/2)*(a + b)*d)
```

3.96.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3665, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^3}{\left(a+b\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{1-\cos^2(c+dx)}{\left(-b\cos^2(c+dx)+a+b\right)^2}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$-\frac{\frac{(a+2b)\int \frac{1}{-b\cos^2(c+dx)+a+b}d\cos(c+dx)}{2b(a+b)}-\frac{a\cos(c+dx)}{2b(a+b)\left(a-b\cos^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}}-\frac{a\cos(c+dx)}{2b(a+b)\left(a-b\cos^2(c+dx)+b\right)}}{d}$$

input
```
Int[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
-((((a + 2*b)*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(2*b^(3/2)*(a +
 b)^(3/2)) - (a*Cos[c + d*x])/(2*b*(a + b)*(a + b - b*Cos[c + d*x]^2)))/d)
```

3.96.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.96.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\frac{a\cos(dx+c)}{2(a+b)b\left(a+b-b\left(\cos^2(dx+c)\right)\right)}-\frac{(a+2b)\,\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{2(a+b)b\sqrt{(a+b)b}}}{d}$
default	$\frac{\frac{a\cos(dx+c)}{2(a+b)b\left(a+b-b\left(\cos^2(dx+c)\right)\right)}-\frac{(a+2b)\,\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{2(a+b)b\sqrt{(a+b)b}}}{d}$
risch	$-\frac{a\left(e^{3i(dx+c)}+e^{i(dx+c)}\right)}{b(a+b)d\left(b\,e^{4i(dx+c)}-4a\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}+b\right)}-\frac{ia\ln\left(e^{2i(dx+c)}+\frac{2i(a+b)e^{i(dx+c)}}{\sqrt{-ab-b^2}}+1\right)}{4\sqrt{-ab-b^2}\,(a+b)db}-\frac{i\ln\left(e^{2i(dx+c)}+\frac{2i}{}\right.}{2\sqrt{-ab-}}$

input
```
int(sin(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*a/(a+b)/b*cos(d*x+c)/(a+b-b*cos(d*x+c)^2)-1/2*(a+2*b)/(a+b)/b/((a
+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/((a+b)*b)^(1/2)))
```

3.96.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(74) = 148.

Time = 0.28 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.94

$$\int \frac{\sin^3(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$$
$$= \left[\frac{\left((ab+2\,b^2)\cos{(dx+c)}^2-a^2-3\,ab-2\,b^2\right)\sqrt{ab+b^2}\log\left(-\frac{b\cos(dx+c)^2-2\,\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)-2\,(a^2b+}{4\,\left((a^2b^3+2\,ab^4+b^5)d\cos{(dx+c)}^2-(a^3b^2+3\,a^2b^3+3\,ab^4+b^5)d\right)}\right.$$

input
```
integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(((a*b + 2*b^2)*cos(d*x + c)^2 - a^2 - 3*a*b - 2*b^2)*sqrt(a*b + b^2)
*log(-(b*cos(d*x + c)^2 - 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d
*x + c)^2 - a - b)) - 2*(a^2*b + a*b^2)*cos(d*x + c))/((a^2*b^3 + 2*a*b^4
+ b^5)*d*cos(d*x + c)^2 - (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d), 1/2*((
(a*b + 2*b^2)*cos(d*x + c)^2 - a^2 - 3*a*b - 2*b^2)*sqrt(-a*b - b^2)*arcta
n(sqrt(-a*b - b^2)*cos(d*x + c)/(a + b)) - (a^2*b + a*b^2)*cos(d*x + c))/(
(a^2*b^3 + 2*a*b^4 + b^5)*d*cos(d*x + c)^2 - (a^3*b^2 + 3*a^2*b^3 + 3*a*b^
4 + b^5)*d)]
```

3.96.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)**2)**2,x)
```

output
```
Timed out
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.34

$$\int \frac{\sin^3(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \frac{\frac{2\,a\cos(dx+c)}{a^2b+2\,ab^2+b^3-(ab^2+b^3)\cos(dx+c)^2} + \frac{(a+2\,b)\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}(ab+b^2)}}{4\,d}$$

input
```
integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/4*(2*a*cos(d*x + c)/(a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^
2) + (a + 2*b)*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sq
rt((a + b)*b)))/(sqrt((a + b)*b)*(a*b + b^2)))/d
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12

$$\int \frac{\sin^3(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \frac{(a+2\,b)\arctan\left(\frac{b\cos(dx+c)}{\sqrt{-ab-b^2}}\right)}{2\,(ab+b^2)\sqrt{-ab-b^2}d} - \frac{a\cos(dx+c)}{2\left(b\cos(dx+c)^2-a-b\right)(ab+b^2)d}$$

input
```
integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(a + 2*b)*arctan(b*cos(d*x + c)/sqrt(-a*b - b^2))/((a*b + b^2)*sqrt(-a
*b - b^2)*d) - 1/2*a*cos(d*x + c)/((b*cos(d*x + c)^2 - a - b)*(a*b + b^2)*
d)
```

3.96.9 Mupad [B] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \frac{a\,\cos(c+d\,x)}{2\,b\,d\,(a+b)\,\left(-b\cos(c+d\,x)^2+a+b\right)} - \frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos(c+d\,x)}{\sqrt{a+b}}\right)\,(a+2\,b)}{2\,b^{3/2}\,d\,(a+b)^{3/2}}$$

input
```
int(sin(c + d*x)^3/(a + b*sin(c + d*x)^2)^2,x)
```

output
```
(a*cos(c + d*x))/(2*b*d*(a + b)*(a + b - b*cos(c + d*x)^2)) - (atanh((b^(1
/2)*cos(c + d*x))/(a + b)^(1/2))*(a + 2*b))/(2*b^(3/2)*d*(a + b)^(3/2))
```

3.96. $\int \frac{\sin^3(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$

3.97 $\int \frac{\sin(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$

3.97.1 Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \frac{\sin(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}d} - \frac{\cos(c+dx)}{2(a+b)d\,(a+b-b\cos^2(c+dx))}$$

output
```
-1/2*cos(d*x+c)/(a+b)/d/(a+b-b*cos(d*x+c)^2)-1/2*arctanh(cos(d*x+c)*b^(1/2
)/(a+b)^(1/2))/(a+b)^(3/2)/d/b^(1/2)
```

3.97.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.01

$$\int \frac{\sin(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \frac{\frac{\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}\sqrt{b}} - \frac{2\cos(c+dx)}{2a+b-b\cos(2(c+dx))}}{2(a+b)d}$$

input
```
Integrate[Sin[c + d*x]/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]]/(Sqrt[-a - b]
*Sqrt[b]) + ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]]/(S
qrt[-a - b]*Sqrt[b]) - (2*Cos[c + d*x])/(2*a + b - b*Cos[2*(c + d*x)]))/(2
*(a + b)*d)
```

3.97.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3665, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(c+dx)}{\left(a+b\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{ 3665}$$

$$-\frac{\int \frac{1}{\left(-b\cos^2(c+dx)+a+b\right)^2}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{ 215}$$

$$-\frac{\frac{\int \frac{1}{-b\cos^2(c+dx)+a+b}d\cos(c+dx)}{2(a+b)}+\frac{\cos(c+dx)}{2(a+b)\left(a-b\cos^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}}+\frac{\cos(c+dx)}{2(a+b)\left(a-b\cos^2(c+dx)+b\right)}}{d}$$

input
```
Int[Sin[c + d*x]/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
-((ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)) +
 Cos[c + d*x]/(2*(a + b)*(a + b - b*Cos[c + d*x]^2)))/d)
```

3.97.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.97.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-\frac{\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^2(dx+c)\right)\right)}-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}}{d}$
default	$\frac{-\frac{\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^2(dx+c)\right)\right)}-\frac{\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}}{d}$
risch	$\frac{e^{3i(dx+c)}+e^{i(dx+c)}}{(a+b)d\left(b\,e^{4i(dx+c)}-4a\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}+b\right)}+\frac{i\ln\left(e^{2i(dx+c)}-\frac{2i(a+b)e^{i(dx+c)}}{\sqrt{-ab-b^2}}+1\right)}{4\sqrt{-ab-b^2}\,(a+b)d}-\frac{i\ln\left(e^{2i(dx+c)}+\frac{2i(a+b}{\sqrt{-}}\right.}{4\sqrt{-ab-b^2}\,(a}$

input
```
int(sin(d*x+c)/(a+b*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*cos(d*x+c)/(a+b)/(a+b-b*cos(d*x+c)^2)-1/2/(a+b)/((a+b)*b)^(1/2)*
arctanh(b*cos(d*x+c)/((a+b)*b)^(1/2)))
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.81

$$\int \frac{\sin(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$$
$$= \left[\frac{\left(b\cos{(dx+c)}^2-a-b\right)\sqrt{ab+b^2}\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)+2\left(ab+b^2\right)\cos{(dx+c)}}{4\left((a^2b^2+2ab^3+b^4)d\cos{(dx+c)}^2-(a^3b+3a^2b^2+3ab^3+b^4)d\right)}, \frac{(b\,c}{}\right.$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*((b*cos(d*x + c)^2 - a - b)*sqrt(a*b + b^2)*log(-(b*cos(d*x + c)^2 -
2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) + 2*(a
*b + b^2)*cos(d*x + c))/((a^2*b^2 + 2*a*b^3 + b^4)*d*cos(d*x + c)^2 - (a^3
*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d), 1/2*((b*cos(d*x + c)^2 - a - b)*sqrt(-
a*b - b^2)*arctan(sqrt(-a*b - b^2)*cos(d*x + c)/(a + b)) + (a*b + b^2)*cos
(d*x + c))/((a^2*b^2 + 2*a*b^3 + b^4)*d*cos(d*x + c)^2 - (a^3*b + 3*a^2*b^
2 + 3*a*b^3 + b^4)*d)]
```

3.97.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)**2)**2,x)
```

output
```
Timed out
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \frac{\sin(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{\frac{2\cos(dx+c)}{(ab+b^2)\cos(dx+c)^2-a^2-2ab-b^2}+\frac{\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}(a+b)}}{4d}$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/4*(2*cos(d*x + c)/((a*b + b^2)*cos(d*x + c)^2 - a^2 - 2*a*b - b^2) + log
((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt((a + b)*b)))/(s
qrt((a + b)*b)*(a + b)))/d
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07

$$\int \frac{\sin(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{\arctan\left(\frac{b\cos(dx+c)}{\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}(a+b)d}+\frac{\cos(dx+c)}{2\left(b\cos(dx+c)^2-a-b\right)(a+b)d}$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*arctan(b*cos(d*x + c)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*(a + b)*d) +
 1/2*cos(d*x + c)/((b*cos(d*x + c)^2 - a - b)*(a + b)*d)
```

3.97.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{\sin(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\cos(c+d\,x)}{2\,d\,(a+b)\,\left(-b\cos(c+d\,x)^2+a+b\right)}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos(c+d\,x)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,d\,(a+b)^{3/2}}$$

input
```
int(sin(c + d*x)/(a + b*sin(c + d*x)^2)^2,x)
```

output
```
- cos(c + d*x)/(2*d*(a + b)*(a + b - b*cos(c + d*x)^2)) - atanh((b^(1/2)*c
os(c + d*x))/(a + b)^(1/2))/(2*b^(1/2)*d*(a + b)^(3/2))
```

3.98 $\int \frac{\csc(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2} dx$

3.98.1 Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{\csc(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2} dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{a^2d} + \frac{\sqrt{b}(3a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}d} + \frac{b\cos(c+dx)}{2a(a+b)d\left(a+b-b\cos^2(c+dx)\right)}$$

output
```
-arctanh(cos(d*x+c))/a^2/d+1/2*b*cos(d*x+c)/a/(a+b)/d/(a+b-b*cos(d*x+c)^2)
+1/2*(3*a+2*b)*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a^2/(a+b)^(
3/2)/d
```

3.98.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.88

$$\int \frac{\csc(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2} dx = \frac{\frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}} + \frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}} + 2\left(\frac{ab\cos(c+dx)}{(a+b)(2a+b-b\cos(2(c+dx)))} - \log(\cos\right.}{2a^2d}$$

input
```
Integrate[Csc[c + d*x]/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
((Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-
a - b]])/(-a - b)^(3/2) + (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]
*Tan[(c + d*x)/2])/Sqrt[-a - b]])/(-a - b)^(3/2) + 2*((a*b*Cos[c + d*x])/(
(a + b)*(2*a + b - b*Cos[2*(c + d*x)])) - Log[Cos[(c + d*x)/2]] + Log[Sin[
(c + d*x)/2]]))/(2*a^2*d)
```

3.98.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3665, 316, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)\left(a+b\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{1}{(1-\cos^2(c+dx))(-b\cos^2(c+dx)+a+b)^2}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$-\frac{-\frac{\int -\frac{b\cos^2(c+dx)+2a+b}{\left(1-\cos^2(c+dx)\right)\left(-b\cos^2(c+dx)+a+b\right)}d\cos(c+dx)}{2a(a+b)}-\frac{b\cos(c+dx)}{2a(a+b)(a-b\cos^2(c+dx)+b)}}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int \frac{b\cos^2(c+dx)+2a+b}{\left(1-\cos^2(c+dx)\right)\left(-b\cos^2(c+dx)+a+b\right)}d\cos(c+dx)}{2a(a+b)}-\frac{b\cos(c+dx)}{2a(a+b)(a-b\cos^2(c+dx)+b)}}{d}$$

$$\Big\downarrow \text{397}$$

$$-\frac{\frac{\frac{2(a+b)\int \frac{1}{1-\cos^2(c+dx)}d\cos(c+dx)}{a}-\frac{b(3a+2b)\int \frac{1}{-b\cos^2(c+dx)+a+b}d\cos(c+dx)}{a}}{2a(a+b)}-\frac{b\cos(c+dx)}{2a(a+b)(a-b\cos^2(c+dx)+b)}}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\frac{2(a+b)\text{arctanh}(\cos(c+dx))}{a}-\frac{b(3a+2b)\int \frac{1}{-b\cos^2(c+dx)+a+b}d\cos(c+dx)}{a}}{2a(a+b)}-\frac{b\cos(c+dx)}{2a(a+b)(a-b\cos^2(c+dx)+b)}}{d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{\frac{2(a+b)\text{arctanh}(\cos(c+dx))}{a}-\frac{\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b\cos(c+dx)}{2a(a+b)(a-b\cos^2(c+dx)+b)}}{d}$$

input
```
Int[Csc[c + d*x]/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
-((((2*(a + b)*ArcTanh[Cos[c + d*x]])/a - (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sq
rt[b]*Cos[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b)) - (b*Cos[
c + d*x])/(2*a*(a + b)*(a + b - b*Cos[c + d*x]^2)))/d)
```

3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.98.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{-\frac{\ln(1+\cos(dx+c))}{2a^2}+\frac{\ln(\cos(dx+c)-1)}{2a^2}+\frac{b\left(\frac{a\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^2(dx+c)\right)\right)}+\frac{(3a+2b)\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}}{d}$
default	$\frac{-\frac{\ln(1+\cos(dx+c))}{2a^2}+\frac{\ln(\cos(dx+c)-1)}{2a^2}+\frac{b\left(\frac{a\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^2(dx+c)\right)\right)}+\frac{(3a+2b)\operatorname{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}}{d}$
risch	$-\frac{b\left(e^{3i(dx+c)}+e^{i(dx+c)}\right)}{a(a+b)d\left(b\,e^{4i(dx+c)}-4a\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}+b\right)}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{d\,a^2}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{d\,a^2}+\frac{3i\sqrt{-(a+b)b}\,\ln\left(e^{2i}\right.}{}$

input
```
int(csc(d*x+c)/(a+b*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a^2*ln(1+cos(d*x+c))+1/2/a^2*ln(cos(d*x+c)-1)+1/a^2*b*(1/2*a/(a+
b)*cos(d*x+c)/(a+b-b*cos(d*x+c)^2)+1/2*(3*a+2*b)/(a+b)/((a+b)*b)^(1/2)*arc
tanh(b*cos(d*x+c)/((a+b)*b)^(1/2))))
```

3.98.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(94) = 188$.

Time = 0.31 (sec) , antiderivative size = 455, normalized size of antiderivative = 4.42

$$\int \frac{\csc(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$$
$$= \left[-\frac{2\,ab\cos\left(dx+c\right) - \left(\left(3\,ab+2\,b^2\right)\cos\left(dx+c\right)^2 - 3\,a^2 - 5\,ab - 2\,b^2\right)\sqrt{\frac{b}{a+b}}\log\left(\frac{b\cos(dx+c)^2+2\,(a+b)\sqrt{\frac{b}{a+b}}\cos}{b\cos(dx+c)^2-a-b}\right.}{4}\right.$$

$$-\frac{ab\cos\left(dx+c\right) + \left(\left(3\,ab+2\,b^2\right)\cos\left(dx+c\right)^2 - 3\,a^2 - 5\,ab - 2\,b^2\right)\sqrt{-\frac{b}{a+b}}\arctan\left(\sqrt{-\frac{b}{a+b}}\cos\left(dx+c\right)\right)}{2\,\left(\left(a^3b\right.\right.}$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*b*cos(d*x + c) - ((3*a*b + 2*b^2)*cos(d*x + c)^2 - 3*a^2 - 5*a*
b - 2*b^2)*sqrt(b/(a + b))*log((b*cos(d*x + c)^2 + 2*(a + b)*sqrt(b/(a + b
))*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) + 2*((a*b + b^2)*cos(
d*x + c)^2 - a^2 - 2*a*b - b^2)*log(1/2*cos(d*x + c) + 1/2) - 2*((a*b + b^
2)*cos(d*x + c)^2 - a^2 - 2*a*b - b^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^3
*b + a^2*b^2)*d*cos(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d), -1/2*(a*b*c
os(d*x + c) + ((3*a*b + 2*b^2)*cos(d*x + c)^2 - 3*a^2 - 5*a*b - 2*b^2)*sqr
t(-b/(a + b))*arctan(sqrt(-b/(a + b))*cos(d*x + c)) + ((a*b + b^2)*cos(d*x
 + c)^2 - a^2 - 2*a*b - b^2)*log(1/2*cos(d*x + c) + 1/2) - ((a*b + b^2)*co
s(d*x + c)^2 - a^2 - 2*a*b - b^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^3*b +
a^2*b^2)*d*cos(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d)]
```

3.98.6 Sympy [F]

$$\int \frac{\csc(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \int \frac{\csc(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)**2)**2,x)
```

output
```
Integral(csc(c + d*x)/(a + b*sin(c + d*x)**2)**2, x)
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.45

$$\int \frac{\csc(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{\frac{2\,b\cos(dx+c)}{a^3+2\,a^2b+ab^2-(a^2b+ab^2)\cos(dx+c)^2} - \frac{(3\,ab+2\,b^2)\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{(a^3+a^2b)\sqrt{(a+b)b}} - \frac{2\log(\cos(dx+c)+1)}{a^2} + \frac{2\log(\cos(dx+c)-1)}{a^2}}{4\,d}$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/4*(2*b*cos(d*x + c)/(a^3 + 2*a^2*b + a*b^2 - (a^2*b + a*b^2)*cos(d*x + c
)^2) - (3*a*b + 2*b^2)*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x +
 c) + sqrt((a + b)*b)))/((a^3 + a^2*b)*sqrt((a + b)*b)) - 2*log(cos(d*x +
c) + 1)/a^2 + 2*log(cos(d*x + c) - 1)/a^2)/d
```

3.98.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(94) = 188.

Time = 0.43 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.39

$$\int \frac{\csc(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\frac{(3\,ab+2\,b^2)\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{(a^3+a^2b)\sqrt{-ab-b^2}} - \frac{2\left(ab-\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{(a^3+a^2b)\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)} - \frac{\log\left(\frac{|-\cos(d}{|\cos(dx}\right.}{a^2}\cdots}{2\,d}$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((3*a*b + 2*b^2)*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*co
s(d*x + c) + sqrt(-a*b - b^2)))/((a^3 + a^2*b)*sqrt(-a*b - b^2)) - 2*(a*b
- a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b^2*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1))/((a^3 + a^2*b)*(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2)) - log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))
/a^2)/d
```

3.98.9 Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 2039, normalized size of antiderivative = 19.80

$$\int \frac{\csc(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(c + d*x)*(a + b*sin(c + d*x)^2)^2),x)
```

output
```
(atan(((((cos(c + d*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4
+ a^2*b^2)) + ((b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2
*a^4*b + a^5 + a^3*b^2) - (cos(c + d*x)*(b*(a + b)^3)^(1/2)*(3*a + 2*b)*(3
2*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2
*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a
^5 + a^2*b^3 + 3*a^3*b^2)))*(b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*
b + a^5 + a^2*b^3 + 3*a^3*b^2)) + (((cos(c + d*x)*(20*a*b^4 + 8*b^5 + 13*a
^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((b*(a + b)^3)^(1/2)*((2*a^4*b^4
+ 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*b^2) + (cos(c + d*x)*(b*(a +
 b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^
2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(
3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(b*(a + b)^3)^(1/2)
*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))/(((3*a*b^3)/2
+ b^4)/(2*a^4*b + a^5 + a^3*b^2) - (((cos(c + d*x)*(20*a*b^4 + 8*b^5 + 13*
a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) + ((b*(a + b)^3)^(1/2)*((2*a^4*b^4
 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*b^2) - (cos(c + d*x)*(b*(a
+ b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b
^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*
(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(b*(a + b)^3)^(1/2
)*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) + (((cos(c + d...
```

3.99 $\int \frac{\csc^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

3.99.1 Optimal result

Integrand size = 23, antiderivative size = 153

$$
\begin{aligned}
\int \frac{\csc^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = & -\frac{(a-4b)\text{arctanh}(\cos(c+dx))}{2a^3d} \\
& -\frac{b^{3/2}(5a+4b)\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^3(a+b)^{3/2}d} \\
& -\frac{b(a+2b)\cos(c+dx)}{2a^2(a+b)d\left(a+b-b\cos^2(c+dx)\right)} \\
& -\frac{\cot(c+dx)\csc(c+dx)}{2ad\left(a+b-b\cos^2(c+dx)\right)}
\end{aligned}
$$

output
```
-1/2*(a-4*b)*arctanh(cos(d*x+c))/a^3/d-1/2*b^(3/2)*(5*a+4*b)*arctanh(cos(d
*x+c)*b^(1/2)/(a+b)^(1/2))/a^3/(a+b)^(3/2)/d-1/2*b*(a+2*b)*cos(d*x+c)/a^2/
(a+b)/d/(a+b-b*cos(d*x+c)^2)-1/2*cot(d*x+c)*csc(d*x+c)/a/d/(a+b-b*cos(d*x+
c)^2)
```

3.99.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.55

$$\int \frac{\csc^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$$

$$= \frac{(-2a-b+b\cos(2(c+dx)))\csc^3(c+dx)\left(\frac{8ab^2\cot(c+dx)}{a+b}+\frac{4b^{3/2}(5a+4b)\arctan\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)(2a+b-b\cos(2(c-}{(-a-b)^{3/2}}\right.}{}$$

input
```
Integrate[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
((-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^3*((8*a*b^2*Cot[c + d*x])/(a
 + b) + (4*b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2
])/Sqrt[-a - b]]*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x])/(-a - b)^(3/
2) + (4*b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/
Sqrt[-a - b]]*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x])/(-a - b)^(3/2)
+ a*(2*a + b - b*Cos[2*(c + d*x)])*Csc[(c + d*x)/2]^2*Csc[c + d*x] + 4*(a
- 4*b)*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]*Log[Cos[(c + d*x)/2]] -
 4*(a - 4*b)*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]*Log[Sin[(c + d*x)
/2]] - a*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]*Sec[(c + d*x)/2]^2))/
(32*a^3*d*(b + a*Csc[c + d*x]^2)^2)
```

3.99.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3665, 316, 402, 27, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(c+dx)^3\left(a+b\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{1}{(1-\cos^2(c+dx))^2(-b\cos^2(c+dx)+a+b)^2}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$-\frac{\frac{\int \frac{-3b\cos^2(c+dx)+a-b}{\left(1-\cos^2(c+dx)\right)\left(-b\cos^2(c+dx)+a+b\right)^2}d\cos(c+dx)}{2a}+\frac{\cos(c+dx)}{2a(1-\cos^2(c+dx))(a-b\cos^2(c+dx)+b)}}{d}$$

$$\Big\downarrow \text{402}$$

$$-\frac{\frac{\frac{b(a+2b)\cos(c+dx)}{a(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{\int -\frac{2\left(a^2-2ba-2b^2-b(a+2b)\cos^2(c+dx)\right)}{\left(1-\cos^2(c+dx)\right)\left(-b\cos^2(c+dx)+a+b\right)}d\cos(c+dx)}{2a(a+b)}}{2a}+\frac{\cos(c+dx)}{2a(1-\cos^2(c+dx))(a-b\cos^2(c+dx)+b)}}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{\int \frac{a^2-2ba-2b^2-b(a+2b)\cos^2(c+dx)}{\left(1-\cos^2(c+dx)\right)\left(-b\cos^2(c+dx)+a+b\right)}d\cos(c+dx)}{a(a+b)}+\frac{b(a+2b)\cos(c+dx)}{a(a+b)\left(a-b\cos^2(c+dx)+b\right)}}{2a}+\frac{\cos(c+dx)}{2a(1-\cos^2(c+dx))(a-b\cos^2(c+dx)+b)}}{d}$$

$$\Big\downarrow \text{397}$$

$$-\frac{\frac{\frac{\frac{b^2(5a+4b)\int \frac{1}{-b\cos^2(c+dx)+a+b}d\cos(c+dx)}{a}+\frac{(a-4b)(a+b)\int \frac{1}{1-\cos^2(c+dx)}d\cos(c+dx)}{a}}{a(a+b)}+\frac{b(a+2b)\cos(c+dx)}{a(a+b)\left(a-b\cos^2(c+dx)+b\right)}}{2a}+\frac{\cos(c+dx)}{2a(1-\cos^2(c+dx))(a-b\cos^2(c+dx)+b)}}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\frac{\frac{b^2(5a+4b)\int \frac{1}{-b\cos^2(c+dx)+a+b}d\cos(c+dx)}{a}+\frac{(a-4b)(a+b)\text{arctanh}(\cos(c+dx))}{a}}{a(a+b)}+\frac{b(a+2b)\cos(c+dx)}{a(a+b)\left(a-b\cos^2(c+dx)+b\right)}}{2a}+\frac{\cos(c+dx)}{2a(1-\cos^2(c+dx))(a-b\cos^2(c+dx)+b)}}{d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{\frac{\frac{b^{3/2}(5a+4b)\text{arctanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}+\frac{(a-4b)(a+b)\text{arctanh}(\cos(c+dx))}{a}}{a(a+b)}+\frac{b(a+2b)\cos(c+dx)}{a(a+b)\left(a-b\cos^2(c+dx)+b\right)}}{2a}+\frac{\cos(c+dx)}{2a(1-\cos^2(c+dx))(a-b\cos^2(c+dx)+b)}}{d}$$

3.99. $\int \frac{\csc^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

input
```
Int[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
-((Cos[c + d*x]/(2*a*(1 - Cos[c + d*x]^2)*(a + b - b*Cos[c + d*x]^2)) + ((
((a - 4*b)*(a + b)*ArcTanh[Cos[c + d*x]])/a + (b^(3/2)*(5*a + 4*b)*ArcTanh
[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a*(a + b)) + (b*(a
 + 2*b)*Cos[c + d*x])/(a*(a + b)*(a + b - b*Cos[c + d*x]^2)))/(2*a))/d)
```

3.99.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.99.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{-\frac{b^{2}\left(\frac{a\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^{2}(dx+c)\right)\right)}+\frac{(5a+4b)\,\mathrm{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^{3}}+\frac{1}{4a^{2}(1+\cos(dx+c))}+\frac{(-a+4b)\ln(1+\cos(dx+c))}{4a^{3}}+\frac{\cdots}{4a^{2}(\mathrm{co}\cdots}}{d}$
default	$\frac{-\frac{b^{2}\left(\frac{a\cos(dx+c)}{2(a+b)\left(a+b-b\left(\cos^{2}(dx+c)\right)\right)}+\frac{(5a+4b)\,\mathrm{arctanh}\left(\frac{b\cos(dx+c)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^{3}}+\frac{1}{4a^{2}(1+\cos(dx+c))}+\frac{(-a+4b)\ln(1+\cos(dx+c))}{4a^{3}}+\frac{\cdots}{4a^{2}(\mathrm{co}\cdots}}{d}$
risch	$\frac{ab\,\mathrm{e}^{7i(dx+c)}+2b^{2}\mathrm{e}^{7i(dx+c)}-4a^{2}\mathrm{e}^{5i(dx+c)}-5ab\,\mathrm{e}^{5i(dx+c)}-2b^{2}\mathrm{e}^{5i(dx+c)}-4a^{2}\mathrm{e}^{3i(dx+c)}-5ab\,\mathrm{e}^{3i(dx+c)}-2b^{2}\mathrm{e}^{3i(dx+c)}+ab\cdots}{d\,a^{2}\left(\mathrm{e}^{2i(dx+c)}-1\right)^{2}(a+b)\left(b\,\mathrm{e}^{4i(dx+c)}-4a\,\mathrm{e}^{2i(dx+c)}-2b\,\mathrm{e}^{2i(dx+c)}+b\right)}$

input
```
int(csc(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b^2/a^3*(1/2*a/(a+b)*cos(d*x+c)/(a+b-b*cos(d*x+c)^2)+1/2*(5*a+4*b)/(
a+b)/((a+b)*b)^(1/2)*arctanh(b*cos(d*x+c)/((a+b)*b)^(1/2)))+1/4/a^2/(1+cos
(d*x+c))+1/4/a^3*(-a+4*b)*ln(1+cos(d*x+c))+1/4/a^2/(cos(d*x+c)-1)+1/4*(a-4
*b)/a^3*ln(cos(d*x+c)-1))
```

3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(143) = 286$.

Time = 0.35 (sec) , antiderivative size = 838, normalized size of antiderivative = 5.48

$$\int \frac{\csc^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$$
$$= \left[\frac{2\left(a^2b+2\,ab^2\right)\cos\left(dx+c\right)^3+\left(\left(5\,ab^2+4\,b^3\right)\cos\left(dx+c\right)^4+5\,a^2b+9\,ab^2+4\,b^3-\left(5\,a^2b+14\,ab^2+8\,b^3\right.\right.}{}\right.$$

input
```
integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(a^2*b + 2*a*b^2)*cos(d*x + c)^3 + ((5*a*b^2 + 4*b^3)*cos(d*x + c)
^4 + 5*a^2*b + 9*a*b^2 + 4*b^3 - (5*a^2*b + 14*a*b^2 + 8*b^3)*cos(d*x + c)
^2)*sqrt(b/(a + b))*log(-(b*cos(d*x + c)^2 - 2*(a + b)*sqrt(b/(a + b))*cos
(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - 2*(a^3 + 2*a^2*b + 2*a*b^
2)*cos(d*x + c) - ((a^2*b - 3*a*b^2 - 4*b^3)*cos(d*x + c)^4 + a^3 - 2*a^2*
b - 7*a*b^2 - 4*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*cos(d*x + c)^2)*log
(1/2*cos(d*x + c) + 1/2) + ((a^2*b - 3*a*b^2 - 4*b^3)*cos(d*x + c)^4 + a^3
 - 2*a^2*b - 7*a*b^2 - 4*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*cos(d*x +
c)^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^4*b + a^3*b^2)*d*cos(d*x + c)^4 -
(a^5 + 3*a^4*b + 2*a^3*b^2)*d*cos(d*x + c)^2 + (a^5 + 2*a^4*b + a^3*b^2)*d
), 1/4*(2*(a^2*b + 2*a*b^2)*cos(d*x + c)^3 + 2*((5*a*b^2 + 4*b^3)*cos(d*x
+ c)^4 + 5*a^2*b + 9*a*b^2 + 4*b^3 - (5*a^2*b + 14*a*b^2 + 8*b^3)*cos(d*x
+ c)^2)*sqrt(-b/(a + b))*arctan(sqrt(-b/(a + b))*cos(d*x + c)) - 2*(a^3 +
2*a^2*b + 2*a*b^2)*cos(d*x + c) - ((a^2*b - 3*a*b^2 - 4*b^3)*cos(d*x + c)^
4 + a^3 - 2*a^2*b - 7*a*b^2 - 4*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*cos
(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + ((a^2*b - 3*a*b^2 - 4*b^3)*cos(
d*x + c)^4 + a^3 - 2*a^2*b - 7*a*b^2 - 4*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8
*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^4*b + a^3*b^2)*d*c
os(d*x + c)^4 - (a^5 + 3*a^4*b + 2*a^3*b^2)*d*cos(d*x + c)^2 + (a^5 + 2*a^
4*b + a^3*b^2)*d)]
```

3.99.6 Sympy [F]

$$\int \frac{\csc^3(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \int \frac{\csc^3(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$$

input
```
integrate(csc(d*x+c)**3/(a+b*sin(d*x+c)**2)**2,x)
```

output
```
Integral(csc(c + d*x)**3/(a + b*sin(c + d*x)**2)**2, x)
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.46

$$\int \frac{\csc^3(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \frac{\frac{(5ab^2+4b^3)\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{(a^4+a^3b)\sqrt{(a+b)b}} + \frac{2\left((ab+2b^2)\cos(dx+c)^3-(a^2+2ab+2b^2)\cos(dx+c)\right)}{(a^3b+a^2b^2)\cos(dx+c)^4+a^4+2a^3b+a^2b^2-(a^4+3a^3b+2a^2b^2)\cos(dx+c)^2} - \frac{(a-4b)\log(\cos(dx+c}{a^3}}{4d}$$

input
```
integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/4*((5*a*b^2 + 4*b^3)*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x +
 c) + sqrt((a + b)*b)))/((a^4 + a^3*b)*sqrt((a + b)*b)) + 2*((a*b + 2*b^2)
*cos(d*x + c)^3 - (a^2 + 2*a*b + 2*b^2)*cos(d*x + c))/((a^3*b + a^2*b^2)*c
os(d*x + c)^4 + a^4 + 2*a^3*b + a^2*b^2 - (a^4 + 3*a^3*b + 2*a^2*b^2)*cos(
d*x + c)^2) - (a - 4*b)*log(cos(d*x + c) + 1)/a^3 + (a - 4*b)*log(cos(d*x
+ c) - 1)/a^3)/d
```

3.99.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(143) = 286$.

Time = 0.42 (sec) , antiderivative size = 512, normalized size of antiderivative = 3.35

$$\int \frac{\csc^3(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \frac{\frac{12(5ab^2+4b^3)\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{(a^4+a^3b)\sqrt{-ab-b^2}} + \frac{3a^3+3a^2b-\frac{8a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{12a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{28ab^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{7a^3(\cos(d}{(\cos(dx-}}{(a^4+a^3b)\left(\frac{a(\cos}{\cos}\right.}}{}$$

input
```
integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/24*(12*(5*a*b^2 + 4*b^3)*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^
2)*cos(d*x + c) + sqrt(-a*b - b^2)))/((a^4 + a^3*b)*sqrt(-a*b - b^2)) + (3
*a^3 + 3*a^2*b - 8*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^2*b*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 28*a*b^2*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) + 7*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - a^2*b*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 16*a*b^2*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 + 16*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2*a^3
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 6*a^2*b*(cos(d*x + c) - 1)^3/
(cos(d*x + c) + 1)^3 + 8*a*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/
((a^4 + a^3*b)*(a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2 - 4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 + a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)) + 6*(a - 4*b)*log(abs(
-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 - 3*(cos(d*x + c) - 1)/(a^2*
(cos(d*x + c) + 1)))/d
```

3.99.9 Mupad [B] (verification not implemented)

Time = 15.46 (sec) , antiderivative size = 2338, normalized size of antiderivative = 15.28

$$\int \frac{\csc^3(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(c + d*x)^3*(a + b*sin(c + d*x)^2)^2),x)
```

output
```
- ((cos(c + d*x)*(2*a*b + a^2 + 2*b^2))/(2*a^2*(a + b)) - (b*cos(c + d*x)^
3*(a + 2*b))/(2*a^2*(a + b)))/(d*(a + b + b*cos(c + d*x)^4 - cos(c + d*x)^
2*(a + 2*b))) - (atan((((a - 4*b)*((cos(c + d*x)*(64*a*b^6 + 32*b^7 + 26*a
^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) + (((4*a^6*b^
5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) - (cos(c
+ d*x)*(a - 4*b)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a
^3*(2*a^5*b + a^6 + a^4*b^2)))*(a - 4*b))/(4*a^3))*1i)/(4*a^3) + ((a - 4*b
)*((cos(c + d*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(
2*(2*a^5*b + a^6 + a^4*b^2)) - (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^
9*b^2)/(2*a^7*b + a^8 + a^6*b^2) + (cos(c + d*x)*(a - 4*b)*(32*a^6*b^5 + 8
0*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b + a^6 + a^4*b^2)))*(
a - 4*b))/(4*a^3))*1i)/(4*a^3))/((12*a*b^6 + 8*b^7 + (3*a^2*b^5)/2 - (5*a^
3*b^4)/4)/(2*a^7*b + a^8 + a^6*b^2) - ((a - 4*b)*((cos(c + d*x)*(64*a*b^6
+ 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)
) + (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6
*b^2) - (cos(c + d*x)*(a - 4*b)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16
*a^9*b^2))/(8*a^3*(2*a^5*b + a^6 + a^4*b^2)))*(a - 4*b))/(4*a^3)))/(4*a^3)
 + ((a - 4*b)*((cos(c + d*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 +
 a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) - (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^
8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) + (cos(c + d*x)*(a - 4*b)*...
```

3.99. $\int \frac{\csc^3(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$

3.100 $\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

3.100.1 Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{(4a-b)x}{2b^3}+\frac{a^{3/2}(4a+5b)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2b^3(a+b)^{3/2}d} - \frac{a(2a+b)\tan(c+dx)}{2b^2(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\sin^2(c+dx)\tan(c+dx)}{2bd\left(a+(a+b)\tan^2(c+dx)\right)}$$

output
```
-1/2*(4*a-b)*x/b^3+1/2*a^(3/2)*(4*a+5*b)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(
1/2))/b^3/(a+b)^(3/2)/d-1/2*a*(2*a+b)*tan(d*x+c)/b^2/(a+b)/d/(a+(a+b)*tan(
d*x+c)^2)-1/2*sin(d*x+c)^2*tan(d*x+c)/b/d/(a+(a+b)*tan(d*x+c)^2)
```

3.100.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.72

$$\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{-2(4a-b)(c+dx)+\frac{2a^{3/2}(4a+5b)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}}+b\left(-1-\frac{2a^2}{(a+b)(2a+b-b\cos(2(c+dx)))}\right)\sin(2(c+dx))}{4b^3d}$$

input
```
Integrate[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
(-2*(4*a - b)*(c + d*x) + (2*a^(3/2)*(4*a + 5*b)*ArcTan[(Sqrt[a + b]*Tan[c
 + d*x])/Sqrt[a]])/(a + b)^(3/2) + b*(-1 - (2*a^2)/((a + b)*(2*a + b - b*C
os[2*(c + d*x)])))*Sin[2*(c + d*x)])/(4*b^3*d)
```

3.100.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3666, 372, 440, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(c+dx)^6}{\left(a+b\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \; 3666$$

$$\frac{\int \frac{\tan^6(c+dx)}{(\tan^2(c+dx)+1)^2((a+b)\tan^2(c+dx)+a)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \; 372$$

$$\frac{\frac{\int \frac{\tan^2(c+dx)\left(3a-(a-b)\tan^2(c+dx)\right)}{\left(\tan^2(c+dx)+1\right)\left((a+b)\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{2b}-\frac{\tan^3(c+dx)}{2b(\tan^2(c+dx)+1)((a+b)\tan^2(c+dx)+a)}}{d}$$

$$\Big\downarrow \; 440$$

$$\frac{\frac{\frac{\int \frac{2\left(a(2a+b)-\left(2a^2+2ba-b^2\right)\tan^2(c+dx)\right)}{\left(\tan^2(c+dx)+1\right)\left((a+b)\tan^2(c+dx)+a\right)}d\tan(c+dx)}{2b(a+b)}-\frac{a(2a+b)\tan(c+dx)}{b(a+b)\left((a+b)\tan^2(c+dx)+a\right)}}{2b}-\frac{\tan^3(c+dx)}{2b(\tan^2(c+dx)+1)((a+b)\tan^2(c+dx)+a)}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{a(2a+b)-\left(2a^2+2ba-b^2\right)\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left((a+b)\tan^2(c+dx)+a\right)}d\tan(c+dx)}{b(a+b)}-\frac{a(2a+b)\tan(c+dx)}{b(a+b)\left((a+b)\tan^2(c+dx)+a\right)}}{2b}-\frac{\tan^3(c+dx)}{2b(\tan^2(c+dx)+1)((a+b)\tan^2(c+dx)+a)}}{d}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{\frac{a^2(4a+5b)\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{b}-\frac{(4a-b)(a+b)\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)}{b}}{b(a+b)}-\frac{a(2a+b)\tan(c+dx)}{b(a+b)\left((a+b)\tan^2(c+dx)+a\right)}}{2b}-\frac{\tan^3(c+dx)}{2b(\tan^2(c+dx)+1)((a+b)\tan^2(c+dx)+a)}}{d}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{\frac{a^2(4a+5b)\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{b}-\frac{(4a-b)(a+b)\arctan(\tan(c+dx))}{b}}{b(a+b)}-\frac{a(2a+b)\tan(c+dx)}{b(a+b)\left((a+b)\tan^2(c+dx)+a\right)}}{2b}-\frac{\tan^3(c+dx)}{2b(\tan^2(c+dx)+1)((a+b)\tan^2(c+dx)+a)}}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\frac{a^{3/2}(4a+5b)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b\sqrt{a+b}}-\frac{(4a-b)(a+b)\arctan(\tan(c+dx))}{b}}{b(a+b)}-\frac{a(2a+b)\tan(c+dx)}{b(a+b)\left((a+b)\tan^2(c+dx)+a\right)}}{2b}-\frac{\tan^3(c+dx)}{2b(\tan^2(c+dx)+1)((a+b)\tan^2(c+dx)+a)}}{d}$$

input
```
Int[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
(-1/2*Tan[c + d*x]^3/(b*(1 + Tan[c + d*x]^2)*(a + (a + b)*Tan[c + d*x]^2))
 + ((-(((4*a - b)*(a + b)*ArcTan[Tan[c + d*x]])/b) + (a^(3/2)*(4*a + 5*b)*
ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(b*Sqrt[a + b]))/(b*(a + b)) -
 (a*(2*a + b)*Tan[c + d*x])/(b*(a + b)*(a + (a + b)*Tan[c + d*x]^2)))/(2*b
))/d
```

3.100.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.100. $\int \frac{\sin^6(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$

3.100.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{a^2\left(-\frac{b\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)}+\frac{(4a+5b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{b^3}-\frac{\frac{\tan(dx+c)b}{2+2\left(\tan^2(dx+c)\right)}+\frac{(4a-b)\arctan(\tan}{2}}{b^3}}{d}$
default	$\frac{\frac{a^2\left(-\frac{b\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)}+\frac{(4a+5b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{b^3}-\frac{\frac{\tan(dx+c)b}{2+2\left(\tan^2(dx+c)\right)}+\frac{(4a-b)\arctan(\tan}{2}}{b^3}}{d}$
risch	$-\frac{2ax}{b^3}+\frac{x}{2b^2}+\frac{i e^{2i(dx+c)}}{8b^2d}-\frac{i e^{-2i(dx+c)}}{8b^2d}+\frac{i a^2\left(2a\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}-b\right)}{b^3(a+b)d\left(-b\,e^{4i(dx+c)}+4a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-b\right)}-\frac{\sqrt{-a(a+}}{}$

input
```
int(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^3*a^2*(-1/2*b/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)+
1/2*(4*a+5*b)/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2
)))-1/b^3*(1/2*b*tan(d*x+c)/(1+tan(d*x+c)^2)+1/2*(4*a-b)*arctan(tan(d*x+c)
)))
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 623, normalized size of antiderivative = 4.21

$$\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$$

$$= \left[-\frac{4\left(4a^2b+3ab^2-b^3\right)dx\cos(dx+c)^2-4\left(4a^3+7a^2b+2ab^2-b^3\right)dx+\left(4a^3+9a^2b+5ab^2-(4a^2b\right.}{}\right.$$

$$-\frac{2\left(4a^2b+3ab^2-b^3\right)dx\cos(dx+c)^2-2\left(4a^3+7a^2b+2ab^2-b^3\right)dx-\left(4a^3+9a^2b+5ab^2-(4a^2b\right.}{4\left((ab\right.}$$

input
```
integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*(4*a^2*b + 3*a*b^2 - b^3)*d*x*cos(d*x + c)^2 - 4*(4*a^3 + 7*a^2*b
 + 2*a*b^2 - b^3)*d*x + (4*a^3 + 9*a^2*b + 5*a*b^2 - (4*a^2*b + 5*a*b^2)*c
os(d*x + c)^2)*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4
- 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + 3*a*b + b^2)*cos(d*
x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c)
 + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 +
 a^2 + 2*a*b + b^2)) + 4*((a*b^2 + b^3)*cos(d*x + c)^3 - (2*a^2*b + 2*a*b^
2 + b^3)*cos(d*x + c))*sin(d*x + c))/((a*b^4 + b^5)*d*cos(d*x + c)^2 - (a^
2*b^3 + 2*a*b^4 + b^5)*d), -1/4*(2*(4*a^2*b + 3*a*b^2 - b^3)*d*x*cos(d*x +
 c)^2 - 2*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*d*x - (4*a^3 + 9*a^2*b + 5*a*b
^2 - (4*a^2*b + 5*a*b^2)*cos(d*x + c)^2)*sqrt(a/(a + b))*arctan(1/2*((2*a
+ b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d*x + c))
) + 2*((a*b^2 + b^3)*cos(d*x + c)^3 - (2*a^2*b + 2*a*b^2 + b^3)*cos(d*x +
c))*sin(d*x + c))/((a*b^4 + b^5)*d*cos(d*x + c)^2 - (a^2*b^3 + 2*a*b^4 + b
^5)*d)]
```

3.100.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**6/(a+b*sin(d*x+c)**2)**2,x)
```

output
```
Timed out
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.22

$$\int \frac{\sin^6(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \frac{\frac{(4\,a^3+5\,a^2b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(ab^3+b^4)\sqrt{(a+b)a}} - \frac{(2\,a^2+2\,ab+b^2)\tan(dx+c)^3+(2\,a^2+ab)\tan(dx+c)}{(a^2b^2+2\,ab^3+b^4)\tan(dx+c)^4+a^2b^2+ab^3+(2\,a^2b^2+3\,ab^3+b^4)\tan(dx+c)^2} - \frac{(dx+c)(4\,a-b)}{b^3}}{2\,d}$$

input
```
integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*((4*a^3 + 5*a^2*b)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a*b^
3 + b^4)*sqrt((a + b)*a)) - ((2*a^2 + 2*a*b + b^2)*tan(d*x + c)^3 + (2*a^2
 + a*b)*tan(d*x + c))/((a^2*b^2 + 2*a*b^3 + b^4)*tan(d*x + c)^4 + a^2*b^2
+ a*b^3 + (2*a^2*b^2 + 3*a*b^3 + b^4)*tan(d*x + c)^2) - (d*x + c)*(4*a - b
)/b^3)/d
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.51

$$\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$$
$$= \frac{\frac{(4\,a^3+5\,a^2b)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{(ab^3+b^4)\sqrt{a^2+ab}} - \frac{2\,a^2\tan(dx+c)^3+2\,ab\tan(dx+c)^3+b^2\tan(dx+c)^3+2\,a^2\tan(dx}{\left(a\tan(dx+c)^4+b\tan(dx+c)^4+2\,a\tan(dx+c)^2+b\tan(dx+c}}{2\,d}$$

input
```
integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*((4*a^3 + 5*a^2*b)*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arct
an((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a*b^3 + b^4)*sqrt
(a^2 + a*b)) - (2*a^2*tan(d*x + c)^3 + 2*a*b*tan(d*x + c)^3 + b^2*tan(d*x
+ c)^3 + 2*a^2*tan(d*x + c) + a*b*tan(d*x + c))/((a*tan(d*x + c)^4 + b*tan
(d*x + c)^4 + 2*a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a*b^2 + b^3)) -
(d*x + c)*(4*a - b)/b^3)/d
```

3.100.9 Mupad [B] (verification not implemented)

Time = 16.77 (sec) , antiderivative size = 2295, normalized size of antiderivative = 15.51

$$\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^6/(a + b*sin(c + d*x)^2)^2,x)
```

output
```
(atan((((a + (5*b)/4)*(-a^3*(a + b)^3)^(1/2)*((tan(c + d*x)*(96*a^5*b - 4*
a*b^5 + 32*a^6 + b^6 - 10*a^2*b^4 + 20*a^3*b^3 + 90*a^4*b^2))/(2*(a*b^4 +
b^5)) + (((2*a*b^9 + 8*a^2*b^8 + 10*a^3*b^7 + 4*a^4*b^6)/(a*b^6 + b^7) + (
tan(c + d*x)*(a + (5*b)/4)*(-a^3*(a + b)^3)^(1/2)*(80*a*b^9 + 16*b^10 + 14
4*a^2*b^8 + 112*a^3*b^7 + 32*a^4*b^6))/(2*(a*b^4 + b^5)*(3*a*b^5 + b^6 + 3
*a^2*b^4 + a^3*b^3)))*(a + (5*b)/4)*(-a^3*(a + b)^3)^(1/2))/(3*a*b^5 + b^6
 + 3*a^2*b^4 + a^3*b^3))*1i)/(3*a*b^5 + b^6 + 3*a^2*b^4 + a^3*b^3) + ((a +
 (5*b)/4)*(-a^3*(a + b)^3)^(1/2)*((tan(c + d*x)*(96*a^5*b - 4*a*b^5 + 32*a
^6 + b^6 - 10*a^2*b^4 + 20*a^3*b^3 + 90*a^4*b^2))/(2*(a*b^4 + b^5)) - (((2
*a*b^9 + 8*a^2*b^8 + 10*a^3*b^7 + 4*a^4*b^6)/(a*b^6 + b^7) - (tan(c + d*x)
*(a + (5*b)/4)*(-a^3*(a + b)^3)^(1/2)*(80*a*b^9 + 16*b^10 + 144*a^2*b^8 +
112*a^3*b^7 + 32*a^4*b^6))/(2*(a*b^4 + b^5)*(3*a*b^5 + b^6 + 3*a^2*b^4 + a
^3*b^3)))*(a + (5*b)/4)*(-a^3*(a + b)^3)^(1/2))/(3*a*b^5 + b^6 + 3*a^2*b^4
 + a^3*b^3))*1i)/(3*a*b^5 + b^6 + 3*a^2*b^4 + a^3*b^3))/((16*a^5*b + 8*a^6
 + (5*a^2*b^4)/4 - (13*a^3*b^3)/2 + (3*a^4*b^2)/2)/(a*b^6 + b^7) + ((a + (
5*b)/4)*(-a^3*(a + b)^3)^(1/2)*((tan(c + d*x)*(96*a^5*b - 4*a*b^5 + 32*a^6
 + b^6 - 10*a^2*b^4 + 20*a^3*b^3 + 90*a^4*b^2))/(2*(a*b^4 + b^5)) + (((2*a
*b^9 + 8*a^2*b^8 + 10*a^3*b^7 + 4*a^4*b^6)/(a*b^6 + b^7) + (tan(c + d*x)*(
a + (5*b)/4)*(-a^3*(a + b)^3)^(1/2)*(80*a*b^9 + 16*b^10 + 144*a^2*b^8 + 11
2*a^3*b^7 + 32*a^4*b^6))/(2*(a*b^4 + b^5)*(3*a*b^5 + b^6 + 3*a^2*b^4 + ...
```

3.100. $\int \frac{\sin^6(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$

3.101 $\int \frac{\sin^4(c+dx)}{(a+b\sin^2(c+dx))^2} \, dx$

3.101.1 Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \frac{\sin^4(c+dx)}{(a+b\sin^2(c+dx))^2} \, dx = \frac{x}{b^2} - \frac{\sqrt{a}(2a+3b)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2b^2(a+b)^{3/2}d} + \frac{a\tan(c+dx)}{2b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)}$$

output
```
x/b^2-1/2*(2*a+3*b)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))*a^(1/2)/b^2/(a+
b)^(3/2)/d+1/2*a*tan(d*x+c)/b/(a+b)/d/(a+(a+b)*tan(d*x+c)^2)
```

3.101.2 Mathematica [A] (verified)

Time = 11.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \frac{\sin^4(c+dx)}{(a+b\sin^2(c+dx))^2} \, dx = \frac{2(c+dx) - \frac{\sqrt{a}(2a+3b)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}} + \frac{ab\sin(2(c+dx))}{(a+b)(2a+b-b\cos(2(c+dx)))}}{2b^2d}$$

input
```
Integrate[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
(2*(c + d*x) - (Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt
[a]])/(a + b)^(3/2) + (a*b*Sin[2*(c + d*x)])/((a + b)*(2*a + b - b*Cos[2*(
c + d*x)])))/(2*b^2*d)
```

3.101.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3666, 372, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^4}{\left(a+b\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{3666}$$

$$\frac{\int \frac{\tan^4(c+dx)}{(\tan^2(c+dx)+1)((a+b)\tan^2(c+dx)+a)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{a\tan(c+dx)}{2b(a+b)((a+b)\tan^2(c+dx)+a)}-\frac{\int \frac{a-(a+2b)\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left((a+b)\tan^2(c+dx)+a\right)}d\tan(c+dx)}{2b(a+b)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{a\tan(c+dx)}{2b(a+b)((a+b)\tan^2(c+dx)+a)}-\frac{\frac{a(2a+3b)\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{b}-\frac{2(a+b)\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)}{b}}{2b(a+b)}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{a\tan(c+dx)}{2b(a+b)((a+b)\tan^2(c+dx)+a)}-\frac{\frac{a(2a+3b)\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{b}-\frac{2(a+b)\arctan(\tan(c+dx))}{b}}{2b(a+b)}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{a\tan(c+dx)}{2b(a+b)((a+b)\tan^2(c+dx)+a)} - \frac{\frac{\sqrt{a}(2a+3b)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b\sqrt{a+b}} - \frac{2(a+b)\arctan(\tan(c+dx))}{b}}{2b(a+b)}}{d}$$

input
```
Int[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
(-1/2*((-2*(a + b)*ArcTan[Tan[c + d*x]])/b + (Sqrt[a]*(2*a + 3*b)*ArcTan[(
Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(b*Sqrt[a + b]))/(b*(a + b)) + (a*Tan[
c + d*x])/(2*b*(a + b)*(a + (a + b)*Tan[c + d*x]^2)))/d
```

3.101.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.101.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{-\frac{a\left(-\frac{b\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)}+\frac{(2a+3b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{b^2}+\frac{\arctan(\tan(dx+c))}{b^2}}{d}$
default	$\frac{-\frac{a\left(-\frac{b\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)}+\frac{(2a+3b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{b^2}+\frac{\arctan(\tan(dx+c))}{b^2}}{d}$
risch	$\frac{x}{b^2}-\frac{ia\left(2a\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}-b\right)}{b^2(a+b)d\left(-b\,e^{4i(dx+c)}+4a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-b\right)}+\frac{\sqrt{-a(a+b)}\,a\ln\left(e^{2i(dx+c)}-\frac{2i\sqrt{-a(a+b)}+2a+b}{b}\right)}{2(a+b)^2 d\,b^2}+$

input
```
int(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a/b^2*(-1/2*b/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)+1/2
*(2*a+3*b)/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
+1/b^2*arctan(tan(d*x+c)))
```

3.101.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(81) = 162.

Time = 0.33 (sec) , antiderivative size = 492, normalized size of antiderivative = 5.29

$$\int \frac{\sin^4(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$$

$$= \left[\frac{8\,(ab+b^2)dx\cos(dx+c)^2-4\,ab\cos(dx+c)\sin(dx+c)-8\,(a^2+2\,ab+b^2)dx+\left((2\,ab+3\,b^2)\cos(dx\right.}{8}\right.$$

input
```
integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(8*(a*b + b^2)*d*x*cos(d*x + c)^2 - 4*a*b*cos(d*x + c)*sin(d*x + c) -
 8*(a^2 + 2*a*b + b^2)*d*x + ((2*a*b + 3*b^2)*cos(d*x + c)^2 - 2*a^2 - 5*a
*b - 3*b^2)*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2
*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(d*x +
 c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) +
a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^
2 + 2*a*b + b^2)))/((a*b^3 + b^4)*d*cos(d*x + c)^2 - (a^2*b^2 + 2*a*b^3 +
b^4)*d), 1/4*(4*(a*b + b^2)*d*x*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*
x + c) - 4*(a^2 + 2*a*b + b^2)*d*x + ((2*a*b + 3*b^2)*cos(d*x + c)^2 - 2*a
^2 - 5*a*b - 3*b^2)*sqrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(d*x + c)^2 -
 a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d*x + c))))/((a*b^3 + b^4)*d*c
os(d*x + c)^2 - (a^2*b^2 + 2*a*b^3 + b^4)*d)]
```

3.101.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**4/(a+b*sin(d*x+c)**2)**2,x)
```

output
```
Timed out
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.17

$$\int \frac{\sin^4(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \frac{\frac{a\tan(dx+c)}{a^2b+ab^2+(a^2b+2\,ab^2+b^3)\tan(dx+c)^2} - \frac{(2\,a^2+3\,ab)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(ab^2+b^3)\sqrt{(a+b)a}} + \frac{2\,(dx+c)}{b^2}}{2\,d}$$

input
```
integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(a*tan(d*x + c)/(a^2*b + a*b^2 + (a^2*b + 2*a*b^2 + b^3)*tan(d*x + c)^
2) - (2*a^2 + 3*a*b)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a*b^2
+ b^3)*sqrt((a + b)*a)) + 2*(d*x + c)/b^2)/d
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.51

$$\int \frac{\sin^4(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(2a^2+3ab)}{(ab^2+b^3)\sqrt{a^2+ab}}-\frac{a\tan(dx+c)}{\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)(ab+b^2)}-\frac{2(dx+c)}{b^2}}{2d}$$

input
```
integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c
) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*(2*a^2 + 3*a*b)/((a*b^2 + b^3)*sqrt(
a^2 + a*b)) - a*tan(d*x + c)/((a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a
*b + b^2)) - 2*(d*x + c)/b^2)/d
```

3.101.9 Mupad [B] (verification not implemented)

Time = 15.21 (sec) , antiderivative size = 1959, normalized size of antiderivative = 21.06

$$\int \frac{\sin^4(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^4/(a + b*sin(c + d*x)^2)^2,x)
```

output
```
(a*tan(c + d*x))/(2*d*(a + tan(c + d*x)^2*(a + b))*(a*b + b^2)) - atan((((
((2*a*b^6 + 4*a^2*b^5 + 2*a^3*b^4)*1i)/(2*(a*b^3 + b^4)) - (tan(c + d*x)*(
80*a*b^7 + 16*b^8 + 144*a^2*b^6 + 112*a^3*b^5 + 32*a^4*b^4))/(8*b^2*(a*b^2
 + b^3)))/(2*b^2) + (tan(c + d*x)*(16*a*b^3 + 28*a^3*b + 8*a^4 + 4*b^4 + 3
3*a^2*b^2))/(4*(a*b^2 + b^3)))/b^2 - ((((2*a*b^6 + 4*a^2*b^5 + 2*a^3*b^4)*
1i)/(2*(a*b^3 + b^4)) + (tan(c + d*x)*(80*a*b^7 + 16*b^8 + 144*a^2*b^6 + 1
12*a^3*b^5 + 32*a^4*b^4))/(8*b^2*(a*b^2 + b^3)))/(2*b^2) - (tan(c + d*x)*(
16*a*b^3 + 28*a^3*b + 8*a^4 + 4*b^4 + 33*a^2*b^2))/(4*(a*b^2 + b^3)))/b^2)
/((3*a*b^2 + (7*a^2*b)/2 + a^3)/(a*b^3 + b^4) + (((((2*a*b^6 + 4*a^2*b^5 +
 2*a^3*b^4)*1i)/(2*(a*b^3 + b^4)) - (tan(c + d*x)*(80*a*b^7 + 16*b^8 + 144
*a^2*b^6 + 112*a^3*b^5 + 32*a^4*b^4))/(8*b^2*(a*b^2 + b^3)))*1i)/(2*b^2) +
 (tan(c + d*x)*(16*a*b^3 + 28*a^3*b + 8*a^4 + 4*b^4 + 33*a^2*b^2)*1i)/(4*(
a*b^2 + b^3)))/b^2 + (((((2*a*b^6 + 4*a^2*b^5 + 2*a^3*b^4)*1i)/(2*(a*b^3 +
 b^4)) + (tan(c + d*x)*(80*a*b^7 + 16*b^8 + 144*a^2*b^6 + 112*a^3*b^5 + 32
*a^4*b^4))/(8*b^2*(a*b^2 + b^3)))*1i)/(2*b^2) - (tan(c + d*x)*(16*a*b^3 +
28*a^3*b + 8*a^4 + 4*b^4 + 33*a^2*b^2)*1i)/(4*(a*b^2 + b^3)))/b^2))/(b^2*d
) - (atan((((-a*(a + b)^3)^(1/2)*((tan(c + d*x)*(16*a*b^3 + 28*a^3*b + 8*a
^4 + 4*b^4 + 33*a^2*b^2))/(2*(a*b^2 + b^3)) - (((2*a*b^6 + 4*a^2*b^5 + 2*a
^3*b^4)/(a*b^3 + b^4) - (tan(c + d*x)*(-a*(a + b)^3)^(1/2)*(2*a + 3*b)*(80
*a*b^7 + 16*b^8 + 144*a^2*b^6 + 112*a^3*b^5 + 32*a^4*b^4))/(8*(a*b^2 + ...
```

3.101. $\int \frac{\sin^4(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$

3.102 $\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$

3.102.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \frac{\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}(a+b)^{3/2}d} - \frac{\cos(c+dx)\sin(c+dx)}{2(a+b)d\,(a+b\sin^2(c+dx))}$$

output
```
-1/2*cos(d*x+c)*sin(d*x+c)/(a+b)/d/(a+b*sin(d*x+c)^2)+1/2*arctan((a+b)^(1/
2)*tan(d*x+c)/a^(1/2))/(a+b)^(3/2)/d/a^(1/2)
```

3.102.2 Mathematica [A] (verified)

Time = 11.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \frac{\frac{\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}} - \frac{\sin(2(c+dx))}{(a+b)(2a+b-b\cos(2(c+dx)))}}{2d}$$

input
```
Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
(ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*(a + b)^(3/2)) - Sin[
2*(c + d*x)]/((a + b)*(2*a + b - b*Cos[2*(c + d*x)])))/(2*d)
```

3.102.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3652, 27, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^2}{\left(a+b\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{3652}$$

$$\frac{\int \frac{a}{b\sin^2(c+dx)+a}dx}{2a(a+b)}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a+b)\left(a+b\sin^2(c+dx)\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{b\sin^2(c+dx)+a}dx}{2(a+b)}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a+b)\left(a+b\sin^2(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{b\sin(c+dx)^2+a}dx}{2(a+b)}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a+b)\left(a+b\sin^2(c+dx)\right)}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{2d(a+b)}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a+b)\left(a+b\sin^2(c+dx)\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}d(a+b)^{3/2}}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a+b)\left(a+b\sin^2(c+dx)\right)}$$

input
```
Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(2*Sqrt[a]*(a + b)^(3/2)*d) - (
Cos[c + d*x]*Sin[c + d*x])/(2*(a + b)*d*(a + b*Sin[c + d*x]^2))
```

3.102.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.102.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)}+\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}}{d}$
default	$\frac{-\frac{\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)}+\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}}{d}$
risch	$\frac{i\left(2a\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}-b\right)}{b(a+b)d\left(-b\,e^{4i(dx+c)}+4a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-b\right)}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia^2+2iab-2a\sqrt{-a^2-ab}-b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{4\sqrt{-a^2-ab}\,(a+b)d}-\frac{\ln\left(e\right.}{}$

input
```
int(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)+1/2/(a+b)/(a*
(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
```

3.102.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(66) = 132$.

Time = 0.28 (sec) , antiderivative size = 419, normalized size of antiderivative = 5.37

$$\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$$
$$= \left[\frac{4\,(a^2+ab)\cos(dx+c)\sin(dx+c) - \left(b\cos(dx+c)^2 - a - b\right)\sqrt{-a^2-ab}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4-2\,(4}{\cdots}\right.}{8\left((a^3b+2\,a^2b^2+ab^3)d\cos(dx+c)^2-(a^4+3\cdots\right.}\right.$$

input
```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*(a^2 + a*b)*cos(d*x + c)*sin(d*x + c) - (b*cos(d*x + c)^2 - a - b)
*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5
*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*cos(d*x
 + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c
)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)))/((a^3*b + 2*a^2*
b^2 + a*b^3)*d*cos(d*x + c)^2 - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d), 1/
4*(2*(a^2 + a*b)*cos(d*x + c)*sin(d*x + c) - (b*cos(d*x + c)^2 - a - b)*sq
rt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*
b)*cos(d*x + c)*sin(d*x + c))))/((a^3*b + 2*a^2*b^2 + a*b^3)*d*cos(d*x + c
)^2 - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d)]
```

3.102.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**2)**2,x)
```

output
```
Timed out
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = -\frac{\frac{\tan(dx+c)}{(a^2+2\,ab+b^2)\tan(dx+c)^2+a^2+ab} - \frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}(a+b)}}{2\,d}$$

input
```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(tan(d*x + c)/((a^2 + 2*a*b + b^2)*tan(d*x + c)^2 + a^2 + a*b) - arct
an((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a + b)))/d
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.40

$$\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$$
$$= \frac{\frac{\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}(a+b)} - \frac{\tan(dx+c)}{\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)(a+b)}}{2\,d}$$

input
```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
 + b*tan(d*x + c))/sqrt(a^2 + a*b)))/(sqrt(a^2 + a*b)*(a + b)) - tan(d*x +
 c)/((a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a + b)))/d
```

3.102.9 Mupad [B] (verification not implemented)

Time = 13.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)^2}{4\,\sqrt{a}\,(a+b)^{3/2}}\right)}{2\,\sqrt{a}\,d\,(a+b)^{3/2}} - \frac{\tan(c+d\,x)}{2\,d\,\left((a+b)\,\tan(c+d\,x)^2+a\right)\,(a+b)}$$

input
```
int(sin(c + d*x)^2/(a + b*sin(c + d*x)^2)^2,x)
```

output
```
atan((tan(c + d*x)*(2*a + 2*b)^2)/(4*a^(1/2)*(a + b)^(3/2)))/(2*a^(1/2)*d*
(a + b)^(3/2)) - tan(c + d*x)/(2*d*(a + tan(c + d*x)^2*(a + b))*(a + b))
```

3.102. $\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$

3.103 $\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

3.103.1 Optimal result

Integrand size = 14, antiderivative size = 87

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}d}+\frac{b\cos(c+dx)\sin(c+dx)}{2a(a+b)d\left(a+b\sin^2(c+dx)\right)}$$

output
```
1/2*(2*a+b)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/(a+b)^(3/2)/d+1
/2*b*cos(d*x+c)*sin(d*x+c)/a/(a+b)/d/(a+b*sin(d*x+c)^2)
```

3.103.2 Mathematica [A] (verified)

Time = 11.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{\frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}}+\frac{\sqrt{a}b\sin(2(c+dx))}{(a+b)(2a+b-b\cos(2(c+dx)))}}{2a^{3/2}d}$$

input
```
Integrate[(a + b*Sin[c + d*x]^2)^(-2),x]
```

output
```
(((2*a + b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a + b)^(3/2) + (S
qrt[a]*b*Sin[2*(c + d*x)])/((a + b)*(2*a + b - b*Cos[2*(c + d*x)])))/(2*a^
(3/2)*d)
```

3.103.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3663, 25, 27, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{3663}$$

$$\frac{b\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}-\frac{\int -\frac{2a+b}{b\sin^2(c+dx)+a}dx}{2a(a+b)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2a+b}{b\sin^2(c+dx)+a}dx}{2a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(2a+b)\int \frac{1}{b\sin^2(c+dx)+a}dx}{2a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(2a+b)\int \frac{1}{b\sin(c+dx)^2+a}dx}{2a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}$$

$$\Big\downarrow \text{3660}$$

$$\frac{(2a+b)\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{2ad(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{3/2}}+\frac{b\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}$$

input
```
Int[(a + b*Sin[c + d*x]^2)^(-2),x]
```

output
```
((2*a + b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^
(3/2)*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a + b)*d*(a + b*Sin[c + d*x
]^2))
```

3.103.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.103.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{b\tan(dx+c)}{2a(a+b)\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)}+\frac{(2a+b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2a(a+b)\sqrt{a(a+b)}}}{d}$
default	$\frac{\frac{b\tan(dx+c)}{2a(a+b)\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)}+\frac{(2a+b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2a(a+b)\sqrt{a(a+b)}}}{d}$
risch	$-\frac{i(2a\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}-b)}{a(a+b)d(-b\,e^{4i(dx+c)}+4a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-b)}-\frac{\ln\left(e^{2i(dx+c)}-\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,(a+b)d}-$ ln

input
```
int(1/(a+b*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*b/a/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)+1/2*(2*a+b
)/a/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
```

3.103.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(75) = 150$.

Time = 0.29 (sec) , antiderivative size = 463, normalized size of antiderivative = 5.32

$$\int \frac{1}{(a+b\sin^2(c+dx))^2}\,dx$$

$$= \left[-\frac{4\left(a^2b+ab^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(\left(2\,ab+b^2\right)\cos\left(dx+c\right)^2-2\,a^2-3\,ab-b^2\right)\sqrt{-a^2-ab}\log\left(\right.}{8\left(\left(a^4b+2\,a^3b^2+a^2b^3\right)d\cos\left(dx+\right.\right.}\right.$$

$$-\frac{2\left(a^2b+ab^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(\left(2\,ab+b^2\right)\cos\left(dx+c\right)^2-2\,a^2-3\,ab-b^2\right)\sqrt{a^2+ab}\arctan\left(\right.}{4\left(\left(a^4b+2\,a^3b^2+a^2b^3\right)d\cos\left(dx+c\right)^2-\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3\right)d\right)}$$

input
```
integrate(1/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*(a^2*b + a*b^2)*cos(d*x + c)*sin(d*x + c) + ((2*a*b + b^2)*cos(d*
x + c)^2 - 2*a^2 - 3*a*b - b^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2
)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*c
os(d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2
+ 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 +
2*a*b + b^2)))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cos(d*x + c)^2 - (a^5 + 3*
a^4*b + 3*a^3*b^2 + a^2*b^3)*d), -1/4*(2*(a^2*b + a*b^2)*cos(d*x + c)*sin(
d*x + c) + ((2*a*b + b^2)*cos(d*x + c)^2 - 2*a^2 - 3*a*b - b^2)*sqrt(a^2 +
 a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d
*x + c)*sin(d*x + c))))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cos(d*x + c)^2 -
(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d)]
```

3.103.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*sin(d*x+c)**2)**2,x)
```

output
```
Timed out
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^2}\,dx=\frac{\frac{b\tan(dx+c)}{a^3+a^2b+(a^3+2\,a^2b+ab^2)\tan(dx+c)^2}+\frac{(2\,a+b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}(a^2+ab)}}{2\,d}$$

input
```
integrate(1/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(b*tan(d*x + c)/(a^3 + a^2*b + (a^3 + 2*a^2*b + a*b^2)*tan(d*x + c)^2)
 + (2*a + b)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)
*(a^2 + a*b)))/d
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+b\sin^2(c+dx))^2}\,dx = \frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(2a+b)}{(a^2+ab)^{\frac{3}{2}}}+\frac{b\tan(dx+c)}{\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)(a^2+ab)}}{2d}$$

input
```
integrate(1/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
 + b*tan(d*x + c))/sqrt(a^2 + a*b)))*(2*a + b)/(a^2 + a*b)^(3/2) + b*tan(d
*x + c)/((a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a^2 + a*b)))/d
```

3.103.9 Mupad [B] (verification not implemented)

Time = 13.45 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+b\sin^2(c+dx))^2}\,dx = \frac{\operatorname{atan}\left(\frac{\tan(c+dx)\,(2a+2b)}{2\sqrt{a}\,\sqrt{a+b}}\right)\,(2a+b)}{2a^{3/2}\,d\,(a+b)^{3/2}} + \frac{b\tan(c+dx)}{2ad\,\left((a+b)\,\tan(c+dx)^2+a\right)\,(a+b)}$$

input
```
int(1/(a + b*sin(c + d*x)^2)^2,x)
```

output
```
(atan((tan(c + d*x)*(2*a + 2*b))/(2*a^(1/2)*(a + b)^(1/2)))*(2*a + b))/(2*
a^(3/2)*d*(a + b)^(3/2)) + (b*tan(c + d*x))/(2*a*d*(a + tan(c + d*x)^2*(a
+ b))*(a + b))
```

3.104 $\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

3.104.1 Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{b(4a+3b)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a+b)^{3/2}d} - \frac{\cot(c+dx)}{ad\left(a+b\sin^2(c+dx)\right)} - \frac{(2ab+3b^2)\cos(c+dx)\sin(c+dx)}{2a^2(a+b)d\left(a+b\sin^2(c+dx)\right)}$$

output
```
-1/2*b*(4*a+3*b)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(5/2)/(a+b)^(3/2
)/d-cot(d*x+c)/a/d/(a+b*sin(d*x+c)^2)-1/2*(2*a*b+3*b^2)*cos(d*x+c)*sin(d*x
+c)/a^2/(a+b)/d/(a+b*sin(d*x+c)^2)
```

3.104.2 Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.22

$$\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{(2a+b-b\cos(2(c+dx)))\left(b(4a+3b)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)(2a+b-b\cos(2(c+dx)))+\sqrt{a}\sqrt{a+b}\right.}{8a^{5/2}(a+b)^{3/2}d\left(b+a\csc^2(c+dx\right.}$$

input
```
Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
-1/8*((2*a + b - b*Cos[2*(c + d*x)])*(b*(4*a + 3*b)*ArcTan[(Sqrt[a + b]*Ta
n[c + d*x])/Sqrt[a]]*(2*a + b - b*Cos[2*(c + d*x)]) + Sqrt[a]*Sqrt[a + b]*
(4*a^2 + 6*a*b + 3*b^2 - b*(2*a + 3*b)*Cos[2*(c + d*x)])*Cot[c + d*x])*Csc
[c + d*x]^4)/(a^(5/2)*(a + b)^(3/2)*d*(b + a*Csc[c + d*x]^2)^2)
```

3.104.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3666, 365, 25, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(c+dx)^2\left(a+b\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \ 3666$$

$$\frac{\int \frac{\cot^2(c+dx)\left(\tan^2(c+dx)+1\right)^2}{\left((a+b)\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \ 365$$

$$\frac{\frac{\int -\frac{-a\tan^2(c+dx)+a+3b}{\left((a+b)\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{a}-\frac{\cot(c+dx)}{a\left((a+b)\tan^2(c+dx)+a\right)}}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{-\frac{\int \frac{-a\tan^2(c+dx)+a+3b}{\left((a+b)\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{a}-\frac{\cot(c+dx)}{a\left((a+b)\tan^2(c+dx)+a\right)}}{d}$$

$$\Big\downarrow \ 298$$

$$\frac{-\frac{\frac{b(4a+3b)\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{2a(a+b)}+\frac{\left(2a^2+4ab+3b^2\right)\tan(c+dx)}{2a(a+b)\left((a+b)\tan^2(c+dx)+a\right)}}{a}-\frac{\cot(c+dx)}{a\left((a+b)\tan^2(c+dx)+a\right)}}{d}$$

$$\Big\downarrow \ 218$$

$$-\frac{\frac{\frac{b(4a+3b)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}+\frac{(2a^2+4ab+3b^2)\tan(c+dx)}{2a(a+b)\left((a+b)\tan^2(c+dx)+a\right)}}{a}-\frac{\cot(c+dx)}{a((a+b)\tan^2(c+dx)+a)}}{d}$$

input
```
Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
(-(Cot[c + d*x]/(a*(a + (a + b)*Tan[c + d*x]^2))) - ((b*(4*a + 3*b)*ArcTan
[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(3/2)) + ((2*a^2
+ 4*a*b + 3*b^2)*Tan[c + d*x])/(2*a*(a + b)*(a + (a + b)*Tan[c + d*x]^2)))
/a)/d
```

3.104.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.104.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{-\frac{1}{a^2\tan(dx+c)}-\frac{b\left(\frac{b\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)}+\frac{(4a+3b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^2}}{d}$
default	$\frac{-\frac{1}{a^2\tan(dx+c)}-\frac{b\left(\frac{b\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)}+\frac{(4a+3b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^2}}{d}$
risch	$-\frac{i\left(-4ab\,e^{4i(dx+c)}-3b^2e^{4i(dx+c)}+8a^2e^{2i(dx+c)}+14ab\,e^{2i(dx+c)}+6b^2e^{2i(dx+c)}-2ab-3b^2\right)}{a^2(a+b)d\left(-b\,e^{4i(dx+c)}+4a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-b\right)\left(e^{2i(dx+c)}-1\right)}+\frac{\ln\left(e^{2i(dx+c)}-\frac{2ia^2+2ia\cdots}{\cdots}\right.}{\sqrt{-a^2-\cdots}}$

input
```
int(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^2/tan(d*x+c)-1/a^2*b*(1/2*b/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+tan
(d*x+c)^2*b+a)+1/2*(4*a+3*b)/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)
/(a*(a+b))^(1/2))))
```

3.104.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(115) = 230$.

Time = 0.30 (sec) , antiderivative size = 588, normalized size of antiderivative = 4.63

$$\int \frac{\csc^2(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx$$

$$= \left[-\frac{4\,(2\,a^3b+5\,a^2b^2+3\,ab^3)\cos{(dx+c)}^3-\left(4\,a^2b+7\,ab^2+3\,b^3-(4\,ab^2+3\,b^3)\cos{(dx+c)}^2\right)\sqrt{-a^2-a}}{8\,\left((a^5b+\right.}\right.$$

$$-\frac{2\,(2\,a^3b+5\,a^2b^2+3\,ab^3)\cos{(dx+c)}^3+\left(4\,a^2b+7\,ab^2+3\,b^3-(4\,ab^2+3\,b^3)\cos{(dx+c)}^2\right)\sqrt{a^2+ab}\,\mathrm{ar}}{4\,\left((a^5b+2\,a^4b^2+a^3b^3)d\cos{(dx+c)}^2-(a^6+3\right.}$$

input
```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*(2*a^3*b + 5*a^2*b^2 + 3*a*b^3)*cos(d*x + c)^3 - (4*a^2*b + 7*a*b
^2 + 3*b^3 - (4*a*b^2 + 3*b^3)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^
2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 -
 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(
d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x
+ c)^2 + a^2 + 2*a*b + b^2))*sin(d*x + c) - 4*(2*a^4 + 6*a^3*b + 7*a^2*b^2
 + 3*a*b^3)*cos(d*x + c))/(((a^5*b + 2*a^4*b^2 + a^3*b^3)*d*cos(d*x + c)^2
 - (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d)*sin(d*x + c)), -1/4*(2*(2*a^3*
b + 5*a^2*b^2 + 3*a*b^3)*cos(d*x + c)^3 + (4*a^2*b + 7*a*b^2 + 3*b^3 - (4*
a*b^2 + 3*b^3)*cos(d*x + c)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d
*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x +
c) - 2*(2*a^4 + 6*a^3*b + 7*a^2*b^2 + 3*a*b^3)*cos(d*x + c))/(((a^5*b + 2*
a^4*b^2 + a^3*b^3)*d*cos(d*x + c)^2 - (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3
)*d)*sin(d*x + c))]
```

3.104.6 Sympy [F]

$$\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx=\int \frac{\csc^2\left(c+dx\right)}{\left(a+b\sin^2\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**2)**2,x)
```

output
```
Integral(csc(c + d*x)**2/(a + b*sin(c + d*x)**2)**2, x)
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05

$$\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\frac{(4\,ab+3\,b^2)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(a^3+a^2b)\sqrt{(a+b)a}}+\frac{(2\,a^2+4\,ab+3\,b^2)\tan(dx+c)^2+2\,a^2+2\,ab}{(a^4+2\,a^3b+a^2b^2)\tan(dx+c)^3+(a^4+a^3b)\tan(dx+c)}}{2\,d}$$

input
```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*((4*a*b + 3*b^2)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a^3 +
 a^2*b)*sqrt((a + b)*a)) + ((2*a^2 + 4*a*b + 3*b^2)*tan(d*x + c)^2 + 2*a^2
 + 2*a*b)/((a^4 + 2*a^3*b + a^2*b^2)*tan(d*x + c)^3 + (a^4 + a^3*b)*tan(d*
x + c)))/d
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.41

$$\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(4\,ab+3\,b^2)}{(a^3+a^2b)\sqrt{a^2+ab}}+\frac{2\,a^2\tan(dx+c)^2+4\,ab\tan(dx+c)^2+3\,b^2\tan(dx+c)^2+2\,a^2+2}{\left(a\tan(dx+c)^3+b\tan(dx+c)^3+a\tan(dx+c)\right)(a^3+a^2b)}}{2\,d}$$

input
```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c
) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*(4*a*b + 3*b^2)/((a^3 + a^2*b)*sqrt(
a^2 + a*b)) + (2*a^2*tan(d*x + c)^2 + 4*a*b*tan(d*x + c)^2 + 3*b^2*tan(d*x
 + c)^2 + 2*a^2 + 2*a*b)/((a*tan(d*x + c)^3 + b*tan(d*x + c)^3 + a*tan(d*x
 + c))*(a^3 + a^2*b)))/d
```

3.104.9 Mupad [B] (verification not implemented)

Time = 13.73 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.04

$$\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\frac{1}{a}+\frac{\tan(c+d\,x)^2\,\left(2\,a^2+4\,a\,b+3\,b^2\right)}{2\,a^2\,(a+b)}}{d\,\left((a+b)\,\tan\left(c+d\,x\right)^3+a\tan\left(c+d\,x\right)\right)} - \frac{b\,\mathrm{atan}\left(\frac{b\tan(c+d\,x)\,\left(a^3+b\,a^2\right)\,(4\,a+3\,b)}{a^{5/2}\,\sqrt{a+b}\,(3\,b^2+4\,a\,b)}\right)\,(4\,a+3\,b)}{2\,a^{5/2}\,d\,(a+b)^{3/2}}$$

input
```
int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x)^2)^2),x)
```

output
```
- (1/a + (tan(c + d*x)^2*(4*a*b + 2*a^2 + 3*b^2))/(2*a^2*(a + b)))/(d*(a*t
an(c + d*x) + tan(c + d*x)^3*(a + b))) - (b*atan((b*tan(c + d*x)*(a^2*b +
a^3)*(4*a + 3*b))/(a^(5/2)*(a + b)^(1/2)*(4*a*b + 3*b^2)))*(4*a + 3*b))/(2
*a^(5/2)*d*(a + b)^(3/2))
```

3.105 $\int \frac{\csc^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

3.105.1 Optimal result

Integrand size = 23, antiderivative size = 162

$$\int \frac{\csc^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{b^2(6a+5b)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a+b)^{3/2}d} - \frac{(2a^2-ab-5b^2)\cot(c+dx)}{2a^3(a+b)d} - \frac{(2a+5b)\cot^3(c+dx)}{6a^2(a+b)d} + \frac{b\csc^3(c+dx)\sec(c+dx)}{2a(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)}$$

output
```
1/2*b^2*(6*a+5*b)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(7/2)/(a+b)^(3/
2)/d-1/2*(2*a^2-a*b-5*b^2)*cot(d*x+c)/a^3/(a+b)/d-1/6*(2*a+5*b)*cot(d*x+c)
^3/a^2/(a+b)/d+1/2*b*csc(d*x+c)^3*sec(d*x+c)/a/(a+b)/d/(a+(a+b)*tan(d*x+c)
^2)
```

3.105.2 Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

$$\int \frac{\csc^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{(-2a-b+b\cos(2(c+dx)))\csc^4(c+dx)\left(\frac{3b^2(6a+5b)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)(-2a-b+b\cos(2(c+dx)))}{(a+b)^{3/2}} + 4\sqrt{a}(a-3b)\right.}{24a^{7/2}d}$$

input
```
Integrate[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
((-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^4*((3*b^2*(6*a + 5*b)*ArcTan
[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]*(-2*a - b + b*Cos[2*(c + d*x)]))/(a +
 b)^(3/2) + 4*Sqrt[a]*(a - 3*b)*(2*a + b - b*Cos[2*(c + d*x)])*Cot[c + d*x
] + 2*a^(3/2)*(2*a + b - b*Cos[2*(c + d*x)])*Cot[c + d*x]*Csc[c + d*x]^2 -
 (3*Sqrt[a]*b^3*Sin[2*(c + d*x)])/(a + b)))/(24*a^(7/2)*d*(b + a*Csc[c + d
*x]^2)^2)
```

3.105.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3666, 370, 25, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^4\left(a+b\sin(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{3666}$$

$$\frac{\int \frac{\cot^4(c+dx)\left(\tan^2(c+dx)+1\right)^3}{\left((a+b)\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{370}$$

$$\frac{\frac{b\left(\tan^2(c+dx)+1\right)^2\cot^3(c+dx)}{2a(a+b)\left((a+b)\tan^2(c+dx)+a\right)}-\frac{\int -\frac{\cot^4(c+dx)\left(\tan^2(c+dx)+1\right)\left((2a+b)\tan^2(c+dx)+2a+5b\right)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{2a(a+b)}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\cot^4(c+dx)\left(\tan^2(c+dx)+1\right)\left((2a+b)\tan^2(c+dx)+2a+5b\right)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{2a(a+b)}+\frac{b\left(\tan^2(c+dx)+1\right)^2\cot^3(c+dx)}{2a(a+b)\left((a+b)\tan^2(c+dx)+a\right)}}{d}$$

$$\Big\downarrow \text{437}$$

$$\frac{\frac{\int \left(\frac{(2a+5b)\cot^4(c+dx)}{a}+\frac{\left(2a^2-ba-5b^2\right)\cot^2(c+dx)}{a^2}+\frac{b^2(6a+5b)}{a^2\left((a+b)\tan^2(c+dx)+a\right)}\right)d\tan(c+dx)}{2a(a+b)}+\frac{b\left(\tan^2(c+dx)+1\right)^2\cot^3(c+dx)}{2a(a+b)\left((a+b)\tan^2(c+dx)+a\right)}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\frac{b^2(6a+5b)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}}-\frac{\left(2a^2-ab-5b^2\right)\cot(c+dx)}{a^2}-\frac{(2a+5b)\cot^3(c+dx)}{3a}}{2a(a+b)}+\frac{b\left(\tan^2(c+dx)+1\right)^2\cot^3(c+dx)}{2a(a+b)\left((a+b)\tan^2(c+dx)+a\right)}}{d}$$

input
```
Int[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^2)^2,x]
```

output
```
(((b^2*(6*a + 5*b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(5/2)*Sq
rt[a + b]) - ((2*a^2 - a*b - 5*b^2)*Cot[c + d*x])/a^2 - ((2*a + 5*b)*Cot[c
 + d*x]^3)/(3*a))/(2*a*(a + b)) + (b*Cot[c + d*x]^3*(1 + Tan[c + d*x]^2)^2
)/(2*a*(a + b)*(a + (a + b)*Tan[c + d*x]^2)))/d
```

3.105.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.105.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{\frac{b^2\left(\frac{b\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)}+\frac{(6a+5b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^3}-\frac{1}{3a^2\tan(dx+c)^3}-\frac{a-2b}{a^3\tan(dx+c)}}{d}$
default	$\frac{\frac{b^2\left(\frac{b\tan(dx+c)}{2(a+b)\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)}+\frac{(6a+5b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^3}-\frac{1}{3a^2\tan(dx+c)^3}-\frac{a-2b}{a^3\tan(dx+c)}}{d}$
risch	$\frac{i\left(-18a\,b^2e^{8i(dx+c)}-15b^3e^{8i(dx+c)}+36a^2b\,e^{6i(dx+c)}+102a\,b^2e^{6i(dx+c)}+60b^3e^{6i(dx+c)}+48a^3e^{4i(dx+c)}-20a^2b\,e^{4i(dx+c)}\right.}{3d\,a^3\left(e^{2i(dx+c)}-1\right)^3(a+b)\left(-b\,e^{4i(\right.}}$

input
```
int(csc(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2/a^3*(1/2*b/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)+1/2
*(6*a+5*b)/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
-1/3/a^2/tan(d*x+c)^3-(a-2*b)/a^3/tan(d*x+c))
```

3.105.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(146) = 292$.

Time = 0.30 (sec) , antiderivative size = 843, normalized size of antiderivative = 5.20

$$\int \frac{\csc^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$$
$$= \left[-\frac{4\left(4a^4b-4a^3b^2-23a^2b^3-15ab^4\right)\cos(dx+c)^5-8\left(2a^5+3a^4b-12a^3b^2-28a^2b^3-15ab^4\right)\cos(d\right.}{}\right.$$
$$-\frac{2\left(4a^4b-4a^3b^2-23a^2b^3-15ab^4\right)\cos(dx+c)^5-4\left(2a^5+3a^4b-12a^3b^2-28a^2b^3-15ab^4\right)\cos(dx}{12\left((a^6b\right.}$$

input
```
integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/24*(4*(4*a^4*b - 4*a^3*b^2 - 23*a^2*b^3 - 15*a*b^4)*cos(d*x + c)^5 - 8
*(2*a^5 + 3*a^4*b - 12*a^3*b^2 - 28*a^2*b^3 - 15*a*b^4)*cos(d*x + c)^3 + 3
*((6*a*b^3 + 5*b^4)*cos(d*x + c)^4 + 6*a^2*b^2 + 11*a*b^3 + 5*b^4 - (6*a^2
*b^2 + 17*a*b^3 + 10*b^4)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8
*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((
2*a + b)*cos(d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x +
 c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^
2 + a^2 + 2*a*b + b^2))*sin(d*x + c) + 12*(2*a^5 + 2*a^4*b - 6*a^3*b^2 - 1
1*a^2*b^3 - 5*a*b^4)*cos(d*x + c))/(((a^6*b + 2*a^5*b^2 + a^4*b^3)*d*cos(d
*x + c)^4 - (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cos(d*x + c)^2 + (a^
7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d)*sin(d*x + c)), -1/12*(2*(4*a^4*b - 4
*a^3*b^2 - 23*a^2*b^3 - 15*a*b^4)*cos(d*x + c)^5 - 4*(2*a^5 + 3*a^4*b - 12
*a^3*b^2 - 28*a^2*b^3 - 15*a*b^4)*cos(d*x + c)^3 + 3*((6*a*b^3 + 5*b^4)*co
s(d*x + c)^4 + 6*a^2*b^2 + 11*a*b^3 + 5*b^4 - (6*a^2*b^2 + 17*a*b^3 + 10*b
^4)*cos(d*x + c)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 -
 a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) + 6*(2*a
^5 + 2*a^4*b - 6*a^3*b^2 - 11*a^2*b^3 - 5*a*b^4)*cos(d*x + c))/(((a^6*b +
2*a^5*b^2 + a^4*b^3)*d*cos(d*x + c)^4 - (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4
*b^3)*d*cos(d*x + c)^2 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d)*sin(d*x
+ c))]
```

3.105.6 Sympy [F]

$$\int \frac{\csc^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx=\int \frac{\csc^4{(c+dx)}}{\left(a+b\sin^2{(c+dx)}\right)^2}\,dx$$

input
```
integrate(csc(d*x+c)**4/(a+b*sin(d*x+c)**2)**2,x)
```

output
```
Integral(csc(c + d*x)**4/(a + b*sin(c + d*x)**2)**2, x)
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.06

$$\int \frac{\csc^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$$
$$=\frac{\frac{3\,(6\,ab^2+5\,b^3)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(a^4+a^3b)\sqrt{(a+b)a}}-\frac{3\,(2\,a^3-6\,ab^2-5\,b^3)\tan(dx+c)^4+2\,a^3+2\,a^2b+2\,(4\,a^3-a^2b-5\,ab^2)\tan(dx+c)^2}{(a^5+2\,a^4b+a^3b^2)\tan(dx+c)^5+(a^5+a^4b)\tan(dx+c)^3}}{6\,d}$$

input
```
integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/6*(3*(6*a*b^2 + 5*b^3)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a^
4 + a^3*b)*sqrt((a + b)*a)) - (3*(2*a^3 - 6*a*b^2 - 5*b^3)*tan(d*x + c)^4
+ 2*a^3 + 2*a^2*b + 2*(4*a^3 - a^2*b - 5*a*b^2)*tan(d*x + c)^2)/((a^5 + 2*
a^4*b + a^3*b^2)*tan(d*x + c)^5 + (a^5 + a^4*b)*tan(d*x + c)^3))/d
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.07

$$\int \frac{\csc^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$$
$$=\frac{\frac{3\,b^3\tan(dx+c)}{(a^4+a^3b)\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)}+\frac{3\,(6\,ab^2+5\,b^3)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{(a^4+a^3b)\sqrt{a^2+ab}}-\frac{2\left(3\,a\tan(dx\right.}{a}}{6\,d}$$

input
```
integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/6*(3*b^3*tan(d*x + c)/((a^4 + a^3*b)*(a*tan(d*x + c)^2 + b*tan(d*x + c)^
2 + a)) + 3*(6*a*b^2 + 5*b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b)
 + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a^4 + a^3*
b)*sqrt(a^2 + a*b)) - 2*(3*a*tan(d*x + c)^2 - 6*b*tan(d*x + c)^2 + a)/(a^3
*tan(d*x + c)^3))/d
```

3.105.9 Mupad [B] (verification not implemented)

Time = 14.87 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.01

$$\int \frac{\csc^4(c+dx)}{(a+b\sin^2(c+dx))^2}\,dx = \frac{b^2\,\mathrm{atan}\left(\frac{b^2\tan(c+d\,x)\,(a^4+b\,a^3)\,(6\,a+5\,b)}{a^{7/2}\,(5\,b^3+6\,a\,b^2)\,\sqrt{a+b}}\right)\,(6\,a+5\,b)}{2\,a^{7/2}\,d\,(a+b)^{3/2}} - \frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\,(4\,a-5\,b)}{3\,a^2}-\frac{\tan(c+d\,x)^4\,(-2\,a^3+6\,a\,b^2+5\,b^3)}{2\,a^3\,(a+b)}}{d\,\left((a+b)\,\tan\left(c+d\,x\right)^5+a\tan\left(c+d\,x\right)^3\right)}$$

input
```
int(1/(sin(c + d*x)^4*(a + b*sin(c + d*x)^2)^2),x)
```

output
```
(b^2*atan((b^2*tan(c + d*x)*(a^3*b + a^4)*(6*a + 5*b))/(a^(7/2)*(6*a*b^2 +
 5*b^3)*(a + b)^(1/2)))*(6*a + 5*b))/(2*a^(7/2)*d*(a + b)^(3/2)) - (1/(3*a
) + (tan(c + d*x)^2*(4*a - 5*b))/(3*a^2) - (tan(c + d*x)^4*(6*a*b^2 - 2*a^
3 + 5*b^3))/(2*a^3*(a + b)))/(d*(tan(c + d*x)^5*(a + b) + a*tan(c + d*x)^3
))
```

3.106 $\int \frac{\sin^6(c+dx)}{(a+b\sin^2(c+dx))^3} dx$

3.106.1 Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{\sin^6(c+dx)}{(a+b\sin^2(c+dx))^3} dx = \frac{x}{b^3} - \frac{\sqrt{a}(8a^2+20ab+15b^2)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8b^3(a+b)^{5/2}d} + \frac{a\tan^3(c+dx)}{4b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} + \frac{a(4a+7b)\tan(c+dx)}{8b^2(a+b)^2d\left(a+(a+b)\tan^2(c+dx)\right)}$$

output
```
x/b^3-1/8*(8*a^2+20*a*b+15*b^2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))*a^(
1/2)/b^3/(a+b)^(5/2)/d+1/4*a*tan(d*x+c)^3/b/(a+b)/d/(a+(a+b)*tan(d*x+c)^2)
^2+1/8*a*(4*a+7*b)*tan(d*x+c)/b^2/(a+b)^2/d/(a+(a+b)*tan(d*x+c)^2)
```

3.106.2 Mathematica [A] (verified)

Time = 12.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91

$$\int \frac{\sin^6(c+dx)}{(a+b\sin^2(c+dx))^3} dx = \frac{8(c+dx) - \frac{\sqrt{a}(8a^2+20ab+15b^2)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{5/2}} + \frac{ab(8a^2+20ab+9b^2-3b(2a+3b)\cos(2(c+dx)))\sin(2(c+dx))}{(a+b)^2(2a+b-b\cos(2(c+dx)))^2}}{8b^3d}$$

input
```
Integrate[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2)^3,x]
```

output
```
(8*(c + d*x) - (Sqrt[a]*(8*a^2 + 20*a*b + 15*b^2)*ArcTan[(Sqrt[a + b]*Tan[
c + d*x])/Sqrt[a]])/(a + b)^(5/2) + (a*b*(8*a^2 + 20*a*b + 9*b^2 - 3*b*(2*
a + 3*b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/((a + b)^2*(2*a + b - b*Cos[2
*(c + d*x)])^2))/(8*b^3*d)
```

3.106.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3666, 372, 440, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^6}{\left(a+b\sin(c+dx)^2\right)^3}dx$$

$$\Big\downarrow \text{3666}$$

$$\frac{\int \frac{\tan^6(c+dx)}{(\tan^2(c+dx)+1)((a+b)\tan^2(c+dx)+a)^3}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{a\tan^3(c+dx)}{4b(a+b)((a+b)\tan^2(c+dx)+a)^2}-\frac{\int \frac{\tan^2(c+dx)\left(3a-(a+4b)\tan^2(c+dx)\right)}{\left(\tan^2(c+dx)+1\right)\left((a+b)\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{4b(a+b)}}{d}$$

$$\Big\downarrow \text{440}$$

$$\frac{\frac{a\tan^3(c+dx)}{4b(a+b)((a+b)\tan^2(c+dx)+a)^2}-\frac{\frac{\int \frac{a(4a+7b)-\left(4a^2+9ba+8b^2\right)\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left((a+b)\tan^2(c+dx)+a\right)}d\tan(c+dx)}{2b(a+b)}-\frac{a(4a+7b)\tan(c+dx)}{2b(a+b)\left((a+b)\tan^2(c+dx)+a\right)}}{4b(a+b)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{a\tan^3(c+dx)}{4b(a+b)((a+b)\tan^2(c+dx)+a)^2}-\frac{\frac{\frac{a\left(8a^2+20ab+15b^2\right)\int\frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{b}-\frac{8(a+b)^2\int\frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)}{b}}{2b(a+b)}-\frac{a(4a+7b)\tan(c+dx)}{2b(a+b)\left((a+b)\tan^2(c+dx)+a\right)}}{4b(a+b)}}{d}$$

$\Big\downarrow$ 216

$$\frac{\frac{a\tan^3(c+dx)}{4b(a+b)((a+b)\tan^2(c+dx)+a)^2}-\frac{\frac{\frac{a\left(8a^2+20ab+15b^2\right)\int\frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{b}-\frac{8(a+b)^2\arctan(\tan(c+dx))}{b}}{2b(a+b)}-\frac{a(4a+7b)\tan(c+dx)}{2b(a+b)\left((a+b)\tan^2(c+dx)+a\right)}}{4b(a+b)}}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{a\tan^3(c+dx)}{4b(a+b)((a+b)\tan^2(c+dx)+a)^2}-\frac{\frac{\frac{\sqrt{a}\left(8a^2+20ab+15b^2\right)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b\sqrt{a+b}}-\frac{8(a+b)^2\arctan(\tan(c+dx))}{b}}{2b(a+b)}-\frac{a(4a+7b)\tan(c+dx)}{2b(a+b)\left((a+b)\tan^2(c+dx)+a\right)}}{4b(a+b)}}{d}$$

input
```
Int[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2)^3,x]
```

output
```
((a*Tan[c + d*x]^3)/(4*b*(a + b)*(a + (a + b)*Tan[c + d*x]^2)^2) - (((-8*(
a + b)^2*ArcTan[Tan[c + d*x]])/b + (Sqrt[a]*(8*a^2 + 20*a*b + 15*b^2)*ArcT
an[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(b*Sqrt[a + b]))/(2*b*(a + b)) - (
a*(4*a + 7*b)*Tan[c + d*x])/(2*b*(a + b)*(a + (a + b)*Tan[c + d*x]^2)))/(4
*b*(a + b)))/d
```

3.106.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.106. $\int \frac{\sin^6(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx$

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.106. $\int \frac{\sin^6(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx$

3.106.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(dx+c))}{b^3}-\frac{a\left(\frac{-\frac{(4a+9b)b\left(\tan^3(dx+c)\right)}{8(a+b)}-\frac{ab(4a+7b)\tan(dx+c)}{8\left(a^2+2ab+b^2\right)}}{\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^2}+\frac{\left(8a^2+20ab+15b^2\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{a(a+b)}}\right)}{b^3}}{d}$
default	$\frac{\frac{\arctan(\tan(dx+c))}{b^3}-\frac{a\left(\frac{-\frac{(4a+9b)b\left(\tan^3(dx+c)\right)}{8(a+b)}-\frac{ab(4a+7b)\tan(dx+c)}{8\left(a^2+2ab+b^2\right)}}{\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^2}+\frac{\left(8a^2+20ab+15b^2\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{a(a+b)}}\right)}{b^3}}{d}$
risch	$\frac{x}{b^3}-\frac{ia\left(-16a^2b\,e^{6i(dx+c)}-28a\,b^2e^{6i(dx+c)}-9b^3e^{6i(dx+c)}+48a^3e^{4i(dx+c)}+120a^2b\,e^{4i(dx+c)}+90a\,b^2e^{4i(dx+c)}+27b^3e^{4\cdots}\right.}{4b^3(a+b)^2d\left(-b\,e^{4i(dx+c)}+4a\,e^{2i(dx+c)}+2b\,e^{2i(dx\cdots}\right.}$

input
```
int(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^3*arctan(tan(d*x+c))-a/b^3*((-1/8*(4*a+9*b)*b/(a+b)*tan(d*x+c)^3-
1/8*a*b*(4*a+7*b)/(a^2+2*a*b+b^2)*tan(d*x+c))/(a*tan(d*x+c)^2+tan(d*x+c)^2
*b+a)^2+1/8*(8*a^2+20*a*b+15*b^2)/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctan((
a+b)*tan(d*x+c)/(a*(a+b))^(1/2))))
```

3.106.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(134) = 268.

Time = 0.31 (sec) , antiderivative size = 950, normalized size of antiderivative = 6.42

$$\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(32*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cos(d*x + c)^4 - 64*(a^3*b + 3*a^2
*b^2 + 3*a*b^3 + b^4)*d*x*cos(d*x + c)^2 + 32*(a^4 + 4*a^3*b + 6*a^2*b^2 +
 4*a*b^3 + b^4)*d*x + ((8*a^2*b^2 + 20*a*b^3 + 15*b^4)*cos(d*x + c)^4 + 8*
a^4 + 36*a^3*b + 63*a^2*b^2 + 50*a*b^3 + 15*b^4 - 2*(8*a^3*b + 28*a^2*b^2
+ 35*a*b^3 + 15*b^4)*cos(d*x + c)^2)*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b
+ b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2
 + 3*a*b + b^2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a
/(a + b))*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b +
 b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)) - 4*(3*(2*a^2*b^2 + 3*a*b^3)*co
s(d*x + c)^3 - (4*a^3*b + 13*a^2*b^2 + 9*a*b^3)*cos(d*x + c))*sin(d*x + c)
)/((a^2*b^5 + 2*a*b^6 + b^7)*d*cos(d*x + c)^4 - 2*(a^3*b^4 + 3*a^2*b^5 + 3
*a*b^6 + b^7)*d*cos(d*x + c)^2 + (a^4*b^3 + 4*a^3*b^4 + 6*a^2*b^5 + 4*a*b^
6 + b^7)*d), 1/16*(16*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cos(d*x + c)^4 - 32*(a
^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*d*x*cos(d*x + c)^2 + 16*(a^4 + 4*a^3*b +
 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x + ((8*a^2*b^2 + 20*a*b^3 + 15*b^4)*cos(d*x
 + c)^4 + 8*a^4 + 36*a^3*b + 63*a^2*b^2 + 50*a*b^3 + 15*b^4 - 2*(8*a^3*b +
 28*a^2*b^2 + 35*a*b^3 + 15*b^4)*cos(d*x + c)^2)*sqrt(a/(a + b))*arctan(1/
2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d
*x + c))) - 2*(3*(2*a^2*b^2 + 3*a*b^3)*cos(d*x + c)^3 - (4*a^3*b + 13*a^2*
b^2 + 9*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^2*b^5 + 2*a*b^6 + b^7)*d...
```

3.106.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(sin(d*x+c)**6/(a+b*sin(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.58

$$\int \frac{\sin^6(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx = -\frac{\frac{(8a^3+20a^2b+15ab^2)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(a^2b^3+2ab^4+b^5)\sqrt{(a+b)a}} - \frac{(4a^3+13a^2b+9ab^2)\tan(dx+c)^3+(4a^3+7a^2b)\tan(dx+c)}{a^4b^2+2a^3b^3+a^2b^4+(a^4b^2+4a^3b^3+6a^2b^4+4ab^5+b^6)\tan(dx+c)^4+2(a^4b^2+3a^3b^3+3a^2}}{8d}$$

input
```
integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((8*a^3 + 20*a^2*b + 15*a*b^2)*arctan((a + b)*tan(d*x + c)/sqrt((a +
b)*a))/((a^2*b^3 + 2*a*b^4 + b^5)*sqrt((a + b)*a)) - ((4*a^3 + 13*a^2*b +
9*a*b^2)*tan(d*x + c)^3 + (4*a^3 + 7*a^2*b)*tan(d*x + c))/(a^4*b^2 + 2*a^3
*b^3 + a^2*b^4 + (a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*tan(d*x
 + c)^4 + 2*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*tan(d*x + c)^2) - 8*
(d*x + c)/b^3)/d
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.51

$$\int \frac{\sin^6(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx = -\frac{\frac{(8a^3+20a^2b+15ab^2)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{(a^2b^3+2ab^4+b^5)\sqrt{a^2+ab}} - \frac{4a^3\tan(dx+c)^3+13a^2b\tan(dx+c)^3+9ab^2\tan(dx}{(a^2b^2+2ab^3+b^4)\left(a\tan(dx+c)^2\right.}}{8d}$$

input
```
integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((8*a^3 + 20*a^2*b + 15*a*b^2)*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a
+ 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a^2*
b^3 + 2*a*b^4 + b^5)*sqrt(a^2 + a*b)) - (4*a^3*tan(d*x + c)^3 + 13*a^2*b*t
an(d*x + c)^3 + 9*a*b^2*tan(d*x + c)^3 + 4*a^3*tan(d*x + c) + 7*a^2*b*tan(
d*x + c))/((a^2*b^2 + 2*a*b^3 + b^4)*(a*tan(d*x + c)^2 + b*tan(d*x + c)^2
+ a)^2) - 8*(d*x + c)/b^3)/d
```

3.106.9 Mupad [B] (verification not implemented)

Time = 17.72 (sec) , antiderivative size = 3189, normalized size of antiderivative = 21.55

$$\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^6/(a + b*sin(c + d*x)^2)^3,x)
```

output
```
((tan(c + d*x)^3*(9*a*b + 4*a^2))/(8*(a*b^2 + b^3)) + (a*tan(c + d*x)*(7*a
*b + 4*a^2))/(8*(2*a*b^3 + b^4 + a^2*b^2)))/(d*(tan(c + d*x)^4*(2*a*b + a^
2 + b^2) + a^2 + tan(c + d*x)^2*(2*a*b + 2*a^2))) - atan((((((7*a*b^10)/2
 + (25*a^2*b^9)/2 + (33*a^3*b^8)/2 + (19*a^4*b^7)/2 + 2*a^5*b^6)*1i)/(2*(3
*a*b^8 + b^9 + 3*a^2*b^7 + a^3*b^6)) - (tan(c + d*x)*(1792*a*b^11 + 256*b^
12 + 5120*a^2*b^10 + 7680*a^3*b^9 + 6400*a^4*b^8 + 2816*a^5*b^7 + 512*a^6*
b^6))/(128*b^3*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)))/(2*b^3) + (tan(c +
d*x)*(384*a*b^5 + 704*a^5*b + 128*a^6 + 64*b^6 + 1185*a^2*b^4 + 1880*a^3*b
^3 + 1600*a^4*b^2))/(64*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)))/b^3 - ((((
(7*a*b^10)/2 + (25*a^2*b^9)/2 + (33*a^3*b^8)/2 + (19*a^4*b^7)/2 + 2*a^5*b^
6)*1i)/(2*(3*a*b^8 + b^9 + 3*a^2*b^7 + a^3*b^6)) + (tan(c + d*x)*(1792*a*b
^11 + 256*b^12 + 5120*a^2*b^10 + 7680*a^3*b^9 + 6400*a^4*b^8 + 2816*a^5*b^
7 + 512*a^6*b^6))/(128*b^3*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)))/(2*b^3)
 - (tan(c + d*x)*(384*a*b^5 + 704*a^5*b + 128*a^6 + 64*b^6 + 1185*a^2*b^4
+ 1880*a^3*b^3 + 1600*a^4*b^2))/(64*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4))
)/b^3)/(((15*a*b^4)/4 + (19*a^4*b)/4 + a^5 + (295*a^2*b^3)/32 + (19*a^3*b^
2)/2)/(3*a*b^8 + b^9 + 3*a^2*b^7 + a^3*b^6) + ((((((7*a*b^10)/2 + (25*a^2*
b^9)/2 + (33*a^3*b^8)/2 + (19*a^4*b^7)/2 + 2*a^5*b^6)*1i)/(2*(3*a*b^8 + b^
9 + 3*a^2*b^7 + a^3*b^6)) - (tan(c + d*x)*(1792*a*b^11 + 256*b^12 + 5120*a
^2*b^10 + 7680*a^3*b^9 + 6400*a^4*b^8 + 2816*a^5*b^7 + 512*a^6*b^6))/(1...
```

3.107 $\int \frac{\sin^4(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx$

3.107.1 Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \frac{\sin^4(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx = \frac{3\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}(a+b)^{5/2}d} - \frac{\tan^3(c+dx)}{4(a+b)d\,(a+(a+b)\tan^2(c+dx))^2} - \frac{3\tan(c+dx)}{8(a+b)^2d\,(a+(a+b)\tan^2(c+dx))}$$

output
```
3/8*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/(a+b)^(5/2)/d/a^(1/2)-1/4*tan(d
*x+c)^3/(a+b)/d/(a+(a+b)*tan(d*x+c)^2)^2-3/8*tan(d*x+c)/(a+b)^2/d/(a+(a+b)
*tan(d*x+c)^2)
```

3.107.2 Mathematica [A] (verified)

Time = 11.50 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int \frac{\sin^4(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx = \frac{\frac{3\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}} + \frac{(-8a-5b+(2a+5b)\cos(2(c+dx)))\sin(2(c+dx))}{(a+b)^2(2a+b-b\cos(2(c+dx)))^2}}{8d}$$

input
```
Integrate[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2)^3,x]
```

output
```
((3*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2)) +
((-8*a - 5*b + (2*a + 5*b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/((a + b)^2*
(2*a + b - b*Cos[2*(c + d*x)])^2))/(8*d)
```

3.107.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3666, 252, 252, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(c+dx)^4}{\left(a+b\sin(c+dx)^2\right)^3}dx$$

$$\Big\downarrow \text{ 3666}$$

$$\frac{\int \frac{\tan^4(c+dx)}{\left((a+b)\tan^2(c+dx)+a\right)^3}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\frac{3\int \frac{\tan^2(c+dx)}{\left((a+b)\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{4(a+b)}-\frac{\tan^3(c+dx)}{4(a+b)\left((a+b)\tan^2(c+dx)+a\right)^2}}{d}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\frac{3\left(\frac{\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{2(a+b)}-\frac{\tan(c+dx)}{2(a+b)\left((a+b)\tan^2(c+dx)+a\right)}\right)}{4(a+b)}-\frac{\tan^3(c+dx)}{4(a+b)\left((a+b)\tan^2(c+dx)+a\right)^2}}{d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}(a+b)^{3/2}}-\frac{\tan(c+dx)}{2(a+b)\left((a+b)\tan^2(c+dx)+a\right)}\right)}{4(a+b)}-\frac{\tan^3(c+dx)}{4(a+b)\left((a+b)\tan^2(c+dx)+a\right)^2}}{d}$$

input
```
Int[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2)^3,x]
```

output
```
(-1/4*Tan[c + d*x]^3/((a + b)*(a + (a + b)*Tan[c + d*x]^2)^2) + (3*(ArcTan
[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(2*Sqrt[a]*(a + b)^(3/2)) - Tan[c + d
*x]/(2*(a + b)*(a + (a + b)*Tan[c + d*x]^2))))/(4*(a + b)))/d
```

3.107.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.107.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{-\frac{5(\tan^3(dx+c))}{8(a+b)}-\frac{3a\tan(dx+c)}{8(a^2+2ab+b^2)}}{\left(a(\tan^2(dx+c))+(\tan^2(dx+c))b+a\right)^2}+\frac{3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8(a^2+2ab+b^2)\sqrt{a(a+b)}}}{d}$
default	$\frac{\frac{-\frac{5(\tan^3(dx+c))}{8(a+b)}-\frac{3a\tan(dx+c)}{8(a^2+2ab+b^2)}}{\left(a(\tan^2(dx+c))+(\tan^2(dx+c))b+a\right)^2}+\frac{3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8(a^2+2ab+b^2)\sqrt{a(a+b)}}}{d}$
risch	$\frac{i(-8a^2b\,e^{6i(dx+c)}-16a\,b^2e^{6i(dx+c)}-5b^3e^{6i(dx+c)}+16a^3e^{4i(dx+c)}+56a^2b\,e^{4i(dx+c)}+46a\,b^2e^{4i(dx+c)}+15b^3e^{4i(dx+c)}-8}{4b^2(a+b)^2d\left(-b\,e^{4i(dx+c)}+4a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-b\right)^2}$

input
```
int(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*((-5/8/(a+b)*tan(d*x+c)^3-3/8*a/(a^2+2*a*b+b^2)*tan(d*x+c))/(a*tan(d*x
+c)^2+tan(d*x+c)^2*b+a)^2+3/8/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctan((a+b)
*tan(d*x+c)/(a*(a+b))^(1/2)))
```

3.107.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(96) = 192.

Time = 0.33 (sec) , antiderivative size = 683, normalized size of antiderivative = 6.21

$$\int \frac{\sin^4(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx$$

$$= \left[-\frac{3\left(b^2\cos(dx+c)^4-2(ab+b^2)\cos(dx+c)^2+a^2+2ab+b^2\right)\sqrt{-a^2-ab}\log\left(\frac{(8a^2+8ab+b^2)\cos(dx+c)^4-2}{}\right.}{32\left((a^4b^2+3a^3b^3+3a^2b^4+ab^5)d\cos(dx+\right.}\right.$$

$$-\frac{3\left(b^2\cos(dx+c)^4-2(ab+b^2)\cos(dx+c)^2+a^2+2ab+b^2\right)\sqrt{a^2+ab}\arctan\left(\frac{(2a+b)\cos(dx+c)^2-a-b}{2\sqrt{a^2+ab}\cos(dx+c)\sin(dx+}\right.}{16\left((a^4b^2+3a^3b^3+3a^2b^4+ab^5)d\cos(dx+c)^4-2(a^5b+4a^4b^2+6a^3b^3+4a^2b^4\right.}$$

input
```
integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(3*(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b
 + b^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*
a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*
cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(
d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)) - 4*((2*a^
3 + 7*a^2*b + 5*a*b^2)*cos(d*x + c)^3 - 5*(a^3 + 2*a^2*b + a*b^2)*cos(d*x
+ c))*sin(d*x + c))/((a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*d*cos(d*x +
 c)^4 - 2*(a^5*b + 4*a^4*b^2 + 6*a^3*b^3 + 4*a^2*b^4 + a*b^5)*d*cos(d*x +
c)^2 + (a^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*d), -
1/16*(3*(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b +
 b^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(
a^2 + a*b)*cos(d*x + c)*sin(d*x + c))) - 2*((2*a^3 + 7*a^2*b + 5*a*b^2)*co
s(d*x + c)^3 - 5*(a^3 + 2*a^2*b + a*b^2)*cos(d*x + c))*sin(d*x + c))/((a^4
*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*d*cos(d*x + c)^4 - 2*(a^5*b + 4*a^4*
b^2 + 6*a^3*b^3 + 4*a^2*b^4 + a*b^5)*d*cos(d*x + c)^2 + (a^6 + 5*a^5*b + 1
0*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*d)]
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**4/(a+b*sin(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.44

$$\int \frac{\sin^4(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx = -\frac{\frac{5\,(a+b)\tan(dx+c)^3+3\,a\tan(dx+c)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\tan(dx+c)^4+a^4+2\,a^3b+a^2b^2+2\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\tan(dx+c)^2}-\frac{3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a^2+2\,ab+b^2)}}{8\,d}$$

input
```
integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((5*(a + b)*tan(d*x + c)^3 + 3*a*tan(d*x + c))/((a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4)*tan(d*x + c)^4 + a^4 + 2*a^3*b + a^2*b^2 + 2*(a^4 +
 3*a^3*b + 3*a^2*b^2 + a*b^3)*tan(d*x + c)^2) - 3*arctan((a + b)*tan(d*x +
 c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a^2 + 2*a*b + b^2)))/d
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.38

$$\int \frac{\sin^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{(a^2+2ab+b^2)\sqrt{a^2+ab}}-\frac{5a\tan(dx+c)^3+5b\tan(dx+c)^3+3a\tan(dx+c)}{\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)^2(a^2+2ab+b^2)}}{8d}$$

input
```
integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/8*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x +
c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a^2 + 2*a*b + b^2)*sqrt(a^2 + a*b
)) - (5*a*tan(d*x + c)^3 + 5*b*tan(d*x + c)^3 + 3*a*tan(d*x + c))/((a*tan(
d*x + c)^2 + b*tan(d*x + c)^2 + a)^2*(a^2 + 2*a*b + b^2)))/d
```

3.107.9 Mupad [B] (verification not implemented)

Time = 14.53 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.35

$$\int \frac{\sin^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{3\,\mathrm{atan}\left(\frac{3\tan(c+d\,x)\,(2\,a+2\,b)\,\left(\frac{8\,a^2}{3}+\frac{16\,a\,b}{3}+\frac{8\,b^2}{3}\right)}{16\,\sqrt{a}\,(a+b)^{5/2}}\right)}{8\,\sqrt{a}\,d\,(a+b)^{5/2}} - \frac{\frac{5\tan(c+d\,x)^3}{8\,(a+b)}+\frac{3\,a\tan(c+d\,x)}{8\,(a^2+2\,a\,b+b^2)}}{d\,\left(\tan\left(c+d\,x\right)^4\,\left(a^2+2\,a\,b+b^2\right)+a^2+\tan\left(c+d\,x\right)^2\,\left(2\,a^2+2\,b\,a\right)\right)}$$

input
```
int(sin(c + d*x)^4/(a + b*sin(c + d*x)^2)^3,x)
```

output
```
(3*atan((3*tan(c + d*x)*(2*a + 2*b)*((16*a*b)/3 + (8*a^2)/3 + (8*b^2)/3))/
(16*a^(1/2)*(a + b)^(5/2))))/(8*a^(1/2)*d*(a + b)^(5/2)) - ((5*tan(c + d*x
)^3)/(8*(a + b)) + (3*a*tan(c + d*x))/(8*(2*a*b + a^2 + b^2)))/(d*(tan(c +
 d*x)^4*(2*a*b + a^2 + b^2) + a^2 + tan(c + d*x)^2*(2*a*b + 2*a^2)))
```

3.107. $\int \frac{\sin^4(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx$

3.108 $\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx$

3.108.1 Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx = \frac{(4a+b)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a+b)^{5/2}d} - \frac{\cos(c+dx)\sin(c+dx)}{4(a+b)d\,(a+b\sin^2(c+dx))^2} - \frac{(2a-b)\cos(c+dx)\sin(c+dx)}{8a(a+b)^2d\,(a+b\sin^2(c+dx))}$$

output
```
1/8*(4*a+b)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/(a+b)^(5/2)/d-1
/4*cos(d*x+c)*sin(d*x+c)/(a+b)/d/(a+b*sin(d*x+c)^2)^2-1/8*(2*a-b)*cos(d*x+
c)*sin(d*x+c)/a/(a+b)^2/d/(a+b*sin(d*x+c)^2)
```

3.108.2 Mathematica [A] (verified)

Time = 11.64 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx = \frac{\frac{(4a+b)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)^{5/2}} - \frac{(8a^2+4ab-b^2+b(-2a+b)\cos(2(c+dx)))\sin(2(c+dx))}{a(a+b)^2(2a+b-b\cos(2(c+dx)))^2}}{8d}$$

input
```
Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2)^3,x]
```

output
```
(((4*a + b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(3/2)*(a + b)^(
5/2)) - ((8*a^2 + 4*a*b - b^2 + b*(-2*a + b)*Cos[2*(c + d*x)])*Sin[2*(c +
d*x)])/(a*(a + b)^2*(2*a + b - b*Cos[2*(c + d*x)])^2))/(8*d)
```

3.108.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3652, 3042, 3652, 27, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(c+dx)^2}{\left(a+b\sin(c+dx)^2\right)^3}dx$$

$$\Big\downarrow \ 3652$$

$$\frac{\int \frac{2a\sin^2(c+dx)+a}{\left(b\sin^2(c+dx)+a\right)^2}dx}{4a(a+b)}-\frac{\sin(c+dx)\cos(c+dx)}{4d(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{2a\sin(c+dx)^2+a}{\left(b\sin(c+dx)^2+a\right)^2}dx}{4a(a+b)}-\frac{\sin(c+dx)\cos(c+dx)}{4d(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

$$\Big\downarrow \ 3652$$

$$\frac{\frac{\int \frac{a(4a+b)}{b\sin^2(c+dx)+a}dx}{2a(a+b)}-\frac{(2a-b)\sin(c+dx)\cos(c+dx)}{2d(a+b)\left(a+b\sin^2(c+dx)\right)}}{4a(a+b)}-\frac{\sin(c+dx)\cos(c+dx)}{4d(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{(4a+b)\int \frac{1}{b\sin^2(c+dx)+a}dx}{2(a+b)}-\frac{(2a-b)\sin(c+dx)\cos(c+dx)}{2d(a+b)\left(a+b\sin^2(c+dx)\right)}}{4a(a+b)}-\frac{\sin(c+dx)\cos(c+dx)}{4d(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(4a+b)\int \frac{1}{b\sin(c+dx)^2+a}dx}{2(a+b)}-\frac{(2a-b)\sin(c+dx)\cos(c+dx)}{2d(a+b)(a+b\sin^2(c+dx))}}{4a(a+b)}-\frac{\sin(c+dx)\cos(c+dx)}{4d(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\frac{(4a+b)\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{2d(a+b)}-\frac{(2a-b)\sin(c+dx)\cos(c+dx)}{2d(a+b)(a+b\sin^2(c+dx))}}{4a(a+b)}-\frac{\sin(c+dx)\cos(c+dx)}{4d(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(4a+b)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}d(a+b)^{3/2}}-\frac{(2a-b)\sin(c+dx)\cos(c+dx)}{2d(a+b)(a+b\sin^2(c+dx))}}{4a(a+b)}-\frac{\sin(c+dx)\cos(c+dx)}{4d(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

input
```
Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2)^3,x]
```

output
```
-1/4*(Cos[c + d*x]*Sin[c + d*x])/((a + b)*d*(a + b*Sin[c + d*x]^2)^2) + ((
(4*a + b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2*Sqrt[a]*(a + b)^(
3/2)*d) - ((2*a - b)*Cos[c + d*x]*Sin[c + d*x])/(2*(a + b)*d*(a + b*Sin[c
+ d*x]^2)))/(4*a*(a + b))
```

3.108.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.108. $\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx$

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.108.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{-\frac{(4a-b)\left(\tan^3(dx+c)\right)}{8a(a+b)}-\frac{(4a+b)\tan(dx+c)}{8\left(a^2+2ab+b^2\right)}}{\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^2}+\frac{(4a+b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)a\sqrt{a(a+b)}}}{d}$
default	$\frac{\frac{-\frac{(4a-b)\left(\tan^3(dx+c)\right)}{8a(a+b)}-\frac{(4a+b)\tan(dx+c)}{8\left(a^2+2ab+b^2\right)}}{\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^2}+\frac{(4a+b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)a\sqrt{a(a+b)}}}{d}$
risch	$\frac{i\left(4a\,b^2 e^{6i(dx+c)}+b^3 e^{6i(dx+c)}+16a^3 e^{4i(dx+c)}+8a^2 b\,e^{4i(dx+c)}-2a\,b^2 e^{4i(dx+c)}-3b^3 e^{4i(dx+c)}-16a^2 b\,e^{2i(dx+c)}-4a\,b^2 e^{2i\cdots}\right.}{4ba(a+b)^2 d\left(-b\,e^{4i(dx+c)}+4a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-b\right)^2}$

input
```
int(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*((-1/8*(4*a-b)/a/(a+b)*tan(d*x+c)^3-1/8*(4*a+b)/(a^2+2*a*b+b^2)*tan(d*
x+c))/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2+1/8*(4*a+b)/(a^2+2*a*b+b^2)/a/(a
*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
```

3.108.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(117) = 234$.

Time = 0.32 (sec) , antiderivative size = 771, normalized size of antiderivative = 5.89

$$\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx$$
$$= \left[-\frac{\left((4\,ab^2+b^3)\cos{(dx+c)}^4+4\,a^3+9\,a^2b+6\,ab^2+b^3-2\,(4\,a^2b+5\,ab^2+b^3)\cos{(dx+c)}^2\right)\sqrt{-a^2-ab}}{32\,\left((a^5b^2+3\,a^4b^3+3\,a^3b^4+\right.}\right.$$
$$-\frac{\left((4\,ab^2+b^3)\cos{(dx+c)}^4+4\,a^3+9\,a^2b+6\,ab^2+b^3-2\,(4\,a^2b+5\,ab^2+b^3)\cos{(dx+c)}^2\right)\sqrt{a^2+ab}\,\mathrm{ar}}{16\,\left((a^5b^2+3\,a^4b^3+3\,a^3b^4+a^2b^5)d\cos{(dx+c)}^4-2\,(a^6b+4\,a^5b^2\right.}$$

input
```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(((4*a*b^2 + b^3)*cos(d*x + c)^4 + 4*a^3 + 9*a^2*b + 6*a*b^2 + b^3
- 2*(4*a^2*b + 5*a*b^2 + b^3)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2
 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 +
4*((2*a + b)*cos(d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d
*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x +
 c)^2 + a^2 + 2*a*b + b^2)) - 4*((2*a^3*b + a^2*b^2 - a*b^3)*cos(d*x + c)^
3 - (4*a^4 + 7*a^3*b + 2*a^2*b^2 - a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^
5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*d*cos(d*x + c)^4 - 2*(a^6*b + 4*a
^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*d*cos(d*x + c)^2 + (a^7 + 5*a^6*
b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d), -1/16*(((4*a*b^2 +
b^3)*cos(d*x + c)^4 + 4*a^3 + 9*a^2*b + 6*a*b^2 + b^3 - 2*(4*a^2*b + 5*a*b
^2 + b^3)*cos(d*x + c)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x +
c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c))) - 2*((2*a^3*b +
 a^2*b^2 - a*b^3)*cos(d*x + c)^3 - (4*a^4 + 7*a^3*b + 2*a^2*b^2 - a*b^3)*c
os(d*x + c))*sin(d*x + c))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*d*
cos(d*x + c)^4 - 2*(a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*d
*cos(d*x + c)^2 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a
^2*b^5)*d)]
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.46

$$\int \frac{\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{\frac{(4a+b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(a^3+2a^2b+ab^2)\sqrt{(a+b)a}} - \frac{(4a^2+3ab-b^2)\tan(dx+c)^3+(4a^2+ab)\tan(dx+c)}{a^5+2a^4b+a^3b^2+(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4)\tan(dx+c)^4+2(a^5+3a^4b+3a^3b^2+a^2b^3)\tan(dx+c)^2}}{8d}$$

input
```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((4*a + b)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a^3 + 2*a^2*
b + a*b^2)*sqrt((a + b)*a)) - ((4*a^2 + 3*a*b - b^2)*tan(d*x + c)^3 + (4*a
^2 + a*b)*tan(d*x + c))/(a^5 + 2*a^4*b + a^3*b^2 + (a^5 + 4*a^4*b + 6*a^3*
b^2 + 4*a^2*b^3 + a*b^4)*tan(d*x + c)^4 + 2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a
^2*b^3)*tan(d*x + c)^2))/d
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.46

$$\int \frac{\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(4a+b)}{(a^3+2a^2b+ab^2)\sqrt{a^2+ab}} - \frac{4a^2\tan(dx+c)^3+3ab\tan(dx+c)^3-b^2\tan(dx+c)^3+4a^2\tan(dx+c)+}{(a^3+2a^2b+ab^2)\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)^2}}{8d}$$

input
```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/8*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
 + b*tan(d*x + c))/sqrt(a^2 + a*b)))*(4*a + b)/((a^3 + 2*a^2*b + a*b^2)*sq
rt(a^2 + a*b)) - (4*a^2*tan(d*x + c)^3 + 3*a*b*tan(d*x + c)^3 - b^2*tan(d*
x + c)^3 + 4*a^2*tan(d*x + c) + a*b*tan(d*x + c))/((a^3 + 2*a^2*b + a*b^2)
*(a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)^2))/d
```

3.108.9 Mupad [B] (verification not implemented)

Time = 14.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.21

$$\int \frac{\sin^2(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx = \frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)\,\left(a^2+2\,a\,b+b^2\right)}{2\,\sqrt{a}\,{(a+b)}^{5/2}}\right)\,(4\,a+b)}{8\,a^{3/2}\,d\,{(a+b)}^{5/2}} - \frac{\frac{\tan(c+d\,x)\,(4\,a+b)}{8\,(a^2+2\,a\,b+b^2)}+\frac{{\tan(c+d\,x)}^3\,(4\,a-b)}{8\,a\,(a+b)}}{d\,\left({\tan\left(c+d\,x\right)}^4\,\left(a^2+2\,a\,b+b^2\right)+a^2+{\tan\left(c+d\,x\right)}^2\,\left(2\,a^2+2\,b\,a\right)\right)}$$

input
```
int(sin(c + d*x)^2/(a + b*sin(c + d*x)^2)^3,x)
```

output
```
(atan((tan(c + d*x)*(2*a + 2*b)*(2*a*b + a^2 + b^2))/(2*a^(1/2)*(a + b)^(5
/2)))*(4*a + b))/(8*a^(3/2)*d*(a + b)^(5/2)) - ((tan(c + d*x)*(4*a + b))/(
8*(2*a*b + a^2 + b^2)) + (tan(c + d*x)^3*(4*a - b))/(8*a*(a + b)))/(d*(tan
(c + d*x)^4*(2*a*b + a^2 + b^2) + a^2 + tan(c + d*x)^2*(2*a*b + 2*a^2)))
```

3.109 $\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$

3.109.1 Optimal result

Integrand size = 14, antiderivative size = 144

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}d} + \frac{b\cos(c+dx)\sin(c+dx)}{4a(a+b)d\left(a+b\sin^2(c+dx)\right)^2} + \frac{3b(2a+b)\cos(c+dx)\sin(c+dx)}{8a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)}$$

output
```
1/8*(8*a^2+8*a*b+3*b^2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(5/2)/(a+
b)^(5/2)/d+1/4*b*cos(d*x+c)*sin(d*x+c)/a/(a+b)/d/(a+b*sin(d*x+c)^2)^2+3/8*
b*(2*a+b)*cos(d*x+c)*sin(d*x+c)/a^2/(a+b)^2/d/(a+b*sin(d*x+c)^2)
```

3.109.2 Mathematica [A] (verified)

Time = 11.72 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{\frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{5/2}} + \frac{\sqrt{a}b(16a^2+16ab+3b^2-3b(2a+b)\cos(2(c+dx)))\sin(2(c+dx))}{(a+b)^2(2a+b-b\cos(2(c+dx)))^2}}{8a^{5/2}d}$$

input
```
Integrate[(a + b*Sin[c + d*x]^2)^(-3),x]
```

output
```
(((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a +
 b)^(5/2) + (Sqrt[a]*b*(16*a^2 + 16*a*b + 3*b^2 - 3*b*(2*a + b)*Cos[2*(c +
 d*x)])*Sin[2*(c + d*x)])/((a + b)^2*(2*a + b - b*Cos[2*(c + d*x)])^2))/(8
*a^(5/2)*d)
```

3.109.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 3663, 25, 3042, 3652, 27, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\sin(c+dx)^2)^3}dx$$

$$\Big\downarrow \text{3663}$$

$$\frac{b\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}-\frac{\int -\frac{-2b\sin^2(c+dx)+4a+3b}{\left(b\sin^2(c+dx)+a\right)^2}dx}{4a(a+b)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-2b\sin^2(c+dx)+4a+3b}{\left(b\sin^2(c+dx)+a\right)^2}dx}{4a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-2b\sin(c+dx)^2+4a+3b}{\left(b\sin(c+dx)^2+a\right)^2}dx}{4a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

$$\Big\downarrow \text{3652}$$

$$\frac{\frac{\int \frac{8a^2+8ba+3b^2}{b\sin^2(c+dx)+a}dx}{2a(a+b)}+\frac{3b(2a+b)\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}}{4a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(8a^2+8ab+3b^2)\int \frac{1}{b\sin^2(c+dx)+a}dx}{2a(a+b)}+\frac{3b(2a+b)\sin(c+dx)\cos(c+dx)}{2ad(a+b)(a+b\sin^2(c+dx))}}{4a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(8a^2+8ab+3b^2)\int \frac{1}{b\sin(c+dx)^2+a}dx}{2a(a+b)}+\frac{3b(2a+b)\sin(c+dx)\cos(c+dx)}{2ad(a+b)(a+b\sin^2(c+dx))}}{4a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\frac{(8a^2+8ab+3b^2)\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{2ad(a+b)}+\frac{3b(2a+b)\sin(c+dx)\cos(c+dx)}{2ad(a+b)(a+b\sin^2(c+dx))}}{4a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{3/2}}+\frac{3b(2a+b)\sin(c+dx)\cos(c+dx)}{2ad(a+b)(a+b\sin^2(c+dx))}}{4a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

input
```
Int[(a + b*Sin[c + d*x]^2)^(-3),x]
```

output
```
(b*Cos[c + d*x]*Sin[c + d*x])/(4*a*(a + b)*d*(a + b*Sin[c + d*x]^2)^2) + (
((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2*a^
(3/2)*(a + b)^(3/2)*d) + (3*b*(2*a + b)*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a
 + b)*d*(a + b*Sin[c + d*x]^2)))/(4*a*(a + b))
```

3.109.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.109.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{\frac{\frac{(8a+3b)b\left(\tan^3(dx+c)\right)}{8a^2(a+b)}+\frac{b(8a+5b)\tan(dx+c)}{8a\left(a^2+2ab+b^2\right)}}{\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^2}+\frac{\left(8a^2+8ab+3b^2\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8a^2\left(a^2+2ab+b^2\right)\sqrt{a(a+b)}}}{d}$
default	$\frac{\frac{\frac{(8a+3b)b\left(\tan^3(dx+c)\right)}{8a^2(a+b)}+\frac{b(8a+5b)\tan(dx+c)}{8a\left(a^2+2ab+b^2\right)}}{\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^2}+\frac{\left(8a^2+8ab+3b^2\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8a^2\left(a^2+2ab+b^2\right)\sqrt{a(a+b)}}}{d}$
risch	$-\frac{i\left(-8a^2b\,e^{6i(dx+c)}-8a\,b^2e^{6i(dx+c)}-3b^3e^{6i(dx+c)}+48a^3e^{4i(dx+c)}+72a^2b\,e^{4i(dx+c)}+42a\,b^2e^{4i(dx+c)}+9b^3e^{4i(dx+c)}-4\right.}{4a^2(a+b)^2d\left(-b\,e^{4i(dx+c)}+4a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-b\right)^2}$

input
```
int(1/(a+b*sin(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*((1/8*(8*a+3*b)/a^2*b/(a+b)*tan(d*x+c)^3+1/8*b*(8*a+5*b)/a/(a^2+2*a*b+
b^2)*tan(d*x+c))/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2+1/8*(8*a^2+8*a*b+3*b^
2)/a^2/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(
1/2)))
```

3.109.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(130) = 260$.

Time = 0.30 (sec) , antiderivative size = 843, normalized size of antiderivative = 5.85

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$$

$$= \left[-\frac{\left((8\,a^2b^2+8\,ab^3+3\,b^4)\cos\left(dx+c\right)^4+8\,a^4+24\,a^3b+27\,a^2b^2+14\,ab^3+3\,b^4-2\,(8\,a^3b+16\,a^2b^2+11\right.}{32\,((}\right.$$

$$-\frac{\left((8\,a^2b^2+8\,ab^3+3\,b^4)\cos\left(dx+c\right)^4+8\,a^4+24\,a^3b+27\,a^2b^2+14\,ab^3+3\,b^4-2\,(8\,a^3b+16\,a^2b^2+11\right.}{16\,\left((a^6b^2+3\,a^5b^3+3\,a^4b^4+a^3b^5)d\cos\left(dx\right.\right.}$$

input
```
integrate(1/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cos(d*x + c)^4 + 8*a^4 + 24*a^3*b +
 27*a^2*b^2 + 14*a*b^3 + 3*b^4 - 2*(8*a^3*b + 16*a^2*b^2 + 11*a*b^3 + 3*b^
4)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c
)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3
 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2
)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))
 + 4*(3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(d*x + c)^3 - (8*a^4*b + 19*a^3
*b^2 + 14*a^2*b^3 + 3*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 + 3*a^5
*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cos(d*x + c)^4 - 2*(a^7*b + 4*a^6*b^2 + 6*a^
5*b^3 + 4*a^4*b^4 + a^3*b^5)*d*cos(d*x + c)^2 + (a^8 + 5*a^7*b + 10*a^6*b^
2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d), -1/16*(((8*a^2*b^2 + 8*a*b^3 + 3
*b^4)*cos(d*x + c)^4 + 8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4 -
2*(8*a^3*b + 16*a^2*b^2 + 11*a*b^3 + 3*b^4)*cos(d*x + c)^2)*sqrt(a^2 + a*b
)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x +
 c)*sin(d*x + c))) + 2*(3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(d*x + c)^3 -
 (8*a^4*b + 19*a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cos(d*x + c))*sin(d*x + c))
/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cos(d*x + c)^4 - 2*(a^7*b
+ 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d*cos(d*x + c)^2 + (a^8 + 5
*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d)]
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sin(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.47

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$$
$$= \frac{\frac{(8\,a^2+8\,ab+3\,b^2)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(a^4+2\,a^3b+a^2b^2)\sqrt{(a+b)a}} + \frac{(8\,a^2b+11\,ab^2+3\,b^3)\tan(dx+c)^3+(8\,a^2b+5\,ab^2)\tan(dx+c)}{a^6+2\,a^5b+a^4b^2+(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4)\tan(dx+c)^4+2\,(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\tan(}}{8\,d}$$

input
```
integrate(1/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((8*a^2 + 8*a*b + 3*b^2)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/
((a^4 + 2*a^3*b + a^2*b^2)*sqrt((a + b)*a)) + ((8*a^2*b + 11*a*b^2 + 3*b^3
)*tan(d*x + c)^3 + (8*a^2*b + 5*a*b^2)*tan(d*x + c))/(a^6 + 2*a^5*b + a^4*
b^2 + (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*tan(d*x + c)^4 + 2
*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*tan(d*x + c)^2))/d
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.47

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(8\,a^2+8\,ab+3\,b^2)}{(a^4+2\,a^3b+a^2b^2)\sqrt{a^2+ab}}+\frac{8\,a^2b\tan(dx+c)^3+11\,ab^2\tan(dx+c)^3+3\,b^3\tan(dx+c)^3+8}{(a^4+2\,a^3b+a^2b^2)\left(a\tan(dx+c)^2+b\tan(\right.}}{8\,d}$$

input
```
integrate(1/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/8*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
 + b*tan(d*x + c))/sqrt(a^2 + a*b)))*(8*a^2 + 8*a*b + 3*b^2)/((a^4 + 2*a^3
*b + a^2*b^2)*sqrt(a^2 + a*b)) + (8*a^2*b*tan(d*x + c)^3 + 11*a*b^2*tan(d*
x + c)^3 + 3*b^3*tan(d*x + c)^3 + 8*a^2*b*tan(d*x + c) + 5*a*b^2*tan(d*x +
 c))/((a^4 + 2*a^3*b + a^2*b^2)*(a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)^
2))/d
```

3.109.9 Mupad [B] (verification not implemented)

Time = 14.12 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{\frac{\tan(c+d\,x)^3\,(3\,b^2+8\,a\,b)}{8\,a^2\,(a+b)}+\frac{\tan(c+d\,x)\,(5\,b^2+8\,a\,b)}{8\,a\,(a^2+2\,a\,b+b^2)}}{d\,\left({\tan\left(c+d\,x\right)}^4\,\left(a^2+2\,a\,b+b^2\right)+a^2+{\tan\left(c+d\,x\right)}^2\,\left(2\,a^2+2\,b\,a\right)\right)} + \frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)\,(a^2+2\,a\,b+b^2)}{2\,\sqrt{a}\,(a+b)^{5/2}}\right)\,\left(8\,a^2+8\,a\,b+3\,b^2\right)}{8\,a^{5/2}\,d\,{\left(a+b\right)}^{5/2}}$$

input
```
int(1/(a + b*sin(c + d*x)^2)^3,x)
```

output
```
((tan(c + d*x)^3*(8*a*b + 3*b^2))/(8*a^2*(a + b)) + (tan(c + d*x)*(8*a*b +
 5*b^2))/(8*a*(2*a*b + a^2 + b^2)))/(d*(tan(c + d*x)^4*(2*a*b + a^2 + b^2)
 + a^2 + tan(c + d*x)^2*(2*a*b + 2*a^2))) + (atan((tan(c + d*x)*(2*a + 2*b
)*(2*a*b + a^2 + b^2))/(2*a^(1/2)*(a + b)^(5/2)))*(8*a*b + 8*a^2 + 3*b^2))
/(8*a^(5/2)*d*(a + b)^(5/2))
```

3.109. $\int \frac{1}{(a+b\sin^2(c+dx))^3}\,dx$

3.110 $\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$

3.110.1 Optimal result

Integrand size = 23, antiderivative size = 196

$$\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = -\frac{3b(8a^2+12ab+5b^2)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a+b)^{5/2}d} - \frac{(2a+3b)(4a+5b)\cot(c+dx)}{8a^3(a+b)^2d} + \frac{b\csc(c+dx)\sec^3(c+dx)}{4a(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} + \frac{b\cot(c+dx)\left(4a+5b+(4a+b)\tan^2(c+dx)\right)}{8a^2(a+b)^2d\left(a+(a+b)\tan^2(c+dx)\right)}$$

output
```
-3/8*b*(8*a^2+12*a*b+5*b^2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(7/2)
/(a+b)^(5/2)/d-1/8*(2*a+3*b)*(4*a+5*b)*cot(d*x+c)/a^3/(a+b)^2/d+1/4*b*csc(
d*x+c)*sec(d*x+c)^3/a/(a+b)/d/(a+(a+b)*tan(d*x+c)^2)^2+1/8*b*cot(d*x+c)*(4
*a+5*b+(4*a+b)*tan(d*x+c)^2)/a^2/(a+b)^2/d/(a+(a+b)*tan(d*x+c)^2)
```

3.110.2 Mathematica [A] (verified)

Time = 2.82 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.09

$$\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$$
$$= \frac{(-2a-b+b\cos(2(c+dx)))\csc^6(c+dx)\left(\frac{3b(8a^2+12ab+5b^2)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)(2a+b-b\cos(2(c+dx)))^2}{(a+b)^{5/2}}+8\sqrt{a}(2a\right.}{64a^{7/2}d\left(b+a\csc\right.}$$

input
```
Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2)^3,x]
```

output
```
((-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^6*((3*b*(8*a^2 + 12*a*b + 5*
b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]*(2*a + b - b*Cos[2*(c + d*
x)])^2)/(a + b)^(5/2) + 8*Sqrt[a]*(2*a + b - b*Cos[2*(c + d*x)])^2*Cot[c +
 d*x] + (4*a^(3/2)*b^2*Sin[2*(c + d*x)])/(a + b) + (Sqrt[a]*b^2*(10*a + 7*
b)*(2*a + b - b*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(a + b)^2))/(64*a^(7/2
)*d*(b + a*Csc[c + d*x]^2)^3)
```

3.110.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 3666, 370, 25, 439, 25, 359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin(c+dx)^2\left(a+b\sin(c+dx)^2\right)^3}dx$$
$$\Big\downarrow \; 3666$$
$$\frac{\int \frac{\cot^2(c+dx)\left(\tan^2(c+dx)+1\right)^3}{\left((a+b)\tan^2(c+dx)+a\right)^3}d\tan(c+dx)}{d}$$
$$\Big\downarrow \; 370$$

$$\frac{\frac{b\left(\tan^2(c+dx)+1\right)^2\cot(c+dx)}{4a(a+b)((a+b)\tan^2(c+dx)+a)^2}-\frac{\int -\frac{\cot^2(c+dx)\left(\tan^2(c+dx)+1\right)\left((4a+b)\tan^2(c+dx)+4a+5b\right)}{\left((a+b)\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{4a(a+b)}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\cot^2(c+dx)\left(\tan^2(c+dx)+1\right)\left((4a+b)\tan^2(c+dx)+4a+5b\right)}{\left((a+b)\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{4a(a+b)}+\frac{b\left(\tan^2(c+dx)+1\right)^2\cot(c+dx)}{4a(a+b)((a+b)\tan^2(c+dx)+a)^2}}{d}$$

$\Big\downarrow$ 439

$$\frac{\frac{\frac{b\cot(c+dx)\left((4a+b)\tan^2(c+dx)+4a+5b\right)}{2a(a+b)\left((a+b)\tan^2(c+dx)+a\right)}-\frac{\int -\frac{\cot^2(c+dx)\left((2a+b)(4a+b)\tan^2(c+dx)+(2a+3b)(4a+5b)\right)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{2a(a+b)}}{4a(a+b)}+\frac{b\left(\tan^2(c+dx)+1\right)^2\cot(c+dx)}{4a(a+b)((a+b)\tan^2(c+dx)+a)^2}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{\cot^2(c+dx)\left((2a+b)(4a+b)\tan^2(c+dx)+(2a+3b)(4a+5b)\right)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{2a(a+b)}+\frac{b\cot(c+dx)\left((4a+b)\tan^2(c+dx)+4a+5b\right)}{2a(a+b)\left((a+b)\tan^2(c+dx)+a\right)}}{4a(a+b)}+\frac{b\left(\tan^2(c+dx)+1\right)^2\cot(c+dx)}{4a(a+b)((a+b)\tan^2(c+dx)+a)^2}}{d}$$

$\Big\downarrow$ 359

$$\frac{\frac{\frac{-\frac{3b\left(8a^2+12ab+5b^2\right)\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{a}-\frac{(2a+3b)(4a+5b)\cot(c+dx)}{a}}{2a(a+b)}+\frac{b\cot(c+dx)\left((4a+b)\tan^2(c+dx)+4a+5b\right)}{2a(a+b)\left((a+b)\tan^2(c+dx)+a\right)}}{4a(a+b)}+\frac{b\left(\tan^2(c+dx)+1\right)^2\cot(c+dx)}{4a(a+b)((a+b)\tan^2(c+dx)+a)^2}}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{-\frac{3b\left(8a^2+12ab+5b^2\right)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}}-\frac{(2a+3b)(4a+5b)\cot(c+dx)}{a}}{2a(a+b)}+\frac{b\cot(c+dx)\left((4a+b)\tan^2(c+dx)+4a+5b\right)}{2a(a+b)\left((a+b)\tan^2(c+dx)+a\right)}}{4a(a+b)}+\frac{b\left(\tan^2(c+dx)+1\right)^2\cot(c+dx)}{4a(a+b)((a+b)\tan^2(c+dx)+a)^2}}{d}$$

input
```
Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2)^3,x]
```

3.110. $\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$

output
```
((b*Cot[c + d*x]*(1 + Tan[c + d*x]^2)^2)/(4*a*(a + b)*(a + (a + b)*Tan[c +
 d*x]^2)^2) + (((-3*b*(8*a^2 + 12*a*b + 5*b^2)*ArcTan[(Sqrt[a + b]*Tan[c +
 d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a + b]) - ((2*a + 3*b)*(4*a + 5*b)*Cot[c +
d*x])/a)/(2*a*(a + b)) + (b*Cot[c + d*x]*(4*a + 5*b + (4*a + b)*Tan[c + d*
x]^2))/(2*a*(a + b)*(a + (a + b)*Tan[c + d*x]^2)))/(4*a*(a + b)))/d
```

3.110.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

3.110. $\int \frac{\csc^2(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.110.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\frac{(12a+7b)b\left(\tan^3(dx+c)\right)}{8a+8b}+\frac{3ab(4a+3b)\tan(dx+c)}{8\left(a^2+2ab+b^2\right)}}{\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^2}+\frac{3\left(8a^2+12ab+5b^2\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{a(a+b)}}\right)}{a^3}-\frac{1}{a^3\tan(dx+c)}}{d}$
default	$\frac{-\frac{b\left(\frac{\frac{(12a+7b)b\left(\tan^3(dx+c)\right)}{8a+8b}+\frac{3ab(4a+3b)\tan(dx+c)}{8\left(a^2+2ab+b^2\right)}}{\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^2}+\frac{3\left(8a^2+12ab+5b^2\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{a(a+b)}}\right)}{a^3}-\frac{1}{a^3\tan(dx+c)}}{d}$
risch	$-\frac{i\left(24a^2b^2e^{8i(dx+c)}+36a\,b^3e^{8i(dx+c)}+15b^4e^{8i(dx+c)}-144a^3b\,e^{6i(dx+c)}-312a^2b^2e^{6i(dx+c)}-234a\,b^3e^{6i(dx+c)}-60b^4e^{6i}\right.}{}$

input
```
int(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b/a^3*((1/8*(12*a+7*b)*b/(a+b)*tan(d*x+c)^3+3/8*a*b*(4*a+3*b)/(a^2+2
*a*b+b^2)*tan(d*x+c))/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2+3/8*(8*a^2+12*a*
b+5*b^2)/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))
^(1/2)))-1/a^3/tan(d*x+c))
```

3.110.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(180) = 360$.

Time = 0.36 (sec) , antiderivative size = 1003, normalized size of antiderivative = 5.12

$$\int \frac{\csc^2(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(4*(8*a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 + 15*a*b^5)*cos(d*x + c)^5
- 4*(16*a^5*b + 76*a^4*b^2 + 137*a^3*b^3 + 107*a^2*b^4 + 30*a*b^5)*cos(d*x
 + c)^3 + 3*(8*a^4*b + 28*a^3*b^2 + 37*a^2*b^3 + 22*a*b^4 + 5*b^5 + (8*a^2
*b^3 + 12*a*b^4 + 5*b^5)*cos(d*x + c)^4 - 2*(8*a^3*b^2 + 20*a^2*b^3 + 17*a
*b^4 + 5*b^5)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*
cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a + b)*cos
(d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 +
2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*
a*b + b^2))*sin(d*x + c) + 4*(8*a^6 + 40*a^5*b + 92*a^4*b^2 + 111*a^3*b^3
+ 66*a^2*b^4 + 15*a*b^5)*cos(d*x + c))/(((a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4
+ a^4*b^5)*d*cos(d*x + c)^4 - 2*(a^8*b + 4*a^7*b^2 + 6*a^6*b^3 + 4*a^5*b^4
 + a^4*b^5)*d*cos(d*x + c)^2 + (a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 +
5*a^5*b^4 + a^4*b^5)*d)*sin(d*x + c)), -1/16*(2*(8*a^4*b^2 + 34*a^3*b^3 +
41*a^2*b^4 + 15*a*b^5)*cos(d*x + c)^5 - 2*(16*a^5*b + 76*a^4*b^2 + 137*a^3
*b^3 + 107*a^2*b^4 + 30*a*b^5)*cos(d*x + c)^3 - 3*(8*a^4*b + 28*a^3*b^2 +
37*a^2*b^3 + 22*a*b^4 + 5*b^5 + (8*a^2*b^3 + 12*a*b^4 + 5*b^5)*cos(d*x + c
)^4 - 2*(8*a^3*b^2 + 20*a^2*b^3 + 17*a*b^4 + 5*b^5)*cos(d*x + c)^2)*sqrt(a
^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*c
os(d*x + c)*sin(d*x + c)))*sin(d*x + c) + 2*(8*a^6 + 40*a^5*b + 92*a^4*b^2
 + 111*a^3*b^3 + 66*a^2*b^4 + 15*a*b^5)*cos(d*x + c))/(((a^7*b^2 + 3*a^...
```

3.110.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.38

$$\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx =$$

$$-\frac{\frac{3\,(8\,a^2b+12\,ab^2+5\,b^3)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{(a+b)a}}+\frac{(8\,a^4+32\,a^3b+60\,a^2b^2+51\,ab^3+15\,b^4)\tan(dx+c)^4+8\,a^4+16\,a^3b+8\,a^2b^2+(16\,a^4+48\,a^3b\cdots}{(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4)\tan(dx+c)^5+2\,(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)\tan(dx+c)^3+\cdots}}{8\,d}$$

input

```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

output

```
-1/8*(3*(8*a^2*b + 12*a*b^2 + 5*b^3)*arctan((a + b)*tan(d*x + c)/sqrt((a +
 b)*a))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*a)) + ((8*a^4 + 32*a^3*b +
 60*a^2*b^2 + 51*a*b^3 + 15*b^4)*tan(d*x + c)^4 + 8*a^4 + 16*a^3*b + 8*a^2
*b^2 + (16*a^4 + 48*a^3*b + 60*a^2*b^2 + 25*a*b^3)*tan(d*x + c)^2)/((a^7 +
 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*tan(d*x + c)^5 + 2*(a^7 + 3*a^
6*b + 3*a^5*b^2 + a^4*b^3)*tan(d*x + c)^3 + (a^7 + 2*a^6*b + a^5*b^2)*tan(
d*x + c)))/d
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.18

$$\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx =$$

$$-\frac{\frac{3\,(8\,a^2b+12\,ab^2+5\,b^3)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{a^2+ab}}+\frac{12\,a^2b^2\tan(dx+c)^3+19\,ab^3\tan(dx+c)^3+7\,b^4\tan(\cdots}{(a^5+2\,a^4b+a^3b^2)\left(a\tan(dx+\cdots\right.}}{8\,d}$$

input

```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

output

```
-1/8*(3*(8*a^2*b + 12*a*b^2 + 5*b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a
 + 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a^5
 + 2*a^4*b + a^3*b^2)*sqrt(a^2 + a*b)) + (12*a^2*b^2*tan(d*x + c)^3 + 19*a
*b^3*tan(d*x + c)^3 + 7*b^4*tan(d*x + c)^3 + 12*a^2*b^2*tan(d*x + c) + 9*a
*b^3*tan(d*x + c))/((a^5 + 2*a^4*b + a^3*b^2)*(a*tan(d*x + c)^2 + b*tan(d*
x + c)^2 + a)^2) + 8/(a^3*tan(d*x + c)))/d
```

3.110.9 Mupad [B] (verification not implemented)

Time = 15.80 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.28

$$\int \frac{\csc^2(c+dx)}{(a+b\sin^2(c+dx))^3}\,dx$$
$$= -\frac{\frac{1}{a}+\frac{\tan(c+dx)^4\,(8\,a^3+24\,a^2\,b+36\,a\,b^2+15\,b^3)}{8\,a^3\,(a+b)}+\frac{\tan(c+dx)^2\,(16\,a^3+48\,a^2\,b+60\,a\,b^2+25\,b^3)}{8\,a^2\,(a^2+2\,a\,b+b^2)}}{d\,\left(\tan(c+d\,x)^5\,(a^2+2\,a\,b+b^2)+a^2\tan(c+d\,x)+\tan(c+d\,x)^3\,(2\,a^2+2\,b\,a)\right)} - \frac{3\,b\,\mathrm{atan}\left(\frac{3\,b\tan(c+d\,x)\,(a^5+2\,a^4\,b+a^3\,b^2)\,(8\,a^2+12\,a\,b+5\,b^2)}{a^{7/2}\,(a+b)^{3/2}\,(24\,a^2\,b+36\,a\,b^2+15\,b^3)}\right)\,(8\,a^2+12\,a\,b+5\,b^2)}{8\,a^{7/2}\,d\,(a+b)^{5/2}}$$

input
```
int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x)^2)^3),x)
```

output
```
- (1/a + (tan(c + d*x)^4*(36*a*b^2 + 24*a^2*b + 8*a^3 + 15*b^3))/(8*a^3*(a
 + b)) + (tan(c + d*x)^2*(60*a*b^2 + 48*a^2*b + 16*a^3 + 25*b^3))/(8*a^2*(
2*a*b + a^2 + b^2)))/(d*(tan(c + d*x)^5*(2*a*b + a^2 + b^2) + a^2*tan(c +
d*x) + tan(c + d*x)^3*(2*a*b + 2*a^2))) - (3*b*atan((3*b*tan(c + d*x)*(2*a
^4*b + a^5 + a^3*b^2)*(12*a*b + 8*a^2 + 5*b^2))/(a^(7/2)*(a + b)^(3/2)*(36
*a*b^2 + 24*a^2*b + 15*b^3)))*(12*a*b + 8*a^2 + 5*b^2))/(8*a^(7/2)*d*(a +
b)^(5/2))
```

3.111 $\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^4}\,dx$

3.111.1 Optimal result

Integrand size = 14, antiderivative size = 206

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^4}\,dx = \frac{(2a+b)\left(8a^2+8ab+5b^2\right)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}(a+b)^{7/2}d} + \frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+dx)\right)^3} + \frac{5b(2a+b)\cos(c+dx)\sin(c+dx)}{24a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^2} + \frac{b\left(44a^2+44ab+15b^2\right)\cos(c+dx)\sin(c+dx)}{48a^3(a+b)^3d\left(a+b\sin^2(c+dx)\right)}$$

output
```
1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^
(7/2)/(a+b)^(7/2)/d+1/6*b*cos(d*x+c)*sin(d*x+c)/a/(a+b)/d/(a+b*sin(d*x+c)^
2)^3+5/24*b*(2*a+b)*cos(d*x+c)*sin(d*x+c)/a^2/(a+b)^2/d/(a+b*sin(d*x+c)^2)
^2+1/48*b*(44*a^2+44*a*b+15*b^2)*cos(d*x+c)*sin(d*x+c)/a^3/(a+b)^3/d/(a+b*
sin(d*x+c)^2)
```

3.111.2 Mathematica [A] (verified)

Time = 11.75 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^4}\,dx$$
$$= \frac{\frac{3(16a^3+24a^2b+18ab^2+5b^3)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{7/2}}+\frac{32a^{5/2}b\sin(2(c+dx))}{(a+b)(2a+b-b\cos(2(c+dx)))^3}+\frac{20a^{3/2}b(2a+b)\sin(2(c+dx))}{(a+b)^2(2a+b-b\cos(2(c+dx)))^2}+\frac{\sqrt{a}b(44a^2+44}{(a+b)^3(2a}}{48a^{7/2}d}$$

input
```
Integrate[(a + b*Sin[c + d*x]^2)^(-4),x]
```

output
```
((3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*ArcTan[(Sqrt[a + b]*Tan[c + d*x
])/Sqrt[a]])/(a + b)^(7/2) + (32*a^(5/2)*b*Sin[2*(c + d*x)])/((a + b)*(2*a
 + b - b*Cos[2*(c + d*x)])^3) + (20*a^(3/2)*b*(2*a + b)*Sin[2*(c + d*x)])/
((a + b)^2*(2*a + b - b*Cos[2*(c + d*x)])^2) + (Sqrt[a]*b*(44*a^2 + 44*a*b
 + 15*b^2)*Sin[2*(c + d*x)])/((a + b)^3*(2*a + b - b*Cos[2*(c + d*x)])))/(
48*a^(7/2)*d)
```

3.111.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {3042, 3663, 25, 3042, 3652, 3042, 3652, 27, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^4}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\left(a+b\sin(c+dx)^2\right)^4}dx$$
$$\Big\downarrow \text{ 3663}$$
$$\frac{b\sin(c+dx)\cos(c+dx)}{6ad(a+b)\left(a+b\sin^2(c+dx)\right)^3}-\frac{\int -\frac{-4b\sin^2(c+dx)+6a+5b}{\left(b\sin^2(c+dx)+a\right)^3}dx}{6a(a+b)}$$
$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-4b\sin^2(c+dx)+6a+5b}{\left(b\sin^2(c+dx)+a\right)^3}dx}{6a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{6ad(a+b)\left(a+b\sin^2(c+dx)\right)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{-4b\sin(c+dx)^2+6a+5b}{\left(b\sin(c+dx)^2+a\right)^3}dx}{6a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{6ad(a+b)\left(a+b\sin^2(c+dx)\right)^3}$$

$$\Big\downarrow \text{ 3652}$$

$$\frac{\frac{\int \frac{24a^2+34ba+15b^2-10b(2a+b)\sin^2(c+dx)}{\left(b\sin^2(c+dx)+a\right)^2}dx}{4a(a+b)}+\frac{5b(2a+b)\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}}{6a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{6ad(a+b)\left(a+b\sin^2(c+dx)\right)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{24a^2+34ba+15b^2-10b(2a+b)\sin(c+dx)^2}{\left(b\sin(c+dx)^2+a\right)^2}dx}{4a(a+b)}+\frac{5b(2a+b)\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}}{6a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{6ad(a+b)\left(a+b\sin^2(c+dx)\right)^3}$$

$$\Big\downarrow \text{ 3652}$$

$$\frac{\frac{\frac{\int \frac{3(2a+b)\left(8a^2+8ba+5b^2\right)}{b\sin^2(c+dx)+a}dx}{2a(a+b)}+\frac{b\left(44a^2+44ab+15b^2\right)\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}}{4a(a+b)}+\frac{5b(2a+b)\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}}{6a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{6ad(a+b)\left(a+b\sin^2(c+dx)\right)^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{3(2a+b)\left(8a^2+8ab+5b^2\right)\int \frac{1}{b\sin^2(c+dx)+a}dx}{2a(a+b)}+\frac{b\left(44a^2+44ab+15b^2\right)\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}}{4a(a+b)}+\frac{5b(2a+b)\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}}{6a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{6ad(a+b)\left(a+b\sin^2(c+dx)\right)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\frac{3(2a+b)\left(8a^2+8ab+5b^2\right)\int \frac{1}{b\sin(c+dx)^2+a}dx}{2a(a+b)}+\frac{b\left(44a^2+44ab+15b^2\right)\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}}{4a(a+b)}+\frac{5b(2a+b)\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}}{6a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{6ad(a+b)\left(a+b\sin^2(c+dx)\right)^3}$$

3.111. $\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^4}dx$

$$\Big\downarrow \text{ 3660}$$

$$\frac{\frac{\frac{3(2a+b)\left(8a^2+8ab+5b^2\right)\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{2ad(a+b)}+\frac{b\left(44a^2+44ab+15b^2\right)\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}}{4a(a+b)}+\frac{5b(2a+b)\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}}{6a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{6ad(a+b)\left(a+b\sin^2(c+dx)\right)^3}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\frac{b\left(44a^2+44ab+15b^2\right)\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}+\frac{3(2a+b)\left(8a^2+8ab+5b^2\right)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{3/2}}}{4a(a+b)}+\frac{5b(2a+b)\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}}{6a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{6ad(a+b)\left(a+b\sin^2(c+dx)\right)^3}$$

input
```
Int[(a + b*Sin[c + d*x]^2)^(-4),x]
```

output
```
(b*Cos[c + d*x]*Sin[c + d*x])/(6*a*(a + b)*d*(a + b*Sin[c + d*x]^2)^3) + (
(5*b*(2*a + b)*Cos[c + d*x]*Sin[c + d*x])/(4*a*(a + b)*d*(a + b*Sin[c + d*
x]^2)^2) + ((3*(2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*ArcTan[(Sqrt[a + b]*Tan[c
 + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(3/2)*d) + (b*(44*a^2 + 44*a*b + 15*
b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a + b)*d*(a + b*Sin[c + d*x]^2)))/(4
*a*(a + b)))/(6*a*(a + b))
```

3.111.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.111.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{\frac{\left(24a^2+18ab+5b^2\right)b\left(\tan^5(dx+c)\right)}{16a^3(a+b)}+\frac{\left(18a^2+18ab+5b^2\right)b\left(\tan^3(dx+c)\right)}{6a^2\left(a^2+2ab+b^2\right)}+\frac{b\left(24a^2+30ab+11b^2\right)\tan(dx+c)}{16a\left(a^3+3a^2b+3a\,b^2+b^3\right)}}{\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^3}+\frac{(16a^3+24a^2b+18a\,b^2+5 \ldots}{16a^3(a^3+3a^2 \ldots}}{d}$
default	$\frac{\frac{\frac{\left(24a^2+18ab+5b^2\right)b\left(\tan^5(dx+c)\right)}{16a^3(a+b)}+\frac{\left(18a^2+18ab+5b^2\right)b\left(\tan^3(dx+c)\right)}{6a^2\left(a^2+2ab+b^2\right)}+\frac{b\left(24a^2+30ab+11b^2\right)\tan(dx+c)}{16a\left(a^3+3a^2b+3a\,b^2+b^3\right)}}{\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^3}+\frac{(16a^3+24a^2b+18a\,b^2+5 \ldots}{16a^3(a^3+3a^2 \ldots}}{d}$
risch	Expression too large to display

input
```
int(1/(a+b*sin(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*((1/16*(24*a^2+18*a*b+5*b^2)/a^3*b/(a+b)*tan(d*x+c)^5+1/6*(18*a^2+18*a
*b+5*b^2)/a^2*b/(a^2+2*a*b+b^2)*tan(d*x+c)^3+1/16*b*(24*a^2+30*a*b+11*b^2)
/a/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(d*x+c))/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)
^3+1/16*(16*a^3+24*a^2*b+18*a*b^2+5*b^3)/a^3/(a^3+3*a^2*b+3*a*b^2+b^3)/(a*
(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
```

3.111.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(190) = 380$.

Time = 0.34 (sec) , antiderivative size = 1361, normalized size of antiderivative = 6.61

$$\int \frac{1}{(a+b\sin^2(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(d*x+c)^2)^4,x, algorithm="fricas")
```

output
```
[-1/192*(3*((16*a^3*b^3 + 24*a^2*b^4 + 18*a*b^5 + 5*b^6)*cos(d*x + c)^6 -
16*a^6 - 72*a^5*b - 138*a^4*b^2 - 147*a^3*b^3 - 93*a^2*b^4 - 33*a*b^5 - 5*
b^6 - 3*(16*a^4*b^2 + 40*a^3*b^3 + 42*a^2*b^4 + 23*a*b^5 + 5*b^6)*cos(d*x
+ c)^4 + 3*(16*a^5*b + 56*a^4*b^2 + 82*a^3*b^3 + 65*a^2*b^4 + 28*a*b^5 + 5
*b^6)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x
+ c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c
)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b +
b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^
2)) + 4*((44*a^4*b^3 + 88*a^3*b^4 + 59*a^2*b^5 + 15*a*b^6)*cos(d*x + c)^5
- 2*(54*a^5*b^2 + 157*a^4*b^3 + 167*a^3*b^4 + 79*a^2*b^5 + 15*a*b^6)*cos(d
*x + c)^3 + 3*(24*a^6*b + 90*a^5*b^2 + 131*a^4*b^3 + 93*a^3*b^4 + 33*a^2*b
^5 + 5*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 + 4*a^7*b^4 + 6*a^6*b^
5 + 4*a^5*b^6 + a^4*b^7)*d*cos(d*x + c)^6 - 3*(a^9*b^2 + 5*a^8*b^3 + 10*a^
7*b^4 + 10*a^6*b^5 + 5*a^5*b^6 + a^4*b^7)*d*cos(d*x + c)^4 + 3*(a^10*b + 6
*a^9*b^2 + 15*a^8*b^3 + 20*a^7*b^4 + 15*a^6*b^5 + 6*a^5*b^6 + a^4*b^7)*d*c
os(d*x + c)^2 - (a^11 + 7*a^10*b + 21*a^9*b^2 + 35*a^8*b^3 + 35*a^7*b^4 +
21*a^6*b^5 + 7*a^5*b^6 + a^4*b^7)*d), -1/96*(3*((16*a^3*b^3 + 24*a^2*b^4 +
 18*a*b^5 + 5*b^6)*cos(d*x + c)^6 - 16*a^6 - 72*a^5*b - 138*a^4*b^2 - 147*
a^3*b^3 - 93*a^2*b^4 - 33*a*b^5 - 5*b^6 - 3*(16*a^4*b^2 + 40*a^3*b^3 + 42*
a^2*b^4 + 23*a*b^5 + 5*b^6)*cos(d*x + c)^4 + 3*(16*a^5*b + 56*a^4*b^2 +...
```

3.111.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sin(d*x+c)**2)**4,x)
```

output
```
Timed out
```

3.111.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.83

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^4}\,dx$$
$$= \frac{\frac{3\left(16a^3+24a^2b+18ab^2+5b^3\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\left(a^6+3a^5b+3a^4b^2+a^3b^3\right)\sqrt{(a+b)a}} + \frac{3\left(24a^4b+66a^3b^2+65a^2b^3+28ab^4+5b^5\right)\tan(dx+c)^5+8\cdots}{a^9+3a^8b+3a^7b^2+a^6b^3+\left(a^9+6a^8b+15a^7b^2+20a^6b^3+15a^5b^4+6a^4b^5+a^3b^6\right)\tan(\cdots}}{48d}$$

input
```
integrate(1/(a+b*sin(d*x+c)^2)^4,x, algorithm="maxima")
```

output
```
1/48*(3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*arctan((a + b)*tan(d*x + c)
/sqrt((a + b)*a))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt((a + b)*a))
+ (3*(24*a^4*b + 66*a^3*b^2 + 65*a^2*b^3 + 28*a*b^4 + 5*b^5)*tan(d*x + c)^
5 + 8*(18*a^4*b + 36*a^3*b^2 + 23*a^2*b^3 + 5*a*b^4)*tan(d*x + c)^3 + 3*(2
4*a^4*b + 30*a^3*b^2 + 11*a^2*b^3)*tan(d*x + c))/(a^9 + 3*a^8*b + 3*a^7*b^
2 + a^6*b^3 + (a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^
4*b^5 + a^3*b^6)*tan(d*x + c)^6 + 3*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b
^3 + 5*a^5*b^4 + a^4*b^5)*tan(d*x + c)^4 + 3*(a^9 + 4*a^8*b + 6*a^7*b^2 +
4*a^6*b^3 + a^5*b^4)*tan(d*x + c)^2))/d
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.67

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^4}\,dx$$
$$=\frac{\frac{3\left(16a^3+24a^2b+18ab^2+5b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\left(a^6+3a^5b+3a^4b^2+a^3b^3\right)\sqrt{a^2+ab}}+\frac{72a^4b\tan(dx+c)^5+198a^3b^2\tan(dx+c)^5+195}{}}{}$$

input

```
integrate(1/(a+b*sin(d*x+c)^2)^4,x, algorithm="giac")
```

output

```
1/48*(3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*(pi*floor((d*x + c)/pi + 1/
2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*
b)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt(a^2 + a*b)) + (72*a^4*b*t
an(d*x + c)^5 + 198*a^3*b^2*tan(d*x + c)^5 + 195*a^2*b^3*tan(d*x + c)^5 +
84*a*b^4*tan(d*x + c)^5 + 15*b^5*tan(d*x + c)^5 + 144*a^4*b*tan(d*x + c)^3
 + 288*a^3*b^2*tan(d*x + c)^3 + 184*a^2*b^3*tan(d*x + c)^3 + 40*a*b^4*tan(
d*x + c)^3 + 72*a^4*b*tan(d*x + c) + 90*a^3*b^2*tan(d*x + c) + 33*a^2*b^3*
tan(d*x + c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*(a*tan(d*x + c)^2 + b
*tan(d*x + c)^2 + a)^3))/d
```

3.111.9 Mupad [B] (verification not implemented)

Time = 15.29 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.65

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^4}\,dx$$
$$=\frac{\frac{\tan(c+dx)\left(24a^2b+30ab^2+11b^3\right)}{16a\left(a^3+3a^2b+3ab^2+b^3\right)}+\frac{\tan(c+dx)^3\left(18a^2b+18ab^2+5b^3\right)}{6a^2\left(a^2+2ab+b^2\right)}+\frac{\tan(c+dx)^5\left(24a^2b+18ab^2+5b^3\right)}{16a^3(a+b)}}{d\left(\tan(c+dx)^6\left(a^3+3a^2b+3ab^2+b^3\right)+\tan(c+dx)^2\left(3a^3+3ba^2\right)+\tan(c+dx)^4\left(3a^3+6a^2b+\right.\right.}$$
$$+\frac{\operatorname{atan}\left(\frac{\tan(c+dx)\,(2a+b)\,(2a+2b)\left(8a^2+8ab+5b^2\right)\left(a^3+3a^2b+3ab^2+b^3\right)}{2\sqrt{a}\,(a+b)^{7/2}\left(16a^3+24a^2b+18ab^2+5b^3\right)}\right)(2a+b)\left(8a^2+8ab+5b^2\right)}{16a^{7/2}d\,(a+b)^{7/2}}$$

input

```
int(1/(a + b*sin(c + d*x)^2)^4,x)
```

output
```
((tan(c + d*x)*(30*a*b^2 + 24*a^2*b + 11*b^3))/(16*a*(3*a*b^2 + 3*a^2*b +
a^3 + b^3)) + (tan(c + d*x)^3*(18*a*b^2 + 18*a^2*b + 5*b^3))/(6*a^2*(2*a*b
 + a^2 + b^2)) + (tan(c + d*x)^5*(18*a*b^2 + 24*a^2*b + 5*b^3))/(16*a^3*(a
 + b)))/(d*(tan(c + d*x)^6*(3*a*b^2 + 3*a^2*b + a^3 + b^3) + tan(c + d*x)^
2*(3*a^2*b + 3*a^3) + tan(c + d*x)^4*(3*a*b^2 + 6*a^2*b + 3*a^3) + a^3)) +
 (atan((tan(c + d*x)*(2*a + b)*(2*a + 2*b)*(8*a*b + 8*a^2 + 5*b^2)*(3*a*b^
2 + 3*a^2*b + a^3 + b^3))/(2*a^(1/2)*(a + b)^(7/2)*(18*a*b^2 + 24*a^2*b +
16*a^3 + 5*b^3)))*(2*a + b)*(8*a*b + 8*a^2 + 5*b^2))/(16*a^(7/2)*d*(a + b)
^(7/2))
```

3.111. $\int \frac{1}{(a+b\sin^2(c+dx))^4}\,dx$

3.112 $\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^5}\,dx$

3.112.1 Optimal result

Integrand size = 14, antiderivative size = 279

$$\begin{aligned}\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^5}\,dx &= \frac{(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{128a^{9/2}(a+b)^{9/2}d} \\ &+ \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} + \frac{7b(2a+b)\cos(c+dx)\sin(c+dx)}{48a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^3} \\ &+ \frac{b(104a^2+104ab+35b^2)\cos(c+dx)\sin(c+dx)}{192a^3(a+b)^3d\left(a+b\sin^2(c+dx)\right)^2} \\ &+ \frac{5b(2a+b)\left(40a^2+40ab+21b^2\right)\cos(c+dx)\sin(c+dx)}{384a^4(a+b)^4d\left(a+b\sin^2(c+dx)\right)}\end{aligned}$$

output
```
1/128*(128*a^4+256*a^3*b+288*a^2*b^2+160*a*b^3+35*b^4)*arctan((a+b)^(1/2)*
tan(d*x+c)/a^(1/2))/a^(9/2)/(a+b)^(9/2)/d+1/8*b*cos(d*x+c)*sin(d*x+c)/a/(a
+b)/d/(a+b*sin(d*x+c)^2)^4+7/48*b*(2*a+b)*cos(d*x+c)*sin(d*x+c)/a^2/(a+b)^
2/d/(a+b*sin(d*x+c)^2)^3+1/192*b*(104*a^2+104*a*b+35*b^2)*cos(d*x+c)*sin(d
*x+c)/a^3/(a+b)^3/d/(a+b*sin(d*x+c)^2)^2+5/384*b*(2*a+b)*(40*a^2+40*a*b+21
*b^2)*cos(d*x+c)*sin(d*x+c)/a^4/(a+b)^4/d/(a+b*sin(d*x+c)^2)
```

3.112.2 Mathematica [A] (verified)

Time = 11.90 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+b\sin^2(c+dx))^5}\,dx$$
$$= \frac{\frac{24(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{9/2}} + \frac{2\sqrt{a}b(24576a^6+73728a^5b+97280a^4b^2+71680a^3b^3+32272a^2b^4+8720a}{}}{}$$

input
```
Integrate[(a + b*Sin[c + d*x]^2)^(-5),x]
```

output
```
((24*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*ArcTan[(Sqrt
[a + b]*Tan[c + d*x])/Sqrt[a]])/(a + b)^(9/2) + (2*Sqrt[a]*b*(24576*a^6 +
73728*a^5*b + 97280*a^4*b^2 + 71680*a^3*b^3 + 32272*a^2*b^4 + 8720*a*b^5 +
 1050*b^6 - b*(27648*a^5 + 69120*a^4*b + 73616*a^3*b^2 + 41304*a^2*b^3 + 1
2310*a*b^4 + 1575*b^5)*Cos[2*(c + d*x)] + 2*b^2*(2816*a^4 + 5632*a^3*b + 4
816*a^2*b^2 + 2000*a*b^3 + 315*b^4)*Cos[4*(c + d*x)] - 400*a^3*b^3*Cos[6*(
c + d*x)] - 600*a^2*b^4*Cos[6*(c + d*x)] - 410*a*b^5*Cos[6*(c + d*x)] - 10
5*b^6*Cos[6*(c + d*x)])*Sin[2*(c + d*x)])/((a + b)^4*(2*a + b - b*Cos[2*(c
 + d*x)])^4))/(3072*a^(9/2)*d)
```

3.112.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {3042, 3663, 25, 3042, 3652, 3042, 3652, 3042, 3652, 27, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sin^2(c+dx))^5}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{(a+b\sin(c+dx)^2)^5}dx$$
$$\Big\downarrow \; 3663$$

$$\frac{b\sin(c+dx)\cos(c+dx)}{8ad(a+b)\left(a+b\sin^2(c+dx)\right)^4}-\frac{\int -\frac{-6b\sin^2(c+dx)+8a+7b}{\left(b\sin^2(c+dx)+a\right)^4}dx}{8a(a+b)}$$

$\downarrow$ 25

$$\frac{\int \frac{-6b\sin^2(c+dx)+8a+7b}{\left(b\sin^2(c+dx)+a\right)^4}dx}{8a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{8ad(a+b)\left(a+b\sin^2(c+dx)\right)^4}$$

$\downarrow$ 3042

$$\frac{\int \frac{-6b\sin(c+dx)^2+8a+7b}{\left(b\sin(c+dx)^2+a\right)^4}dx}{8a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{8ad(a+b)\left(a+b\sin^2(c+dx)\right)^4}$$

$\downarrow$ 3652

$$\frac{\frac{\int \frac{48a^2+76ba+35b^2-28b(2a+b)\sin^2(c+dx)}{\left(b\sin^2(c+dx)+a\right)^3}dx}{6a(a+b)}+\frac{7b(2a+b)\sin(c+dx)\cos(c+dx)}{6ad(a+b)\left(a+b\sin^2(c+dx)\right)^3}}{8a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{8ad(a+b)\left(a+b\sin^2(c+dx)\right)^4}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{48a^2+76ba+35b^2-28b(2a+b)\sin(c+dx)^2}{\left(b\sin(c+dx)^2+a\right)^3}dx}{6a(a+b)}+\frac{7b(2a+b)\sin(c+dx)\cos(c+dx)}{6ad(a+b)\left(a+b\sin^2(c+dx)\right)^3}}{8a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{8ad(a+b)\left(a+b\sin^2(c+dx)\right)^4}$$

$\downarrow$ 3652

$$\frac{\frac{\frac{\int \frac{192a^3+392ba^2+340b^2a+105b^3-2b\left(104a^2+104ba+35b^2\right)\sin^2(c+dx)}{\left(b\sin^2(c+dx)+a\right)^2}dx}{4a(a+b)}+\frac{b\left(104a^2+104ab+35b^2\right)\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}}{6a(a+b)}+\frac{7b(2a+b)\sin(c+dx)\cos(c+dx)}{6ad(a+b)\left(a+b\sin^2(c+dx)\right)^3}}{8a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{8ad(a+b)\left(a+b\sin^2(c+dx)\right)^4}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{192a^3+392ba^2+340b^2a+105b^3-2b\left(104a^2+104ba+35b^2\right)\sin(c+dx)^2}{\left(b\sin(c+dx)^2+a\right)^2}dx}{4a(a+b)}+\frac{b\left(104a^2+104ab+35b^2\right)\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}}{6a(a+b)}+\frac{7b(2a+b)\sin(c+dx)\cos(c+dx)}{6ad(a+b)\left(a+b\sin^2(c+dx)\right)^3}}{8a(a+b)}+\frac{b\sin(c+dx)\cos(c+dx)}{8ad(a+b)\left(a+b\sin^2(c+dx)\right)^4}$$

3.112. $\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^5}dx$

$$\Big\downarrow \; 3652$$

$$\frac{\frac{\frac{\frac{\int \frac{3\left(128a^4+256ba^3+288b^2a^2+160b^3a+35b^4\right)}{b\sin^2(c+dx)+a}dx}{2a(a+b)}+\frac{5b(2a+b)\left(40a^2+40ab+21b^2\right)\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}}{4a(a+b)}+\frac{b\left(104a^2+104ab+35b^2\right)\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}}{6a(a+b)}+\frac{7b(2a+b\cdots}{6ad(a+\cdots}}{8a(a+b)}$$

$$\frac{b\sin(c+dx)\cos(c+dx)}{8ad(a+b)\left(a+b\sin^2(c+dx)\right)^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\frac{3\left(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4\right)\int \frac{1}{b\sin^2(c+dx)+a}dx}{2a(a+b)}+\frac{5b(2a+b)\left(40a^2+40ab+21b^2\right)\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}}{4a(a+b)}+\frac{b\left(104a^2+104ab+35b^2\right)\sin(c+dx)\cos(c+d\cdots}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}}{6a(a+b)}\cdots}{8a(a+b)}$$

$$\frac{b\sin(c+dx)\cos(c+dx)}{8ad(a+b)\left(a+b\sin^2(c+dx)\right)^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\frac{3\left(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4\right)\int \frac{1}{b\sin(c+dx)^2+a}dx}{2a(a+b)}+\frac{5b(2a+b)\left(40a^2+40ab+21b^2\right)\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}}{4a(a+b)}+\frac{b\left(104a^2+104ab+35b^2\right)\sin(c+dx)\cos(c+d\cdots}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}}{6a(a+b)}\cdots}{8a(a+b)}$$

$$\frac{b\sin(c+dx)\cos(c+dx)}{8ad(a+b)\left(a+b\sin^2(c+dx)\right)^4}$$

$$\Big\downarrow \; 3660$$

$$\frac{\frac{\frac{\frac{3\left(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4\right)\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{2ad(a+b)}+\frac{5b(2a+b)\left(40a^2+40ab+21b^2\right)\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}}{4a(a+b)}+\frac{b\left(104a^2+104ab+35b^2\right)\sin(\cdots}{4ad(a+b)\left(a+b\sin^2(\cdots\right.}}{6a(a+b)}\cdots}{8a(a+b)}$$

$$\frac{b\sin(c+dx)\cos(c+dx)}{8ad(a+b)\left(a+b\sin^2(c+dx)\right)^4}$$

$$\Big\downarrow \; 218$$

3.112. $\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^5}\,dx$

$$\frac{\frac{\frac{b(104a^2+104ab+35b^2)\sin(c+dx)\cos(c+dx)}{4ad(a+b)(a+b\sin^2(c+dx))^2}+\frac{\frac{5b(2a+b)(40a^2+40ab+21b^2)\sin(c+dx)\cos(c+dx)}{2ad(a+b)(a+b\sin^2(c+dx))}+\frac{3(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4)\arctan\left(\frac{\sqrt{a+b}\tan(c}{\sqrt{a}}\right.}{2a^{3/2}d(a+b)^{3/2}}}{4a(a+b)}}{6a(a+b)}}{8a(a+b)}$$

$$\frac{b\sin(c+dx)\cos(c+dx)}{8ad(a+b)\left(a+b\sin^2(c+dx)\right)^4}$$

input
```
Int[(a + b*Sin[c + d*x]^2)^(-5),x]
```

output
```
(b*Cos[c + d*x]*Sin[c + d*x])/(8*a*(a + b)*d*(a + b*Sin[c + d*x]^2)^4) + (
(7*b*(2*a + b)*Cos[c + d*x]*Sin[c + d*x])/(6*a*(a + b)*d*(a + b*Sin[c + d*
x]^2)^3) + ((b*(104*a^2 + 104*a*b + 35*b^2)*Cos[c + d*x]*Sin[c + d*x])/(4*
a*(a + b)*d*(a + b*Sin[c + d*x]^2)^2) + ((3*(128*a^4 + 256*a^3*b + 288*a^2
*b^2 + 160*a*b^3 + 35*b^4)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2*
a^(3/2)*(a + b)^(3/2)*d) + (5*b*(2*a + b)*(40*a^2 + 40*a*b + 21*b^2)*Cos[c
 + d*x]*Sin[c + d*x])/(2*a*(a + b)*d*(a + b*Sin[c + d*x]^2)))/(4*a*(a + b)
))/(6*a*(a + b)))/(8*a*(a + b))
```

3.112.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.112.4 Maple [A] (verified)

Time = 3.55 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{b\left(256a^3+288a^2b+160a\,b^2+35b^3\right)\left(\tan^7(dx+c)\right)}{128a^4(a+b)}+\frac{\left(2304a^3+3168a^2b+1760a\,b^2+385b^3\right)b\left(\tan^5(dx+c)\right)}{384a^3\left(a^2+2ab+b^2\right)}+\frac{\left(2304a^3+3744a^2b+2336a\,b\cdots\right.}{384a^2\left(a^3+3a^2\cdots\right.}}{\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^4}$
default	$\frac{\frac{b\left(256a^3+288a^2b+160a\,b^2+35b^3\right)\left(\tan^7(dx+c)\right)}{128a^4(a+b)}+\frac{\left(2304a^3+3168a^2b+1760a\,b^2+385b^3\right)b\left(\tan^5(dx+c)\right)}{384a^3\left(a^2+2ab+b^2\right)}+\frac{\left(2304a^3+3744a^2b+2336a\,b\cdots\right.}{384a^2\left(a^3+3a^2\cdots\right.}}{\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^4}$
risch	Expression too large to display

input
```
int(1/(a+b*sin(d*x+c)^2)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*((1/128*b*(256*a^3+288*a^2*b+160*a*b^2+35*b^3)/a^4/(a+b)*tan(d*x+c)^7+
1/384*(2304*a^3+3168*a^2*b+1760*a*b^2+385*b^3)/a^3*b/(a^2+2*a*b+b^2)*tan(d
*x+c)^5+1/384*(2304*a^3+3744*a^2*b+2336*a*b^2+511*b^3)/a^2*b/(a^3+3*a^2*b+
3*a*b^2+b^3)*tan(d*x+c)^3+1/128*b*(256*a^3+480*a^2*b+352*a*b^2+93*b^3)/a/(
a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*tan(d*x+c))/(a*tan(d*x+c)^2+tan(d*x+c)^
2*b+a)^4+1/128*(128*a^4+256*a^3*b+288*a^2*b^2+160*a*b^3+35*b^4)/a^4/(a^4+4
*a^3*b+6*a^2*b^2+4*a*b^3+b^4)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(
a+b))^(1/2)))
```

3.112.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 958 vs. $2(261) = 522$.

Time = 0.38 (sec) , antiderivative size = 2017, normalized size of antiderivative = 7.23

$$\int \frac{1}{(a + b\sin^2(c + dx))^5}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(d*x+c)^2)^5,x, algorithm="fricas")
```

output
```
[-1/1536*(3*((128*a^4*b^4 + 256*a^3*b^5 + 288*a^2*b^6 + 160*a*b^7 + 35*b^8
)*cos(d*x + c)^8 + 128*a^8 + 768*a^7*b + 2080*a^6*b^2 + 3360*a^5*b^3 + 355
5*a^4*b^4 + 2508*a^3*b^5 + 1138*a^2*b^6 + 300*a*b^7 + 35*b^8 - 4*(128*a^5*
b^3 + 384*a^4*b^4 + 544*a^3*b^5 + 448*a^2*b^6 + 195*a*b^7 + 35*b^8)*cos(d*
x + c)^6 + 6*(128*a^6*b^2 + 512*a^5*b^3 + 928*a^4*b^4 + 992*a^3*b^5 + 643*
a^2*b^6 + 230*a*b^7 + 35*b^8)*cos(d*x + c)^4 - 4*(128*a^7*b + 640*a^6*b^2
+ 1440*a^5*b^3 + 1920*a^4*b^4 + 1635*a^3*b^5 + 873*a^2*b^6 + 265*a*b^7 + 3
5*b^8)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x
 + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x +
c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b +
 b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b
^2)) + 4*(5*(80*a^5*b^4 + 200*a^4*b^5 + 202*a^3*b^6 + 103*a^2*b^7 + 21*a*b
^8)*cos(d*x + c)^7 - (1408*a^6*b^3 + 4824*a^5*b^4 + 6724*a^4*b^5 + 4923*a^
3*b^6 + 1930*a^2*b^7 + 315*a*b^8)*cos(d*x + c)^5 + (1728*a^7*b^2 + 7456*a^
6*b^3 + 13370*a^5*b^4 + 12969*a^4*b^5 + 7327*a^3*b^6 + 2315*a^2*b^7 + 315*
a*b^8)*cos(d*x + c)^3 - 3*(256*a^8*b + 1312*a^7*b^2 + 2848*a^6*b^3 + 3427*
a^5*b^4 + 2508*a^4*b^5 + 1138*a^3*b^6 + 300*a^2*b^7 + 35*a*b^8)*cos(d*x +
c))*sin(d*x + c))/((a^10*b^4 + 5*a^9*b^5 + 10*a^8*b^6 + 10*a^7*b^7 + 5*a^6
*b^8 + a^5*b^9)*d*cos(d*x + c)^8 - 4*(a^11*b^3 + 6*a^10*b^4 + 15*a^9*b^5 +
 20*a^8*b^6 + 15*a^7*b^7 + 6*a^6*b^8 + a^5*b^9)*d*cos(d*x + c)^6 + 6*(a...
```

3.112.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^5}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sin(d*x+c)**2)**5,x)
```

output
```
Timed out
```

3.112.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(261) = 522$.

Time = 0.40 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.11

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^5}\,dx = \frac{\frac{3\,(128\,a^4+256\,a^3b+288\,a^2b^2+160\,ab^3+35\,b^4)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(a^8+4\,a^7b+6\,a^6b^2+4\,a^5b^3+a^4b^4)\sqrt{(a+b)a}} + \frac{3\,(256\,a^6b+1056\,a^5b^2+1792\,a^4b^3+1635\,a^3b^4}{a^{12}+4\,a^{11}b+6\,a^{10}b^2+4\,a^9b^3+a^8b^4+(a^{12}+8\,a^{11}b+28\,a^{10}b^2+56\,a^9b^3}}{}$$

input
```
integrate(1/(a+b*sin(d*x+c)^2)^5,x, algorithm="maxima")
```

output
```
1/384*(3*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*arctan((
a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b
^3 + a^4*b^4)*sqrt((a + b)*a)) + (3*(256*a^6*b + 1056*a^5*b^2 + 1792*a^4*b
^3 + 1635*a^3*b^4 + 873*a^2*b^5 + 265*a*b^6 + 35*b^7)*tan(d*x + c)^7 + (23
04*a^6*b + 7776*a^5*b^2 + 10400*a^4*b^3 + 7073*a^3*b^4 + 2530*a^2*b^5 + 38
5*a*b^6)*tan(d*x + c)^5 + (2304*a^6*b + 6048*a^5*b^2 + 6080*a^4*b^3 + 2847
*a^3*b^4 + 511*a^2*b^5)*tan(d*x + c)^3 + 3*(256*a^6*b + 480*a^5*b^2 + 352*
a^4*b^3 + 93*a^3*b^4)*tan(d*x + c))/(a^12 + 4*a^11*b + 6*a^10*b^2 + 4*a^9*
b^3 + a^8*b^4 + (a^12 + 8*a^11*b + 28*a^10*b^2 + 56*a^9*b^3 + 70*a^8*b^4 +
 56*a^7*b^5 + 28*a^6*b^6 + 8*a^5*b^7 + a^4*b^8)*tan(d*x + c)^8 + 4*(a^12 +
 7*a^11*b + 21*a^10*b^2 + 35*a^9*b^3 + 35*a^8*b^4 + 21*a^7*b^5 + 7*a^6*b^6
 + a^5*b^7)*tan(d*x + c)^6 + 6*(a^12 + 6*a^11*b + 15*a^10*b^2 + 20*a^9*b^3
 + 15*a^8*b^4 + 6*a^7*b^5 + a^6*b^6)*tan(d*x + c)^4 + 4*(a^12 + 5*a^11*b +
 10*a^10*b^2 + 10*a^9*b^3 + 5*a^8*b^4 + a^7*b^5)*tan(d*x + c)^2))/d
```

3.112.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. 2(261) = 522.

Time = 0.38 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.88

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^5}\,dx$$
$$= \frac{\frac{3\,(128\,a^4+256\,a^3b+288\,a^2b^2+160\,ab^3+35\,b^4)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{(a^8+4\,a^7b+6\,a^6b^2+4\,a^5b^3+a^4b^4)\sqrt{a^2+ab}}+\frac{768\,a^6b\tan(dx+c)^7+3168\,a^5b^2}{}}{}$$

input
```
integrate(1/(a+b*sin(d*x+c)^2)^5,x, algorithm="giac")
```

output
```
1/384*(3*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*(pi*floo
r((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x
+ c))/sqrt(a^2 + a*b)))/((a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4)
*sqrt(a^2 + a*b)) + (768*a^6*b*tan(d*x + c)^7 + 3168*a^5*b^2*tan(d*x + c)^
7 + 5376*a^4*b^3*tan(d*x + c)^7 + 4905*a^3*b^4*tan(d*x + c)^7 + 2619*a^2*b
^5*tan(d*x + c)^7 + 795*a*b^6*tan(d*x + c)^7 + 105*b^7*tan(d*x + c)^7 + 23
04*a^6*b*tan(d*x + c)^5 + 7776*a^5*b^2*tan(d*x + c)^5 + 10400*a^4*b^3*tan(
d*x + c)^5 + 7073*a^3*b^4*tan(d*x + c)^5 + 2530*a^2*b^5*tan(d*x + c)^5 + 3
85*a*b^6*tan(d*x + c)^5 + 2304*a^6*b*tan(d*x + c)^3 + 6048*a^5*b^2*tan(d*x
 + c)^3 + 6080*a^4*b^3*tan(d*x + c)^3 + 2847*a^3*b^4*tan(d*x + c)^3 + 511*
a^2*b^5*tan(d*x + c)^3 + 768*a^6*b*tan(d*x + c) + 1440*a^5*b^2*tan(d*x + c
) + 1056*a^4*b^3*tan(d*x + c) + 279*a^3*b^4*tan(d*x + c))/((a^8 + 4*a^7*b
+ 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4)*(a*tan(d*x + c)^2 + b*tan(d*x + c)^2 +
a)^4))/d
```

3.112.9 Mupad [B] (verification not implemented)

Time = 16.12 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.61

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^5}\,dx$$
$$= \frac{\frac{\tan(c+d\,x)\,(256\,a^3\,b+480\,a^2\,b^2+352\,a\,b^3+93\,b^4)}{128\,a\,(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4)}+\frac{\tan(c+d\,x)^3\,(2304\,a^3\,b+3744\,a^2\,b^2+2336\,a\,b^3+511\,b^4)}{384\,a^2\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}+\frac{\tan(c+d\,x)^5\,(2304}{38}}{d\,\left(\tan(c+d\,x)^4\,(6\,a^4+12\,a^3\,b+6\,a^2\,b^2)+\tan(c+d\,x)^2\,(4\,a^4+4\,b\,a^3)+\tan(c+d\,x)^6\,(4\,a^4+12\,a^3\,b+\right.}$$
$$+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)\,(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4)}{2\,\sqrt{a}\,(a+b)^{9/2}}\right)\,(128\,a^4+256\,a^3\,b+288\,a^2\,b^2+160\,a\,b^3+35\,b^4)}{128\,a^{9/2}\,d\,(a+b)^{9/2}}$$

input
```
int(1/(a + b*sin(c + d*x)^2)^5,x)
```

output
```
((tan(c + d*x)*(352*a*b^3 + 256*a^3*b + 93*b^4 + 480*a^2*b^2))/(128*a*(4*a
*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + (tan(c + d*x)^3*(2336*a*b^3 + 2
304*a^3*b + 511*b^4 + 3744*a^2*b^2))/(384*a^2*(3*a*b^2 + 3*a^2*b + a^3 + b
^3)) + (tan(c + d*x)^5*(1760*a*b^3 + 2304*a^3*b + 385*b^4 + 3168*a^2*b^2))
/(384*a^3*(2*a*b + a^2 + b^2)) + (tan(c + d*x)^7*(160*a*b^3 + 256*a^3*b +
35*b^4 + 288*a^2*b^2))/(128*a^4*(a + b)))/(d*(tan(c + d*x)^4*(12*a^3*b + 6
*a^4 + 6*a^2*b^2) + tan(c + d*x)^2*(4*a^3*b + 4*a^4) + tan(c + d*x)^6*(4*a
*b^3 + 12*a^3*b + 4*a^4 + 12*a^2*b^2) + a^4 + tan(c + d*x)^8*(4*a*b^3 + 4*
a^3*b + a^4 + b^4 + 6*a^2*b^2))) + (atan((tan(c + d*x)*(2*a + 2*b)*(4*a*b^
3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2))/(2*a^(1/2)*(a + b)^(9/2)))*(160*a*b^
3 + 256*a^3*b + 128*a^4 + 35*b^4 + 288*a^2*b^2))/(128*a^(9/2)*d*(a + b)^(9
/2))
```

3.112. $\int \frac{1}{(a+b\sin^2(c+dx))^5}\,dx$

3.113 $\int \frac{\sin(x)}{\sqrt{1+\sin^2(x)}}\,dx$

3.113.1 Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \frac{\sin(x)}{\sqrt{1+\sin^2(x)}}\,dx = -\arcsin\left(\frac{\cos(x)}{\sqrt{2}}\right)$$

output
```
-arcsin(1/2*cos(x)*2^(1/2))
```

3.113.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.64

$$\int \frac{\sin(x)}{\sqrt{1+\sin^2(x)}}\,dx = i\log\left(i\sqrt{2}\cos(x)+\sqrt{3-\cos(2x)}\right)$$

input
```
Integrate[Sin[x]/Sqrt[1 + Sin[x]^2],x]
```

output
```
I*Log[I*Sqrt[2]*Cos[x] + Sqrt[3 - Cos[2*x]]]
```

3.113.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3665, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\sqrt{\sin^2(x)+1}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(x)}{\sqrt{\sin(x)^2+1}}dx$$
$$\Big\downarrow \text{ 3665}$$
$$-\int \frac{1}{\sqrt{2-\cos^2(x)}}d\cos(x)$$
$$\Big\downarrow \text{ 223}$$
$$-\arcsin\left(\frac{\cos(x)}{\sqrt{2}}\right)$$

input
```
Int[Sin[x]/Sqrt[1 + Sin[x]^2],x]
```

output
```
-ArcSin[Cos[x]/Sqrt[2]]
```

3.113.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.113.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(10) = 20$.

Time = 0.68 (sec) , antiderivative size = 33, normalized size of antiderivative = 3.00

method	result	size
default	$\frac{\sqrt{(1+\sin^2(x))(\cos^2(x))}\,\arcsin(\sin^2(x))}{2\cos(x)\sqrt{1+\sin^2(x)}}$	33

input
```
int(sin(x)/(1+sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((1+sin(x)^2)*cos(x)^2)^(1/2)*arcsin(sin(x)^2)/cos(x)/(1+sin(x)^2)^(1/
2)
```

3.113.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(10) = 20$.

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 5.18

$$\int \frac{\sin(x)}{\sqrt{1+\sin^2(x)}}\,dx = \frac{1}{2}\arctan\left(-\frac{\cos(x)\sin(x) - (\cos(x)^3 - \cos(x))\sqrt{-\cos(x)^2+2}}{\cos(x)^4 - 3\cos(x)^2 + 1}\right) - \frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

input
```
integrate(sin(x)/(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*arctan(-(cos(x)*sin(x) - (cos(x)^3 - cos(x))*sqrt(-cos(x)^2 + 2))/(cos
(x)^4 - 3*cos(x)^2 + 1)) - 1/2*arctan(sin(x)/cos(x))
```

3.113.6 Sympy [F]

$$\int \frac{\sin(x)}{\sqrt{1+\sin^2(x)}}\,dx = \int \frac{\sin(x)}{\sqrt{\sin^2(x)+1}}\,dx$$

input
```
integrate(sin(x)/(1+sin(x)**2)**(1/2),x)
```

output
```
Integral(sin(x)/sqrt(sin(x)**2 + 1), x)
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\sin(x)}{\sqrt{1+\sin^2(x)}}\,dx = -\arcsin\left(\frac{1}{2}\sqrt{2}\cos(x)\right)$$

input
```
integrate(sin(x)/(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(1/2*sqrt(2)*cos(x))
```

3.113.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{\sin(x)}{\sqrt{1+\sin^2(x)}}\,dx = -\frac{1}{2}\sqrt{-\cos(x)^2+2}\cos(x) - \arcsin\left(\frac{1}{2}\sqrt{2}\cos(x)\right)$$

input
```
integrate(sin(x)/(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-cos(x)^2 + 2)*cos(x) - arcsin(1/2*sqrt(2)*cos(x))
```

3.113.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int \frac{\sin(x)}{\sqrt{1+\sin^2(x)}}\,dx = \ln\left(\sqrt{\sin(x)^2+1}+\cos(x)\ 1\mathrm{i}\right)\ 1\mathrm{i}$$

input
```
int(sin(x)/(sin(x)^2 + 1)^(1/2),x)
```

output
```
log(cos(x)*1i + (sin(x)^2 + 1)^(1/2))*1i
```

3.114 $\int \sin(x)\sqrt{1+\sin^2(x)}\,dx$

3.114.1 Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \sin(x)\sqrt{1+\sin^2(x)}\,dx = -\arcsin\left(\frac{\cos(x)}{\sqrt{2}}\right) - \frac{1}{2}\cos(x)\sqrt{2-\cos^2(x)}$$

output
```
-arcsin(1/2*cos(x)*2^(1/2))-1/2*cos(x)*(2-cos(x)^2)^(1/2)
```

3.114.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.77

$$\int \sin(x)\sqrt{1+\sin^2(x)}\,dx = -\frac{\cos(x)\sqrt{3-\cos(2x)}}{2\sqrt{2}} + i\log\left(i\sqrt{2}\cos(x)+\sqrt{3-\cos(2x)}\right)$$

input
```
Integrate[Sin[x]*Sqrt[1 + Sin[x]^2],x]
```

output
```
-1/2*(Cos[x]*Sqrt[3 - Cos[2*x]])/Sqrt[2] + I*Log[I*Sqrt[2]*Cos[x] + Sqrt[3
 - Cos[2*x]]]
```

3.114.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3665, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sqrt{\sin^2(x)+1}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(x)\sqrt{\sin(x)^2+1}dx$$

$$\Big\downarrow \ 3665$$

$$-\int \sqrt{2-\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \ 211$$

$$-\int \frac{1}{\sqrt{2-\cos^2(x)}}d\cos(x)-\frac{1}{2}\sqrt{2-\cos^2(x)}\cos(x)$$

$$\Big\downarrow \ 223$$

$$-\arcsin\left(\frac{\cos(x)}{\sqrt{2}}\right)-\frac{1}{2}\cos(x)\sqrt{2-\cos^2(x)}$$

input
```
Int[Sin[x]*Sqrt[1 + Sin[x]^2],x]
```

output
```
-ArcSin[Cos[x]/Sqrt[2]] - (Cos[x]*Sqrt[2 - Cos[x]^2])/2
```

3.114.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.114.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.70

method	result	size
default	$-\frac{\sqrt{\left(1+\sin^{2}\left(x\right)\right)\left(\cos^{2}\left(x\right)\right)}\,\left(\arcsin\left(\cos^{2}\left(x\right)-1\right)+\sqrt{-\left(\cos^{4}\left(x\right)\right)+2\left(\cos^{2}\left(x\right)\right)}\right)}{2\cos\left(x\right)\sqrt{1+\sin^{2}\left(x\right)}}$	51

input
```
int(sin(x)*(1+sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((1+sin(x)^2)*cos(x)^2)^(1/2)*(arcsin(cos(x)^2-1)+(-cos(x)^4+2*cos(x)
^2)^(1/2))/cos(x)/(1+sin(x)^2)^(1/2)
```

3.114.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(25) = 50.

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.37

$$\begin{aligned}
&\int \sin(x)\sqrt{1+\sin^2(x)}\,dx \\
&= -\frac{1}{2}\sqrt{-\cos\left(x\right)^{2}+2}\cos\left(x\right) \\
&\quad+\frac{1}{2}\arctan\left(-\frac{\cos\left(x\right)\sin\left(x\right)-\left(\cos\left(x\right)^{3}-\cos\left(x\right)\right)\sqrt{-\cos\left(x\right)^{2}+2}}{\cos\left(x\right)^{4}-3\,\cos\left(x\right)^{2}+1}\right) \\
&\quad-\frac{1}{2}\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)}\right)
\end{aligned}$$

input
```
integrate(sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-cos(x)^2 + 2)*cos(x) + 1/2*arctan(-(cos(x)*sin(x) - (cos(x)^3 -
 cos(x))*sqrt(-cos(x)^2 + 2))/(cos(x)^4 - 3*cos(x)^2 + 1)) - 1/2*arctan(si
n(x)/cos(x))
```

3.114.6 Sympy [F]

$$\int \sin(x)\sqrt{1+\sin^2(x)}\,dx = \int \sqrt{\sin^2(x)+1}\sin(x)\,dx$$

input
```
integrate(sin(x)*(1+sin(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(sin(x)**2 + 1)*sin(x), x)
```

3.114.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \sin(x)\sqrt{1+\sin^2(x)}\,dx = -\frac{1}{2}\sqrt{-\cos(x)^2+2}\cos(x) - \arcsin\left(\frac{1}{2}\sqrt{2}\cos(x)\right)$$

input
```
integrate(sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(-cos(x)^2 + 2)*cos(x) - arcsin(1/2*sqrt(2)*cos(x))
```

3.114.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \sin(x)\sqrt{1+\sin^2(x)}\,dx = -\frac{1}{2}\sqrt{-\cos(x)^2+2}\cos(x) - \arcsin\left(\frac{1}{2}\sqrt{2}\cos(x)\right)$$

input
```
integrate(sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-cos(x)^2 + 2)*cos(x) - arcsin(1/2*sqrt(2)*cos(x))
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \sin(x)\sqrt{1+\sin^2(x)}\,dx = \int \sin(x)\,\sqrt{\sin(x)^2+1}\,dx$$

input
```
int(sin(x)*(sin(x)^2 + 1)^(1/2),x)
```

output
```
int(sin(x)*(sin(x)^2 + 1)^(1/2), x)
```

3.115 $\int \frac{\sin(7+3x)}{\sqrt{3+\sin^2(7+3x)}}\,dx$

3.115.1 Optimal result

Integrand size = 21, antiderivative size = 15

$$\int \frac{\sin(7+3x)}{\sqrt{3+\sin^2(7+3x)}}\,dx = -\frac{1}{3}\arcsin\left(\frac{1}{2}\cos(7+3x)\right)$$

output
```
-1/3*arcsin(1/2*cos(7+3*x))
```

3.115.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \frac{\sin(7+3x)}{\sqrt{3+\sin^2(7+3x)}}\,dx = \frac{1}{3}i\log\left(i\sqrt{2}\cos(7+3x)+\sqrt{7-\cos(2(7+3x))}\right)$$

input
```
Integrate[Sin[7 + 3*x]/Sqrt[3 + Sin[7 + 3*x]^2],x]
```

output
```
(I/3)*Log[I*Sqrt[2]*Cos[7 + 3*x] + Sqrt[7 - Cos[2*(7 + 3*x)]]]
```

3.115.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3665, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3x+7)}{\sqrt{\sin^2(3x+7)+3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(3x+7)}{\sqrt{\sin(3x+7)^2+3}}dx$$

$$\Big\downarrow \text{ 3665}$$

$$-\frac{1}{3}\int \frac{1}{\sqrt{4-\cos^2(3x+7)}}d\cos(3x+7)$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{1}{3}\arcsin\left(\frac{1}{2}\cos(3x+7)\right)$$

input
```
Int[Sin[7 + 3*x]/Sqrt[3 + Sin[7 + 3*x]^2],x]
```

output
```
-1/3*ArcSin[Cos[7 + 3*x]/2]
```

3.115.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.115.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(11) = 22$.

Time = 0.70 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.80

method	result	size
default	$-\frac{\sqrt{\left(3+\sin^2(7+3x)\right)\left(\cos^2(7+3x)\right)}\,\arcsin\left(-1+\frac{\left(\cos^2(7+3x)\right)}{2}\right)}{6\cos(7+3x)\sqrt{3+\sin^2(7+3x)}}$	57

input
```
int(sin(7+3*x)/(3+sin(7+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*((3+sin(7+3*x)^2)*cos(7+3*x)^2)^(1/2)*arcsin(-1+1/2*cos(7+3*x)^2)/cos
(7+3*x)/(3+sin(7+3*x)^2)^(1/2)
```

3.115.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(11) = 22$.

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 6.27

$$\int \frac{\sin(7+3x)}{\sqrt{3+\sin^2(7+3x)}}\,dx$$
$$= \frac{1}{6}\arctan\left(-\frac{4\cos(3x+7)\sin(3x+7)-\left(\cos(3x+7)^3-2\cos(3x+7)\right)\sqrt{-\cos(3x+7)^2+4}}{\cos(3x+7)^4-8\cos(3x+7)^2+4}\right)$$
$$-\frac{1}{6}\arctan\left(\frac{\sin(3x+7)}{\cos(3x+7)}\right)$$

input
```
integrate(sin(7+3*x)/(3+sin(7+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*arctan(-(4*cos(3*x + 7)*sin(3*x + 7) - (cos(3*x + 7)^3 - 2*cos(3*x + 7
))*sqrt(-cos(3*x + 7)^2 + 4))/(cos(3*x + 7)^4 - 8*cos(3*x + 7)^2 + 4)) - 1
/6*arctan(sin(3*x + 7)/cos(3*x + 7))
```

3.115.6 Sympy [F]

$$\int \frac{\sin(7+3x)}{\sqrt{3+\sin^2(7+3x)}}\,dx = \int \frac{\sin(3x+7)}{\sqrt{\sin^2(3x+7)+3}}\,dx$$

input
```
integrate(sin(7+3*x)/(3+sin(7+3*x)**2)**(1/2),x)
```

output
```
Integral(sin(3*x + 7)/sqrt(sin(3*x + 7)**2 + 3), x)
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\sin(7+3x)}{\sqrt{3+\sin^2(7+3x)}}\,dx = -\frac{1}{3}\arcsin\left(\frac{1}{2}\cos(3x+7)\right)$$

input
```
integrate(sin(7+3*x)/(3+sin(7+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*arcsin(1/2*cos(3*x + 7))
```

3.115.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(11) = 22.

Time = 0.53 (sec) , antiderivative size = 48, normalized size of antiderivative = 3.20

$$\int \frac{\sin(7+3x)}{\sqrt{3+\sin^2(7+3x)}}\,dx = \frac{2}{3}\arctan\left(-\frac{1}{2}\sqrt{3}\tan\left(\frac{3}{2}x+\frac{7}{2}\right)^2-\frac{1}{2}\sqrt{3}+\frac{1}{2}\sqrt{3\tan\left(\frac{3}{2}x+\frac{7}{2}\right)^4+10\tan\left(\frac{3}{2}x+\frac{7}{2}\right)^2+3}\right)$$

input
```
integrate(sin(7+3*x)/(3+sin(7+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
2/3*arctan(-1/2*sqrt(3)*tan(3/2*x + 7/2)^2 - 1/2*sqrt(3) + 1/2*sqrt(3*tan(
3/2*x + 7/2)^4 + 10*tan(3/2*x + 7/2)^2 + 3))
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(7+3x)}{\sqrt{3+\sin^2(7+3x)}}\,dx = \int \frac{\sin(3\,x+7)}{\sqrt{\sin(3\,x+7)^2+3}}\,dx$$

input
```
int(sin(3*x + 7)/(sin(3*x + 7)^2 + 3)^(1/2),x)
```

output
```
int(sin(3*x + 7)/(sin(3*x + 7)^2 + 3)^(1/2), x)
```

3.116 $\int \left(a-a\sin^2(x)\right)^{5/2}\,dx$

3.116.1 Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \left(a-a\sin^2(x)\right)^{5/2}\,dx=\frac{8}{15}a^2\sqrt{a\cos^2(x)}\tan(x)+\frac{4}{15}a\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)$$

output
```
4/15*a*(a*cos(x)^2)^(3/2)*tan(x)+1/5*(a*cos(x)^2)^(5/2)*tan(x)+8/15*a^2*(a
*cos(x)^2)^(1/2)*tan(x)
```

3.116.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

$$\int \left(a-a\sin^2(x)\right)^{5/2}\,dx=\frac{1}{15}a^2\sqrt{a\cos^2(x)}\left(15-10\sin^2(x)+3\sin^4(x)\right)\tan(x)$$

input
```
Integrate[(a - a*Sin[x]^2)^(5/2),x]
```

output
```
(a^2*Sqrt[a*Cos[x]^2]*(15 - 10*Sin[x]^2 + 3*Sin[x]^4)*Tan[x])/15
```

3.116.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {3042, 3655, 3042, 3682, 3042, 3682, 3042, 3686, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sin^2(x)\right)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin(x)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \left(a\cos^2(x)\right)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{4}{5}a\int \left(a\cos^2(x)\right)^{3/2}dx+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\int \left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}dx+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3682}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\int \sqrt{a\cos^2(x)}dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}\right)+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}\right)+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3686}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\sec(x)\sqrt{a\cos^2(x)}\int \cos(x)dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}\right)+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\sec(x)\sqrt{a\cos^2(x)}\int\sin\left(x+\frac{\pi}{2}\right)dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}\right)+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}+\frac{4}{5}a\left(\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}+\frac{2}{3}a\tan(x)\sqrt{a\cos^2(x)}\right)$$

input
```
Int[(a - a*Sin[x]^2)^(5/2),x]
```

output
```
((a*Cos[x]^2)^(5/2)*Tan[x])/5 + (4*a*((2*a*Sqrt[a*Cos[x]^2]*Tan[x])/3 + ((
a*Cos[x]^2)^(3/2)*Tan[x])/3))/5
```

3.116.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.116.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result
default	$\frac{\cos(x)a^3\sin(x)(3(\cos^4(x))+4(\cos^2(x))+8)}{15\sqrt{a(\cos^2(x))}}$
risch	$-\frac{ia^2e^{6ix}\sqrt{a(e^{2ix}+1)^2e^{-2ix}}}{160(e^{2ix}+1)}-\frac{5ia^2e^{2ix}\sqrt{a(e^{2ix}+1)^2e^{-2ix}}}{16(e^{2ix}+1)}+\frac{5ia^2\sqrt{a(e^{2ix}+1)^2e^{-2ix}}}{16(e^{2ix}+1)}+\frac{5ia^2e^{-2ix}\sqrt{a(e^{2ix}+1)^2e^{-2ix}}}{96(e^{2ix}+1)}-$ 11

input
```
int((a-a*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*cos(x)*a^3*sin(x)*(3*cos(x)^4+4*cos(x)^2+8)/(a*cos(x)^2)^(1/2)
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \left(a-a\sin^2(x)\right)^{5/2}\,dx=\frac{\left(3\,a^2\cos\left(x\right)^4+4\,a^2\cos\left(x\right)^2+8\,a^2\right)\sqrt{a\cos\left(x\right)^2}\sin\left(x\right)}{15\,\cos\left(x\right)}$$

input
```
integrate((a-a*sin(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(3*a^2*cos(x)^4 + 4*a^2*cos(x)^2 + 8*a^2)*sqrt(a*cos(x)^2)*sin(x)/cos
(x)
```

3.116.6 Sympy [F(-1)]

Timed out.

$$\int \left(a-a\sin^2(x)\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a-a*sin(x)**2)**(5/2),x)
```

output
```
Timed out
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \left(a-a\sin^2(x)\right)^{5/2}\,dx=\frac{1}{240}\left(3\,a^2\sin\left(5\,x\right)+25\,a^2\sin\left(3\,x\right)+150\,a^2\sin\left(x\right)\right)\sqrt{a}$$

input
```
integrate((a-a*sin(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/240*(3*a^2*sin(5*x) + 25*a^2*sin(3*x) + 150*a^2*sin(x))*sqrt(a)
```

3.116.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(41) = 82.

Time = 0.38 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.58

$$\int \left(a-a\sin^2(x)\right)^{5/2}\,dx=$$
$$-\frac{2\left(15\,a^{\frac{5}{2}}\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^4\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^4-1\right)-40\,a^{\frac{5}{2}}\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^4-1\right)+\right.}{15\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^5}$$

input
```
integrate((a-a*sin(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-2/15*(15*a^(5/2)*(1/tan(1/2*x) + tan(1/2*x))^4*sgn(tan(1/2*x)^4 - 1) - 40
*a^(5/2)*(1/tan(1/2*x) + tan(1/2*x))^2*sgn(tan(1/2*x)^4 - 1) + 48*a^(5/2)*
sgn(tan(1/2*x)^4 - 1))/(1/tan(1/2*x) + tan(1/2*x))^5
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \left(a-a\sin^2(x)\right)^{5/2}\,dx=\int \left(a-a\sin\left(x\right)^2\right)^{5/2}\,dx$$

input
```
int((a - a*sin(x)^2)^(5/2),x)
```

output
```
int((a - a*sin(x)^2)^(5/2), x)
```

3.117 $\int (a - a\sin^2(x))^{3/2}\, dx$

3.117.1 Optimal result

Integrand size = 13, antiderivative size = 34

$$\int (a - a\sin^2(x))^{3/2}\, dx = \frac{2}{3}a\sqrt{a\cos^2(x)}\tan(x) + \frac{1}{3}(a\cos^2(x))^{3/2}\tan(x)$$

output
```
1/3*(a*cos(x)^2)^(3/2)*tan(x)+2/3*a*(a*cos(x)^2)^(1/2)*tan(x)
```

3.117.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int (a - a\sin^2(x))^{3/2}\, dx = -\frac{1}{3}a\sqrt{a\cos^2(x)}(-3 + \sin^2(x))\tan(x)$$

input
```
Integrate[(a - a*Sin[x]^2)^(3/2),x]
```

output
```
-1/3*(a*Sqrt[a*Cos[x]^2]*(-3 + Sin[x]^2)*Tan[x])
```

3.117.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3042, 3655, 3042, 3682, 3042, 3686, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\sin^2(x)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin(x)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \left(a\cos^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{2}{3}a\int \sqrt{a\cos^2(x)}dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3686}$$

$$\frac{2}{3}a\sec(x)\sqrt{a\cos^2(x)}\int \cos(x)dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a\sec(x)\sqrt{a\cos^2(x)}\int \sin\left(x+\frac{\pi}{2}\right)dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}+\frac{2}{3}a\tan(x)\sqrt{a\cos^2(x)}$$

input
```
Int[(a - a*Sin[x]^2)^(3/2),x]
```

output
```
(2*a*Sqrt[a*Cos[x]^2]*Tan[x])/3 + ((a*Cos[x]^2)^(3/2)*Tan[x])/3
```

3.117.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.117.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{\cos(x) a^{2} \sin(x) \left(\sin^{2}(x)-3\right)}{3 \sqrt{a \left(\cos^{2}(x)\right)}}$	24
risch	$-\frac{i a \,{\mathrm e}^{4 i x} \sqrt{a \left({\mathrm e}^{2 i x}+1\right)^{2} {\mathrm e}^{-2 i x}}}{24 \left({\mathrm e}^{2 i x}+1\right)}-\frac{3 i a \,{\mathrm e}^{2 i x} \sqrt{a \left({\mathrm e}^{2 i x}+1\right)^{2} {\mathrm e}^{-2 i x}}}{8 \left({\mathrm e}^{2 i x}+1\right)}+\frac{3 i a \sqrt{a \left({\mathrm e}^{2 i x}+1\right)^{2} {\mathrm e}^{-2 i x}}}{8 \left({\mathrm e}^{2 i x}+1\right)}+\frac{i a \,{\mathrm e}^{-2 i x} \sqrt{a \left({\mathrm e}^{2 i x}+1\right)^{2} {\mathrm e}^{-2 i x}}}{24 \,{\mathrm e}^{2 i x}+24}$	141

input
```
int((a-a*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*cos(x)*a^2*sin(x)*(sin(x)^2-3)/(a*cos(x)^2)^(1/2)
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \left(a-a\sin^2(x)\right)^{3/2}\,dx=\frac{\left(a\cos\left(x\right)^{2}+2\,a\right)\sqrt{a\cos\left(x\right)^{2}}\sin\left(x\right)}{3\,\cos\left(x\right)}$$

input
```
integrate((a-a*sin(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(a*cos(x)^2 + 2*a)*sqrt(a*cos(x)^2)*sin(x)/cos(x)
```

3.117.6 Sympy [F]

$$\int \left(a-a\sin^2(x)\right)^{3/2}\,dx=\int \left(-a\sin^{2}\left(x\right)+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a-a*sin(x)**2)**(3/2),x)
```

output
```
Integral((-a*sin(x)**2 + a)**(3/2), x)
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.50

$$\int \left(a-a\sin^2(x)\right)^{3/2}\,dx=\frac{1}{12}\left(a\sin\left(3\,x\right)+9\,a\sin\left(x\right)\right)\sqrt{a}$$

input
```
integrate((a-a*sin(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/12*(a*sin(3*x) + 9*a*sin(x))*sqrt(a)
```

3.117.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int \left(a-a\sin^2(x)\right)^{3/2}\,dx=-\frac{2\left(3\,a^{\frac{3}{2}}\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^{4}-1\right)-4\,a^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^{4}-1\right)\right)}{3\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^{3}}$$

input
```
integrate((a-a*sin(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-2/3*(3*a^(3/2)*(1/tan(1/2*x) + tan(1/2*x))^2*sgn(tan(1/2*x)^4 - 1) - 4*a^
(3/2)*sgn(tan(1/2*x)^4 - 1))/(1/tan(1/2*x) + tan(1/2*x))^3
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \left(a-a\sin^2(x)\right)^{3/2}\,dx=\int {\left(a-a\,{\sin\left(x\right)}^2\right)}^{3/2}\,dx$$

input
```
int((a - a*sin(x)^2)^(3/2),x)
```

output
```
int((a - a*sin(x)^2)^(3/2), x)
```

3.118 $\int \sqrt{a-a\sin^2(x)}\,dx$

3.118.1 Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \sqrt{a-a\sin^2(x)}\,dx=\sqrt{a\cos^2(x)}\tan(x)$$

output
```
(a*cos(x)^2)^(1/2)*tan(x)
```

3.118.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{a-a\sin^2(x)}\,dx=\sqrt{a\cos^2(x)}\tan(x)$$

input
```
Integrate[Sqrt[a - a*Sin[x]^2],x]
```

output
```
Sqrt[a*Cos[x]^2]*Tan[x]
```

3.118.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 3655, 3042, 3686, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-a\sin^2(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sqrt{a-a\sin(x)^2}dx$$
$$\Big\downarrow \text{ 3655}$$
$$\int \sqrt{a\cos^2(x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}dx$$
$$\Big\downarrow \text{ 3686}$$
$$\sec(x)\sqrt{a\cos^2(x)}\int\cos(x)dx$$
$$\Big\downarrow \text{ 3042}$$
$$\sec(x)\sqrt{a\cos^2(x)}\int\sin\left(x+\frac{\pi}{2}\right)dx$$
$$\Big\downarrow \text{ 3117}$$
$$\tan(x)\sqrt{a\cos^2(x)}$$

input
```
Int[Sqrt[a - a*Sin[x]^2],x]
```

output
```
Sqrt[a*Cos[x]^2]*Tan[x]
```

3.118.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.118.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{a\cos(x)\sin(x)}{\sqrt{a(\cos^2(x))}}$	15
risch	$-\frac{i\sqrt{a(e^{2ix}+1)^2 e^{-2ix}}\, e^{2ix}}{2(e^{2ix}+1)}+\frac{i\sqrt{a(e^{2ix}+1)^2 e^{-2ix}}}{2\, e^{2ix}+2}$	67

input
```
int((a-a*sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*cos(x)*sin(x)/(a*cos(x)^2)^(1/2)
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sqrt{a - a\sin^2(x)}\, dx = \frac{\sqrt{a\cos{(x)}^2}\sin{(x)}}{\cos{(x)}}$$

input
```
integrate((a-a*sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*cos(x)^2)*sin(x)/cos(x)
```

3.118.6 Sympy [F]

$$\int \sqrt{a - a\sin^2(x)}\, dx = \int \sqrt{-a\sin^2{(x)} + a}\, dx$$

input
```
integrate((a-a*sin(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(-a*sin(x)**2 + a), x)
```

3.118.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.46

$$\int \sqrt{a - a\sin^2(x)}\, dx = \sqrt{a}\sin{(x)}$$

input
```
integrate((a-a*sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a)*sin(x)
```

3.118.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int \sqrt{a-a\sin^2(x)}\,dx = -\frac{2\sqrt{a}\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^4-1\right)}{\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)}$$

input
```
integrate((a-a*sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(a)*sgn(tan(1/2*x)^4 - 1)/(1/tan(1/2*x) + tan(1/2*x))
```

3.118.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.54

$$\int \sqrt{a-a\sin^2(x)}\,dx = \frac{\sqrt{2}\,\sqrt{a}\,\sqrt{\cos(2\,x)+1}\,(\cos(2\,x)-1+\sin(2\,x)\;1\mathrm{i})}{2\,\left(\cos(2\,x)\;1\mathrm{i}-\sin(2\,x)+1\mathrm{i}\right)}$$

input
```
int((a - a*sin(x)^2)^(1/2),x)
```

output
```
(2^(1/2)*a^(1/2)*(cos(2*x) + 1)^(1/2)*(cos(2*x) + sin(2*x)*1i - 1))/(2*(co
s(2*x)*1i - sin(2*x) + 1i))
```

3.119 $\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx$

3.119.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx = \frac{\operatorname{arctanh}(\sin(x))\cos(x)}{\sqrt{a\cos^2(x)}}$$

output
```
arctanh(sin(x))*cos(x)/(a*cos(x)^2)^(1/2)
```

3.119.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx = \frac{\operatorname{arctanh}(\sin(x))\cos(x)}{\sqrt{a\cos^2(x)}}$$

input
```
Integrate[1/Sqrt[a - a*Sin[x]^2],x]
```

output
```
(ArcTanh[Sin[x]]*Cos[x])/Sqrt[a*Cos[x]^2]
```

3.119.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 3655, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-a\sin^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a-a\sin(x)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{1}{\sqrt{a\cos^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cos(x)\int \sec(x)dx}{\sqrt{a\cos^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(x)\int \csc\left(x+\frac{\pi}{2}\right)dx}{\sqrt{a\cos^2(x)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cos(x)\text{arctanh}(\sin(x))}{\sqrt{a\cos^2(x)}}$$

input
```
Int[1/Sqrt[a - a*Sin[x]^2],x]
```

output
```
(ArcTanh[Sin[x]]*Cos[x])/Sqrt[a*Cos[x]^2]
```

3.119.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.119.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

method	result	size
default	$\frac{\operatorname{am}^{-1}(x\|1)}{\sec(x)\sqrt{a(\cos^2(x))}\,\operatorname{csgn}(\cos(x))}$	22
risch	$-\frac{2\ln(e^{ix}-i)\cos(x)}{\sqrt{a(e^{2ix}+1)^2 e^{-2ix}}}+\frac{2\ln(e^{ix}+i)\cos(x)}{\sqrt{a(e^{2ix}+1)^2 e^{-2ix}}}$	64

input
```
int(1/(a-a*sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/sec(x)/(a*cos(x)^2)^(1/2)/csgn(cos(x))*InverseJacobiAM(x,1)
```

3.119.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 4.06

$$\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx = \left[-\frac{\sqrt{a\cos(x)^2}\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)}{2\,a\cos(x)}, -\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{a\cos(x)^2}\sqrt{-a}\sin(x)}{a\cos(x)}\right)}{a}\right]$$

input
```
integrate(1/(a-a*sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(a*cos(x)^2)*log(-(sin(x) - 1)/(sin(x) + 1))/(a*cos(x)), -sqrt(-
a)*arctan(sqrt(a*cos(x)^2)*sqrt(-a)*sin(x)/(a*cos(x)))/a]
```

3.119.6 Sympy [F]

$$\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{-a\sin^2(x)+a}}\,dx$$

input
```
integrate(1/(a-a*sin(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(-a*sin(x)**2 + a), x)
```

3.119.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(14) = 28$.

Time = 0.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx = \frac{\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)-\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)}{2\sqrt{a}}$$

input
```
integrate(1/(a-a*sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2
*sin(x) + 1))/sqrt(a)
```

3.119.8 Giac [F]

$$\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{-a\sin{(x)}^2+a}}\,dx$$

input
```
integrate(1/(a-a*sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-a*sin(x)^2 + a), x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx = \int \frac{1}{\sqrt{a-a\sin{(x)}^2}}\,dx$$

input
```
int(1/(a - a*sin(x)^2)^(1/2),x)
```

output
```
int(1/(a - a*sin(x)^2)^(1/2), x)
```

3.120 $\int \frac{1}{(a-a\sin^2(x))^{3/2}} \, dx$

3.120.1 Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \frac{1}{(a-a\sin^2(x))^{3/2}} \, dx = \frac{\operatorname{arctanh}(\sin(x))\cos(x)}{2a\sqrt{a\cos^2(x)}} + \frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

output
```
1/2*arctanh(sin(x))*cos(x)/a/(a*cos(x)^2)^(1/2)+1/2*tan(x)/a/(a*cos(x)^2)^
(1/2)
```

3.120.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a-a\sin^2(x))^{3/2}} \, dx = \frac{\operatorname{arctanh}(\sin(x))\cos(x)+\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

input
```
Integrate[(a - a*Sin[x]^2)^(-3/2),x]
```

output
```
(ArcTanh[Sin[x]]*Cos[x] + Tan[x])/(2*a*Sqrt[a*Cos[x]^2])
```

3.120.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3042, 3655, 3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a-a\sin(x)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3655}$$

$$\int \frac{1}{\left(a\cos^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3683}$$

$$\frac{\int \frac{1}{\sqrt{a\cos^2(x)}}dx}{2a}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}}dx}{2a}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

$$\Big\downarrow \text{ 3686}$$

$$\frac{\cos(x)\int \sec(x)dx}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cos(x)\int \csc\left(x+\frac{\pi}{2}\right)dx}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\cos(x)\text{arctanh}(\sin(x))}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

input
```
Int[(a - a*Sin[x]^2)^(-3/2),x]
```

output
```
(ArcTanh[Sin[x]]*Cos[x])/(2*a*Sqrt[a*Cos[x]^2]) + Tan[x]/(2*a*Sqrt[a*Cos[x
]^2])
```

3.120.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.120.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{(-\ln(1+\sin(x))+\ln(\sin(x)-1))(\cos^2(x))-2\sin(x)}{4a\cos(x)\sqrt{a(\cos^2(x))}}$	41
risch	$-\frac{i(\mathrm{e}^{2ix}-1)}{a(\mathrm{e}^{2ix}+1)\sqrt{a(\mathrm{e}^{2ix}+1)^2\mathrm{e}^{-2ix}}}-\frac{\ln(\mathrm{e}^{ix}-i)\cos(x)}{a\sqrt{a(\mathrm{e}^{2ix}+1)^2\mathrm{e}^{-2ix}}}+\frac{\ln(\mathrm{e}^{ix}+i)\cos(x)}{a\sqrt{a(\mathrm{e}^{2ix}+1)^2\mathrm{e}^{-2ix}}}$	109

input
```
int(1/(a-a*sin(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a*((-ln(1+sin(x))+ln(sin(x)-1))*cos(x)^2-2*sin(x))/cos(x)/(a*cos(x)^2
)^(1/2)
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a-a\sin^2(x))^{3/2}}\,dx = -\frac{\sqrt{a\cos(x)^2}\left(\cos(x)^2\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)-2\sin(x)\right)}{4a^2\cos(x)^3}$$

input
```
integrate(1/(a-a*sin(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(a*cos(x)^2)*(cos(x)^2*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*sin(x)
)/(a^2*cos(x)^3)
```

3.120.6 Sympy [F]

$$\int \frac{1}{(a-a\sin^2(x))^{3/2}}\,dx = \int \frac{1}{(-a\sin^2(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a-a*sin(x)**2)**(3/2),x)
```

output
```
Integral((-a*sin(x)**2 + a)**(-3/2), x)
```

3.120.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(34) = 68.

Time = 0.42 (sec) , antiderivative size = 304, normalized size of antiderivative = 7.24

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^{3/2}}\,dx = \frac{4\left(\sin\left(3\,x\right)-\sin\left(x\right)\right)\cos\left(4\,x\right)+\left(2\left(2\,\cos\left(2\,x\right)+1\right)\cos\left(4\,x\right)+\cos\left(4\,x\right)^2+4\,\cos\right.}{}$$

input
```
integrate(1/(a-a*sin(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(4*(sin(3*x) - sin(x))*cos(4*x) + (2*(2*cos(2*x) + 1)*cos(4*x) + cos(4
*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4
*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - (2*(2*cos(2*x) +
1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x)
 + 4*sin(2*x)^2 + 4*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
- 4*(cos(3*x) - cos(x))*sin(4*x) + 4*(2*cos(2*x) + 1)*sin(3*x) - 8*cos(3*x
)*sin(2*x) + 8*cos(x)*sin(2*x) - 8*cos(2*x)*sin(x) - 4*sin(x))/((a*cos(4*x
)^2 + 4*a*cos(2*x)^2 + a*sin(4*x)^2 + 4*a*sin(4*x)*sin(2*x) + 4*a*sin(2*x)
^2 + 2*(2*a*cos(2*x) + a)*cos(4*x) + 4*a*cos(2*x) + a)*sqrt(a))
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^{3/2}}\,dx = -\frac{\frac{\log\left(\left|-\sqrt{a}\tan(x)+\sqrt{a\tan(x)^2+a}\right|\right)}{\sqrt{a}}-\frac{\sqrt{a\tan(x)^2+a}\tan(x)}{a}}{2\,a}$$

input
```
integrate(1/(a-a*sin(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(log(abs(-sqrt(a)*tan(x) + sqrt(a*tan(x)^2 + a)))/sqrt(a) - sqrt(a*ta
n(x)^2 + a)*tan(x)/a)/a
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a - a\sin^2(x))^{3/2}}\, dx = \int \frac{1}{\left(a - a\sin{(x)}^2\right)^{3/2}}\, dx$$

input
```
int(1/(a - a*sin(x)^2)^(3/2),x)
```

output
```
int(1/(a - a*sin(x)^2)^(3/2), x)
```

3.121 $\int \frac{1}{(a-a\sin^2(x))^{5/2}}\,dx$

3.121.1 Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{1}{(a-a\sin^2(x))^{5/2}}\,dx = \frac{3\text{arctanh}(\sin(x))\cos(x)}{8a^2\sqrt{a\cos^2(x)}} + \frac{\tan(x)}{4a\,(a\cos^2(x))^{3/2}} + \frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}$$

output
```
3/8*arctanh(sin(x))*cos(x)/a^2/(a*cos(x)^2)^(1/2)+1/4*tan(x)/a/(a*cos(x)^2
)^(3/2)+3/8*tan(x)/a^2/(a*cos(x)^2)^(1/2)
```

3.121.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59

$$\int \frac{1}{(a-a\sin^2(x))^{5/2}}\,dx = \frac{3\text{arctanh}(\sin(x))\cos(x) + (3+2\sec^2(x))\tan(x)}{8a^2\sqrt{a\cos^2(x)}}$$

input
```
Integrate[(a - a*Sin[x]^2)^(-5/2),x]
```

output
```
(3*ArcTanh[Sin[x]]*Cos[x] + (3 + 2*Sec[x]^2)*Tan[x])/(8*a^2*Sqrt[a*Cos[x]^
2])
```

3.121.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {3042, 3655, 3042, 3683, 3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-a\sin^2(x))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a-a\sin(x)^2)^{5/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{1}{(a\cos^2(x))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3683}$$

$$\frac{3\int \frac{1}{(a\cos^2(x))^{3/2}}dx}{4a}+\frac{\tan(x)}{4a(a\cos^2(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{\left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}}dx}{4a}+\frac{\tan(x)}{4a(a\cos^2(x))^{3/2}}$$

$$\Big\downarrow \text{3683}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{a\cos^2(x)}}dx}{2a}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}\right)}{4a}+\frac{\tan(x)}{4a(a\cos^2(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}}dx}{2a}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}\right)}{4a}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

$\Big\downarrow$ 3686

$$\frac{3\left(\frac{\cos(x)\int \sec(x)dx}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}\right)}{4a}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{\cos(x)\int \csc\left(x+\frac{\pi}{2}\right)dx}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}\right)}{4a}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

$\Big\downarrow$ 4257

$$\frac{3\left(\frac{\cos(x)\text{arctanh}(\sin(x))}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}\right)}{4a}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

input
```
Int[(a - a*Sin[x]^2)^(-5/2),x]
```

output
```
Tan[x]/(4*a*(a*Cos[x]^2)^(3/2)) + (3*((ArcTanh[Sin[x]]*Cos[x])/(2*a*Sqrt[a
*Cos[x]^2]) + Tan[x]/(2*a*Sqrt[a*Cos[x]^2])))/(4*a)
```

3.121.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.121.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{(3\ln(1+\sin(x))-3\ln(\sin(x)-1))(\cos^4(x))+6(\cos^2(x))\sin(x)+4\sin(x)}{16a^2(1+\sin(x))(\sin(x)-1)\cos(x)\sqrt{a(\cos^2(x))}}$	63
risch	$-\frac{i(3\,\mathrm{e}^{6ix}+11\,\mathrm{e}^{4ix}-11\,\mathrm{e}^{2ix}-3)}{4a^2(\mathrm{e}^{2ix}+1)^3\sqrt{a(\mathrm{e}^{2ix}+1)^2\mathrm{e}^{-2ix}}}+\frac{3\ln(\mathrm{e}^{ix}+i)\cos(x)}{4a^2\sqrt{a(\mathrm{e}^{2ix}+1)^2\mathrm{e}^{-2ix}}}-\frac{3\ln(\mathrm{e}^{ix}-i)\cos(x)}{4a^2\sqrt{a(\mathrm{e}^{2ix}+1)^2\mathrm{e}^{-2ix}}}$	126

input
```
int(1/(a-a*sin(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/a^2*((3*ln(1+sin(x))-3*ln(sin(x)-1))*cos(x)^4+6*cos(x)^2*sin(x)+4*si
n(x))/(1+sin(x))/(sin(x)-1)/cos(x)/(a*cos(x)^2)^(1/2)
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a-a\sin^2(x))^{5/2}}\,dx = -\frac{\left(3\cos(x)^4\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)-2\left(3\cos(x)^2+2\right)\sin(x)\right)\sqrt{a\cos(x)^2}}{16\,a^3\cos(x)^5}$$

input
```
integrate(1/(a-a*sin(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/16*(3*cos(x)^4*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*(3*cos(x)^2 + 2)*sin
(x))*sqrt(a*cos(x)^2)/(a^3*cos(x)^5)
```

3.121.6 Sympy [F]

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^{5/2}}\,dx=\int \frac{1}{\left(-a\sin^2{(x)}+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a-a*sin(x)**2)**(5/2),x)
```

output
```
Integral((-a*sin(x)**2 + a)**(-5/2), x)
```

3.121.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(49) = 98$.

Time = 0.60 (sec) , antiderivative size = 933, normalized size of antiderivative = 15.30

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/(a-a*sin(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/16*(4*(3*sin(7*x) + 11*sin(5*x) - 11*sin(3*x) - 3*sin(x))*cos(8*x) - 24*
(2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*cos(7*x) + 16*(11*sin(5*x) - 11*sin
(3*x) - 3*sin(x))*cos(6*x) - 88*(3*sin(4*x) + 2*sin(2*x))*cos(5*x) - 24*(1
1*sin(3*x) + 3*sin(x))*cos(4*x) + 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*
x) + 1)*cos(8*x) + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) +
 16*cos(6*x)^2 + 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x
)^2 + 4*(2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*
(3*sin(4*x) + 2*sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*si
n(4*x)*sin(2*x) + 16*sin(2*x)^2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2
+ 2*sin(x) + 1) - 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*cos(8*x)
 + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*x)^2 +
 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*sin(6
*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x) + 2*
sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x)
+ 16*sin(2*x)^2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
- 4*(3*cos(7*x) + 11*cos(5*x) - 11*cos(3*x) - 3*cos(x))*sin(8*x) + 12*(4*c
os(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*sin(7*x) - 16*(11*cos(5*x) - 11*cos
(3*x) - 3*cos(x))*sin(6*x) + 44*(6*cos(4*x) + 4*cos(2*x) + 1)*sin(5*x) + 2
4*(11*cos(3*x) + 3*cos(x))*sin(4*x) - 44*(4*cos(2*x) + 1)*sin(3*x) + 176*c
os(3*x)*sin(2*x) + 48*cos(x)*sin(2*x) - 48*cos(2*x)*sin(x) - 12*sin(x))...
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a-a\sin^2(x))^{5/2}}\,dx = -\frac{5\left(\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)\right)^3-\frac{12}{\tan\left(\frac{1}{2}x\right)}-12\tan\left(\frac{1}{2}x\right)}{4\left(\left(\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)\right)^2-4\right)^2a^{\frac{5}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^4-1\right)}$$

input
```
integrate(1/(a-a*sin(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/4*(5*(1/tan(1/2*x) + tan(1/2*x))^3 - 12/tan(1/2*x) - 12*tan(1/2*x))/(((
1/tan(1/2*x) + tan(1/2*x))^2 - 4)^2*a^(5/2)*sgn(tan(1/2*x)^4 - 1))
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-a\sin^2(x))^{5/2}}\,dx = \int \frac{1}{\left(a-a\sin(x)^2\right)^{5/2}}\,dx$$

input
```
int(1/(a - a*sin(x)^2)^(5/2),x)
```

output
```
int(1/(a - a*sin(x)^2)^(5/2), x)
```

3.122 $\int \sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.122.1 Optimal result

Integrand size = 25, antiderivative size = 125

$$\int \sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{(a-3b)(a+b)\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8b^{3/2}f} + \frac{(a-3b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{8bf} - \frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{3/2}}{4bf}$$

output
```
1/8*(a-3*b)*(a+b)*arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/b^
(3/2)/f-1/4*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(3/2)/b/f+1/8*(a-3*b)*cos(f*x+
e)*(a+b-b*cos(f*x+e)^2)^(1/2)/b/f
```

3.122.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.95

$$\int \sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\frac{\cos(e+fx)\sqrt{2a+b-b\cos(2(e+fx))}(-a-4b+b\cos(2(e+fx)))}{\sqrt{2}b} + \frac{(a+b)(-a+3b)\log\left(\sqrt{2}\sqrt{-b}\cos(e+fx)+\sqrt{2a+b-b\cos(2(e+fx))}\right)}{(-b)^{3/2}}}{8f}$$

input
```
Integrate[Sin[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
((Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*(-a - 4*b + b*Cos[2*(e +
 f*x)]))/(Sqrt[2]*b) + ((a + b)*(-a + 3*b)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*
x] + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]])/(-b)^(3/2))/(8*f)
```

3.122.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3665, 299, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^3\sqrt{a+b\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)\sqrt{-b\cos^2(e+fx)+a+b}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{299}$$

$$-\frac{\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4b}-\frac{(a-3b)\int \sqrt{-b\cos^2(e+fx)+a+b}d\cos(e+fx)}{4b}}{f}$$

$$\Big\downarrow \text{211}$$

$$-\frac{\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4b}-\frac{(a-3b)\left(\frac{1}{2}(a+b)\int \frac{1}{\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)+\frac{1}{2}\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}\right)}{4b}}{f}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4b}-\frac{(a-3b)\left(\frac{1}{2}(a+b)\int \frac{1}{\frac{b\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}+1}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}+\frac{1}{2}\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}\right)}{4b}}{f}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4b}-\frac{(a-3b)\left(\frac{(a+b)\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2\sqrt{b}}+\frac{1}{2}\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}\right)}{4b}}{f}$$

input
```
Int[Sin[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-(((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(3/2))/(4*b) - ((a - 3*b)*(((a
 + b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/(2*Sq
rt[b]) + (Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/2))/(4*b))/f)
```

3.122.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.122.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(109) = 218$.

Time = 1.13 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.49

method	result
default	$-\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\left(-4\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\,b^{\frac{5}{2}}(\cos^2(fx+e))+10\sqrt{-b(\cos^4(fx+e))+(a+b)}\right.}{}$

input
```
int(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(-4*(-b*cos(f*x+e)^4+(a+b)*c
os(f*x+e)^2)^(1/2)*b^(5/2)*cos(f*x+e)^2+10*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+
e)^2)^(1/2)*b^(5/2)+2*a*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^(3/2)
+arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x
+e)^2)^(1/2))*a^2*b-2*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f
*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))*a*b^2-3*b^3*arctan(1/2*(-2*b*cos(f*x+e)
^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)))/b^(5/2)/cos(f
*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 501, normalized size of antiderivative = 4.01

$$\int \sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$= \left[\frac{(a^2-2ab-3b^2)\sqrt{-b}\log\left(128\,b^4\cos(fx+e)^8-256\,(ab^3+b^4)\cos(fx+e)^6+160\,(a^2b^2+2\,ab^3+b^4)\right.}{}\right.$$

$$-\frac{(a^2-2ab-3b^2)\sqrt{b}\arctan\left(\frac{\left(8\,b^2\cos(fx+e)^4-8\,(ab+b^2)\cos(fx+e)^2+a^2+2\,ab+b^2\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{b}}{4\left(2\,b^3\cos(fx+e)^5-3\,(ab^2+b^3)\cos(fx+e)^3+(a^2b+2\,ab^2+b^3)\cos(fx+e)\right)}\right)-4\,(2\,b^2\,c}{32\,b^2f}$$

input
```
integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/64*((a^2 - 2*a*b - 3*b^2)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*
b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 +
 a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b
^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos
(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) +
8*(2*b^2*cos(f*x + e)^3 - (a*b + 5*b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)
^2 + a + b))/(b^2*f), -1/32*((a^2 - 2*a*b - 3*b^2)*sqrt(b)*arctan(1/4*(8*b
^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3
)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) - 4*(2*b^2*cos(f
*x + e)^3 - (a*b + 5*b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(
b^2*f)]
```

3.122.6 Sympy [F(-1)]

Timed out.

$$\int \sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.122.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.41

$$\int \sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$= \frac{\frac{(a+b)a\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} + \frac{(a+b)\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} - \frac{4\,a\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} - 4\sqrt{b}\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right) - 4\sqrt{-b\cos}}{8\,f}$$

input
```
integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/8*((a + b)*a*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/b^(3/2) + (a + b)*ar
csin(b*cos(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 4*a*arcsin(b*cos(f*x + e)/s
qrt((a + b)*b))/sqrt(b) - 4*sqrt(b)*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))
 - 4*sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e) - 2*(-b*cos(f*x + e)^2 +
 a + b)^(3/2)*cos(f*x + e)/b + sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*cos
(f*x + e)/b)/f
```

3.122.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2404 vs. 2(109) = 218.

Time = 0.50 (sec) , antiderivative size = 2404, normalized size of antiderivative = 19.23

$$\int \sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/4*((a^2 - 2*a*b - 3*b^2)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f
*x + 1/2*e)^2 + a) + sqrt(a))/sqrt(b))/b^(3/2) - 2*((sqrt(a)*tan(1/2*f*x +
 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4
*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a^2 - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
- sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2
*f*x + 1/2*e)^2 + a))^7*a*b - 3*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*t
an(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2
*e)^2 + a))^7*b^2 + 7*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)
)^6*a^(5/2) + 18*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*a
^(3/2)*b - 21*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt
(a)*b^2 + 21*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)
^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^3 +
 122*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a
*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^2*b + 121*(
sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1
/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*b^2 - 44*(sqrt...
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sin(e+f\,x)^3\,\sqrt{b\sin(e+f\,x)^2+a}\,dx$$

input
```
int(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.123 $\int \sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.123.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{(a+b)\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2\sqrt{b}f} - \frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f}$$

output
```
-1/2*(a+b)*arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f/b^(1/2)
-1/2*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/f
```

3.123.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19

$$\int \sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\sqrt{2}\cos(e+fx)\sqrt{2a+b-b\cos(2(e+fx))}+\frac{2(a+b)\log\left(\sqrt{2}\sqrt{-b}\cos(e+fx)+\sqrt{2a+b-b\cos(2(e+fx))}\right)}{\sqrt{-b}}}{4f}$$

input
```
Integrate[Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-1/4*(Sqrt[2]*Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]] + (2*(a + b)
*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]])/
Sqrt[-b])/f
```

3.123.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3665, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)\sqrt{a+b\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \sqrt{-b\cos^2(e+fx)+a+b}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{211}$$

$$-\frac{\frac{1}{2}(a+b)\int \frac{1}{\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)+\frac{1}{2}\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{f}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{1}{2}(a+b)\int \frac{1}{\frac{b\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}+1}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}+\frac{1}{2}\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{f}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{(a+b)\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2\sqrt{b}}+\frac{1}{2}\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{f}$$

input
```
Int[Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-((((a + b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])
/(2*Sqrt[b]) + (Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/2)/f)
```

3.123.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.123.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(66) = 132.

Time = 0.78 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.36

method	result
default	$-\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\left(-b\arctan\left(\frac{-2b\left(\cos^2(fx+e)\right)+a+b}{2\sqrt{b}\sqrt{-b\left(\cos^4(fx+e)\right)+(a+b)\left(\cos^2(fx+e)\right)}}\right)+2\sqrt{b}\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos}\right.}{4\sqrt{b}\cos(fx+e)\sqrt{a+b(\sin^2(fx+e))}\,f}$

input
```
int(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(-b*arctan(1/2*(-2*b*cos(f*x+
e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))+2*b^(1/2)*(-
b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)-arctan(1/2*(-2*b*cos(f*x+e)^2+a+b
)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))*a)/b^(1/2)/cos(f*x+e
)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.123.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(66) = 132$.

Time = 0.37 (sec) , antiderivative size = 433, normalized size of antiderivative = 5.55

$$\int \sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$= \left[-\frac{8\sqrt{-b\cos(fx+e)^2+a+b}\,b\cos(fx+e)+(a+b)\sqrt{-b}\log\left(128\,b^4\cos(fx+e)^8-256\,(ab^3+b^4)\cos\right.}{}\right.$$

input
```
integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) + (a + b)*sqrt(-b
)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2
*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3
*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 +
b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-
b*cos(f*x + e)^2 + a + b)*sqrt(-b)))/(b*f), 1/8*((a + b)*sqrt(b)*arctan(1/
4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2
)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2
 + b^3)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) - 4*sqrt(-
b*cos(f*x + e)^2 + a + b)*b*cos(f*x + e))/(b*f)]
```

3.123.6 Sympy [F]

$$\int \sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2{(e+fx)}}\sin{(e+fx)}\,dx$$

input
```
integrate(sin(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*sin(e + f*x), x)
```

3.123.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$= -\frac{\frac{a\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} + \sqrt{b}\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right) + \sqrt{-b\cos{(fx+e)}^2+a+b}\cos{(fx+e)}}{2f}$$

input
```
integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(a*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/sqrt(b) + sqrt(b)*arcsin(b*
cos(f*x + e)/sqrt((a + b)*b)) + sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x +
e))/f
```

3.123.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 723 vs. 2(66) = 132.

Time = 0.41 (sec) , antiderivative size = 723, normalized size of antiderivative = 9.27

$$\int \sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
((a + b)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)
+ sqrt(a))/sqrt(b))/sqrt(b) + 2*((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/
2*e)^2 + a))^3*a - (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3
*b + 3*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2
*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2) + 5
*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan
(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b + 3*(sq
rt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2
*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2 + 9*(sqrt(a)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e
)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b + 4*(sqrt(a)*tan(1/2*f*x + 1/2*
e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*ta
n(1/2*f*x + 1/2*e)^2 + a))*b^2 + a^(5/2) + 3*a^(3/2)*b + 4*sqrt(a)*b^2)/((
sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1
/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e
)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) + a + 4*b)^2)/f
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sin(e+f\,x)\,\sqrt{b\sin(e+f\,x)^2+a}\,dx$$

input
```
int(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.124 $\int \csc(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.124.1 Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \csc(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f} - \frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f}$$

output
```
-arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))*a^(1/2)/f-arctan(c
os(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))*b^(1/2)/f
```

3.124.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.19

$$\int \csc(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)+\sqrt{-b}\log\left(\sqrt{2}\sqrt{-b}\cos(e+fx)+\sqrt{2a+b-b\cos(2(e+fx))}\right)}{f}$$

input
```
Integrate[Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-(Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(
e + f*x)]]]) + Sqrt[-b]*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b -
 b*Cos[2*(e + f*x)]]])/f
```

3.124.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 3665, 301, 224, 216, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin(e+fx)^2}}{\sin(e+fx)}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{\sqrt{-b\cos^2(e+fx)+a+b}}{1-\cos^2(e+fx)}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{301}$$

$$-\frac{b\int \frac{1}{\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)+a\int \frac{1}{(1-\cos^2(e+fx))\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{224}$$

$$-\frac{a\int \frac{1}{(1-\cos^2(e+fx))\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)+b\int \frac{1}{\frac{b\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}+1}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}}{f}$$

$$\Big\downarrow \text{216}$$

$$-\frac{a\int \frac{1}{(1-\cos^2(e+fx))\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)+\sqrt{b}\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}$$

$$\Big\downarrow \text{291}$$

$$-\frac{a\int \frac{1}{1-\frac{a\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}+\sqrt{b}\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)+\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}$$

input
```
Int[Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-((Sqrt[b]*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]] +
 Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/f
)
```

3.124.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.124.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(71) = 142$.

Time = 1.01 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.10

method	result
default	$\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\left(\sqrt{b}\arctan\left(\frac{-2b(\cos^2(fx+e))+a+b}{2\sqrt{b}\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}}\right)-\sqrt{a}\ln\left(\frac{-(a-b)(\cos^2(fx+e))-2\sqrt{a}\sqrt{-}}{c}\right.\right.}{2\cos(fx+e)\sqrt{a+b(\sin^2(fx+e))}\,f}$

input
```
int(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(b^(1/2)*arctan(1/2*(-2*b*cos(
f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))-a^(1/2)*
ln((-(a-b)*cos(f*x+e)^2-2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/
2)-a-b)/(cos(f*x+e)^2-1)))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.124.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(71) = 142$.

Time = 0.48 (sec) , antiderivative size = 1158, normalized size of antiderivative = 13.95

$$\int \csc(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)
^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^
2 - 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b
 + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x
 + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) + 2*sqrt(a)*log(2*((a^2 -
 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*
((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a
 + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)
))/f, 1/8*(4*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b
*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*
x + e))) + sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x
 + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6
*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x
+ e)^2 - 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(
a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*co
s(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)))/f, 1/4*(sqrt(b)*arc
tan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b
 + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*
(a*b^2 + b^3)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) +...
```

3.124.6 Sympy [F]

$$\int \csc(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2(e+fx)}\csc(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*csc(e + f*x), x)
```

3.124.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.59

$$\int \csc(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx =$$

$$-\frac{2\sqrt{b}\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{ab+b^2}}\right)+\sqrt{a}\log\left(b-\frac{\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a}}{\cos(fx+e)-1}-\frac{a}{\cos(fx+e)-1}\right)-\sqrt{a}\log\left(-b+\frac{\sqrt{-b\cos(fx+e}}{\cos(fx+}\right.}{2f}$$

input
```
integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*sqrt(b)*arcsin(b*cos(f*x + e)/sqrt(a*b + b^2)) + sqrt(a)*log(b - s
qrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x + e) - 1) - a/(cos(f*x + e
) - 1)) - sqrt(a)*log(-b + sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*
x + e) + 1) + a/(cos(f*x + e) + 1)))/f
```

3.124.8 Giac [F(-2)]

Exception generated.

$$\int \csc(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \frac{\sqrt{b\sin(e+f\,x)^2+a}}{\sin(e+f\,x)}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x),x)
```

output
```
int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x), x)
```

3.125 $\int \csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.125.1 Optimal result

Integrand size = 25, antiderivative size = 84

$$\int \csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2\sqrt{a}f} - \frac{\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{2f}$$

output
```
-1/2*(a+b)*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f/a^(1/2
)-1/2*cot(f*x+e)*csc(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/f
```

3.125.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19

$$\int \csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$= \frac{-2(a+b)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right) - \sqrt{2}\sqrt{a}\sqrt{2a+b-b\cos(2(e+fx))}\cot(e+fx)\csc(e+fx)}{4\sqrt{a}f}$$

input
```
Integrate[Csc[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-2*(a + b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*
(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Cot[e +
f*x]*Csc[e + f*x])/(4*Sqrt[a]*f)
```

3.125.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3665, 292, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin(e+fx)^2}}{\sin(e+fx)^3}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{\sqrt{-b\cos^2(e+fx)+a+b}}{\left(1-\cos^2(e+fx)\right)^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{292}$$

$$-\frac{\frac{1}{2}(a+b)\int \frac{1}{\left(1-\cos^2(e+fx)\right)\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)+\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2\left(1-\cos^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\frac{1}{2}(a+b)\int \frac{1}{1-\frac{a\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}+\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2\left(1-\cos^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2\sqrt{a}}+\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2\left(1-\cos^2(e+fx)\right)}}{f}$$

input
```
Int[Csc[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-((((a + b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]
)/(2*Sqrt[a]) + (Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(2*(1 - Cos[
e + f*x]^2)))/f)
```

3.125.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.125.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(72) = 144$.

Time = 0.93 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.70

method	result
default	$-\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\left(a\ln\left(\frac{(a-b)\left(\cos^2(fx+e)\right)+2\sqrt{a}\sqrt{-b\left(\cos^4(fx+e)\right)+(a+b)\left(\cos^2(fx+e)\right)+a+b}}{\sin(fx+e)^2}\right)(\sin^2(fx+e))+b\,\mathrm{l}}{4\sqrt{a}\,\sin(fx+e)^2\cos(fx-}$

input
```
int(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(a*ln(((a-b)*cos(f*x+e)^2+2*a
^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f
*x+e)^2+b*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+
e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^2+2*a^(1/2)*(cos(f*x+e)^2*(a+b*s
in(f*x+e)^2))^(1/2))/a^(1/2)/sin(f*x+e)^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1
/2)/f
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 338, normalized size of antiderivative = 4.02

$$\int \csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$= \left[\frac{4\sqrt{-b\cos(fx+e)^2+a+b}a\cos(fx+e)+\left((a+b)\cos(fx+e)^2-a-b\right)\sqrt{a}\log\left(\frac{2\left((a^2-6ab+b^2)\cos(fx\right.}{}\right.}{8\left(af\cos(fx+e)^2\right.}\right.$$

input
```
integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e) + ((a + b)*cos(f*x
+ e)^2 - a - b)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a
^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos
(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(c
os(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(a*f*cos(f*x + e)^2 - a*f), 1/4*((
(a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)
^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 -
 (a^2 + a*b)*cos(f*x + e))) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x
+ e))/(a*f*cos(f*x + e)^2 - a*f)]
```

3.125.6 Sympy [F]

$$\int \csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2(e+fx)}\csc^3(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*csc(e + f*x)**3, x)
```

3.125.7 Maxima [F]

$$\int \csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin(fx+e)^2+a}\,\csc(fx+e)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^3, x)
```

3.125.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(72) = 144$.

Time = 0.49 (sec) , antiderivative size = 434, normalized size of antiderivative = 5.17

$$\int \csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$= \frac{\frac{4(a+b)\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{2\left(a^{\frac{3}{2}}+\sqrt{a}b\right)\log\left(\left|-\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\right.\right.\right.}{}}{}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*(4*(a + b)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*
x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a
))/sqrt(-a))/sqrt(-a) - 2*(a^(3/2) + sqrt(a)*b)*log(abs(-(sqrt(a)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^
2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a - a^(3/2) - 2*sqrt(a)*b))/a + sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
1/2*e)^2 + a) + 2*((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a
 + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a
*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b + a^(3/2))/((
sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1
/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a))/f
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx=\int \frac{\sqrt{b\sin\left(e+f\,x\right)^2+a}}{\sin\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^3,x)
```

output
```
int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^3, x)
```

3.126 $\int \csc^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.126.1 Optimal result

Integrand size = 25, antiderivative size = 143

$$\int \csc^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{(3a-b)(a+b)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8a^{3/2}f} - \frac{(3a-b)\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{8af} - \frac{(a+b-b\cos^2(e+fx))^{3/2}\cot(e+fx)\csc^3(e+fx)}{4af}$$

output
```
-1/8*(3*a-b)*(a+b)*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/
a^(3/2)/f-1/4*(a+b-b*cos(f*x+e)^2)^(3/2)*cot(f*x+e)*csc(f*x+e)^3/a/f-1/8*(
3*a-b)*cot(f*x+e)*csc(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/a/f
```

3.126.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int \csc^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{(-6a^2-4ab+2b^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right) - \sqrt{2}\sqrt{a}\sqrt{2a+b-b\cos(2(e+fx))}\cot(e+fx)\csc(e-}{16a^{3/2}f}$$

input
```
Integrate[Csc[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
((-6*a^2 - 4*a*b + 2*b^2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a
+ b - b*Cos[2*(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a + b - b*Cos[2*(e + f
*x)]]*Cot[e + f*x]*Csc[e + f*x]*(3*a + b + 2*a*Csc[e + f*x]^2))/(16*a^(3/2
)*f)
```

3.126.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3665, 296, 292, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{a+b\sin(e+fx)^2}}{\sin(e+fx)^5}dx$$

$$\Big\downarrow \ 3665$$

$$-\frac{\int \frac{\sqrt{-b\cos^2(e+fx)+a+b}}{(1-\cos^2(e+fx))^3}d\cos(e+fx)}{f}$$

$$\Big\downarrow \ 296$$

$$-\frac{\frac{(3a-b)\int \frac{\sqrt{-b\cos^2(e+fx)+a+b}}{\left(1-\cos^2(e+fx)\right)^2}d\cos(e+fx)}{4a}+\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4a(1-\cos^2(e+fx))^2}}{f}$$

$$\Big\downarrow \ 292$$

$$-\frac{\frac{(3a-b)\left(\frac{1}{2}(a+b)\int \frac{1}{\left(1-\cos^2(e+fx)\right)\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)+\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2\left(1-\cos^2(e+fx)\right)}\right)}{4a}+\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4a(1-\cos^2(e+fx))^2}}{f}$$

$$\Big\downarrow \ 291$$

$$-\frac{\frac{(3a-b)\left(\frac{1}{2}(a+b)\int \frac{1}{1-\frac{a\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}+\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2\left(1-\cos^2(e+fx)\right)}\right)}{4a}+\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4a(1-\cos^2(e+fx))^2}}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{(3a-b)\left(\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2\sqrt{a}}+\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2\left(1-\cos^2(e+fx)\right)}\right)}{4a}+\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4a(1-\cos^2(e+fx))^2}}{f}$$

input
```
Int[Csc[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-(((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(3/2))/(4*a*(1 - Cos[e + f*x]^
2)^2) + ((3*a - b)*(((a + b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b
*Cos[e + f*x]^2]])/(2*Sqrt[a]) + (Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]
^2])/(2*(1 - Cos[e + f*x]^2))))/(4*a))/f)
```

3.126.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.126.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(127) = 254$.

Time = 1.10 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.65

method	result
default	$-\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\left(3a^3\ln\left(\frac{(a-b)\left(\cos^2(fx+e)\right)+2\sqrt{a}\sqrt{-b\left(\cos^4(fx+e)\right)+(a+b)\left(\cos^2(fx+e)\right)}+a+b}{\sin(fx+e)^2}\right)(\sin^4(fx+e))+$

input
```
int(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(3*a^3*ln(((a-b)*cos(f*x+e)^
2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*
sin(f*x+e)^4+2*b*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*c
os(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^4*a^2-ln(((a-b)*cos(f*x+e
)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2
)*b^2*sin(f*x+e)^4*a+6*sin(f*x+e)^2*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2
)*a^(5/2)+2*sin(f*x+e)^2*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*a^(3/2)*b
+4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*a^(5/2))/sin(f*x+e)^4/a^(5/2)/c
os(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.64

$$\int \csc^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$= \left[-\frac{\left((3a^2+2ab-b^2)\cos(fx+e)^4-2(3a^2+2ab-b^2)\cos(fx+e)^2+3a^2+2ab-b^2\right)\sqrt{a}\log\left(\frac{2\left((a^2-\right.}{}\right.}{}\right.$$

input
```
integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*(((3*a^2 + 2*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 2*a*b - b^2)*co
s(f*x + e)^2 + 3*a^2 + 2*a*b - b^2)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos
(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x
+ e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a
^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((3*a^2 + a
*b)*cos(f*x + e)^3 - (5*a^2 + a*b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 +
a + b))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f), 1/16*(((3
*a^2 + 2*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^
2 + 3*a^2 + 2*a*b - b^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a
+ b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 +
 a*b)*cos(f*x + e))) + 2*((3*a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + a*b)*cos
(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^4 - 2*a^2*
f*cos(f*x + e)^2 + a^2*f)]
```

3.126.6 Sympy [F]

$$\int \csc^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2(e+fx)}\csc^5(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**5*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*csc(e + f*x)**5, x)
```

3.126.7 Maxima [F]

$$\int \csc^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin(fx+e)^2+a}\csc(fx+e)^5\,dx$$

input
```
integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^5, x)
```

3.126.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 914 vs. $2(127) = 254$.

Time = 0.57 (sec) , antiderivative size = 914, normalized size of antiderivative = 6.39

$$\int \csc^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a)*(tan(1/2*f*x + 1/2*e)^2 + (7*a + 2*b)/a) + 8*(3*a
^2 + 2*a*b - b^2)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
+ a))/sqrt(-a))/(sqrt(-a)*a) - 4*(3*a^(5/2) + 2*a^(3/2)*b - sqrt(a)*b^2)*l
og(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a - a^(3/2)
- 2*sqrt(a)*b))/a^2 + 4*(4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/
2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
 + a))^3*a^2 + 8*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a
*b + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2
*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2 + 5*(sq
rt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2
*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2) + 4*(sqrt(a)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x +
 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b - 2*(sqrt(a)*tan(
1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2
*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3 - 4*(sqrt(a)*tan(1/2*f*x + 1/
2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4...
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \csc^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx=\int \frac{\sqrt{b\sin\left(e+f\,x\right)^2+a}}{\sin\left(e+f\,x\right)^5}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^5,x)
```

output
```
int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^5, x)
```

3.127 $\int \sin^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.127.1 Optimal result

Integrand size = 25, antiderivative size = 259

$$\begin{aligned}
&\int \sin^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx \\
&= -\frac{(a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} \\
&\quad -\frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{5f} \\
&\quad -\frac{(2a^2-3ab-8b^2)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15b^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} \\
&\quad +\frac{2a(a-2b)(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{15b^2f\sqrt{a+b\sin^2(e+fx)}}
\end{aligned}$$

output
```
-1/15*(a+4*b)*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f-1/5*cos(f
*x+e)*sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2)/f-1/15*(2*a^2-3*a*b-8*b^2)*Ell
ipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f
*x+e)^2)^(1/2)/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+2/15*a*(a-2*b)*(a+b)*Ellip
ticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x
+e)^2/a)^(1/2)/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.127.2 Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.77

$$\int \sin^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$= \frac{-16a(2a^2-3ab-8b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+32a(a^2-ab-2b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,\text{Ellipt}}{240b^2f\sqrt{2a+}}$$

input
```
Integrate[Sin[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-16*a*(2*a^2 - 3*a*b - 8*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Elli
pticE[e + f*x, -(b/a)] + 32*a*(a^2 - a*b - 2*b^2)*Sqrt[(2*a + b - b*Cos[2*
(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] - Sqrt[2]*b*(8*a^2 + 48*a*b + 25
*b^2 - 4*b*(4*a + 7*b)*Cos[2*(e + f*x)] + 3*b^2*Cos[4*(e + f*x)])*Sin[2*(e
 + f*x)])/(240*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.127.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 3667, 380, 444, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^4\sqrt{a+b\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3667}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sin^4(e+fx)\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{380}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{5}\int\frac{\sin^2(e+fx)((a+4b)\sin^2(e+fx)+3a)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\frac{1}{5}\sin^3(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{\right.}{f}$$

$\downarrow$ 444

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{5}\left(\frac{\int\frac{a(a+4b)-\left(2a^2-3ba-8b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3b}-\frac{(a+4b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+\right.\right.}{3b}}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{5}\left(\frac{\frac{2a(a-2b)(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{\left(2a^2-3ab-8b^2\right)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx}{b}}{3b}\right.\right.}{f}$$

$\downarrow$ 323

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{5}\left(\frac{\frac{2a(a-2b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(2a^2-3ab-8b^2\right)\int\frac{\sqrt{b\sin^2(e}}{\sqrt{1-\sin^2}}}{b}}{3b}\right.\right.$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{5}\left(\frac{\frac{2a(a-2b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(2a^2-3ab-8b^2\right)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+}{b}}{3b}\right.\right.}{f}$$

$\downarrow$ 330

3.127. $\int\sin^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{5}\left(\frac{\frac{2a(a-2b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(2a^2-3ab-8b^2\right)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2}{}}}{\sqrt{1-\sin}}}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{3b}\right.\right.$$

$\downarrow$ 327

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{5}\left(\frac{\frac{2a(a-2b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(2a^2-3ab-8b^2\right)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\right.}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{3b}\right.\right.$$

input
```
Int[Sin[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-1/5*(Sin[e + f*x]^3*Sqrt[1 - Sin[e +
f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2]) + (-1/3*((a + 4*b)*Sin[e + f*x]*Sqrt[1
 - Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/b + (-(((2*a^2 - 3*a*b - 8*
b^2)*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(
b*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) + (2*a*(a - 2*b)*(a + b)*EllipticF[ArcS
in[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Si
n[e + f*x]^2]))/(3*b))/5))/f
```

3.127.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.127. $\int \sin^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.127.4 Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.59

method	result
default	$\frac{3b^3\left(\sin^7(fx+e)\right)+4a\,b^2\left(\sin^5(fx+e)\right)+b^3\left(\sin^5(fx+e)\right)+2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^3-2a^2\sqrt{\cos(}}{}$

input
```
int(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(3*b^3*sin(f*x+e)^7+4*a*b^2*sin(f*x+e)^5+b^3*sin(f*x+e)^5+2*(cos(f*x+
e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a^3-2*a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(
sin(f*x+e),(-1/a*b)^(1/2))*b-4*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/
a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-2*(cos(f*x+e)^2)^(1/2)*(
(a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3+3*(co
s(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*
b)^(1/2))*a^2*b+8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Ellipt
icE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2+a^2*b*sin(f*x+e)^3-4*b^3*sin(f*x+e)^3
-a^2*b*sin(f*x+e)-4*a*b^2*sin(f*x+e))/b^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1
/2)/f
```

3.127.5 Fricas [F]

$$\int \sin^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\sin{(fx+e)}^4\,dx$$

input
```
integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b*cos(f*x + e)^2 +
a + b), x)
```

3.127.6 Sympy [F(-1)]

Timed out.

$$\int \sin^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.127.7 Maxima [F]

$$\int \sin^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\sin{(fx+e)}^4\,dx$$

input
```
integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

3.127.8 Giac [F]

$$\int \sin^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\sin{(fx+e)}^4\,dx$$

input
```
integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \sin^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx=\int \sin\left(e+f\,x\right)^4\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.128 $\int \sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.128.1 Optimal result

Integrand size = 25, antiderivative size = 159

$$\int \sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{(a+2b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3bf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{a(a+b)\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3bf\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-1/3*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+1/3*(a+2*b)*(cos(f*x
+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)
^2)^(1/2)/b/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*a*(a+b)*(cos(f*x+e)^2)^(1/2)/
cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/b
/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.128.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00

$$\int \sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$=\frac{2\sqrt{2}a(a+2b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-2\sqrt{2}a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\operatorname{EllipticF}\left(e+fx,-\frac{b}{a}\right)}{6\sqrt{2}bf\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Sin[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(2*Sqrt[2]*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e
+ f*x, -(b/a)] - 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a
]*EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e
+ f*x)])/(6*Sqrt[2]*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.128.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3649, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sin(e+fx)^2\sqrt{a+b\sin(e+fx)^2}dx$$
$$\Big\downarrow \text{ 3649}$$
$$\frac{1}{3}\int \frac{(a+2b)\sin^2(e+fx)+a}{\sqrt{b\sin^2(e+fx)+a}}dx-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{1}{3}\int \frac{(a+2b)\sin(e+fx)^2+a}{\sqrt{b\sin(e+fx)^2+a}}dx-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$
$$\Big\downarrow \text{ 3651}$$

$$\frac{1}{3}\left(\frac{(a+2b)\int \sqrt{b\sin^2(e+fx)+a}dx}{b}-\frac{a(a+b)\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}dx}{b}\right)-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{(a+2b)\int \sqrt{b\sin(e+fx)^2+a}dx}{b}-\frac{a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}\right)-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$\big\downarrow$ 3657

$$\frac{1}{3}\left(\frac{(a+2b)\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin^2(e+fx)}{a}+1}dx}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}\right)-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{(a+2b)\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin(e+fx)^2}{a}+1}dx}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}\right)-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$\big\downarrow$ 3656

$$\frac{1}{3}\left(\frac{(a+2b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}\right)-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$\big\downarrow$ 3662

$$\frac{1}{3}\left(\frac{(a+2b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}dx}{b\sqrt{a+b\sin^2(e+fx)}}\right)-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$\big\downarrow$ 3042

3.128. $\int \sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

$$\frac{1}{3}\left(\frac{(a+2b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin(e+fx)^2}{a}+1}}dx}{b\sqrt{a+b\sin^2(e+fx)}}\right)-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$\Big\downarrow$ 3661

$$\frac{1}{3}\left(\frac{(a+2b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}\right)-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

input
```
Int[Sin[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-1/3*(Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f + (((a + 2*b
)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*f*Sqrt[1 + (b*
Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Si
n[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2]))/3
```

3.128.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.128.4 Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.67

method	result
default	$-\frac{-b^2(\sin^5(fx+e))+\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2+a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin\right.}{}$

input
```
int(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-b^2*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*
EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(
f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-(cos(f*x+e)^2)^(
1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2
-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),
(-1/a*b)^(1/2))*a*b-a*b*sin(f*x+e)^3+b^2*sin(f*x+e)^3+a*b*sin(f*x+e))/b/co
s(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.128.5 Fricas [F]

$$\int \sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\sin{(fx+e)}^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*(cos(f*x + e)^2 - 1), x)
```

3.128.6 Sympy [F]

$$\int \sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2{(e+fx)}}\sin^2{(e+fx)}\,dx$$

input
```
integrate(sin(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*sin(e + f*x)**2, x)
```

3.128.7 Maxima [F]

$$\int \sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\sin{(fx+e)}^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

3.128.8 Giac [F]

$$\int \sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\sin{(fx+e)}^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sin{(e+f\,x)}^2\,\sqrt{b\sin{(e+f\,x)}^2+a}\,dx$$

input
```
int(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.129 $\int \sqrt{a+b\sin^2(e+fx)}\,dx$

3.129.1 Optimal result

Integrand size = 16, antiderivative size = 51

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

output
```
(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*si
n(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)
```

3.129.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \frac{a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)])/(f*S
qrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.129.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin^2(e+fx)}{a}+1}dx}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin(e+fx)^2}{a}+1}dx}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

input
```
Int[Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin
[e + f*x]^2)/a])
```

3.129.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.129.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

method	result	size
default	$\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,E\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)}{\cos(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}\,f}$	71

input
```
int((a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.129.5 Fricas [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin\left(fx+e\right)^2+a}\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-b*cos(f*x + e)^2 + a + b), x)
```

3.129.6 Sympy [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2{(e+fx)}}\,dx$$

input
```
integrate((a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2), x)
```

3.129.7 Maxima [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

3.129.8 Giac [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \begin{cases} \frac{\sqrt{a}\,\mathrm{E}\left(e+f\,x\middle|-\frac{b}{a}\right)}{f} & \text{if } 0<a \\ \int \sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx & \text{if } \neg 0<a \end{cases}$$

input
```
int((a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
piecewise(0 < a, (a^(1/2)*ellipticE(e + f*x, -b/a))/f, ~0 < a, int((a + b*
sin(e + f*x)^2)^(1/2), x))
```

3.130 $\int \csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.130.1 Optimal result

Integrand size = 25, antiderivative size = 174

$$\begin{aligned}&\int \csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx\\&=-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}\\&\quad-\frac{\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\\&\quad+\frac{(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

output
```
-cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-EllipticE(sin(f*x+e),(-b/a)^(1/2))*
sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)
^2/a)^(1/2)+(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e
)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.130.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.79

$$\int \csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$= \frac{-\sqrt{2}(2a+b-b\cos(2(e+fx)))\cot(e+fx)-2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+2(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}}{2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-(Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x]) - 2*a*Sqrt[(2*a +
b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2*(a + b)*Sqrt[(2*
a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(2*f*Sqrt[2*a +
 b - b*Cos[2*(e + f*x)]])
```

3.130.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3667, 377, 27, 326, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{a+b\sin(e+fx)^2}}{\sin(e+fx)^2}dx$$

$$\Big\downarrow \ 3667$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\csc^2(e+fx)\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \ 377$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\int \frac{b\sqrt{1-\sin^2(e+fx)}}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(b\int \frac{\sqrt{1-\sin^2(e+fx)}}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}\right.}{f}$$

$\downarrow$ 326

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(b\left(\frac{(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)-\sqrt{1-\sin\right.}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(b\left(\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right.\right.}{f}$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(b\left(\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)-\sqrt{1\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(b\left(\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{a+b\sin^2(e+fx)}\int \frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.\right.}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(b\left(\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\right.}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.\right.}{f}$$

input
```
Int[Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

3.130. $\int \csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-(Csc[e + f*x]*Sqrt[1 - Sin[e + f*x]^2
]*Sqrt[a + b*Sin[e + f*x]^2]) + b*(-((EllipticE[ArcSin[Sin[e + f*x]], -(b/
a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) + ((a
+ b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a
])/(b*Sqrt[a + b*Sin[e + f*x]^2]))))/f
```

3.130.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.130.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

method	result
default	$\frac{b(\cos^4(fx+e))+(-a-b)(\cos^2(fx+e))+\sin(fx+e)\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\,(F(\sin(fx+e),\sqrt{-\frac{b}{a}})a+F(\sin(fx+}{\sin(fx+e)\cos(fx+e)\sqrt{a+b(\sin^2(fx+e))}\,f}$

input
```
int(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*cos(f*x+e)^4+(-a-b)*cos(f*x+e)^2+sin(f*x+e)*(cos(f*x+e)^2)^(1/2)*(-b/a*
cos(f*x+e)^2+(a+b)/a)^(1/2)*(EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a+Ellipt
icF(sin(f*x+e),(-1/a*b)^(1/2))*b-EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a))/
sin(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.130.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 626, normalized size of antiderivative = 3.60

$$\int \csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx =$$

$$-\frac{2\,(-2i\,a-i\,b)\sqrt{-b}\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}F\left(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))\right)\mid\frac{8\,a^2+8}{}\right.}{}$$

input
```
integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(2*(-2*I*a - I*b)*sqrt(-b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)
/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f
*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a
^2 + a*b)/b^2))/b^2)*sin(f*x + e) + 2*(2*I*a + I*b)*sqrt(-b)*sqrt((2*b*sqr
t((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a
*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b +
b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2)*sin(f*x + e) + 2*sqrt(-b
*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) + (2*I*sqrt(-b)*b*sqrt((a^2 + a*b)
/b^2)*sin(f*x + e) + (2*I*a + I*b)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((
a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)
/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2
 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (-2*I*sqrt(-b)*b*sqrt((a^
2 + a*b)/b^2)*sin(f*x + e) + (-2*I*a - I*b)*sqrt(-b)*sin(f*x + e))*sqrt((2
*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a
^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*
a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/(b*f*sin(f*x + e)
)
```

3.130.6 Sympy [F]

$$\int \csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2\left(e+fx\right)}\csc^2\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*csc(e + f*x)**2, x)
```

3.130.7 Maxima [F]

$$\int \csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\csc{(fx+e)}^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^2, x)
```

3.130.8 Giac [F]

$$\int \csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\csc{(fx+e)}^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^2, x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \frac{\sqrt{b\sin{(e+f\,x)}^2+a}}{\sin{(e+f\,x)}^2}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^2,x)
```

output
```
int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^2, x)
```

3.131 $\int \csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.131.1 Optimal result

Integrand size = 25, antiderivative size = 234

$$\int \csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{(2a+b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{2(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-1/3*(2*a+b)*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a/f-1/3*cot(f*x+e)*csc(f*
x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/f-1/3*(2*a+b)*EllipticE(sin(f*x+e),(-b/a)^
(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a/f/(1+b*s
in(f*x+e)^2/a)^(1/2)+2/3*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+
e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1
/2)
```

3.131.2 Mathematica [A] (verified)

Time = 2.21 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.80

$$\int \csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$= \frac{\frac{(4(2a^2+4ab+b^2)\cos(2(e+fx))-(2a+b)(8a+3b+b\cos(4(e+fx))))\cot(e+fx)\csc^2(e+fx)}{2\sqrt{2}} - 2a(2a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E(e+}{6af\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Csc[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(((4*(2*a^2 + 4*a*b + b^2)*Cos[2*(e + f*x)] - (2*a + b)*(8*a + 3*b + b*Cos
[4*(e + f*x)]))*Cot[e + f*x]*Csc[e + f*x]^2)/(2*Sqrt[2]) - 2*a*(2*a + b)*S
qrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 4*a*(a
+ b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(6
*a*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.131.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3667, 377, 445, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin(e+fx)^2}}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \text{3667}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\csc^4(e+fx)\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{377}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\int \frac{\csc^2(e+fx)\left(b\sin^2(e+fx)+2a+b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\frac{1}{3}\sqrt{1-\sin^2(e+fx)}\csc^3(e+fx)\sqrt{a}\right.}{f}$$

$\downarrow$ 445

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(-\frac{\int -\frac{b\left(a-(2a+b)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{(2a+b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+}}{a}\right.\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{\int \frac{b\left(a-(2a+b)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{(2a+b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}\right.\right.}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{b\int \frac{a-(2a+b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{(2a+b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx}}{a}\right.\right.}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{b\left(\frac{2a(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{(2a+b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)}{a}-(2a+\right.\right.}{f}$$

$\downarrow$ 323

3.131. $\int \csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{b\left(\frac{2a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(}{b}\right.}{a}\right.\right.}{f}$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{b\left(\frac{2a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)}{a}-\frac{(2\right.\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{b\left(\frac{2a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\,s}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.}{a}\right.\right.}{}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{b\left(\frac{2a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\right.}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.}{a}\right.\right.}{f}$$

input

```
Int[Csc[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

3.131. $\int \csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-1/3*(Csc[e + f*x]^3*Sqrt[1 - Sin[e +
f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2]) + (-(((2*a + b)*Csc[e + f*x]*Sqrt[1 -
Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/a) + (b*(-(((2*a + b)*Elliptic
E[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b
*Sin[e + f*x]^2)/a])) + (2*a*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a
)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2])))/a)/3))
/f
```

3.131.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.131. $\int \csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.131.4 Maple [A] (verified)

Time = 2.46 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.46

method	result
default	$2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2\left(\sin^3(fx+e)\right)+2b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\Big(\sin(fx+e$

input
```
int(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+2*b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*
x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-2*(co
s(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*
b)^(1/2))*a^2*sin(f*x+e)^3-(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b*sin(f*x+e)^3+2*a*b*sin(f*x+e)^
6+b^2*sin(f*x+e)^6+2*a^2*sin(f*x+e)^4-b^2*sin(f*x+e)^4-a^2*sin(f*x+e)^2-2*
a*b*sin(f*x+e)^2-a^2)/a/sin(f*x+e)^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.131.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 947, normalized size of antiderivative = 4.05

$$\int \csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*((2*((-2*I*a*b - I*b^2)*cos(f*x + e)^2 + 2*I*a*b + I*b^2)*sqrt(-b)*sqr
t((a^2 + a*b)/b^2)*sin(f*x + e) - ((4*I*a^2 + 4*I*a*b + I*b^2)*cos(f*x + e
)^2 - 4*I*a^2 - 4*I*a*b - I*b^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^
2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b
^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 -
 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((2*I*a*b + I*b^2)*cos(f
*x + e)^2 - 2*I*a*b - I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) -
 ((-4*I*a^2 - 4*I*a*b - I*b^2)*cos(f*x + e)^2 + 4*I*a^2 + 4*I*a*b + I*b^2)
*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elli
ptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e)
- I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b
)/b^2))/b^2) - 2*(2*((-I*a*b - I*b^2)*cos(f*x + e)^2 + I*a*b + I*b^2)*sqrt
(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + ((-2*I*a^2 - I*a*b)*cos(f*x + e)
^2 + 2*I*a^2 + I*a*b)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^
2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a +
 b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b +
 b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*(2*((I*a*b + I*b^2)*cos(f*x + e)^2 -
 I*a*b - I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + ((2*I*a^2 +
I*a*b)*cos(f*x + e)^2 - 2*I*a^2 - I*a*b)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*
sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a...
```

3.131.6 Sympy [F]

$$\int \csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2{(e+fx)}}\csc^4{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*csc(e + f*x)**4, x)
```

3.131.7 Maxima [F]

$$\int \csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\csc{(fx+e)}^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^4, x)
```

3.131.8 Giac [F]

$$\int \csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\csc{(fx+e)}^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^4, x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \frac{\sqrt{b\sin{(e+f\,x)}^2+a}}{\sin{(e+f\,x)}^4}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^4,x)
```

output
```
int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^4, x)
```

3.132 $\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.132.1 Optimal result

Integrand size = 25, antiderivative size = 169

$$\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{(a-5b)(a+b)^2\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{16b^{3/2}f}+\frac{(a-5b)(a+b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{16bf}+\frac{(a-5b)\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{3/2}}{24bf}-\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{5/2}}{6bf}$$

output
```
1/16*(a-5*b)*(a+b)^2*arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))
/b^(3/2)/f+1/24*(a-5*b)*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(3/2)/b/f-1/6*cos(
f*x+e)*(a+b-b*cos(f*x+e)^2)^(5/2)/b/f+1/16*(a-5*b)*(a+b)*cos(f*x+e)*(a+b-b
*cos(f*x+e)^2)^(1/2)/b/f
```

3.132.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.90

$$\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{-\frac{\cos(e+fx)\sqrt{2a+b-b\cos(2(e+fx))}\left(3a^2+29ab+23b^2-b(7a+9b)\cos(2(e+fx))+b^2\cos(4(e+fx))\right)}{3\sqrt{2}b}+\frac{(a+b)^2(-a+5b)\log\left(\sqrt{\ldots}\right.}{\ldots}}{16f}$$

input
```
Integrate[Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-1/3*(Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*(3*a^2 + 29*a*b + 2
3*b^2 - b*(7*a + 9*b)*Cos[2*(e + f*x)] + b^2*Cos[4*(e + f*x)]))/(Sqrt[2]*b
) + ((a + b)^2*(-a + 5*b)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b
 - b*Cos[2*(e + f*x)]]])/(-b)^(3/2))/(16*f)
```

3.132.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 3665, 299, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^3\left(a+b\sin(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)\left(-b\cos^2(e+fx)+a+b\right)^{3/2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{299}$$

$$-\frac{\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{5/2}}{6b}-\frac{(a-5b)\int \left(-b\cos^2(e+fx)+a+b\right)^{3/2}d\cos(e+fx)}{6b}}{f}$$

$$\Big\downarrow 211$$

$$-\frac{\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{5/2}}{6b}-\frac{(a-5b)\left(\frac{3}{4}(a+b)\int\sqrt{-b\cos^2(e+fx)+a+b}d\cos(e+fx)+\frac{1}{4}\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}\right)}{6b}}{f}$$

$$\Big\downarrow 211$$

$$-\frac{\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{5/2}}{6b}-\frac{(a-5b)\left(\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int\frac{1}{\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)+\frac{1}{2}\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}\right)+\frac{1}{4}\cos\cdots}{6b}}{f}$$

$$\Big\downarrow 224$$

$$-\frac{\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{5/2}}{6b}-\frac{(a-5b)\left(\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int\frac{1}{\frac{b\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}+1}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}+\frac{1}{2}\cos(e+fx)\sqrt{a-b\cos^2(e+f\cdots}{6b}}{f}$$

$$\Big\downarrow 216$$

$$-\frac{\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{5/2}}{6b}-\frac{(a-5b)\left(\frac{3}{4}(a+b)\left(\frac{(a+b)\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2\sqrt{b}}+\frac{1}{2}\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}\right)+\frac{1}{4}\cos(e+f\cdots}{6b}}{f}$$

input
```
Int[Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-(((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(5/2))/(6*b) - ((a - 5*b)*((Co
s[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(3/2))/4 + (3*(a + b)*(((a + b)*ArcT
an[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/(2*Sqrt[b]) + (
Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/2))/4))/(6*b))/f)
```

3.132. $\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.132.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.132.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(149) = 298$.

Time = 1.08 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.64

method	result
default	$-\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\left(16b^{\frac{7}{2}}\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\,(\cos^4(fx+e))-4b^{\frac{5}{2}}\sqrt{-b(\cos^4(fx+e))+(a+b\right.}{}$

input
```
int(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(16*b^(7/2)*(-b*cos(f*x+e)^4
+(a+b)*cos(f*x+e)^2)^(1/2)*cos(f*x+e)^4-4*b^(5/2)*(-b*cos(f*x+e)^4+(a+b)*c
os(f*x+e)^2)^(1/2)*(13*b+7*a)*cos(f*x+e)^2+66*b^(7/2)*(-b*cos(f*x+e)^4+(a+
b)*cos(f*x+e)^2)^(1/2)+72*a*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^(
5/2)+6*a^2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^(3/2)+3*arctan(1/2
*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2
))*a^3*b-9*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+
b)*cos(f*x+e)^2)^(1/2))*a^2*b^2-27*b^3*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/
b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))*a-15*b^4*arctan(1/2*(-
2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)))
/b^(5/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.132.5 Fricas [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.43

$$\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\left(a^3-3a^2b-9ab^2-5b^3\right)\sqrt{-b}\log\left(128b^4\cos(fx+e)^8-256\left(ab^3+b^4\right)\cos(fx+e)^6+\right.}{}\right.$$

$$-\frac{3\left(a^3-3a^2b-9ab^2-5b^3\right)\sqrt{b}\arctan\left(\frac{\left(8b^2\cos(fx+e)^4-8\left(ab+b^2\right)\cos(fx+e)^2+a^2+2ab+b^2\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{b}}{4\left(2b^3\cos(fx+e)^5-3\left(ab^2+b^3\right)\cos(fx+e)^3+\left(a^2b+2ab^2+b^3\right)\cos(fx+e)\right)}\right)+}{}$$

input
```
integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/384*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(-b)*log(128*b^4*cos(f*x +
 e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*c
os(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*
a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^7 - 24*(a
*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a +
b)*sqrt(-b)) - 8*(8*b^3*cos(f*x + e)^5 - 2*(7*a*b^2 + 13*b^3)*cos(f*x + e)
^3 + 3*(a^2*b + 12*a*b^2 + 11*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 +
a + b))/(b^2*f), -1/192*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(b)*arcta
n(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b +
 b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a
*b^2 + b^3)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) + 4*(8
*b^3*cos(f*x + e)^5 - 2*(7*a*b^2 + 13*b^3)*cos(f*x + e)^3 + 3*(a^2*b + 12*
a*b^2 + 11*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(b^2*f)]
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int \sin^3(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.47

$$\int \sin^3(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \frac{\frac{3(a+b)^2 a \arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} + \frac{3(a+b)^2 \arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} - \frac{18(a+b)a \arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} - 18(a+b)\sqrt{b}\,\mathrm{arc}\cdots}{\cdots}$$

input
```
integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/48*(3*(a + b)^2*a*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/b^(3/2) + 3*(a
+ b)^2*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 18*(a + b)*a*arcsi
n(b*cos(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 18*(a + b)*sqrt(b)*arcsin(b*co
s(f*x + e)/sqrt((a + b)*b)) - 12*(-b*cos(f*x + e)^2 + a + b)^(3/2)*cos(f*x
 + e) - 18*sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*cos(f*x + e) - 8*(-b*co
s(f*x + e)^2 + a + b)^(5/2)*cos(f*x + e)/b + 2*(-b*cos(f*x + e)^2 + a + b)
^(3/2)*(a + b)*cos(f*x + e)/b + 3*sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)^
2*cos(f*x + e)/b)/f
```

3.132.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5251 vs. $2(149) = 298$.

Time = 0.93 (sec) , antiderivative size = 5251, normalized size of antiderivative = 31.07

$$\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/24*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*arctan(-1/2*(sqrt(a)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
+ 4*b*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/sqrt(b))/b^(3/2) - 2*(3*(sqrt
(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f
*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^11*a^3 - 9*(sqrt(a)*tan(1
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*
e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^11*a^2*b - 27*(sqrt(a)*tan(1/2*f*x
 + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^11*a*b^2 - 15*(sqrt(a)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*t
an(1/2*f*x + 1/2*e)^2 + a))^11*b^3 + 33*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f
*x + 1/2*e)^2 + a))^10*a^(7/2) + 93*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))^10*a^(5/2)*b - 297*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
1/2*e)^2 + a))^10*a^(3/2)*b^2 - 165*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))^10*sqrt(a)*b^3 + 165*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f...
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \sin^3(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \int \sin(e+f\,x)^3 \left(b\sin(e+f\,x)^2+a\right)^{3/2} dx$$

input
```
int(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.133 $\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.133.1 Optimal result

Integrand size = 23, antiderivative size = 114

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=-\frac{3(a+b)^2\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8\sqrt{b}f}-\frac{3(a+b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{8f}-\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{3/2}}{4f}$$

output
```
-1/4*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(3/2)/f-3/8*(a+b)^2*arctan(cos(f*x+e)
*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f/b^(1/2)-3/8*(a+b)*cos(f*x+e)*(a+b-b
*cos(f*x+e)^2)^(1/2)/f
```

3.133.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=-\frac{\frac{\cos(e+fx)\sqrt{2a+b-b\cos(2(e+fx))}(5a+4b-b\cos(2(e+fx)))}{\sqrt{2}}+\frac{3(a+b)^2\log\left(\sqrt{2}\sqrt{-b}\cos(e+fx)+\sqrt{2a+b-b\cos(2(e+fx))}\right)}{\sqrt{-b}}}{8f}$$

input
```
Integrate[Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-1/8*((Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*(5*a + 4*b - b*Cos[
2*(e + f*x)]))/Sqrt[2] + (3*(a + b)^2*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] +
Sqrt[2*a + b - b*Cos[2*(e + f*x)]]])/Sqrt[-b])/f
```

3.133.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3665, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)\left(a+b\sin(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \left(-b\cos^2(e+fx)+a+b\right)^{3/2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{211}$$

$$-\frac{\frac{3}{4}(a+b)\int \sqrt{-b\cos^2(e+fx)+a+b}d\cos(e+fx)+\frac{1}{4}\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{f}$$

$$\Big\downarrow \text{211}$$

$$-\frac{\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)+\frac{1}{2}\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}\right)+\frac{1}{4}\cos(e+fx)}{f}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{\frac{b\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}+1}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}+\frac{1}{2}\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}\right)+\frac{1}{4}\cos(e}{f}$$

$$\downarrow 216$$

$$-\frac{\frac{3}{4}(a+b)\left(\frac{(a+b)\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2\sqrt{b}}+\frac{1}{2}\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}\right)+\frac{1}{4}\cos(e+fx)\left(a-b\cos^2(e\right.}{f}$$

input
```
Int[Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-(((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(3/2))/4 + (3*(a + b)*(((a + b
)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/(2*Sqrt[b
]) + (Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/2))/4)/f)
```

3.133.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.133.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(98) = 196.

Time = 1.12 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.71

method	result
default	$-\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\left(-4b^{\frac{3}{2}}\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\,(\cos^2(fx+e))+10b^{\frac{3}{2}}\sqrt{-b(\cos^4(fx+e))+(a+\right.}{}$

input
```
int(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(-4*b^(3/2)*(-b*cos(f*x+e)^4
+(a+b)*cos(f*x+e)^2)^(1/2)*cos(f*x+e)^2+10*b^(3/2)*(-b*cos(f*x+e)^4+(a+b)*
cos(f*x+e)^2)^(1/2)+10*a*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^(1/2
)-3*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(
f*x+e)^2)^(1/2))*a^2-6*b*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*co
s(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))*a-3*b^2*arctan(1/2*(-2*b*cos(f*x+e)^
2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)))/b^(1/2)/cos(f*
x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 495, normalized size of antiderivative = 4.34

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \left[-\frac{3\left(a^2+2ab+b^2\right)\sqrt{-b}\log\left(128b^4\cos(fx+e)^8-256\left(ab^3+b^4\right)\cos(fx+e)^6+160\left(a^2b^2\right.\right.}{}\right.$$

input
```
integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/64*(3*(a^2 + 2*a*b + b^2)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a
*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4
+ a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*
b^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*co
s(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) -
 8*(2*b^2*cos(f*x + e)^3 - 5*(a*b + b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e
)^2 + a + b))/(b*f), 1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(b)*arctan(1/4*(8*b^2
*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-
b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*
cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) + 4*(2*b^2*cos(f*x
 + e)^3 - 5*(a*b + b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(b*
f)]
```

3.133.6 Sympy [F(-1)]

Timed out.

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx =$$

$$-\frac{\frac{3\,(a+b)a\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} + 3\,(a+b)\sqrt{b}\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right) + 2\left(-b\cos\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}\cos\left(fx+e\right)+3}{8\,f}$$

input
```
integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/8*(3*(a + b)*a*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/sqrt(b) + 3*(a +
b)*sqrt(b)*arcsin(b*cos(f*x + e)/sqrt((a + b)*b)) + 2*(-b*cos(f*x + e)^2 +
 a + b)^(3/2)*cos(f*x + e) + 3*sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*cos
(f*x + e))/f
```

3.133.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2403 vs. $2(98) = 196$.

Time = 0.63 (sec) , antiderivative size = 2403, normalized size of antiderivative = 21.08

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*(3*(a^2 + 2*a*b + b^2)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*
x + 1/2*e)^2 + a) + sqrt(a))/sqrt(b))/sqrt(b) + 2*(5*(sqrt(a)*tan(1/2*f*x
+ 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a^2 - 6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/
2*f*x + 1/2*e)^2 + a))^7*a*b - 3*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/
2*e)^2 + a))^7*b^2 + 35*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
a))^6*a^(5/2) + 22*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6
*a^(3/2)*b - 21*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2
*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*sq
rt(a)*b^2 + 105*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2
*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^
3 + 246*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^2*b + 10
5*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*ta
n(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*b^2 - 44*(s...
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \sin\left(e+f\,x\right)\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

input
```
int(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.134 $\int \csc(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} \, dx$

3.134.1 Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \csc(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} \, dx = -\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2f} - \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f} - \frac{b\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f}$$

output
```
-a^(3/2)*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f-1/2*(3*a
+b)*arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*b*
cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/f
```

3.134.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.16

$$\int \csc(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} \, dx = -\frac{4a^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right) + \sqrt{2}b\cos(e+fx)\sqrt{2a+b-b\cos(2(e+fx))} - 2\sqrt{-b}(3a+b)\log\left(\sqrt{\right.}{4f}$$

input
```
Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-1/4*(4*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Co
s[2*(e + f*x)]]] + Sqrt[2]*b*Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)
]] - 2*Sqrt[-b]*(3*a + b)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b
 - b*Cos[2*(e + f*x)]]])/f
```

3.134.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3665, 318, 25, 398, 224, 216, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^{3/2}}{\sin(e+fx)}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{\left(-b\cos^2(e+fx)+a+b\right)^{3/2}}{1-\cos^2(e+fx)}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{318}$$

$$-\frac{\frac{1}{2}b\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}-\frac{1}{2}\int -\frac{(a+b)(2a+b)-b(3a+b)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{2}\int \frac{(a+b)(2a+b)-b(3a+b)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)+\frac{1}{2}b\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{f}$$

$$\Big\downarrow \text{398}$$

$$-\frac{\frac{1}{2}\left(2a^2\int \frac{1}{\left(1-\cos^2(e+fx)\right)\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)+b(3a+b)\int \frac{1}{\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)\right)+\frac{1}{2}b\cos(e}{f}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{1}{2}\left(2a^2\int\frac{1}{(1-\cos^2(e+fx))\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)+b(3a+b)\int\frac{1}{\frac{b\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}+1}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}\right)+\frac{1}{2}}{f}$$

$\big\downarrow$ 216

$$-\frac{\frac{1}{2}\left(2a^2\int\frac{1}{(1-\cos^2(e+fx))\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)+\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\cos(e+f}{f}$$

$\big\downarrow$ 291

$$-\frac{\frac{1}{2}\left(2a^2\int\frac{1}{1-\frac{a\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}+\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\cos(e+fx)\sqrt{a}}{f}$$

$\big\downarrow$ 219

$$-\frac{\frac{1}{2}\left(2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)+\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\cos(e+fx)\sqrt{a-b\cos^2(e}}{f}$$

input
```
Int[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-(((Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e +
 f*x]^2]] + 2*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e
+ f*x]^2]])/2 + (b*Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/2)/f)
```

3.134.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.134. $\int \csc(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx$

3.134.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(104) = 208$.

Time = 1.13 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.09

method	result
default	$\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\left(b^{\frac{3}{2}}\arctan\left(\frac{-2b\left(\cos^2(fx+e)\right)+a+b}{2\sqrt{b}\sqrt{-b\left(\cos^4(fx+e)\right)+(a+b)\left(\cos^2(fx+e)\right)}}\right)-2a^{\frac{3}{2}}\ln\left(\frac{-(a-b)\left(\cos^2(fx+e)\right)-2\sqrt{a}\sqrt{-}}{c}\right.\right.}{4\cos(fx-}$

input
```
int(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(b^(3/2)*arctan(1/2*(-2*b*cos(
f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))-2*a^(3/2
)*ln((-(a-b)*cos(f*x+e)^2-2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(
1/2)-a-b)/(cos(f*x+e)^2-1))+3*b^(1/2)*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b
^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))*a-2*b*(-b*cos(f*x+e)^4+
(a+b)*cos(f*x+e)^2)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.134.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(104) = 208$.

Time = 0.58 (sec) , antiderivative size = 1282, normalized size of antiderivative = 10.51

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) - (3*a + b)*sqrt(
-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a
^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 - 8*(16*b
^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2
+ b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) - 4*a^(3/2)*log(2*((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*co
s(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(
a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/f, 1/16*
(8*sqrt(-a)*a*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e)))
 - 8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) + (3*a + b)*sqrt(-b)*l
og(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^
2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 - 8*(16*b^3*co
s(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3
)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*c
os(f*x + e)^2 + a + b)*sqrt(-b)))/f, 1/8*((3*a + b)*sqrt(b)*arctan(1/4*(8*
b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*...
```

3.134.6 Sympy [F]

$$\int \csc(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \int \left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}} \csc\left(e+fx\right) dx$$

input
```
integrate(csc(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sin(e + f*x)**2)**(3/2)*csc(e + f*x), x)
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.47

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx =$$

$$-\frac{3\,a\sqrt{b}\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{ab+b^2}}\right)+b^{\frac{3}{2}}\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{ab+b^2}}\right)+\sqrt{-b\cos\left(fx+e\right)^2+a+b}b\cos\left(fx+e\right)+a^{\frac{3}{2}}\log\left(b\cdots\right.}{2\,f}$$

input
```
integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(3*a*sqrt(b)*arcsin(b*cos(f*x + e)/sqrt(a*b + b^2)) + b^(3/2)*arcsin(
b*cos(f*x + e)/sqrt(a*b + b^2)) + sqrt(-b*cos(f*x + e)^2 + a + b)*b*cos(f*
x + e) + a^(3/2)*log(b - sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x
+ e) - 1) - a/(cos(f*x + e) - 1)) - a^(3/2)*log(-b + sqrt(-b*cos(f*x + e)^
2 + a + b)*sqrt(a)/(cos(f*x + e) + 1) + a/(cos(f*x + e) + 1)))/f
```

3.134.8 Giac [F(-2)]

Exception generated.

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x),x)
```

output
```
int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x), x)
```

3.135 $\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.135.1 Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f}-\frac{\sqrt{a}(a+3b)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2f}-\frac{a\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{2f}$$

output
```
-b^(3/2)*arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f-1/2*(a+3*
b)*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))*a^(1/2)/f-1/2*a*
cot(f*x+e)*csc(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/f
```

3.135.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.15

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{2\sqrt{a}(a+3b)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)+\sqrt{2}a\sqrt{2a+b-b\cos(2(e+fx))}\cot(e+fx)\csc(e+fx)+4}{4f}$$

input
```
Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-1/4*(2*Sqrt[a]*(a + 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a
+ b - b*Cos[2*(e + f*x)]]] + Sqrt[2]*a*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*
Cot[e + f*x]*Csc[e + f*x] + 4*(-b)^(3/2)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x]
 + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]])/f
```

3.135.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 3665, 315, 25, 398, 224, 216, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^{3/2}}{\sin(e+fx)^3}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{\left(-b\cos^2(e+fx)+a+b\right)^{3/2}}{\left(1-\cos^2(e+fx)\right)^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{315}$$

$$-\frac{\frac{a\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2\left(1-\cos^2(e+fx)\right)}-\frac{1}{2}\int -\frac{(a+b)(a+2b)-2b^2\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{2}\int \frac{(a+b)(a+2b)-2b^2\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)+\frac{a\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2\left(1-\cos^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \text{398}$$

$$-\frac{\frac{1}{2}\left(2b^2\int \frac{1}{\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)+a(a+3b)\int \frac{1}{\left(1-\cos^2(e+fx)\right)\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)\right)+\frac{a\cos(e+f}{2(}}{f}$$

$$\Big\downarrow 224$$

$$-\frac{\frac{1}{2}\left(2b^2\int \frac{1}{\frac{b\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}+1}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}+a(a+3b)\int \frac{1}{(1-\cos^2(e+fx))\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)\right)+\frac{a}{}}{f}$$

$$\Big\downarrow 216$$

$$-\frac{\frac{1}{2}\left(a(a+3b)\int \frac{1}{(1-\cos^2(e+fx))\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)+2b^{3/2}\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)\right)+\frac{a\cos(e+fx)\sqrt{a}}{2(1-\cos}}{f}$$

$$\Big\downarrow 291$$

$$-\frac{\frac{1}{2}\left(a(a+3b)\int \frac{1}{1-\frac{a\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}+2b^{3/2}\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)\right)+\frac{a\cos(e+fx)\sqrt{a-b\cos^2(}}{2(1-\cos^2(e+fx)}}{f}$$

$$\Big\downarrow 219$$

$$-\frac{\frac{1}{2}\left(2b^{3/2}\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)+\sqrt{a}(a+3b)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)\right)+\frac{a\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2(1-\cos^2(e+fx))}}{f}$$

input
```
Int[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-(((2*b^(3/2)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]
] + Sqrt[a]*(a + 3*b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e
+ f*x]^2]])/2 + (a*Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(2*(1 - Co
s[e + f*x]^2)))/f)
```

3.135.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.135. $\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.135.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(110) = 220$.

Time = 1.16 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.24

method	result
default	$\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\left(2b^{\frac{3}{2}}\arctan\left(\frac{2b\left(\sin^2(fx+e)\right)+a-b}{2\sqrt{b}\sqrt{\left(\cos^2(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)}}\right)(\sin^2(fx+e))-a^{\frac{3}{2}}\ln\left(\frac{(a-b)\left(\cos^2(fx+e)\right)}{}\right.\right.}{}$

input
```
int(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(2*b^(3/2)*arctan(1/2/b^(1/2)*
(2*b*sin(f*x+e)^2+a-b)/(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))*sin(f*x+e)
^2-a^(3/2)*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x
+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^2-3*a^(1/2)*b*ln(((a-b)*cos(f*x
+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)
^2)*sin(f*x+e)^2-2*a*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))/sin(f*x+e)^2
/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.135.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(110) = 220$.

Time = 0.59 (sec) , antiderivative size = 1449, normalized size of antiderivative = 11.32

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e) + (b*cos(f*x + e)^2
 - b)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)
^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^
2 - 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b
 + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x
 + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) + ((a + 3*b)*cos(f*x + e)
^2 - a - 3*b)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2
 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f
*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos
(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(f*cos(f*x + e)^2 - f), 1/8*(2*((a +
 3*b)*cos(f*x + e)^2 - a - 3*b)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)
^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 -
 (a^2 + a*b)*cos(f*x + e))) + 4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x
+ e) + (b*cos(f*x + e)^2 - b)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a
*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4
+ a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*
b^3 + b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*co
s(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)...
```

3.135.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(csc(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.135.7 Maxima [F]

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^3, x)
```

3.135.8 Giac [F(-2)]

Exception generated.

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^3,x)
```

output
```
int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^3, x)
```

3.136 $\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.136.1 Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{3(a+b)^2\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8\sqrt{a}f} - \frac{3(a+b)\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{8f} - \frac{\left(a+b-b\cos^2(e+fx)\right)^{3/2}\cot(e+fx)\csc^3(e+fx)}{4f}$$

output
```
-1/4*(a+b-b*cos(f*x+e)^2)^(3/2)*cot(f*x+e)*csc(f*x+e)^3/f-3/8*(a+b)^2*arct
anh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f/a^(1/2)-3/8*(a+b)*cot
(f*x+e)*csc(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/f
```

3.136.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.89

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{\frac{6(a+b)^2\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)}{\sqrt{a}} + \sqrt{2}\sqrt{2a+b-b\cos(2(e+fx))}\cot(e+fx)\csc(e+fx)\,(3a+5b+2a}{16f}$$

input
```
Integrate[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-1/16*((6*(a + b)^2*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b -
b*Cos[2*(e + f*x)]]])/Sqrt[a] + Sqrt[2]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
*Cot[e + f*x]*Csc[e + f*x]*(3*a + 5*b + 2*a*Csc[e + f*x]^2))/f
```

3.136.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3665, 292, 292, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^{3/2}}{\sin(e+fx)^5}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{\left(-b\cos^2(e+fx)+a+b\right)^{3/2}}{\left(1-\cos^2(e+fx)\right)^3}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{292}$$

$$-\frac{\frac{3}{4}(a+b)\int \frac{\sqrt{-b\cos^2(e+fx)+a+b}}{\left(1-\cos^2(e+fx)\right)^2}d\cos(e+fx)+\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4\left(1-\cos^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \text{292}$$

$$-\frac{\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{\left(1-\cos^2(e+fx)\right)\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)+\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2\left(1-\cos^2(e+fx)\right)}\right)+\frac{\cos(e+fx)(a-b\,c}{4(1-\cos^2}}{f}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{1-\frac{a\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}+\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2\left(1-\cos^2(e+fx)\right)}\right)+\frac{\cos(e+fx)(a-b\cos^2(e+f}{4(1-\cos^2(e+fx)}}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{3}{4}(a+b)\left(\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2\sqrt{a}}+\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2(1-\cos^2(e+fx))}\right)+\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4(1-\cos^2(e+fx))^2}}{f}$$

input
```
Int[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-(((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(3/2))/(4*(1 - Cos[e + f*x]^2)
^2) + (3*(a + b)*(((a + b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*C
os[e + f*x]^2]])/(2*Sqrt[a]) + (Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2
])/(2*(1 - Cos[e + f*x]^2))))/4)/f)
```

3.136.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.136.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(112) = 224$.

Time = 1.10 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.94

method	result
default	$-\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\left(3a^2\ln\left(\frac{(a-b)\left(\cos^2(fx+e)\right)+2\sqrt{a}\sqrt{-b\left(\cos^4(fx+e)\right)+(a+b)\left(\cos^2(fx+e)\right)}+a+b}{\sin(fx+e)^2}\right)(\sin^4(fx+e))+$

input
```
int(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(3*a^2*ln(((a-b)*cos(f*x+e)^
2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*
sin(f*x+e)^4+6*a*b*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)
*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^4+3*b^2*ln(((a-b)*cos(f
*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+
e)^2)*sin(f*x+e)^4+6*a^(3/2)*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*sin(f
*x+e)^2+10*b*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*a^(1/2)*sin(f*x+e)^2+
4*a^(3/2)*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))/a^(1/2)/sin(f*x+e)^4/co
s(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 484, normalized size of antiderivative = 3.78

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\left((a^2+2ab+b^2)\cos(fx+e)^4-2(a^2+2ab+b^2)\cos(fx+e)^2+a^2+2ab+b^2\right)\sqrt{a}\log}{}\right.$$

input
```
integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(3*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f
*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x
+ e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^
3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 +
2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((3*a^2 + 5*a*b)
*cos(f*x + e)^3 - 5*(a^2 + a*b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a +
 b))/(a*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f), 1/16*(3*((a^2 + 2*
a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2
*a*b + b^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*
cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x
 + e))) + 2*((3*a^2 + 5*a*b)*cos(f*x + e)^3 - 5*(a^2 + a*b)*cos(f*x + e))*
sqrt(-b*cos(f*x + e)^2 + a + b))/(a*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^
2 + a*f)]
```

3.136.6 Sympy [F(-1)]

Timed out.

$$\int \csc^5(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**5*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.136.7 Maxima [F]

$$\int \csc^5(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \csc\left(fx+e\right)^5 \, dx$$

input
```
integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^5, x)
```

3.136.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(112) = 224$.

Time = 0.72 (sec) , antiderivative size = 912, normalized size of antiderivative = 7.12

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a)*(a*tan(1/2*f*x + 1/2*e)^2 + (7*a^2 + 10*a*b)/a) +
 24*(a^2 + 2*a*b + b^2)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*t
an(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2
*e)^2 + a))/sqrt(-a))/sqrt(-a) - 12*(a^(5/2) + 2*a^(3/2)*b + sqrt(a)*b^2)*
log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a - a^(3/2)
 - 2*sqrt(a)*b))/a + 4*(4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
+ a))^3*a^2 + 12*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a
*b + 10*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2 + 5*(s
qrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/
2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2) + 8*(sqrt(a)
*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b - 2*(sqrt(a)*tan
(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/
2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3 - 8*(sqrt(a)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + ...
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^5}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^5,x)
```

output
```
int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^5, x)
```

3.136. $\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.137 $\int \csc^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.137.1 Optimal result

Integrand size = 25, antiderivative size = 197

$$\begin{aligned}\int \csc^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx =& -\frac{(5a-b)(a+b)^2\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{16a^{3/2}f} \\ &- \frac{(5a-b)(a+b)\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{16af} \\ &- \frac{(5a-b)\left(a+b-b\cos^2(e+fx)\right)^{3/2}\cot(e+fx)\csc^3(e+fx)}{24af} \\ &- \frac{\left(a+b-b\cos^2(e+fx)\right)^{5/2}\cot(e+fx)\csc^5(e+fx)}{6af}\end{aligned}$$

output
```
-1/16*(5*a-b)*(a+b)^2*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2
))/a^(3/2)/f-1/24*(5*a-b)*(a+b-b*cos(f*x+e)^2)^(3/2)*cot(f*x+e)*csc(f*x+e)
^3/a/f-1/6*(a+b-b*cos(f*x+e)^2)^(5/2)*cot(f*x+e)*csc(f*x+e)^5/a/f-1/16*(5*
a-b)*(a+b)*cot(f*x+e)*csc(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/a/f
```

3.137.2 Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.82

$$\int \csc^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{-6(5a-b)(a+b)^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)-\sqrt{2}\sqrt{a}\sqrt{2a+b-b\cos(2(e+fx))}\csc^2(e$$

input
```
Integrate[Csc[e + f*x]^7*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-6*(5*a - b)*(a + b)^2*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + 
b - b*Cos[2*(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a + b - b*Cos[2*(e + f*x
)]]*Csc[e + f*x]^2*((15*a^2 + 22*a*b + 3*b^2)*Cos[e + f*x] + 2*a*Cot[e + f
*x]*Csc[e + f*x]*(5*a + 7*b + 4*a*Csc[e + f*x]^2)))/(96*a^(3/2)*f)
```

3.137.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 3665, 296, 292, 292, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^{3/2}}{\sin(e+fx)^7}dx$$

$$\Big\downarrow \; 3665$$

$$-\frac{\int \frac{\left(-b\cos^2(e+fx)+a+b\right)^{3/2}}{\left(1-\cos^2(e+fx)\right)^4}d\cos(e+fx)}{f}$$

$$\Big\downarrow \; 296$$

$$-\frac{\frac{(5a-b)\int \frac{\left(-b\cos^2(e+fx)+a+b\right)^{3/2}}{\left(1-\cos^2(e+fx)\right)^3}d\cos(e+fx)}{6a}+\frac{\cos(e+fx)(a-b\cos^2(e+fx)+b)^{5/2}}{6a(1-\cos^2(e+fx))^3}}{f}$$

$$\Big\downarrow \text{ 292}$$

$$-\frac{\frac{(5a-b)\left(\frac{3}{4}(a+b)\int \frac{\sqrt{-b\cos^2(e+fx)+a+b}}{\left(1-\cos^2(e+fx)\right)^2}d\cos(e+fx)+\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4\left(1-\cos^2(e+fx)\right)^2}\right)}{6a}+\frac{\cos(e+fx)(a-b\cos^2(e+fx)+b)^{5/2}}{6a(1-\cos^2(e+fx))^3}}{f}$$

$$\Big\downarrow \text{ 292}$$

$$-\frac{\frac{(5a-b)\left(\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{\left(1-\cos^2(e+fx)\right)\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)+\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2\left(1-\cos^2(e+fx)\right)}\right)+\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4\left(1-\cos^2(e+fx)\right)^2}}{6a}}{f}$$

$$\Big\downarrow \text{ 291}$$

$$-\frac{\frac{(5a-b)\left(\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{1-\frac{a\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}+\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2\left(1-\cos^2(e+fx)\right)}\right)+\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4\left(1-\cos^2(e+fx)\right)^2}\right)}{6a}}{f}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\frac{(5a-b)\left(\frac{3}{4}(a+b)\left(\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2\sqrt{a}}+\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2\left(1-\cos^2(e+fx)\right)}\right)+\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4\left(1-\cos^2(e+fx)\right)^2}\right)}{6a}+\frac{\cos(e+fx)(a\ldots}{6a(1-\ldots}}{f}$$

input
```
Int[Csc[e + f*x]^7*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-(((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(5/2))/(6*a*(1 - Cos[e + f*x]^
2)^3) + ((5*a - b)*((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(3/2))/(4*(1
- Cos[e + f*x]^2)^2) + (3*(a + b)*(((a + b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])
/Sqrt[a + b - b*Cos[e + f*x]^2]])/(2*Sqrt[a]) + (Cos[e + f*x]*Sqrt[a + b -
 b*Cos[e + f*x]^2])/(2*(1 - Cos[e + f*x]^2))))/4))/(6*a))/f)
```

3.137. $\int \csc^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.137.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.137.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(177) = 354$.

Time = 1.39 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.87

method	result
default	$-\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\left(15a^4\ln\left(\frac{(a-b)\left(\cos^2(fx+e)\right)+2\sqrt{a}\sqrt{-b\left(\cos^4(fx+e)\right)+(a+b)\left(\cos^2(fx+e)\right)+a+b}}{\sin(fx+e)^2}\right)(\sin^6(fx+e))}{}$

input
```
int(csc(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(15*a^4*ln(((a-b)*cos(f*x+e)
^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)
*sin(f*x+e)^6+27*a^3*b*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(
a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^6+9*b^2*ln(((a-b)*c
os(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(
f*x+e)^2)*sin(f*x+e)^6*a^2-3*b^3*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(
f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^6*a+30*si
n(f*x+e)^4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*a^(7/2)+44*sin(f*x+e)^4
*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*a^(5/2)*b+6*sin(f*x+e)^4*(cos(f*x
+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*a^(3/2)*b^2+20*sin(f*x+e)^2*(cos(f*x+e)^2*
(a+b*sin(f*x+e)^2))^(1/2)*a^(7/2)+28*sin(f*x+e)^2*(cos(f*x+e)^2*(a+b*sin(f
*x+e)^2))^(1/2)*a^(5/2)*b+16*a^(7/2)*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/
2))/sin(f*x+e)^6/a^(5/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.137.5 Fricas [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 752, normalized size of antiderivative = 3.82

$$\int \csc^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \left[-\frac{3\left((5a^3+9a^2b+3ab^2-b^3)\cos(fx+e)^6-3(5a^3+9a^2b+3ab^2-b^3)\cos(fx+e)^4-\right.}{}\right.$$

input
```
integrate(csc(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/192*(3*((5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^6 - 3*(5*a^3 +
9*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - 5*a^3 - 9*a^2*b - 3*a*b^2 + b^3
+ 3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^2)*sqrt(a)*log(2*((a^2
- 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 + 4
*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 +
a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1
)) - 4*((15*a^3 + 22*a^2*b + 3*a*b^2)*cos(f*x + e)^5 - 2*(20*a^3 + 29*a^2*
b + 3*a*b^2)*cos(f*x + e)^3 + 3*(11*a^3 + 12*a^2*b + a*b^2)*cos(f*x + e))*
sqrt(-b*cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^6 - 3*a^2*f*cos(f*x +
 e)^4 + 3*a^2*f*cos(f*x + e)^2 - a^2*f), 1/96*(3*((5*a^3 + 9*a^2*b + 3*a*b
^2 - b^3)*cos(f*x + e)^6 - 3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e
)^4 - 5*a^3 - 9*a^2*b - 3*a*b^2 + b^3 + 3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3
)*cos(f*x + e)^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sq
rt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*c
os(f*x + e))) + 2*((15*a^3 + 22*a^2*b + 3*a*b^2)*cos(f*x + e)^5 - 2*(20*a^
3 + 29*a^2*b + 3*a*b^2)*cos(f*x + e)^3 + 3*(11*a^3 + 12*a^2*b + a*b^2)*cos
(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^6 - 3*a^2*
f*cos(f*x + e)^4 + 3*a^2*f*cos(f*x + e)^2 - a^2*f)]
```

3.137.6 Sympy [F(-1)]

Timed out.

$$\int \csc^7(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**7*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.137.7 Maxima [F]

$$\int \csc^7(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \csc\left(fx+e\right)^7 dx$$

input
```
integrate(csc(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^7, x)
```

3.137.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1623 vs. $2(177) = 354$.

Time = 0.94 (sec) , antiderivative size = 1623, normalized size of antiderivative = 8.24

$$\int \csc^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/384*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*ta
n(1/2*f*x + 1/2*e)^2 + a)*((a*tan(1/2*f*x + 1/2*e)^2 + (8*a^3 + 7*a^2*b)/a
^2)*tan(1/2*f*x + 1/2*e)^2 + (37*a^3 + 51*a^2*b + 6*a*b^2)/a^2) + 24*(5*a^
3 + 9*a^2*b + 3*a*b^2 - b^3)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
+ 1/2*e)^2 + a))/sqrt(-a))/(sqrt(-a)*a) - 12*(5*a^(7/2) + 9*a^(5/2)*b + 3*
a^(3/2)*b^2 - sqrt(a)*b^3)*log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))*a - a^(3/2) - 2*sqrt(a)*b))/a^2 + 2*(45*(sqrt(a)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^3 + 132*(sqrt(a)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*t
an(1/2*f*x + 1/2*e)^2 + a))^5*a^2*b + 108*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
- sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2
*f*x + 1/2*e)^2 + a))^5*a*b^2 + 12*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
1/2*e)^2 + a))^5*b^3 + 63*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
+ a))^4*a^(7/2) + 120*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +...
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \csc^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^7}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^7,x)
```

output
```
int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^7, x)
```

3.137. $\int \csc^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.138 $\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.138.1 Optimal result

Integrand size = 25, antiderivative size = 325

$$\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{(a^2+11ab+8b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf} - \frac{2(4a+3b)\cos(e+fx)\sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35f} - \frac{b\cos(e+fx)\sin^5(e+fx)\sqrt{a+b\sin^2(e+fx)}}{7f} - \frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35b^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{a(a+b)\left(2a^2-5ab-8b^2\right)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{35b^2f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-1/35*(a^2+11*a*b+8*b^2)*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/
f-2/35*(4*a+3*b)*cos(f*x+e)*sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2)/f-1/7*b*
cos(f*x+e)*sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2)/f-2/35*(a+2*b)*(a^2-4*a*b
-4*b^2)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)
*(a+b*sin(f*x+e)^2)^(1/2)/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/35*a*(a+b)*(2
*a^2-5*a*b-8*b^2)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e
)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.138.2 Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.77

$$\int \sin^4(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \frac{-128a(a^3-2a^2b-12ab^2-8b^3)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+64a(2a^3-3a^2b-13ab$$

input
```
Integrate[Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-128*a*(a^3 - 2*a^2*b - 12*a*b^2 - 8*b^3)*Sqrt[(2*a + b - b*Cos[2*(e + f*
x)])/a]*EllipticE[e + f*x, -(b/a)] + 64*a*(2*a^3 - 3*a^2*b - 13*a*b^2 - 8*
b^3)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + S
qrt[2]*b*(-32*a^3 - 496*a^2*b - 684*a*b^2 - 250*b^3 + b*(144*a^2 + 480*a*b
 + 299*b^2)*Cos[2*(e + f*x)] - 2*b^2*(26*a + 27*b)*Cos[4*(e + f*x)] + 5*b^
3*Cos[6*(e + f*x)])*Sin[2*(e + f*x)])/(2240*b^2*f*Sqrt[2*a + b - b*Cos[2*(
e + f*x)]])
```

3.138.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3667, 379, 25, 444, 27, 444, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^4 \left(a+b\sin(e+fx)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{3667}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sin^4(e+fx)\left(b\sin^2(e+fx)+a\right)^{3/2}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{379}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{1}{7}\int -\frac{\sin^4(e+fx)(2b(4a+3b)\sin^2(e+fx)+a(7a+5b))}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\frac{1}{7}b\sqrt{1-\sin^2(e+fx)}\sin\cdots\right.}{f}$$

$\big\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{7}\int \frac{\sin^4(e+fx)(2b(4a+3b)\sin^2(e+fx)+a(7a+5b))}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\frac{1}{7}b\sin^5(e+fx)\sqrt{1-\sin^2(e\cdots}\right.}{f}$$

$\big\downarrow$ 444

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{7}\left(\frac{\int \frac{3b\sin^2(e+fx)\left(\left(a^2+11ba+8b^2\right)\sin^2(e+fx)+2a(4a+3b)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{5b}-\frac{2}{5}(4a+3b)\sin^3(e+fx)\sqrt{\cdots}\right.\right.}{f}$$

$\big\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\int \frac{\sin^2(e+fx)\left(\left(a^2+11ba+8b^2\right)\sin^2(e+fx)+2a(4a+3b)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\frac{2}{5}(4a+3b)\sin^3(e+\cdots\right.\right.}{f}$$

$\big\downarrow$ 444

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\int \frac{a\left(a^2+11ba+8b^2\right)-2(a+2b)\left(a^2-4ba-4b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3b}-\frac{\left(a^2+11ab+8b^2\right)\sin(e+fx)\sqrt{1\cdots}}{3\cdots}\right.\right.\right.$$

$\big\downarrow$ 399

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\frac{a(a+b)\left(2a^2-5ab-8b^2\right)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\int \frac{\sqrt{b\sin^2(\cdots}}{\sqrt{1-\sin\cdots}}}{b}}{3b}\right.\right.\right.$$

$\big\downarrow$ 323

3.138. $\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\frac{a(a+b)\left(2a^2-5ab-8b^2\right)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\left(a^2-4a\right.}{}}{3b}\right.\right.\right.$$

$\downarrow$ 321

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\frac{a(a+b)\left(2a^2-5ab-8b^2\right)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\int\frac{\sqrt{b\sin\ldots}}{\sqrt{1-\ldots}}}{b}}{3b}\right.\right.\right.$$

$\downarrow$ 330

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\frac{a(a+b)\left(2a^2-5ab-8b^2\right)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{a+b\,\mathrm{s}\ldots}}{b\sqrt{\ldots}}}{3b}\right.\right.\right.$$

$\downarrow$ 327

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\frac{a(a+b)\left(2a^2-5ab-8b^2\right)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{a+b\,\mathrm{s}\ldots}}{b\sqrt{\frac{b\sin\ldots}{}}}}{3b}\right.\right.\right.$$

input
```
Int[Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

3.138. $\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-1/7*(b*Sin[e + f*x]^5*Sqrt[1 - Sin[e
+ f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2]) + ((-2*(4*a + 3*b)*Sin[e + f*x]^3*Sq
rt[1 - Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/5 + (3*(-1/3*((a^2 + 11
*a*b + 8*b^2)*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x
]^2])/b + ((-2*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*EllipticE[ArcSin[Sin[e + f*
x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e + f*x]^2)/a]
) + (a*(a + b)*(2*a^2 - 5*a*b - 8*b^2)*EllipticF[ArcSin[Sin[e + f*x]], -(b
/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2]))/(3*b)
))/5)/7))/f
```

3.138.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.138. $\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.138.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(299) = 598$.

Time = 4.04 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.85

method	result
default	$\frac{5b^4(\sin^9(fx+e))+13ab^3(\sin^7(fx+e))+b^4(\sin^7(fx+e))+9a^2b^2(\sin^5(fx+e))+4ab^3(\sin^5(fx+e))+2b^4(\sin^5(fx+e))+2\sqrt{\frac{\cos(2fx+2}{2}}}{}$

input
```
int(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(5*b^4*sin(f*x+e)^9+13*a*b^3*sin(f*x+e)^7+b^4*sin(f*x+e)^7+9*a^2*b^2*
sin(f*x+e)^5+4*a*b^3*sin(f*x+e)^5+2*b^4*sin(f*x+e)^5+2*(cos(f*x+e)^2)^(1/2
)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^4-3*
(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1
/a*b)^(1/2))*a^3*b-13*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*El
lipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2-8*(cos(f*x+e)^2)^(1/2)*((a+b*si
n(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3-2*(cos(f*x
+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1
/2))*a^4+4*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin
(f*x+e),(-1/a*b)^(1/2))*a^3*b+24*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/
a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2+16*(cos(f*x+e)^2)^(1
/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^
3+a^3*b*sin(f*x+e)^3+2*a^2*b^2*sin(f*x+e)^3-9*a*b^3*sin(f*x+e)^3-8*b^4*sin
(f*x+e)^3-a^3*b*sin(f*x+e)-11*a^2*b^2*sin(f*x+e)-8*a*b^3*sin(f*x+e))/b^2/c
os(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.138.5 Fricas [F]

$$\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}\sin(fx+e)^4\,dx$$

input
```
integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(b*cos(f*x + e)^6 - (a + 3*b)*cos(f*x + e)^4 + (2*a + 3*b)*cos(f
*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b), x)
```

3.138.6 Sympy [F(-1)]

Timed out.

$$\int \sin^4(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.138.7 Maxima [F]

$$\int \sin^4(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \sin\left(fx+e\right)^4 \, dx$$

input
```
integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

3.138.8 Giac [F]

$$\int \sin^4(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \sin\left(fx+e\right)^4 \, dx$$

input
```
integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \sin\left(e+f\,x\right)^4\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

input
```
int(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.139 $\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.139.1 Optimal result

Integrand size = 25, antiderivative size = 218

$$\begin{aligned}\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = & -\frac{(3a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f} \\ & -\frac{\cos(e+fx)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f} \\ & +\frac{(3a^2+13ab+8b^2)\,E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{15bf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} \\ & -\frac{a(a+b)(3a+4b)\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{15bf\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

output
```
-1/5*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/f-1/15*(3*a+4*b)*cos(f
*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+1/15*(3*a^2+13*a*b+8*b^2)*(cos
(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*
x+e)^2)^(1/2)/b/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/15*a*(a+b)*(3*a+4*b)*(cos(f
*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+
e)^2/a)^(1/2)/b/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.139.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.92

$$\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{16a(3a^2+13ab+8b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-16a(3a^2+7ab+4b^2)\sqrt{\frac{2a+b-b\cos}{a}}}{2}$$

input
```
Integrate[Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(16*a*(3*a^2 + 13*a*b + 8*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Elli
pticE[e + f*x, -(b/a)] - 16*a*(3*a^2 + 7*a*b + 4*b^2)*Sqrt[(2*a + b - b*Co
s[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] - Sqrt[2]*b*(48*a^2 + 68*a*b
 + 25*b^2 - 4*b*(9*a + 7*b)*Cos[2*(e + f*x)] + 3*b^2*Cos[4*(e + f*x)])*Sin
[2*(e + f*x)])/(240*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.139.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 3649, 3042, 3649, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(e+fx)^2\left(a+b\sin(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3649}$$

$$\frac{1}{5}\int \sqrt{b\sin^2(e+fx)+a}\left((3a+4b)\sin^2(e+fx)+a\right)dx-\frac{\sin(e+fx)\cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\int \sqrt{b\sin(e+fx)^2+a}\left((3a+4b)\sin(e+fx)^2+a\right)dx-\frac{\sin(e+fx)\cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}$$

↓ 3649

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{(3a^2+13ba+8b^2)\sin^2(e+fx)+2a(3a+2b)}{\sqrt{b\sin^2(e+fx)+a}}dx-\frac{(3a+4b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx}}{3f}\right.$$
$$\frac{\sin(e+fx)\cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}$$

↓ 3042

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{(3a^2+13ba+8b^2)\sin(e+fx)^2+2a(3a+2b)}{\sqrt{b\sin(e+fx)^2+a}}dx-\frac{(3a+4b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx}}{3f}\right.$$
$$\frac{\sin(e+fx)\cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}$$

↓ 3651

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{(3a^2+13ab+8b^2)\int\sqrt{b\sin^2(e+fx)+a}dx}{b}-\frac{a(a+b)(3a+4b)\int\frac{1}{\sqrt{b\sin^2(e+fx)+a}}dx}{b}\right)-\frac{(3a+4b)\sin(e}{}\right.$$
$$\frac{\sin(e+fx)\cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}$$

↓ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{(3a^2+13ab+8b^2)\int\sqrt{b\sin(e+fx)^2+a}dx}{b}-\frac{a(a+b)(3a+4b)\int\frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}\right)-\frac{(3a+4b)\sin(e}{}\right.$$
$$\frac{\sin(e+fx)\cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}$$

↓ 3657

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{(3a^2+13ab+8b^2)\sqrt{a+b\sin^2(e+fx)}\int\sqrt{\frac{b\sin^2(e+fx)}{a}+1}dx}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)(3a+4b)\int\frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}\right)-\right.$$
$$\frac{\sin(e+fx)\cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}$$

↓ 3042

3.139. $\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(3a^2+13ab+8b^2\right)\sqrt{a+b\sin^2(e+fx)}\int\sqrt{\frac{b\sin(e+fx)^2}{a}+1}dx}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)(3a+4b)\int\frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}\right)-\right.$$
$$\frac{\sin(e+fx)\cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}$$

$\downarrow$ 3656

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(3a^2+13ab+8b^2\right)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)(3a+4b)\int\frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}\right)-\frac{(3a+\right.$$
$$\frac{\sin(e+fx)\cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}$$

$\downarrow$ 3662

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(3a^2+13ab+8b^2\right)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)(3a+4b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin^2(e+fx}{a}}}}{b\sqrt{a+b\sin^2(e+fx)}}\right.\right.$$
$$\frac{\sin(e+fx)\cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(3a^2+13ab+8b^2\right)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)(3a+4b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin(e+fx)^2}{a}}}}{b\sqrt{a+b\sin^2(e+fx)}}\right.\right.$$
$$\frac{\sin(e+fx)\cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}$$

$\downarrow$ 3661

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(3a^2+13ab+8b^2\right)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)(3a+4b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(e\right.}{bf\sqrt{a+b\sin^2(e+fx)}}\right.\right.$$
$$\frac{\sin(e+fx)\cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}$$

input
```
Int[Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

3.139. $\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

output
```
-1/5*(Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/f + (-1/3*((
3*a + 4*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f + (((3*
a^2 + 13*a*b + 8*b^2)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2
])/(b*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (a*(a + b)*(3*a + 4*b)*EllipticF
[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sin[e +
f*x]^2]))/3)/5
```

3.139.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

3.139.4 Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.97

method	result
default	$-\frac{-3b^3\left(\sin^7(fx+e)\right)-9a\,b^2\left(\sin^5(fx+e)\right)-b^3\left(\sin^5(fx+e)\right)+3\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^3+7a^2\sqrt{}}{}$

input
```
int(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-3*b^3*sin(f*x+e)^7-9*a*b^2*sin(f*x+e)^5-b^3*sin(f*x+e)^5+3*(cos(f*
x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(
1/2))*a^3+7*a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Elliptic
F(sin(f*x+e),(-1/a*b)^(1/2))*b+4*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2
)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-3*(cos(f*x+e)^2)^(1/2)
*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-13*
(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1
/a*b)^(1/2))*a^2*b-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Ell
ipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2-6*a^2*b*sin(f*x+e)^3+5*a*b^2*sin(f
*x+e)^3+4*b^3*sin(f*x+e)^3+6*a^2*b*sin(f*x+e)+4*a*b^2*sin(f*x+e))/b/cos(f*
x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.139.5 Fricas [F]

$$\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cos(f*x + e)^4 - (a + 2*b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos
(f*x + e)^2 + a + b), x)
```

3.139.6 Sympy [F(-1)]

Timed out.

$$\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(sin(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.139.7 Maxima [F]

$$\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

3.139.8 Giac [F]

$$\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \sin\left(e+f\,x\right)^2\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

input
```
int(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.140 $\int (a+b\sin^2(e+fx))^{3/2}\,dx$

3.140.1 Optimal result

Integrand size = 16, antiderivative size = 154

$$\int (a+b\sin^2(e+fx))^{3/2}\,dx = -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{a(a+b)\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-1/3*b*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+2/3*(2*a+b)*(cos(f
*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+
e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*a*(a+b)*(cos(f*x+e)^2)^(1/2)/
cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/f
/(a+b*sin(f*x+e)^2)^(1/2)
```

3.140.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \frac{4\sqrt{2}a(2a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-2\sqrt{2}a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\text{EllipticF}}{6\sqrt{2}f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(4*Sqrt[2]*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e
+ f*x, -(b/a)] - 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a
]*EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e
+ f*x)])/(6*Sqrt[2]*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.140.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {3042, 3659, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin(e+fx)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{3}\int \frac{2b(2a+b)\sin^2(e+fx)+a(3a+b)}{\sqrt{b\sin^2(e+fx)+a}}dx-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{2b(2a+b)\sin(e+fx)^2+a(3a+b)}{\sqrt{b\sin(e+fx)^2+a}}dx-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3651}$$

$$\frac{1}{3}\left(2(2a+b)\int \sqrt{b\sin^2(e+fx)+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}dx\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(2(2a+b)\int \sqrt{b\sin(e+fx)^2+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3657}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin^2(e+fx)}{a}+1}dx}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin(e+fx)^2}{a}+1}dx}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3656}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3662}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}dx}{\sqrt{a+b\sin^2(e+fx)}}\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

3.140. $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\left|-\frac{b}{a}\right.\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin(e+fx)^2}{a}+1}}dx}{\sqrt{a+b\sin^2(e+fx)}}\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3661}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\left|-\frac{b}{a}\right.\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

input
```
Int[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-1/3*(b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f + ((2*(2*a
 + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (
b*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*
Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2]))/3
```

3.140.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.140.4 Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.73

method	result
default	$-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2}{3}-\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)b}{3}+4\sqrt{\frac{\cos(2f}{2}}$

input
```
int((a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/3*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+
e),(-1/a*b)^(1/2))*a^2-1/3*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(
1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b+4/3*(cos(f*x+e)^2)^(1/2)*((a+b
*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2/3*(cos(
f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)
^(1/2))*a*b+1/3*b^2*sin(f*x+e)^5+1/3*a*b*sin(f*x+e)^3-1/3*b^2*sin(f*x+e)^3
-1/3*a*b*sin(f*x+e))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.140.5 Fricas [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^(3/2), x)
```

3.140.6 Sympy [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \int \left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sin(e + f*x)**2)**(3/2), x)
```

3.140.7 Maxima [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.140.8 Giac [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \int \left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2} dx$$

input
```
int((a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int((a + b*sin(e + f*x)^2)^(3/2), x)
```

3.141 $\int \csc^2(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx$

3.141.1 Optimal result

Integrand size = 25, antiderivative size = 181

$$\int \csc^2(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx = -\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{(a-b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{a(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-a*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-(a-b)*EllipticE(sin(f*x+e),(-b/a)
^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*si
n(f*x+e)^2/a)^(1/2)+a*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*
(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.141.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.78

$$\int \csc^2(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx = -\frac{a\left(\sqrt{2}(2a+b-b\cos(2(e+fx)))\cot(e+fx)+2(a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-2(a+b)\sqrt{\cdots}\right.}{2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*(a*(Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x] + 2*(a - b)*S
qrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] - 2*(a +
b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)]))/(f*
Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.141.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 3667, 376, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^{3/2}}{\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{ 3667}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\csc^2(e+fx)\left(b\sin^2(e+fx)+a\right)^{3/2}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 376}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\int \frac{b\left(2a-(a-b)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-a\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+}\right.}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(b\int \frac{2a-(a-b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-a\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+}\right.}{f}$$

$$\Big\downarrow \text{ 399}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(b\left(\frac{a(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{(a-b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)-a\sqrt{}\right.}{f}$$

$\Big\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(b\left(\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin}{b}\right.\right.}{f}$$

$\Big\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(b\left(\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)\right.}{f}$$

$\Big\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(b\left(\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\sqrt{a+b\sin^2(e+fx)}\int \frac{\sqrt{\frac{b\sin^2(e+fx)}{a}}}{\sqrt{1-\sin^2(e+f}}}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.\right.}{f}$$

$\Big\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(b\left(\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+\right.}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.\right.}{f}$$

input
```
Int[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-(a*Csc[e + f*x]*Sqrt[1 - Sin[e + f*x]
^2]*Sqrt[a + b*Sin[e + f*x]^2]) + b*(-(((a - b)*EllipticE[ArcSin[Sin[e + f
*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e + f*x]^2)/a
])) + (a*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e
 + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2]))))/f
```

3.141. $\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.141.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

3.141. $\int \csc^2(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.141.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.96

method	result
default	$\frac{a\left(b\left(\cos^4(fx+e)\right)+(-a-b)\left(\cos^2(fx+e)\right)+\sin(fx+e)\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\left(F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a+F\left(\sin(\right.\right.\right.}{\sin(fx+e)\cos(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}\,f}$

input
```
int(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(b*cos(f*x+e)^4+(-a-b)*cos(f*x+e)^2+sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/
a)^(1/2)*(cos(f*x+e)^2)^(1/2)*(EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a+Elli
pticF(sin(f*x+e),(-1/a*b)^(1/2))*b-EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a+
EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b))/sin(f*x+e)/cos(f*x+e)/(a+b*sin(f*
x+e)^2)^(1/2)/f
```

3.141.5 Fricas [F]

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*csc(f
*x + e)^2, x)
```

3.141.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(csc(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.141.7 Maxima [F]

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^2, x)
```

3.141.8 Giac [F]

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^2, x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^2,x)
```

output
```
int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^2, x)
```

3.142 $\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.142.1 Optimal result

Integrand size = 25, antiderivative size = 236

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=-\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{2(a+2b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}+\frac{(a+b)(2a+3b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-2/3*(a+2*b)*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-1/3*a*cot(f*x+e)*csc(f*
x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/f-2/3*(a+2*b)*EllipticE(sin(f*x+e),(-b/a)^
(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin
(f*x+e)^2/a)^(1/2)+1/3*(a+b)*(2*a+3*b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*
sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+
e)^2)^(1/2)
```

3.142.2 Mathematica [A] (verified)

Time = 3.06 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.85

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\frac{(-8a^2-13ab-6b^2+2(2a^2+7ab+4b^2)\cos(2(e+fx))-b(a+2b)\cos(4(e+fx)))\cot(e+fx)\csc^2(e+fx)}{\sqrt{2}}-4a(a+2b)\sqrt{\frac{2a+b}{}}}{6f\sqrt{2a+b-b}}$$

input
```
Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(((-8*a^2 - 13*a*b - 6*b^2 + 2*(2*a^2 + 7*a*b + 4*b^2)*Cos[2*(e + f*x)] -
b*(a + 2*b)*Cos[4*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2)/Sqrt[2] - 4*a*(
a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)]
 + 2*(2*a^2 + 5*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ellipt
icF[e + f*x, -(b/a)])/(6*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.142.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3667, 376, 445, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^{3/2}}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \text{3667}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\csc^4(e+fx)\left(b\sin^2(e+fx)+a\right)^{3/2}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{376}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\int\frac{\csc^2(e+fx)(b(a+3b)\sin^2(e+fx)+2a(a+2b))}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\frac{1}{3}a\sqrt{1-\sin^2(e+fx)}\csc^3(e-}{f}$$

$\downarrow$ 445

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(-\frac{\int-\frac{ab\left(-2(a+2b)\sin^2(e+fx)+a+3b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-2(a+2b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{\int\frac{ab\left(-2(a+2b)\sin^2(e+fx)+a+3b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-2(a+2b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{}\right.\right.}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(b\int\frac{-2(a+2b)\sin^2(e+fx)+a+3b}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-2(a+2b)\sqrt{1-\sin^2(e+fx)}\csc(e-\right.\right.}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(b\left(\frac{(a+b)(2a+3b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{2(a+2b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+}{b}\right.\right.\right.}{}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(b\left(\frac{(a+b)(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\int\frac{\sqrt{b\sin^2}}{\sqrt{1-\sin}}}{}\right.\right.\right.}{}$$

$\downarrow$ 321

3.142. $\int\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(b\left(\frac{(a+b)(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d}{b}\right.\right.\right.$$

$\downarrow$ 330

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(b\left(\frac{(a+b)(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}\int}{b\sqrt{\frac{b\sin^2}{}}}\right.\right.\right.$$

$\downarrow$ 327

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(b\left(\frac{(a+b)(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}E\Big(}{b\sqrt{\frac{b\sin^2(e+}{a}}}\right.\right.\right.$$

input
```
Int[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-1/3*(a*Csc[e + f*x]^3*Sqrt[1 - Sin[e
+ f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2]) + (-2*(a + 2*b)*Csc[e + f*x]*Sqrt[1
- Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2] + b*((-2*(a + 2*b)*EllipticE[
ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*S
in[e + f*x]^2)/a]) + ((a + b)*(2*a + 3*b)*EllipticF[ArcSin[Sin[e + f*x]],
-(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2])))/3
))/f
```

3.142.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

3.142. $\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.142.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.73

method	result
default	$\frac{2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2\left(\sin^3(fx+e)\right)+5b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\Big(\sin(fx+e}{}$

input
```
int(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+5*b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*
x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3+3*b^2
*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-
1/a*b)^(1/2))*sin(f*x+e)^3-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(
1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3-4*(cos(f*x+e)^2
)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*
a*b*sin(f*x+e)^3+2*a*b*sin(f*x+e)^6+4*b^2*sin(f*x+e)^6+2*a^2*sin(f*x+e)^4+
3*a*b*sin(f*x+e)^4-4*b^2*sin(f*x+e)^4-a^2*sin(f*x+e)^2-5*a*b*sin(f*x+e)^2-
a^2)/sin(f*x+e)^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.142.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 969, normalized size of antiderivative = 4.11

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*((2*((-I*a*b - 2*I*b^2)*cos(f*x + e)^2 + I*a*b + 2*I*b^2)*sqrt(-b)*sqr
t((a^2 + a*b)/b^2)*sin(f*x + e) - ((2*I*a^2 + 5*I*a*b + 2*I*b^2)*cos(f*x +
 e)^2 - 2*I*a^2 - 5*I*a*b - 2*I*b^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt
((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*
b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b
^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((I*a*b + 2*I*b^2)*c
os(f*x + e)^2 - I*a*b - 2*I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x +
e) - ((-2*I*a^2 - 5*I*a*b - 2*I*b^2)*cos(f*x + e)^2 + 2*I*a^2 + 5*I*a*b +
2*I*b^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)
/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f
*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a
^2 + a*b)/b^2))/b^2) + (2*((I*a*b + I*b^2)*cos(f*x + e)^2 - I*a*b - I*b^2)
*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - ((-2*I*a^2 - 7*I*a*b - 3*I*
b^2)*cos(f*x + e)^2 + 2*I*a^2 + 7*I*a*b + 3*I*b^2)*sqrt(-b)*sin(f*x + e))*
sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*
sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a
^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((-I*a
*b - I*b^2)*cos(f*x + e)^2 + I*a*b + I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)
*sin(f*x + e) - ((2*I*a^2 + 7*I*a*b + 3*I*b^2)*cos(f*x + e)^2 - 2*I*a^2 -
7*I*a*b - 3*I*b^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^...
```

3.142.6 Sympy [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.142.7 Maxima [F]

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^4, x)
```

3.142.8 Giac [F]

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^4, x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^4}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^4,x)
```

output
```
int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^4, x)
```

3.143 $\int \left(a+b\sin^2(c+dx)\right)^{5/2}\,dx$

3.143.1 Optimal result

Integrand size = 16, antiderivative size = 210

$$\int \left(a+b\sin^2(c+dx)\right)^{5/2}\,dx=-\frac{4b(2a+b)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin^2(c+dx)}}{15d}-\frac{b\cos(c+dx)\sin(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}+\frac{(23a^2+23ab+8b^2)\,E\left(c+dx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(c+dx)}}{15d\sqrt{1+\frac{b\sin^2(c+dx)}{a}}}-\frac{4a(a+b)(2a+b)\,\text{EllipticF}\left(c+dx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(c+dx)}{a}}}{15d\sqrt{a+b\sin^2(c+dx)}}$$

output
```
-1/5*b*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c)^2)^(3/2)/d-4/15*b*(2*a+b)*cos
(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c)^2)^(1/2)/d+1/15*(23*a^2+23*a*b+8*b^2)*(
cos(d*x+c)^2)^(1/2)/cos(d*x+c)*EllipticE(sin(d*x+c),(-b/a)^(1/2))*(a+b*sin
(d*x+c)^2)^(1/2)/d/(1+b*sin(d*x+c)^2/a)^(1/2)-4/15*a*(a+b)*(2*a+b)*(cos(d*
x+c)^2)^(1/2)/cos(d*x+c)*EllipticF(sin(d*x+c),(-b/a)^(1/2))*(1+b*sin(d*x+c
)^2/a)^(1/2)/d/(a+b*sin(d*x+c)^2)^(1/2)
```

3.143.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.92

$$\int \left(a+b\sin^2(c+dx)\right)^{5/2}\,dx = \frac{16a(23a^2+23ab+8b^2)\sqrt{\frac{2a+b-b\cos(2(c+dx))}{a}}E\left(c+dx\middle|-\frac{b}{a}\right)-64a(2a^2+3ab+b^2)\sqrt{\frac{2a+b-b\cos(\ldots}{a}}\ldots}{2\ldots}$$

input
```
Integrate[(a + b*Sin[c + d*x]^2)^(5/2),x]
```

output
```
(16*a*(23*a^2 + 23*a*b + 8*b^2)*Sqrt[(2*a + b - b*Cos[2*(c + d*x)])/a]*Ell
ipticE[c + d*x, -(b/a)] - 64*a*(2*a^2 + 3*a*b + b^2)*Sqrt[(2*a + b - b*Cos
[2*(c + d*x)])/a]*EllipticF[c + d*x, -(b/a)] - Sqrt[2]*b*(88*a^2 + 88*a*b
+ 25*b^2 - 28*b*(2*a + b)*Cos[2*(c + d*x)] + 3*b^2*Cos[4*(c + d*x)])*Sin[2
*(c + d*x)])/(240*d*Sqrt[2*a + b - b*Cos[2*(c + d*x)]])
```

3.143.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {3042, 3659, 3042, 3649, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin^2(c+dx)\right)^{5/2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a+b\sin(c+dx)^2\right)^{5/2}dx$$

$$\Big\downarrow \ 3659$$

$$\frac{1}{5}\int \sqrt{b\sin^2(c+dx)+a}\left(4b(2a+b)\sin^2(c+dx)+a(5a+b)\right)dx-\frac{b\sin(c+dx)\cos(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\int \sqrt{b\sin(c+dx)^2+a}\left(4b(2a+b)\sin(c+dx)^2+a(5a+b)\right)dx-\frac{b\sin(c+dx)\cos(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}$$

$\downarrow$ 3649

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{b\left(23a^2+23ba+8b^2\right)\sin^2(c+dx)+a\left(15a^2+11ba+4b^2\right)}{\sqrt{b\sin^2(c+dx)+a}}dx-\frac{4b(2a+b)\sin(c+dx)\cos(c+dx)\sqrt{a+b}}{3d}\right.$$
$$\frac{b\sin(c+dx)\cos(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{b\left(23a^2+23ba+8b^2\right)\sin(c+dx)^2+a\left(15a^2+11ba+4b^2\right)}{\sqrt{b\sin(c+dx)^2+a}}dx-\frac{4b(2a+b)\sin(c+dx)\cos(c+dx)\sqrt{a+b}}{3d}\right.$$
$$\frac{b\sin(c+dx)\cos(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}$$

$\downarrow$ 3651

$$\frac{1}{5}\left(\frac{1}{3}\left(\left(23a^2+23ab+8b^2\right)\int \sqrt{b\sin^2(c+dx)+a}dx-4a(a+b)(2a+b)\int \frac{1}{\sqrt{b\sin^2(c+dx)+a}}dx\right)-\frac{4b(2a+b}{}\right.$$
$$\frac{b\sin(c+dx)\cos(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\left(23a^2+23ab+8b^2\right)\int \sqrt{b\sin(c+dx)^2+a}dx-4a(a+b)(2a+b)\int \frac{1}{\sqrt{b\sin(c+dx)^2+a}}dx\right)-\frac{4b(2a+b}{}\right.$$
$$\frac{b\sin(c+dx)\cos(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}$$

$\downarrow$ 3657

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2+23ab+8b^2\right)\sqrt{a+b\sin^2(c+dx)}\int \sqrt{\frac{b\sin^2(c+dx)}{a}+1}dx}{\sqrt{\frac{b\sin^2(c+dx)}{a}+1}}-4a(a+b)(2a+b)\int \frac{1}{\sqrt{b\sin(c+dx)^2+a}}\right.\right.$$
$$\frac{b\sin(c+dx)\cos(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}$$

$\downarrow$ 3042

3.143. $\int \left(a+b\sin^2(c+dx)\right)^{5/2}\,dx$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2+23ab+8b^2\right)\sqrt{a+b\sin^2(c+dx)}\int\sqrt{\frac{b\sin(c+dx)^2}{a}+1}dx}{\sqrt{\frac{b\sin^2(c+dx)}{a}+1}}-4a(a+b)(2a+b)\int\frac{1}{\sqrt{b\sin(c+dx)^2+a}}\right.\right.$$
$$\frac{b\sin(c+dx)\cos(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}$$

$\big\downarrow$ 3656

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2+23ab+8b^2\right)\sqrt{a+b\sin^2(c+dx)}E\left(c+dx\middle|-\frac{b}{a}\right)}{d\sqrt{\frac{b\sin^2(c+dx)}{a}+1}}-4a(a+b)(2a+b)\int\frac{1}{\sqrt{b\sin(c+dx)^2+a}}dx\right)\right.$$
$$\frac{b\sin(c+dx)\cos(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}$$

$\big\downarrow$ 3662

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2+23ab+8b^2\right)\sqrt{a+b\sin^2(c+dx)}E\left(c+dx\middle|-\frac{b}{a}\right)}{d\sqrt{\frac{b\sin^2(c+dx)}{a}+1}}-\frac{4a(a+b)(2a+b)\sqrt{\frac{b\sin^2(c+dx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin^2(c+dx)}{a}}}}{\sqrt{a+b\sin^2(c+dx)}}\right.\right.$$
$$\frac{b\sin(c+dx)\cos(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2+23ab+8b^2\right)\sqrt{a+b\sin^2(c+dx)}E\left(c+dx\middle|-\frac{b}{a}\right)}{d\sqrt{\frac{b\sin^2(c+dx)}{a}+1}}-\frac{4a(a+b)(2a+b)\sqrt{\frac{b\sin^2(c+dx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin(c+dx)}{a}}}}{\sqrt{a+b\sin^2(c+dx)}}\right.\right.$$
$$\frac{b\sin(c+dx)\cos(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}$$

$\big\downarrow$ 3661

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2+23ab+8b^2\right)\sqrt{a+b\sin^2(c+dx)}E\left(c+dx\middle|-\frac{b}{a}\right)}{d\sqrt{\frac{b\sin^2(c+dx)}{a}+1}}-\frac{4a(a+b)(2a+b)\sqrt{\frac{b\sin^2(c+dx)}{a}+1}\,\text{EllipticF}\,(c}{d\sqrt{a+b\sin^2(c+dx)}}\right.\right.$$
$$\frac{b\sin(c+dx)\cos(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}$$

input
```
Int[(a + b*Sin[c + d*x]^2)^(5/2),x]
```

3.143. $\int\left(a+b\sin^2(c+dx)\right)^{5/2}\,dx$

output
```
-1/5*(b*Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x]^2)^(3/2))/d + ((-4*b
*(2*a + b)*Cos[c + d*x]*Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]^2])/(3*d) + (
((23*a^2 + 23*a*b + 8*b^2)*EllipticE[c + d*x, -(b/a)]*Sqrt[a + b*Sin[c + d
*x]^2])/(d*Sqrt[1 + (b*Sin[c + d*x]^2)/a]) - (4*a*(a + b)*(2*a + b)*Ellipt
icF[c + d*x, -(b/a)]*Sqrt[1 + (b*Sin[c + d*x]^2)/a])/(d*Sqrt[a + b*Sin[c +
 d*x]^2]))/3)/5
```

3.143.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

3.143. $\int (a+b\sin^2(c+dx))^{5/2}\,dx$

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.143.4 Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.08

method	result
default	$\frac{-\frac{b^{3}\left(\cos^{6}(dx+c)\right)\sin(dx+c)}{5}+\frac{\left(14a\,b^{2}+10b^{3}\right)\left(\cos^{4}(dx+c)\right)\sin(dx+c)}{15}+\frac{\left(-11a^{2}b-18a\,b^{2}-7b^{3}\right)\left(\cos^{2}(dx+c)\right)\sin(dx+c)}{15}-\frac{8a^{3}\sqrt{\frac{\cos(2dx+2c)}{2}+\frac{1}{2}}}{}}{}$

input
```
int((a+b*sin(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/5*b^3*cos(d*x+c)^6*sin(d*x+c)+1/15*(14*a*b^2+10*b^3)*cos(d*x+c)^4*sin(
d*x+c)+1/15*(-11*a^2*b-18*a*b^2-7*b^3)*cos(d*x+c)^2*sin(d*x+c)-8/15*a^3*(c
os(d*x+c)^2)^(1/2)*(-b/a*cos(d*x+c)^2+(a+b)/a)^(1/2)*EllipticF(sin(d*x+c),
(-1/a*b)^(1/2))-4/5*a^2*(cos(d*x+c)^2)^(1/2)*(-b/a*cos(d*x+c)^2+(a+b)/a)^(
1/2)*EllipticF(sin(d*x+c),(-1/a*b)^(1/2))*b-4/15*a*(cos(d*x+c)^2)^(1/2)*(-
b/a*cos(d*x+c)^2+(a+b)/a)^(1/2)*EllipticF(sin(d*x+c),(-1/a*b)^(1/2))*b^2+2
3/15*(cos(d*x+c)^2)^(1/2)*(-b/a*cos(d*x+c)^2+(a+b)/a)^(1/2)*EllipticE(sin(
d*x+c),(-1/a*b)^(1/2))*a^3+23/15*(cos(d*x+c)^2)^(1/2)*(-b/a*cos(d*x+c)^2+(
a+b)/a)^(1/2)*EllipticE(sin(d*x+c),(-1/a*b)^(1/2))*a^2*b+8/15*(cos(d*x+c)^
2)^(1/2)*(-b/a*cos(d*x+c)^2+(a+b)/a)^(1/2)*EllipticE(sin(d*x+c),(-1/a*b)^(
1/2))*a*b^2)/cos(d*x+c)/(a+b*sin(d*x+c)^2)^(1/2)/d
```

3.143.5 Fricas [F]

$$\int \left(a+b\sin^2(c+dx)\right)^{5/2}\,dx = \int \left(b\sin\left(dx+c\right)^2+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sin(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b
+ b^2)*sqrt(-b*cos(d*x + c)^2 + a + b), x)
```

3.143.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(c+dx)\right)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c)**2)**(5/2),x)
```

output
```
Timed out
```

3.143.7 Maxima [F]

$$\int \left(a+b\sin^2(c+dx)\right)^{5/2}\,dx = \int \left(b\sin\left(dx+c\right)^2+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sin(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^2 + a)^(5/2), x)
```

3.143.8 Giac [F]

$$\int \left(a+b\sin^2(c+dx)\right)^{5/2}\,dx=\int \left(b\sin\left(dx+c\right)^2+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sin(d*x+c)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^2 + a)^(5/2), x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(c+dx)\right)^{5/2}\,dx=\int \left(b\sin\left(c+d\,x\right)^2+a\right)^{5/2}\,dx$$

input
```
int((a + b*sin(c + d*x)^2)^(5/2),x)
```

output
```
int((a + b*sin(c + d*x)^2)^(5/2), x)
```

3.144 $\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.144.1 Optimal result

Integrand size = 25, antiderivative size = 83

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{(a-b)\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2b^{3/2}f} - \frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2bf}$$

output
```
1/2*(a-b)*arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/b^(3/2)/f-
1/2*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/b/f
```

3.144.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{2a+b-b\cos(2(e+fx))}}{2\sqrt{2}bf} + \frac{(a-b)\log\left(\sqrt{2}\sqrt{-b}\cos(e+fx)+\sqrt{2a+b-b\cos(2(e+fx))}\right)}{2\sqrt{-b}bf}$$

input
```
Integrate[Sin[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-1/2*(Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])/(Sqrt[2]*b*f) + ((a
 - b)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*x)
]]])/(2*Sqrt[-b]*b*f)
```

3.144.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3665, 299, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^3}{\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{1-\cos^2(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{299}$$

$$-\frac{\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2b}-\frac{(a-b)\int \frac{1}{\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)}{2b}}{f}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2b}-\frac{(a-b)\int \frac{1}{\frac{b\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}+1}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}}{2b}}{f}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2b}-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2b^{3/2}}}{f}$$

input
```
Int[Sin[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-((-1/2*((a - b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]
^2]])/b^(3/2) + (Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(2*b))/f)
```

3.144.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.144.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(71) = 142$.

Time = 0.91 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.78

method	result
default	$\frac{\sqrt{-(-b(\sin^2(fx+e))-a)(\cos^2(fx+e))}\left(-\frac{\sqrt{-\left(-b\left(\sin^2(fx+e)\right)-a\right)\left(\cos^2(fx+e)\right)}}{2b}+\frac{(-a+b)\arctan\left(\frac{\sqrt{b}\left(\sin^2(fx+e)-\frac{-a+b}{2b}\right)}{\sqrt{-\left(-b\left(\sin^2(fx+e)\right)-a\right)\left(\cos^2(fx+e\right.}}\right.}{4b^{\frac{3}{2}}}\right.}{\cos(fx+e)\sqrt{a+b(\sin^2(fx+e))}\,f}$

input
```
int(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(-1/2/b*(-(-b*sin(f*x+e)^2-a)*co
s(f*x+e)^2)^(1/2)+1/4*(-a+b)/b^(3/2)*arctan(b^(1/2)*(sin(f*x+e)^2-1/2*(-a+
b)/b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)))/cos(f*x+e)/(a+b*sin(f*x+
e)^2)^(1/2)/f
```

3.144.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(71) = 142$.

Time = 0.36 (sec) , antiderivative size = 438, normalized size of antiderivative = 5.28

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

$$= \left[-\frac{8\sqrt{-b\cos(fx+e)^2+a+b}b\cos(fx+e)-(a-b)\sqrt{-b}\log\left(128\,b^4\cos(fx+e)^8-256\,(ab^3+b^4)\cos\right.}{}\right.$$

$$-\frac{(a-b)\sqrt{b}\arctan\left(\frac{\left(8\,b^2\cos(fx+e)^4-8\,(ab+b^2)\cos(fx+e)^2+a^2+2\,ab+b^2\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{b}}{4\left(2\,b^3\cos(fx+e)^5-3\,(ab^2+b^3)\cos(fx+e)^3+(a^2b+2\,ab^2+b^3)\cos(fx+e)\right)}\right)+4\sqrt{-b\cos(fx+\right.}{8\,b^2f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) - (a - b)*sqrt(-b
)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2
*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3
*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 +
b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-
b*cos(f*x + e)^2 + a + b)*sqrt(-b)))/(b^2*f), -1/8*((a - b)*sqrt(b)*arctan
(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b +
b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*
b^2 + b^3)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) + 4*sqr
t(-b*cos(f*x + e)^2 + a + b)*b*cos(f*x + e))/(b^2*f)]
```

3.144.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\frac{a\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} - \frac{\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} - \frac{\sqrt{-b\cos(fx+e)^2+a+b}\cos(fx+e)}{b}}{2f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(a*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/b^(3/2) - arcsin(b*cos(f*x +
 e)/sqrt((a + b)*b))/sqrt(b) - sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e
)/b)/f
```

3.144.8 Giac [F]

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sin(fx+e)^3}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sin(e+f\,x)^3}{\sqrt{b\sin(e+f\,x)^2+a}}\,dx$$

input
```
int(sin(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.145 $\int \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.145.1 Optimal result

Integrand size = 23, antiderivative size = 41

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{\sqrt{b}f}$$

output
```
-arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f/b^(1/2)
```

3.145.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.29

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\log\left(\sqrt{2}\sqrt{-b}\cos(e+fx)+\sqrt{2a+b-b\cos(2(e+fx))}\right)}{\sqrt{-b}f}$$

input
```
Integrate[Sin[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-(Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]]/
(Sqrt[-b]*f))
```

3.145.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3665, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{1}{\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\int \frac{1}{\frac{b\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}+1}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}}{f}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{\sqrt{b}f}$$

input
```
Int[Sin[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-(ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(Sqrt[b]*f
))
```

3.145.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.145.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(35) = 70$.

Time $= 0.65$ (sec) , antiderivative size $= 99$, normalized size of antiderivative $= 2.41$

method	result	size
default	$\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\,\arctan\left(\frac{2b\left(\sin^2(fx+e)\right)+a-b}{2\sqrt{b}\,\sqrt{\left(\cos^2(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)}}\right)}{2\sqrt{b}\,\cos(fx+e)\sqrt{a+b(\sin^2(fx+e))}\,f}$	99

input
```
int(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)/b^(1/2)*arctan(1/2/b^(1/2)*(2*
b*sin(f*x+e)^2+a-b)/(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))/cos(f*x+e)/(a
+b*sin(f*x+e)^2)^(1/2)/f
```

3.145.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(35) = 70.

Time = 0.35 (sec) , antiderivative size = 370, normalized size of antiderivative = 9.02

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$= \left[-\frac{\sqrt{-b}\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+b^4\right)\cos\left(fx+e\right)^6+160\left(a^2b^2+2\,ab^3+b^4\right)\cos\left(fx+e\right)^4+\right.}{}\right.$$

input
```
integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)
^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^
2 + 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b
 + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x
 + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b))/(b*f), 1/4*arctan(1/4*(8*
b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqr
t(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^
3)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)))/(sqrt(b)*f)]
```

3.145.6 Sympy [F]

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sin\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sin(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.59

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}f}$$

input
```
integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/(sqrt(b)*f)
```

3.145.8 Giac [F]

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sin(e+f\,x)}{\sqrt{b\sin(e+f\,x)^2+a}}\,dx$$

input
```
int(sin(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.146 $\int \frac{\csc(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.146.1 Optimal result

Integrand size = 23, antiderivative size = 41

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{\sqrt{a}f}$$

output
```
-arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f/a^(1/2)
```

3.146.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)}{\sqrt{a}f}$$

input
```
Integrate[Csc[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-(ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]
]]/(Sqrt[a]*f))
```

3.146.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3665, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{1}{(1-\cos^2(e+fx))\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\int \frac{1}{1-\frac{a\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

input
```
Int[Csc[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-(ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(Sqrt[a]*
f))
```

3.146.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.146.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(35) = 70$.

Time = 0.88 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.73

method	result	size
default	$-\frac{\sqrt{(\cos^2(fx+e))(a+b(\sin^2(fx+e)))}\,\ln\left(\frac{(a-b)\left(\cos^2(fx+e)\right)+2\sqrt{a}\,\sqrt{-b\left(\cos^4(fx+e)\right)+(a+b)\left(\cos^2(fx+e)\right)}+a+b}{\sin(fx+e)^2}\right)}{2\sqrt{a}\,\cos(fx+e)\sqrt{a+b(\sin^2(fx+e))}\,f}$	112

input
```
int(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)/a^(1/2)*ln(((a-b)*cos(f*x+e)^
2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)/
cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.146.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(35) = 70$.

Time = 0.31 (sec) , antiderivative size = 219, normalized size of antiderivative = 5.34

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \left[\frac{\log\left(\frac{2\left((a^2-6ab+b^2)\cos(fx+e)^4+2\,(3a^2+2ab-b^2)\cos(fx+e)^2-4\left((a-b)\cos(fx+e)^3+(a+b)\cos(fx+e)\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a}+a^2}{\cos(fx+e)^4-2\cos(fx+e)^2+1}\right.}{4\sqrt{a}f}\right.$$

input
```
integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*c
os(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*
cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*c
os(f*x + e)^2 + 1))/(sqrt(a)*f), 1/2*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x
 + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e
)^3 - (a^2 + a*b)*cos(f*x + e)))/(a*f)]
```

3.146.6 Sympy [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\csc(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

3.146.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(35) = 70$.

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.66

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$= -\frac{\frac{\log\left(b-\frac{\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a}}{\cos(fx+e)-1}-\frac{a}{\cos(fx+e)-1}\right)}{\sqrt{a}}-\frac{\log\left(-b+\frac{\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a}}{\cos(fx+e)+1}+\frac{a}{\cos(fx+e)+1}\right)}{\sqrt{a}}}{2f}$$

input
```
integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(log(b - sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x + e) - 1) -
 a/(cos(f*x + e) - 1))/sqrt(a) - log(-b + sqrt(-b*cos(f*x + e)^2 + a + b)*
sqrt(a)/(cos(f*x + e) + 1) + a/(cos(f*x + e) + 1))/sqrt(a))/f
```

3.146.8 Giac [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\csc(fx+e)}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{1}{\sin(e+f\,x)\,\sqrt{b\sin(e+f\,x)^2+a}}\,dx$$

input
```
int(1/(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

3.147 $\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.147.1 Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{(a-b)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{2af}$$

output
```
-1/2*(a-b)*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/a^(3/2)/
f-1/2*cot(f*x+e)*csc(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/a/f
```

3.147.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.15

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{-2(a-b)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right) - \sqrt{2}\sqrt{a}\sqrt{2a+b-b\cos(2(e+fx))}\cot(e+fx)\csc(e+fx)}{4a^{3/2}f}$$

input
```
Integrate[Csc[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-2*(a - b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*
(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Cot[e +
f*x]*Csc[e + f*x])/(4*a^(3/2)*f)
```

3.147.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3665, 296, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^3\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{1}{(1-\cos^2(e+fx))^2\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{296}$$

$$-\frac{\frac{(a-b)\int \frac{1}{\left(1-\cos^2(e+fx)\right)\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)}{2a}+\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2a(1-\cos^2(e+fx))}}{f}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\frac{(a-b)\int \frac{1}{1-\frac{a\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}}{2a}+\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2a(1-\cos^2(e+fx))}}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2a^{3/2}}+\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2a(1-\cos^2(e+fx))}}{f}$$

input
```
Int[Csc[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-((((a - b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]
)/(2*a^(3/2)) + (Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(2*a*(1 - Co
s[e + f*x]^2)))/f)
```

3.147.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.147. $\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.147.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(77) = 154.

Time = 1.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.85

method	result
default	$\frac{\sqrt{-\left(-b\left(\sin^2(fx+e)\right)-a\right)\left(\cos^2(fx+e)\right)}\left(-\frac{\sqrt{-\left(-b\left(\sin^2(fx+e)\right)-a\right)\left(\cos^2(fx+e)\right)}}{2a\sin(fx+e)^2}+\frac{(-a+b)\ln\left(\frac{2a+(-a+b)\left(\sin^2(fx+e)\right)+2\sqrt{a}\sqrt{-\left(-b\left(\mathrm{si}\right.\right.}}{\sin(fx+e)^2}\right.}{4a^{\frac{3}{2}}}\right.}{\cos(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}\,f}$

input
```
int(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(-1/2/a/sin(f*x+e)^2*(-(-b*sin(f
*x+e)^2-a)*cos(f*x+e)^2)^(1/2)+1/4*(-a+b)/a^(3/2)*ln((2*a+(-a+b)*sin(f*x+e
)^2+2*a^(1/2)*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/sin(f*x+e)^2))/co
s(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.90

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$= \left[\frac{4\sqrt{-b\cos{(fx+e)}^2+a+b}a\cos{(fx+e)}-\left((a-b)\cos{(fx+e)}^2-a+b\right)\sqrt{a}\log\left(\frac{2\left(\left(a^2-6ab+b^2\right)\cos(fx\right.}{}\right.}{8\left(a^2f\cos{(fx+e)}^2\right.}\right.$$

input
```
integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e) - ((a - b)*cos(f*x
+ e)^2 - a + b)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a
^2 + 2*a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos
(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(c
os(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(a^2*f*cos(f*x + e)^2 - a^2*f), 1/
4*(((a - b)*cos(f*x + e)^2 - a + b)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x
+ e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)
^3 - (a^2 + a*b)*cos(f*x + e))) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(
f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f)]
```

3.147.6 Sympy [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\csc^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)**3/sqrt(a + b*sin(e + f*x)**2), x)
```

3.147.7 Maxima [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\csc(fx+e)^3}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^3/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.147.8 Giac [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\csc\left(fx+e\right)^3}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^3\,\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(1/(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

3.148 $\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.148.1 Optimal result

Integrand size = 25, antiderivative size = 206

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{2(a-b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3b^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{a(2a-b)\sqrt{\cos^2(e+fx)}\,\mathrm{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3b^2f\sqrt{a+b\sin^2(e+fx)}}$$

output

```
-1/3*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f-2/3*(a-b)*Elliptic
E(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)
^2)^(1/2)/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*a*(2*a-b)*EllipticF(sin(f*x
+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/
2)/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.148.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.79

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$= \frac{-4\sqrt{2}a(a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+2\sqrt{2}a(2a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\operatorname{EllipticF}\left(e+fx,-\right.}{6\sqrt{2}b^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Sin[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-4*Sqrt[2]*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
 f*x, -(b/a)] + 2*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/
a]*EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e
 + f*x)])/(6*Sqrt[2]*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.148.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3667, 381, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(e+fx)^4}{\sqrt{a+b\sin(e+fx)^2}}dx$$
$$\Big\downarrow \text{3667}$$
$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sin^4(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{f}$$
$$\Big\downarrow \text{381}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int \frac{a-2(a-b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3b}-\frac{\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}{3b}\right)}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{a(2a-b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{2(a-b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}}{3b}-\sin(e+fx)\sqrt{1\right.}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{a(2a-b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}}{3b}\right.}{f}$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{a(2a-b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}}{3b}-\sin(e+fx)\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{a(2a-b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\sqrt{a+b\sin^2(e+fx)}\int \frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{3b}\right.}{f}$$

$\downarrow$ 327

3.148. $\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{a(2a-b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{3b}-\right.}{f}$$

input
```
Int[Sin[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-1/3*(Sin[e + f*x]*Sqrt[1 - Sin[e + f*
x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/b + ((-2*(a - b)*EllipticE[ArcSin[Sin[e
+ f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e + f*x]^2
)/a]) + (a*(2*a - b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*S
in[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2]))/(3*b)))/f
```

3.148.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.148.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.30

method	result
default	$\frac{b^2(\sin^5(fx+e))+2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2-a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx\right.}{}$

input
```
int(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b^2*sin(f*x+e)^5+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*
EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(
f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*(cos(f*x+e)^2)
^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a
^2+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e
),(-1/a*b)^(1/2))*a*b+a*b*sin(f*x+e)^3-b^2*sin(f*x+e)^3-a*b*sin(f*x+e))/b^
2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.148.5 Fricas [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b*cos(f*x + e)^2 +
 a + b)/(b*cos(f*x + e)^2 - a - b), x)
```

3.148.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.148.7 Maxima [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.148.8 Giac [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)^4}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(sin(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.149 $\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.149.1 Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{bf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{a\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{bf\sqrt{a+b\sin^2(e+fx)}}$$

output
```
(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*si
n(f*x+e)^2)^(1/2)/b/f/(1+b*sin(f*x+e)^2/a)^(1/2)-a*(cos(f*x+e)^2)^(1/2)/co
s(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/b/f
/(a+b*sin(f*x+e)^2)^(1/2)
```

3.149.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\sqrt{2a+b-b\cos(2(e+fx))}\left(E\left(e+fx\middle|-\frac{b}{a}\right)-\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\right)}{bf\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}}$$

input
```
Integrate[Sin[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*(EllipticE[e + f*x, -(b/a)] - Elliptic
F[e + f*x, -(b/a)]))/(b*f*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a])
```

3.149.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^2}{\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3651}$$

$$\frac{\int \sqrt{b\sin^2(e+fx)+a}dx}{b}-\frac{a\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sin(e+fx)^2+a}dx}{b}-\frac{a\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin^2(e+fx)}{a}+1}dx}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin(e+fx)^2}{a}+1}dx}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}$$

$$\Big\downarrow \text{3656}$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\int\frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}$$

$$\Big\downarrow \text{3662}$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}dx}{b\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin(e+fx)^2}{a}+1}}dx}{b\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3661}$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}$$

input
```
Int[Sin[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*f*Sqrt[1 + (b*S
in[e + f*x]^2)/a]) - (a*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x
]^2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

3.149.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.149.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,\left(F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)-E\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)\right)}{b\cos(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}\,f}$	93

input
```
int(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)/b*(EllipticF(sin(f*x+
e),(-1/a*b)^(1/2))-EllipticE(sin(f*x+e),(-1/a*b)^(1/2)))/cos(f*x+e)/(a+b*s
in(f*x+e)^2)^(1/2)/f
```

3.149.5 Fricas [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx=\int \frac{\sin\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-b*cos(f*x + e)^2 + a + b)*(cos(f*x + e)^2 - 1)/(b*cos(f*x +
 e)^2 - a - b), x)
```

3.149.6 Sympy [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

input
```
integrate(sin(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sin(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

3.149.7 Maxima [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sin(fx+e)^2}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.149.8 Giac [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sin(fx+e)^2}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)^2}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(sin(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.150 $\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.150.1 Optimal result

Integrand size = 16, antiderivative size = 51

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*si
n(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.150.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)}{f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(f*Sqr
t[2*a + b - b*Cos[2*(e + f*x)]])
```

3.150.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3662}$$

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}dx}{\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin(e+fx)^2}{a}+1}}dx}{\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3661}$$

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

input
```
Int[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*
Sin[e + f*x]^2])
```

3.150.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.150.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\sqrt{\frac{a+b\left(\sin^{2}\left(fx+e\right)\right)}{a}}\,\operatorname{am}^{-1}\left(fx+e\middle|\frac{i\sqrt{b}}{\sqrt{a}}\right)}{f\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}}$	52

input
```
int(1/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(a+b*sin(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*InverseJacobiAM(
f*x+e,I/a^(1/2)*b^(1/2))
```

3.150.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 305, normalized size of antiderivative = 5.98

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\left(2i\sqrt{-b}b\sqrt{\frac{a^2+ab}{b^2}}+(-2i\,a-i\,b)\sqrt{-b}\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}F(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}(\cos{(fx+e)}+i\,\mathrm{s}\right.}{}$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-((2*I*sqrt(-b)*b*sqrt((a^2 + a*b)/b^2) + (-2*I*a - I*b)*sqrt(-b))*sqrt((2
*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a
^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*
a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (-2*I*sqrt(-b)*b
*sqrt((a^2 + a*b)/b^2) + (2*I*a + I*b)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b
)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2
*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a
*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/(b^2*f)
```

3.150.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\sin^2{(e+fx)}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sin(e + f*x)**2), x)
```

3.150.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sin{(fx+e)}^2+a}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.150.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sin{(fx+e)}^2+a}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sin{(e+f\,x)}^2+a}}\,dx$$

input
```
int(1/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(1/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.151 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.151.1 Optimal result

Integrand size = 25, antiderivative size = 177

$$\begin{aligned}\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx &= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} \\ &\quad - \frac{\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} \\ &\quad + \frac{\sqrt{\cos^2(e+fx)}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

output
```
-cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a/f-EllipticE(sin(f*x+e),(-b/a)^(1/2)
)*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a/f/(1+b*sin(f*
x+e)^2/a)^(1/2)+EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^
2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.151.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$= \frac{-\sqrt{2}(2a+b-b\cos(2(e+fx)))\cot(e+fx)-2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\ldots}{2af\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Csc[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-(Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x]) - 2*a*Sqrt[(2*a +
b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2*a*Sqrt[(2*a + b
- b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(2*a*f*Sqrt[2*a + b -
 b*Cos[2*(e + f*x)]])
```

3.151.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3667, 382, 25, 27, 389, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(e+fx)^2\sqrt{a+b\sin(e+fx)^2}}dx$$
$$\Big\downarrow \text{3667}$$
$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\csc^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{f}$$
$$\Big\downarrow \text{382}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int -\frac{b\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}\right)}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{\int \frac{b\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}\right)}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{b\int \frac{\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}\right)}{f}$$

$\downarrow$ 389

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{b\left(\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}\right)}{a}-\frac{\sqrt{1-\sin^2(e+fx)}\text{cs}}{}\right.}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{b\left(\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}\right)}{a}-\sqrt{}\right.}{f}$$

$\downarrow$ 321

3.151. $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{b\left(\frac{\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}\right)}{a}-\frac{\sqrt{1-\sin^2(e+fx}}{}\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{b\left(\frac{\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}\right)}{a}-\right.}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{b\left(\frac{\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}\right)}{a}-\frac{\sqrt{1-}}{}\right.}{f}$$

input
```
Int[Csc[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-((Csc[e + f*x]*Sqrt[1 - Sin[e + f*x]^
2]*Sqrt[a + b*Sin[e + f*x]^2])/a) - (b*((EllipticE[ArcSin[Sin[e + f*x]], -
(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (a
*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(
b*Sqrt[a + b*Sin[e + f*x]^2])))/a))/f
```

3.151. $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.151.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

3.151. $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.151.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.79

method	result
default	$\frac{b(\cos^4(fx+e))+(-a-b)(\cos^2(fx+e))+\sin(fx+e)\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,a\left(F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)-E\left(\sin(fx+\right.\right.}{a\sin(fx+e)\cos(fx+e)\sqrt{a+b(\sin^2(fx+e))}\,f}$

input
```
int(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*cos(f*x+e)^4+(-a-b)*cos(f*x+e)^2+sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)
^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(EllipticF(sin(f*x+e),(-1/a*b)^(1/2))-Ellipt
icE(sin(f*x+e),(-1/a*b)^(1/2))))/a/sin(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2
)^(1/2)/f
```

3.151.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 643, normalized size of antiderivative = 3.63

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx =$$

$$-\frac{-4i\sqrt{-b}b\sqrt{\frac{2b\sqrt{\frac{a^2+ab}{b^2}}+2a+b}{b}}\sqrt{\frac{a^2+ab}{b^2}}F\left(\arcsin\left(\sqrt{\frac{2b\sqrt{\frac{a^2+ab}{b^2}}+2a+b}{b}}(\cos(fx+e)+i\sin(fx+e))\right)\,\middle|\,\frac{8a^2+8a\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(-4*I*sqrt(-b)*b*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*sqrt((
a^2 + a*b)/b^2)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a +
b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b +
b^2)*sqrt((a^2 + a*b)/b^2))/b^2)*sin(f*x + e) + 4*I*sqrt(-b)*b*sqrt((2*b*s
qrt((a^2 + a*b)/b^2) + 2*a + b)/b)*sqrt((a^2 + a*b)/b^2)*elliptic_f(arcsin
(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x +
 e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2)*
sin(f*x + e) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) + (2*I*sqr
t(-b)*b*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + (2*I*a + I*b)*sqrt(-b)*sin(f*
x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sq
rt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e)
)), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (
-2*I*sqrt(-b)*b*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + (-2*I*a - I*b)*sqrt(-
b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(
arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin
(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))
/b^2))/(a*b*f*sin(f*x + e))
```

3.151.6 Sympy [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

3.151.7 Maxima [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\csc(fx+e)^2}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.151.8 Giac [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\csc\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^2\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

3.152 $\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.152.1 Optimal result

Integrand size = 25, antiderivative size = 244

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{2(a-b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{(2a-b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3af\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-2/3*(a-b)*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a^2/f-1/3*cot(f*x+e)*csc(f*
x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/a/f-2/3*(a-b)*EllipticE(sin(f*x+e),(-b/a)^
(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^2/f/(1+b
*sin(f*x+e)^2/a)^(1/2)+1/3*(2*a-b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(
f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/a/f/(a+b*sin(f*x+e)
^2)^(1/2)
```

3.152.2 Mathematica [A] (verified)

Time = 2.68 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.80

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$= \frac{\frac{(-8a^2-ab+3b^2+2(2a^2+ab-2b^2)\cos(2(e+fx))+b(-a+b)\cos(4(e+fx)))\cot(e+fx)\csc^2(e+fx)}{\sqrt{2}} - 4a(a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E(}{6a^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Csc[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(((-8*a^2 - a*b + 3*b^2 + 2*(2*a^2 + a*b - 2*b^2)*Cos[2*(e + f*x)] + b*(-a
 + b)*Cos[4*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2)/Sqrt[2] - 4*a*(a - b)
*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2*a*(
2*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)]
)/(6*a^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.152.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3667, 382, 445, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin(e+fx)^4\sqrt{a+b\sin(e+fx)^2}}dx$$
$$\Big\downarrow \; 3667$$
$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\csc^4(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{f}$$
$$\Big\downarrow \; 382$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int \frac{\csc^2(e+fx)\left(b\sin^2(e+fx)+2(a-b)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3a}-\frac{\sqrt{1-\sin^2(e+fx)}\csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a}\right)}{f}$$

$$\Big\downarrow \text{ 445}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{\int -\frac{b\left(a-2(a-b)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{2(a-b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}}{3a}-\frac{\sqrt{1-\sin^2 \cdots}}{\cdots}\right.}{f \cdots}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{\int \frac{b\left(a-2(a-b)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{2(a-b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}}{3a}-\frac{\sqrt{1-\sin^2(\cdots}}{\cdots}\right.}{f \cdots}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{b\int \frac{a-2(a-b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{2(a-b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}}{3a}-\frac{\sqrt{1-\sin^2 \cdots}}{\cdots}\right.}{f \cdots}$$

$$\Big\downarrow \text{ 399}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{b\left(\frac{a(2a-b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{2(a-b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)}{a}-\frac{2(a-b)\sqrt{1-\sin \cdots}}{\cdots}}{3a \cdots}\right.}{f \cdots}$$

$$\Big\downarrow \text{ 323}$$

3.152. $\displaystyle\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{b\left(\frac{a(2a-b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fa}{b}\right.}{a}}{3a}\right.}{f}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{b\left(\frac{a(2a-b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)}{a}-\frac{2(a-b)\sqrt{}}{}}{3a}\right.}{f}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{b\left(\frac{a(2a-b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.}{a}}{3a}\right.}{f}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{b\left(\frac{a(2a-b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right)}{a}}{3a}\right.}{f}$$

3.152. $\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

input
```
Int[Csc[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-1/3*(Csc[e + f*x]^3*Sqrt[1 - Sin[e +
f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/a + ((-2*(a - b)*Csc[e + f*x]*Sqrt[1 -
 Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/a + (b*((-2*(a - b)*EllipticE
[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*
Sin[e + f*x]^2)/a]) + (a*(2*a - b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]
*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2])))/a)/(3*a)
))/f
```

3.152.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

3.152. $\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.152.4 Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.45

method	result
default	$2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2\left(\sin^3(fx+e)\right)-b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\right.$

3.152. $\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

input
```
int(csc(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3-b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+
e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-2*(cos(
f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)
^(1/2))*a^2*sin(f*x+e)^3+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b*sin(f*x+e)^3+2*a*b*sin(f*x+e)^
6-2*b^2*sin(f*x+e)^6+2*a^2*sin(f*x+e)^4-3*a*b*sin(f*x+e)^4+2*b^2*sin(f*x+e
)^4-a^2*sin(f*x+e)^2+a*b*sin(f*x+e)^2-a^2)/a^2/sin(f*x+e)^3/cos(f*x+e)/(a+
b*sin(f*x+e)^2)^(1/2)/f
```

3.152.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 955, normalized size of antiderivative = 3.91

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*((2*((-I*a*b + I*b^2)*cos(f*x + e)^2 + I*a*b - I*b^2)*sqrt(-b)*sqrt((a
^2 + a*b)/b^2)*sin(f*x + e) - ((2*I*a^2 - I*a*b - I*b^2)*cos(f*x + e)^2 -
2*I*a^2 + I*a*b + I*b^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)
/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*
a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*
b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((I*a*b - I*b^2)*cos(f*x + e)^2
- I*a*b + I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - ((-2*I*a^2
+ I*a*b + I*b^2)*cos(f*x + e)^2 + 2*I*a^2 - I*a*b - I*b^2)*sqrt(-b)*sin(f*
x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sq
rt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e)
)), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (
2*((I*a*b - 2*I*b^2)*cos(f*x + e)^2 - I*a*b + 2*I*b^2)*sqrt(-b)*sqrt((a^2
+ a*b)/b^2)*sin(f*x + e) - ((-2*I*a^2 - I*a*b)*cos(f*x + e)^2 + 2*I*a^2 +
I*a*b)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b
)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x
 + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2
 + a*b)/b^2))/b^2) + (2*((-I*a*b + 2*I*b^2)*cos(f*x + e)^2 + I*a*b - 2*I*b
^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - ((2*I*a^2 + I*a*b)*cos(f
*x + e)^2 - 2*I*a^2 - I*a*b)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 +
a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^...
```

3.152.6 Sympy [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\csc^4{(e+fx)}}{\sqrt{a+b\sin^2{(e+fx)}}}\,dx$$

input
```
integrate(csc(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)**4/sqrt(a + b*sin(e + f*x)**2), x)
```

3.152.7 Maxima [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx=\int \frac{\csc\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.152.8 Giac [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx=\int \frac{\csc\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx=\int \frac{1}{\sin\left(e+f\,x\right)^4\,\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(1/(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

3.153 $\int \frac{\sin^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.153.1 Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \frac{\sin^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{b^{3/2}f} + \frac{a\cos(e+fx)}{b(a+b)f\sqrt{a+b-b\cos^2(e+fx)}}$$

output
```
-arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/b^(3/2)/f+a*cos(f*x
+e)/b/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

3.153.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.22

$$\int \frac{\sin^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{\frac{\sqrt{2}ab\cos(e+fx)}{(a+b)\sqrt{2a+b-b\cos(2(e+fx))}} + \sqrt{-b}\log\left(\sqrt{2}\sqrt{-b}\cos(e+fx) + \sqrt{2a+b-b\cos(\right.}{b^2f}$$

input
```
Integrate[Sin[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
((Sqrt[2]*a*b*Cos[e + f*x])/((a + b)*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]) +
 Sqrt[-b]*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e +
f*x)]]])/(b^2*f)
```

3.153.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3665, 298, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^3}{\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{1-\cos^2(e+fx)}{(-b\cos^2(e+fx)+a+b)^{3/2}}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{298}$$

$$-\frac{\frac{\int \frac{1}{\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)}{b}-\frac{a\cos(e+fx)}{b(a+b)\sqrt{a-b\cos^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{\int \frac{1}{\frac{b\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}+1}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}}{b}-\frac{a\cos(e+fx)}{b(a+b)\sqrt{a-b\cos^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{b^{3/2}}-\frac{a\cos(e+fx)}{b(a+b)\sqrt{a-b\cos^2(e+fx)+b}}}{f}$$

input
```
Int[Sin[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-((ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/b^(3/2) -
 (a*Cos[e + f*x])/(b*(a + b)*Sqrt[a + b - b*Cos[e + f*x]^2]))/f)
```

3.153.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.153.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(71) = 142$.

Time = 1.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.97

method	result
default	$\frac{\sqrt{-(-b(\sin^2(fx+e))-a)(\cos^2(fx+e))}\left(\frac{\arctan\left(\frac{\sqrt{b}\left(\sin^2(fx+e)-\frac{-a+b}{2b}\right)}{\sqrt{-\left(-b\left(\sin^2(fx+e)\right)-a\right)\left(\cos^2(fx+e)\right)}}\right)}{2b^{\frac{3}{2}}}+\frac{a\left(\cos^2(fx+e)\right)}{b(a+b)\sqrt{-\left(-b\left(\sin^2(fx+e)\right)-a\right)\left(\cos^2(fx+e)\right)}}\right.}{\cos(fx+e)\sqrt{a+b(\sin^2(fx+e))}\,f}$

input
```
int(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(1/2/b^(3/2)*arctan(b^(1/2)*(sin
(f*x+e)^2-1/2*(-a+b)/b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))+1/b*a*c
os(f*x+e)^2/(a+b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/cos(f*x+e)/(a
+b*sin(f*x+e)^2)^(1/2)/f
```

3.153.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(71) = 142$.

Time = 0.40 (sec) , antiderivative size = 564, normalized size of antiderivative = 7.14

$$\int \frac{\sin^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \left[-\frac{8\sqrt{-b\cos(fx+e)^2+a+b}ab\cos(fx+e)+\left((ab+b^2)\cos(fx+e)^2-\right.}{}\right.$$

$$-\frac{4\sqrt{-b\cos(fx+e)^2+a+b}ab\cos(fx+e)-\left((ab+b^2)\cos(fx+e)^2-a^2-2ab-b^2\right)\sqrt{b}\arctan\left(\frac{(8b^2\cos}{4(}\right.}{4\left((ab^3+b^4)f\cos(fx+e)^2-(a^2b^2+2ab^3+b^4)f\right.}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(8*sqrt(-b*cos(f*x + e)^2 + a + b)*a*b*cos(f*x + e) + ((a*b + b^2)*c
os(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 2
56*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x +
e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 +
 3*a*b^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^
3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-
b)))/((a*b^3 + b^4)*f*cos(f*x + e)^2 - (a^2*b^2 + 2*a*b^3 + b^4)*f), -1/4*
(4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*b*cos(f*x + e) - ((a*b + b^2)*cos(f*x
 + e)^2 - a^2 - 2*a*b - b^2)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*
(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a
 + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (a^
2*b + 2*a*b^2 + b^3)*cos(f*x + e)))/((a*b^3 + b^4)*f*cos(f*x + e)^2 - (a^
2*b^2 + 2*a*b^3 + b^4)*f)]
```

3.153.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \frac{\sin^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = -\frac{\frac{\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} + \frac{\cos(fx+e)}{\sqrt{-b\cos(fx+e)^2+a+b}(a+b)} - \frac{\cos(fx+e)}{\sqrt{-b\cos(fx+e)^2+a+b}b}}{f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-(arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/b^(3/2) + cos(f*x + e)/(sqrt(-b*c
os(f*x + e)^2 + a + b)*(a + b)) - cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a
 + b)*b))/f
```

3.153.8 Giac [F]

$$\int \frac{\sin^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^3}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx=\int \frac{\sin\left(e+f\,x\right)^3}{{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.154 $\int \frac{\sin(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.154.1 Optimal result

Integrand size = 23, antiderivative size = 34

$$\int \frac{\sin(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx=-\frac{\cos(e+fx)}{(a+b)f\sqrt{a+b-b\cos^2(e+fx)}}$$

output
```
-cos(f*x+e)/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

3.154.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \frac{\sin(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx=-\frac{\sqrt{2}\cos(e+fx)}{(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Sin[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-((Sqrt[2]*Cos[e + f*x])/((a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]))
```

3.154.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3665, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(e+fx)}{\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 3665$$

$$-\frac{\int \frac{1}{\left(-b\cos^2(e+fx)+a+b\right)^{3/2}}d\cos(e+fx)}{f}$$

$$\Big\downarrow \ 208$$

$$-\frac{\cos(e+fx)}{f(a+b)\sqrt{a-b\cos^2(e+fx)+b}}$$

input
```
Int[Sin[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-(Cos[e + f*x]/((a + b)*f*Sqrt[a + b - b*Cos[e + f*x]^2]))
```

3.154.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.154.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{\cos(fx+e)}{(a+b)\sqrt{a+b(\sin^2(fx+e))}\, f}$	31

input
```
int(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-cos(f*x+e)/(a+b)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int \frac{\sin(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{\sqrt{-b\cos{(fx+e)}^2+a+b}\cos{(fx+e)}}{(ab+b^2)f\cos{(fx+e)}^2-(a^2+2\,ab+b^2)f}$$

input
```
integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)/((a*b + b^2)*f*cos(f*x + e)^2
 - (a^2 + 2*a*b + b^2)*f)
```

3.154.6 Sympy [F]

$$\int \frac{\sin(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+fx)}{(a+b\sin^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sin(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\sin(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = -\frac{\cos(fx+e)}{\sqrt{-b\cos(fx+e)^2+a+b}(a+b)f}$$

input
```
integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*f)
```

3.154.8 Giac [F]

$$\int \frac{\sin(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.154.9 Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.50

$$\int \frac{\sin(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = -\frac{\sqrt{2}\,\sqrt{2\,a+b-b\,\cos\left(2\,e+2\,f\,x\right)}\,\left(4\,a\,\cos\left(e+f\,x\right)+b\,\cos\left(e+f\,x\right)-b\,\cos\left(3\,e+3\,f\,x\right)\right)}{f\,\left(a+b\right)\,\left(8\,a\,b+8\,a^2+3\,b^2-4\,b^2\,\cos\left(2\,e+2\,f\,x\right)+b^2\,\cos\left(4\,e+4\,f\,x\right)-8\,a\,b\,\cos\left(2\,e+2\,f\,x\right)\right)}$$

input
```
int(sin(e + f*x)/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
-(2^(1/2)*(2*a + b - b*cos(2*e + 2*f*x))^(1/2)*(4*a*cos(e + f*x) + b*cos(e
 + f*x) - b*cos(3*e + 3*f*x)))/(f*(a + b)*(8*a*b + 8*a^2 + 3*b^2 - 4*b^2*c
os(2*e + 2*f*x) + b^2*cos(4*e + 4*f*x) - 8*a*b*cos(2*e + 2*f*x)))
```

3.155 $\int \frac{\csc(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.155.1 Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{\csc(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{a^{3/2}f} + \frac{b\cos(e+fx)}{a(a+b)f\sqrt{a+b-b\cos^2(e+fx)}}$$

output
```
-arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/a^(3/2)/f+b*cos(f*
x+e)/a/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

3.155.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.18

$$\int \frac{\csc(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{-\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right) + \frac{\sqrt{2}\sqrt{a}b\cos(e+fx)}{(a+b)\sqrt{2a+b-b\cos(2(e+fx))}}}{a^{3/2}f}$$

input
```
Integrate[Csc[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]
]] + (Sqrt[2]*Sqrt[a]*b*Cos[e + f*x])/((a + b)*Sqrt[2*a + b - b*Cos[2*(e +
 f*x)]]))/(a^(3/2)*f)
```

3.155.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3665, 296, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(e+fx)\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3665}$$

$$-\frac{\int \frac{1}{(1-\cos^2(e+fx))(-b\cos^2(e+fx)+a+b)^{3/2}}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 296}$$

$$-\frac{\frac{\int \frac{1}{\left(1-\cos^2(e+fx)\right)\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)}{a}-\frac{b\cos(e+fx)}{a(a+b)\sqrt{a-b\cos^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{ 291}$$

$$-\frac{\frac{\int \frac{1}{1-\frac{a\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}}{a}-\frac{b\cos(e+fx)}{a(a+b)\sqrt{a-b\cos^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{a^{3/2}}-\frac{b\cos(e+fx)}{a(a+b)\sqrt{a-b\cos^2(e+fx)+b}}}{f}$$

input
```
Int[Csc[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-((ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/a^(3/2)
- (b*Cos[e + f*x])/(a*(a + b)*Sqrt[a + b - b*Cos[e + f*x]^2]))/f)
```

3.155.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.155.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(71) = 142$.

Time = 1.02 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.09

method	result
default	$\frac{\sqrt{-\left(-b\left(\sin^{2}(fx+e)\right)-a\right)\left(\cos^{2}(fx+e)\right)}\left(-\frac{\ln\left(\frac{2a+(-a+b)\left(\sin^{2}(fx+e)\right)+2\sqrt{a}\sqrt{-\left(-b\left(\sin^{2}(fx+e)\right)-a\right)\left(\cos^{2}(fx+e)\right)}}{\sin(fx+e)^{2}}\right)}{2a^{\frac{3}{2}}}+\frac{b \cdots}{a(a+b)\sqrt{-\left(-b\left(\cdots\right.\right.}} \cdots\right.}{\cos(fx+e)\sqrt{a+b\left(\sin^{2}(fx+e)\right)}\,f}$

input
```
int(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(-1/2/a^(3/2)*ln((2*a+(-a+b)*sin
(f*x+e)^2+2*a^(1/2)*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/sin(f*x+e)^
2)+1/a*b*cos(f*x+e)^2/(a+b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/cos
(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.155.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(71) = 142$.

Time = 0.38 (sec) , antiderivative size = 422, normalized size of antiderivative = 5.34

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \left[-\frac{4\sqrt{-b\cos\left(fx+e\right)^2+a+b}ab\cos\left(fx+e\right)-\left((ab+b^2)\cos\left(fx+e\right)^2-\right.}{}\right.$$

$$-\frac{2\sqrt{-b\cos\left(fx+e\right)^2+a+b}ab\cos\left(fx+e\right)-\left((ab+b^2)\cos\left(fx+e\right)^2-a^2-2\,ab-b^2\right)\sqrt{-a}\arctan\left(-\frac{(}{}\right.}{2\left((a^3b+a^2b^2)f\cos\left(fx+e\right)^2-(a^4+2\,a^3b+a^2b^2)f\right)}$$

input
```
integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*b*cos(f*x + e) - ((a*b + b^2)*c
os(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(
f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x +
 e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^
2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^3*b + a^2*b
^2)*f*cos(f*x + e)^2 - (a^4 + 2*a^3*b + a^2*b^2)*f), -1/2*(2*sqrt(-b*cos(f
*x + e)^2 + a + b)*a*b*cos(f*x + e) - ((a*b + b^2)*cos(f*x + e)^2 - a^2 -
2*a*b - b^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b
*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*
x + e))))/((a^3*b + a^2*b^2)*f*cos(f*x + e)^2 - (a^4 + 2*a^3*b + a^2*b^2)*
f)]
```

3.155.6 Sympy [F]

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\csc\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.155.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(71) = 142$.

Time = 0.34 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.09

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\frac{2\,b^2\cos(fx+e)}{\sqrt{-b\cos(fx+e)^2+a+b}a^2b+\sqrt{-b\cos(fx+e)^2+a+b}ab^2} - \frac{\log\left(b-\frac{\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a}}{\cos(fx+e)-1}-\frac{a}{\cos(fx+e)-}\right.}{a^{\frac{3}{2}}}}{2\,f}$$

input
```
integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*b^2*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*a^2*b + sqrt(-b*c
os(f*x + e)^2 + a + b)*a*b^2) - log(b - sqrt(-b*cos(f*x + e)^2 + a + b)*sq
rt(a)/(cos(f*x + e) - 1) - a/(cos(f*x + e) - 1))/a^(3/2) + log(-b + sqrt(-
b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x + e) + 1) + a/(cos(f*x + e) + 1
))/a^(3/2))/f
```

3.155.8 Giac [F]

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\sin\left(e+f\,x\right)\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2)), x)
```

3.156 $\int \frac{\csc^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

3.156.1 Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = -\frac{(a-3b)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{b(a+3b)\cos(e+fx)}{2a^2(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b-b\cos^2(e+fx)}}$$

output
```
-1/2*(a-3*b)*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/a^(5/2
)/f-1/2*b*(a+3*b)*cos(f*x+e)/a^2/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(1/2)-1/2*co
t(f*x+e)*csc(f*x+e)/a/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

3.156.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{-\frac{(a-3b)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)}{a^{5/2}} + \frac{\left(-2a^2-3ab-3b^2+b(a+3b)\cos(2(e+fx))\right)\cot(e+fx)\ c}{\sqrt{2}a^2(a+b)\sqrt{2a+b-b\cos(2(e+fx))}}}{2f}$$

input
```
Integrate[Csc[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-(((a - 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[
2*(e + f*x)]]])/a^(5/2)) + ((-2*a^2 - 3*a*b - 3*b^2 + b*(a + 3*b)*Cos[2*(e
 + f*x)])*Cot[e + f*x]*Csc[e + f*x])/(Sqrt[2]*a^2*(a + b)*Sqrt[2*a + b - b
*Cos[2*(e + f*x)]]))/(2*f)
```

3.156.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3665, 316, 402, 25, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^3\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{1}{\left(1-\cos^2(e+fx)\right)^2\left(-b\cos^2(e+fx)+a+b\right)^{3/2}}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$-\frac{\frac{\int \frac{-2b\cos^2(e+fx)+a-b}{\left(1-\cos^2(e+fx)\right)\left(-b\cos^2(e+fx)+a+b\right)^{3/2}}d\cos(e+fx)}{2a}+\frac{\cos(e+fx)}{2a(1-\cos^2(e+fx))\sqrt{a-b\cos^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{402}$$

$$-\frac{\frac{\frac{b(a+3b)\cos(e+fx)}{a(a+b)\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\int -\frac{(a-3b)(a+b)}{\left(1-\cos^2(e+fx)\right)\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)}{a(a+b)}}{2a}+\frac{\cos(e+fx)}{2a(1-\cos^2(e+fx))\sqrt{a-b\cos^2(e+fx)+b}}}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\frac{\int \frac{(a-3b)(a+b)}{\left(1-\cos^2(e+fx)\right)\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)}{a(a+b)}+\frac{b(a+3b)\cos(e+fx)}{a(a+b)\sqrt{a-b\cos^2(e+fx)+b}}}{2a}+\frac{\cos(e+fx)}{2a(1-\cos^2(e+fx))\sqrt{a-b\cos^2(e+fx)+b}}}{f}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\frac{(a-3b)\int \frac{1}{\left(1-\cos^2(e+fx)\right)\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)}{a}+\frac{b(a+3b)\cos(e+fx)}{a(a+b)\sqrt{a-b\cos^2(e+fx)+b}}}{2a}+\frac{\cos(e+fx)}{2a(1-\cos^2(e+fx))\sqrt{a-b\cos^2(e+fx)+b}}}{f}$$

$$\Big\downarrow 291$$

$$-\frac{\frac{\frac{(a-3b)\int \frac{1}{1-\frac{a\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}}{a}+\frac{b(a+3b)\cos(e+fx)}{a(a+b)\sqrt{a-b\cos^2(e+fx)+b}}}{2a}+\frac{\cos(e+fx)}{2a(1-\cos^2(e+fx))\sqrt{a-b\cos^2(e+fx)+b}}}{f}$$

$$\Big\downarrow 219$$

$$-\frac{\frac{\frac{(a-3b)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{a^{3/2}}+\frac{b(a+3b)\cos(e+fx)}{a(a+b)\sqrt{a-b\cos^2(e+fx)+b}}}{2a}+\frac{\cos(e+fx)}{2a(1-\cos^2(e+fx))\sqrt{a-b\cos^2(e+fx)+b}}}{f}$$

input
```
Int[Csc[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-((Cos[e + f*x]/(2*a*(1 - Cos[e + f*x]^2)*Sqrt[a + b - b*Cos[e + f*x]^2])
+ (((a - 3*b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2
]])/a^(3/2) + (b*(a + 3*b)*Cos[e + f*x])/(a*(a + b)*Sqrt[a + b - b*Cos[e +
 f*x]^2]))/(2*a))/f)
```

3.156.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.156. $\int \frac{\csc^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.156. $\int \frac{\csc^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

3.156.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(118) = 236$.

Time = 1.25 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.04

method	result
default	$\frac{\sqrt{-(-b(\sin^2(fx+e))-a)(\cos^2(fx+e))}\left(-\frac{\sqrt{-\left(-b\left(\sin^2(fx+e)\right)-a\right)\left(\cos^2(fx+e)\right)}}{2a^2\sin(fx+e)^2}-\frac{\ln\left(\frac{2a+(-a+b)\left(\sin^2(fx+e)\right)+2\sqrt{a}\sqrt{-\left(-b\left(\sin^2(fx+\right.\right.}}{\sin(fx+e)^2}\right.}{4a^{\frac{3}{2}}}\right.}{\cos(fx+e}$

input
```
int(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(-1/2/a^2/sin(f*x+e)^2*(-(-b*sin
(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)-1/4/a^(3/2)*ln((2*a+(-a+b)*sin(f*x+e)^2+2
*a^(1/2)*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/sin(f*x+e)^2)+3/4/a^(5
/2)*b*ln((2*a+(-a+b)*sin(f*x+e)^2+2*a^(1/2)*(-(-b*sin(f*x+e)^2-a)*cos(f*x+
e)^2)^(1/2))/sin(f*x+e)^2)-b^2/a^2*cos(f*x+e)^2/(a+b)/(-(-b*sin(f*x+e)^2-a
)*cos(f*x+e)^2)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.156.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(118) = 236$.

Time = 0.52 (sec) , antiderivative size = 634, normalized size of antiderivative = 4.73

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \left[-\frac{\left((a^2b-2\,ab^2-3\,b^3)\cos\left(fx+e\right)^4+a^3-a^2b-5\,ab^2-3\,b^3-(a^3-7\,ab\right.}{}\right.$$

input
```
integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(((a^2*b - 2*a*b^2 - 3*b^3)*cos(f*x + e)^4 + a^3 - a^2*b - 5*a*b^2 -
 3*b^3 - (a^3 - 7*a*b^2 - 6*b^3)*cos(f*x + e)^2)*sqrt(a)*log(2*((a^2 - 6*a
*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 + 4*((a
- b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b
)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) -
4*((a^2*b + 3*a*b^2)*cos(f*x + e)^3 - (a^3 + 2*a^2*b + 3*a*b^2)*cos(f*x +
e))*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^4*b + a^3*b^2)*f*cos(f*x + e)^4 -
 (a^5 + 3*a^4*b + 2*a^3*b^2)*f*cos(f*x + e)^2 + (a^5 + 2*a^4*b + a^3*b^2)*
f), 1/4*(((a^2*b - 2*a*b^2 - 3*b^3)*cos(f*x + e)^4 + a^3 - a^2*b - 5*a*b^2
 - 3*b^3 - (a^3 - 7*a*b^2 - 6*b^3)*cos(f*x + e)^2)*sqrt(-a)*arctan(-1/2*((
a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a
*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))) + 2*((a^2*b + 3*a*b^2)*cos(
f*x + e)^3 - (a^3 + 2*a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^
2 + a + b))/((a^4*b + a^3*b^2)*f*cos(f*x + e)^4 - (a^5 + 3*a^4*b + 2*a^3*b
^2)*f*cos(f*x + e)^2 + (a^5 + 2*a^4*b + a^3*b^2)*f)]
```

3.156.6 Sympy [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^3(e+fx)}{(a+b\sin^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)**3/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.156.7 Maxima [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^3}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^3/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.156.8 Giac [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^3}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin(e+f\,x)^3\,(b\sin(e+f\,x)^2+a)^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2)), x)
```

3.157 $\int \frac{\sin^6(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.157.1 Optimal result

Integrand size = 25, antiderivative size = 274

$$\int \frac{\sin^6(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{a\cos(e+fx)\sin^3(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(4a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3b^2(a+b)f} - \frac{(8a^2+3ab-2b^2)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3b^3(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{a(8a-b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3b^3f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
a*cos(f*x+e)*sin(f*x+e)^3/b/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(4*a+b)*c
os(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b^2/(a+b)/f-1/3*(8*a^2+3*a*b
-2*b^2)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)
*(a+b*sin(f*x+e)^2)^(1/2)/b^3/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*a*(8*
a-b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1
+b*sin(f*x+e)^2/a)^(1/2)/b^3/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.157.2 Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.72

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{-2\sqrt{2}a(8a^2+3ab-2b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+2\sqrt{2}a(8a^2+7}{6\sqrt{}}$$

input
```
Integrate[Sin[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-2*Sqrt[2]*a*(8*a^2 + 3*a*b - 2*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/
a]*EllipticE[e + f*x, -(b/a)] + 2*Sqrt[2]*a*(8*a^2 + 7*a*b - b^2)*Sqrt[(2*
a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + b*(-8*a^2 - 3*
a*b - b^2 + b*(a + b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(6*Sqrt[2]*b^3*(
a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.157.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3667, 372, 444, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(e+fx)^6}{(a+b\sin(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \; 3667$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sin^6(e+fx)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 372$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin^3(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\int\frac{\sin^2(e+fx)\left(3a-(4a+b)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b(a+b)}\right)}{f}$$

$\downarrow$ 444

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin^3(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{\int-\frac{a(4a+b)-\left(8a^2+3ba-2b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3b}+\frac{(4a+b)\sqrt{1-\sin^2(e+fx)}\,\mathrm{s}\ldots}{\ldots}}{b(a+b)}\ldots\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin^3(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{(4a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}{3b}-\frac{\int\frac{a(4a+b)-\left(8a^2+3ba-2b^2\ldots\right.}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin\ldots}}}{3b}}{b(a+b)}\ldots\right.}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin^3(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{(4a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}{3b}-\frac{a(8a-b)(a+b)\int\frac{\ldots}{\sqrt{1-\sin^2(\ldots}}\ldots}{\ldots}}{b\ldots}\ldots\right.}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin^3(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{(4a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}{3b}-\frac{a(8a-b)(a+b)\sqrt{\frac{b\sin^2(e+f\ldots}{a}}\ldots}{\ldots}}{\ldots}\ldots\right.}{f}$$

$\downarrow$ 321

3.157. $\displaystyle\int\frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin^3(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{(4a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}{3b}-\frac{a(8a-b)(a+b)\sqrt{\frac{b\sin^2(e+f\cdots}{a}}\cdots}{b\sqrt{\cdots}}\cdots}{\cdots}\right)}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin^3(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{(4a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}{3b}-\frac{a(8a-b)(a+b)\sqrt{\frac{b\sin^2(e+f\cdots}{a}}\cdots}{b\sqrt{\cdots}}\cdots}{\cdots}\right)}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin^3(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{(4a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}{3b}-\frac{a(8a-b)(a+b)\sqrt{\frac{b\sin^2(e+f\cdots}{a}}\cdots}{b\sqrt{\cdots}}\cdots}{\cdots}\right)}{f}$$

input
```
Int[Sin[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((a*Sin[e + f*x]^3*Sqrt[1 - Sin[e + f*x
]^2])/(b*(a + b)*Sqrt[a + b*Sin[e + f*x]^2]) - (((4*a + b)*Sin[e + f*x]*Sq
rt[1 - Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b) - (-(((8*a^2 + 3*
a*b - 2*b^2)*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*
x]^2])/(b*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) + (a*(8*a - b)*(a + b)*Elliptic
F[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a
+ b*Sin[e + f*x]^2]))/(3*b))/(b*(a + b))))/f
```

3.157. $\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

3.157.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

3.157. $\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.157.4 Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.48

method	result
default	$\frac{a\,b^{2}\left(\sin^{5}\left(fx+e\right)\right)+b^{3}\left(\sin^{5}\left(fx+e\right)\right)+8\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^{2}\left(fx+e\right)\right)}{a}}\,F\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^{3}+7a^{2}\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\mathrm{s}\right.}{}}}{}$

input
```
int(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*b^2*sin(f*x+e)^5+b^3*sin(f*x+e)^5+8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(
f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+7*a^2*(cos(f*x
+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1
/2))*b-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f
*x+e),(-1/a*b)^(1/2))*b^2-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1
/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-3*(cos(f*x+e)^2)^(1/2)*((a+b*
sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+2*(cos(f
*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a*b^2+4*a^2*b*sin(f*x+e)^3-b^3*sin(f*x+e)^3-4*a^2*b*sin(f*x+e)-a*b^
2*sin(f*x+e))/b^3/(a+b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.157.5 Fricas [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^6}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*sqrt(
-b*cos(f*x + e)^2 + a + b)/(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e
)^2 + a^2 + 2*a*b + b^2), x)
```

3.157.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**6/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.157.7 Maxima [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^6}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.157.8 Giac [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^6}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+f\,x)^6}{(b\sin(e+f\,x)^2+a)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^6/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^6/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.158 $\int \frac{\sin^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.158.1 Optimal result

Integrand size = 25, antiderivative size = 202

$$\int \frac{\sin^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{a\cos(e+fx)\sin(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{(2a+b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{b^2(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{2a\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{b^2f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
a*cos(f*x+e)*sin(f*x+e)/b/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+(2*a+b)*Ellipti
cE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e
)^2)^(1/2)/b^2/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-2*a*EllipticF(sin(f*x+e)
,(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/
b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.158.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.67

$$\int \frac{\sin^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{a\left(2(2a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-4(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\right.}{2b^2(a+b)f\sqrt{2a+b-b\cos(2(e+}}$$

input
```
Integrate[Sin[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(a*(2*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x,
-(b/a)] - 4*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f
*x, -(b/a)] + Sqrt[2]*b*Sin[2*(e + f*x)]))/(2*b^2*(a + b)*f*Sqrt[2*a + b -
 b*Cos[2*(e + f*x)]])
```

3.158.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3667, 372, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^4}{(a+b\sin(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3667}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sin^4(e+fx)}{\sqrt{1-\sin^2(e+fx)}(b\sin^2(e+fx)+a)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\int \frac{a-(2a+b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b(a+b)}\right)}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{2a(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{(2a+b)\int\frac{\sqrt{b\sin^2(e+fx}}{\sqrt{1-\sin^2(e+}}}{b}}{b(a+b)}\right.}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{2a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+}{}}{b(a+b)}\right.}{f}$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{2a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+b)\int\frac{\sqrt{b\sin^2(e}}{\sqrt{1-\sin^2}}}{b}}{b(a+b)}\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{2a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+b)\sqrt{a+b\sin^2(e}}{b}}{b(a+b)}\right.}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{2a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+b)\sqrt{a+b\sin^2(e}}{b\sqrt{}}}{b(a+b)}\right.}{f}$$

3.158. $\displaystyle\int\frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

input
```
Int[Sin[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((a*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^
2])/(b*(a + b)*Sqrt[a + b*Sin[e + f*x]^2]) - (-(((2*a + b)*EllipticE[ArcSi
n[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e
+ f*x]^2)/a])) + (2*a*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt
[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2]))/(b*(a + b))))/
f
```

3.158.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.158.4 Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.19

method	result
default	$-\frac{a\left(2a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)+2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)b}{b^2(a+}$

input
```
int(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-a*(2*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*
x+e),(-1/a*b)^(1/2))+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f
*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))-(cos(f*x+e)^2)^(1/2
)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b+sin(
f*x+e)^3*b-b*sin(f*x+e))/b^2/(a+b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.158.5 Fricas [F]

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sin\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b*cos(f*x + e)^2 +
a + b)/(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b +
b^2), x)
```

3.158.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(sin(f*x+e)**4/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.158.7 Maxima [F]

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sin\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.158.8 Giac [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^4}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+f\,x)^4}{(b\sin(e+f\,x)^2+a)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.159 $\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

3.159.1 Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=-\frac{\cos(e+fx)\sin(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}}-\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{b(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}+\frac{\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{bf\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-cos(f*x+e)*sin(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)-(cos(f*x+e)^2)^(1/
2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)^(1/2)/
b/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*Ellip
ticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/b/f/(a+b*sin(f*x+
e)^2)^(1/2)
```

3.159.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\frac{-\sqrt{2}a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\text{Ell}}{\sqrt{2}b(a+b)f\sqrt{2a+b-b\cos(2(e+fx}}$$

input
```
Integrate[Sin[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-(Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b
/a)]) + Sqrt[2]*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e
 + f*x, -(b/a)] - b*Sin[2*(e + f*x)])/(Sqrt[2]*b*(a + b)*f*Sqrt[2*a + b -
b*Cos[2*(e + f*x)]])
```

3.159.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3652, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^2}{\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3652}$$

$$\frac{\int \frac{a-a\sin^2(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}dx}{a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a-a\sin(e+fx)^2}{\sqrt{b\sin(e+fx)^2+a}}dx}{a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3651}$$

$$\frac{\frac{a(a+b)\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}dx}{b}-\frac{a\int \sqrt{b\sin^2(e+fx)+a}dx}{b}}{a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}-\frac{a\int \sqrt{b\sin(e+fx)^2+a}dx}{b}}{a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3657}$$

$$\frac{\frac{a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}-\frac{a\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin^2(e+fx)}{a}+1}dx}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}-\frac{a\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin(e+fx)^2}{a}+1}dx}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{\frac{a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}-\frac{a\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3662}$$

$$\frac{\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}dx}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{a\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin(e+fx)^2}{a}+1}}dx}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{a\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3661}$$

$$\frac{\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}-\frac{a\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

input
```
Int[Sin[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-((Cos[e + f*x]*Sin[e + f*x])/((a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])) + (-
((a*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*f*Sqrt[1 + (
b*Sin[e + f*x]^2)/a])) + (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b
*Sin[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2]))/(a*(a + b))
```

3.159. $\int \frac{\sin^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.159.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.159. $\int \frac{\sin^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.159.4 Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.25

method	result
default	$\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)+\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)b-a\sqrt{\cos}}{b(a+b)\cos(fx+e)\sqrt{a+b(\sin^2(fx+e))}\,f}$

input
```
int(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),
(-1/a*b)^(1/2))+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Elliptic
F(sin(f*x+e),(-1/a*b)^(1/2))*b-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/
a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))+sin(f*x+e)^3*b-b*sin(f*x+e))
/b/(a+b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.159.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 780, normalized size of antiderivative = 5.10

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{2\sqrt{-b\cos\left(fx+e\right)^2+a+b}b^2\cos\left(fx+e\right)\sin\left(fx+e\right)-2\left(\left(-2i\,ab-i\,b^2\right)\right.}{}$$

input
```
integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(-b*cos(f*x + e)^2 + a + b)*b^2*cos(f*x + e)*sin(f*x + e) - 2*(
(-2*I*a*b - I*b^2)*cos(f*x + e)^2 + 2*I*a^2 + 3*I*a*b + I*b^2)*sqrt(-b)*sq
rt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sq
rt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2
 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*((2*I*a*b
 + I*b^2)*cos(f*x + e)^2 - 2*I*a^2 - 3*I*a*b - I*b^2)*sqrt(-b)*sqrt((2*b*s
qrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 +
 a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b
+ b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-I*b^2*cos(f*x +
 e)^2 + I*a*b + I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((2*I*a*b + I*b^2)
*cos(f*x + e)^2 - 2*I*a^2 - 3*I*a*b - I*b^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2
 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^
2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 -
4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(I*b^2*cos(f*x + e)^2 - I
*a*b - I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((-2*I*a*b - I*b^2)*cos(f*x
 + e)^2 + 2*I*a^2 + 3*I*a*b + I*b^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/
b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
 + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b
 + b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/((a*b^3 + b^4)*f*cos(f*x + e)^2 - (a^
2*b^2 + 2*a*b^3 + b^4)*f)
```

3.159.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.159.7 Maxima [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^2}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.159.8 Giac [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^2}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+f\,x)^2}{(b\sin(e+f\,x)^2+a)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.160 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

3.160.1 Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}}+\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{a(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

output
```
b*cos(f*x+e)*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+(cos(f*x+e)^2)^
(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)^(1/
2)/a/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)
```

3.160.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\frac{2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}b\sin(2(e+fx))}{2a(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

output
```
(2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + S
qrt[2]*b*Sin[2*(e + f*x)])/(2*a*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)
]])
```

3.160.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 3663, 25, 3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 3663$$

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\int -\sqrt{b\sin^2(e+fx)+a}dx}{a(a+b)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \sqrt{b\sin^2(e+fx)+a}dx}{a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \sqrt{b\sin(e+fx)^2+a}dx}{a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \; 3657$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin^2(e+fx)}{a}+1}dx}{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin(e+fx)^2}{a}+1}dx}{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \; 3656$$

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx|-\frac{b}{a}\right)}{af(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

input
```
Int[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

output
```
(b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + (
EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)*f*Sqrt[1
 + (b*Sin[e + f*x]^2)/a])
```

3.160.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.160.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}\,aE\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)+\left(\cos^2(fx+e)\right)\sin(fx+e)b}{a(a+b)\cos(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}\,f}$	103

input
```
int(1/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*EllipticE(sin(f*
x+e),(-1/a*b)^(1/2))+cos(f*x+e)^2*sin(f*x+e)*b)/a/(a+b)/cos(f*x+e)/(a+b*si
n(f*x+e)^2)^(1/2)/f
```

3.160.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 938, normalized size of antiderivative = 9.29

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{-b\cos\left(fx+e\right)^2+a+b}b^3\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(2\left(i\,b^3\cos\left(fx+e\right)^2-i\,ab^2-i\,b^3\right)\sqrt{-b}\sqrt{\frac{a^2+a}{b^2}}\right.}{}$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(-b*cos(f*x + e)^2 + a + b)*b^3*cos(f*x + e)*sin(f*x + e) - (2
*(I*b^3*cos(f*x + e)^2 - I*a*b^2 - I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) -
 (2*I*a^2*b + 3*I*a*b^2 + I*b^3 + (-2*I*a*b^2 - I*b^3)*cos(f*x + e)^2)*sqr
t(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqr
t((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))
), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (2
*(-I*b^3*cos(f*x + e)^2 + I*a*b^2 + I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)
- (-2*I*a^2*b - 3*I*a*b^2 - I*b^3 + (2*I*a*b^2 + I*b^3)*cos(f*x + e)^2)*sq
rt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sq
rt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e)
)), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 2
*(2*(-I*a^2*b - 2*I*a*b^2 - I*b^3 + (I*a*b^2 + I*b^3)*cos(f*x + e)^2)*sqrt
(-b)*sqrt((a^2 + a*b)/b^2) + (2*I*a^3 + 3*I*a^2*b + I*a*b^2 + (-2*I*a^2*b
- I*a*b^2)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
 + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(
cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sq
rt((a^2 + a*b)/b^2))/b^2) + 2*(2*(I*a^2*b + 2*I*a*b^2 + I*b^3 + (-I*a*b^2
- I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + (-2*I*a^3 - 3*I*
a^2*b - I*a*b^2 + (2*I*a^2*b + I*a*b^2)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*
b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt(...
```

3.160.6 Sympy [F]

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{1}{\left(a+b\sin^2{(e+fx)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sin(e + f*x)**2)**(-3/2), x)
```

3.160.7 Maxima [F]

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

3.160.8 Giac [F]

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(1/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.161 $\int \frac{\csc^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.161.1 Optimal result

Integrand size = 25, antiderivative size = 235

$$\begin{aligned}\int \frac{\csc^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx &= \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} \\ &- \frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2(a+b)f} \\ &- \frac{(a+2b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} \\ &+ \frac{\sqrt{\cos^2(e+fx)}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{af\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

output
```
b*cot(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)-(a+2*b)*cot(f*x+e)*(a+b*si
n(f*x+e)^2)^(1/2)/a^2/(a+b)/f-(a+2*b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*s
ec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^2/(a+b)/f/(1+b*s
in(f*x+e)^2/a)^(1/2)+EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*
x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/a/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.161.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.72

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(-2a^2-3ab-2b^2+b(a+2b)\cos(2(e+fx))\right)\cot(e+fx)-\sqrt{2}a(a+2b)\sqrt{}}{\sqrt{2}a^2(a+b)f}$$

input
```
Integrate[Csc[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
((-2*a^2 - 3*a*b - 2*b^2 + b*(a + 2*b)*Cos[2*(e + f*x)])*Cot[e + f*x] - Sq
rt[2]*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x
, -(b/a)] + Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ellip
ticF[e + f*x, -(b/a)])/(Sqrt[2]*a^2*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e +
f*x)]])
```

3.161.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3667, 374, 25, 445, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^2\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3667}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\csc^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{374}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\sqrt{1-\sin^2(e+fx)}\csc(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\int -\frac{\csc^2(e+fx)\left(-b\sin^2(e+fx)+a+2b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a(a+b)}\right)}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int \frac{\csc^2(e+fx)\left(-b\sin^2(e+fx)+a+2b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a(a+b)}+\frac{b\sqrt{1-\sin^2(e+fx)}\csc(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

$\downarrow$ 445

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{\int \frac{b\left((a+2b)\sin^2(e+fx)+a\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{(a+2b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}}{a(a+b)}+\frac{b\sqrt{1-\sin}}{a(a+b)}\right.}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{b\int \frac{(a+2b)\sin^2(e+fx)+a}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{(a+2b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}}{a(a+b)}+\frac{b\sqrt{1-\sin}}{a(a+b}\right.}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{b\left(\frac{(a+2b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}\right)}{a}-\frac{(a+2b)\sqrt{1-}}{}}{a(a+b)}\right.}{f}$$

$\downarrow$ 323

3.161. $\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{b\left(\frac{(a+2b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+f}{b\sqrt{a+b\sin^2(e+fx)}}\right.}{a}}{a(a+b)}\right.}{f}$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{b\left(\frac{(a+2b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}\right)}{a}-\frac{(a+2b)\sqrt{}}{}}{a(a+b)}\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{b\left(\frac{(a+2b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\arcsin(\sin(e+fx)),\right.}{b\sqrt{a+b\sin^2(e+fx)}}\right.}{a}}{a(a+b)}\right.}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{b\left(\frac{(a+2b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}\right)}{a}}{a(a+b)}\right.}{f}$$

3.161. $\int\frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$

input
```
Int[Csc[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((b*Csc[e + f*x]*Sqrt[1 - Sin[e + f*x]^
2])/(a*(a + b)*Sqrt[a + b*Sin[e + f*x]^2]) + (-(((a + 2*b)*Csc[e + f*x]*Sq
rt[1 - Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/a) - (b*(((a + 2*b)*Ell
ipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1
 + (b*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], -(b
/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2])))/a)/(
a*(a + b))))/f
```

3.161.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

3.161. $\int \frac{\csc^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.161. $\int \frac{\csc^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.161.4 Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.85

method	result
default	$\frac{\left(a b+2 b^{2}\right)\left(\cos^{4}(f x+e)\right)+\left(-a^{2}-2 a b-2 b^{2}\right)\left(\cos^{2}(f x+e)\right)+\sin(f x+e) \sqrt{-\frac{b\left(\cos^{2}(f x+e)\right)}{a}+\frac{a+b}{a}}\, \sqrt{\frac{\cos(2 f x+2 e)}{2}+\frac{1}{2}}\, a\left(F\left(\sin(f x+e), \sqrt{-\cdots}\right.\right.}{a^{2} \sin(f x+e)(a+b) \cos(f x+e) \sqrt{a+b\left(\sin^{2}(f x+e)\right)}\cdots}$

input
```
int(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((a*b+2*b^2)*cos(f*x+e)^4+(-a^2-2*a*b-2*b^2)*cos(f*x+e)^2+sin(f*x+e)*(-b/a
*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(EllipticF(sin(f*x+e),
(-1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))*a-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b))/a^2/sin(f
*x+e)/(a+b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.161.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 1075, normalized size of antiderivative = 4.57

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((2*(I*a^2*b + 3*I*a*b^2 + 2*I*b^3 + (-I*a*b^2 - 2*I*b^3)*cos(f*x + e)
^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - (-2*I*a^3 - 7*I*a^2*b -
7*I*a*b^2 - 2*I*b^3 + (2*I*a^2*b + 5*I*a*b^2 + 2*I*b^3)*cos(f*x + e)^2)*sq
rt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*ellipti
c_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I
*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b
^2))/b^2) + (2*(-I*a^2*b - 3*I*a*b^2 - 2*I*b^3 + (I*a*b^2 + 2*I*b^3)*cos(f
*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - (2*I*a^3 + 7*I*a^
2*b + 7*I*a*b^2 + 2*I*b^3 + (-2*I*a^2*b - 5*I*a*b^2 - 2*I*b^3)*cos(f*x + e
)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*
elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x +
 e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 +
 a*b)/b^2))/b^2) - 2*(4*(I*a^2*b + 2*I*a*b^2 + I*b^3 + (-I*a*b^2 - I*b^3)*
cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + (-2*I*a^3 -
3*I*a^2*b - I*a*b^2 + (2*I*a^2*b + I*a*b^2)*cos(f*x + e)^2)*sqrt(-b)*sin(f
*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(s
qrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e
))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) -
2*(4*(-I*a^2*b - 2*I*a*b^2 - I*b^3 + (I*a*b^2 + I*b^3)*cos(f*x + e)^2)*sqr
t(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + (2*I*a^3 + 3*I*a^2*b + I*a*b...
```

3.161.6 Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^2(e+fx)}{(a+b\sin^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)**2/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.161.7 Maxima [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^2}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.161.8 Giac [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^2}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin(e+f\,x)^2\,\left(b\sin(e+f\,x)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2)), x)
```

3.162 $\int \frac{\sin^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.162.1 Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{b^{5/2}f} + \frac{a(3a+5b)\cos(e+fx)}{3b^2(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}} + \frac{a\cos(e+fx)\sin^2(e+fx)}{3b(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}}$$

output
```
-arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/b^(5/2)/f+1/3*a*cos
(f*x+e)*sin(f*x+e)^2/b/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(3/2)+1/3*a*(3*a+5*b)*
cos(f*x+e)/b^2/(a+b)^2/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

3.162.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.97

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\frac{2\sqrt{2}a\cos(e+fx)\left(3a^2+7ab+3b^2-b(2a+3b)\cos(2(e+fx))\right)}{(a+b)^2(2a+b-b\cos(2(e+fx)))^{3/2}} - \frac{3\log\left(\sqrt{2}\sqrt{-b}\cos(e+fx)+\sqrt{2a+b-b\cos(}\right.}{\sqrt{-b}}}{3b^2f}$$

input
```
Integrate[Sin[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
((2*Sqrt[2]*a*Cos[e + f*x]*(3*a^2 + 7*a*b + 3*b^2 - b*(2*a + 3*b)*Cos[2*(e
 + f*x)]))/((a + b)^2*(2*a + b - b*Cos[2*(e + f*x)])^(3/2)) - (3*Log[Sqrt[
2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]])/Sqrt[-b])/
(3*b^2*f)
```

3.162.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 3665, 315, 25, 298, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^5}{\left(a+b\sin(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{\left(1-\cos^2(e+fx)\right)^2}{\left(-b\cos^2(e+fx)+a+b\right)^{5/2}}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{315}$$

$$-\frac{-\frac{\int -\frac{-3(a+b)\cos^2(e+fx)+a+3b}{\left(-b\cos^2(e+fx)+a+b\right)^{3/2}}d\cos(e+fx)}{3b(a+b)}-\frac{a\cos(e+fx)\left(1-\cos^2(e+fx)\right)}{3b(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int \frac{-3(a+b)\cos^2(e+fx)+a+3b}{\left(-b\cos^2(e+fx)+a+b\right)^{3/2}}d\cos(e+fx)}{3b(a+b)}-\frac{a\cos(e+fx)\left(1-\cos^2(e+fx)\right)}{3b(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{298}$$

$$-\frac{\frac{\frac{3(a+b)\int \frac{1}{\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)}{b}-\frac{a(3a+5b)\cos(e+fx)}{b(a+b)\sqrt{a-b\cos^2(e+fx)+b}}}{3b(a+b)}-\frac{a\cos(e+fx)\left(1-\cos^2(e+fx)\right)}{3b(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}}{f}$$

$$\Big\downarrow \; 224$$

$$-\frac{\frac{\frac{3(a+b)\int \frac{1}{\frac{b\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}+1}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}}{b}-\frac{a(3a+5b)\cos(e+fx)}{b(a+b)\sqrt{a-b\cos^2(e+fx)+b}}}{3b(a+b)}-\frac{a\cos(e+fx)(1-\cos^2(e+fx))}{3b(a+b)(a-b\cos^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \; 216$$

$$-\frac{\frac{\frac{3(a+b)\arctan\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{b^{3/2}}-\frac{a(3a+5b)\cos(e+fx)}{b(a+b)\sqrt{a-b\cos^2(e+fx)+b}}}{3b(a+b)}-\frac{a\cos(e+fx)(1-\cos^2(e+fx))}{3b(a+b)(a-b\cos^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[Sin[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
-((-1/3*(a*Cos[e + f*x]*(1 - Cos[e + f*x]^2))/(b*(a + b)*(a + b - b*Cos[e
+ f*x]^2)^(3/2)) + ((3*(a + b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b -
b*Cos[e + f*x]^2]])/b^(3/2) - (a*(3*a + 5*b)*Cos[e + f*x])/(b*(a + b)*Sqrt
[a + b - b*Cos[e + f*x]^2]))/(3*b*(a + b)))/f)
```

3.162.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.162. $\int \frac{\sin^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.162.4 Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.77

method	result
default	$\frac{\sqrt{-\left(-b\left(\sin^2(fx+e)\right)-a\right)\left(\cos^2(fx+e)\right)}\left(\frac{\arctan\left(\frac{\sqrt{b}\left(\sin^2(fx+e)-\frac{-a+b}{2b}\right)}{\sqrt{-\left(-b\left(\sin^2(fx+e)\right)-a\right)\left(\cos^2(fx+e)\right)}}\right)}{2b^{\frac{5}{2}}}-\frac{a^2\left(2b\left(\sin^2(fx+e)\right)+3a+b\right)}{3b^2\sqrt{-\left(-b\left(\sin^2(fx+e)\right)-a\right)\left(\cos^2(fx+e)\right)}\left(\right.}\right.}{\cos(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}\,f}$

input
```
int(sin(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(1/2/b^(5/2)*arctan(b^(1/2)*(sin
(f*x+e)^2-1/2*(-a+b)/b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))-1/3*a^2
/b^2*(2*b*sin(f*x+e)^2+3*a+b)*cos(f*x+e)^2/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e
)^2)^(1/2)/(a+b*sin(f*x+e)^2)/(a^2+2*a*b+b^2)+2*a/b^2*cos(f*x+e)^2/(a+b)/(
-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1
/2)/f
```

3.162.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(123) = 246$.

Time = 0.75 (sec) , antiderivative size = 885, normalized size of antiderivative = 6.46

$$\int \frac{\sin^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \left[-\frac{3\left((a^2b^2+2\,ab^3+b^4)\cos(fx+e)^4+a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4-2\right.}{}\right.$$

input
```
integrate(sin(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4 - 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)
^2)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6
 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^
2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2
+ 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b +
 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x +
 e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) + 8*(2*(2*a^2*b^2 + 3*a*b^3
)*cos(f*x + e)^3 - 3*(a^3*b + 3*a^2*b^2 + 2*a*b^3)*cos(f*x + e))*sqrt(-b*c
os(f*x + e)^2 + a + b))/((a^2*b^5 + 2*a*b^6 + b^7)*f*cos(f*x + e)^4 - 2*(a
^3*b^4 + 3*a^2*b^5 + 3*a*b^6 + b^7)*f*cos(f*x + e)^2 + (a^4*b^3 + 4*a^3*b^
4 + 6*a^2*b^5 + 4*a*b^6 + b^7)*f), 1/12*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cos(
f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 2*(a^3*b + 3*a^2*
b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e
)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x +
e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)
^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) - 4*(2*(2*a^2*b^2 + 3*a*b^3)*c
os(f*x + e)^3 - 3*(a^3*b + 3*a^2*b^2 + 2*a*b^3)*cos(f*x + e))*sqrt(-b*cos(
f*x + e)^2 + a + b))/((a^2*b^5 + 2*a*b^6 + b^7)*f*cos(f*x + e)^4 - 2*(a^3*
b^4 + 3*a^2*b^5 + 3*a*b^6 + b^7)*f*cos(f*x + e)^2 + (a^4*b^3 + 4*a^3*b^...
```

3.162.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

3.162.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(123) = 246$.

Time = 0.37 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.07

$$\int \frac{\sin^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = -\frac{\left(\frac{3\cos(fx+e)^2}{\left(-b\cos(fx+e)^2+a+b\right)^{\frac{3}{2}}b} - \frac{2a}{\left(-b\cos(fx+e)^2+a+b\right)^{\frac{3}{2}}b^2} - \frac{2}{\left(-b\cos(fx+e)^2+a+b\right)^{\frac{3}{2}}b}\right)\cos(fx+e) + \frac{3\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}} + \cdots}{}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*((3*cos(f*x + e)^2/((-b*cos(f*x + e)^2 + a + b)^(3/2)*b) - 2*a/((-b*c
os(f*x + e)^2 + a + b)^(3/2)*b^2) - 2/((-b*cos(f*x + e)^2 + a + b)^(3/2)*b
))*cos(f*x + e) + 3*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 2*cos
(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)^2) + cos(f*x + e)/((-b*
cos(f*x + e)^2 + a + b)^(3/2)*(a + b)) - 3*cos(f*x + e)/(sqrt(-b*cos(f*x +
 e)^2 + a + b)*b^2) + 2*a*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a
 + b)*b^2) - 2*cos(f*x + e)/((-b*cos(f*x + e)^2 + a + b)^(3/2)*b) + 4*cos(
f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*b))/f
```

3.162.8 Giac [F]

$$\int \frac{\sin^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx=\int \frac{\sin(fx+e)^5}{(b\sin(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx=\int \frac{\sin(e+fx)^5}{(b\sin(e+fx)^2+a)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^5/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)^5/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.163 $\int \frac{\sin^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.163.1 Optimal result

Integrand size = 25, antiderivative size = 81

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{2\cos(e+fx)}{3(a+b)^2 f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\cos(e+fx)\sin^2(e+fx)}{3(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}}$$

output
```
-1/3*cos(f*x+e)*sin(f*x+e)^2/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(3/2)-2/3*cos(f*
x+e)/(a+b)^2/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

3.163.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\sqrt{2}\cos(e+fx)(-5a-3b+(a+3b)\cos(2(e+fx)))}{3(a+b)^2 f(2a+b-b\cos(2(e+fx)))^{3/2}}$$

input
```
Integrate[Sin[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[2]*Cos[e + f*x]*(-5*a - 3*b + (a + 3*b)*Cos[2*(e + f*x)]))/(3*(a + b
)^2*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

3.163.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3665, 292, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^3}{\left(a+b\sin(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{1-\cos^2(e+fx)}{\left(-b\cos^2(e+fx)+a+b\right)^{5/2}}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{292}$$

$$-\frac{\frac{2\int \frac{1}{\left(-b\cos^2(e+fx)+a+b\right)^{3/2}}d\cos(e+fx)}{3(a+b)}+\frac{\cos(e+fx)\left(1-\cos^2(e+fx)\right)}{3(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{208}$$

$$-\frac{\frac{2\cos(e+fx)}{3(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}+\frac{\left(1-\cos^2(e+fx)\right)\cos(e+fx)}{3(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}}{f}$$

input
```
Int[Sin[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
-(((Cos[e + f*x]*(1 - Cos[e + f*x]^2))/(3*(a + b)*(a + b - b*Cos[e + f*x]^
2)^(3/2)) + (2*Cos[e + f*x])/(3*(a + b)^2*Sqrt[a + b - b*Cos[e + f*x]^2]))
/f)
```

3.163.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.163.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{\left(a\left(\sin^{2}\left(fx+e\right)\right)+3b\left(\sin^{2}\left(fx+e\right)\right)+2a\right)\cos\left(fx+e\right)}{3\left(a+b\left(\sin^{2}\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(a^{2}+2ab+b^{2}\right)f}$	64

input
```
int(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a*sin(f*x+e)^2+3*b*sin(f*x+e)^2+2*a)*cos(f*x+e)/(a+b*sin(f*x+e)^2)^(
3/2)/(a^2+2*a*b+b^2)/f
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.69

$$\int \frac{\sin^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{((a+3b)\cos(fx+e)^3 - 3(a+b)\cos(fx+e))\sqrt{-b}}{3\left((a^2b^2+2ab^3+b^4)f\cos(fx+e)^4 - 2(a^3b+3a^2b^2+3ab^3+b^4)f\cos(fx\right.}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((a + 3*b)*cos(f*x + e)^3 - 3*(a + b)*cos(f*x + e))*sqrt(-b*cos(f*x +
e)^2 + a + b)/((a^2*b^2 + 2*a*b^3 + b^4)*f*cos(f*x + e)^4 - 2*(a^3*b + 3*a
^2*b^2 + 3*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*
a*b^3 + b^4)*f)
```

3.163.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.49

$$\int \frac{\sin^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = -\frac{\frac{2\cos(fx+e)}{\sqrt{-b\cos(fx+e)^2+a+b}(a+b)^2} + \frac{\cos(fx+e)}{\left(-b\cos(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)} - \frac{\cos(fx+e)}{\left(-b\cos(fx+e)^2+a+b\right)^{\frac{3}{2}}b} + \frac{\cos(fx+e)}{\sqrt{-b\cos(fx+e)^2+a+b}(a+b)b}}{3f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(2*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)^2) + cos(f*x
 + e)/((-b*cos(f*x + e)^2 + a + b)^(3/2)*(a + b)) - cos(f*x + e)/((-b*cos(
f*x + e)^2 + a + b)^(3/2)*b) + cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a +
b)*(a + b)*b))/f
```

3.163.8 Giac [F]

$$\int \frac{\sin^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(fx+e)^3}{(b\sin(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.163.9 Mupad [B] (verification not implemented)

Time = 20.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.17

$$\int \frac{\sin^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{a+b\,\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}\,\left(a+3\,b-10\,a\right.}{3\,f\,{(a+b)}^2\,\left(b-4\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-2\,b\,\mathrm{e}^{e\,2\mathrm{i}+f}\right.}$$

input
```
int(sin(e + f*x)^3/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
(2*exp(e*1i + f*x*1i)*(exp(e*2i + f*x*2i) + 1)*(a + b*((exp(- e*1i - f*x*1
i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*(a + 3*b - 10*a*exp(e*2i +
f*x*2i) + a*exp(e*4i + f*x*4i) - 6*b*exp(e*2i + f*x*2i) + 3*b*exp(e*4i + f
*x*4i)))/(3*f*(a + b)^2*(b - 4*a*exp(e*2i + f*x*2i) - 2*b*exp(e*2i + f*x*2
i) + b*exp(e*4i + f*x*4i))^2)
```

3.164 $\int \frac{\sin(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.164.1 Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{\sin(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{3(a+b)f\,(a+b-b\cos^2(e+fx))^{3/2}} - \frac{2\cos(e+fx)}{3(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}}$$

output
```
-1/3*cos(f*x+e)/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(3/2)-2/3*cos(f*x+e)/(a+b)^2/
f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

3.164.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{\sin(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{2\sqrt{2}\cos(e+fx)(-3a-2b+b\cos(2(e+fx)))}{3(a+b)^2f(2a+b-b\cos(2(e+fx)))^{3/2}}$$

input
```
Integrate[Sin[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(2*Sqrt[2]*Cos[e + f*x]*(-3*a - 2*b + b*Cos[2*(e + f*x)]))/(3*(a + b)^2*f*
(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

3.164.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3665, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(e+fx)}{(a+b\sin(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 3665}$$

$$-\frac{\int \frac{1}{(-b\cos^2(e+fx)+a+b)^{5/2}}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 209}$$

$$-\frac{\frac{2\int \frac{1}{\left(-b\cos^2(e+fx)+a+b\right)^{3/2}}d\cos(e+fx)}{3(a+b)}+\frac{\cos(e+fx)}{3(a+b)(a-b\cos^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{ 208}$$

$$-\frac{\frac{2\cos(e+fx)}{3(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}+\frac{\cos(e+fx)}{3(a+b)(a-b\cos^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[Sin[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
-((Cos[e + f*x]/(3*(a + b)*(a + b - b*Cos[e + f*x]^2)^(3/2)) + (2*Cos[e +
f*x])/(3*(a + b)^2*Sqrt[a + b - b*Cos[e + f*x]^2]))/f)
```

3.164.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.164.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{(2b(\sin^2(fx+e))+3a+b)\cos(fx+e)}{3(a+b(\sin^2(fx+e)))^{\frac{3}{2}}(a^2+2ab+b^2)f}$	55

input
```
int(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(2*b*sin(f*x+e)^2+3*a+b)*cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2)/(a^2+2*a
*b+b^2)/f
```

3.164.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(65) = 130.

Time = 0.38 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.84

$$\int \frac{\sin(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(2\,b\cos\left(fx+e\right)^{3}-3\left(a+b\right)\cos\left(fx+e\right)\right)\sqrt{-b\cos}}{3\left(\left(a^{2}b^{2}+2\,ab^{3}+b^{4}\right)f\cos\left(fx+e\right)^{4}-2\left(a^{3}b+3\,a^{2}b^{2}+3\,ab^{3}+b^{4}\right)f\cos\left(fx\right.\right.}$$

input
```
integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*b*cos(f*x + e)^3 - 3*(a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 +
 a + b)/((a^2*b^2 + 2*a*b^3 + b^4)*f*cos(f*x + e)^4 - 2*(a^3*b + 3*a^2*b^2
 + 3*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
+ b^4)*f)
```

3.164.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{\sin(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{\frac{2\,\cos(fx+e)}{\sqrt{-b\cos(fx+e)^2+a+b}\,(a+b)^2}+\frac{\cos(fx+e)}{\left(-b\cos(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)}}{3\,f}$$

input
```
integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(2*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)^2) + cos(f*x
 + e)/((-b*cos(f*x + e)^2 + a + b)^(3/2)*(a + b)))/f
```

3.164.8 Giac [F]

$$\int \frac{\sin(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sin\left(fx+e\right)}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.164.9 Mupad [B] (verification not implemented)

Time = 20.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.18

$$\int \frac{\sin(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{4\,e^{e\,1i+f\,x\,1i}\,\left(e^{e\,2i+f\,x\,2i}+1\right)\,\sqrt{a+b\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,\left(b-6\,a\,e^{e\,2i+f\cdots}\right.}{3\,f\,\left(a+b\right)^2\,\left(b-4\,a\,e^{e\,2i+f\,x\,2i}-2\,b\,e^{e\,2i+f\,x\,2i}+b\,e^{e\,4i\cdots}\right.}$$

input
```
int(sin(e + f*x)/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
(4*exp(e*1i + f*x*1i)*(exp(e*2i + f*x*2i) + 1)*(a + b*((exp(- e*1i - f*x*1
i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*(b - 6*a*exp(e*2i + f*x*2i)
 - 4*b*exp(e*2i + f*x*2i) + b*exp(e*4i + f*x*4i)))/(3*f*(a + b)^2*(b - 4*a
*exp(e*2i + f*x*2i) - 2*b*exp(e*2i + f*x*2i) + b*exp(e*4i + f*x*4i))^2)
```

3.165 $\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.165.1 Optimal result

Integrand size = 23, antiderivative size = 129

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{a^{5/2}f} + \frac{b\cos(e+fx)}{3a(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} + \frac{b(5a+3b)\cos(e+fx)}{3a^2(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}}$$

output
```
-arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/a^(5/2)/f+1/3*b*co
s(f*x+e)/a/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(3/2)+1/3*b*(5*a+3*b)*cos(f*x+e)/a
^2/(a+b)^2/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

3.165.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)}{a^{5/2}} + \frac{\sqrt{2}b\cos(e+fx)\left(12a^2+13ab+3b^2-b(5a+3b)\cos(2(e+fx))\right)}{3a^2(a+b)^2(2a+b-b\cos(2(e+fx)))^{3/2}}}{f}$$

input
```
Integrate[Csc[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(-(ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)
]]]/a^(5/2)) + (Sqrt[2]*b*Cos[e + f*x]*(12*a^2 + 13*a*b + 3*b^2 - b*(5*a +
 3*b)*Cos[2*(e + f*x)]))/(3*a^2*(a + b)^2*(2*a + b - b*Cos[2*(e + f*x)])^(
3/2)))/f
```

3.165.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 3665, 316, 25, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(e+fx)\left(a+b\sin(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3665}$$

$$-\frac{\int \frac{1}{(1-\cos^2(e+fx))(-b\cos^2(e+fx)+a+b)^{5/2}}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{ 316}$$

$$-\frac{-\frac{\int -\frac{2b\cos^2(e+fx)+3a+b}{\left(1-\cos^2(e+fx)\right)\left(-b\cos^2(e+fx)+a+b\right)^{3/2}}d\cos(e+fx)}{3a(a+b)}-\frac{b\cos(e+fx)}{3a(a+b)(a-b\cos^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\int \frac{2b\cos^2(e+fx)+3a+b}{\left(1-\cos^2(e+fx)\right)\left(-b\cos^2(e+fx)+a+b\right)^{3/2}}d\cos(e+fx)}{3a(a+b)}-\frac{b\cos(e+fx)}{3a(a+b)(a-b\cos^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \text{ 402}$$

$$-\frac{\frac{-\frac{\int -\frac{3(a+b)^2}{\left(1-\cos^2(e+fx)\right)\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)}{a(a+b)}-\frac{b(5a+3b)\cos(e+fx)}{a(a+b)\sqrt{a-b\cos^2(e+fx)+b}}}{3a(a+b)}-\frac{b\cos(e+fx)}{3a(a+b)(a-b\cos^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \, 27$$

$$-\frac{\frac{\frac{3(a+b)\int \frac{1}{\left(1-\cos^2(e+fx)\right)\sqrt{-b\cos^2(e+fx)+a+b}}d\cos(e+fx)}{a}-\frac{b(5a+3b)\cos(e+fx)}{a(a+b)\sqrt{a-b\cos^2(e+fx)+b}}}{3a(a+b)}-\frac{b\cos(e+fx)}{3a(a+b)(a-b\cos^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \, 291$$

$$-\frac{\frac{\frac{3(a+b)\int \frac{1}{1-\frac{a\cos^2(e+fx)}{-b\cos^2(e+fx)+a+b}}d\frac{\cos(e+fx)}{\sqrt{-b\cos^2(e+fx)+a+b}}}{a}-\frac{b(5a+3b)\cos(e+fx)}{a(a+b)\sqrt{a-b\cos^2(e+fx)+b}}}{3a(a+b)}-\frac{b\cos(e+fx)}{3a(a+b)(a-b\cos^2(e+fx)+b)^{3/2}}}{f}$$

$$\Big\downarrow \, 219$$

$$-\frac{\frac{\frac{3(a+b)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{a^{3/2}}-\frac{b(5a+3b)\cos(e+fx)}{a(a+b)\sqrt{a-b\cos^2(e+fx)+b}}}{3a(a+b)}-\frac{b\cos(e+fx)}{3a(a+b)(a-b\cos^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[Csc[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
-((-1/3*(b*Cos[e + f*x])/(a*(a + b)*(a + b - b*Cos[e + f*x]^2)^(3/2)) + ((
3*(a + b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/
a^(3/2) - (b*(5*a + 3*b)*Cos[e + f*x])/(a*(a + b)*Sqrt[a + b - b*Cos[e + f
*x]^2]))/(3*a*(a + b)))/f)
```

3.165.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.165. $\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.165. $\int \frac{\csc(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.165.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(115) = 230$.

Time = 1.36 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.93

method	result
default	$\frac{\sqrt{-\left(-b\left(\sin^{2}\left(fx+e\right)\right)-a\right)\left(\cos^{2}\left(fx+e\right)\right)}\left(-\frac{\ln\left(\frac{2a+\left(-a+b\right)\left(\sin^{2}\left(fx+e\right)\right)+2\sqrt{a}\,\sqrt{-\left(-b\left(\sin^{2}\left(fx+e\right)\right)-a\right)\left(\cos^{2}\left(fx+e\right)\right)}}{\sin\left(fx+e\right)^{2}}\right)}{2a^{\frac{5}{2}}}+\frac{}{3a\sqrt{-\left(-b\left(\sin^{2}\left(\right.\right.\right.}}\right.}{\cos\left(fx+e\right)\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}}$

input
```
int(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(-1/2/a^(5/2)*ln((2*a+(-a+b)*sin
(f*x+e)^2+2*a^(1/2)*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/sin(f*x+e)^
2)+1/3/a*b*(2*b*sin(f*x+e)^2+3*a+b)*cos(f*x+e)^2/(-(-b*sin(f*x+e)^2-a)*cos
(f*x+e)^2)^(1/2)/(a+b*sin(f*x+e)^2)/(a^2+2*a*b+b^2)+b/a^2*cos(f*x+e)^2/(a+
b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2
)^(1/2)/f
```

3.165.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(115) = 230$.

Time = 0.56 (sec) , antiderivative size = 752, normalized size of antiderivative = 5.83

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \left[\frac{3\left(\left(a^2b^2+2\,ab^3+b^4\right)\cos\left(fx+e\right)^4+a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4-2\,(\right.}{}\right.$$

input
```
integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2
*b^2 + 4*a*b^3 + b^4 - 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^
2)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b -
b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sq
rt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4
 - 2*cos(f*x + e)^2 + 1)) - 4*((5*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^3 - 3*(2
*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))
/((a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^4 - 2*(a^6*b + 3*a^5*b^2
+ 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a
^4*b^3 + a^3*b^4)*f), 1/6*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a
^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3
+ b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a +
b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a
*b)*cos(f*x + e))) - 2*((5*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^3 - 3*(2*a^3*b
+ 3*a^2*b^2 + a*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^5*
b^2 + 2*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^4 - 2*(a^6*b + 3*a^5*b^2 + 3*a^4
*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3
+ a^3*b^4)*f)]
```

3.165.6 Sympy [F]

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\csc\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Integral(csc(e + f*x)/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.165.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(115) = 230$.

Time = 0.41 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.37

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\frac{\frac{4\,b^3\cos(fx+e)}{\sqrt{-b\cos(fx+e)^2+a+b}\,a^3b^2+2\sqrt{-b\cos(fx+e)^2+a+b}\,a^2b^3+\sqrt{-b\cos(fx+e)^2+a+b}\,ab^4}+\frac{\cdots}{\left(-b\cos(fx\cdots\right.}}{\cdots}$$

input
```
integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(4*b^3*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*a^3*b^2 + 2*sqrt(
-b*cos(f*x + e)^2 + a + b)*a^2*b^3 + sqrt(-b*cos(f*x + e)^2 + a + b)*a*b^4
) + 2*b^2*cos(f*x + e)/((-b*cos(f*x + e)^2 + a + b)^(3/2)*a^2*b + (-b*cos(
f*x + e)^2 + a + b)^(3/2)*a*b^2) + 6*b^2*cos(f*x + e)/(sqrt(-b*cos(f*x + e
)^2 + a + b)*a^3*b + sqrt(-b*cos(f*x + e)^2 + a + b)*a^2*b^2) - 3*log(b -
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x + e) - 1) - a/(cos(f*x +
e) - 1))/a^(5/2) + 3*log(-b + sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos
(f*x + e) + 1) + a/(cos(f*x + e) + 1))/a^(5/2))/f
```

3.165.8 Giac [F]

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\csc\left(fx+e\right)}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\sin\left(e+f\,x\right)\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(5/2)), x)
```

3.166 $\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.166.1 Optimal result

Integrand size = 25, antiderivative size = 285

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{a\cos(e+fx)\sin^3(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2a(2a+3b)\cos(e+fx)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{(8a^2+13ab+3b^2)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3b^3(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{a(8a+9b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3b^3(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
1/3*a*cos(f*x+e)*sin(f*x+e)^3/b/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*a*(2*
a+3*b)*cos(f*x+e)*sin(f*x+e)/b^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*(8
*a^2+13*a*b+3*b^2)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+
e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/b^3/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1
/2)-1/3*a*(8*a+9*b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x
+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/b^3/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/
2)
```

3.166.2 Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.67

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{a\left(-2a(8a^2+13ab+3b^2)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)+2a(8a^2+17ab+9b^2)\left(\frac{2a+b-b\cos(2(e+fx)}{a}\right.\right.}{6b^3(a+b)^2f(2a+b-b}$$

input
```
Integrate[Sin[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
-1/6*(a*(-2*a*(8*a^2 + 13*a*b + 3*b^2)*((2*a + b - b*Cos[2*(e + f*x)])/a)^
(3/2)*EllipticE[e + f*x, -(b/a)] + 2*a*(8*a^2 + 17*a*b + 9*b^2)*((2*a + b
- b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF[e + f*x, -(b/a)] + Sqrt[2]*b*(-8*
a^2 - 17*a*b - 7*b^2 + b*(5*a + 7*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/
(b^3*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

3.166.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3667, 372, 440, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(e+fx)^6}{\left(a+b\sin(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3667}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sin^6(e+fx)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 372}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin^3(e+fx)\sqrt{1-\sin^2(e+fx)}}{3b(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\int\frac{\sin^2(e+fx)\left(3a-(4a+3b)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3b(a+b)}\right)}{f}$$

$\Big\downarrow$ 440

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin^3(e+fx)\sqrt{1-\sin^2(e+fx)}}{3b(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\int\frac{2a(2a+3b)-\left(8a^2+13ba+3b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b(a+b)}-\frac{2a(2a+3b)\sin(e+fx)\sqrt{\cdots}}{b(a+b)\sqrt{a+b\sin^2\cdots}}}{3b(a+b)}\cdots\right.}{f}$$

$\Big\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin^3(e+fx)\sqrt{1-\sin^2(e+fx)}}{3b(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{a(a+b)(8a+9b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{\left(8a^2+13ab+3b\cdots\right.}{\cdots}}{b(a+b)}}{3b(a+b\cdots}\cdots\right.}{f}$$

$\Big\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin^3(e+fx)\sqrt{1-\sin^2(e+fx)}}{3b(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{a(a+b)(8a+9b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+f\cdots}{b\sqrt{a+b\sin^2(e+fx)}}\cdots}{b(a+b)}}{\cdots}\cdots\right.}{f}$$

$\Big\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin^3(e+fx)\sqrt{1-\sin^2(e+fx)}}{3b(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{a(a+b)(8a+9b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(8a^2+13ab\cdots\right.}{\cdots}}{b(a+b)}}{3b(a\cdots}\cdots\right.}{f}$$

3.166. $\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$\Big\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin^3(e+fx)\sqrt{1-\sin^2(e+fx)}}{3b(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{a(a+b)(8a+9b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(8a^2+13ab\right.}{}}{b(a+b)}}{}\right.}{f}$$

$\Big\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin^3(e+fx)\sqrt{1-\sin^2(e+fx)}}{3b(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{a(a+b)(8a+9b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\left(8a^2+13ab\right.}{b(a+b)}}{}\right.}{f}$$

input
```
Int[Sin[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((a*Sin[e + f*x]^3*Sqrt[1 - Sin[e + f*x
]^2])/(3*b*(a + b)*(a + b*Sin[e + f*x]^2)^(3/2)) - ((-2*a*(2*a + 3*b)*Sin[
e + f*x]*Sqrt[1 - Sin[e + f*x]^2])/(b*(a + b)*Sqrt[a + b*Sin[e + f*x]^2])
+ (-(((8*a^2 + 13*a*b + 3*b^2)*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqr
t[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) + (a*(a + b)*
(8*a + 9*b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*
x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2]))/(b*(a + b)))/(3*b*(a + b))))/f
```

3.166.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.166. $\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

3.166. $\int \frac{\sin^6(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.166.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(263) = 526$.

Time = 3.98 (sec) , antiderivative size = 698, normalized size of antiderivative = 2.45

method	result
default	$-\left((5a b^2+7b^3)(\cos^4(fx+e))\sin(fx+e)+(-4a^2b-11a b^2-7b^3)(\cos^2(fx+e))\sin(fx+e)-\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\sqrt{\frac{\cos(2fx+2e)}{2}+}\right.$

input
```
int(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((5*a*b^2+7*b^3)*cos(f*x+e)^4*sin(f*x+e)+(-4*a^2*b-11*a*b^2-7*b^3)*co
s(f*x+e)^2*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/
2)*b*(8*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+17*EllipticF(sin(f*x+e),(
-1/a*b)^(1/2))*a*b+9*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-8*EllipticE(
sin(f*x+e),(-1/a*b)^(1/2))*a^2-13*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b
-3*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b^2)*cos(f*x+e)^2+8*(cos(f*x+e)^2)
^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a^3+25*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Elliptic
F(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+26*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e
)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2+9*(cos(f*x+e
)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)
^(1/2))*b^3-8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Ellip
ticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-21*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+
e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-16*(cos(f*x
+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*
b)^(1/2))*a*b^2-3*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*E
llipticE(sin(f*x+e),(-1/a*b)^(1/2))*b^3)*a/(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^
2/b^3/cos(f*x+e)/f
```

3.166.5 Fricas [F]

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sin\left(fx+e\right)^6}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*sqrt(-
b*cos(f*x + e)^2 + a + b)/(b^3*cos(f*x + e)^6 - 3*(a*b^2 + b^3)*cos(f*x +
e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2*b + 2*a*b^2 + b^3)*cos(f*x +
 e)^2), x)
```

3.166.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**6/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

3.166.7 Maxima [F]

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sin\left(fx+e\right)^6}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.166.8 Giac [F]

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sin\left(fx+e\right)^6}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+f\,x)^6}{\left(b\sin(e+f\,x)^2+a\right)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^6/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)^6/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.167 $\int \frac{\sin^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.167.1 Optimal result

Integrand size = 25, antiderivative size = 269

$$\int \frac{\sin^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{a\cos(e+fx)\sin(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a+2b)\cos(e+fx)\sin(e+fx)}{3b(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{2(a+2b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3b^2(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{(2a+3b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3b^2(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
1/3*a*cos(f*x+e)*sin(f*x+e)/b/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)-2/3*(a+2*b)
*cos(f*x+e)*sin(f*x+e)/b/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)-2/3*(a+2*b)*El
lipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(
f*x+e)^2)^(1/2)/b^2/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*(2*a+3*b)*Ell
ipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f
*x+e)^2/a)^(1/2)/b^2/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.167.2 Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.68

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{2a^2(a+2b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)-a(2a^2+5ab+3b^2)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}\text{EllipticF}\left(e\right.}{3b^2(a+b)^2f(2a+b-b\cos(2(e+}$$

input
```
Integrate[Sin[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
-1/3*(2*a^2*(a + 2*b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e
 + f*x, -(b/a)] - a*(2*a^2 + 5*a*b + 3*b^2)*((2*a + b - b*Cos[2*(e + f*x)]
)/a)^(3/2)*EllipticF[e + f*x, -(b/a)] - Sqrt[2]*b*(-a^2 - 4*a*b - 2*b^2 +
b*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(b^2*(a + b)^2*f*(2*a + b
- b*Cos[2*(e + f*x)])^(3/2))
```

3.167.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3667, 372, 402, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^4}{\left(a+b\sin(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3667}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sin^4(e+fx)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3b(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\int\frac{a-(2a+3b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3b(a+b)}\right)}{f}$$

$\downarrow$ 402

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3b(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a+2b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\int\frac{a\left(-2(a+2b)\sin^2(e+fx)+a+3b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+}{a(a+b)}}{3b(a+b)}\right.}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3b(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a+2b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\int\frac{-2(a+2b)\sin^2(e+fx)+a+3b}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+}{a+b}}{3b(a+b)}\right.}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3b(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a+2b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{(a+b)(2a+3b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+}}}{b}}{3b(a+b)}\right.}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3b(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a+2b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{(a+b)(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{}{\sqrt{1-\sin^2}}}{b\sqrt{a+b\sin^2(}}}{3b(a}\right.}{f}$$

$\downarrow$ 321

3.167. $\int\frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3b(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a+2b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{(a+b)(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\text{ar}\right.}{b\sqrt{a+b\sin^2(e+fx)}}}{}}{3b(a+b)}\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3b(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a+2b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{(a+b)(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\text{ar}\right.}{b\sqrt{a+b\sin^2(e+fx)}}}{}}{3b(}\right.}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3b(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a+2b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{(a+b)(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\text{ar}\right.}{b\sqrt{a+b\sin^2(e+fx)}}}{}}{3b(a+}\right.}{f}$$

input
```
Int[Sin[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((a*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^
2])/(3*b*(a + b)*(a + b*Sin[e + f*x]^2)^(3/2)) - ((2*(a + 2*b)*Sin[e + f*x
]*Sqrt[1 - Sin[e + f*x]^2])/((a + b)*Sqrt[a + b*Sin[e + f*x]^2]) - ((-2*(a
 + 2*b)*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2]
)/(b*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((a + b)*(2*a + 3*b)*EllipticF[ArcS
in[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Si
n[e + f*x]^2]))/(a + b))/(3*b*(a + b))))/f
```

3.167. $\int \frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.167.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

3.167. $\int \frac{\sin^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.167.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(247) = 494$.

Time = 2.80 (sec) , antiderivative size = 623, normalized size of antiderivative = 2.32

method	result
default	$\frac{(2a\,b^2+4b^3)\left(\cos^4(fx+e)\right)\sin(fx+e)+\left(-a^2b-5a\,b^2-4b^3\right)\left(\cos^2(fx+e)\right)\sin(fx+e)-\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}\,\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,b\Big(2}{}$

input
```
int(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((2*a*b^2+4*b^3)*cos(f*x+e)^4*sin(f*x+e)+(-a^2*b-5*a*b^2-4*b^3)*cos(f*
x+e)^2*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*b
*(2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+5*EllipticF(sin(f*x+e),(-1/a*
b)^(1/2))*a*b+3*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-2*EllipticE(sin(f
*x+e),(-1/a*b)^(1/2))*a^2-4*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b)*cos(
f*x+e)^2+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Elliptic
F(sin(f*x+e),(-1/a*b)^(1/2))*a^3+7*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2
+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+8*(cos(f*x+e)^2
)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1
/2))*a*b^2+3*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Ellipt
icF(sin(f*x+e),(-1/a*b)^(1/2))*b^3-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)
^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-6*(cos(f*x+e)^2
)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1
/2))*a^2*b-4*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Ellipt
icE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2)/(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^2/b^2
/cos(f*x+e)/f
```

3.167.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 1432, normalized size of antiderivative = 5.32

$$\int \frac{\sin^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((2*((-I*a*b^3 - 2*I*b^4)*cos(f*x + e)^4 - I*a^3*b - 4*I*a^2*b^2 - 5*I
*a*b^3 - 2*I*b^4 - 2*(-I*a^2*b^2 - 3*I*a*b^3 - 2*I*b^4)*cos(f*x + e)^2)*sq
rt(-b)*sqrt((a^2 + a*b)/b^2) - ((2*I*a^2*b^2 + 5*I*a*b^3 + 2*I*b^4)*cos(f*
x + e)^4 + 2*I*a^4 + 9*I*a^3*b + 14*I*a^2*b^2 + 9*I*a*b^3 + 2*I*b^4 + 2*(-
2*I*a^3*b - 7*I*a^2*b^2 - 7*I*a*b^3 - 2*I*b^4)*cos(f*x + e)^2)*sqrt(-b))*s
qrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*s
qrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^
2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((I*a*b
^3 + 2*I*b^4)*cos(f*x + e)^4 + I*a^3*b + 4*I*a^2*b^2 + 5*I*a*b^3 + 2*I*b^4
 - 2*(I*a^2*b^2 + 3*I*a*b^3 + 2*I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2
+ a*b)/b^2) - ((-2*I*a^2*b^2 - 5*I*a*b^3 - 2*I*b^4)*cos(f*x + e)^4 - 2*I*a
^4 - 9*I*a^3*b - 14*I*a^2*b^2 - 9*I*a*b^3 - 2*I*b^4 + 2*(2*I*a^3*b + 7*I*a
^2*b^2 + 7*I*a*b^3 + 2*I*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^
2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b
^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 -
 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((I*a*b^3 + I*b^4)*cos(f
*x + e)^4 + I*a^3*b + 3*I*a^2*b^2 + 3*I*a*b^3 + I*b^4 - 2*(I*a^2*b^2 + 2*I
*a*b^3 + I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((-2*I*a^
2*b^2 - 7*I*a*b^3 - 3*I*b^4)*cos(f*x + e)^4 - 2*I*a^4 - 11*I*a^3*b - 19*I*
a^2*b^2 - 13*I*a*b^3 - 3*I*b^4 + 2*(2*I*a^3*b + 9*I*a^2*b^2 + 10*I*a*b^...
```

3.167.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**4/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

3.167.7 Maxima [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(fx+e)^4}{(b\sin(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.167.8 Giac [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(fx+e)^4}{(b\sin(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+f\,x)^4}{(b\sin(e+f\,x)^2+a)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.168 $\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.168.1 Optimal result

Integrand size = 25, antiderivative size = 221

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{\cos(e+fx)\sin(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a-b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3ab(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{\operatorname{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3b(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-1/3*cos(f*x+e)*sin(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)-1/3*(a-b)*cos(
f*x+e)*sin(f*x+e)/a/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(a-b)*(cos(f*x+
e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^
2)^(1/2)/a/b/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*(cos(f*x+e)^2)^(1/2)
/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/
b/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.168.2 Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.79

$$\int \frac{\sin^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{-2a^2(a-b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)+2a^2(a+b)\left(\frac{2a+b-b\cos(2}{a}\right.}{6ab(a+b)}$$

input
```
Integrate[Sin[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(-2*a^2*(a - b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x
, -(b/a)] + 2*a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*Ellipti
cF[e + f*x, -(b/a)] - Sqrt[2]*b*(4*a^2 + a*b - b^2 + b*(-a + b)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/(6*a*b*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*x)]
)^(3/2))
```

3.168.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 3652, 3042, 3652, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^2}{(a+b\sin(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{3652}$$

$$\frac{\int \frac{a\sin^2(e+fx)+a}{(b\sin^2(e+fx)+a)^{3/2}}dx}{3a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a\sin(e+fx)^2+a}{(b\sin(e+fx)^2+a)^{3/2}}dx}{3a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3652}$$

$$\frac{\frac{\int \frac{2a^2-a(a-b)\sin^2(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}dx}{a(a+b)}-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{2a^2-a(a-b)\sin(e+fx)^2}{\sqrt{b\sin(e+fx)^2+a}}dx}{a(a+b)}-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3651}$$

$$\frac{\frac{\frac{a^2(a+b)\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}dx}{b}-\frac{a(a-b)\int \sqrt{b\sin^2(e+fx)+a}dx}{b}}{a(a+b)}-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{a^2(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}-\frac{a(a-b)\int \sqrt{b\sin(e+fx)^2+a}dx}{b}}{a(a+b)}-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3657}$$

$$\frac{\frac{\frac{a^2(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}-\frac{a(a-b)\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin^2(e+fx)}{a}+1}dx}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{a(a+b)}-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{a^2(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}-\frac{a(a-b)\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin(e+fx)^2}{a}+1}dx}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{a(a+b)}-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

3.168. $\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\Big\downarrow \text{3656}$$

$$\frac{\frac{\frac{a^2(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{b}-\frac{a(a-b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\left|-\frac{b}{a}\right.\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{a(a+b)}-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3662}$$

$$\frac{\frac{\frac{a^2(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}dx}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{a(a-b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\left|-\frac{b}{a}\right.\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{a(a+b)}-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{a^2(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin(e+fx)^2}{a}+1}}dx}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{a(a-b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\left|-\frac{b}{a}\right.\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{a(a+b)}-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3661}$$

$$\frac{\frac{\frac{a^2(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}-\frac{a(a-b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\left|-\frac{b}{a}\right.\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{a(a+b)}-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}-\frac{\sin(e+fx)\cos(e+fx)}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

input
```
Int[Sin[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

3.168. $\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

output
```
-1/3*(Cos[e + f*x]*Sin[e + f*x])/((a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
+ (-(((a - b)*Cos[e + f*x]*Sin[e + f*x])/((a + b)*f*Sqrt[a + b*Sin[e + f*x
]^2])) + (-((a*(a - b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^
2])/(b*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) + (a^2*(a + b)*EllipticF[e + f*x
, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2])
)/(a*(a + b)))/(3*a*(a + b))
```

3.168.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

3.168. $\int \frac{\sin^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.168.4 Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.19

method	result
default	$\frac{(a\,b^2-b^3)\left(\cos^4(fx+e)\right)\sin(fx+e)+\left(-2a^2b-a\,b^2+b^3\right)\left(\cos^2(fx+e)\right)\sin(fx+e)-\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}\,ab\Big(F\Big(}{}$

input
```
int(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((a*b^2-b^3)*cos(f*x+e)^4*sin(f*x+e)+(-2*a^2*b-a*b^2+b^3)*cos(f*x+e)^2
*sin(f*x+e)-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*b*(El
lipticF(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*
b-EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticE(sin(f*x+e),(-1/a*b)^(1/
2))*b)*cos(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)
*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos
(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+(cos(f
*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/
a*b)^(1/2))*a*b^2-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*E
llipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3+(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x
+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2)/(a+b*sin(
f*x+e)^2)^(3/2)/(a+b)^2/a/b/cos(f*x+e)/f
```

3.168.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 1400, normalized size of antiderivative = 6.33

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/6*((2*((-I*a*b^3 + I*b^4)*cos(f*x + e)^4 - I*a^3*b - I*a^2*b^2 + I*a*b^3
 + I*b^4 - 2*(-I*a^2*b^2 + I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b
)/b^2) - ((2*I*a^2*b^2 - I*a*b^3 - I*b^4)*cos(f*x + e)^4 + 2*I*a^4 + 3*I*a
^3*b - I*a^2*b^2 - 3*I*a*b^3 - I*b^4 + 2*(-2*I*a^3*b - I*a^2*b^2 + 2*I*a*b
^3 + I*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*
a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*
(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*s
qrt((a^2 + a*b)/b^2))/b^2) + (2*((I*a*b^3 - I*b^4)*cos(f*x + e)^4 + I*a^3*
b + I*a^2*b^2 - I*a*b^3 - I*b^4 - 2*(I*a^2*b^2 - I*b^4)*cos(f*x + e)^2)*sq
rt(-b)*sqrt((a^2 + a*b)/b^2) - ((-2*I*a^2*b^2 + I*a*b^3 + I*b^4)*cos(f*x +
 e)^4 - 2*I*a^4 - 3*I*a^3*b + I*a^2*b^2 + 3*I*a*b^3 + I*b^4 + 2*(2*I*a^3*b
 + I*a^2*b^2 - 2*I*a*b^3 - I*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt
((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*
b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b
^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 4*(((I*a*b^3 + I*b^4)*c
os(f*x + e)^4 + I*a^3*b + 3*I*a^2*b^2 + 3*I*a*b^3 + I*b^4 + 2*(-I*a^2*b^2
- 2*I*a*b^3 - I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + ((-2
*I*a^2*b^2 - I*a*b^3)*cos(f*x + e)^4 - 2*I*a^4 - 5*I*a^3*b - 4*I*a^2*b^2 -
 I*a*b^3 + 2*(2*I*a^3*b + 3*I*a^2*b^2 + I*a*b^3)*cos(f*x + e)^2)*sqrt(-b))
*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((...
```

3.168.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

3.168.7 Maxima [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(fx+e)^2}{(b\sin(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.168.8 Giac [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(fx+e)^2}{(b\sin(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+f\,x)^2}{(b\sin(e+f\,x)^2+a)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.169 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.169.1 Optimal result

Integrand size = 16, antiderivative size = 223

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3a^2(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}-\frac{\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3a(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
1/3*b*cos(f*x+e)*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*b*(2*a+
b)*cos(f*x+e)*sin(f*x+e)/a^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+2/3*(2*a+b
)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*
sin(f*x+e)^2)^(1/2)/a^2/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*(cos(f*x+
e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^
2/a)^(1/2)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.169.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{2a^2(2a+b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)-a^2(a+b)\left(\frac{2a+b-b\cos(2(e-}{a}\right.}{3a^2(a+b}$$

input
```
Integrate[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

output
```
(2*a^2*(2*a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*
x, -(b/a)] - a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*Elliptic
F[e + f*x, -(b/a)] - Sqrt[2]*b*(-5*a^2 - 5*a*b - b^2 + b*(2*a + b)*Cos[2*(
e + f*x)])*Sin[2*(e + f*x)])/(3*a^2*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*
x)])^(3/2))
```

3.169.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 3663, 25, 3042, 3652, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\int -\frac{-b\sin^2(e+fx)+3a+2b}{\left(b\sin^2(e+fx)+a\right)^{3/2}}dx}{3a(a+b)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-b\sin^2(e+fx)+3a+2b}{\left(b\sin^2(e+fx)+a\right)^{3/2}}dx}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-b\sin(e+fx)^2+3a+2b}{(b\sin(e+fx)^2+a)^{3/2}}dx}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3652}$$

$$\frac{\frac{\int \frac{2b(2a+b)\sin^2(e+fx)+a(3a+b)}{\sqrt{b\sin^2(e+fx)+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{2b(2a+b)\sin(e+fx)^2+a(3a+b)}{\sqrt{b\sin(e+fx)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3651}$$

$$\frac{\frac{2(2a+b)\int \sqrt{b\sin^2(e+fx)+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(2a+b)\int \sqrt{b\sin(e+fx)^2+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3657}$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin^2(e+fx)}{a}+1}dx}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

3.169. $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\int\sqrt{\frac{b\sin(e+fx)^2}{a}+1}dx}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-a(a+b)\int\frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \; 3656$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-a(a+b)\int\frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \; 3662$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}dx}{\sqrt{a+b\sin^2(e+fx)}}}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin(e+fx)^2}{a}+1}}dx}{\sqrt{a+b\sin^2(e+fx)}}}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \; 3661$$

$$\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}}{a(a+b)}}{3a(a+b)}$$

input

```
Int[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

3.169. $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

output
```
(b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
 + ((2*b*(2*a + b)*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[
e + f*x]^2]) + ((2*(2*a + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e +
 f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*
x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2]))
/(a*(a + b)))/(3*a*(a + b))
```

3.169.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.169. $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.169.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(245) = 490$.

Time = 1.82 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.45

method	result
default	$-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2b\left(\sin^2(fx+e)\right)+\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e\right.}{}$

input
```
int(1/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+
e),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e
)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2-4*(c
os(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a^2*b*sin(f*x+e)^2-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a
)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2+4*a*b^2*si
n(f*x+e)^5+2*b^3*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^
(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+a^2*(cos(f*x+e)^2)^(1/2)*((
a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-4*(cos(f
*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a^3-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(s
in(f*x+e),(-1/a*b)^(1/2))*a^2*b+5*a^2*b*sin(f*x+e)^3-a*b^2*sin(f*x+e)^3-2*
b^3*sin(f*x+e)^3-5*a^2*b*sin(f*x+e)-3*a*b^2*sin(f*x+e))/(a+b*sin(f*x+e)^2)
^(3/2)/a^2/(a+b)^2/cos(f*x+e)/f
```

3.169.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 1531, normalized size of antiderivative = 6.87

$$\int \frac{1}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((2*(2*I*a^3*b^2 + 5*I*a^2*b^3 + 4*I*a*b^4 + I*b^5 + (2*I*a*b^4 + I*b^
5)*cos(f*x + e)^4 - 2*(2*I*a^2*b^3 + 3*I*a*b^4 + I*b^5)*cos(f*x + e)^2)*sq
rt(-b)*sqrt((a^2 + a*b)/b^2) - (-4*I*a^4*b - 12*I*a^3*b^2 - 13*I*a^2*b^3 -
 6*I*a*b^4 - I*b^5 + (-4*I*a^2*b^3 - 4*I*a*b^4 - I*b^5)*cos(f*x + e)^4 + 2
*(4*I*a^3*b^2 + 8*I*a^2*b^3 + 5*I*a*b^4 + I*b^5)*cos(f*x + e)^2)*sqrt(-b))
*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b
*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*
a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-2*I
*a^3*b^2 - 5*I*a^2*b^3 - 4*I*a*b^4 - I*b^5 + (-2*I*a*b^4 - I*b^5)*cos(f*x
+ e)^4 - 2*(-2*I*a^2*b^3 - 3*I*a*b^4 - I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqr
t((a^2 + a*b)/b^2) - (4*I*a^4*b + 12*I*a^3*b^2 + 13*I*a^2*b^3 + 6*I*a*b^4
+ I*b^5 + (4*I*a^2*b^3 + 4*I*a*b^4 + I*b^5)*cos(f*x + e)^4 + 2*(-4*I*a^3*b
^2 - 8*I*a^2*b^3 - 5*I*a*b^4 - I*b^5)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*
sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2
+ a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b
 + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-3*I*a^4*b - 11
*I*a^3*b^2 - 15*I*a^2*b^3 - 9*I*a*b^4 - 2*I*b^5 + (-3*I*a^2*b^3 - 5*I*a*b^
4 - 2*I*b^5)*cos(f*x + e)^4 - 2*(-3*I*a^3*b^2 - 8*I*a^2*b^3 - 7*I*a*b^4 -
2*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - (-6*I*a^5 - 17*I
*a^4*b - 17*I*a^3*b^2 - 7*I*a^2*b^3 - I*a*b^4 + (-6*I*a^3*b^2 - 5*I*a^2...
```

3.169.6 Sympy [F]

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\left(a+b\sin^2{(e+fx)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Integral((a + b*sin(e + f*x)**2)**(-5/2), x)
```

3.169.7 Maxima [F]

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

3.169.8 Giac [F]

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(1/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.170 $\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.170.1 Optimal result

Integrand size = 25, antiderivative size = 322

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(3a+2b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{(3a^2+13ab+8b^2)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3(a+b)^2f} - \frac{(3a^2+13ab+8b^2)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{(3a+4b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
1/3*b*cot(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*b*(3*a+2*b)*cot(f*
x+e)/a^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(3*a^2+13*a*b+8*b^2)*cot(f
*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a^3/(a+b)^2/f-1/3*(3*a^2+13*a*b+8*b^2)*Elli
pticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*
x+e)^2)^(1/2)/a^3/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*(3*a+4*b)*Ellip
ticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x
+e)^2/a)^(1/2)/a^2/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.170.2 Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.66

$$\int \frac{\csc^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{4a^2\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}\left(-\left((3a^2+13ab+8b^2)\,E\left(e+fx\middle|-\frac{b}{a}\right)\right)+(3a^2+\right.}{}$$

input
```
Integrate[Csc[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(4*a^2*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*(-((3*a^2 + 13*a*b + 8*b^2
)*EllipticE[e + f*x, -(b/a)]) + (3*a^2 + 7*a*b + 4*b^2)*EllipticF[e + f*x,
 -(b/a)]) - 2*Sqrt[2]*(3*(a + b)^2*(2*a + b - b*Cos[2*(e + f*x)])^2*Cot[e
+ f*x] + 2*a*b^2*(a + b)*Sin[2*(e + f*x)] + b^2*(7*a + 5*b)*(2*a + b - b*C
os[2*(e + f*x)])*Sin[2*(e + f*x)]))/(12*a^3*(a + b)^2*f*(2*a + b - b*Cos[2
*(e + f*x)])^(3/2))
```

3.170.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 3667, 374, 25, 441, 25, 445, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^2\,(a+b\sin(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{3667}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\csc^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\,(b\sin^2(e+fx)+a)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{374}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\sqrt{1-\sin^2(e+fx)}\csc(e+fx)}{3a(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\int -\frac{\csc^2(e+fx)\left(-3b\sin^2(e+fx)+3a+4b\right)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}\right)}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int \frac{\csc^2(e+fx)\left(-3b\sin^2(e+fx)+3a+4b\right)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}+\frac{b\sqrt{1-\sin^2(e+fx)}\csc(e+fx)}{3a(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{f}$$

$\downarrow$ 441

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{2b(3a+2b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\int -\frac{\csc^2(e+fx)\left(3a^2+13ba+8b^2-2b(3a+2b)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a(a+b)}}{3a(a+b)}+\frac{b\sqrt{1-\cdots}}{3a(a\cdots}\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{\int \frac{\csc^2(e+fx)\left(3a^2+13ba+8b^2-2b(3a+2b)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a(a+b)}+\frac{2b(3a+2b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sqrt{1-\cdots}}{3a(a+b\cdots}\right.}{f}$$

$\downarrow$ 445

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{-\frac{\int \frac{b\left(\left(3a^2+13ba+8b^2\right)\sin^2(e+fx)+2a(3a+2b)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{\left(3a^2+13ab+8b^2\right)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(\cdots}}{a}}{a(a+b)}}{3a(a+b)}\right.\cdots}{f}$$

$\downarrow$ 27

3.170. $\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{\frac{b\int\frac{\left(3a^2+13ba+8b^2\right)\sin^2(e+fx)+2a(3a+2b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{\left(3a^2+13ab+8b^2\right)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+\cdots}}{a}}{a(a+b)}}{3a(a+b)}\cdots\right.}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{b\left(\frac{\left(3a^2+13ab+8b^2\right)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a(a+b)(3a+4b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+f\cdots}{b}\right.}{a}\cdots}{a(a+b)}}{3a(a+b)}\cdots\right.}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{b\left(\frac{\left(3a^2+13ab+8b^2\right)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a(a+b)(3a+4b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e\cdots}{a}}}}{b\sqrt{a+b\sin^2(e+fx)}}\right.}{a}\cdots}{a(a+b)}}{3\cdots}\cdots\right.}{\cdots}$$

$\downarrow$ 321

3.170. $\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{b\left(\frac{\left(3a^2+13ab+8b^2\right)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a(a+b)(3a+4b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\right.}{b\sqrt{a+b\sin^2(e+fx)}}\right.}{a}}{a(a+b)}}{3a(a+b}\right.}{f}$$

$\downarrow$ 330

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{b\left(\frac{\left(3a^2+13ab+8b^2\right)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)(3a+4b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\right.}{b\sqrt{a+b\sin^2(e+fx)}}\right.}{a}}{a(a+b)}\right.$$

$\downarrow$ 327

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{b\left(\frac{\left(3a^2+13ab+8b^2\right)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)(3a+4b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin\right.}{b\sqrt{a+b\sin^2(e+fx)}}\right.}{a}}{a(a+b)}}{3a}\right.$$

input
```
Int[Csc[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

3.170. $\int\frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((b*Csc[e + f*x]*Sqrt[1 - Sin[e + f*x]^
2])/(3*a*(a + b)*(a + b*Sin[e + f*x]^2)^(3/2)) + ((2*b*(3*a + 2*b)*Csc[e +
 f*x]*Sqrt[1 - Sin[e + f*x]^2])/(a*(a + b)*Sqrt[a + b*Sin[e + f*x]^2]) + (
-(((3*a^2 + 13*a*b + 8*b^2)*Csc[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a +
 b*Sin[e + f*x]^2])/a) - (b*(((3*a^2 + 13*a*b + 8*b^2)*EllipticE[ArcSin[Si
n[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e + f*
x]^2)/a]) - (a*(a + b)*(3*a + 4*b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]
*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2])))/a)/(a*(a
 + b)))/(3*a*(a + b))))/f
```

3.170.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

3.170. $\int \frac{\csc^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.170. $\int \frac{\csc^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.170.4 Maple [A] (verified)

Time = 3.70 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.64

method	result
default	$\frac{\left(-3a^2b^2-13a\,b^3-8b^4\right)\left(\cos^6(fx+e)\right)+\left(6a^3b+26a^2b^2+38a\,b^3+16b^4\right)\left(\cos^4(fx+e)\right)-\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}\,ab\Big(3}{}$

input
```
int(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-3*a^2*b^2-13*a*b^3-8*b^4)*cos(f*x+e)^6+(6*a^3*b+26*a^2*b^2+38*a*b^3
+16*b^4)*cos(f*x+e)^4-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/
2)*a*b*(3*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+7*EllipticF(sin(f*x+e),
(-1/a*b)^(1/2))*a*b+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-3*EllipticE
(sin(f*x+e),(-1/a*b)^(1/2))*a^2-13*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*
b-8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b^2)*cos(f*x+e)^2*sin(f*x+e)+(-3*
a^4-12*a^3*b-26*a^2*b^2-25*a*b^3-8*b^4)*cos(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*
(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*(3*EllipticF(sin(f*x+e),(-1/a*b)^(1/2)
)*a^3+10*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+11*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a*b^2+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^3-3*Ellip
ticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-16*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)
)*a^2*b-21*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2-8*EllipticE(sin(f*x+
e),(-1/a*b)^(1/2))*b^3)*sin(f*x+e))/(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^2/sin(f
*x+e)/a^3/cos(f*x+e)/f
```

3.170.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 1719, normalized size of antiderivative = 5.34

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/6*((2*(-3*I*a^4*b - 19*I*a^3*b^2 - 37*I*a^2*b^3 - 29*I*a*b^4 - 8*I*b^5 +
 (-3*I*a^2*b^3 - 13*I*a*b^4 - 8*I*b^5)*cos(f*x + e)^4 - 2*(-3*I*a^3*b^2 -
16*I*a^2*b^3 - 21*I*a*b^4 - 8*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 +
a*b)/b^2)*sin(f*x + e) - (6*I*a^5 + 41*I*a^4*b + 93*I*a^3*b^2 + 95*I*a^2*b
^3 + 45*I*a*b^4 + 8*I*b^5 + (6*I*a^3*b^2 + 29*I*a^2*b^3 + 29*I*a*b^4 + 8*I
*b^5)*cos(f*x + e)^4 + 2*(-6*I*a^4*b - 35*I*a^3*b^2 - 58*I*a^2*b^3 - 37*I*
a*b^4 - 8*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^
2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b
^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 -
 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(3*I*a^4*b + 19*I*a^3*b^
2 + 37*I*a^2*b^3 + 29*I*a*b^4 + 8*I*b^5 + (3*I*a^2*b^3 + 13*I*a*b^4 + 8*I*
b^5)*cos(f*x + e)^4 - 2*(3*I*a^3*b^2 + 16*I*a^2*b^3 + 21*I*a*b^4 + 8*I*b^5
)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - (-6*I*a^5
- 41*I*a^4*b - 93*I*a^3*b^2 - 95*I*a^2*b^3 - 45*I*a*b^4 - 8*I*b^5 + (-6*I*
a^3*b^2 - 29*I*a^2*b^3 - 29*I*a*b^4 - 8*I*b^5)*cos(f*x + e)^4 + 2*(6*I*a^4
*b + 35*I*a^3*b^2 + 58*I*a^2*b^3 + 37*I*a*b^4 + 8*I*b^5)*cos(f*x + e)^2)*s
qrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*ellipt
ic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) -
I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/
b^2))/b^2) - 4*((-9*I*a^4*b - 35*I*a^3*b^2 - 51*I*a^2*b^3 - 33*I*a*b^4 ...
```

3.170.6 Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\csc^2(e+fx)}{(a+b\sin^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Integral(csc(e + f*x)**2/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.170.7 Maxima [F]

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\csc\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.170.8 Giac [F]

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\csc\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\sin\left(e+f\,x\right)^2\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(5/2)), x)
```

3.171 $\int (d\sin(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p \,dx$

3.171.1 Optimal result

Integrand size = 25, antiderivative size = 122

$$\int (d\sin(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p \,dx =$$

$$-\frac{d\,\text{AppellF1}\left(\frac{1}{2},\frac{1-m}{2},-p,\frac{3}{2},\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+}{a+b}\right.}{f}$$

output
```
-d*AppellF1(1/2,-1/2*m+1/2,-p,3/2,cos(f*x+e)^2,b*cos(f*x+e)^2/(a+b))*cos(f
*x+e)*(a+b-b*cos(f*x+e)^2)^p*(d*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(-1/2*m+
1/2)/f/((1-b*cos(f*x+e)^2/(a+b))^p)
```

3.171.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

$$\int (d\sin(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p \,dx$$

$$=\frac{\text{AppellF1}\left(\frac{1+m}{2},\frac{1}{2},-p,\frac{3+m}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}(d\sin(e+fx))^m\left(a+b\sin^2(e+fx\right.}{f(1+m)}$$

input
```
Integrate[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(AppellF1[(1 + m)/2, 1/2, -p, (3 + m)/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]
^2)/a)]*Sqrt[Cos[e + f*x]^2]*(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p*T
an[e + f*x])/(f*(1 + m)*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

3.171.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3668, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sin(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sin(e+fx))^m \left(a+b\sin(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{3668}$$

$$-\frac{d\sin^2(e+fx)^{\frac{1-m}{2}}(d\sin(e+fx))^{m-1}\int \left(1-\cos^2(e+fx)\right)^{\frac{m-1}{2}}\left(-b\cos^2(e+fx)+a+b\right)^p d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$-\frac{d\sin^2(e+fx)^{\frac{1-m}{2}}(d\sin(e+fx))^{m-1}\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\int \left(1-\cos^2(e+fx)\right)^{\frac{m-1}{2}}\left(1\right.}{f}$$

$$\Big\downarrow \text{333}$$

$$-\frac{d\cos(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(d\sin(e+fx))^{m-1}\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},\frac{1-}{}\right.}{f}$$

input
```
Int[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
-((d*AppellF1[1/2, (1 - m)/2, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/
(a + b)]*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*(d*Sin[e + f*x])^(-1 +
m)*(Sin[e + f*x]^2)^((1 - m)/2))/(f*(1 - (b*Cos[e + f*x]^2)/(a + b))^p))
```

3.171.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

3.171.4 Maple [F]

$$\int (d\sin (fx+e))^m \left(a+b\left(\sin^2 (fx+e)\right)\right)^p dx$$

input
```
int((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x)
```

3.171.5 Fricas [F]

$$\int (d\sin(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p\,dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p (d\sin\left(fx+e\right))^m\,dx$$

input
```
integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*(d*sin(f*x + e))^m, x)
```

3.171.6 Sympy [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p\,dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))**m*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.171.7 Maxima [F]

$$\int (d\sin(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p\,dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p (d\sin\left(fx+e\right))^m\,dx$$

input
```
integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

3.171.8 Giac [F]

$$\int (d\sin(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p (d\sin\left(fx+e\right))^m \, dx$$

input
```
integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p \, dx = \int (d\,\sin\left(e+f\,x\right))^m \left(b\sin\left(e+f\,x\right)^2+a\right)^p dx$$

input
```
int((d*sin(e + f*x))^m*(a + b*sin(e + f*x)^2)^p,x)
```

output
```
int((d*sin(e + f*x))^m*(a + b*sin(e + f*x)^2)^p, x)
```

3.172 $\int \sin^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.172.1 Optimal result

Integrand size = 23, antiderivative size = 220

$$\int \sin^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$
$$=\frac{(3a-2b(2+p))\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)}$$
$$-\frac{\left(3a^2-4ab(1+p)+4b^2(2+3p+p^2)\right)\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\text{Hyperg}}{b^2f(3+2p)(5+2p)}$$
$$-\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}\sin^2(e+fx)}{bf(5+2p)}$$

output
```
(3*a-2*b*(2+p))*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(p+1)/b^2/f/(4*p^2+16*p+15
)-(3*a^2-4*a*b*(p+1)+4*b^2*(p^2+3*p+2))*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^p*
hypergeom([1/2, -p],[3/2],b*cos(f*x+e)^2/(a+b))/b^2/f/(4*p^2+16*p+15)/((1-
b*cos(f*x+e)^2/(a+b))^p)-cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(p+1)*sin(f*x+e)^
2/b/f/(5+2*p)
```

3.172.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.58 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.45

$$\int \sin^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$
$$=\frac{\text{AppellF1}\left(3,\frac{1}{2},-p,4,\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\sin^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{a+b\sin}{}\right.}{6f}$$

input
```
Integrate[Sin[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(AppellF1[3, 1/2, -p, 4, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[Cos
[e + f*x]^2]*Sin[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(6*f*((
a + b*Sin[e + f*x]^2)/a)^p)
```

3.172.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3665, 318, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sin(e+fx)^5\left(a+b\sin(e+fx)^2\right)^p\,dx$$
$$\Big\downarrow \text{ 3665}$$
$$-\frac{\int \left(1-\cos^2(e+fx)\right)^2\left(-b\cos^2(e+fx)+a+b\right)^p\,d\cos(e+fx)}{f}$$
$$\Big\downarrow \text{ 318}$$
$$-\frac{\frac{\cos(e+fx)\left(1-\cos^2(e+fx)\right)\left(a-b\cos^2(e+fx)+b\right)^{p+1}}{b(2p+5)}-\frac{\int \left(-b\cos^2(e+fx)+a+b\right)^p\left(-\left((3a-2b(p+2))\cos^2(e+fx)\right)+a-2b(p+2)\right)d\cos(e+fx)}{b(2p+5)}}{f}$$

$$\Big\downarrow \; 299$$

$$-\frac{\frac{\cos(e+fx)\left(1-\cos^2(e+fx)\right)\left(a-b\cos^2(e+fx)+b\right)^{p+1}}{b(2p+5)}-\frac{\frac{(3a-2b(p+2))\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{p+1}}{b(2p+3)}-\frac{\left(3a^2-4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\int\left(-b\cos\cdots\right.}{b(2p+3)}}{b(2p+5)}}{f}$$

$$\Big\downarrow \; 238$$

$$-\frac{\frac{\cos(e+fx)\left(1-\cos^2(e+fx)\right)\left(a-b\cos^2(e+fx)+b\right)^{p+1}}{b(2p+5)}-\frac{\frac{(3a-2b(p+2))\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{p+1}}{b(2p+3)}-\frac{\left(3a^2-4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\left(a-b\cos\cdots\right.}{\cdots}}{b(2p+5)}}{f}$$

$$\Big\downarrow \; 237$$

$$-\frac{\frac{\cos(e+fx)\left(1-\cos^2(e+fx)\right)\left(a-b\cos^2(e+fx)+b\right)^{p+1}}{b(2p+5)}-\frac{\frac{(3a-2b(p+2))\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{p+1}}{b(2p+3)}-\frac{\left(3a^2-4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cos(e+f\cdots}{\cdots}}{b(2p+\cdots}}{f}$$

input
```
Int[Sin[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
-(((Cos[e + f*x]*(1 - Cos[e + f*x]^2)*(a + b - b*Cos[e + f*x]^2)^(1 + p))/
(b*(5 + 2*p)) - (((3*a - 2*b*(2 + p))*Cos[e + f*x]*(a + b - b*Cos[e + f*x]
^2)^(1 + p))/(b*(3 + 2*p)) - ((3*a^2 - 4*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^
2))*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3
/2, (b*Cos[e + f*x]^2)/(a + b)])/(b*(3 + 2*p)*(1 - (b*Cos[e + f*x]^2)/(a +
 b))^p))/(b*(5 + 2*p)))/f)
```

3.172.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.172.4 Maple [F]

$$\int \left(\sin^5\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p dx$$

input
```
int(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

3.172.5 Fricas [F]

$$\int \sin^5(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^5 \, dx$$

input
```
integrate(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(-b*cos(f*x + e)^2 + a +
b)^p*sin(f*x + e), x)
```

3.172.6 Sympy [F(-1)]

Timed out.

$$\int \sin^5(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.172.7 Maxima [F]

$$\int \sin^5(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^5 \, dx$$

input
```
integrate(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

3.172.8 Giac [F]

$$\int \sin^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^5\,dx$$

input
```
integrate(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int \sin^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \sin\left(e+f\,x\right)^5\,\left(b\sin\left(e+f\,x\right)^2+a\right)^p\,dx$$

input
```
int(sin(e + f*x)^5*(a + b*sin(e + f*x)^2)^p,x)
```

output
```
int(sin(e + f*x)^5*(a + b*sin(e + f*x)^2)^p, x)
```

3.173 $\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.173.1 Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=-\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{bf(3+2p)}+\frac{(a-2b(1+p))\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2}\right.}{bf(3+2p)}$$

output
```
-cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(p+1)/b/f/(3+2*p)+(a-2*b*(p+1))*cos(f*x+e
)*(a+b-b*cos(f*x+e)^2)^p*hypergeom([1/2, -p],[3/2],b*cos(f*x+e)^2/(a+b))/b
/f/(3+2*p)/((1-b*cos(f*x+e)^2/(a+b))^p)
```

3.173.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.41 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$
$$=\frac{\text{AppellF1}\left(2,\frac{1}{2},-p,3,\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{a+b\sin\right.}{4f}$$

input
```
Integrate[Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(AppellF1[2, 1/2, -p, 3, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[Cos
[e + f*x]^2]*Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(4*f*((
a + b*Sin[e + f*x]^2)/a)^p)
```

3.173.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3665, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^3\left(a+b\sin(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int\left(1-\cos^2(e+fx)\right)\left(-b\cos^2(e+fx)+a+b\right)^p d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{299}$$

$$-\frac{\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{p+1}}{b(2p+3)}-\frac{(a-2b(p+1))\int\left(-b\cos^2(e+fx)+a+b\right)^p d\cos(e+fx)}{b(2p+3)}}{f}$$

$$\Big\downarrow \text{238}$$

$$-\frac{\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{p+1}}{b(2p+3)}-\frac{(a-2b(p+1))\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\int\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^p d\cos(e+fx)}{b(2p+3)}}{f}$$

$$\Big\downarrow \text{237}$$

$$-\frac{\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{p+1}}{b(2p+3)}-\frac{(a-2b(p+1))\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{b\cos^2}{\ldots}\right.}{b(2p+3)}}{f}$$

input
```
Int[Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
-(((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(1 + p))/(b*(3 + 2*p)) - ((a -
 2*b*(1 + p))*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[
1/2, -p, 3/2, (b*Cos[e + f*x]^2)/(a + b)])/(b*(3 + 2*p)*(1 - (b*Cos[e + f*
x]^2)/(a + b))^p))/f)
```

3.173.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.173. $\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.173.4 Maple [F]

$$\int \left(\sin^3(fx+e)\right) \left(a+b\left(\sin^2(fx+e)\right)\right)^p dx$$

input
```
int(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

3.173.5 Fricas [F]

$$\int \sin^3(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^3 \, dx$$

input
```
integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*(-b*cos(f*x + e)^2 + a + b)^p*sin(f*x + e),
 x)
```

3.173.6 Sympy [F(-1)]

Timed out.

$$\int \sin^3(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.173.7 Maxima [F]

$$\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^3\,dx$$

input
```
integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

3.173.8 Giac [F]

$$\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^3\,dx$$

input
```
integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

3.173.9 Mupad [F(-1)]

Timed out.

$$\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \sin\left(e+f\,x\right)^3\left(b\sin\left(e+f\,x\right)^2+a\right)^p\,dx$$

input
```
int(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^p,x)
```

output
```
int(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^p, x)
```

3.174 $\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.174.1 Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

output
```
-cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^p*hypergeom([1/2, -p],[3/2],b*cos(f*x+e)^
2/(a+b))/f/((1-b*cos(f*x+e)^2/(a+b))^p)
```

3.174.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

input
```
Integrate[Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
-((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/
2, (b*Cos[e + f*x]^2)/(a + b)])/(f*(1 - (b*Cos[e + f*x]^2)/(a + b))^p))
```

3.174.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3665, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(e+fx)\left(a+b\sin(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \; 3665$$

$$-\frac{\int \left(-b\cos^2(e+fx)+a+b\right)^p d\cos(e+fx)}{f}$$

$$\Big\downarrow \; 238$$

$$-\frac{\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\int \left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^p d\cos(e+fx)}{f}$$

$$\Big\downarrow \; 237$$

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
-((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/
2, (b*Cos[e + f*x]^2)/(a + b)])/(f*(1 - (b*Cos[e + f*x]^2)/(a + b))^p))
```

3.174.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.174.4 Maple [F]

$$\int \sin(fx+e)\left(a+b\left(\sin^2(fx+e)\right)\right)^p dx$$

input
```
int(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

3.174.5 Fricas [F]

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin(fx+e)^2+a\right)^p\sin(fx+e)\,dx$$

input
```
integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*sin(f*x + e), x)
```

3.174.6 Sympy [F(-1)]

Timed out.

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(sin(f*x+e)*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.174.7 Maxima [F]

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)\,dx$$

input
```
integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

3.174.8 Giac [F]

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)\,dx$$

input
```
integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \sin\left(e+f\,x\right)\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^p\,dx$$

input
```
int(sin(e + f*x)*(a + b*sin(e + f*x)^2)^p,x)
```

output
```
int(sin(e + f*x)*(a + b*sin(e + f*x)^2)^p, x)
```

3.175 $\int \csc(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx$

3.175.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \csc(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = -\frac{\operatorname{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}}{f}$$

output
```
-AppellF1(1/2,1,-p,3/2,cos(f*x+e)^2,b*cos(f*x+e)^2/(a+b))*cos(f*x+e)*(a+b-
b*cos(f*x+e)^2)^p/f/((1-b*cos(f*x+e)^2/(a+b))^p)
```

3.175.2 Mathematica [F]

$$\int \csc(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \csc(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx$$

input
```
Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^p, x]
```

3.175.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3665, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^p}{\sin(e+fx)}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{\left(-b\cos^2(e+fx)+a+b\right)^p}{1-\cos^2(e+fx)}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$-\frac{\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\int \frac{\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^p}{1-\cos^2(e+fx)}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{333}$$

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
-((AppellF1[1/2, 1, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(a + b)]*C
os[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p)/(f*(1 - (b*Cos[e + f*x]^2)/(a +
b))^p))
```

3.175.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.175.4 Maple [F]

$$\int \csc(fx+e)\left(a+b\left(\sin^2(fx+e)\right)\right)^p dx$$

input
```
int(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

3.175.5 Fricas [F]

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e), x)
```

3.175.6 Sympy [F]

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(a+b\sin^2\left(e+fx\right)\right)^p\csc\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*sin(e + f*x)**2)**p*csc(e + f*x), x)
```

3.175.7 Maxima [F]

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

3.175.8 Giac [F]

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\sin\left(e+f\,x\right)}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^p/sin(e + f*x),x)
```

output
```
int((a + b*sin(e + f*x)^2)^p/sin(e + f*x), x)
```

3.176 $\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.176.1 Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{AppellF1}\left(\frac{1}{2},2,-p,\frac{3}{2},\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

output
```
-AppellF1(1/2,2,-p,3/2,cos(f*x+e)^2,b*cos(f*x+e)^2/(a+b))*cos(f*x+e)*(a+b-
b*cos(f*x+e)^2)^p/f/((1-b*cos(f*x+e)^2/(a+b))^p)
```

3.176.2 Mathematica [F]

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

input
```
Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p, x]
```

3.176.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3665, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^p}{\sin(e+fx)^3}dx$$

$$\Big\downarrow \; 3665$$

$$-\frac{\int \frac{\left(-b\cos^2(e+fx)+a+b\right)^p}{\left(1-\cos^2(e+fx)\right)^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \; 334$$

$$-\frac{\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\int \frac{\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^p}{\left(1-\cos^2(e+fx)\right)^2}d\cos(e+fx)}{f}$$

$$\Big\downarrow \; 333$$

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},2,-p,\frac{3}{2},\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
-((AppellF1[1/2, 2, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(a + b)]*C
os[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p)/(f*(1 - (b*Cos[e + f*x]^2)/(a +
b))^p))
```

3.176.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.176.4 Maple [F]

$$\int \left(\csc^3(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)^p dx$$

input
```
int(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

3.176.5 Fricas [F]

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e)^3, x)
```

3.176.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(csc(f*x+e)**3*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.176.7 Maxima [F]

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

3.176.8 Giac [F]

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\sin\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^3,x)
```

output
```
int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^3, x)
```

3.177 $\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.177.1 Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\operatorname{AppellF1}\left(\frac{1}{2},3,-p,\frac{3}{2},\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

output
```
-AppellF1(1/2,3,-p,3/2,cos(f*x+e)^2,b*cos(f*x+e)^2/(a+b))*cos(f*x+e)*(a+b-
b*cos(f*x+e)^2)^p/f/((1-b*cos(f*x+e)^2/(a+b))^p)
```

3.177.2 Mathematica [F]

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

input
```
Integrate[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
Integrate[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p, x]
```

3.177.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3665, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^p}{\sin(e+fx)^5}dx$$

$$\Big\downarrow \text{3665}$$

$$-\frac{\int \frac{\left(-b\cos^2(e+fx)+a+b\right)^p}{\left(1-\cos^2(e+fx)\right)^3}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$-\frac{\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\int \frac{\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^p}{\left(1-\cos^2(e+fx)\right)^3}d\cos(e+fx)}{f}$$

$$\Big\downarrow \text{333}$$

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},3,-p,\frac{3}{2},\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
-((AppellF1[1/2, 3, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(a + b)]*C
os[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p)/(f*(1 - (b*Cos[e + f*x]^2)/(a +
b))^p))
```

3.177.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.177.4 Maple [F]

$$\int \left(\csc^5(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)^p dx$$

input
```
int(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

3.177.5 Fricas [F]

$$\int \csc^5(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^5 \, dx$$

input
```
integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e)^5, x)
```

3.177.6 Sympy [F(-1)]

Timed out.

$$\int \csc^5(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**5*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.177.7 Maxima [F]

$$\int \csc^5(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^5 \, dx$$

input
```
integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^5, x)
```

3.177.8 Giac [F]

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^5\,dx$$

input
```
integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^5, x)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\sin\left(e+f\,x\right)^5}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^5,x)
```

output
```
int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^5, x)
```

3.178 $\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.178.1 Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\text{AppellF1}\left(\frac{5}{2},\frac{1}{2},-p,\frac{7}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\right.}{5f}$$

output
```
1/5*AppellF1(5/2,1/2,-p,7/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*sin(f*x+e)^4*(
a+b*sin(f*x+e)^2)^p*(cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f/((1+b*sin(f*x+e)^2/a
)^p)
```

3.178.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\text{AppellF1}\left(\frac{5}{2},\frac{1}{2},-p,\frac{7}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{a+bs}{}\right.}{5f}$$

input
```
Integrate[Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(AppellF1[5/2, 1/2, -p, 7/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt
[Cos[e + f*x]^2]*Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(5*
f*((a + b*Sin[e + f*x]^2)/a)^p)
```

3.178.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3667, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(e+fx)^4\left(a+b\sin(e+fx)^2\right)^p dx$$

$$\Big\downarrow \; 3667$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sin^4(e+fx)\left(b\sin^2(e+fx)+a\right)^p}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 395$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\int \frac{\sin^4(e+fx)\left(\frac{b\sin^2(e+fx)}{a}+1\right)^p}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 394$$

$$\frac{\sin^4(e+fx)\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{5}{2},\frac{1}{2},-p,\frac{7}{2},\sin^2(e+\right.}{5f}$$

input
```
Int[Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(AppellF1[5/2, 1/2, -p, 7/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt
[Cos[e + f*x]^2]*Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(5*
f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

3.178.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.178.4 Maple [F]

$$\int \left(\sin^{4}\left(fx+e\right)\right) \left(a+b\left(\sin^{2}\left(fx+e\right)\right)\right)^{p} dx$$

input
```
int(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

3.178.5 Fricas [F]

$$\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^4\,dx$$

input
```
integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(-b*cos(f*x + e)^2 + a +
b)^p, x)
```

3.178.6 Sympy [F(-1)]

Timed out.

$$\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(sin(f*x+e)**4*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.178.7 Maxima [F]

$$\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^4\,dx$$

input
```
integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

3.178.8 Giac [F]

$$\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^4\,dx$$

input
```
integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \sin\left(e+f\,x\right)^4\left(b\sin\left(e+f\,x\right)^2+a\right)^p\,dx$$

input
```
int(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^p,x)
```

output
```
int(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^p, x)
```

3.179 $\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.179.1 Optimal result

Integrand size = 23, antiderivative size = 99

$$\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\text{AppellF1}\left(\frac{3}{2},2+p,-p,\frac{5}{2},-\tan^2(e+fx),-\frac{(a+b)\tan^2(e+fx)}{a}\right)\sec^2(e+fx)^p\left(a+b\sin^2(e+fx)\right)^p\tan^3(e+\ldots}{3f}$$

output
```
1/3*AppellF1(3/2,2+p,-p,5/2,-tan(f*x+e)^2,-(a+b)*tan(f*x+e)^2/a)*(sec(f*x+
e)^2)^p*(a+b*sin(f*x+e)^2)^p*tan(f*x+e)^3/f/((1+(a+b)*tan(f*x+e)^2/a)^p)
```

3.179.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03

$$\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\text{AppellF1}\left(\frac{3}{2},\frac{1}{2},-p,\frac{5}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{a+b\sin\ldots}{}\right.}{3f}$$

input
```
Integrate[Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(AppellF1[3/2, 1/2, -p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt
[Cos[e + f*x]^2]*Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(3*
f*((a + b*Sin[e + f*x]^2)/a)^p)
```

3.179.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3653, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^2\left(a+b\sin(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \text{3653}$$

$$\frac{\sec^2(e+fx)^p\left(a+b\sin^2(e+fx)\right)^p\left((a+b)\tan^2(e+fx)+a\right)^{-p}\int \tan^2(e+fx)\left(\tan^2(e+fx)+1\right)^{-p-2}\left((a+b)\right.}{f}$$

$$\Big\downarrow \text{395}$$

$$\frac{\sec^2(e+fx)^p\left(a+b\sin^2(e+fx)\right)^p\left(\frac{(a+b)\tan^2(e+fx)}{a}+1\right)^{-p}\int \tan^2(e+fx)\left(\tan^2(e+fx)+1\right)^{-p-2}\left(\frac{(a+b)\tan^2(e+}{a}\right.}{f}$$

$$\Big\downarrow \text{394}$$

$$\frac{\tan^3(e+fx)\sec^2(e+fx)^p\left(a+b\sin^2(e+fx)\right)^p\left(\frac{(a+b)\tan^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},p+2,-p,\frac{5}{2},-\tan^2(e+f\right.}{3f}$$

input
```
Int[Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(AppellF1[3/2, 2 + p, -p, 5/2, -Tan[e + f*x]^2, -(((a + b)*Tan[e + f*x]^2)
/a)]*(Sec[e + f*x]^2)^p*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x]^3)/(3*f*(1 +
 ((a + b)*Tan[e + f*x]^2)/a)^p)
```

3.179.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3653
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff*(a + b*Sin[e + f*x]^2)^p*((Sec[e + f*x]^2)^p/(f*(a + (a + b)*Tan[e +
 f*x]^2)^p))   Subst[Int[(a + (a + b)*ff^2*x^2)^p*((A + (A + B)*ff^2*x^2)/(
1 + ff^2*x^2)^(p + 2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
A, B}, x] &&  !IntegerQ[p]
```

3.179.4 Maple [F]

$$\int \left(\sin^2 (fx+e)\right) \left(a+b\left(\sin^2 (fx+e)\right)\right)^p dx$$

input
```
int(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

3.179.5 Fricas [F]

$$\int \sin^2(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^2 \, dx$$

input
```
integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*(-b*cos(f*x + e)^2 + a + b)^p, x)
```

3.179.6 Sympy [F(-1)]

Timed out.

$$\int \sin^2(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**2*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.179.7 Maxima [F]

$$\int \sin^2(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^2 \, dx$$

input
```
integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

3.179.8 Giac [F]

$$\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \sin\left(e+f\,x\right)^2\left(b\sin\left(e+f\,x\right)^2+a\right)^p\,dx$$

input
```
int(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^p,x)
```

output
```
int(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^p, x)
```

3.180 $\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.180.1 Optimal result

Integrand size = 23, antiderivative size = 97

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx =$$
$$-\frac{\text{AppellF1}\left(-\frac{1}{2},\frac{1}{2},-p,\frac{1}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\csc(e+fx)\sec(e+fx)\left(a+b\sin^2(\right.}{f}$$

output
```
-AppellF1(-1/2,1/2,-p,1/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*csc(f*x+e)*sec(f
*x+e)*(a+b*sin(f*x+e)^2)^p*(cos(f*x+e)^2)^(1/2)/f/((1+b*sin(f*x+e)^2/a)^p)
```

3.180.2 Mathematica [F]

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

input
```
Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p, x]
```

3.180.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3667, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^p}{\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3667}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\csc^2(e+fx)\left(b\sin^2(e+fx)+a\right)^p}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{395}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\int \frac{\csc^2(e+fx)\left(\frac{b\sin^2(e+fx)}{a}+1\right)^p}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{394}$$

$$-\frac{\sqrt{\cos^2(e+fx)}\csc(e+fx)\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},\frac{1}{2},-p,\frac{1}{2},\sin^2(e$$

input
```
Int[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
-((AppellF1[-1/2, 1/2, -p, 1/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*S
qrt[Cos[e + f*x]^2]*Csc[e + f*x]*Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(f
*(1 + (b*Sin[e + f*x]^2)/a)^p))
```

3.180.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.180.4 Maple [F]

$$\int \left(\csc^2 (fx+e)\right) \left(a+b\left(\sin^2 (fx+e)\right)\right)^p dx$$

input
```
int(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

3.180.5 Fricas [F]

$$\int \csc^2(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^2 \, dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e)^2, x)
```

3.180.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**2*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.180.7 Maxima [F]

$$\int \csc^2(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^2 \, dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

3.180.8 Giac [F]

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\sin\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^2,x)
```

output
```
int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^2, x)
```

3.181 $\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.181.1 Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx =$$
$$-\frac{\operatorname{AppellF1}\left(-\frac{3}{2},\frac{1}{2},-p,-\frac{1}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\csc^3(e+fx)\sec(e+fx)\left(a+b\sin\right.}{3f}$$

output
```
-1/3*AppellF1(-3/2,1/2,-p,-1/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*csc(f*x+e)^
3*sec(f*x+e)*(a+b*sin(f*x+e)^2)^p*(cos(f*x+e)^2)^(1/2)/f/((1+b*sin(f*x+e)^
2/a)^p)
```

3.181.2 Mathematica [F]

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

input
```
Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p, x]
```

3.181.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3667, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^p}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \ 3667$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\csc^4(e+fx)\left(b\sin^2(e+fx)+a\right)^p}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \ 395$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\int \frac{\csc^4(e+fx)\left(\frac{b\sin^2(e+fx)}{a}+1\right)^p}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \ 394$$

$$-\frac{\sqrt{\cos^2(e+fx)}\csc^3(e+fx)\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(-\frac{3}{2},\frac{1}{2},-p,-\frac{1}{2},\sin\right.}{3f}$$

input
```
Int[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
-1/3*(AppellF1[-3/2, 1/2, -p, -1/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a
)]*Sqrt[Cos[e + f*x]^2]*Csc[e + f*x]^3*Sec[e + f*x]*(a + b*Sin[e + f*x]^2)
^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

3.181.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3667
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/Sqrt[1 - ff^2*x^2]), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.181.4 Maple [F]

$$\int \left(\csc^{4}\left(f x+e\right)\right) \left(a+b \left(\sin^{2}\left(f x+e\right)\right)\right)^{p}d x$$

input
```
int(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

3.181.5 Fricas [F]

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e)^4, x)
```

3.181.6 Sympy [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(csc(f*x+e)**4*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.181.7 Maxima [F]

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

3.181.8 Giac [F]

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\sin\left(e+f\,x\right)^4}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^4,x)
```

output
```
int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^4, x)
```

3.182 $\int \frac{\sin^7(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.182.1 Optimal result

Integrand size = 23, antiderivative size = 335

$$\int \frac{\sin^7(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{3x}{8b} + \frac{2(-1)^{2/3}a^{5/3}\arctan\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^{7/3}d} - \frac{2a^{5/3}\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^{7/3}d} + \frac{2\sqrt[3]{-1}a^{5/3}\arctan\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}\,b^{7/3}d} + \frac{a\cos(c+dx)}{b^2d} - \frac{3\cos(c+dx)\sin(c+dx)}{8bd} - \frac{\cos(c+dx)\sin^3(c+dx)}{4bd}$$

output
```
3/8*x/b+a*cos(d*x+c)/b^2/d-3/8*cos(d*x+c)*sin(d*x+c)/b/d-1/4*cos(d*x+c)*si
n(d*x+c)^3/b/d-2/3*a^(5/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^
(2/3)-b^(2/3))^(1/2))/b^(7/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*(-1)^(1/3)*a^(
5/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^
(1/3)*b^(2/3))^(1/2))/b^(7/3)/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)+2/3*(-1
)^(2/3)*a^(5/3)*arctan(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a^
(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/b^(7/3)/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1
/2)
```

3.182.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.56 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.65

$$\int \frac{\sin^7(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{96a\cos(c+dx) - 32a^2\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&, \frac{-2\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\log}{}\right.}{}$$

input
```
Integrate[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^3),x]
```

output
```
(96*a*Cos[c + d*x] - 32*a^2*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*
I)*b*#1^4 + I*b*#1^6 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
#1)]*#1^2 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*
b*#1^2 + b*#1^4) & ] + 3*b*(12*(c + d*x) - 8*Sin[2*(c + d*x)] + Sin[4*(c +
 d*x)]))/(96*b^2*d)
```

3.182.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^7}{a+b\sin(c+dx)^3}dx$$

$$\Big\downarrow \text{3699}$$

$$\int \left(\frac{a^2\sin(c+dx)}{b^2\left(a+b\sin^3(c+dx)\right)}-\frac{a\sin(c+dx)}{b^2}+\frac{\sin^4(c+dx)}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^{5/3}\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{7/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{-1}a^{5/3}\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3b^{7/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}+\frac{2(-1)^{2/3}a^{5/3}\arctan\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{7/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{a\cos(c+dx)}{b^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4bd}-\frac{3\sin(c+dx)\cos(c+dx)}{8bd}+\frac{3x}{8b}$$

input
```
Int[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^3),x]
```

output
```
(3*x)/(8*b) - (2*a^(5/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[
a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(7/3)*d) + (2*(-1)^(1/3)
*a^(5/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/
3) + (-1)^(1/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^(7/3)*d
) + (2*(-1)^(2/3)*a^(5/3)*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)
*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) -
 (-1)^(2/3)*b^(2/3)]*b^(7/3)*d) + (a*Cos[c + d*x])/(b^2*d) - (3*Cos[c + d*
x]*Sin[c + d*x])/(8*b*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b*d)
```

3.182.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.182.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.92 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.66

method	result
derivativedivides	$-\frac{4\left(\frac{-\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{16}-\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{11\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{16}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{2}+\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{16}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}\cdots\right.}{b^2}\cdots$
default	$-\frac{4\left(\frac{-\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{16}-\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{11\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{16}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{2}+\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{16}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}\cdots\right.}{b^2}\cdots$
risch	$\frac{3x}{8b}+\frac{a\,\mathrm{e}^{i(dx+c)}}{2d\,b^2}+\frac{a\,\mathrm{e}^{-i(dx+c)}}{2b^2d}+\frac{i\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^2b^{14}d^6-729b^{16}d^6\right)\_Z^6-3981312a^4b^{10}d^4\_Z^4-4398046511104a^{10}\cdots\right.}\cdots\right.}{\cdots}$

input
```
int(sin(d*x+c)^7/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-4/b^2*((-3/16*tan(1/2*d*x+1/2*c)^7*b-1/2*a*tan(1/2*d*x+1/2*c)^6-11/1
6*tan(1/2*d*x+1/2*c)^5*b-3/2*tan(1/2*d*x+1/2*c)^4*a+11/16*tan(1/2*d*x+1/2*
c)^3*b-3/2*tan(1/2*d*x+1/2*c)^2*a+3/16*b*tan(1/2*d*x+1/2*c)-1/2*a)/(1+tan(
1/2*d*x+1/2*c)^2)^4-3/16*b*arctan(tan(1/2*d*x+1/2*c)))+2/3*a^2/b^2*sum((_R
^3+_R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf
(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

3.182.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.43 (sec) , antiderivative size = 21338, normalized size of antiderivative = 63.70

$$\int \frac{\sin^7(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.182.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**7/(a+b*sin(d*x+c)**3),x)
```

output
```
Timed out
```

3.182.7 Maxima [F]

$$\int \frac{\sin^7(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sin{(dx+c)}^7}{b\sin{(dx+c)}^3+a}\,dx$$

input
```
integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
-1/32*(32*b^2*d*integrate(-4*(3*a^2*b*cos(4*d*x + 4*c)^2 + 3*a^2*b*cos(2*d
*x + 2*c)^2 + 3*a^2*b*sin(4*d*x + 4*c)^2 + 8*a^3*cos(2*d*x + 2*c)*sin(3*d*
x + 3*c) - 8*a^3*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 3*a^2*b*sin(2*d*x + 2
*c)^2 - a^2*b*cos(2*d*x + 2*c) - (a^2*b*cos(4*d*x + 4*c) - a^2*b*cos(2*d*x
 + 2*c))*cos(6*d*x + 6*c) - (6*a^2*b*cos(2*d*x + 2*c) + 8*a^3*sin(3*d*x +
3*c) - a^2*b)*cos(4*d*x + 4*c) - (a^2*b*sin(4*d*x + 4*c) - a^2*b*sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c) + 2*(4*a^3*cos(3*d*x + 3*c) - 3*a^2*b*sin(2*d*x
+ 2*c))*sin(4*d*x + 4*c))/(b^4*cos(6*d*x + 6*c)^2 + 9*b^4*cos(4*d*x + 4*c)
^2 + 64*a^2*b^2*cos(3*d*x + 3*c)^2 + 9*b^4*cos(2*d*x + 2*c)^2 + b^4*sin(6*
d*x + 6*c)^2 + 9*b^4*sin(4*d*x + 4*c)^2 + 64*a^2*b^2*sin(3*d*x + 3*c)^2 -
48*a*b^3*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 9*b^4*sin(2*d*x + 2*c)^2 - 6*
b^4*cos(2*d*x + 2*c) + b^4 - 2*(3*b^4*cos(4*d*x + 4*c) - 3*b^4*cos(2*d*x +
 2*c) - 8*a*b^3*sin(3*d*x + 3*c) + b^4)*cos(6*d*x + 6*c) - 6*(3*b^4*cos(2*
d*x + 2*c) + 8*a*b^3*sin(3*d*x + 3*c) - b^4)*cos(4*d*x + 4*c) - 2*(8*a*b^3
*cos(3*d*x + 3*c) + 3*b^4*sin(4*d*x + 4*c) - 3*b^4*sin(2*d*x + 2*c))*sin(6
*d*x + 6*c) + 6*(8*a*b^3*cos(3*d*x + 3*c) - 3*b^4*sin(2*d*x + 2*c))*sin(4*
d*x + 4*c) + 16*(3*a*b^3*cos(2*d*x + 2*c) - a*b^3)*sin(3*d*x + 3*c)), x) -
 12*b*d*x - 32*a*cos(d*x + c) - b*sin(4*d*x + 4*c) + 8*b*sin(2*d*x + 2*c))
/(b^2*d)
```

3.182.8 Giac [F]

$$\int \frac{\sin^7(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sin(dx+c)^7}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)^7/(b*sin(d*x + c)^3 + a), x)
```

3.182.9 Mupad [B] (verification not implemented)

Time = 15.45 (sec) , antiderivative size = 1978, normalized size of antiderivative = 5.90

$$\int \frac{\sin^7(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^7/(a + b*sin(c + d*x)^3),x)
```

output
```
symsum(log((150994944*a^12*b^3*sin(c/2 + (d*x)/2) - 56623104*a^13*b^2*cos(
c/2 + (d*x)/2) - 12582912*a^15*cos(c/2 + (d*x)/2) + 679477248*root(729*a^2
*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)*a^11*b^5*sin(c/2
 + (d*x)/2) + 679477248*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^1
0*z^4 + a^10, z, k)^2*a^9*b^8*cos(c/2 + (d*x)/2) - 42467328*root(729*a^2*b
^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^2*a^11*b^6*cos(c/2
 + (d*x)/2) - 402653184*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^1
0*z^4 + a^10, z, k)^2*a^13*b^4*cos(c/2 + (d*x)/2) + 4586471424*root(729*a^
2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^3*a^8*b^10*cos(
c/2 + (d*x)/2) - 503316480*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*
b^10*z^4 + a^10, z, k)^3*a^12*b^6*cos(c/2 + (d*x)/2) + 1911029760*root(729
*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^4*a^7*b^12*c
os(c/2 + (d*x)/2) + 1774190592*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*
a^4*b^10*z^4 + a^10, z, k)^4*a^9*b^10*cos(c/2 + (d*x)/2) - 301989888*root(
729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^4*a^11*b^
8*cos(c/2 + (d*x)/2) - 18345885696*root(729*a^2*b^14*z^6 - 729*b^16*z^6 +
243*a^4*b^10*z^4 + a^10, z, k)^5*a^4*b^16*cos(c/2 + (d*x)/2) + 17199267840
*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^5*a
^6*b^14*cos(c/2 + (d*x)/2) + 32614907904*root(729*a^2*b^14*z^6 - 729*b^16*
z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^5*a^8*b^12*cos(c/2 + (d*x)/2) + 91...
```

3.183 $\int \frac{\sin^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.183.1 Optimal result

Integrand size = 23, antiderivative size = 273

$$\int \frac{\sin^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{x}{2b} - \frac{2a\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^{5/3}d} + \frac{2a\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}\,b^{5/3}d} + \frac{2a\text{arctanh}\left(\frac{\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}\,b^{5/3}d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd}$$

output
```
1/2*x/b-1/2*cos(d*x+c)*sin(d*x+c)/b/d-2/3*a*arctan((b^(1/3)+a^(1/3)*tan(1/
2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/b^(5/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2
/3*a*arctanh((b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/((-1)^(1/3)*a
^(2/3)+b^(2/3))^(1/2))/b^(5/3)/d/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2)+2/3*a*
arctanh((b^(1/3)-(-1)^(1/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/
3)+b^(2/3))^(1/2))/b^(5/3)/d/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2)
```

3.183.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.93

$$\int \frac{\sin^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$= \frac{6(c+dx) - 2ia\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&, \frac{2\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-i\log\left(1-2\cos(c-\right.}{\ldots}\right.}{\ldots}$$

input
```
Integrate[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

output
```
(6*(c + d*x) - (2*I)*a*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*
#1^4 + I*b*#1^6 & , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1
- 2*Cos[c + d*x]*#1 + #1^2] - 4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#
1^2 + (2*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]
/(Cos[c + d*x] - #1)]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*
#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ] - 3*Sin[2*(c + d*x)])/(12*b*d)
```

3.183.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin(c+dx)^5}{a+b\sin(c+dx)^3}dx$$
$$\Big\downarrow \; 3699$$
$$\int \left(\frac{\sin^2(c+dx)}{b}-\frac{a\sin^2(c+dx)}{b\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2a\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3b^{5/3}d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}+\frac{2a\text{arctanh}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3b^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}+\frac{x}{2b}$$

input
```
Int[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

output
```
x/(2*b) - (2*a*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) -
b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(5/3)*d) + (2*a*ArcTanh[(b^(1/3) -
 (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)
]])/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]*b^(5/3)*d) + (2*a*ArcTanh[(b^
(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-1)^(1/3)*a^(2/3) + b^(
2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]*b^(5/3)*d) - (Cos[c + d*x]*S
in[c + d*x])/(2*b*d)
```

3.183.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.183. $\int \frac{\sin^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.183.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.52

method	result
derivativedivides	$\frac{\frac{8\left(\frac{\left(\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b}-\frac{4a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2}{\_R^5a+2\cdots}\right.}{3b}}{d}$
default	$\frac{\frac{8\left(\frac{\left(\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b}-\frac{4a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2}{\_R^5a+2\cdots}\right.}{3b}}{d}$
risch	$\frac{x}{2b}-\frac{i\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^2b^{10}d^6-729b^{12}d^6\right)\_Z^6-248832a^2b^8d^4\_Z^4-28311552a^4b^4d^2\_Z^2-1073741824a^6\right)}\_R\ln\left(e^{i(dx\cdots}\right.\right.}{\cdots}$

input
```
int(sin(d*x+c)^5/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(8/b*((1/8*tan(1/2*d*x+1/2*c)^3-1/8*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x
+1/2*c)^2)^2+1/8*arctan(tan(1/2*d*x+1/2*c)))-4/3/b*a*sum(_R^2/(_R^5*a+2*_R
^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*
_Z^3*b+3*_Z^2*a+a))
```

3.183.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 29175, normalized size of antiderivative = 106.87

$$\int \frac{\sin^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.183.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**5/(a+b*sin(d*x+c)**3),x)
```

output
```
Timed out
```

3.183.7 Maxima [F]

$$\int \frac{\sin^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sin{(dx+c)}^5}{b\sin{(dx+c)}^3+a}\,dx$$

input
```
integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
-1/4*(4*b*d*integrate(2*(16*a^2*cos(3*d*x + 3*c)^2 + 16*a^2*sin(3*d*x + 3*
c)^2 + 3*a*b*cos(d*x + c)*sin(2*d*x + 2*c) - 3*a*b*cos(2*d*x + 2*c)*sin(d*
x + c) + a*b*sin(d*x + c) - (a*b*sin(5*d*x + 5*c) - 2*a*b*sin(3*d*x + 3*c)
 + a*b*sin(d*x + c))*cos(6*d*x + 6*c) - (8*a^2*cos(3*d*x + 3*c) + 3*a*b*si
n(4*d*x + 4*c) - 3*a*b*sin(2*d*x + 2*c))*cos(5*d*x + 5*c) - 3*(2*a*b*sin(3
*d*x + 3*c) - a*b*sin(d*x + c))*cos(4*d*x + 4*c) - 2*(4*a^2*cos(d*x + c) +
 3*a*b*sin(2*d*x + 2*c))*cos(3*d*x + 3*c) + (a*b*cos(5*d*x + 5*c) - 2*a*b*
cos(3*d*x + 3*c) + a*b*cos(d*x + c))*sin(6*d*x + 6*c) + (3*a*b*cos(4*d*x +
 4*c) - 3*a*b*cos(2*d*x + 2*c) - 8*a^2*sin(3*d*x + 3*c) + a*b)*sin(5*d*x +
 5*c) + 3*(2*a*b*cos(3*d*x + 3*c) - a*b*cos(d*x + c))*sin(4*d*x + 4*c) + 2
*(3*a*b*cos(2*d*x + 2*c) - 4*a^2*sin(d*x + c) - a*b)*sin(3*d*x + 3*c))/(b^
3*cos(6*d*x + 6*c)^2 + 9*b^3*cos(4*d*x + 4*c)^2 + 64*a^2*b*cos(3*d*x + 3*c
)^2 + 9*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(6*d*x + 6*c)^2 + 9*b^3*sin(4*d*x
+ 4*c)^2 + 64*a^2*b*sin(3*d*x + 3*c)^2 - 48*a*b^2*cos(3*d*x + 3*c)*sin(2*d
*x + 2*c) + 9*b^3*sin(2*d*x + 2*c)^2 - 6*b^3*cos(2*d*x + 2*c) + b^3 - 2*(3
*b^3*cos(4*d*x + 4*c) - 3*b^3*cos(2*d*x + 2*c) - 8*a*b^2*sin(3*d*x + 3*c)
+ b^3)*cos(6*d*x + 6*c) - 6*(3*b^3*cos(2*d*x + 2*c) + 8*a*b^2*sin(3*d*x +
3*c) - b^3)*cos(4*d*x + 4*c) - 2*(8*a*b^2*cos(3*d*x + 3*c) + 3*b^3*sin(4*d
*x + 4*c) - 3*b^3*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(8*a*b^2*cos(3*d*
x + 3*c) - 3*b^3*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) + 16*(3*a*b^2*cos(2...
```

3.183.8 Giac [F]

$$\int \frac{\sin^5(c+dx)}{a+b\sin^3(c+dx)}\,dx=\int \frac{\sin{(dx+c)}^5}{b\sin{(dx+c)}^3+a}\,dx$$

input
```
integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.183.9 Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 1962, normalized size of antiderivative = 7.19

$$\int \frac{\sin^5(c+dx)}{a+b\sin^3(c+dx)}\,dx=\text{Too large to display}$$

input
```
int(sin(c + d*x)^5/(a + b*sin(c + d*x)^3),x)
```

output
```
tan(c/2 + (d*x)/2)^3/(b*d + 2*b*d*tan(c/2 + (d*x)/2)^2 + b*d*tan(c/2 + (d*
x)/2)^4) - tan(c/2 + (d*x)/2)/(b*d + 2*b*d*tan(c/2 + (d*x)/2)^2 + b*d*tan(
c/2 + (d*x)/2)^4) + symsum(log((134217728*a^9*b^2 - 16777216*a^11 - 402653
184*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^
2 + a^6, z, k)*a^8*b^4 + 50331648*a^10*b*tan(c/2 + (d*x)/2) - 2415919104*r
oot(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a
^6, z, k)^2*a^7*b^6 + 914358272*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243
*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^2*a^9*b^4 + 7247757312*root(729
*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z,
k)^3*a^6*b^8 - 478150656*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^
8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^3*a^8*b^6 + 10871635968*root(729*a^2*b
^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^4*a
^5*b^10 - 21214789632*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z
^4 - 27*a^4*b^4*z^2 + a^6, z, k)^4*a^7*b^8 - 301989888*root(729*a^2*b^10*z
^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^4*a^9*b^
6 - 32614907904*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 2
7*a^4*b^4*z^2 + a^6, z, k)^5*a^4*b^12 + 59567505408*root(729*a^2*b^10*z^6
- 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^5*a^6*b^10
+ 4529848320*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a
^4*b^4*z^2 + a^6, z, k)^5*a^8*b^8 + 55717134336*root(729*a^2*b^10*z^6 -...
```

3.184 $\int \frac{\sin^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.184.1 Optimal result

Integrand size = 23, antiderivative size = 259

$$\int \frac{\sin^3(c+dx)}{a+b\sin^3(c+dx)}\,dx=\frac{x}{b}-\frac{2\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}bd}-\frac{2\sqrt[3]{a}\arctan\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}bd}+\frac{2\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}bd}$$

output
```
x/b-2/3*a^(1/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/
3))^(1/2))/b/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*a^(1/3)*arctan(((-1)^(2/3)*b^(1
/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/b/d/(a
^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)+2/3*a^(1/3)*arctan((-1)^(1/3)*(b^(1/3)+(-
1)^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/b
/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

3.184.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \$\text{Aborted}$$

input
```
Integrate[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

output
```
$Aborted
```

3.184.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{a+b\sin^3(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^3}{a+b\sin(c+dx)^3}dx$$

$$\Big\downarrow \text{3699}$$

$$\int \left(\frac{1}{b}-\frac{a}{b\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}+\frac{2\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{x}{b}$$

input
```
Int[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

output
```
x/b - (2*a^(1/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3)
- b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b*d) - (2*a^(1/3)*ArcTan[((-1)^(2/
3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]]
)/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b*d) + (2*a^(1/3)*ArcTan[((-1)^(1/
3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2
/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*b*d)
```

3.184.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.184.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.78 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.40

method	result
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3b}}{d}$
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}-\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3b}}{d}$
risch	$\frac{x}{b}+\frac{i\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^2b^6d^6-729b^8d^6\right)\_Z^6-15552a^2b^4d^4\_Z^4+110592a^2b^2d^2\_Z^2-262144a^2\right)}\_R\ln\left(\mathrm{e}^{i(dx+c)}+\left(\frac{243a}{1}\right.\right.\right.}{}$

3.184. $\int \frac{\sin^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

input
```
int(sin(d*x+c)^3/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b*arctan(tan(1/2*d*x+1/2*c))-1/3/b*a*sum((_R^4+2*_R^2+1)/(_R^5*a+2*
_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+
8*_Z^3*b+3*_Z^2*a+a)))
```

3.184.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.25 (sec) , antiderivative size = 29221, normalized size of antiderivative = 112.82

$$\int \frac{\sin^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.184.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)**3),x)
```

output
```
Timed out
```

3.184.7 Maxima [F]

$$\int \frac{\sin^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sin(dx+c)^3}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
(8*a*b*integrate(-(8*a*cos(3*d*x + 3*c)^2 - b*cos(3*d*x + 3*c)*sin(6*d*x +
 6*c) + 3*b*cos(3*d*x + 3*c)*sin(4*d*x + 4*c) + b*cos(6*d*x + 6*c)*sin(3*d
*x + 3*c) - 3*b*cos(4*d*x + 4*c)*sin(3*d*x + 3*c) + 8*a*sin(3*d*x + 3*c)^2
 - 3*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + (3*b*cos(2*d*x + 2*c) - b)*sin(
3*d*x + 3*c))/(b^3*cos(6*d*x + 6*c)^2 + 9*b^3*cos(4*d*x + 4*c)^2 + 64*a^2*
b*cos(3*d*x + 3*c)^2 + 9*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(6*d*x + 6*c)^2 +
 9*b^3*sin(4*d*x + 4*c)^2 + 64*a^2*b*sin(3*d*x + 3*c)^2 - 48*a*b^2*cos(3*d
*x + 3*c)*sin(2*d*x + 2*c) + 9*b^3*sin(2*d*x + 2*c)^2 - 6*b^3*cos(2*d*x +
2*c) + b^3 - 2*(3*b^3*cos(4*d*x + 4*c) - 3*b^3*cos(2*d*x + 2*c) - 8*a*b^2*
sin(3*d*x + 3*c) + b^3)*cos(6*d*x + 6*c) - 6*(3*b^3*cos(2*d*x + 2*c) + 8*a
*b^2*sin(3*d*x + 3*c) - b^3)*cos(4*d*x + 4*c) - 2*(8*a*b^2*cos(3*d*x + 3*c
) + 3*b^3*sin(4*d*x + 4*c) - 3*b^3*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*
(8*a*b^2*cos(3*d*x + 3*c) - 3*b^3*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) + 16*
(3*a*b^2*cos(2*d*x + 2*c) - a*b^2)*sin(3*d*x + 3*c)), x) + x)/b
```

3.184.8 Giac [F]

$$\int \frac{\sin^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sin(dx+c)^3}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.184.9 Mupad [B] (verification not implemented)

Time = 14.44 (sec) , antiderivative size = 1672, normalized size of antiderivative = 6.46

$$\int \frac{\sin^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input

```
int(sin(c + d*x)^3/(a + b*sin(c + d*x)^3),x)
```

output

```
symsum(log(134217728*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4
+ 27*a^2*b^2*z^2 + a^2, z, k)*a^7*tan(c/2 + (d*x)/2) - 268435456*root(729*
a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^
2*a^7*b - 1073741824*a^6*tan(c/2 + (d*x)/2) + 4831838208*root(729*a^2*b^6*
z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^2*a^5*b^
3 + 33722204160*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*
a^2*b^2*z^2 + a^2, z, k)^3*a^6*b^3 + 15703474176*root(729*a^2*b^6*z^6 - 72
9*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^4*a^5*b^5 - 4831
838208*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z
^2 + a^2, z, k)^4*a^7*b^3 - 130459631616*root(729*a^2*b^6*z^6 - 729*b^8*z^
6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^5*a^4*b^7 + 154014842880
*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a
^2, z, k)^5*a^6*b^5 + 35332816896*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243
*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^6*a^5*b^7 - 21743271936*root(72
9*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k
)^6*a^7*b^5 - 130459631616*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^
4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^7*a^4*b^9 + 122305904640*root(729*a^2*
b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^7*a^
6*b^7 + 2013265920*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 +
27*a^2*b^2*z^2 + a^2, z, k)*a^6*b - 3221225472*root(729*a^2*b^6*z^6 - 7...
```

3.185 $\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.185.1 Optimal result

Integrand size = 21, antiderivative size = 267

$$\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{2(-1)^{2/3}\arctan\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\sqrt[3]{b}d} - \frac{2\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{a^{2/3}-b^{2/3}}\sqrt[3]{b}d} + \frac{2\sqrt[3]{-1}\arctan\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}\sqrt[3]{b}d}$$

output
```
-2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/
a^(1/3)/b^(1/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*(-1)^(1/3)*arctan(((-1)^(2/3
)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/
a^(1/3)/b^(1/3)/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)+2/3*(-1)^(2/3)*arctan
(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/
3))^(1/2))/a^(1/3)/b^(1/3)/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

3.185.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 11.05 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.64

$$\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx = -\frac{\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&,\frac{-2\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)}{b-4ia\#1}\right.}{3d}$$

input
```
Integrate[Sin[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

output
```
-1/3*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 &
, (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*
#1 + #1^2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - I*Log[1 - 2
*Cos[c + d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ]/d
```

3.185.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)}{a+b\sin(c+dx)^3}dx$$

$$\Big\downarrow \text{3699}$$

$$\int \left(-\frac{1}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}-\frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)}+\frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\sin(c\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{-1}\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}+\frac{2(-1)^{2/3}\arctan\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

input
```
Int[Sin[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

output
```
(-2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/
(3*a^(1/3)*Sqrt[a^(2/3) - b^(2/3)]*b^(1/3)*d) + (2*(-1)^(1/3)*ArcTan[((-1)
^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/
3)]])/(3*a^(1/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^(1/3)*d) + (2*(-1)^(
2/3)*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/S
qrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(1/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b
^(2/3)]*b^(1/3)*d)
```

3.185.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.185.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.29

method	result
derivativedivides	$\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}+3a\_Z^{4}+8b\_Z^{3}+3a\_Z^{2}+a\right)}\frac{\left(\_R^{3}+\_R\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a+2\_R^{3}a+4\_R^{2}b+\_Ra}\right)}{3d}$
default	$\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}+3a\_Z^{4}+8b\_Z^{3}+3a\_Z^{2}+a\right)}\frac{\left(\_R^{3}+\_R\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a+2\_R^{3}a+4\_R^{2}b+\_Ra}\right)}{3d}$
risch	$-\frac{i\left(\sum_{\_R=\mathrm{RootOf}\left(-64+\left(729a^{4}b^{2}d^{6}-729a^{2}b^{4}d^{6}\right)\_Z^{6}-972a^{2}b^{2}d^{4}\_Z^{4}\right)}\_R\ln\left(\mathrm{e}^{i(dx+c)}+\left(-\frac{243id^{5}b^{2}a^{5}}{32a^{2}+32b^{2}}+\frac{243id^{5}b^{4}a^{3}}{32a^{2}+32b^{2}}\right)\_R\right.\right.}{}$

input
```
int(sin(d*x+c)/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*sum((_R^3+_R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-
_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

3.185.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 18879, normalized size of antiderivative = 70.71

$$\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.185.6 Sympy [F]

$$\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)**3),x)
```

output
```
Integral(sin(c + d*x)/(a + b*sin(c + d*x)**3), x)
```

3.185.7 Maxima [F]

$$\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sin(dx+c)}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/(b*sin(d*x + c)^3 + a), x)
```

3.185.8 Giac [F]

$$\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sin(dx+c)}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)/(b*sin(d*x + c)^3 + a), x)
```

3.185.9 Mupad [B] (verification not implemented)

Time = 15.87 (sec) , antiderivative size = 652, normalized size of antiderivative = 2.44

$$\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$= \frac{\sum_{k=1}^{6} \ln\left(-8192\,a^3\,b + \mathrm{root}(729\,a^4\,b^2\,d^6 - 729\,a^2\,b^4\,d^6 + 243\,a^2\,b^2\,d^4 + 1, d, k)^2\,a^3\,b^3\,294912 + \mathrm{root}(729\,a^4\right.}{}$$

input
```
int(sin(c + d*x)/(a + b*sin(c + d*x)^3),x)
```

output
```
symsum(log(294912*root(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4
 + 1, d, k)^2*a^3*b^3 - 8192*a^3*b + 1548288*root(729*a^4*b^2*d^6 - 729*a^
2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)^3*a^4*b^3 + 1990656*root(729*a^4*b^
2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)^4*a^5*b^3 - 7962624*r
oot(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)^5*a^4*b
^5 + 5971968*root(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1,
 d, k)^5*a^6*b^3 + 65536*a^2*b^2*tan(c/2 + (d*x)/2) + 196608*root(729*a^4*
b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)*a^3*b^2*tan(c/2 + (
d*x)/2) + 294912*root(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4
+ 1, d, k)^2*a^4*b^2*tan(c/2 + (d*x)/2) - 1769472*root(729*a^4*b^2*d^6 - 7
29*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)^3*a^3*b^4*tan(c/2 + (d*x)/2) +
 221184*root(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k
)^3*a^5*b^2*tan(c/2 + (d*x)/2) + 2654208*root(729*a^4*b^2*d^6 - 729*a^2*b^
4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)^4*a^4*b^4*tan(c/2 + (d*x)/2) - 1990656*
root(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)^5*a^5*
b^4*tan(c/2 + (d*x)/2))*root(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b
^2*d^4 + 1, d, k), k, 1, 6)/d
```

3.186 $\int \frac{\csc(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.186.1 Optimal result

Integrand size = 21, antiderivative size = 264

$$\int \frac{\csc(c+dx)}{a+b\sin^3(c+dx)}\,dx = -\frac{2\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a\sqrt{a^{2/3}-b^{2/3}}\,d} - \frac{\operatorname{arctanh}(\cos(c+dx))}{ad} + \frac{2\sqrt[3]{b}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right)}{3a\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}\,d} + \frac{2\sqrt[3]{b}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3a\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}\,d}$$

output
```
-arctanh(cos(d*x+c))/a/d-2/3*b^(1/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1
/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*b^(1/3)*ar
ctanh((b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)+
b^(2/3))^(1/2))/a/d/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2)+2/3*b^(1/3)*arctanh
((b^(1/3)-(-1)^(1/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)+b^(2
/3))^(1/2))/a/d/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2)
```

3.186.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00

$$\int \frac{\csc(c+dx)}{a+b\sin^3(c+dx)}\,dx =$$

$$-\frac{6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-6\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+ib\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\right.}{}$$

input
```
Integrate[Csc[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

output
```
-1/6*(6*Log[Cos[(c + d*x)/2]] - 6*Log[Sin[(c + d*x)/2]] + I*b*RootSum[(-I)
*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & , (2*ArcTan[Sin[c
 + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 4*Arc
Tan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (2*I)*Log[1 - 2*Cos[c + d*x]*
#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*Log[
1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1
^5) & ])/(a*d)
```

3.186.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)\left(a+b\sin(c+dx)^3\right)}dx$$

$$\Big\downarrow \text{3699}$$

$$\int \left(\frac{\csc(c+dx)}{a}-\frac{b\sin^2(c+dx)}{a\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3ad\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{b}\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3ad\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}+\frac{2\sqrt[3]{b}\text{arctanh}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3ad\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}-\frac{\text{arctanh}(\cos(c+dx))}{ad}$$

input
```
Int[Csc[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

output
```
(-2*b^(1/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(
2/3)]])/(3*a*Sqrt[a^(2/3) - b^(2/3)]*d) - ArcTanh[Cos[c + d*x]]/(a*d) + (2
*b^(1/3)*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[-((-
1)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*a*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]*
d) + (2*b^(1/3)*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sq
rt[(-1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*a*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]
*d)
```

3.186.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.186.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.36

method	result
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{4b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}}{d}$
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{4b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}}{d}$
risch	$2i\left(\sum_{\_R=\mathrm{RootOf}\left(\left(46656a^8d^6-46656b^2a^6d^6\right)\_Z^6-3888b^2a^4d^4\_Z^4-108a^2b^2d^2\_Z^2-b^2\right)}\_R\ln\left(\mathrm{e}^{i(dx+c)}+\left(-\right.\right.\right.$

input
```
int(csc(d*x+c)/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a*ln(tan(1/2*d*x+1/2*c))-4/3/a*b*sum(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b
+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2
*a+a)))
```

3.186.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.77 (sec) , antiderivative size = 29139, normalized size of antiderivative = 110.38

$$\int \frac{\csc(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.186.6 Sympy [F]

$$\int \frac{\csc(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\csc{(c+dx)}}{a+b\sin^3{(c+dx)}}\,dx$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)**3),x)
```

output
```
Integral(csc(c + d*x)/(a + b*sin(c + d*x)**3), x)
```

3.186.7 Maxima [F]

$$\int \frac{\csc(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\csc{(dx+c)}}{b\sin{(dx+c)}^3+a}\,dx$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
-1/2*(2*a*d*integrate(2*(16*a*b*cos(3*d*x + 3*c)^2 + 16*a*b*sin(3*d*x + 3*
c)^2 + 3*b^2*cos(d*x + c)*sin(2*d*x + 2*c) - 3*b^2*cos(2*d*x + 2*c)*sin(d*
x + c) + b^2*sin(d*x + c) - (b^2*sin(5*d*x + 5*c) - 2*b^2*sin(3*d*x + 3*c)
 + b^2*sin(d*x + c))*cos(6*d*x + 6*c) - (8*a*b*cos(3*d*x + 3*c) + 3*b^2*si
n(4*d*x + 4*c) - 3*b^2*sin(2*d*x + 2*c))*cos(5*d*x + 5*c) - 3*(2*b^2*sin(3
*d*x + 3*c) - b^2*sin(d*x + c))*cos(4*d*x + 4*c) - 2*(4*a*b*cos(d*x + c) +
 3*b^2*sin(2*d*x + 2*c))*cos(3*d*x + 3*c) + (b^2*cos(5*d*x + 5*c) - 2*b^2*
cos(3*d*x + 3*c) + b^2*cos(d*x + c))*sin(6*d*x + 6*c) + (3*b^2*cos(4*d*x +
 4*c) - 3*b^2*cos(2*d*x + 2*c) - 8*a*b*sin(3*d*x + 3*c) + b^2)*sin(5*d*x +
 5*c) + 3*(2*b^2*cos(3*d*x + 3*c) - b^2*cos(d*x + c))*sin(4*d*x + 4*c) + 2
*(3*b^2*cos(2*d*x + 2*c) - 4*a*b*sin(d*x + c) - b^2)*sin(3*d*x + 3*c))/(a*
b^2*cos(6*d*x + 6*c)^2 + 9*a*b^2*cos(4*d*x + 4*c)^2 + 64*a^3*cos(3*d*x + 3
*c)^2 + 9*a*b^2*cos(2*d*x + 2*c)^2 + a*b^2*sin(6*d*x + 6*c)^2 + 9*a*b^2*si
n(4*d*x + 4*c)^2 + 64*a^3*sin(3*d*x + 3*c)^2 - 48*a^2*b*cos(3*d*x + 3*c)*s
in(2*d*x + 2*c) + 9*a*b^2*sin(2*d*x + 2*c)^2 - 6*a*b^2*cos(2*d*x + 2*c) +
a*b^2 - 2*(3*a*b^2*cos(4*d*x + 4*c) - 3*a*b^2*cos(2*d*x + 2*c) - 8*a^2*b*s
in(3*d*x + 3*c) + a*b^2)*cos(6*d*x + 6*c) - 6*(3*a*b^2*cos(2*d*x + 2*c) +
8*a^2*b*sin(3*d*x + 3*c) - a*b^2)*cos(4*d*x + 4*c) - 2*(8*a^2*b*cos(3*d*x
+ 3*c) + 3*a*b^2*sin(4*d*x + 4*c) - 3*a*b^2*sin(2*d*x + 2*c))*sin(6*d*x +
6*c) + 6*(8*a^2*b*cos(3*d*x + 3*c) - 3*a*b^2*sin(2*d*x + 2*c))*sin(4*d*...
```

3.186.8 Giac [F]

$$\int \frac{\csc(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\csc(dx+c)}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.186.9 Mupad [B] (verification not implemented)

Time = 15.20 (sec) , antiderivative size = 1439, normalized size of antiderivative = 5.45

$$\int \frac{\csc(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(c + d*x)*(a + b*sin(c + d*x)^3)),x)
```

output
```
symsum(log(98304*b^5 + 1048576*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^
4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)*b^6*tan(c/2 + (d*x)/2) - 98304*roo
t(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2,
z, k)^2*a^2*b^5 + 5898240*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2
*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^3*a^3*b^5 - 7962624*root(729*a^6*b^2*z^
6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^4*a^4*b^5
- 663552*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2
*z^2 - b^2, z, k)^4*a^6*b^3 - 5308416*root(729*a^6*b^2*z^6 - 729*a^8*z^6 -
 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^5*a^5*b^5 + 10616832*root(7
29*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z,
k)^5*a^7*b^3 + 7962624*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^
4 + 27*a^2*b^2*z^2 - b^2, z, k)^6*a^6*b^5 - 9953280*root(729*a^6*b^2*z^6 -
 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^6*a^8*b^3 - 5
89824*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^
2 - b^2, z, k)*a*b^5 - 24576*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*
b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)*a^2*b^4*tan(c/2 + (d*x)/2) - 3145728
*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b
^2, z, k)^2*a*b^6*tan(c/2 + (d*x)/2) + 466944*root(729*a^6*b^2*z^6 - 729*a
^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^2*a^3*b^4*tan(c/2 +
 (d*x)/2) - 18874368*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z...
```

3.187 $\int \frac{\csc^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.187.1 Optimal result

Integrand size = 23, antiderivative size = 287

$$\int \frac{\csc^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = -\frac{2b\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}\sqrt{a^{2/3}-b^{2/3}}d} - \frac{2b\arctan\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{5/3}\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}d} + \frac{2b\arctan\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}d} - \frac{\operatorname{arctanh}(\cos(c+dx))}{2ad} - \frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

output
```
-1/2*arctanh(cos(d*x+c))/a/d-1/2*cot(d*x+c)*csc(d*x+c)/a/d-2/3*b*arctan((b
^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a^(5/3)/d/(a^(
2/3)-b^(2/3))^(1/2)-2/3*b*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1
/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(5/3)/d/(a^(2/3)+(-1)^(1/3)*b
^(2/3))^(1/2)+2/3*b*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*
d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(5/3)/d/(a^(2/3)-(-1)^(2
/3)*b^(2/3))^(1/2)
```

3.187.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.63

$$\int \frac{\csc^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{16ib\text{RootSum}\left[-b+3b\#1^2-8ia\#1^3-3b\#1^4+b\#1^6\&,\frac{2\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1-i\log\left(1-2\cos(c+dx)\#1+\#1\right.}{b-4ia\#1-2b\#1^2+b\#1^4}\right.}{24ac}$$

input
```
Integrate[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

output
```
((16*I)*b*RootSum[-b + 3*b*#1^2 - (8*I)*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (2*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 - I*Log[1 - 2*Cos[c + d*x]*#1
+ #1^2]*#1)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ] - 3*(Csc[(c + d*x)/2]
^2 + 4*Log[Cos[(c + d*x)/2]] - 4*Log[Sin[(c + d*x)/2]] - Sec[(c + d*x)/2]^
2))/(24*a*d)
```

3.187.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(c+dx)^3\left(a+b\sin(c+dx)^3\right)}dx$$

$$\Big\downarrow \ 3699$$

$$\int \left(\frac{\csc^3(c+dx)}{a}-\frac{b}{a\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2b\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2b\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}+\frac{2b\arctan\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{\text{arctanh}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

input
```
Int[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

output
```
(-2*b*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]]
)/(3*a^(5/3)*Sqrt[a^(2/3) - b^(2/3)]*d) - (2*b*ArcTan[((-1)^(2/3)*b^(1/3)
+ a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*a^(5/3
)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) + (2*b*ArcTan[((-1)^(1/3)*(b^(1/3)
 + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)
]])/(3*a^(5/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) - ArcTanh[Cos[c + d*x
]]/(2*a*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

3.187.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.187.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.49 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.47

method	result
derivativedivides	$\frac{\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}-\frac{\cdots}{8a\tan\cdots}}{d}$
default	$\frac{\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}-\frac{\cdots}{8a\tan\cdots}}{d}$
risch	$\frac{e^{3i(dx+c)}+e^{i(dx+c)}}{da\left(e^{2i(dx+c)}-1\right)^2}-8i\left(\sum_{\_R=\mathrm{RootOf}\left(\left(191102976a^{12}d^6-191102976a^{10}b^2d^6\right)\_Z^6-995328a^8b^2d^4\_Z^4+1728a^4b^4d\cdots\right.}\cdots\right.$

input
```
int(csc(d*x+c)^3/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*tan(1/2*d*x+1/2*c)^2/a-1/3/a*b*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^3
*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z
^3*b+3*_Z^2*a+a))-1/8/a/tan(1/2*d*x+1/2*c)^2+1/2/a*ln(tan(1/2*d*x+1/2*c)))
```

3.187.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.83 (sec) , antiderivative size = 29431, normalized size of antiderivative = 102.55

$$\int \frac{\csc^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.187.6 Sympy [F]

$$\int \frac{\csc^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\csc^3{(c+dx)}}{a+b\sin^3{(c+dx)}}\,dx$$

input
```
integrate(csc(d*x+c)**3/(a+b*sin(d*x+c)**3),x)
```

output
```
Integral(csc(c + d*x)**3/(a + b*sin(c + d*x)**3), x)
```

3.187.7 Maxima [F]

$$\int \frac{\csc^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\csc{(dx+c)}^3}{b\sin{(dx+c)}^3+a}\,dx$$

input
```
integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/4*(4*(cos(3*d*x + 3*c) + cos(d*x + c))*cos(4*d*x + 4*c) - 4*(2*cos(2*d*x
 + 2*c) - 1)*cos(3*d*x + 3*c) - 8*cos(2*d*x + 2*c)*cos(d*x + c) + 32*(a*b*
d*cos(4*d*x + 4*c)^2 + 4*a*b*d*cos(2*d*x + 2*c)^2 + a*b*d*sin(4*d*x + 4*c)
^2 - 4*a*b*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*b*d*sin(2*d*x + 2*c)^
2 - 4*a*b*d*cos(2*d*x + 2*c) + a*b*d - 2*(2*a*b*d*cos(2*d*x + 2*c) - a*b*d
)*cos(4*d*x + 4*c))*integrate(-(8*a*cos(3*d*x + 3*c)^2 - b*cos(3*d*x + 3*c
)*sin(6*d*x + 6*c) + 3*b*cos(3*d*x + 3*c)*sin(4*d*x + 4*c) + b*cos(6*d*x +
 6*c)*sin(3*d*x + 3*c) - 3*b*cos(4*d*x + 4*c)*sin(3*d*x + 3*c) + 8*a*sin(3
*d*x + 3*c)^2 - 3*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + (3*b*cos(2*d*x + 2
*c) - b)*sin(3*d*x + 3*c))/(a*b^2*cos(6*d*x + 6*c)^2 + 9*a*b^2*cos(4*d*x +
 4*c)^2 + 64*a^3*cos(3*d*x + 3*c)^2 + 9*a*b^2*cos(2*d*x + 2*c)^2 + a*b^2*s
in(6*d*x + 6*c)^2 + 9*a*b^2*sin(4*d*x + 4*c)^2 + 64*a^3*sin(3*d*x + 3*c)^2
 - 48*a^2*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 9*a*b^2*sin(2*d*x + 2*c)^2
 - 6*a*b^2*cos(2*d*x + 2*c) + a*b^2 - 2*(3*a*b^2*cos(4*d*x + 4*c) - 3*a*b^
2*cos(2*d*x + 2*c) - 8*a^2*b*sin(3*d*x + 3*c) + a*b^2)*cos(6*d*x + 6*c) -
6*(3*a*b^2*cos(2*d*x + 2*c) + 8*a^2*b*sin(3*d*x + 3*c) - a*b^2)*cos(4*d*x
+ 4*c) - 2*(8*a^2*b*cos(3*d*x + 3*c) + 3*a*b^2*sin(4*d*x + 4*c) - 3*a*b^2*
sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(8*a^2*b*cos(3*d*x + 3*c) - 3*a*b^2
*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) + 16*(3*a^2*b*cos(2*d*x + 2*c) - a^2*b
)*sin(3*d*x + 3*c)), x) + (2*(2*cos(2*d*x + 2*c) - 1)*cos(4*d*x + 4*c) ...
```

3.187.8 Giac [F]

$$\int \frac{\csc^3(c+dx)}{a+b\sin^3(c+dx)}\,dx=\int \frac{\csc(dx+c)^3}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.187.9 Mupad [B] (verification not implemented)

Time = 15.06 (sec) , antiderivative size = 1573, normalized size of antiderivative = 5.48

$$\int \frac{\csc^3(c+dx)}{a+b\sin^3(c+dx)}\,dx=\text{Too large to display}$$

input
```
int(1/(sin(c + d*x)^3*(a + b*sin(c + d*x)^3)),x)
```

output
```
symsum(log(-(65536*a*b^9 - 262144*b^10*tan(c/2 + (d*x)/2) - 131072*root(72
9*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z,
 k)*a^2*b^9 - 61440*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4
 - 27*a^4*b^4*z^2 - b^6, z, k)*a^4*b^7 + 860160*root(729*a^10*b^2*z^6 - 72
9*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^2*a^5*b^7 - 324
4032*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z
^2 - b^6, z, k)^3*a^6*b^7 - 1105920*root(729*a^10*b^2*z^6 - 729*a^12*z^6 -
 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^3*a^8*b^5 + 3538944*root(72
9*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z,
 k)^4*a^7*b^7 + 3870720*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2
*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^4*a^9*b^5 + 663552*root(729*a^10*b^2*z^
6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^5*a^10*b^
5 - 4976640*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^
4*b^4*z^2 - b^6, z, k)^5*a^12*b^3 - 7962624*root(729*a^10*b^2*z^6 - 729*a^
12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^6*a^11*b^5 + 995328
0*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2
- b^6, z, k)^6*a^13*b^3 + 24576*a^2*b^8*tan(c/2 + (d*x)/2) + 540672*root(7
29*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z
, k)*a^3*b^8*tan(c/2 + (d*x)/2) - 7077888*root(729*a^10*b^2*z^6 - 729*a^12
*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^2*a^4*b^8*tan(c/2 ...
```

3.188 $\int \frac{\csc^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.188.1 Optimal result

Integrand size = 23, antiderivative size = 344

$$\int \frac{\csc^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{2(-1)^{2/3}b^{5/3}\arctan\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{7/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} - \frac{2b^{5/3}\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{7/3}\sqrt{a^{2/3}-b^{2/3}}\,d} + \frac{2\sqrt[3]{-1}b^{5/3}\arctan\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{7/3}\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}\,d} - \frac{3\,\text{arctanh}(\cos(c+dx))}{8ad} + \frac{b\cot(c+dx)}{a^2d} - \frac{3\cot(c+dx)\csc(c+dx)}{8ad} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad}$$

output
```
-3/8*arctanh(cos(d*x+c))/a/d+b*cot(d*x+c)/a^2/d-3/8*cot(d*x+c)*csc(d*x+c)/
a/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d-2/3*b^(5/3)*arctan((b^(1/3)+a^(1/3)*ta
n(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a^(7/3)/d/(a^(2/3)-b^(2/3))^(1/
2)+2/3*(-1)^(1/3)*b^(5/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1
/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(7/3)/d/(a^(2/3)+(-1)^(1/3)*b
^(2/3))^(1/2)+2/3*(-1)^(2/3)*b^(5/3)*arctan(((-1)^(1/3)*b^(1/3)-a^(1/3)*ta
n(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(7/3)/d/(a^(2/3)-(
-1)^(2/3)*b^(2/3))^(1/2)
```

3.188.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.23 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.84

$$\int \frac{\csc^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$= \frac{-64b^2\text{RootSum}\left[-b+3b\#1^2-8ia\#1^3-3b\#1^4+b\#1^6\&,\frac{-2\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\log\left(1-2\cos(c+dx)\#1+\#1\right.}{b-4ic}\right.}{}$$

input
```
Integrate[Csc[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

output
```
(-64*b^2*RootSum[-b + 3*b*#1^2 - (8*I)*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (-2*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*#1 + #
1^2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - I*Log[1 - 2*Cos[c
 + d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ] + 3*(32*
b*Cot[(c + d*x)/2] - 6*a*Csc[(c + d*x)/2]^2 - a*Csc[(c + d*x)/2]^4 - 24*a*
Log[Cos[(c + d*x)/2]] + 24*a*Log[Sin[(c + d*x)/2]] + 6*a*Sec[(c + d*x)/2]^
2 + a*Sec[(c + d*x)/2]^4 - 32*b*Tan[(c + d*x)/2]))/(192*a^2*d)
```

3.188.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(c+dx)^5\left(a+b\sin(c+dx)^3\right)}dx$$
$$\Big\downarrow \text{3699}$$

$$\int \left(\frac{b^2\sin(c+dx)}{a^2\left(a+b\sin^3(c+dx)\right)}-\frac{b\csc^2(c+dx)}{a^2}+\frac{\csc^5(c+dx)}{a}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2b^{5/3}\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{7/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{-1}b^{5/3}\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{7/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}+\frac{2(-1)^{2/3}b^{5/3}\arctan\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{7/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{b\cot(c+dx)}{a^2d}-\frac{3\text{arctanh}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

input
```
Int[Csc[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

output
```
(-2*b^(5/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(
2/3)]])/(3*a^(7/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*(-1)^(1/3)*b^(5/3)*ArcT
an[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/
3)*b^(2/3)]])/(3*a^(7/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) + (2*(-1)^(
2/3)*b^(5/3)*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x
)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(7/3)*Sqrt[a^(2/3) - (-1)
^(2/3)*b^(2/3)]*d) - (3*ArcTanh[Cos[c + d*x]])/(8*a*d) + (b*Cot[c + d*x])/
(a^2*d) - (3*Cot[c + d*x]*Csc[c + d*x])/(8*a*d) - (Cot[c + d*x]*Csc[c + d*
x]^3)/(4*a*d)
```

3.188.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.188.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.04 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.57

method	result
derivativedivides	$\frac{\frac{\frac{(\tan^4(\frac{dx}{2}+\frac{c}{2}))a}{4}+2(\tan^2(\frac{dx}{2}+\frac{c}{2}))a-8b\tan(\frac{dx}{2}+\frac{c}{2})}{16a^2}+\frac{2b^2\left(\sum_{\_R=\text{RootOf}(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a)}\frac{(\_R^3+\_R}{\_R^5a+2\_}\right.}{3a^2}}{d}$
default	$\frac{\frac{\frac{(\tan^4(\frac{dx}{2}+\frac{c}{2}))a}{4}+2(\tan^2(\frac{dx}{2}+\frac{c}{2}))a-8b\tan(\frac{dx}{2}+\frac{c}{2})}{16a^2}+\frac{2b^2\left(\sum_{\_R=\text{RootOf}(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a)}\frac{(\_R^3+\_R}{\_R^5a+2\_}\right.}{3a^2}}{d}$
risch	$\frac{3a\,e^{7i(dx+c)}-11a\,e^{5i(dx+c)}+8ib\,e^{6i(dx+c)}-11a\,e^{3i(dx+c)}-24ib\,e^{4i(dx+c)}+3a\,e^{i(dx+c)}+24ib\,e^{2i(dx+c)}-8ib}{4a^2d(e^{2i(dx+c)}-1)^4}+32i\Big($

input
```
int(csc(d*x+c)^5/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/16/a^2*(1/4*tan(1/2*d*x+1/2*c)^4*a+2*tan(1/2*d*x+1/2*c)^2*a-8*b*tan
(1/2*d*x+1/2*c))+2/3*b^2/a^2*sum((_R^3+_R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)
*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
-1/64/a/tan(1/2*d*x+1/2*c)^4-1/8/a/tan(1/2*d*x+1/2*c)^2+3/8/a*ln(tan(1/2*d
*x+1/2*c))+1/2/a^2*b/tan(1/2*d*x+1/2*c))
```

3.188.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 131.01 (sec) , antiderivative size = 21564, normalized size of antiderivative = 62.69

$$\int \frac{\csc^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.188.6 Sympy [F]

$$\int \frac{\csc^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\csc^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**5/(a+b*sin(d*x+c)**3),x)
```

output
```
Integral(csc(c + d*x)**5/(a + b*sin(c + d*x)**3), x)
```

3.188.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.188.8 Giac [F]

$$\int \frac{\csc^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\csc(dx+c)^5}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)^5/(b*sin(d*x + c)^3 + a), x)
```

3.188.9 Mupad [B] (verification not implemented)

Time = 15.22 (sec) , antiderivative size = 1560, normalized size of antiderivative = 4.53

$$\int \frac{\csc^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(c + d*x)^5*(a + b*sin(c + d*x)^3)),x)
```

output
```
symsum(log((262144*b^14*tan(c/2 + (d*x)/2) - 3072*a^3*b^11 + 155648*root(7
29*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)*a^4*b^11 -
 393216*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z,
 k)^2*a^5*b^11 + 774144*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^
4*z^4 - b^10, z, k)^2*a^7*b^9 - 2064384*root(729*a^14*b^2*z^6 - 729*a^16*z
^6 - 243*a^10*b^4*z^4 - b^10, z, k)^3*a^8*b^9 + 2073600*root(729*a^14*b^2*
z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^3*a^10*b^7 - 9510912*r
oot(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^4*a^1
1*b^7 + 2737152*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 -
b^10, z, k)^4*a^13*b^5 + 10616832*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 2
43*a^10*b^4*z^4 - b^10, z, k)^5*a^12*b^7 - 10285056*root(729*a^14*b^2*z^6
- 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^5*a^14*b^5 + 3732480*root(
729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^5*a^16*b^
3 + 7962624*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10
, z, k)^6*a^15*b^5 - 9953280*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^
10*b^4*z^4 - b^10, z, k)^6*a^17*b^3 + 98304*a^2*b^12*tan(c/2 + (d*x)/2) -
262144*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z,
k)*a^3*b^12*tan(c/2 + (d*x)/2) + 165888*root(729*a^14*b^2*z^6 - 729*a^16*z
^6 - 243*a^10*b^4*z^4 - b^10, z, k)*a^5*b^10*tan(c/2 + (d*x)/2) - 1327104*
root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^2...
```

3.189 $\int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.189.1 Optimal result

Integrand size = 23, antiderivative size = 293

$$\int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}\,dx = -\frac{ax}{b^2}+\frac{2a^{4/3}\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}b^2d}+\frac{2a^{4/3}\arctan\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}b^2d}-\frac{2a^{4/3}\arctan\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}b^2d}-\frac{\cos(c+dx)}{bd}+\frac{\cos^3(c+dx)}{3bd}$$

output
```
-a*x/b^2-cos(d*x+c)/b/d+1/3*cos(d*x+c)^3/b/d+2/3*a^(4/3)*arctan((b^(1/3)+a
^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/b^2/d/(a^(2/3)-b^(2/3)
)^(1/2)+2/3*a^(4/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))
/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/b^2/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1
/2)-2/3*a^(4/3)*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+
1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/b^2/d/(a^(2/3)-(-1)^(2/3)*b^(2
/3))^(1/2)
```

3.189.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.56

$$\int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}\,dx =$$

$$-\frac{12ac+12adx+9b\cos(c+dx)-b\cos(3(c+dx))+8ia^2\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+\right.}{12b^2d}$$

input
```
Integrate[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^3),x]
```

output
```
-1/12*(12*a*c + 12*a*d*x + 9*b*Cos[c + d*x] - b*Cos[3*(c + d*x)] + (8*I)*a
^2*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & ,
(2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 - I*Log[1 - 2*Cos[c + d*x]*
#1 + #1^2]*#1)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ])/(b^2*d)
```

3.189.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^6}{a+b\sin(c+dx)^3}dx$$

$$\Big\downarrow \text{3699}$$

$$\int \left(\frac{a^2}{b^2\left(a+b\sin^3(c+dx)\right)}-\frac{a}{b^2}+\frac{\sin^3(c+dx)}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^{4/3}\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2a^{4/3}\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}-\frac{2a^{4/3}\arctan\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{ax}{b^2}+\frac{\cos^3(c+dx)}{3bd}-\frac{\cos(c+dx)}{bd}$$

input
```
Int[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^3),x]
```

output
```
-((a*x)/b^2) + (2*a^(4/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt
[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^2*d) + (2*a^(4/3)*ArcTa
n[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3
)*b^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^2*d) - (2*a^(4/3)*Arc
Tan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2
/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^2*d) -
 Cos[c + d*x]/(b*d) + Cos[c + d*x]^3/(3*b*d)
```

3.189.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.189. $\int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.189.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.51 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.49

method	result
derivativedivides	$\frac{-\frac{2\left(\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{2b}{3}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^2}+\frac{a^2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R\cdots\right.}{\_R^5a+2\cdots}\cdots\right.}{3b^2}}{d}$
default	$\frac{-\frac{2\left(\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{2b}{3}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^2}+\frac{a^2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R\cdots\right.}{\_R^5a+2\cdots}\cdots\right.}{3b^2}}{d}$
risch	$-\frac{ax}{b^2}-\frac{3\,\mathrm{e}^{i(dx+c)}}{8bd}-\frac{3\,\mathrm{e}^{-i(dx+c)}}{8bd}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^2b^{12}d^6-729b^{14}d^6\right)\_Z^6+995328a^4b^8d^4\_Z^4+452984832a^6b^4d^2\cdots\right.}\cdots\right.}{\cdots}$

input
```
int(sin(d*x+c)^6/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/b^2*((2*b*tan(1/2*d*x+1/2*c)^2+2/3*b)/(1+tan(1/2*d*x+1/2*c)^2)^3+a
*arctan(tan(1/2*d*x+1/2*c)))+1/3*a^2/b^2*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^
3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_
Z^3*b+3*_Z^2*a+a)))
```

3.189.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 29350, normalized size of antiderivative = 100.17

$$\int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.189.6 Sympy [F]

$$\int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

input
```
integrate(sin(d*x+c)**6/(a+b*sin(d*x+c)**3),x)
```

output
```
Integral(sin(c + d*x)**6/(a + b*sin(c + d*x)**3), x)
```

3.189.7 Maxima [F]

$$\int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sin(dx+c)^6}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
-1/12*(96*a^2*b^2*d*integrate(-(8*a*cos(3*d*x + 3*c)^2 - b*cos(3*d*x + 3*c
)*sin(6*d*x + 6*c) + 3*b*cos(3*d*x + 3*c)*sin(4*d*x + 4*c) + b*cos(6*d*x +
 6*c)*sin(3*d*x + 3*c) - 3*b*cos(4*d*x + 4*c)*sin(3*d*x + 3*c) + 8*a*sin(3
*d*x + 3*c)^2 - 3*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + (3*b*cos(2*d*x + 2
*c) - b)*sin(3*d*x + 3*c))/(b^4*cos(6*d*x + 6*c)^2 + 9*b^4*cos(4*d*x + 4*c
)^2 + 64*a^2*b^2*cos(3*d*x + 3*c)^2 + 9*b^4*cos(2*d*x + 2*c)^2 + b^4*sin(6
*d*x + 6*c)^2 + 9*b^4*sin(4*d*x + 4*c)^2 + 64*a^2*b^2*sin(3*d*x + 3*c)^2 -
 48*a*b^3*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 9*b^4*sin(2*d*x + 2*c)^2 - 6
*b^4*cos(2*d*x + 2*c) + b^4 - 2*(3*b^4*cos(4*d*x + 4*c) - 3*b^4*cos(2*d*x
+ 2*c) - 8*a*b^3*sin(3*d*x + 3*c) + b^4)*cos(6*d*x + 6*c) - 6*(3*b^4*cos(2
*d*x + 2*c) + 8*a*b^3*sin(3*d*x + 3*c) - b^4)*cos(4*d*x + 4*c) - 2*(8*a*b^
3*cos(3*d*x + 3*c) + 3*b^4*sin(4*d*x + 4*c) - 3*b^4*sin(2*d*x + 2*c))*sin(
6*d*x + 6*c) + 6*(8*a*b^3*cos(3*d*x + 3*c) - 3*b^4*sin(2*d*x + 2*c))*sin(4
*d*x + 4*c) + 16*(3*a*b^3*cos(2*d*x + 2*c) - a*b^3)*sin(3*d*x + 3*c)), x)
+ 12*a*d*x - b*cos(3*d*x + 3*c) + 9*b*cos(d*x + c))/(b^2*d)
```

3.189.8 Giac [F]

$$\int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sin(dx+c)^6}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.189.9 Mupad [B] (verification not implemented)

Time = 15.21 (sec) , antiderivative size = 1800, normalized size of antiderivative = 6.14

$$\int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^6/(a + b*sin(c + d*x)^3),x)
```

output
```
symsum(log((1073741824*a^13*tan(c/2 + (d*x)/2) + 2013265920*root(729*a^2*b
^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)*a^1
2*b^2 - 4831838208*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4
+ 27*a^6*b^4*z^2 + a^8, z, k)^2*a^10*b^5 + 268435456*root(729*a^2*b^12*z^6
 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^2*a^12*b^3
 + 33722204160*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27
*a^6*b^4*z^2 + a^8, z, k)^3*a^10*b^6 - 15703474176*root(729*a^2*b^12*z^6 -
 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^4*a^8*b^9 +
4831838208*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6
*b^4*z^2 + a^8, z, k)^4*a^10*b^7 - 130459631616*root(729*a^2*b^12*z^6 - 72
9*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^5*a^6*b^12 + 15
4014842880*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6
*b^4*z^2 + a^8, z, k)^5*a^8*b^10 - 35332816896*root(729*a^2*b^12*z^6 - 729
*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^6*a^6*b^13 + 217
43271936*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b
^4*z^2 + a^8, z, k)^6*a^8*b^11 - 130459631616*root(729*a^2*b^12*z^6 - 729*
b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^7*a^4*b^16 + 1223
05904640*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b
^4*z^2 + a^8, z, k)^7*a^6*b^14 - 3221225472*root(729*a^2*b^12*z^6 - 729*b^
14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)*a^11*b^3*tan(c/2...
```

3.190 $\int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.190.1 Optimal result

Integrand size = 23, antiderivative size = 281

$$\int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = -\frac{2(-1)^{2/3}a^{2/3}\arctan\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^{4/3}d} + \frac{2a^{2/3}\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^{4/3}d} - \frac{2\sqrt[3]{-1}a^{2/3}\arctan\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}\,b^{4/3}d} - \frac{\cos(c+dx)}{bd}$$

output
```
-cos(d*x+c)/b/d+2/3*a^(2/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a
^(2/3)-b^(2/3))^(1/2))/b^(4/3)/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*(-1)^(1/3)*a^
(2/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)
^(1/3)*b^(2/3))^(1/2))/b^(4/3)/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)-2/3*(-
1)^(2/3)*a^(2/3)*arctan(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a
^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/b^(4/3)/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(
1/2)
```

3.190.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.66

$$\int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{-3\cos(c+dx)+a\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&,\frac{-2\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\log\left(1-2\right.}{}\right.}{3bd}$$

input
```
Integrate[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

output
```
(-3*Cos[c + d*x] + a*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1
^4 + I*b*#1^6 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1
^2 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1^2
+ b*#1^4) & ])/(3*b*d)
```

3.190.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(c+dx)^4}{a+b\sin(c+dx)^3}dx$$

$$\Big\downarrow \ 3699$$

$$\int \left(\frac{\sin(c+dx)}{b}-\frac{a\sin(c+dx)}{b\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^{2/3}\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\sqrt[3]{-1}a^{2/3}\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}-\frac{2(-1)^{2/3}a^{2/3}\arctan\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{\cos(c+dx)}{bd}$$

input
```
Int[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

output
```
(2*a^(2/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2
/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(4/3)*d) - (2*(-1)^(1/3)*a^(2/3)*ArcTa
n[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3
)*b^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^(4/3)*d) - (2*(-1)^(2
/3)*a^(2/3)*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)
/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^
(2/3)]*b^(4/3)*d) - Cos[c + d*x]/(b*d)
```

3.190.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.190.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.37

method	result
derivativedivides	$\frac{-\frac{2}{b\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3+\_R\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3b}}{d}$
default	$\frac{-\frac{2}{b\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3+\_R\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3b}}{d}$
risch	$-\frac{e^{i(dx+c)}}{2bd}-\frac{e^{-i(dx+c)}}{2bd}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^2b^8d^6-729b^{10}d^6\right)\_Z^6+62208a^2b^6d^4\_Z^4+16777216a^4\right)}\_R\ln\left(e^{i(dx+c)}\right.\right.}{}$

input
```
int(sin(d*x+c)^4/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/b/(1+tan(1/2*d*x+1/2*c)^2)-2/3/b*a*sum((_R^3+_R)/(_R^5*a+2*_R^3*a+
4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*
b+3*_Z^2*a+a)))
```

3.190.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 21185, normalized size of antiderivative = 75.39

$$\int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.190.6 Sympy [F]

$$\int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

input
```
integrate(sin(d*x+c)**4/(a+b*sin(d*x+c)**3),x)
```

output
```
Integral(sin(c + d*x)**4/(a + b*sin(c + d*x)**3), x)
```

3.190.7 Maxima [F]

$$\int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sin(dx+c)^4}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
(b*d*integrate(-4*(3*a*b*cos(4*d*x + 4*c)^2 + 3*a*b*cos(2*d*x + 2*c)^2 + 3
*a*b*sin(4*d*x + 4*c)^2 + 8*a^2*cos(2*d*x + 2*c)*sin(3*d*x + 3*c) - 8*a^2*
cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 3*a*b*sin(2*d*x + 2*c)^2 - a*b*cos(2*d
*x + 2*c) - (a*b*cos(4*d*x + 4*c) - a*b*cos(2*d*x + 2*c))*cos(6*d*x + 6*c)
 - (6*a*b*cos(2*d*x + 2*c) + 8*a^2*sin(3*d*x + 3*c) - a*b)*cos(4*d*x + 4*c
) - (a*b*sin(4*d*x + 4*c) - a*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 2*(4*
a^2*cos(3*d*x + 3*c) - 3*a*b*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))/(b^3*cos(
6*d*x + 6*c)^2 + 9*b^3*cos(4*d*x + 4*c)^2 + 64*a^2*b*cos(3*d*x + 3*c)^2 +
9*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(6*d*x + 6*c)^2 + 9*b^3*sin(4*d*x + 4*c)
^2 + 64*a^2*b*sin(3*d*x + 3*c)^2 - 48*a*b^2*cos(3*d*x + 3*c)*sin(2*d*x + 2
*c) + 9*b^3*sin(2*d*x + 2*c)^2 - 6*b^3*cos(2*d*x + 2*c) + b^3 - 2*(3*b^3*c
os(4*d*x + 4*c) - 3*b^3*cos(2*d*x + 2*c) - 8*a*b^2*sin(3*d*x + 3*c) + b^3)
*cos(6*d*x + 6*c) - 6*(3*b^3*cos(2*d*x + 2*c) + 8*a*b^2*sin(3*d*x + 3*c) -
 b^3)*cos(4*d*x + 4*c) - 2*(8*a*b^2*cos(3*d*x + 3*c) + 3*b^3*sin(4*d*x + 4
*c) - 3*b^3*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(8*a*b^2*cos(3*d*x + 3*
c) - 3*b^3*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) + 16*(3*a*b^2*cos(2*d*x + 2*
c) - a*b^2)*sin(3*d*x + 3*c)), x) - cos(d*x + c))/(b*d)
```

3.190.8 Giac [F]

$$\int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sin(dx+c)^4}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)^4/(b*sin(d*x + c)^3 + a), x)
```

3.190.9 Mupad [B] (verification not implemented)

Time = 15.32 (sec) , antiderivative size = 665, normalized size of antiderivative = 2.37

$$\int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\sum_{k=1}^{6} \ln\left(8192\,a^8\,b^5 - \mathrm{root}(729\,a^2\,b^8\,d^6 - 729\,b^{10}\,d^6 + 243\,a^2\,b^6\,d^4 + a^4, d, k)^2\,a^6\,b^9\,294912 + \mathrm{root}(729\,a^2\,b^8\right.}{} - \frac{2}{b\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + b\,d}$$

input
```
int(sin(c + d*x)^4/(a + b*sin(c + d*x)^3),x)
```

output
```
symsum(log(8192*a^8*b^5 - 294912*root(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243
*a^2*b^6*d^4 + a^4, d, k)^2*a^6*b^9 + 1548288*root(729*a^2*b^8*d^6 - 729*b
^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)^3*a^6*b^10 - 1990656*root(729*a^2*b
^8*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)^4*a^6*b^11 - 7962624*
root(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)^5*a^4*b
^14 + 5971968*root(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4,
 d, k)^5*a^6*b^12 - 65536*a^7*b^6*tan(c/2 + (d*x)/2) + 196608*root(729*a^2
*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)*a^7*b^7*tan(c/2 + (
d*x)/2) - 294912*root(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a
^4, d, k)^2*a^7*b^8*tan(c/2 + (d*x)/2) - 1769472*root(729*a^2*b^8*d^6 - 72
9*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)^3*a^5*b^11*tan(c/2 + (d*x)/2) +
221184*root(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)^
3*a^7*b^9*tan(c/2 + (d*x)/2) - 2654208*root(729*a^2*b^8*d^6 - 729*b^10*d^6
 + 243*a^2*b^6*d^4 + a^4, d, k)^4*a^5*b^12*tan(c/2 + (d*x)/2) - 1990656*ro
ot(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)^5*a^5*b^1
3*tan(c/2 + (d*x)/2))*root(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^
4 + a^4, d, k), k, 1, 6)/d - 2/(b*d + b*d*tan(c/2 + (d*x)/2)^2)
```

3.191 $\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.191.1 Optimal result

Integrand size = 23, antiderivative size = 240

$$\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{2\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}b^{2/3}d} - \frac{2\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}b^{2/3}d} - \frac{2\text{arctanh}\left(\frac{\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}b^{2/3}d}$$

output
```
2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/b
^(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*arctanh((b^(1/3)+(-1)^(2/3)*a^(1/3)*t
an(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2))/b^(2/3)/d/((-1)^(1/
3)*a^(2/3)+b^(2/3))^(1/2)-2/3*arctanh((b^(1/3)-(-1)^(1/3)*a^(1/3)*tan(1/2*
d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2))/b^(2/3)/d/(-(-1)^(2/3)*a^
(2/3)+b^(2/3))^(1/2)
```

3.191.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 11.04 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.96

$$\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{i\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&,\frac{2\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)-4}{}\right.}{}$$

input
```
Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

output
```
((I/6)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6
& , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x]
*#1 + #1^2] - 4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (2*I)*Log[
1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
 #1)]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^
2 - 2*b*#1^3 + b*#1^5) & ])/d
```

3.191.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(c+dx)^2}{a+b\sin(c+dx)^3}dx$$

$$\Big\downarrow \; 3699$$

$$\int \left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}+\frac{1}{3b^{2/3}\left(\sqrt[3]{b}\sin(c+dx)-\sqrt[3]{-1}\sqrt[3]{a}\right)}+\frac{1}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3b^{2/3}d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}-\frac{2\text{arctanh}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3b^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}$$

input
```
Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

output
```
(2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(
3*Sqrt[a^(2/3) - b^(2/3)]*b^(2/3)*d) - (2*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^
(1/3)*Tan[(c + d*x)/2])/Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*Sqrt[-(
(-1)^(2/3)*a^(2/3)) + b^(2/3)]*b^(2/3)*d) - (2*ArcTanh[(b^(1/3) + (-1)^(2/
3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*Sqrt[
(-1)^(1/3)*a^(2/3) + b^(2/3)]*b^(2/3)*d)
```

3.191.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.191.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.81 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.32

method	result
derivativedivides	$\frac{4\left(\sum_{\_R=\operatorname{RootOf}\left(a\_Z^{6}+3a\_Z^{4}+8b\_Z^{3}+3a\_Z^{2}+a\right)}\frac{\_R^{2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a+2\_R^{3}a+4\_R^{2}b+\_Ra}\right)}{3d}$
default	$\frac{4\left(\sum_{\_R=\operatorname{RootOf}\left(a\_Z^{6}+3a\_Z^{4}+8b\_Z^{3}+3a\_Z^{2}+a\right)}\frac{\_R^{2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a+2\_R^{3}a+4\_R^{2}b+\_Ra}\right)}{3d}$
risch	$-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(4096+\left(729a^{2}b^{4}d^{6}-729b^{6}d^{6}\right)\_Z^{6}+3888b^{4}d^{4}\_Z^{4}-6912b^{2}d^{2}\_Z^{2}\right)}\_R\ln\left(\mathrm{e}^{i(dx+c)}+\left(-\frac{243}{1024}b^{3}d^{5}a^{2}+\frac{243}{1024}\right.\right.\right.}{4}$

input
```
int(sin(d*x+c)^2/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
4/3/d*sum(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_
R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

3.191.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 25253, normalized size of antiderivative = 105.22

$$\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.191.6 Sympy [F]

$$\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

input
```
integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**3),x)
```

output
```
Integral(sin(c + d*x)**2/(a + b*sin(c + d*x)**3), x)
```

3.191.7 Maxima [F]

$$\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sin(dx+c)^2}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

3.191.8 Giac [F]

$$\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sin(dx+c)^2}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

3.191.9 Mupad [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.46

$$\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$= \frac{\sum_{k=1}^{6} \ln\left(-\frac{8192\,a^4\left(-729\,a^2\,b^3-81\,a^2\,b^2\,\mathrm{root}\left(d^6-27\,b^2\,d^4+243\,b^4\,d^2+729\,b^4\,(a^2-b^2),d,k\right)+243\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b^4+324\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b^3\,\mathrm{ro}\right.}{}\right.}{}$$

input
```
int(sin(c + d*x)^2/(a + b*sin(c + d*x)^3),x)
```

output
```
symsum(log(-(8192*a^4*(12*b*root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729*b^4*
(a^2 - b^2), d, k)^4 + 324*b^4*root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729*b
^4*(a^2 - b^2), d, k) + 972*b^5 + 4*root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 +
729*b^4*(a^2 - b^2), d, k)^5 - 729*a^2*b^3 - 72*b^2*root(d^6 - 27*b^2*d^4
+ 243*b^4*d^2 + 729*b^4*(a^2 - b^2), d, k)^3 - 216*b^3*root(d^6 - 27*b^2*d
^4 + 243*b^4*d^2 + 729*b^4*(a^2 - b^2), d, k)^2 - 81*a^2*b^2*root(d^6 - 27
*b^2*d^4 + 243*b^4*d^2 + 729*b^4*(a^2 - b^2), d, k) + 243*a*b^4*tan(c/2 +
(d*x)/2) + 3*a*tan(c/2 + (d*x)/2)*root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 72
9*b^4*(a^2 - b^2), d, k)^4 + 162*a*b^2*tan(c/2 + (d*x)/2)*root(d^6 - 27*b^
2*d^4 + 243*b^4*d^2 + 729*b^4*(a^2 - b^2), d, k)^2 + 36*a*b*tan(c/2 + (d*x
)/2)*root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729*b^4*(a^2 - b^2), d, k)^3 +
324*a*tan(c/2 + (d*x)/2)*b^3*root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729*b^4
*(a^2 - b^2), d, k)))/root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729*b^4*(a^2 -
 b^2), d, k)^5)*root(729*a^2*b^4*d^6 - 729*b^6*d^6 + 243*b^4*d^4 - 27*b^2*
d^2 + 1, d, k), k, 1, 6)/d
```

3.192 $\int \frac{1}{a+b\sin^3(c+dx)}\,dx$

3.192.1 Optimal result

Integrand size = 14, antiderivative size = 245

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx = \frac{2\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}d} + \frac{2\arctan\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}d} - \frac{2\arctan\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}d}$$

output
```
2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a
^(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*ta
n(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(2/3)/d/(a^(2/3)+(
-1)^(1/3)*b^(2/3))^(1/2)-2/3*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(2/3)*a^(1/3)
*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(2/3)/d/(a^(2/3
)-(-1)^(2/3)*b^(2/3))^(1/2)
```

3.192.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx=\$\text{Aborted}$$

input
```
Integrate[(a + b*Sin[c + d*x]^3)^(-1),x]
```

output
```
$Aborted
```

3.192.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin^3(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{a+b\sin(c+dx)^3}dx$$

$$\Big\downarrow \; 3692$$

$$\int \left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)}-\frac{1}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)-\sqrt[3]{a}\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\sin(c+\right.}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{2\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}-\frac{2\arctan\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

input
```
Int[(a + b*Sin[c + d*x]^3)^(-1),x]
```

output
```
(2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(
3*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*ArcTan[((-1)^(2/3)*b^(1/3) + a^(
1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*a^(2/3)*Sqr
t[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)
^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*
a^(2/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*d)
```

3.192.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.192.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.34

method	result
derivativedivides	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}}{3d}$
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}}{3d}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+\left(729a^6d^6-729a^4b^2d^6\right)\_Z^6+243a^4d^4\_Z^4+27a^2d^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{i(dx+c)}+\left(-\frac{486d^5a^6}{b}+48\right.\right.$

input
```
int(1/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1
/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

3.192.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 25429, normalized size of antiderivative = 103.79

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.192.6 Sympy [F]

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx = \int \frac{1}{a+b\sin^3\left(c+dx\right)}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c)**3),x)
```

output
```
Integral(1/(a + b*sin(c + d*x)**3), x)
```

3.192.7 Maxima [F]

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx = \int \frac{1}{b\sin\left(dx+c\right)^3+a}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate(1/(b*sin(d*x + c)^3 + a), x)
```

3.192.8 Giac [F]

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx = \int \frac{1}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(1/(b*sin(d*x + c)^3 + a), x)
```

3.192.9 Mupad [B] (verification not implemented)

Time = 15.55 (sec) , antiderivative size = 609, normalized size of antiderivative = 2.49

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx = \frac{\sum_{k=1}^{6}\ln\left(-\frac{8192\,a\,b^3\left(-729\,a^5+243\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^4\,b-324\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^4\,\mathrm{root}\left(d^6+27\,a^2\,d^4+243\,a^4\,d^2+729\,a^4\left(a^2-b^2\right),d,k\right)+972\,a^3\,b^2+a\right.}{}\right.}{}$$

input
```
int(1/(a + b*sin(c + d*x)^3),x)
```

output
```
symsum(log(-(8192*a*b^3*(972*a^3*b^2 - 729*a^5 - 9*a*root(d^6 + 27*a^2*d^4
 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^4 - 162*a^3*root(d^6 + 27*a^2*
d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^2 - 4*tan(c/2 + (d*x)/2)*ro
ot(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5 + 243*a^4
*b*tan(c/2 + (d*x)/2) - 324*tan(c/2 + (d*x)/2)*a^4*root(d^6 + 27*a^2*d^4 +
 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k) + 24*b*tan(c/2 + (d*x)/2)*root(d
^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^4 - 72*a^2*tan(
c/2 + (d*x)/2)*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2),
d, k)^3 + 36*a*b*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2)
, d, k)^3 + 243*b*a^3*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 -
 b^2), d, k) + 648*tan(c/2 + (d*x)/2)*a^2*b^2*root(d^6 + 27*a^2*d^4 + 243*
a^4*d^2 + 729*a^4*(a^2 - b^2), d, k) + 216*a^2*b*tan(c/2 + (d*x)/2)*root(d
^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^2))/root(d^6 +
27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5)*root(729*a^4*b^2*
d^6 - 729*a^6*d^6 - 243*a^4*d^4 - 27*a^2*d^2 - 1, d, k), k, 1, 6)/d
```

3.193 $\int \frac{\csc^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.193.1 Optimal result

Integrand size = 23, antiderivative size = 281

$$\int \frac{\csc^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = -\frac{2(-1)^{2/3}b^{2/3}\arctan\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} + \frac{2b^{2/3}\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{4/3}\sqrt{a^{2/3}-b^{2/3}}\,d} - \frac{2\sqrt[3]{-1}b^{2/3}\arctan\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{4/3}\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}\,d} - \frac{\cot(c+dx)}{ad}$$

output
```
-cot(d*x+c)/a/d+2/3*b^(2/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a
^(2/3)-b^(2/3))^(1/2))/a^(4/3)/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*(-1)^(1/3)*b^
(2/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)
^(1/3)*b^(2/3))^(1/2))/a^(4/3)/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)-2/3*(-
1)^(2/3)*b^(2/3)*arctan(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a
^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(4/3)/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(
1/2)
```

3.193.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.37 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.70

$$\int \frac{\csc^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$= \frac{-3\cot\left(\frac{1}{2}(c+dx)\right)+2b\text{RootSum}\left[-b+3b\#1^2-8ia\#1^3-3b\#1^4+b\#1^6\&,\frac{-2\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\log(\cdots}{\cdots}\right.}{6ad}$$

input
```
Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

output
```
(-3*Cot[(c + d*x)/2] + 2*b*RootSum[-b + 3*b*#1^2 - (8*I)*a*#1^3 - 3*b*#1^4
 + b*#1^6 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*C
os[c + d*x]*#1 + #1^2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 -
 I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*
#1^4) & ] + 3*Tan[(c + d*x)/2])/(6*a*d)
```

3.193.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(c+dx)^2\left(a+b\sin(c+dx)^3\right)}dx$$
$$\Big\downarrow \text{3699}$$
$$\int \left(\frac{\csc^2(c+dx)}{a}-\frac{b\sin(c+dx)}{a\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$\downarrow \text{2009}$$

$$\frac{2b^{2/3}\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\sqrt[3]{-1}b^{2/3}\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}-\frac{2(-1)^{2/3}b^{2/3}\arctan\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{\cot(c+dx)}{ad}$$

input
```
Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

output
```
(2*b^(2/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2
/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) - b^(2/3)]*d) - (2*(-1)^(1/3)*b^(2/3)*ArcTa
n[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3
)*b^(2/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*(-1)^(2
/3)*b^(2/3)*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)
/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) - (-1)^
(2/3)*b^(2/3)]*d) - Cot[c + d*x]/(a*d)
```

3.193.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.193.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.41

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{2b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3+\_R\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}-\frac{1}{2a\tan\left(\frac{dx}{2}+\right.}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{2b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3+\_R\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}-\frac{1}{2a\tan\left(\frac{dx}{2}+\right.}}{d}$
risch	$-\frac{2i}{ad\left(e^{2i(dx+c)}-1\right)}-4\left(\sum_{\_R=\mathrm{RootOf}\left(\left(2985984a^{10}d^6-2985984a^8b^2d^6\right)\_Z^6+62208a^6b^2d^4\_Z^4+b^4\right)}\_R\ln\left(e^{i(d}\right.\right.$

input
```
int(csc(d*x+c)^2/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*tan(1/2*d*x+1/2*c)/a-2/3/a*b*sum((_R^3+_R)/(_R^5*a+2*_R^3*a+4*_R^
2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_
Z^2*a+a))-1/2/a/tan(1/2*d*x+1/2*c))
```

3.193.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 21243, normalized size of antiderivative = 75.60

$$\int \frac{\csc^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.193.6 Sympy [F]

$$\int \frac{\csc^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\csc^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**3),x)
```

output
```
Integral(csc(c + d*x)**2/(a + b*sin(c + d*x)**3), x)
```

3.193.7 Maxima [F]

$$\int \frac{\csc^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\csc(dx+c)^2}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
((a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 - 2*a*d*cos(2*d*x + 2*c)
 + a*d)*integrate(-4*(3*b^2*cos(4*d*x + 4*c)^2 + 3*b^2*cos(2*d*x + 2*c)^2
+ 3*b^2*sin(4*d*x + 4*c)^2 + 8*a*b*cos(2*d*x + 2*c)*sin(3*d*x + 3*c) - 8*a
*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 3*b^2*sin(2*d*x + 2*c)^2 - b^2*cos(
2*d*x + 2*c) - (b^2*cos(4*d*x + 4*c) - b^2*cos(2*d*x + 2*c))*cos(6*d*x + 6
*c) - (6*b^2*cos(2*d*x + 2*c) + 8*a*b*sin(3*d*x + 3*c) - b^2)*cos(4*d*x +
4*c) - (b^2*sin(4*d*x + 4*c) - b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 2*
(4*a*b*cos(3*d*x + 3*c) - 3*b^2*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))/(a*b^2
*cos(6*d*x + 6*c)^2 + 9*a*b^2*cos(4*d*x + 4*c)^2 + 64*a^3*cos(3*d*x + 3*c)
^2 + 9*a*b^2*cos(2*d*x + 2*c)^2 + a*b^2*sin(6*d*x + 6*c)^2 + 9*a*b^2*sin(4
*d*x + 4*c)^2 + 64*a^3*sin(3*d*x + 3*c)^2 - 48*a^2*b*cos(3*d*x + 3*c)*sin(
2*d*x + 2*c) + 9*a*b^2*sin(2*d*x + 2*c)^2 - 6*a*b^2*cos(2*d*x + 2*c) + a*b
^2 - 2*(3*a*b^2*cos(4*d*x + 4*c) - 3*a*b^2*cos(2*d*x + 2*c) - 8*a^2*b*sin(
3*d*x + 3*c) + a*b^2)*cos(6*d*x + 6*c) - 6*(3*a*b^2*cos(2*d*x + 2*c) + 8*a
^2*b*sin(3*d*x + 3*c) - a*b^2)*cos(4*d*x + 4*c) - 2*(8*a^2*b*cos(3*d*x + 3
*c) + 3*a*b^2*sin(4*d*x + 4*c) - 3*a*b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c
) + 6*(8*a^2*b*cos(3*d*x + 3*c) - 3*a*b^2*sin(2*d*x + 2*c))*sin(4*d*x + 4*
c) + 16*(3*a^2*b*cos(2*d*x + 2*c) - a^2*b)*sin(3*d*x + 3*c)), x) - 2*sin(2
*d*x + 2*c))/(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 - 2*a*d*cos(
2*d*x + 2*c) + a*d)
```

3.193.8 Giac [F]

$$\int \frac{\csc^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\csc(dx+c)^2}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(csc(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

3.193.9 Mupad [B] (verification not implemented)

Time = 14.46 (sec) , antiderivative size = 697, normalized size of antiderivative = 2.48

$$\int \frac{\csc^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$= \frac{\left(\sum_{k=1}^{6} \ln\left(8192\,a^7\,b^6 - \mathrm{root}(729\,a^8\,b^2\,d^6 - 729\,a^{10}\,d^6 - 243\,a^6\,b^2\,d^4 - b^4, d, k\right)^2 a^9\,b^6\,294912 + \mathrm{root}(729\,a^8$$

input
```
int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x)^3)),x)
```

output
```
(symsum(log(8192*a^7*b^6 - 294912*root(729*a^8*b^2*d^6 - 729*a^10*d^6 - 24
3*a^6*b^2*d^4 - b^4, d, k)^2*a^9*b^6 + 1548288*root(729*a^8*b^2*d^6 - 729*
a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k)^3*a^11*b^5 - 1990656*root(729*a^8*
b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k)^4*a^13*b^4 - 7962624
*root(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k)^5*a^13
*b^5 + 5971968*root(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4
, d, k)^5*a^15*b^3 - 65536*a^6*b^7*tan(c/2 + (d*x)/2) + 196608*root(729*a^
8*b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k)*a^8*b^6*tan(c/2 +
(d*x)/2) - 294912*root(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 -
b^4, d, k)^2*a^10*b^5*tan(c/2 + (d*x)/2) - 1769472*root(729*a^8*b^2*d^6 -
729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k)^3*a^10*b^6*tan(c/2 + (d*x)/2)
+ 221184*root(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k
)^3*a^12*b^4*tan(c/2 + (d*x)/2) - 2654208*root(729*a^8*b^2*d^6 - 729*a^10*
d^6 - 243*a^6*b^2*d^4 - b^4, d, k)^4*a^12*b^5*tan(c/2 + (d*x)/2) - 1990656
*root(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k)^5*a^14
*b^4*tan(c/2 + (d*x)/2))*root(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2
*d^4 - b^4, d, k), k, 1, 6) - 1/(2*a*tan(c/2 + (d*x)/2)) + tan(c/2 + (d*x)
/2)/(2*a))/d
```

3.194 $\int \frac{\csc^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.194.1 Optimal result

Integrand size = 23, antiderivative size = 296

$$\int \frac{\csc^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{2b^{4/3}\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^2\sqrt{a^{2/3}-b^{2/3}}d}+\frac{b\,\text{arctanh}(\cos(c+dx))}{a^2d} - \frac{2b^{4/3}\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right)}{3a^2\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}d} - \frac{2b^{4/3}\text{arctanh}\left(\frac{\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3a^2\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}d} - \frac{\cot(c+dx)}{ad} - \frac{\cot^3(c+dx)}{3ad}$$

output
```
b*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a/d-1/3*cot(d*x+c)^3/a/d+2/3*b^(4/3
)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a^2
/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*b^(4/3)*arctanh((b^(1/3)+(-1)^(2/3)*a^(1/3)
*tan(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2))/a^2/d/((-1)^(1/3)
*a^(2/3)+b^(2/3))^(1/2)-2/3*b^(4/3)*arctanh((b^(1/3)-(-1)^(1/3)*a^(1/3)*ta
n(1/2*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2))/a^2/d/(-(-1)^(2/3)*
a^(2/3)+b^(2/3))^(1/2)
```

3.194.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.45 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.12

$$\int \frac{\csc^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$= \frac{-8a\cot\left(\frac{1}{2}(c+dx)\right)+24b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-24b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4ib^2\text{RootSum}\left[-b+3b\#1^{2}\right.}{}$$

input
```
Integrate[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

output
```
(-8*a*Cot[(c + d*x)/2] + 24*b*Log[Cos[(c + d*x)/2]] - 24*b*Log[Sin[(c + d*
x)/2]] + (4*I)*b^2*RootSum[-b + 3*b*#1^2 - (8*I)*a*#1^3 - 3*b*#1^4 + b*#1^
6 & , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*
x]*#1 + #1^2] - 4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (2*I)*Lo
g[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
 - #1)]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#
1^2 - 2*b*#1^3 + b*#1^5) & ] + 8*a*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - (a*
Csc[(c + d*x)/2]^4*Sin[c + d*x])/2 + 8*a*Tan[(c + d*x)/2])/(24*a^2*d)
```

3.194.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(c+dx)^4\left(a+b\sin(c+dx)^3\right)}dx$$

$$\Big\downarrow \text{ 3699}$$

$$\int \left(\frac{b^2\sin^2(c+dx)}{a^2\left(a+b\sin^3(c+dx)\right)}-\frac{b\csc(c+dx)}{a^2}+\frac{\csc^4(c+dx)}{a}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\text{arctanh}(\cos(c+dx))}{a^2d}+\frac{2b^{4/3}\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^2d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2b^{4/3}\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3a^2d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}-\frac{2b^{4/3}\text{arctanh}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3a^2d\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}-\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}$$

input
```
Int[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

output
```
(2*b^(4/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2
/3)]])/(3*a^2*Sqrt[a^(2/3) - b^(2/3)]*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*
d) - (2*b^(4/3)*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sq
rt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*a^2*Sqrt[-((-1)^(2/3)*a^(2/3)) +
b^(2/3)]*d) - (2*b^(4/3)*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*
x)/2])/Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*a^2*Sqrt[(-1)^(1/3)*a^(2/3)
 + b^(2/3)]*d) - Cot[c + d*x]/(a*d) - Cot[c + d*x]^3/(3*a*d)
```

3.194.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3699
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.194.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.55

method	result
derivativedivides	$\frac{\frac{\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3}+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{1}{24a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{3}{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{4b^2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4\cdots\right.}\right.}{\cdots}}{d}$
default	$\frac{\frac{\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3}+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{1}{24a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{3}{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{4b^2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4\cdots\right.}\right.}{\cdots}}{d}$
risch	$\frac{4i\left(3\,\mathrm{e}^{2i(dx+c)}-1\right)}{3da\left(\mathrm{e}^{2i(dx+c)}-1\right)^3}-\frac{b\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{a^2d}+16\left(\sum_{\_R=\mathrm{RootOf}\left(\left(12230590464a^{14}d^6-12230590464a^{12}b^2d^6\right)\_Z^6+1592\cdots\right.}\right.$

input
```
int(csc(d*x+c)^4/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8/a*(1/3*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c))-1/24/a/tan(1/2*
d*x+1/2*c)^3-3/8/a/tan(1/2*d*x+1/2*c)-1/a^2*b*ln(tan(1/2*d*x+1/2*c))+4/3*b
^2/a^2*sum(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),
_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a)))
```

3.194.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.05 (sec) , antiderivative size = 29423, normalized size of antiderivative = 99.40

$$\int \frac{\csc^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.194.6 Sympy [F]

$$\int \frac{\csc^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\csc^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**4/(a+b*sin(d*x+c)**3),x)
```

output
```
Integral(csc(c + d*x)**4/(a + b*sin(c + d*x)**3), x)
```

3.194.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.194.8 Giac [F]

$$\int \frac{\csc^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\csc(dx+c)^4}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.194.9 Mupad [B] (verification not implemented)

Time = 16.27 (sec) , antiderivative size = 1503, normalized size of antiderivative = 5.08

$$\int \frac{\csc^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(c + d*x)^4*(a + b*sin(c + d*x)^3)),x)
```

output
```
symsum(log((98304*b^11 + 589824*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243
*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)*a^2*b^10 - 98304*root(729*a^12*
b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^2*a
^4*b^9 - 5898240*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 +
27*a^4*b^6*z^2 - b^8, z, k)^3*a^6*b^8 - 7962624*root(729*a^12*b^2*z^6 - 72
9*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^4*a^8*b^7 - 663
552*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^
2 - b^8, z, k)^4*a^10*b^5 + 5308416*root(729*a^12*b^2*z^6 - 729*a^14*z^6 -
 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^5*a^10*b^6 - 10616832*root(
729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8,
z, k)^5*a^12*b^4 + 7962624*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*
b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^6*a^12*b^5 - 9953280*root(729*a^12*b
^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^6*a^
14*b^3 + 24576*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27
*a^4*b^6*z^2 - b^8, z, k)*a^3*b^9*tan(c/2 + (d*x)/2) - 3145728*root(729*a^
12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^
2*a^3*b^10*tan(c/2 + (d*x)/2) + 466944*root(729*a^12*b^2*z^6 - 729*a^14*z^
6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^2*a^5*b^8*tan(c/2 + (d*x
)/2) + 18874368*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 2
7*a^4*b^6*z^2 - b^8, z, k)^3*a^5*b^9*tan(c/2 + (d*x)/2) + 3981312*root(...
```

3.195 $\int \frac{\sin^9(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.195.1 Optimal result

Integrand size = 24, antiderivative size = 177

$$\int \frac{\sin^9(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{a^{3/2}\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}b^{9/4}d} - \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}b^{9/4}d} + \frac{(a+b)\cos(c+dx)}{b^2d} - \frac{2\cos^3(c+dx)}{3bd} + \frac{\cos^5(c+dx)}{5bd}$$

output
```
(a+b)*cos(d*x+c)/b^2/d-2/3*cos(d*x+c)^3/b/d+1/5*cos(d*x+c)^5/b/d-1/2*a^(3/
2)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/b^(9/4)/d/(a^(1/2)-b
^(1/2))^(1/2)-1/2*a^(3/2)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/
2))/b^(9/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

3.195.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.67 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.29

$$\int \frac{\sin^9(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$= \frac{\cos(c+dx)(120a+89b-28b\cos(2(c+dx))+3b\cos(4(c+dx)))+60ia^2\text{RootSum}\left[b-4b\#1^2-16a\#1\right.}{}$$

input
```
Integrate[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4),x]
```

output
```
(Cos[c + d*x]*(120*a + 89*b - 28*b*Cos[2*(c + d*x)] + 3*b*Cos[4*(c + d*x)]
) + (60*I)*a^2*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*
#1^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + I*Log[1 - 2*Cos
[c + d*x]*#1 + #1^2]*#1 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3
- I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*
b*#1^4 + b*#1^6) & ])/(120*b^2*d)
```

3.195.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3694, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^9(c+dx)}{a-b\sin^4(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^9}{a-b\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{3694}$$

$$-\frac{\int \frac{\left(1-\cos^2(c+dx)\right)^4}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{1484}$$

$$-\frac{\int \left(-\frac{\cos^4(c+dx)}{b}+\frac{2\cos^2(c+dx)}{b}-\frac{a+b}{b^2}+\frac{a^2}{b^2\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)}\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{a^{3/2}\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{9/4}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{9/4}\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(a+b)\cos(c+dx)}{b^2}-\frac{\cos^5(c+dx)}{5b}+\frac{2\cos^3(c+dx)}{3b}}{d}$$

input
```
Int[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4),x]
```

output
```
-(((a^(3/2)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqr
t[Sqrt[a] - Sqrt[b]]*b^(9/4)) + (a^(3/2)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sq
rt[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(9/4)) - ((a + b)*Cos
[c + d*x])/b^2 + (2*Cos[c + d*x]^3)/(3*b) - Cos[c + d*x]^5/(5*b))/d)
```

3.195.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.195.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{\frac{\frac{b\left(\cos^5(dx+c)\right)}{5}-\frac{2b\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)a+\cos(dx+c)b}{b^2}+\frac{a^2\left(-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{b}}{d}$
default	$\frac{\frac{\frac{b\left(\cos^5(dx+c)\right)}{5}-\frac{2b\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)a+\cos(dx+c)b}{b^2}+\frac{a^2\left(-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{b}}{d}$
risch	$\frac{a\,e^{i(dx+c)}}{2d\,b^2}+\frac{5\,e^{i(dx+c)}}{16bd}+\frac{e^{-i(dx+c)}a}{2b^2d}+\frac{5\,e^{-i(dx+c)}}{16bd}-\frac{i\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a\,b^9d^4-b^{10}d^4\right)\_Z^4-32768a^3b^5d^2\_Z^2-2684\right.}\right.}{}$

input
```
int(sin(d*x+c)^9/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^2*(1/5*b*cos(d*x+c)^5-2/3*b*cos(d*x+c)^3+cos(d*x+c)*a+cos(d*x+c)*
b)+a^2/b*(-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/
(((a*b)^(1/2)+b)*b)^(1/2))-1/2/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arcta
n(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))))
```

3.195.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 872 vs. 2(133) = 266.

Time = 0.34 (sec) , antiderivative size = 872, normalized size of antiderivative = 4.93

$$\int \frac{\sin^9(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$= \frac{12\,b\cos(dx+c)^5-15\,b^2d\sqrt{-\frac{(ab^4-b^5)\sqrt{\frac{a^7}{(a^2b^9-2\,ab^{10}+b^{11})d^4}}d^2+a^3}{(ab^4-b^5)d^2}}\log\left(a^5\cos(dx+c)+\left(a^4b^2d-(ab^7-b^8)\sqrt{\right.\right.}{}$$

input
```
integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/60*(12*b*cos(d*x + c)^5 - 15*b^2*d*sqrt(-((a*b^4 - b^5)*sqrt(a^7/((a^2*b
^9 - 2*a*b^10 + b^11)*d^4))*d^2 + a^3)/((a*b^4 - b^5)*d^2))*log(a^5*cos(d*
x + c) + (a^4*b^2*d - (a*b^7 - b^8)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)*
d^4))*d^3)*sqrt(-((a*b^4 - b^5)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)*d^4)
)*d^2 + a^3)/((a*b^4 - b^5)*d^2))) + 15*b^2*d*sqrt(((a*b^4 - b^5)*sqrt(a^7
/((a^2*b^9 - 2*a*b^10 + b^11)*d^4))*d^2 - a^3)/((a*b^4 - b^5)*d^2))*log(a^
5*cos(d*x + c) - (a^4*b^2*d + (a*b^7 - b^8)*sqrt(a^7/((a^2*b^9 - 2*a*b^10
+ b^11)*d^4))*d^3)*sqrt(((a*b^4 - b^5)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^1
1)*d^4))*d^2 - a^3)/((a*b^4 - b^5)*d^2))) + 15*b^2*d*sqrt(-((a*b^4 - b^5)*
sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)*d^4))*d^2 + a^3)/((a*b^4 - b^5)*d^2)
)*log(-a^5*cos(d*x + c) + (a^4*b^2*d - (a*b^7 - b^8)*sqrt(a^7/((a^2*b^9 -
2*a*b^10 + b^11)*d^4))*d^3)*sqrt(-((a*b^4 - b^5)*sqrt(a^7/((a^2*b^9 - 2*a*
b^10 + b^11)*d^4))*d^2 + a^3)/((a*b^4 - b^5)*d^2))) - 15*b^2*d*sqrt(((a*b^
4 - b^5)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)*d^4))*d^2 - a^3)/((a*b^4 -
b^5)*d^2))*log(-a^5*cos(d*x + c) - (a^4*b^2*d + (a*b^7 - b^8)*sqrt(a^7/((a
^2*b^9 - 2*a*b^10 + b^11)*d^4))*d^3)*sqrt(((a*b^4 - b^5)*sqrt(a^7/((a^2*b^
9 - 2*a*b^10 + b^11)*d^4))*d^2 - a^3)/((a*b^4 - b^5)*d^2))) - 40*b*cos(d*x
 + c)^3 + 60*(a + b)*cos(d*x + c))/(b^2*d)
```

3.195.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^9(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**9/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.195.7 Maxima [F]

$$\int \frac{\sin^9(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\sin(dx+c)^9}{b\sin(dx+c)^4-a}\,dx$$

input
```
integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/240*(240*b^2*d*integrate(8*(4*a^2*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) +
2*(8*a^3 - 3*a^2*b)*cos(3*d*x + 3*c)*sin(4*d*x + 4*c) - 2*(8*a^3 - 3*a^2*b
)*cos(4*d*x + 4*c)*sin(3*d*x + 3*c) - (a^2*b*sin(5*d*x + 5*c) - a^2*b*sin(
3*d*x + 3*c))*cos(8*d*x + 8*c) + 4*(a^2*b*sin(5*d*x + 5*c) - a^2*b*sin(3*d
*x + 3*c))*cos(6*d*x + 6*c) - 2*(2*a^2*b*sin(2*d*x + 2*c) + (8*a^3 - 3*a^2
*b)*sin(4*d*x + 4*c))*cos(5*d*x + 5*c) + (a^2*b*cos(5*d*x + 5*c) - a^2*b*c
os(3*d*x + 3*c))*sin(8*d*x + 8*c) - 4*(a^2*b*cos(5*d*x + 5*c) - a^2*b*cos(
3*d*x + 3*c))*sin(6*d*x + 6*c) + (4*a^2*b*cos(2*d*x + 2*c) - a^2*b + 2*(8*
a^3 - 3*a^2*b)*cos(4*d*x + 4*c))*sin(5*d*x + 5*c) - (4*a^2*b*cos(2*d*x + 2
*c) - a^2*b)*sin(3*d*x + 3*c))/(b^4*cos(8*d*x + 8*c)^2 + 16*b^4*cos(6*d*x
+ 6*c)^2 + 16*b^4*cos(2*d*x + 2*c)^2 + b^4*sin(8*d*x + 8*c)^2 + 16*b^4*sin
(6*d*x + 6*c)^2 + 16*b^4*sin(2*d*x + 2*c)^2 - 8*b^4*cos(2*d*x + 2*c) + b^4
 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*b^4)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b^2 -
48*a*b^3 + 9*b^4)*sin(4*d*x + 4*c)^2 + 16*(8*a*b^3 - 3*b^4)*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) - 2*(4*b^4*cos(6*d*x + 6*c) + 4*b^4*cos(2*d*x + 2*c) -
 b^4 + 2*(8*a*b^3 - 3*b^4)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*b^4*c
os(2*d*x + 2*c) - b^4 + 2*(8*a*b^3 - 3*b^4)*cos(4*d*x + 4*c))*cos(6*d*x +
6*c) - 4*(8*a*b^3 - 3*b^4 - 4*(8*a*b^3 - 3*b^4)*cos(2*d*x + 2*c))*cos(4*d*
x + 4*c) - 4*(2*b^4*sin(6*d*x + 6*c) + 2*b^4*sin(2*d*x + 2*c) + (8*a*b^3 -
 3*b^4)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*b^4*sin(2*d*x + 2*c)...
```

3.195.8 Giac [F]

$$\int \frac{\sin^9(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\sin(dx+c)^9}{b\sin(dx+c)^4-a}\,dx$$

input
```
integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

3.195.9 Mupad [B] (verification not implemented)

Time = 14.43 (sec) , antiderivative size = 1067, normalized size of antiderivative = 6.03

$$\int \frac{\sin^9(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\cos(c+d\,x)^5}{5\,b\,d} - \frac{2\cos(c+d\,x)^3}{3\,b\,d} + \frac{\cos(c+d\,x)\,\left(\frac{a-b}{b^2}+\frac{2}{b}\right)}{d}$$

$$+\frac{\mathrm{atan}\left(-\frac{a^4\,\cos(c+d\,x)\,\sqrt{-\frac{\sqrt{a^7\,b^9}}{16\,\left(a\,b^9-b^{10}\right)}-\frac{a^3\,b^5}{16\,\left(a\,b^9-b^{10}\right)}}\,8\mathrm{i}}{\frac{2\,a^6\,b^7}{a\,b^9-b^{10}}+\frac{2\,a^3\,b^2\,\sqrt{a^7\,b^9}}{a\,b^9-b^{10}}} + \frac{a^4\,b^9\,\cos(c+d\,x)\,\sqrt{-\frac{\sqrt{a^7\,b^9}}{16\,\left(a\,b^9-b^{10}\right)}-\frac{a^3\,b^5}{16\,\left(a\,b^9-b^{10}\right)}}\,8\mathrm{i}}{\frac{2\,a^6\,b^{16}}{a\,b^9-b^{10}}-\frac{2\,a^7\,b^{15}}{a\,b^9-b^{10}}+\frac{2\,a^3\,b^{11}\,\sqrt{a^7\,b^9}}{a\,b^9-b^{10}}-\frac{2\,a^4\,b^{10}\,\sqrt{a^7\,b^9}}{a\,b^9-b^{10}}} + \frac{a\,b^4\,\cos(c+d\,x)\,\sqrt{\cdots}}{\frac{2\,a^6\,b^{16}}{a\,b^9-b^{10}}-\frac{2\,a^7}{a\,b^9}\cdots}\right.}{d}$$

$$-\frac{\mathrm{atan}\left(\frac{a^4\,\cos(c+d\,x)\,\sqrt{\frac{\sqrt{a^7\,b^9}}{16\,\left(a\,b^9-b^{10}\right)}-\frac{a^3\,b^5}{16\,\left(a\,b^9-b^{10}\right)}}\,8\mathrm{i}}{\frac{2\,a^6\,b^7}{a\,b^9-b^{10}}-\frac{2\,a^3\,b^2\,\sqrt{a^7\,b^9}}{a\,b^9-b^{10}}} - \frac{a^4\,b^9\,\cos(c+d\,x)\,\sqrt{\frac{\sqrt{a^7\,b^9}}{16\,\left(a\,b^9-b^{10}\right)}-\frac{a^3\,b^5}{16\,\left(a\,b^9-b^{10}\right)}}\,8\mathrm{i}}{\frac{2\,a^6\,b^{16}}{a\,b^9-b^{10}}-\frac{2\,a^7\,b^{15}}{a\,b^9-b^{10}}-\frac{2\,a^3\,b^{11}\,\sqrt{a^7\,b^9}}{a\,b^9-b^{10}}+\frac{2\,a^4\,b^{10}\,\sqrt{a^7\,b^9}}{a\,b^9-b^{10}}} + \frac{a\,b^4\,\cos(c+d\,x)\,\sqrt{\frac{\cdots}{16\,(\cdots}}}{\frac{2\,a^6\,b^{16}}{a\,b^9-b^{10}}-\frac{2\,a^7\,b^{15}}{a\,b^9-b^{10}}\cdots}\right.}{d}$$

input
```
int(sin(c + d*x)^9/(a - b*sin(c + d*x)^4),x)
```

output
```
(atan((a^4*b^9*cos(c + d*x)*(- (a^7*b^9)^(1/2)/(16*(a*b^9 - b^10)) - (a^3*
b^5)/(16*(a*b^9 - b^10)))^(1/2)*8i)/((2*a^6*b^16)/(a*b^9 - b^10) - (2*a^7*
b^15)/(a*b^9 - b^10) + (2*a^3*b^11*(a^7*b^9)^(1/2))/(a*b^9 - b^10) - (2*a^
4*b^10*(a^7*b^9)^(1/2))/(a*b^9 - b^10)) - (a^4*cos(c + d*x)*(- (a^7*b^9)^(
1/2)/(16*(a*b^9 - b^10)) - (a^3*b^5)/(16*(a*b^9 - b^10)))^(1/2)*8i)/((2*a^
6*b^7)/(a*b^9 - b^10) + (2*a^3*b^2*(a^7*b^9)^(1/2))/(a*b^9 - b^10)) + (a*b
^4*cos(c + d*x)*(- (a^7*b^9)^(1/2)/(16*(a*b^9 - b^10)) - (a^3*b^5)/(16*(a*
b^9 - b^10)))^(1/2)*(a^7*b^9)^(1/2)*8i)/((2*a^6*b^16)/(a*b^9 - b^10) - (2*
a^7*b^15)/(a*b^9 - b^10) + (2*a^3*b^11*(a^7*b^9)^(1/2))/(a*b^9 - b^10) - (
2*a^4*b^10*(a^7*b^9)^(1/2))/(a*b^9 - b^10)))*(-((a^7*b^9)^(1/2) + a^3*b^5)
/(16*(a*b^9 - b^10)))^(1/2)*2i)/d - (atan((a^4*cos(c + d*x)*((a^7*b^9)^(1/
2)/(16*(a*b^9 - b^10)) - (a^3*b^5)/(16*(a*b^9 - b^10)))^(1/2)*8i)/((2*a^6*
b^7)/(a*b^9 - b^10) - (2*a^3*b^2*(a^7*b^9)^(1/2))/(a*b^9 - b^10)) - (a^4*b
^9*cos(c + d*x)*((a^7*b^9)^(1/2)/(16*(a*b^9 - b^10)) - (a^3*b^5)/(16*(a*b^
9 - b^10)))^(1/2)*8i)/((2*a^6*b^16)/(a*b^9 - b^10) - (2*a^7*b^15)/(a*b^9 -
 b^10) - (2*a^3*b^11*(a^7*b^9)^(1/2))/(a*b^9 - b^10) + (2*a^4*b^10*(a^7*b^
9)^(1/2))/(a*b^9 - b^10)) + (a*b^4*cos(c + d*x)*((a^7*b^9)^(1/2)/(16*(a*b^
9 - b^10)) - (a^3*b^5)/(16*(a*b^9 - b^10)))^(1/2)*(a^7*b^9)^(1/2)*8i)/((2*
a^6*b^16)/(a*b^9 - b^10) - (2*a^7*b^15)/(a*b^9 - b^10) - (2*a^3*b^11*(a^7*
b^9)^(1/2))/(a*b^9 - b^10) + (2*a^4*b^10*(a^7*b^9)^(1/2))/(a*b^9 - b^10...
```

3.196 $\int \frac{\sin^7(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.196.1 Optimal result

Integrand size = 24, antiderivative size = 148

$$\int \frac{\sin^7(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{a\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}b^{7/4}d}+\frac{a\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}b^{7/4}d}+\frac{\cos(c+dx)}{bd}-\frac{\cos^3(c+dx)}{3bd}$$

output
```
cos(d*x+c)/b/d-1/3*cos(d*x+c)^3/b/d-1/2*a*arctan(b^(1/4)*cos(d*x+c)/(a^(1/
2)-b^(1/2))^(1/2))/b^(7/4)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*a*arctanh(b^(1/4)
*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/b^(7/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

3.196.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.30 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.09

$$\int \frac{\sin^7(c+dx)}{a-b\sin^4(c+dx)}\,dx$$
$$= \frac{18\cos(c+dx)-2\cos(3(c+dx))-3ia\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-2\text{ar}}{}\right.}{}$$

input
```
Integrate[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4),x]
```

output
```
(18*Cos[c + d*x] - 2*Cos[3*(c + d*x)] - (3*I)*a*RootSum[b - 4*b*#1^2 - 16*
a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c +
 d*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 6*ArcTan[Sin[c + d*x]
/(Cos[c + d*x] - #1)]*#1^2 - (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2
- 6*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (3*I)*Log[1 - 2*Cos[c
+ d*x]*#1 + #1^2]*#1^4 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 -
 I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3
- 3*b*#1^5 + b*#1^7) & ])/(24*b*d)
```

3.196.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3694, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(c+dx)}{a-b\sin^4(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^7}{a-b\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{3694}$$

$$-\frac{\int \frac{\left(1-\cos^2(c+dx)\right)^3}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{1484}$$

$$-\frac{\int \left(\frac{\cos^2(c+dx)}{b}-\frac{1}{b}+\frac{a-a\cos^2(c+dx)}{b\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)}\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{a\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{7/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{a\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{7/4}\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\cos^3(c+dx)}{3b}-\frac{\cos(c+dx)}{b}}{d}$$

input
```
Int[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4),x]
```

output
```
-(((a*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt
[a] - Sqrt[b]]*b^(7/4)) - (a*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] +
 Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(7/4)) - Cos[c + d*x]/b + Cos[c +
 d*x]^3/(3*b))/d)
```

3.196.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.196.4 Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{-\frac{\frac{\left(\cos^3(dx+c)\right)}{3}-\cos(dx+c)}{b}-a\left(\frac{\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}-b\right)b}}-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{d}$
default	$\frac{-\frac{\frac{\left(\cos^3(dx+c)\right)}{3}-\cos(dx+c)}{b}-a\left(\frac{\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}-b\right)b}}-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{d}$
risch	$\frac{3\,e^{i(dx+c)}}{8bd}+\frac{3\,e^{-i(dx+c)}}{8bd}+\frac{i\left(\sum_{\_R=\text{RootOf}\left(\left(a\,b^7d^4-b^8d^4\right)\_Z^4-2048a^2b^4d^2\_Z^2-1048576a^4\right)}\_R\ln\left(e^{2i(dx+c)}+\left(\left(-\right.\right.\right.\right.}{128}$

input
```
int(sin(d*x+c)^7/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b*(1/3*cos(d*x+c)^3-cos(d*x+c))-a*(1/2/b/(((a*b)^(1/2)-b)*b)^(1/2)
*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/2/b/(((a*b)^(1/2)+b)*b)^
(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))))
```

3.196.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 849 vs. 2(110) = 220.

Time = 0.34 (sec) , antiderivative size = 849, normalized size of antiderivative = 5.74

$$\int \frac{\sin^7(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$=\frac{3\,bd\sqrt{-\frac{(ab^3-b^4)d^2\sqrt{\frac{a^5}{\left(a^2b^7-2\,ab^8+b^9\right)d^4}}+a^2}{(ab^3-b^4)d^2}}\log\left(a^3\cos(dx+c)+\left(a^2b^2d-(ab^5-b^6)d^3\sqrt{\frac{a^5}{(a^2b^7-2\,ab^8+b^9)d^4}}\right)\sqrt{-\frac{(}{}}\right.}{}$$

input
```
integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/12*(3*b*d*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d
^4)) + a^2)/((a*b^3 - b^4)*d^2))*log(a^3*cos(d*x + c) + (a^2*b^2*d - (a*b^
5 - b^6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(-((a*b^3 - b^
4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b^3 - b^4)*d^2
))) - 3*b*d*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^
4)) - a^2)/((a*b^3 - b^4)*d^2))*log(a^3*cos(d*x + c) - (a^2*b^2*d + (a*b^5
 - b^6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(((a*b^3 - b^4)
*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))
) - 3*b*d*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4
)) + a^2)/((a*b^3 - b^4)*d^2))*log(-a^3*cos(d*x + c) + (a^2*b^2*d - (a*b^5
 - b^6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(-((a*b^3 - b^4
)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b^3 - b^4)*d^2)
)) + 3*b*d*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4
)) - a^2)/((a*b^3 - b^4)*d^2))*log(-a^3*cos(d*x + c) - (a^2*b^2*d + (a*b^5
 - b^6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(((a*b^3 - b^4)
*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))
) - 4*cos(d*x + c)^3 + 12*cos(d*x + c))/(b*d)
```

3.196.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**7/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.196.7 Maxima [F]

$$\int \frac{\sin^7(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\sin(dx+c)^7}{b\sin(dx+c)^4-a}\,dx$$

input
```
integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
-1/12*(12*b*d*integrate(-2*(12*a*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) - 4*a
*b*cos(d*x + c)*sin(2*d*x + 2*c) + 4*a*b*cos(2*d*x + 2*c)*sin(d*x + c) - a
*b*sin(d*x + c) + (a*b*sin(7*d*x + 7*c) - 3*a*b*sin(5*d*x + 5*c) + 3*a*b*s
in(3*d*x + 3*c) - a*b*sin(d*x + c))*cos(8*d*x + 8*c) + 2*(2*a*b*sin(6*d*x
+ 6*c) + 2*a*b*sin(2*d*x + 2*c) + (8*a^2 - 3*a*b)*sin(4*d*x + 4*c))*cos(7*
d*x + 7*c) + 4*(3*a*b*sin(5*d*x + 5*c) - 3*a*b*sin(3*d*x + 3*c) + a*b*sin(
d*x + c))*cos(6*d*x + 6*c) - 6*(2*a*b*sin(2*d*x + 2*c) + (8*a^2 - 3*a*b)*s
in(4*d*x + 4*c))*cos(5*d*x + 5*c) - 2*(3*(8*a^2 - 3*a*b)*sin(3*d*x + 3*c)
- (8*a^2 - 3*a*b)*sin(d*x + c))*cos(4*d*x + 4*c) - (a*b*cos(7*d*x + 7*c) -
 3*a*b*cos(5*d*x + 5*c) + 3*a*b*cos(3*d*x + 3*c) - a*b*cos(d*x + c))*sin(8
*d*x + 8*c) - (4*a*b*cos(6*d*x + 6*c) + 4*a*b*cos(2*d*x + 2*c) - a*b + 2*(
8*a^2 - 3*a*b)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) - 4*(3*a*b*cos(5*d*x + 5
*c) - 3*a*b*cos(3*d*x + 3*c) + a*b*cos(d*x + c))*sin(6*d*x + 6*c) + 3*(4*a
*b*cos(2*d*x + 2*c) - a*b + 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c))*sin(5*d*x
+ 5*c) + 2*(3*(8*a^2 - 3*a*b)*cos(3*d*x + 3*c) - (8*a^2 - 3*a*b)*cos(d*x +
 c))*sin(4*d*x + 4*c) - 3*(4*a*b*cos(2*d*x + 2*c) - a*b)*sin(3*d*x + 3*c))
/(b^3*cos(8*d*x + 8*c)^2 + 16*b^3*cos(6*d*x + 6*c)^2 + 16*b^3*cos(2*d*x +
2*c)^2 + b^3*sin(8*d*x + 8*c)^2 + 16*b^3*sin(6*d*x + 6*c)^2 + 16*b^3*sin(2
*d*x + 2*c)^2 - 8*b^3*cos(2*d*x + 2*c) + b^3 + 4*(64*a^2*b - 48*a*b^2 + 9*
b^3)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*sin(4*d*x + 4...
```

3.196.8 Giac [F]

$$\int \frac{\sin^7(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\sin{(dx+c)}^7}{b\sin{(dx+c)}^4-a}\,dx$$

input
```
integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

3.196.9 Mupad [B] (verification not implemented)

Time = 13.80 (sec) , antiderivative size = 1119, normalized size of antiderivative = 7.56

$$\int \frac{\sin^7(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\cos(c+d\,x)}{b\,d} - \frac{\cos(c+d\,x)^3}{3\,b\,d}$$
$$+ \frac{\operatorname{atan}\left(-\frac{a^3\cos(c+d\,x)\sqrt{-\frac{\sqrt{a^5\,b^7}}{16\,\left(a\,b^7-b^8\right)}-\frac{a^2\,b^4}{16\,\left(a\,b^7-b^8\right)}}\,8\mathrm{i}}{\frac{2\,a^4}{b^2}+\frac{2\,a^4\,b^6}{a\,b^7-b^8}+\frac{2\,a^2\,b^2\,\sqrt{a^5\,b^7}}{a\,b^7-b^8}} + \frac{a^3\,b^8\cos(c+d\,x)\sqrt{-\frac{\sqrt{a^5\,b^7}}{16\,\left(a\,b^7-b^8\right)}-\frac{a^2\,b^4}{16\,\left(a\,b^7-b^8\right)}}\,8\mathrm{i}}{2\,a^4\,b^6-2\,a^5\,b^5+\frac{2\,a^4\,b^{14}}{a\,b^7-b^8}-\frac{2\,a^5\,b^{13}}{a\,b^7-b^8}+\frac{2\,a^2\,b^{10}\,\sqrt{a^5\,b^7}}{a\,b^7-b^8}-\frac{2\,a^3\,b^9\,\sqrt{a^5\,b^7}}{a\,b^7-b^8}} + \frac{a\ldots}{2\,a^4\,b^6\ldots}\right)\ldots}{d}$$
$$- \frac{\operatorname{atan}\left(\frac{a^3\cos(c+d\,x)\sqrt{\frac{\sqrt{a^5\,b^7}}{16\,\left(a\,b^7-b^8\right)}-\frac{a^2\,b^4}{16\,\left(a\,b^7-b^8\right)}}\,8\mathrm{i}}{\frac{2\,a^4}{b^2}+\frac{2\,a^4\,b^6}{a\,b^7-b^8}-\frac{2\,a^2\,b^2\,\sqrt{a^5\,b^7}}{a\,b^7-b^8}} - \frac{a^3\,b^8\cos(c+d\,x)\sqrt{\frac{\sqrt{a^5\,b^7}}{16\,\left(a\,b^7-b^8\right)}-\frac{a^2\,b^4}{16\,\left(a\,b^7-b^8\right)}}\,8\mathrm{i}}{2\,a^4\,b^6-2\,a^5\,b^5+\frac{2\,a^4\,b^{14}}{a\,b^7-b^8}-\frac{2\,a^5\,b^{13}}{a\,b^7-b^8}-\frac{2\,a^2\,b^{10}\,\sqrt{a^5\,b^7}}{a\,b^7-b^8}+\frac{2\,a^3\,b^9\,\sqrt{a^5\,b^7}}{a\,b^7-b^8}} + \frac{a\,b^4\ldots}{2\,a^4\,b^6-2\ldots}\right)\ldots}{d}$$

input
```
int(sin(c + d*x)^7/(a - b*sin(c + d*x)^4),x)
```

output
```
cos(c + d*x)/(b*d) - cos(c + d*x)^3/(3*b*d) + (atan((a^3*b^8*cos(c + d*x)*
(- (a^5*b^7)^(1/2)/(16*(a*b^7 - b^8)) - (a^2*b^4)/(16*(a*b^7 - b^8)))^(1/2
)*8i)/(2*a^4*b^6 - 2*a^5*b^5 + (2*a^4*b^14)/(a*b^7 - b^8) - (2*a^5*b^13)/(
a*b^7 - b^8) + (2*a^2*b^10*(a^5*b^7)^(1/2))/(a*b^7 - b^8) - (2*a^3*b^9*(a^
5*b^7)^(1/2))/(a*b^7 - b^8)) - (a^3*cos(c + d*x)*(- (a^5*b^7)^(1/2)/(16*(a
*b^7 - b^8)) - (a^2*b^4)/(16*(a*b^7 - b^8)))^(1/2)*8i)/((2*a^4)/b^2 + (2*a
^4*b^6)/(a*b^7 - b^8) + (2*a^2*b^2*(a^5*b^7)^(1/2))/(a*b^7 - b^8)) + (a*b^
4*cos(c + d*x)*(- (a^5*b^7)^(1/2)/(16*(a*b^7 - b^8)) - (a^2*b^4)/(16*(a*b^
7 - b^8)))^(1/2)*(a^5*b^7)^(1/2)*8i)/(2*a^4*b^6 - 2*a^5*b^5 + (2*a^4*b^14)
/(a*b^7 - b^8) - (2*a^5*b^13)/(a*b^7 - b^8) + (2*a^2*b^10*(a^5*b^7)^(1/2))
/(a*b^7 - b^8) - (2*a^3*b^9*(a^5*b^7)^(1/2))/(a*b^7 - b^8)))*(-((a^5*b^7)^
(1/2) + a^2*b^4)/(16*(a*b^7 - b^8)))^(1/2)*2i)/d - (atan((a^3*cos(c + d*x)
*((a^5*b^7)^(1/2)/(16*(a*b^7 - b^8)) - (a^2*b^4)/(16*(a*b^7 - b^8)))^(1/2)
*8i)/((2*a^4)/b^2 + (2*a^4*b^6)/(a*b^7 - b^8) - (2*a^2*b^2*(a^5*b^7)^(1/2)
)/(a*b^7 - b^8)) - (a^3*b^8*cos(c + d*x)*((a^5*b^7)^(1/2)/(16*(a*b^7 - b^8
)) - (a^2*b^4)/(16*(a*b^7 - b^8)))^(1/2)*8i)/(2*a^4*b^6 - 2*a^5*b^5 + (2*a
^4*b^14)/(a*b^7 - b^8) - (2*a^5*b^13)/(a*b^7 - b^8) - (2*a^2*b^10*(a^5*b^7
)^(1/2))/(a*b^7 - b^8) + (2*a^3*b^9*(a^5*b^7)^(1/2))/(a*b^7 - b^8)) + (a*b
^4*cos(c + d*x)*((a^5*b^7)^(1/2)/(16*(a*b^7 - b^8)) - (a^2*b^4)/(16*(a*b^7
 - b^8)))^(1/2)*(a^5*b^7)^(1/2)*8i)/(2*a^4*b^6 - 2*a^5*b^5 + (2*a^4*b^1...
```

3.197 $\int \frac{\sin^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.197.1 Optimal result

Integrand size = 24, antiderivative size = 138

$$\int \frac{\sin^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}b^{5/4}d} - \frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}b^{5/4}d} + \frac{\cos(c+dx)}{bd}$$

output
```
cos(d*x+c)/b/d-1/2*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*a^(1
/2)/b^(5/4)/d/(a^(1/2)-b^(1/2))^(1/2)-1/2*arctanh(b^(1/4)*cos(d*x+c)/(a^(1
/2)+b^(1/2))^(1/2))*a^(1/2)/b^(5/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

3.197.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.82 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.43

$$\int \frac{\sin^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{2\cos(c+dx) + ia\text{RootSum}\left[b - 4b\#1^2 - 16a\#1^4 + 6b\#1^4 - 4b\#1^6 + b\#1^8\&, \frac{-2\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1+}{}\right.}{2bd}$$

input
```
Integrate[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

output
```
(2*Cos[c + d*x] + I*a*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1
^6 + b*#1^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + I*Log[1
- 2*Cos[c + d*x]*#1 + #1^2]*#1 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
]*#1^3 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1
^2 - 3*b*#1^4 + b*#1^6) & ])/(2*b*d)
```

3.197.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3694, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(c+dx)}{a-b\sin^4(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(c+dx)^5}{a-b\sin(c+dx)^4}dx$$

$$\Big\downarrow \ 3694$$

$$-\frac{\int \frac{\left(1-\cos^2(c+dx)\right)^2}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{d}$$

$$\Big\downarrow \ 1484$$

$$-\frac{\int \left(\frac{a}{b\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)}-\frac{1}{b}\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\frac{\sqrt{a}\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{5/4}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{5/4}\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cos(c+dx)}{b}}{d}$$

input
```
Int[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

output
```
-(((Sqrt[a]*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqr
t[Sqrt[a] - Sqrt[b]]*b^(5/4)) + (Sqrt[a]*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sq
rt[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(5/4)) - Cos[c + d*x]
/b)/d)
```

3.197.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.197.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)}{b}+a\left(-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{d}$
default	$\frac{\frac{\cos(dx+c)}{b}+a\left(-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{d}$
risch	$\frac{e^{i(dx+c)}}{2bd}+\frac{e^{-i(dx+c)}}{2bd}-\frac{i\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a\,b^{5}d^{4}-b^{6}d^{4}\right)\_Z^{4}-128a\,d^{2}\_Z^{2}b^{3}-4096a^{2}\right)}\_R\ln\left(e^{2i(dx+c)}+\left(\left(-\frac{ib^{4}d^{3}}{256a}+\frac{i}{2}\right.\right.\right.\right.}{32}$

input
```
int(sin(d*x+c)^5/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(cos(d*x+c)/b+a*(-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(co
s(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/2/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^
(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))))
```

3.197.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 815 vs. $2(98) = 196$.

Time = 0.36 (sec) , antiderivative size = 815, normalized size of antiderivative = 5.91

$$\int \frac{\sin^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{bd\sqrt{-\frac{(ab^2-b^3)d^2\sqrt{\frac{a^3}{\left(a^2b^5-2\,ab^6+b^7\right)d^4}}+a}{(ab^2-b^3)d^2}}\log\left(a^2\cos\left(dx+c\right)-\left((ab^4-b^5)d^3\sqrt{\frac{a^3}{(a^2b^5-2\,ab^6+b^7)d^4}}-a^2bd\right)\sqrt{-\frac{(a}{}}\right.}{}$$

input
```
integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/4*(b*d*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4
)) + a)/((a*b^2 - b^3)*d^2))*log(a^2*cos(d*x + c) - ((a*b^4 - b^5)*d^3*sqr
t(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2*b*d)*sqrt(-((a*b^2 - b^3)*d^2
*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2))) - b*
d*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a)/(
(a*b^2 - b^3)*d^2))*log(a^2*cos(d*x + c) - ((a*b^4 - b^5)*d^3*sqrt(a^3/((a
^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2*b*d)*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/
((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2))) - b*d*sqrt(-((
a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((a*b^2 -
b^3)*d^2))*log(-a^2*cos(d*x + c) - ((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 -
 2*a*b^6 + b^7)*d^4)) - a^2*b*d)*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b
^5 - 2*a*b^6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2))) + b*d*sqrt(((a*b^2 -
b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2
))*log(-a^2*cos(d*x + c) - ((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6
 + b^7)*d^4)) + a^2*b*d)*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*
b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2))) - 4*cos(d*x + c))/(b*d)
```

3.197.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**5/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.197.7 Maxima [F]

$$\int \frac{\sin^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\sin\left(dx+c\right)^5}{b\sin\left(dx+c\right)^4-a}\,dx$$

input
```
integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
(b*d*integrate(8*(4*a*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 2*(8*a^2 - 3*a
*b)*cos(3*d*x + 3*c)*sin(4*d*x + 4*c) - 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c)
*sin(3*d*x + 3*c) - (a*b*sin(5*d*x + 5*c) - a*b*sin(3*d*x + 3*c))*cos(8*d*
x + 8*c) + 4*(a*b*sin(5*d*x + 5*c) - a*b*sin(3*d*x + 3*c))*cos(6*d*x + 6*c
) - 2*(2*a*b*sin(2*d*x + 2*c) + (8*a^2 - 3*a*b)*sin(4*d*x + 4*c))*cos(5*d*
x + 5*c) + (a*b*cos(5*d*x + 5*c) - a*b*cos(3*d*x + 3*c))*sin(8*d*x + 8*c)
- 4*(a*b*cos(5*d*x + 5*c) - a*b*cos(3*d*x + 3*c))*sin(6*d*x + 6*c) + (4*a*
b*cos(2*d*x + 2*c) - a*b + 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c))*sin(5*d*x +
 5*c) - (4*a*b*cos(2*d*x + 2*c) - a*b)*sin(3*d*x + 3*c))/(b^3*cos(8*d*x +
8*c)^2 + 16*b^3*cos(6*d*x + 6*c)^2 + 16*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(8
*d*x + 8*c)^2 + 16*b^3*sin(6*d*x + 6*c)^2 + 16*b^3*sin(2*d*x + 2*c)^2 - 8*
b^3*cos(2*d*x + 2*c) + b^3 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*cos(4*d*x + 4
*c)^2 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a*b^2 -
 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^3*cos(6*d*x + 6*c) + 4*
b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(8*d
*x + 8*c) + 8*(4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*
x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^2 - 3*b^3 - 4*(8*a*b^2 - 3*b^3)*cos(
2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^3*sin(6*d*x + 6*c) + 2*b^3*sin(2*d
*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*b
^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(6*d*x + 6...
```

3.197.8 Giac [F]

$$\int \frac{\sin^5(c+dx)}{a-b\sin^4(c+dx)}\,dx=\int -\frac{\sin\left(dx+c\right)^5}{b\sin\left(dx+c\right)^4-a}\,dx$$

input
```
integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

3.197.9 Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 1001, normalized size of antiderivative = 7.25

$$\int \frac{\sin^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\cos(c+d\,x)}{b\,d} - \frac{2\,\mathrm{atanh}\left(\frac{8\,a^2\,b^7\,\cos(c+d\,x)\sqrt{-\frac{\sqrt{a^3\,b^5}}{16\,\left(a\,b^5-b^6\right)}-\frac{a\,b^3}{16\,\left(a\,b^5-b^6\right)}}}{\frac{2\,a^3\,b^{11}}{a\,b^5-b^6}-\frac{2\,a^4\,b^{10}}{a\,b^5-b^6}+\frac{2\,a^2\,b^8\,\sqrt{a^3\,b^5}}{a\,b^5-b^6}-\frac{2\,a^3\,b^7\,\sqrt{a^3\,b^5}}{a\,b^5-b^6}} - \frac{8\,a^2\,b\,\cos(c+d\,x)\sqrt{-\frac{\sqrt{a^3\,b^5}}{16\,\left(a\,b^5-b^6\right)}-\frac{a\,b^3}{16\,\left(a\,b^5-b^6\right)}}}{\frac{2\,a^3\,b^5}{a\,b^5-b^6}+\frac{2\,a^2\,b^2\,\sqrt{a^3\,b^5}}{a\,b^5-b^6}} + \frac{8\,a\,b^4\,\cos(c+d\,x)\sqrt{-\cdots}}{\frac{2\,a^3\,b^{11}}{a\,b^5-b^6}-\frac{2\,a^4\,b^{10}}{a\,b^5-b^6}\cdots}\cdots\right)\cdots}{d} + \frac{2\,\mathrm{atanh}\left(\frac{8\,a^2\,b\,\cos(c+d\,x)\sqrt{\frac{\sqrt{a^3\,b^5}}{16\,\left(a\,b^5-b^6\right)}-\frac{a\,b^3}{16\,\left(a\,b^5-b^6\right)}}}{\frac{2\,a^3\,b^5}{a\,b^5-b^6}-\frac{2\,a^2\,b^2\,\sqrt{a^3\,b^5}}{a\,b^5-b^6}} - \frac{8\,a^2\,b^7\,\cos(c+d\,x)\sqrt{\frac{\sqrt{a^3\,b^5}}{16\,\left(a\,b^5-b^6\right)}-\frac{a\,b^3}{16\,\left(a\,b^5-b^6\right)}}}{\frac{2\,a^3\,b^{11}}{a\,b^5-b^6}-\frac{2\,a^4\,b^{10}}{a\,b^5-b^6}-\frac{2\,a^2\,b^8\,\sqrt{a^3\,b^5}}{a\,b^5-b^6}+\frac{2\,a^3\,b^7\,\sqrt{a^3\,b^5}}{a\,b^5-b^6}} + \frac{8\,a\,b^4\,\cos(c+d\,x)\sqrt{\frac{\sqrt{\cdots}}{16\,(\cdots}}}{\frac{2\,a^3\,b^{11}}{a\,b^5-b^6}-\frac{2\,a^4\,b^{10}}{a\,b^5-b^6}-\cdots}\cdots\right)\cdots}{d}$$

input
```
int(sin(c + d*x)^5/(a - b*sin(c + d*x)^4),x)
```

output
```
cos(c + d*x)/(b*d) - (2*atanh((8*a^2*b^7*cos(c + d*x)*(- (a^3*b^5)^(1/2)/(
16*(a*b^5 - b^6)) - (a*b^3)/(16*(a*b^5 - b^6)))^(1/2))/((2*a^3*b^11)/(a*b^
5 - b^6) - (2*a^4*b^10)/(a*b^5 - b^6) + (2*a^2*b^8*(a^3*b^5)^(1/2))/(a*b^5
 - b^6) - (2*a^3*b^7*(a^3*b^5)^(1/2))/(a*b^5 - b^6)) - (8*a^2*b*cos(c + d*
x)*(- (a^3*b^5)^(1/2)/(16*(a*b^5 - b^6)) - (a*b^3)/(16*(a*b^5 - b^6)))^(1/
2))/((2*a^3*b^5)/(a*b^5 - b^6) + (2*a^2*b^2*(a^3*b^5)^(1/2))/(a*b^5 - b^6)
) + (8*a*b^4*cos(c + d*x)*(- (a^3*b^5)^(1/2)/(16*(a*b^5 - b^6)) - (a*b^3)/
(16*(a*b^5 - b^6)))^(1/2)*(a^3*b^5)^(1/2))/((2*a^3*b^11)/(a*b^5 - b^6) - (
2*a^4*b^10)/(a*b^5 - b^6) + (2*a^2*b^8*(a^3*b^5)^(1/2))/(a*b^5 - b^6) - (2
*a^3*b^7*(a^3*b^5)^(1/2))/(a*b^5 - b^6)))*(-((a^3*b^5)^(1/2) + a*b^3)/(16*
(a*b^5 - b^6)))^(1/2))/d + (2*atanh((8*a^2*b*cos(c + d*x)*((a^3*b^5)^(1/2)
/(16*(a*b^5 - b^6)) - (a*b^3)/(16*(a*b^5 - b^6)))^(1/2))/((2*a^3*b^5)/(a*b
^5 - b^6) - (2*a^2*b^2*(a^3*b^5)^(1/2))/(a*b^5 - b^6)) - (8*a^2*b^7*cos(c
+ d*x)*((a^3*b^5)^(1/2)/(16*(a*b^5 - b^6)) - (a*b^3)/(16*(a*b^5 - b^6)))^(
1/2))/((2*a^3*b^11)/(a*b^5 - b^6) - (2*a^4*b^10)/(a*b^5 - b^6) - (2*a^2*b^
8*(a^3*b^5)^(1/2))/(a*b^5 - b^6) + (2*a^3*b^7*(a^3*b^5)^(1/2))/(a*b^5 - b^
6)) + (8*a*b^4*cos(c + d*x)*((a^3*b^5)^(1/2)/(16*(a*b^5 - b^6)) - (a*b^3)/
(16*(a*b^5 - b^6)))^(1/2)*(a^3*b^5)^(1/2))/((2*a^3*b^11)/(a*b^5 - b^6) - (
2*a^4*b^10)/(a*b^5 - b^6) - (2*a^2*b^8*(a^3*b^5)^(1/2))/(a*b^5 - b^6) + (2
*a^3*b^7*(a^3*b^5)^(1/2))/(a*b^5 - b^6)))*(((a^3*b^5)^(1/2) - a*b^3)/(1...
```

3.198 $\int \frac{\sin^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.198.1 Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{\sin^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{3/4}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{3/4}d}$$

output
```
-1/2*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/b^(3/4)/d/(a^(1/2)
-b^(1/2))^(1/2)+1/2*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/b^
(3/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

3.198.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.61 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.48

$$\int \frac{\sin^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{i\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-2\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\log\left(1-2\cos(c+dx)\#1\right.}{}\right.}{}$$

input
```
Integrate[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

output
```
((-1/8*I)*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8
& , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos[c + d*x
]*#1 + #1^2] + 6*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (3*I)*Log
[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 6*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
- #1)]*#1^4 + (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 2*ArcTan[Sin[
c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#
1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/d
```

3.198.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 3694, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{a-b\sin^4(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(c+dx)^3}{a-b\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{ 3694}$$

$$-\frac{\int \frac{1-\cos^2(c+dx)}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{ 1480}$$

$$-\frac{-\frac{1}{2}\int \frac{1}{-b\cos^2(c+dx)-\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}d\cos(c+dx)-\frac{1}{2}\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{1}{2}\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}}{d}$$

input
```
Int[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

output
```
-((ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[Sqrt[a]
- Sqrt[b]]*b^(3/4)) - ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b
]]]/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/4)))/d)
```

3.198.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.198.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{b\left(-\frac{\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}-b\right)b}}+\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{d}$
default	$\frac{b\left(-\frac{\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}-b\right)b}}+\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{d}$
risch	$\frac{i\left(\sum_{\_R=\operatorname{RootOf}\left(-16+\left(a\,b^{3}d^{4}-b^{4}d^{4}\right)\_Z^{4}-8b^{2}d^{2}\_Z^{2}\right)}\_R\ln\left(\mathrm{e}^{2i(dx+c)}+\left(\left(-\frac{1}{4}ia\,b^{2}d^{3}+\frac{1}{4}ib^{3}d^{3}\right)\_R^{3}+2idb\_R\right)\mathrm{e}^{i(dx+\right.\right.}{8}$

input
```
int(sin(d*x+c)^3/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*b*(-1/2/b/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-
b)*b)^(1/2))+1/2/b/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^
(1/2)+b)*b)^(1/2)))
```

3.198.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 703 vs. 2(79) = 158.

Time = 0.33 (sec) , antiderivative size = 703, normalized size of antiderivative = 6.11

$$\int \frac{\sin^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$=\frac{1}{4}\sqrt{-\frac{(ab-b^2)d^2\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}+1}{(ab-b^2)d^2}}\log\left(-\left((ab^2-b^3)d^3\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}-bd\right)\sqrt{-\frac{(ab-b^2)}{}}\right.$$
$$\left.+\cos(dx+c)\right)$$
$$-\frac{1}{4}\sqrt{-\frac{(ab-b^2)d^2\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}+1}{(ab-b^2)d^2}}\log\left(-\left((ab^2-b^3)d^3\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}-bd\right)\sqrt{-\frac{(ab-}{}}\right.$$
$$\left.-\cos(dx+c)\right)$$
$$-\frac{1}{4}\sqrt{\frac{(ab-b^2)d^2\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}-1}{(ab-b^2)d^2}}\log\left(-\left((ab^2-b^3)d^3\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}+bd\right)\sqrt{\frac{(ab-b^2)d}{}}\right.$$
$$\left.+\cos(dx+c)\right)$$
$$+\frac{1}{4}\sqrt{\frac{(ab-b^2)d^2\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}-1}{(ab-b^2)d^2}}\log\left(-\left((ab^2-b^3)d^3\sqrt{\frac{a}{(a^2b^3-2\,ab^4+b^5)d^4}}+bd\right)\sqrt{\frac{(ab-b^2)d}{}}\right.$$
$$\left.-\cos(dx+c)\right)$$

input
```
integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

3.198. $\int \frac{\sin^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

output
```
1/4*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((
a*b - b^2)*d^2))*log(-((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)
*d^4)) - b*d)*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4
)) + 1)/((a*b - b^2)*d^2)) + cos(d*x + c)) - 1/4*sqrt(-((a*b - b^2)*d^2*sq
rt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a*b - b^2)*d^2))*log(-((a*b^2
 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - b*d)*sqrt(-((a*b - b
^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a*b - b^2)*d^2)) -
cos(d*x + c)) - 1/4*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5
)*d^4)) - 1)/((a*b - b^2)*d^2))*log(-((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 -
 2*a*b^4 + b^5)*d^4)) + b*d)*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*
b^4 + b^5)*d^4)) - 1)/((a*b - b^2)*d^2)) + cos(d*x + c)) + 1/4*sqrt(((a*b
- b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1)/((a*b - b^2)*d^2))
*log(-((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + b*d)*sq
rt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1)/((a*b - b
^2)*d^2)) - cos(d*x + c))
```

3.198.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**3/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.198.7 Maxima [F]

$$\int \frac{\sin^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\sin(dx+c)^3}{b\sin(dx+c)^4-a}\,dx$$

input
```
integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
-integrate(sin(d*x + c)^3/(b*sin(d*x + c)^4 - a), x)
```

3.198.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(79) = 158$.

Time = 0.75 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.44

$$\int \frac{\sin^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\sqrt{-b^2-\sqrt{ab}b}\arctan\left(\frac{\cos(dx+c)}{d\sqrt{-\frac{bd^2+\sqrt{(a-b)bd^4+b^2d^4}}{bd^4}}}\right)}{2\left(b+\sqrt{ab}\right)d|b|} + \frac{\sqrt{-b^2+\sqrt{ab}b}\arctan\left(\frac{\cos(dx+c)}{d\sqrt{-\frac{bd^2-\sqrt{(a-b)bd^4+b^2d^4}}{bd^4}}}\right)}{2\left(b-\sqrt{ab}\right)d|b|}$$

input
```
integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
1/2*sqrt(-b^2 - sqrt(a*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(b*d^2 + sqrt((a
 - b)*b*d^4 + b^2*d^4))/(b*d^4))))/((b + sqrt(a*b))*d*abs(b)) + 1/2*sqrt(-
b^2 + sqrt(a*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(b*d^2 - sqrt((a - b)*b*d^
4 + b^2*d^4))/(b*d^4))))/((b - sqrt(a*b))*d*abs(b))
```

3.198.9 Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 976, normalized size of antiderivative = 8.49

$$\int \frac{\sin^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$= \frac{2\,\mathrm{atanh}\left(\frac{8ab^2\cos(c+dx)\sqrt{\frac{\sqrt{ab^3}}{16(ab^3-b^4)}-\frac{b^2}{16(ab^3-b^4)}}}{2ab+\frac{2ab^5}{ab^3-b^4}-\frac{2ab^3\sqrt{ab^3}}{ab^3-b^4}} - \frac{8ab^6\cos(c+dx)\sqrt{\frac{\sqrt{ab^3}}{16(ab^3-b^4)}-\frac{b^2}{16(ab^3-b^4)}}}{2ab^5-2a^2b^4-\frac{2a^2b^8}{ab^3-b^4}+\frac{2ab^9}{ab^3-b^4}+\frac{2a^2b^6\sqrt{ab^3}}{ab^3-b^4}-\frac{2ab^7\sqrt{ab^3}}{ab^3-b^4}} + \frac{8ab^4\cos(\cdots}{2ab^5-2a^2b^4\cdots}\right.}{d}$$

$$- \frac{2\,\mathrm{atanh}\left(\frac{8ab^6\cos(c+dx)\sqrt{-\frac{b^2}{16(ab^3-b^4)}-\frac{\sqrt{ab^3}}{16(ab^3-b^4)}}}{2ab^5-2a^2b^4-\frac{2a^2b^8}{ab^3-b^4}+\frac{2ab^9}{ab^3-b^4}-\frac{2a^2b^6\sqrt{ab^3}}{ab^3-b^4}+\frac{2ab^7\sqrt{ab^3}}{ab^3-b^4}} - \frac{8ab^2\cos(c+dx)\sqrt{-\frac{b^2}{16(ab^3-b^4)}-\frac{\sqrt{ab^3}}{16(ab^3-b^4)}}}{2ab+\frac{2ab^5}{ab^3-b^4}+\frac{2ab^3\sqrt{ab^3}}{ab^3-b^4}} + \frac{8ab^4\cdots}{2ab^5-2a\cdots}\right.}{d}$$

input
```
int(sin(c + d*x)^3/(a - b*sin(c + d*x)^4),x)
```

output
```
(2*atanh((8*a*b^2*cos(c + d*x)*((a*b^3)^(1/2)/(16*(a*b^3 - b^4)) - b^2/(16
*(a*b^3 - b^4)))^(1/2))/(2*a*b + (2*a*b^5)/(a*b^3 - b^4) - (2*a*b^3*(a*b^3
)^(1/2))/(a*b^3 - b^4)) - (8*a*b^6*cos(c + d*x)*((a*b^3)^(1/2)/(16*(a*b^3
- b^4)) - b^2/(16*(a*b^3 - b^4)))^(1/2))/(2*a*b^5 - 2*a^2*b^4 - (2*a^2*b^8
)/(a*b^3 - b^4) + (2*a*b^9)/(a*b^3 - b^4) + (2*a^2*b^6*(a*b^3)^(1/2))/(a*b
^3 - b^4) - (2*a*b^7*(a*b^3)^(1/2))/(a*b^3 - b^4)) + (8*a*b^4*cos(c + d*x)
*(a*b^3)^(1/2)*((a*b^3)^(1/2)/(16*(a*b^3 - b^4)) - b^2/(16*(a*b^3 - b^4)))
^(1/2))/(2*a*b^5 - 2*a^2*b^4 - (2*a^2*b^8)/(a*b^3 - b^4) + (2*a*b^9)/(a*b^
3 - b^4) + (2*a^2*b^6*(a*b^3)^(1/2))/(a*b^3 - b^4) - (2*a*b^7*(a*b^3)^(1/2
))/(a*b^3 - b^4)))*(-(b^2 - (a*b^3)^(1/2))/(16*(a*b^3 - b^4)))^(1/2))/d -
(2*atanh((8*a*b^6*cos(c + d*x)*(- b^2/(16*(a*b^3 - b^4)) - (a*b^3)^(1/2)/(
16*(a*b^3 - b^4)))^(1/2))/(2*a*b^5 - 2*a^2*b^4 - (2*a^2*b^8)/(a*b^3 - b^4)
 + (2*a*b^9)/(a*b^3 - b^4) - (2*a^2*b^6*(a*b^3)^(1/2))/(a*b^3 - b^4) + (2*
a*b^7*(a*b^3)^(1/2))/(a*b^3 - b^4)) - (8*a*b^2*cos(c + d*x)*(- b^2/(16*(a*
b^3 - b^4)) - (a*b^3)^(1/2)/(16*(a*b^3 - b^4)))^(1/2))/(2*a*b + (2*a*b^5)/
(a*b^3 - b^4) + (2*a*b^3*(a*b^3)^(1/2))/(a*b^3 - b^4)) + (8*a*b^4*cos(c +
d*x)*(a*b^3)^(1/2)*(- b^2/(16*(a*b^3 - b^4)) - (a*b^3)^(1/2)/(16*(a*b^3 -
b^4)))^(1/2))/(2*a*b^5 - 2*a^2*b^4 - (2*a^2*b^8)/(a*b^3 - b^4) + (2*a*b^9)
/(a*b^3 - b^4) - (2*a^2*b^6*(a*b^3)^(1/2))/(a*b^3 - b^4) + (2*a*b^7*(a*b^3
)^(1/2))/(a*b^3 - b^4)))*(-(b^2 + (a*b^3)^(1/2))/(16*(a*b^3 - b^4)))^(1...
```

3.198. $\int \frac{\sin^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.199 $\int \frac{\sin(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.199.1 Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{\sin(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt[4]{b}d} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt[4]{b}d}$$

output
```
-1/2*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/b^(1/4)/d/a^(1/2)/
(a^(1/2)-b^(1/2))^(1/2)-1/2*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(
1/2))/b^(1/4)/d/a^(1/2)/(a^(1/2)+b^(1/2))^(1/2)
```

3.199.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.46

$$\int \frac{\sin(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{i\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&, \frac{-2\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1+i\log\left(1-2\cos(c+dx)\#\right.}{-b-}\right.}{2d}$$

input
```
Integrate[Sin[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

output
```
((I/2)*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & ,
 (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + I*Log[1 - 2*Cos[c + d*x
]*#1 + #1^2]*#1 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - I*Log[
1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^4 +
 b*#1^6) & ])/d
```

3.199.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 3694, 1406, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{a-b\sin^4(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(c+dx)}{a-b\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{ 3694}$$

$$-\frac{\int \frac{1}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{ 1406}$$

$$-\frac{\frac{\sqrt{b}\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos(c+dx)}{2\sqrt{a}}-\frac{\sqrt{b}\int \frac{1}{-b\cos^2(c+dx)-\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}d\cos(c+dx)}{2\sqrt{a}}}{d}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{\frac{\sqrt{b}\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos(c+dx)}{2\sqrt{a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}}{d}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\sqrt{\sqrt{a}+\sqrt{b}}}}{d}$$

input
```
Int[Sin[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

output
```
-((ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[a]*Sqrt[
Sqrt[a] - Sqrt[b]]*b^(1/4)) + ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a]
+ Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4)))/d)
```

3.199.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.199.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{b\left(-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{d}$
default	$\frac{b\left(-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{d}$
risch	$-\frac{i\left(\sum_{\_R=\operatorname{RootOf}\left(-1+\left(16a^3b\,d^4-16a^2b^2d^4\right)\_Z^4-8a\,d^2\_Z^2b\right)}\_R\ln\left(\mathrm{e}^{2i(dx+c)}+\left(\left(-16ia^2b\,d^3+16ia\,b^2d^3\right)\_R^3+(4iad\right.\right.\right.}{2}$

input
```
int(sin(d*x+c)/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*b*(-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((
a*b)^(1/2)+b)*b)^(1/2))-1/2/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(c
os(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2)))
```

3.199.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(85) = 170$.

Time = 0.34 (sec) , antiderivative size = 703, normalized size of antiderivative = 5.62

$$\int \frac{\sin(c+dx)}{a-b\sin^4(c+dx)}\,dx =$$

$$-\frac{1}{4}\sqrt{-\frac{(a^2-ab)d^2\sqrt{\frac{1}{(a^3b-2\,a^2b^2+ab^3)d^4}}+1}{(a^2-ab)d^2}}\log\left(-\left((a^2b-ab^2)d^3\sqrt{\frac{1}{(a^3b-2\,a^2b^2+ab^3)d^4}}-ad\right)\sqrt{-\frac{(a}{}}\right.$$

$$\left.+\cos(dx+c)\right)$$

$$+\frac{1}{4}\sqrt{-\frac{(a^2-ab)d^2\sqrt{\frac{1}{(a^3b-2\,a^2b^2+ab^3)d^4}}+1}{(a^2-ab)d^2}}\log\left(-\left((a^2b-ab^2)d^3\sqrt{\frac{1}{(a^3b-2\,a^2b^2+ab^3)d^4}}-ad\right)\sqrt{-\frac{(a}{}}\right.$$

$$\left.-\cos(dx+c)\right)$$

$$+\frac{1}{4}\sqrt{\frac{(a^2-ab)d^2\sqrt{\frac{1}{(a^3b-2\,a^2b^2+ab^3)d^4}}-1}{(a^2-ab)d^2}}\log\left(-\left((a^2b-ab^2)d^3\sqrt{\frac{1}{(a^3b-2\,a^2b^2+ab^3)d^4}}+ad\right)\sqrt{\frac{(a^2-}{}}\right.$$

$$\left.+\cos(dx+c)\right)$$

$$-\frac{1}{4}\sqrt{\frac{(a^2-ab)d^2\sqrt{\frac{1}{(a^3b-2\,a^2b^2+ab^3)d^4}}-1}{(a^2-ab)d^2}}\log\left(-\left((a^2b-ab^2)d^3\sqrt{\frac{1}{(a^3b-2\,a^2b^2+ab^3)d^4}}+ad\right)\sqrt{\frac{(a^2-}{}}\right.$$

$$\left.-\cos(dx+c)\right)$$

input
```
integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

3.199. $\int \frac{\sin(c+dx)}{a-b\sin^4(c+dx)}\,dx$

output
```
-1/4*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)
/((a^2 - a*b)*d^2))*log(-((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 +
 a*b^3)*d^4)) - a*d)*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a
*b^3)*d^4)) + 1)/((a^2 - a*b)*d^2)) + cos(d*x + c)) + 1/4*sqrt(-((a^2 - a*
b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b)*d^2))*l
og(-((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - a*d)*
sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^
2 - a*b)*d^2)) - cos(d*x + c)) + 1/4*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b
- 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2))*log(-((a^2*b - a*b^2)*d
^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + a*d)*sqrt(((a^2 - a*b)*d^2*
sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2)) + cos(d*
x + c)) - 1/4*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^
4)) - 1)/((a^2 - a*b)*d^2))*log(-((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a
^2*b^2 + a*b^3)*d^4)) + a*d)*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*
b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2)) - cos(d*x + c))
```

3.199.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.199.7 Maxima [F]

$$\int \frac{\sin(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\sin(dx+c)}{b\sin(dx+c)^4-a}\,dx$$

input
```
integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
-integrate(sin(d*x + c)/(b*sin(d*x + c)^4 - a), x)
```

3.199.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(85) = 170$.

Time = 0.68 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.46

$$\int \frac{\sin(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\sqrt{ab}\sqrt{-b^2-\sqrt{ab}b}\arctan\left(\frac{\cos(dx+c)}{d\sqrt{-\frac{bd^2+\sqrt{(a-b)bd^4+b^2d^4}}{bd^4}}}\right)}{2\left(ab+\sqrt{ab}a\right)d|b|} + \frac{\sqrt{ab}\sqrt{-b^2+\sqrt{ab}b}\arctan\left(\frac{\cos(dx+c)}{d\sqrt{-\frac{bd^2-\sqrt{(a-b)bd^4+b^2d^4}}{bd^4}}}\right)}{2\left(ab-\sqrt{ab}a\right)d|b|}$$

input
```
integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
-1/2*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(b*d^
2 + sqrt((a - b)*b*d^4 + b^2*d^4))/(b*d^4))))/((a*b + sqrt(a*b)*a)*d*abs(b
)) + 1/2*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(
b*d^2 - sqrt((a - b)*b*d^4 + b^2*d^4))/(b*d^4))))/((a*b - sqrt(a*b)*a)*d*a
bs(b))
```

3.199.9 Mupad [B] (verification not implemented)

Time = 15.33 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.89

$$\begin{aligned}\int \frac{\sin(c+dx)}{a-b\sin^4(c+dx)}\,dx &= \frac{\ln\left(4\,a\,b^3\,\sqrt{\frac{1}{a\,b+\sqrt{a^3\,b}}}-4\,b^3\,\cos\left(c+d\,x\right)+\frac{4\,a\,b^4\,\cos(c+d\,x)}{a\,b+\sqrt{a^3\,b}}\right)\,\sqrt{-\frac{a\,b-\sqrt{a^3\,b}}{16\,(a^3\,b-a^2\,b^2)}}}{d}\\
&+\frac{\ln\left(4\,b^3\,\cos\left(c+d\,x\right)-4\,a\,b^3\,\sqrt{\frac{1}{a\,b-\sqrt{a^3\,b}}}-\frac{4\,a\,b^4\,\cos(c+d\,x)}{a\,b-\sqrt{a^3\,b}}\right)\,\sqrt{-\frac{a\,b+\sqrt{a^3\,b}}{16\,(a^3\,b-a^2\,b^2)}}}{d}\\
&-\frac{\ln\left(4\,b^3\,\cos\left(c+d\,x\right)+4\,a\,b^3\,\sqrt{\frac{1}{a\,b+\sqrt{a^3\,b}}}-\frac{4\,a\,b^4\,\cos(c+d\,x)}{a\,b+\sqrt{a^3\,b}}\right)\,\sqrt{\frac{1}{a\,b+\sqrt{a^3\,b}}}}{4\,d}\\
&-\frac{\ln\left(4\,b^3\,\cos\left(c+d\,x\right)+4\,a\,b^3\,\sqrt{\frac{1}{a\,b-\sqrt{a^3\,b}}}-\frac{4\,a\,b^4\,\cos(c+d\,x)}{a\,b-\sqrt{a^3\,b}}\right)\,\sqrt{\frac{1}{a\,b-\sqrt{a^3\,b}}}}{4\,d}\end{aligned}$$

input
```
int(sin(c + d*x)/(a - b*sin(c + d*x)^4),x)
```

output
```
(log(4*a*b^3*(1/(a*b + (a^3*b)^(1/2)))^(1/2) - 4*b^3*cos(c + d*x) + (4*a*b
^4*cos(c + d*x))/(a*b + (a^3*b)^(1/2)))*(-(a*b - (a^3*b)^(1/2))/(16*(a^3*b
 - a^2*b^2)))^(1/2))/d + (log(4*b^3*cos(c + d*x) - 4*a*b^3*(1/(a*b - (a^3*
b)^(1/2)))^(1/2) - (4*a*b^4*cos(c + d*x))/(a*b - (a^3*b)^(1/2)))*(-(a*b +
(a^3*b)^(1/2))/(16*(a^3*b - a^2*b^2)))^(1/2))/d - (log(4*b^3*cos(c + d*x)
+ 4*a*b^3*(1/(a*b + (a^3*b)^(1/2)))^(1/2) - (4*a*b^4*cos(c + d*x))/(a*b +
(a^3*b)^(1/2)))*(1/(a*b + (a^3*b)^(1/2)))^(1/2))/(4*d) - (log(4*b^3*cos(c
+ d*x) + 4*a*b^3*(1/(a*b - (a^3*b)^(1/2)))^(1/2) - (4*a*b^4*cos(c + d*x))/
(a*b - (a^3*b)^(1/2)))*(1/(a*b - (a^3*b)^(1/2)))^(1/2))/(4*d)
```

3.199. $\int \frac{\sin(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.200 $\int \frac{\csc(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.200.1 Optimal result

Integrand size = 22, antiderivative size = 136

$$\int \frac{\csc(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}-\sqrt{b}}d}-\frac{\operatorname{arctanh}(\cos(c+dx))}{ad}+\frac{\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}+\sqrt{b}}d}$$

output
```
-arctanh(cos(d*x+c))/a/d-1/2*b^(1/4)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^
(1/2))^(1/2))/a/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*b^(1/4)*arctanh(b^(1/4)*cos(
d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a/d/(a^(1/2)+b^(1/2))^(1/2)
```

3.200.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.69 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.34

$$\int \frac{\csc(c+dx)}{a-b\sin^4(c+dx)}\,dx =$$

$$-\frac{8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+ib\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1\right.}{}$$

input
```
Integrate[Csc[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

output
```
-1/8*(8*Log[Cos[(c + d*x)/2]] - 8*Log[Sin[(c + d*x)/2]] + I*b*RootSum[b -
4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*ArcTan[Sin[c +
 d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 6*ArcTa
n[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (3*I)*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2]*#1^2 - 6*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (3*I)*Lo
g[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
 - #1)]*#1^6 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1
^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a*d)
```

3.200.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3694, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{a-b\sin^4(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(c+dx)\left(a-b\sin(c+dx)^4\right)}dx$$
$$\Big\downarrow \text{3694}$$
$$-\frac{\int \frac{1}{(1-\cos^2(c+dx))(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b)}d\cos(c+dx)}{d}$$
$$\Big\downarrow \text{1484}$$
$$-\frac{\int \left(\frac{b-b\cos^2(c+dx)}{a(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b)}-\frac{1}{a(\cos^2(c+dx)-1)}\right)d\cos(c+dx)}{d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\text{arctanh}(\cos(c+dx))}{a}}{d}$$

input
```
Int[Csc[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

output
```
-(((b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a*S
qrt[Sqrt[a] - Sqrt[b]]) + ArcTanh[Cos[c + d*x]]/a - (b^(1/4)*ArcTanh[(b^(1
/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a*Sqrt[Sqrt[a] + Sqrt[b]]))
/d)
```

3.200.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.200.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-\frac{b^2\left(\frac{\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}-b\right)b}}-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{a}+\frac{\ln(\cos(dx+c)-1)}{2a}-\frac{\ln(1+\cos(dx+c))}{2a}}{d}$
default	$\frac{-\frac{b^2\left(\frac{\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}-b\right)b}}-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2b\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{a}+\frac{\ln(\cos(dx+c)-1)}{2a}-\frac{\ln(1+\cos(dx+c))}{2a}}{d}$
risch	$\frac{\ln(e^{i(dx+c)}-1)}{da}-\frac{\ln(e^{i(dx+c)}+1)}{da}+2i\left(\sum_{\_R=\operatorname{RootOf}((4096a^5d^4-4096a^4bd^4)\_Z^4-128a^2bd^2\_Z^2-b)}\_R\ln\left(\right.\right.$

input
```
int(csc(d*x+c)/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a*b^2*(1/2/b/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)
^(1/2)-b)*b)^(1/2))-1/2/b/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(
((a*b)^(1/2)+b)*b)^(1/2)))+1/2/a*ln(cos(d*x+c)-1)-1/2/a*ln(1+cos(d*x+c)))
```

3.200.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(100) = 200$.

Time = 0.37 (sec) , antiderivative size = 773, normalized size of antiderivative = 5.68

$$\int \frac{\csc(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$= \frac{ad\sqrt{-\frac{(a^3-a^2b)d^2\sqrt{\frac{b}{(a^5-2a^4b+a^3b^2)d^4}}+b}{(a^3-a^2b)d^2}}\log\left(b\cos(dx+c)-\left((a^4-a^3b)d^3\sqrt{\frac{b}{(a^5-2a^4b+a^3b^2)d^4}}-abd\right)\sqrt{-\frac{(a^3-a^2}{}}\right.}{}$$

input
```
integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/4*(a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4))
+ b)/((a^3 - a^2*b)*d^2))*log(b*cos(d*x + c) - ((a^4 - a^3*b)*d^3*sqrt(b/(
(a^5 - 2*a^4*b + a^3*b^2)*d^4)) - a*b*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/(
(a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))) - a*d*sqrt(((a^
3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)
*d^2))*log(b*cos(d*x + c) - ((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^
3*b^2)*d^4)) + a*b*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3
*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))) - a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt
(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))*log(-b*cos(d
*x + c) - ((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - a*b
*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b)/
((a^3 - a^2*b)*d^2))) + a*d*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b
 + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))*log(-b*cos(d*x + c) - ((a^4 -
a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + a*b*d)*sqrt(((a^3 - a
^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2)
)) - 2*log(1/2*cos(d*x + c) + 1/2) + 2*log(-1/2*cos(d*x + c) + 1/2))/(a*d)
```

3.200.6 Sympy [F]

$$\int \frac{\csc(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int \frac{\csc(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)/(a-b*sin(d*x+c)**4),x)
```

output
```
Integral(csc(c + d*x)/(a - b*sin(c + d*x)**4), x)
```

3.200.7 Maxima [F]

$$\int \frac{\csc(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\csc(dx+c)}{b\sin(dx+c)^4-a}\,dx$$

input
```
integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
-1/2*(2*a*d*integrate(-2*(12*b^2*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) - 4*b^2
*cos(d*x + c)*sin(2*d*x + 2*c) + 4*b^2*cos(2*d*x + 2*c)*sin(d*x + c) - b^2
*sin(d*x + c) + (b^2*sin(7*d*x + 7*c) - 3*b^2*sin(5*d*x + 5*c) + 3*b^2*sin
(3*d*x + 3*c) - b^2*sin(d*x + c))*cos(8*d*x + 8*c) + 2*(2*b^2*sin(6*d*x +
6*c) + 2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4*c))*cos(7*d*
x + 7*c) + 4*(3*b^2*sin(5*d*x + 5*c) - 3*b^2*sin(3*d*x + 3*c) + b^2*sin(d*
x + c))*cos(6*d*x + 6*c) - 6*(2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin
(4*d*x + 4*c))*cos(5*d*x + 5*c) - 2*(3*(8*a*b - 3*b^2)*sin(3*d*x + 3*c) -
(8*a*b - 3*b^2)*sin(d*x + c))*cos(4*d*x + 4*c) - (b^2*cos(7*d*x + 7*c) - 3
*b^2*cos(5*d*x + 5*c) + 3*b^2*cos(3*d*x + 3*c) - b^2*cos(d*x + c))*sin(8*d
*x + 8*c) - (4*b^2*cos(6*d*x + 6*c) + 4*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*
a*b - 3*b^2)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) - 4*(3*b^2*cos(5*d*x + 5*c
) - 3*b^2*cos(3*d*x + 3*c) + b^2*cos(d*x + c))*sin(6*d*x + 6*c) + 3*(4*b^2
*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*sin(5*d*x +
5*c) + 2*(3*(8*a*b - 3*b^2)*cos(3*d*x + 3*c) - (8*a*b - 3*b^2)*cos(d*x + c
))*sin(4*d*x + 4*c) - 3*(4*b^2*cos(2*d*x + 2*c) - b^2)*sin(3*d*x + 3*c))/(
a*b^2*cos(8*d*x + 8*c)^2 + 16*a*b^2*cos(6*d*x + 6*c)^2 + 16*a*b^2*cos(2*d*
x + 2*c)^2 + a*b^2*sin(8*d*x + 8*c)^2 + 16*a*b^2*sin(6*d*x + 6*c)^2 + 16*a
*b^2*sin(2*d*x + 2*c)^2 - 8*a*b^2*cos(2*d*x + 2*c) + a*b^2 + 4*(64*a^3 - 4
8*a^2*b + 9*a*b^2)*cos(4*d*x + 4*c)^2 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)...
```

3.200.8 Giac [F]

$$\int \frac{\csc(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\csc(dx+c)}{b\sin(dx+c)^4-a}\,dx$$

input
```
integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

3.200.9 Mupad [B] (verification not implemented)

Time = 15.64 (sec) , antiderivative size = 2031, normalized size of antiderivative = 14.93

$$\int \frac{\csc(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(c + d*x)*(a - b*sin(c + d*x)^4)),x)
```

output
```
- (atan((((((((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)*(256*a^4*b^
4 - 192*a^3*b^5 + cos(c + d*x)*(768*a^4*b^5 - 512*a^5*b^4)*((a^2*b + (a^5*
b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)) - 144*a^2*b^5*cos(c + d*x))*((a^2*b +
 (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) + 12*a*b^5)*((a^2*b + (a^5*b)^(1
/2))/(16*(a^4*b - a^5)))^(1/2) + 6*b^5*cos(c + d*x))*((a^2*b + (a^5*b)^(1/
2))/(16*(a^4*b - a^5)))^(1/2)*1i + (((((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b
- a^5)))^(1/2)*(192*a^3*b^5 - 256*a^4*b^4 + cos(c + d*x)*(768*a^4*b^5 - 51
2*a^5*b^4)*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)) - 144*a^2*b
^5*cos(c + d*x))*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) - 12*a
*b^5)*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) + 6*b^5*cos(c + d
*x))*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)*1i)/((((((a^2*b +
(a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)*(256*a^4*b^4 - 192*a^3*b^5 + cos(
c + d*x)*(768*a^4*b^5 - 512*a^5*b^4)*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b -
 a^5)))^(1/2)) - 144*a^2*b^5*cos(c + d*x))*((a^2*b + (a^5*b)^(1/2))/(16*(a
^4*b - a^5)))^(1/2) + 12*a*b^5)*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)
))^(1/2) + 6*b^5*cos(c + d*x))*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5))
)^(1/2) - (((((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)*(192*a^3*b
^5 - 256*a^4*b^4 + cos(c + d*x)*(768*a^4*b^5 - 512*a^5*b^4)*((a^2*b + (a^5
*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)) - 144*a^2*b^5*cos(c + d*x))*((a^2*b
+ (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) - 12*a*b^5)*((a^2*b + (a^5*b...
```

3.201 $\int \frac{\csc^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.201.1 Optimal result

Integrand size = 24, antiderivative size = 184

$$\int \frac{\csc^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{b^{3/4}\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}-\sqrt{b}}d} - \frac{\operatorname{arctanh}(\cos(c+dx))}{2ad} - \frac{b^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}+\sqrt{b}}d} - \frac{1}{4ad(1-\cos(c+dx))} + \frac{1}{4ad(1+\cos(c+dx))}$$

output
```
-1/2*arctanh(cos(d*x+c))/a/d-1/4/a/d/(1-cos(d*x+c))+1/4/a/d/(1+cos(d*x+c))
-1/2*b^(3/4)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^(3/2)/d/
(a^(1/2)-b^(1/2))^(1/2)-1/2*b^(3/4)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^
(1/2))^(1/2))/a^(3/2)/d/(a^(1/2)+b^(1/2))^(1/2)
```

3.201.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.86 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.32

$$\int \frac{\csc^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$= \frac{-\csc^2\left(\frac{1}{2}(c+dx)\right)-4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4ib\text{RootSum}\left[b-4b\#1^2-16a\#\right.}{}$$

input
```
Integrate[Csc[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

output
```
(-Csc[(c + d*x)/2]^2 - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Sin[(c + d*x)/2]] +
 (4*I)*b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 &
 , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + I*Log[1 - 2*Cos[c + d
*x]*#1 + #1^2]*#1 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - I*Lo
g[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^4
 + b*#1^6) & ] + Sec[(c + d*x)/2]^2)/(8*a*d)
```

3.201.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3694, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(c+dx)^3\left(a-b\sin(c+dx)^4\right)}dx$$

$$\Big\downarrow \ 3694$$

$$-\frac{\int \frac{1}{(1-\cos^2(c+dx))^2(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b)}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{1484}$$

$$-\frac{\int \left(\frac{b}{a(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b)}-\frac{1}{2a(\cos^2(c+dx)-1)}+\frac{1}{4a(\cos(c+dx)-1)^2}+\frac{1}{4a(\cos(c+dx)+1)^2}\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{b^{3/4}\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\text{arctanh}(\cos(c+dx))}{2a}+\frac{1}{4a(1-\cos(c+dx))}-\frac{1}{4a(\cos(c+dx)+1)}}{d}$$

input
```
Int[Csc[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

output
```
-(((b^(3/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a^(
3/2)*Sqrt[Sqrt[a] - Sqrt[b]]) + ArcTanh[Cos[c + d*x]]/(2*a) + (b^(3/4)*Arc
Tanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(3/2)*Sqrt[Sqrt
[a] + Sqrt[b]]) + 1/(4*a*(1 - Cos[c + d*x])) - 1/(4*a*(1 + Cos[c + d*x])))
/d)
```

3.201.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.201. $\int \frac{\csc^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.201.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{1}{4a(\cos(dx+c)-1)}+\frac{\ln(\cos(dx+c)-1)}{4a}+\frac{1}{4a(1+\cos(dx+c))}-\frac{\ln(1+\cos(dx+c))}{4a}+\frac{b^2\left(-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{a}}{d}$
default	$\frac{\frac{1}{4a(\cos(dx+c)-1)}+\frac{\ln(\cos(dx+c)-1)}{4a}+\frac{1}{4a(1+\cos(dx+c))}-\frac{\ln(1+\cos(dx+c))}{4a}+\frac{b^2\left(-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{a}}{d}$
risch	$\frac{e^{3i(dx+c)}+e^{i(dx+c)}}{da\left(e^{2i(dx+c)}-1\right)^2}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{2da}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2da}-8i\left(\sum_{\_R=\operatorname{RootOf}\left(\left(1048576a^7d^4-1048576a^6b\,d^4\right)\_Z^4\right.}\right.$

input
```
int(csc(d*x+c)^3/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4/a/(cos(d*x+c)-1)+1/4/a*ln(cos(d*x+c)-1)+1/4/a/(1+cos(d*x+c))-1/4/
a*ln(1+cos(d*x+c))+1/a*b^2*(-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arc
tanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/2/(a*b)^(1/2)/(((a*b)^(1/2)
-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))))
```

3.201.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. 2(136) = 272.

Time = 0.41 (sec) , antiderivative size = 924, normalized size of antiderivative = 5.02

$$\int \frac{\csc^3(c+dx)}{a-b\sin^4(c+dx)}\,dx =$$

$$-\frac{\left(ad\cos(dx+c)^2-ad\right)\sqrt{-\frac{(a^4-a^3b)d^2\sqrt{\frac{b^3}{(a^7-2a^6b+a^5b^2)d^4}}+b^2}{(a^4-a^3b)d^2}}\log\left(b^2\cos(dx+c)-\left((a^5-a^4b)d^3\sqrt{\frac{b^3}{(a^7-2a^6b\cdots}}\right.\right.}{}$$

input
```
integrate(csc(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/4*((a*d*cos(d*x + c)^2 - a*d)*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 -
 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*log(b^2*cos(d*x + c)
 - ((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - a^2*b*d)
*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)
/((a^4 - a^3*b)*d^2))) - (a*d*cos(d*x + c)^2 - a*d)*sqrt(((a^4 - a^3*b)*d^
2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))*lo
g(b^2*cos(d*x + c) - ((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2
)*d^4)) + a^2*b*d)*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*
b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))) - (a*d*cos(d*x + c)^2 - a*d)*sqrt(
-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4
 - a^3*b)*d^2))*log(-b^2*cos(d*x + c) - ((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7
- 2*a^6*b + a^5*b^2)*d^4)) - a^2*b*d)*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^3/((
a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))) + (a*d*cos(d*x
 + c)^2 - a*d)*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)
*d^4)) - b^2)/((a^4 - a^3*b)*d^2))*log(-b^2*cos(d*x + c) - ((a^5 - a^4*b)*
d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + a^2*b*d)*sqrt(((a^4 - a^3*
b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2
))) + (cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(d*x + c)^2 -
 1)*log(-1/2*cos(d*x + c) + 1/2) - 2*cos(d*x + c))/(a*d*cos(d*x + c)^2 - a
*d)
```

3.201.6 Sympy [F]

$$\int \frac{\csc^3(c+dx)}{a-b\sin^4(c+dx)}\,dx=\int \frac{\csc^3{(c+dx)}}{a-b\sin^4{(c+dx)}}\,dx$$

input
```
integrate(csc(d*x+c)**3/(a-b*sin(d*x+c)**4),x)
```

output
```
Integral(csc(c + d*x)**3/(a - b*sin(c + d*x)**4), x)
```

3.201.7 Maxima [F]

$$\int \frac{\csc^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\csc(dx+c)^3}{b\sin(dx+c)^4-a}\,dx$$

input
```
integrate(csc(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/4*(4*(cos(3*d*x + 3*c) + cos(d*x + c))*cos(4*d*x + 4*c) - 4*(2*cos(2*d*x
 + 2*c) - 1)*cos(3*d*x + 3*c) - 8*cos(2*d*x + 2*c)*cos(d*x + c) + 4*(a*d*c
os(4*d*x + 4*c)^2 + 4*a*d*cos(2*d*x + 2*c)^2 + a*d*sin(4*d*x + 4*c)^2 - 4*
a*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*d*sin(2*d*x + 2*c)^2 - 4*a*d*c
os(2*d*x + 2*c) + a*d - 2*(2*a*d*cos(2*d*x + 2*c) - a*d)*cos(4*d*x + 4*c))
*integrate(8*(4*b^2*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 2*(8*a*b - 3*b^2)*
cos(3*d*x + 3*c)*sin(4*d*x + 4*c) - 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c)*sin
(3*d*x + 3*c) - (b^2*sin(5*d*x + 5*c) - b^2*sin(3*d*x + 3*c))*cos(8*d*x +
8*c) + 4*(b^2*sin(5*d*x + 5*c) - b^2*sin(3*d*x + 3*c))*cos(6*d*x + 6*c) -
2*(2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4*c))*cos(5*d*x +
5*c) + (b^2*cos(5*d*x + 5*c) - b^2*cos(3*d*x + 3*c))*sin(8*d*x + 8*c) - 4*
(b^2*cos(5*d*x + 5*c) - b^2*cos(3*d*x + 3*c))*sin(6*d*x + 6*c) + (4*b^2*co
s(2*d*x + 2*c) - b^2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*sin(5*d*x + 5*c
) - (4*b^2*cos(2*d*x + 2*c) - b^2)*sin(3*d*x + 3*c))/(a*b^2*cos(8*d*x + 8*
c)^2 + 16*a*b^2*cos(6*d*x + 6*c)^2 + 16*a*b^2*cos(2*d*x + 2*c)^2 + a*b^2*s
in(8*d*x + 8*c)^2 + 16*a*b^2*sin(6*d*x + 6*c)^2 + 16*a*b^2*sin(2*d*x + 2*c
)^2 - 8*a*b^2*cos(2*d*x + 2*c) + a*b^2 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*c
os(4*d*x + 4*c)^2 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*sin(4*d*x + 4*c)^2 + 1
6*(8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*a*b^2*cos(6
*d*x + 6*c) + 4*a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*...
```

3.201.8 Giac [F]

$$\int \frac{\csc^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\csc(dx+c)^3}{b\sin(dx+c)^4-a}\,dx$$

input
```
integrate(csc(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

3.201.9 Mupad [B] (verification not implemented)

Time = 15.16 (sec) , antiderivative size = 2779, normalized size of antiderivative = 15.10

$$\int \frac{\csc^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(c + d*x)^3*(a - b*sin(c + d*x)^4)),x)
```

output
```
(atan(cos(c + d*x)*1i)*1i)/(d*(2*a - 2*a*cos(c + d*x)^2)) - cos(c + d*x)/(
d*(2*a - 2*a*cos(c + d*x)^2)) - (atan(cos(c + d*x)*1i)*cos(c + d*x)^2*1i)/
(d*(2*a - 2*a*cos(c + d*x)^2)) + (a*atan((a^13*cos(c + d*x)*(((a^7*b^3)^(1
/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*2048i + a^10*b*cos(c + d*x)*(((a
^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(3/2)*64i - a^12*b*cos(c + d
*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*7168i - a^4*b^
5*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*8i
+ a^5*b^4*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(
1/2)*12i - a^7*b^2*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 1
6*a^7))^(1/2)*4i + a^7*b^4*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a
^6*b - 16*a^7))^(3/2)*320i - a^8*b^3*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*
b^2)/(16*a^6*b - 16*a^7))^(3/2)*576i + a^9*b^2*cos(c + d*x)*(((a^7*b^3)^(1
/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(3/2)*192i - a^10*b^3*cos(c + d*x)*(((
a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*3072i + a^11*b^2*cos(
c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*8192i)/(2
*b^3*(a^7*b^3)^(1/2) + a^3*b^5 + a^5*b^3 - a*b^2*(a^7*b^3)^(1/2) + a^2*b*(
a^7*b^3)^(1/2)))*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*4
i)/(d*(2*a - 2*a*cos(c + d*x)^2)) + (a*atan((a^13*cos(c + d*x)*(-((a^7*b^3
)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*2048i + a^10*b*cos(c + d*x)*
(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(3/2)*64i - a^12*b*c...
```

3.202 $\int \frac{\csc^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.202.1 Optimal result

Integrand size = 24, antiderivative size = 229

$$\int \frac{\csc^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{b^{5/4}\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}d} - \frac{(3a+8b)\text{arctanh}(\cos(c+dx))}{8a^2d} + \frac{b^{5/4}\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}+\sqrt{b}}d} - \frac{1}{16ad(1-\cos(c+dx))^2} - \frac{3}{16ad(1-\cos(c+dx))} + \frac{1}{16ad(1+\cos(c+dx))^2} + \frac{3}{16ad(1+\cos(c+dx))}$$

output
```
-1/8*(3*a+8*b)*arctanh(cos(d*x+c))/a^2/d-1/16/a/d/(1-cos(d*x+c))^2-3/16/a/
d/(1-cos(d*x+c))+1/16/a/d/(1+cos(d*x+c))^2+3/16/a/d/(1+cos(d*x+c))-1/2*b^(
5/4)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^2/d/(a^(1/2)-b^(
1/2))^(1/2)+1/2*b^(5/4)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2)
)/a^2/d/(a^(1/2)+b^(1/2))^(1/2)
```

3.202.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.33 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.79

$$\int \frac{\csc^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$= \frac{-6a\csc^2\left(\frac{1}{2}(c+dx)\right)-a\csc^4\left(\frac{1}{2}(c+dx)\right)-24a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-64b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+24a\,\mathrm{l}}{}$$

input
```
Integrate[Csc[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

output
```
(-6*a*Csc[(c + d*x)/2]^2 - a*Csc[(c + d*x)/2]^4 - 24*a*Log[Cos[(c + d*x)/2
]] - 64*b*Log[Cos[(c + d*x)/2]] + 24*a*Log[Sin[(c + d*x)/2]] + 64*b*Log[Si
n[(c + d*x)/2]] - (8*I)*b^2*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 -
4*b*#1^6 + b*#1^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log
[1 - 2*Cos[c + d*x]*#1 + #1^2] + 6*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
]*#1^2 - (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 6*ArcTan[Sin[c + d
*x]/(Cos[c + d*x] - #1)]*#1^4 + (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1
^4 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*Log[1 - 2*Cos[c +
 d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7)
 & ] + 6*a*Sec[(c + d*x)/2]^2 + a*Sec[(c + d*x)/2]^4)/(64*a^2*d)
```

3.202.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3694, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(c+dx)^5\left(a-b\sin(c+dx)^4\right)}dx$$

$$\Big\downarrow \text{3694}$$

$$-\frac{\int \frac{1}{(1-\cos^2(c+dx))^3(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b)}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{1484}$$

$$-\frac{\int \left(-\frac{(\cos^2(c+dx)-1)b^2}{a^2(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b)}+\frac{-3a-8b}{8a^2(\cos^2(c+dx)-1)}+\frac{3}{16a(\cos(c+dx)-1)^2}+\frac{3}{16a(\cos(c+dx)+1)^2}-\frac{1}{8a(\cos(c+dx)-1)^3}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{b^{5/4}\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{b^{5/4}\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{(3a+8b)\text{arctanh}(\cos(c+dx))}{8a^2}+\frac{3}{16a(1-\cos(c+dx))}-\frac{3}{16a(\cos(c+dx)}}{d}$$

input
```
Int[Csc[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

output
```
-(((b^(5/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a^2
*Sqrt[Sqrt[a] - Sqrt[b]]) + ((3*a + 8*b)*ArcTanh[Cos[c + d*x]])/(8*a^2) -
(b^(5/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^2*S
qrt[Sqrt[a] + Sqrt[b]]) + 1/(16*a*(1 - Cos[c + d*x])^2) + 3/(16*a*(1 - Cos
[c + d*x])) - 1/(16*a*(1 + Cos[c + d*x])^2) - 3/(16*a*(1 + Cos[c + d*x])))
/d)
```

3.202.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.202.4 Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{1}{16a(1+\cos(dx+c))^2}+\frac{3}{16a(1+\cos(dx+c))}+\frac{(-3a-8b)\ln(1+\cos(dx+c))}{16a^2}-\frac{b^3\left(\frac{\arctan\left(\frac{\cos(dx+c)b}{\sqrt{(\sqrt{ab}-b)b}}\right)}{2b\sqrt{(\sqrt{ab}-b)b}}-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{(\sqrt{ab}+b)b}}\right)}{2b\sqrt{(\sqrt{ab}+b)b}}\right)}{a^2}-}{d}$
default	$\frac{\frac{1}{16a(1+\cos(dx+c))^2}+\frac{3}{16a(1+\cos(dx+c))}+\frac{(-3a-8b)\ln(1+\cos(dx+c))}{16a^2}-\frac{b^3\left(\frac{\arctan\left(\frac{\cos(dx+c)b}{\sqrt{(\sqrt{ab}-b)b}}\right)}{2b\sqrt{(\sqrt{ab}-b)b}}-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{(\sqrt{ab}+b)b}}\right)}{2b\sqrt{(\sqrt{ab}+b)b}}\right)}{a^2}-}{d}$
risch	$\frac{3\,e^{7i(dx+c)}-11\,e^{5i(dx+c)}-11\,e^{3i(dx+c)}+3\,e^{i(dx+c)}}{4da\left(e^{2i(dx+c)}-1\right)^4}+\frac{3\ln\left(e^{i(dx+c)}-1\right)}{8da}+\frac{b\ln\left(e^{i(dx+c)}-1\right)}{a^2d}-\frac{3\ln\left(e^{i(dx+c)}+1\right)}{8da}-$

input
```
int(csc(d*x+c)^5/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/16/a/(1+cos(d*x+c))^2+3/16/a/(1+cos(d*x+c))+1/16/a^2*(-3*a-8*b)*ln(
1+cos(d*x+c))-b^3/a^2*(1/2/b/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b
/(((a*b)^(1/2)-b)*b)^(1/2))-1/2/b/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*
x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2)))-1/16/a/(cos(d*x+c)-1)^2+3/16/a/(cos(d*x
+c)-1)+1/16*(3*a+8*b)/a^2*ln(cos(d*x+c)-1))
```

3.202.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1089 vs. $2(179) = 358$.

Time = 0.43 (sec) , antiderivative size = 1089, normalized size of antiderivative = 4.76

$$\int \frac{\csc^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/16*(6*a*cos(d*x + c)^3 + 4*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^
2 + a^2*d)*sqrt(-((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^
4)) + b^3)/((a^5 - a^4*b)*d^2))*log(b^4*cos(d*x + c) + (a^2*b^3*d - (a^7 -
 a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqrt(-((a^5 - a^4*b
)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^3)/((a^5 - a^4*b)*d^2)
)) - 4*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(((a^5
- a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - b^3)/((a^5 - a^4*
b)*d^2))*log(b^4*cos(d*x + c) - (a^2*b^3*d + (a^7 - a^6*b)*d^3*sqrt(b^5/((
a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqrt(((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2
*a^8*b + a^7*b^2)*d^4)) - b^3)/((a^5 - a^4*b)*d^2))) - 4*(a^2*d*cos(d*x +
c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(-((a^5 - a^4*b)*d^2*sqrt(b^5/(
(a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^3)/((a^5 - a^4*b)*d^2))*log(-b^4*cos(d
*x + c) + (a^2*b^3*d - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^
2)*d^4)))*sqrt(-((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4
)) + b^3)/((a^5 - a^4*b)*d^2))) + 4*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*
x + c)^2 + a^2*d)*sqrt(((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b
^2)*d^4)) - b^3)/((a^5 - a^4*b)*d^2))*log(-b^4*cos(d*x + c) - (a^2*b^3*d +
 (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqrt(((a^5 -
 a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - b^3)/((a^5 - a^4*b
)*d^2))) - 10*a*cos(d*x + c) - ((3*a + 8*b)*cos(d*x + c)^4 - 2*(3*a + 8...
```

3.202.6 Sympy [F]

$$\int \frac{\csc^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int \frac{\csc^5{(c+dx)}}{a-b\sin^4{(c+dx)}}\,dx$$

input
```
integrate(csc(d*x+c)**5/(a-b*sin(d*x+c)**4),x)
```

output
```
Integral(csc(c + d*x)**5/(a - b*sin(c + d*x)**4), x)
```

3.202.7 Maxima [F]

$$\int \frac{\csc^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\csc(dx+c)^5}{b\sin(dx+c)^4-a}\,dx$$

input
```
integrate(csc(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
-1/16*(48*a*cos(2*d*x + 2*c)*cos(d*x + c) - 176*a*sin(3*d*x + 3*c)*sin(2*d
*x + 2*c) + 48*a*sin(2*d*x + 2*c)*sin(d*x + c) - 4*(3*a*cos(7*d*x + 7*c) -
 11*a*cos(5*d*x + 5*c) - 11*a*cos(3*d*x + 3*c) + 3*a*cos(d*x + c))*cos(8*d
*x + 8*c) + 12*(4*a*cos(6*d*x + 6*c) - 6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*
x + 2*c) - a)*cos(7*d*x + 7*c) - 16*(11*a*cos(5*d*x + 5*c) + 11*a*cos(3*d*
x + 3*c) - 3*a*cos(d*x + c))*cos(6*d*x + 6*c) + 44*(6*a*cos(4*d*x + 4*c) -
 4*a*cos(2*d*x + 2*c) + a)*cos(5*d*x + 5*c) + 24*(11*a*cos(3*d*x + 3*c) -
3*a*cos(d*x + c))*cos(4*d*x + 4*c) - 44*(4*a*cos(2*d*x + 2*c) - a)*cos(3*d
*x + 3*c) - 12*a*cos(d*x + c) + 16*(a^2*d*cos(8*d*x + 8*c)^2 + 16*a^2*d*co
s(6*d*x + 6*c)^2 + 36*a^2*d*cos(4*d*x + 4*c)^2 + 16*a^2*d*cos(2*d*x + 2*c)
^2 + a^2*d*sin(8*d*x + 8*c)^2 + 16*a^2*d*sin(6*d*x + 6*c)^2 + 36*a^2*d*sin
(4*d*x + 4*c)^2 - 48*a^2*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*d*si
n(2*d*x + 2*c)^2 - 8*a^2*d*cos(2*d*x + 2*c) + a^2*d - 2*(4*a^2*d*cos(6*d*x
 + 6*c) - 6*a^2*d*cos(4*d*x + 4*c) + 4*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos
(8*d*x + 8*c) - 8*(6*a^2*d*cos(4*d*x + 4*c) - 4*a^2*d*cos(2*d*x + 2*c) + a
^2*d)*cos(6*d*x + 6*c) - 12*(4*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(4*d*x +
 4*c) - 4*(2*a^2*d*sin(6*d*x + 6*c) - 3*a^2*d*sin(4*d*x + 4*c) + 2*a^2*d*s
in(2*d*x + 2*c))*sin(8*d*x + 8*c) - 16*(3*a^2*d*sin(4*d*x + 4*c) - 2*a^2*d
*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-2*(12*b^3*cos(3*d*x + 3*c)
*sin(2*d*x + 2*c) - 4*b^3*cos(d*x + c)*sin(2*d*x + 2*c) + 4*b^3*cos(2*d...
```

3.202.8 Giac [F]

$$\int \frac{\csc^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\csc(dx+c)^5}{b\sin(dx+c)^4-a}\,dx$$

input
```
integrate(csc(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
sage0*x
```

3.202.9 Mupad [B] (verification not implemented)

Time = 15.24 (sec) , antiderivative size = 3692, normalized size of antiderivative = 16.12

$$\int \frac{\csc^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(c + d*x)^5*(a - b*sin(c + d*x)^4)),x)
```

output
```
(atan(((((768*a^3*b^8 - 144*a^5*b^6)/(64*a^5) + (((10240*a^8*b^5 - 12288*a
^7*b^6 + 6144*a^9*b^4)/(64*a^5) - (cos(c + d*x)*(12288*a^8*b^5 - 8192*a^9*
b^4)*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))/(16*a^4))*(((
a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) + (cos(c + d*x)*(2304*
a^4*b^7 + 768*a^5*b^6 + 144*a^6*b^5))/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^
3)/(16*(a^8*b - a^9)))^(1/2))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^
9)))^(1/2) - (cos(c + d*x)*(48*a*b^8 + 96*b^9 + 9*a^2*b^7))/(16*a^4))*(((a
^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2)*1i - (((768*a^3*b^8 - 1
44*a^5*b^6)/(64*a^5) + (((10240*a^8*b^5 - 12288*a^7*b^6 + 6144*a^9*b^4)/(6
4*a^5) + (cos(c + d*x)*(12288*a^8*b^5 - 8192*a^9*b^4)*(((a^9*b^5)^(1/2) +
a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^3)
/(16*(a^8*b - a^9)))^(1/2) - (cos(c + d*x)*(2304*a^4*b^7 + 768*a^5*b^6 + 1
44*a^6*b^5))/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1
/2))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) + (cos(c + d*x
)*(48*a*b^8 + 96*b^9 + 9*a^2*b^7))/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^3)/
(16*(a^8*b - a^9)))^(1/2)*1i)/((((768*a^3*b^8 - 144*a^5*b^6)/(64*a^5) + ((
(10240*a^8*b^5 - 12288*a^7*b^6 + 6144*a^9*b^4)/(64*a^5) - (cos(c + d*x)*(1
2288*a^8*b^5 - 8192*a^9*b^4)*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9
)))^(1/2))/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2
) + (cos(c + d*x)*(2304*a^4*b^7 + 768*a^5*b^6 + 144*a^6*b^5))/(16*a^4))...
```

3.203 $\int \frac{\sin^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.203.1 Optimal result

Integrand size = 24, antiderivative size = 184

$$\int \frac{\sin^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{5x}{8b}-\frac{(a+b)x}{b^2}+\frac{a^{5/4}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}b^2d}+\frac{a^{5/4}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}b^2d}+\frac{5\cos(c+dx)\sin(c+dx)}{8bd}-\frac{\cos^3(c+dx)\sin(c+dx)}{4bd}$$

output
```
5/8*x/b-(a+b)*x/b^2+5/8*cos(d*x+c)*sin(d*x+c)/b/d-1/4*cos(d*x+c)^3*sin(d*x
+c)/b/d+1/2*a^(5/4)*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/b^2
/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*a^(5/4)*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(
d*x+c)/a^(1/4))/b^2/d/(a^(1/2)+b^(1/2))^(1/2)
```

3.203.2 Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.93

$$\int \frac{\sin^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{4(8a+3b)(c+dx)-\frac{16a^{3/2}\arctan\left(\frac{(\sqrt{a}+\sqrt{b})\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{16a^{3/2}\text{arctanh}\left(\frac{(\sqrt{a}-\sqrt{b})\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}-8b\sin(2(c+dx))}{32b^2d}$$

input
```
Integrate[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

output
```
-1/32*(4*(8*a + 3*b)*(c + d*x) - (16*a^(3/2)*ArcTan[((Sqrt[a] + Sqrt[b])*T
an[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + (16*a
^(3/2)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b
]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] - 8*b*Sin[2*(c + d*x)] + b*Sin[4*(c + d*x)
])/(b^2*d)
```

3.203.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3696, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^8(c+dx)}{a-b\sin^4(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(c+dx)^8}{a-b\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{ 3696}$$

$$\frac{\int \frac{\tan^8(c+dx)}{(\tan^2(c+dx)+1)^3((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 1610}$$

$$\frac{\int \left(\frac{(\tan^2(c+dx)+1)a^2}{b^2((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}+\frac{-a-b}{b^2(\tan^2(c+dx)+1)}+\frac{2}{b(\tan^2(c+dx)+1)^2}-\frac{1}{b(\tan^2(c+dx)+1)^3}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{a^{5/4}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a^{5/4}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(a+b)\arctan(\tan(c+dx))}{b^2}+\frac{5\arctan(\tan(c+dx))}{8b}+\frac{5\tan(c+dx}{8b(\tan^2(c+dx}}{d}$$

input
```
Int[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

output
```
((5*ArcTan[Tan[c + d*x]])/(8*b) - ((a + b)*ArcTan[Tan[c + d*x]])/b^2 + (a^
(5/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt
[a] - Sqrt[b]]*b^2) + (a^(5/4)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x
])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^2) - Tan[c + d*x]/(4*b*(1 + Tan[
c + d*x]^2)^2) + (5*Tan[c + d*x])/(8*b*(1 + Tan[c + d*x]^2)))/d
```

3.203.3.1 Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

3.203. $\int \frac{\sin^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.203.4 Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{-\frac{\frac{-\frac{5\left(\tan^3(dx+c)\right)b}{8}-\frac{3\tan(dx+c)b}{8}}{\left(1+\tan^2(dx+c)\right)^2}+\frac{(8a+3b)\arctan(\tan(dx+c))}{8}}{b^2}+\frac{a^2(a-b)\left(\frac{\left(\sqrt{ab}+b\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-b\right)\text{ar}}{2\sqrt{ab}\,(}\right.}{b^2}}{d}$
default	$\frac{-\frac{\frac{-\frac{5\left(\tan^3(dx+c)\right)b}{8}-\frac{3\tan(dx+c)b}{8}}{\left(1+\tan^2(dx+c)\right)^2}+\frac{(8a+3b)\arctan(\tan(dx+c))}{8}}{b^2}+\frac{a^2(a-b)\left(\frac{\left(\sqrt{ab}+b\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-b\right)\text{ar}}{2\sqrt{ab}\,(}\right.}{b^2}}{d}$
risch	$-\frac{ax}{b^2}-\frac{3x}{8b}-\frac{i\mathrm{e}^{2i(dx+c)}}{8bd}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8bd}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a\,b^8d^4-b^9d^4\right)\_Z^4+8192a^3b^4d^2\_Z^2+16777216a^5\right)}\_R\ln\right.}{}$

input
```
int(sin(d*x+c)^8/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^2*((-5/8*tan(d*x+c)^3*b-3/8*tan(d*x+c)*b)/(1+tan(d*x+c)^2)^2+1/8
*(8*a+3*b)*arctan(tan(d*x+c)))+a^2/b^2*(a-b)*(1/2*((a*b)^(1/2)+b)/(a*b)^(1
/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1
/2)+a)*(a-b))^(1/2))+1/2*((a*b)^(1/2)-b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a
)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))))
```

3.203.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1311 vs. 2(142) = 284.

Time = 0.43 (sec) , antiderivative size = 1311, normalized size of antiderivative = 7.12

$$\int \frac{\sin^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/8*(b^2*d*sqrt(-((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d
^4)) + a^3)/((a*b^4 - b^5)*d^2))*log(1/4*a^3*cos(d*x + c)^2 - 1/4*a^3 - 1/
4*(2*(a^2*b^3 - a*b^4)*d^2*cos(d*x + c)^2 - (a^2*b^3 - a*b^4)*d^2)*sqrt(a^
5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + 1/2*(a^2*b^2*d*cos(d*x + c)*sin(d*x +
 c) - (a*b^5 - b^6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4))*cos(d*x
+ c)*sin(d*x + c))*sqrt(-((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 +
 b^9)*d^4)) + a^3)/((a*b^4 - b^5)*d^2))) - b^2*d*sqrt(-((a*b^4 - b^5)*d^2*
sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^3)/((a*b^4 - b^5)*d^2))*log(
1/4*a^3*cos(d*x + c)^2 - 1/4*a^3 - 1/4*(2*(a^2*b^3 - a*b^4)*d^2*cos(d*x +
c)^2 - (a^2*b^3 - a*b^4)*d^2)*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) -
1/2*(a^2*b^2*d*cos(d*x + c)*sin(d*x + c) - (a*b^5 - b^6)*d^3*sqrt(a^5/((a^
2*b^7 - 2*a*b^8 + b^9)*d^4))*cos(d*x + c)*sin(d*x + c))*sqrt(-((a*b^4 - b^
5)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^3)/((a*b^4 - b^5)*d^2
))) - b^2*d*sqrt(((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^
4)) - a^3)/((a*b^4 - b^5)*d^2))*log(-1/4*a^3*cos(d*x + c)^2 + 1/4*a^3 - 1/
4*(2*(a^2*b^3 - a*b^4)*d^2*cos(d*x + c)^2 - (a^2*b^3 - a*b^4)*d^2)*sqrt(a^
5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + 1/2*(a^2*b^2*d*cos(d*x + c)*sin(d*x +
 c) + (a*b^5 - b^6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4))*cos(d*x
+ c)*sin(d*x + c))*sqrt(((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 +
b^9)*d^4)) - a^3)/((a*b^4 - b^5)*d^2))) + b^2*d*sqrt(((a*b^4 - b^5)*d^2...
```

3.203.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**8/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.203.7 Maxima [F]

$$\int \frac{\sin^8(c+dx)}{a-b\sin^4(c+dx)}\,dx=\int -\frac{\sin\left(dx+c\right)^8}{b\sin\left(dx+c\right)^4-a}\,dx$$

input
```
integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
-1/32*(512*a^2*b^2*d*integrate((b*cos(8*d*x + 8*c)*cos(4*d*x + 4*c) - 4*b*
cos(6*d*x + 6*c)*cos(4*d*x + 4*c) - 2*(8*a - 3*b)*cos(4*d*x + 4*c)^2 + b*s
in(8*d*x + 8*c)*sin(4*d*x + 4*c) - 4*b*sin(6*d*x + 6*c)*sin(4*d*x + 4*c) -
 2*(8*a - 3*b)*sin(4*d*x + 4*c)^2 - 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
- (4*b*cos(2*d*x + 2*c) - b)*cos(4*d*x + 4*c))/(b^4*cos(8*d*x + 8*c)^2 + 1
6*b^4*cos(6*d*x + 6*c)^2 + 16*b^4*cos(2*d*x + 2*c)^2 + b^4*sin(8*d*x + 8*c
)^2 + 16*b^4*sin(6*d*x + 6*c)^2 + 16*b^4*sin(2*d*x + 2*c)^2 - 8*b^4*cos(2*
d*x + 2*c) + b^4 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*b^4)*cos(4*d*x + 4*c)^2 +
4*(64*a^2*b^2 - 48*a*b^3 + 9*b^4)*sin(4*d*x + 4*c)^2 + 16*(8*a*b^3 - 3*b^4
)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^4*cos(6*d*x + 6*c) + 4*b^4*co
s(2*d*x + 2*c) - b^4 + 2*(8*a*b^3 - 3*b^4)*cos(4*d*x + 4*c))*cos(8*d*x + 8
*c) + 8*(4*b^4*cos(2*d*x + 2*c) - b^4 + 2*(8*a*b^3 - 3*b^4)*cos(4*d*x + 4*
c))*cos(6*d*x + 6*c) - 4*(8*a*b^3 - 3*b^4 - 4*(8*a*b^3 - 3*b^4)*cos(2*d*x
+ 2*c))*cos(4*d*x + 4*c) - 4*(2*b^4*sin(6*d*x + 6*c) + 2*b^4*sin(2*d*x + 2
*c) + (8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*b^4*sin
(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c)), x)
+ 4*(8*a + 3*b)*d*x + b*sin(4*d*x + 4*c) - 8*b*sin(2*d*x + 2*c))/(b^2*d)
```

3.203.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(142) = 284$.

Time = 0.74 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.51

$$\int \frac{\sin^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$$
$$=\frac{\frac{4\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}a^3-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}a^2b-\sqrt{a^2-ab+\sqrt{ab}(a-b)}ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(dx+c)}{\sqrt{\frac{ab^2+\sqrt{a^2b^4-\left(ab^2-b^3\right)ab^2}}{ab^2-b^3}}}\right)\right)|-a}{3a^4b^2-12a^3b^3+14a^2b^4-4ab^5-b^6}}{}$$

input
```
integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
1/8*(4*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3 - 6*sqrt(a^2 - a*b + sqr
t(a*b)*(a - b))*a^2*b - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^2)*(pi*flo
or((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b^2 + sqrt(a^2*b^4 -
(a*b^2 - b^3)*a*b^2))/(a*b^2 - b^3))))*abs(-a + b)/(3*a^4*b^2 - 12*a^3*b^3
 + 14*a^2*b^4 - 4*a*b^5 - b^6) + 4*(3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*
a^3 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b - sqrt(a^2 - a*b - sqrt(
a*b)*(a - b))*a*b^2)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/s
qrt((a*b^2 - sqrt(a^2*b^4 - (a*b^2 - b^3)*a*b^2))/(a*b^2 - b^3))))*abs(-a
+ b)/(3*a^4*b^2 - 12*a^3*b^3 + 14*a^2*b^4 - 4*a*b^5 - b^6) - (d*x + c)*(8*
a + 3*b)/b^2 + (5*tan(d*x + c)^3 + 3*tan(d*x + c))/((tan(d*x + c)^2 + 1)^2
*b))/d
```

3.203.9 Mupad [B] (verification not implemented)

Time = 17.02 (sec) , antiderivative size = 5022, normalized size of antiderivative = 27.29

$$\int \frac{\sin^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^8/(a - b*sin(c + d*x)^4),x)
```

output

```
(atan((((((2048*a^3*b^10 + 8192*a^4*b^9 - 22528*a^5*b^8 + 12288*a^6*b^7)/
(64*b^5) - (tan(c + d*x)*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))
^(1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/
(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (tan(c
 + d*x)*(432*a^2*b^9 + 1584*a^3*b^8 - 880*a^4*b^7 - 5488*a^5*b^6 + 2048*a^
6*b^5 + 2304*a^7*b^4))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8
- b^9)))^(1/2) - (144*a^3*b^8 + 624*a^4*b^7 + 112*a^5*b^6 - 1648*a^6*b^5 +
 1536*a^7*b^4 - 768*a^8*b^3)/(64*b^5))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(
a*b^8 - b^9)))^(1/2) + (tan(c + d*x)*(9*a^4*b^5 - 96*a^9 - 336*a^8*b + 93*
a^5*b^4 + 259*a^6*b^3 + 71*a^7*b^2))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^
4)/(16*(a*b^8 - b^9)))^(1/2)*1i - (((((2048*a^3*b^10 + 8192*a^4*b^9 - 2252
8*a^5*b^8 + 12288*a^6*b^7)/(64*b^5) + (tan(c + d*x)*(-((a^5*b^9)^(1/2) + a
^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288
*a^4*b^9 + 12288*a^5*b^8))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*
b^8 - b^9)))^(1/2) + (tan(c + d*x)*(432*a^2*b^9 + 1584*a^3*b^8 - 880*a^4*b
^7 - 5488*a^5*b^6 + 2048*a^6*b^5 + 2304*a^7*b^4))/(16*b^4))*(-((a^5*b^9)^(
1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (144*a^3*b^8 + 624*a^4*b^7 + 1
12*a^5*b^6 - 1648*a^6*b^5 + 1536*a^7*b^4 - 768*a^8*b^3)/(64*b^5))*(-((a^5*
b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (tan(c + d*x)*(9*a^4*b^5
 - 96*a^9 - 336*a^8*b + 93*a^5*b^4 + 259*a^6*b^3 + 71*a^7*b^2))/(16*b^4...
```

3.203. $\int \frac{\sin^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.204 $\int \frac{\sin^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.204.1 Optimal result

Integrand size = 24, antiderivative size = 155

$$\int \frac{\sin^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{x}{2b}+\frac{a^{3/4}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}b^{3/2}d} - \frac{a^{3/4}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}b^{3/2}d}+\frac{\cos(c+dx)\sin(c+dx)}{2bd}$$

output
```
-1/2*x/b+1/2*cos(d*x+c)*sin(d*x+c)/b/d+1/2*a^(3/4)*arctan((a^(1/2)-b^(1/2)
)^(1/2)*tan(d*x+c)/a^(1/4))/b^(3/2)/d/(a^(1/2)-b^(1/2))^(1/2)-1/2*a^(3/4)*
arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/b^(3/2)/d/(a^(1/2)+b^(1
/2))^(1/2)
```

3.204.2 Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01

$$\int \frac{\sin^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{-2\sqrt{b}(c+dx)-\frac{2a\arctan\left(\frac{(\sqrt{a}+\sqrt{b})\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}-\frac{2a\,\mathrm{arctanh}\left(\frac{(\sqrt{a}-\sqrt{b})\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+\sqrt{b}\sin(2(c+dx))}{4b^{3/2}d}$$

input
```
Integrate[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

output
```
(-2*Sqrt[b]*(c + d*x) - (2*a*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqr
t[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - (2*a*ArcTanh[((Sqrt[a
] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*
Sqrt[b]] + Sqrt[b]*Sin[2*(c + d*x)])/(4*b^(3/2)*d)
```

3.204.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3696, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(c+dx)}{a-b\sin^4(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^6}{a-b\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\tan^6(c+dx)}{(\tan^2(c+dx)+1)^2((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1610}$$

$$\frac{\int \left(\frac{a\tan^2(c+dx)}{b((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}-\frac{1}{b(\tan^2(c+dx)+1)}+\frac{1}{b(\tan^2(c+dx)+1)^2}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^{3/4}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{a^{3/4}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\arctan(\tan(c+dx))}{2b}+\frac{\tan(c+dx)}{2b(\tan^2(c+dx)+1)}}{d}$$

input
```
Int[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

output
```
(-1/2*ArcTan[Tan[c + d*x]]/b + (a^(3/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Ta
n[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(3/2)) - (a^(3/4)*ArcTa
n[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[
b]]*b^(3/2)) + Tan[c + d*x]/(2*b*(1 + Tan[c + d*x]^2)))/d
```

3.204.3.1 Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

3.204.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.31 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10

method	result
risch	$-\frac{x}{2b}-\frac{i e^{2i(dx+c)}}{8bd}+\frac{i e^{-2i(dx+c)}}{8bd}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a\,b^{6}d^{4}-b^{7}d^{4}\right)\_Z^{4}+512a^{2}b^{3}d^{2}\_Z^{2}+65536a^{3}\right)} \_R\ln\left(e^{2i(dx+c)}+\right.\right.}{64}$
derivativedivides	$\frac{-\frac{-\frac{\tan(dx+c)}{2\left(1+\tan^{2}(dx+c)\right)}+\frac{\arctan(\tan(dx+c))}{2}}{b}+\frac{a(a-b)\left(\frac{\left(\sqrt{ab}+a\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2(a-b)\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-a\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+}{\sqrt{\left(\sqrt{ab}-a\right)(a-}}\right.}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right.}{b}}{d}$
default	$\frac{-\frac{-\frac{\tan(dx+c)}{2\left(1+\tan^{2}(dx+c)\right)}+\frac{\arctan(\tan(dx+c))}{2}}{b}+\frac{a(a-b)\left(\frac{\left(\sqrt{ab}+a\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2(a-b)\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-a\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+}{\sqrt{\left(\sqrt{ab}-a\right)(a-}}\right.}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right.}{b}}{d}$

input
```
int(sin(d*x+c)^6/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/b-1/8*I/b/d*exp(2*I*(d*x+c))+1/8*I/b/d*exp(-2*I*(d*x+c))-1/64*sum(_
R*ln(exp(2*I*(d*x+c))+(1/2048*I/a*b^4*d^3-1/2048*I/a^2*b^5*d^3)*_R^3+(-1/1
28*b^2*d^2+1/128/a*b^3*d^2)*_R^2+1/4*I*d*b*_R-2/b*a-1),_R=RootOf((a*b^6*d^
4-b^7*d^4)*_Z^4+512*a^2*b^3*d^2*_Z^2+65536*a^3))
```

3.204.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1275 vs. 2(111) = 222.

Time = 0.45 (sec) , antiderivative size = 1275, normalized size of antiderivative = 8.23

$$\int \frac{\sin^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/8*(b*d*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4
)) + a^2)/((a*b^3 - b^4)*d^2))*log(1/4*a^2*cos(d*x + c)^2 - 1/4*a^2 - 1/4*
(2*(a^2*b^2 - a*b^3)*d^2*cos(d*x + c)^2 - (a^2*b^2 - a*b^3)*d^2)*sqrt(a^3/
((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + 1/2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b
^5 - 2*a*b^6 + b^7)*d^4))*cos(d*x + c)*sin(d*x + c) - a^2*b*d*cos(d*x + c)
*sin(d*x + c))*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7
)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))) - b*d*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a
^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))*log(1/4*a^
2*cos(d*x + c)^2 - 1/4*a^2 - 1/4*(2*(a^2*b^2 - a*b^3)*d^2*cos(d*x + c)^2 -
 (a^2*b^2 - a*b^3)*d^2)*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - 1/2*((
a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4))*cos(d*x + c)*si
n(d*x + c) - a^2*b*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a*b^3 - b^4)*d^2*s
qrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))) + b*
d*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2)
/((a*b^3 - b^4)*d^2))*log(-1/4*a^2*cos(d*x + c)^2 + 1/4*a^2 - 1/4*(2*(a^2*
b^2 - a*b^3)*d^2*cos(d*x + c)^2 - (a^2*b^2 - a*b^3)*d^2)*sqrt(a^3/((a^2*b^
5 - 2*a*b^6 + b^7)*d^4)) + 1/2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a
*b^6 + b^7)*d^4))*cos(d*x + c)*sin(d*x + c) + a^2*b*d*cos(d*x + c)*sin(d*x
 + c))*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) -
 a^2)/((a*b^3 - b^4)*d^2))) - b*d*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^...
```

3.204.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**6/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.204.7 Maxima [F]

$$\int \frac{\sin^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\sin{(dx+c)}^6}{b\sin{(dx+c)}^4-a}\,dx$$

input
```
integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/4*(4*b*d*integrate(-4*(4*a*b*cos(6*d*x + 6*c)^2 + 4*a*b*cos(2*d*x + 2*c)
^2 + 4*a*b*sin(6*d*x + 6*c)^2 + 4*a*b*sin(2*d*x + 2*c)^2 - 4*(8*a^2 - 3*a*
b)*cos(4*d*x + 4*c)^2 - a*b*cos(2*d*x + 2*c) - 4*(8*a^2 - 3*a*b)*sin(4*d*x
 + 4*c)^2 + 2*(8*a^2 - 7*a*b)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (a*b*cos
(6*d*x + 6*c) - 2*a*b*cos(4*d*x + 4*c) + a*b*cos(2*d*x + 2*c))*cos(8*d*x +
 8*c) + (8*a*b*cos(2*d*x + 2*c) - a*b + 2*(8*a^2 - 7*a*b)*cos(4*d*x + 4*c)
)*cos(6*d*x + 6*c) + 2*(a*b + (8*a^2 - 7*a*b)*cos(2*d*x + 2*c))*cos(4*d*x
+ 4*c) - (a*b*sin(6*d*x + 6*c) - 2*a*b*sin(4*d*x + 4*c) + a*b*sin(2*d*x +
2*c))*sin(8*d*x + 8*c) + 2*(4*a*b*sin(2*d*x + 2*c) + (8*a^2 - 7*a*b)*sin(4
*d*x + 4*c))*sin(6*d*x + 6*c))/(b^3*cos(8*d*x + 8*c)^2 + 16*b^3*cos(6*d*x
+ 6*c)^2 + 16*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(8*d*x + 8*c)^2 + 16*b^3*sin
(6*d*x + 6*c)^2 + 16*b^3*sin(2*d*x + 2*c)^2 - 8*b^3*cos(2*d*x + 2*c) + b^3
 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b - 48*a
*b^2 + 9*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) - 2*(4*b^3*cos(6*d*x + 6*c) + 4*b^3*cos(2*d*x + 2*c) - b^3
 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*b^3*cos(2
*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c)
 - 4*(8*a*b^2 - 3*b^3 - 4*(8*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))*cos(4*d*x +
4*c) - 4*(2*b^3*sin(6*d*x + 6*c) + 2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b
^3)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*b^3*sin(2*d*x + 2*c) + (...
```

3.204.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(111) = 222$.

Time = 0.78 (sec) , antiderivative size = 695, normalized size of antiderivative = 4.48

$$\int \frac{\sin^6(c+dx)}{a-b\sin^4(c+dx)}\,dx =$$

$$-\frac{\frac{dx+c}{b} + \frac{\left(\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}a^2-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}ab-\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}b^2\right)b^2|-a+b|-\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{\cdots}\right.\right.}{(3a^5b^2-15a^4b^3+26a^3\cdots}}{\cdots}$$

input
```
integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
-1/2*((d*x + c)/b + ((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2
- 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) - (3*sqrt(a^2 - a*b + sqr
t(a*b)*(a - b))*sqrt(a*b)*a^3*b - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sq
rt(a*b)*a^2*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3)*abs
(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b +
sqrt(a^2*b^2 - (a*b - b^2)*a*b))/(a*b - b^2))))/((3*a^5*b^2 - 15*a^4*b^3 +
 26*a^3*b^4 - 18*a^2*b^5 + 3*a*b^6 + b^7)*abs(b)) - ((3*sqrt(a^2 - a*b - s
qrt(a*b)*(a - b))*sqrt(a*b)*a^2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sq
rt(a*b)*a*b - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-
a + b) - (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 6*sqrt(a
^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 - sqrt(a^2 - a*b - sqrt(a*
b)*(a - b))*sqrt(a*b)*a*b^3)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) +
arctan(tan(d*x + c)/sqrt((a*b - sqrt(a^2*b^2 - (a*b - b^2)*a*b))/(a*b - b^
2))))/((3*a^5*b^2 - 15*a^4*b^3 + 26*a^3*b^4 - 18*a^2*b^5 + 3*a*b^6 + b^7)*
abs(b)) - tan(d*x + c)/((tan(d*x + c)^2 + 1)*b))/d
```

3.204.9 Mupad [B] (verification not implemented)

Time = 16.09 (sec) , antiderivative size = 1273, normalized size of antiderivative = 8.21

$$\int \frac{\sin^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^6/(a - b*sin(c + d*x)^4),x)
```

output
```
sin(2*c + 2*d*x)/(4*b*d) - (atan((a*b^7*sin(c + d*x)*(((a^3*b^7)^(1/2) - a
^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*4i - b^12*sin(c + d*x)*(((a^3*b^7)^(1/2
) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(5/2)*3072i - b^10*sin(c + d*x)*(((a^3*b
^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(3/2)*192i + a*b^9*sin(c + d*x)*
(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(3/2)*192i + a^2*b^6*sin
(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*24i + a^
3*b^5*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)
*4i + a^4*b^4*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7
))^(1/2)*8i + a^2*b^8*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6
- 16*b^7))^(3/2)*448i + a^3*b^7*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/
(16*a*b^6 - 16*b^7))^(3/2)*320i + a^2*b^10*sin(c + d*x)*(((a^3*b^7)^(1/2)
- a^2*b^3)/(16*a*b^6 - 16*b^7))^(5/2)*3072i)/(a^2*b^5*cos(c + d*x) + a^3*b
^4*cos(c + d*x) - a^4*b^3*cos(c + d*x) - a^2*cos(c + d*x)*(a^3*b^7)^(1/2)
+ 2*a*b*cos(c + d*x)*(a^3*b^7)^(1/2)))*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*
b^6 - 16*b^7))^(1/2)*2i)/d - (atan((a*b^7*sin(c + d*x)*(-((a^3*b^7)^(1/2)
+ a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*4i - b^12*sin(c + d*x)*(-((a^3*b^7)^
(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(5/2)*3072i - b^10*sin(c + d*x)*(-((
a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(3/2)*192i + a*b^9*sin(c +
d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(3/2)*192i + a^2*b
^6*sin(c + d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2...
```

3.204. $\int \frac{\sin^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.205 $\int \frac{\sin^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.205.1 Optimal result

Integrand size = 24, antiderivative size = 127

$$\int \frac{\sin^4(c+dx)}{a-b\sin^4(c+dx)}\,dx=-\frac{x}{b}+\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}bd}+\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}bd}$$

output
```
-x/b+1/2*a^(1/4)*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/b/d/(a
^(1/2)-b^(1/2))^(1/2)+1/2*a^(1/4)*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c
)/a^(1/4))/b/d/(a^(1/2)+b^(1/2))^(1/2)
```

3.205.2 Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13

$$\int \frac{\sin^4(c+dx)}{a-b\sin^4(c+dx)}\,dx=\frac{-2(c+dx)+\frac{\sqrt{a}\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}}{2bd}$$

input
```
Integrate[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

output
```
(-2*(c + d*x) + (Sqrt[a]*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a
+ Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - (Sqrt[a]*ArcTanh[((Sqrt[a
] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*
Sqrt[b]])/(2*b*d)
```

3.205.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3696, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{a-b\sin^4(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^4}{a-b\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\tan^4(c+dx)}{(\tan^2(c+dx)+1)((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1610}$$

$$\frac{\int \left(\frac{a(\tan^2(c+dx)+1)}{b((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}-\frac{1}{b(\tan^2(c+dx)+1)}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\arctan(\tan(c+dx))}{b}}{d}$$

input
```
Int[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

output
```
(-(ArcTan[Tan[c + d*x]]/b) + (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[
c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b) + (a^(1/4)*ArcTan[(Sqrt[
Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b))/
d
```

3.205.3.1 Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

3.205.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.72 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{x}{b}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a\,b^4d^4-b^5d^4\right)\_Z^4+32a\,b^2d^2\_Z^2+256a\right)}\_R\ln\left(\mathrm{e}^{2i(dx+c)}+\left(\frac{1}{32}ia\,b^2d^3-\frac{1}{32}ib^3d^3\right)\_R^3+\left(-\frac{1}{8}b\,d\right.\right.\right.}{16}$
derivativedivides	$\frac{-\frac{\arctan(\tan(dx+c))}{b}+\frac{a(a-b)\left(\frac{\left(\sqrt{ab}+b\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{b}}{d}$
default	$\frac{-\frac{\arctan(\tan(dx+c))}{b}+\frac{a(a-b)\left(\frac{\left(\sqrt{ab}+b\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{b}}{d}$

input
```
int(sin(d*x+c)^4/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
-x/b+1/16*sum(_R*ln(exp(2*I*(d*x+c))+(1/32*I*a*b^2*d^3-1/32*I*b^3*d^3)*_R^
3+(-1/8*b*d^2*a+1/8*b^2*d^2)*_R^2+(1/2*I*a*d+1/2*I*d*b)*_R-2/b*a-1),_R=Roo
tOf((a*b^4*d^4-b^5*d^4)*_Z^4+32*a*b^2*d^2*_Z^2+256*a))
```

3.205.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1125 vs. $2(91) = 182$.

Time = 0.40 (sec) , antiderivative size = 1125, normalized size of antiderivative = 8.86

$$\int \frac{\sin^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/8*(b*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) +
a)/((a*b^2 - b^3)*d^2))*log(1/4*cos(d*x + c)^2 + 1/2*((a*b^2 - b^3)*d^3*sq
rt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4))*cos(d*x + c)*sin(d*x + c) - b*d*cos(
d*x + c)*sin(d*x + c))*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4
 + b^5)*d^4)) + a)/((a*b^2 - b^3)*d^2)) - 1/4*(2*(a*b - b^2)*d^2*cos(d*x +
 c)^2 - (a*b - b^2)*d^2)*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1/4) -
b*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + a)/((
a*b^2 - b^3)*d^2))*log(1/4*cos(d*x + c)^2 - 1/2*((a*b^2 - b^3)*d^3*sqrt(a/
((a^2*b^3 - 2*a*b^4 + b^5)*d^4))*cos(d*x + c)*sin(d*x + c) - b*d*cos(d*x +
 c)*sin(d*x + c))*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^
5)*d^4)) + a)/((a*b^2 - b^3)*d^2)) - 1/4*(2*(a*b - b^2)*d^2*cos(d*x + c)^2
 - (a*b - b^2)*d^2)*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1/4) + b*sqr
t(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2
- b^3)*d^2))*log(-1/4*cos(d*x + c)^2 + 1/2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2
*b^3 - 2*a*b^4 + b^5)*d^4))*cos(d*x + c)*sin(d*x + c) + b*d*cos(d*x + c)*s
in(d*x + c))*sqrt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4
)) - a)/((a*b^2 - b^3)*d^2)) - 1/4*(2*(a*b - b^2)*d^2*cos(d*x + c)^2 - (a*
b - b^2)*d^2)*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1/4) - b*sqrt(((a*
b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)
*d^2))*log(-1/4*cos(d*x + c)^2 - 1/2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^...
```

3.205.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**4/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.205.7 Maxima [F]

$$\int \frac{\sin^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\sin\left(dx+c\right)^4}{b\sin\left(dx+c\right)^4-a}\,dx$$

input
```
integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
-(16*a*b*integrate((b*cos(8*d*x + 8*c)*cos(4*d*x + 4*c) - 4*b*cos(6*d*x +
6*c)*cos(4*d*x + 4*c) - 2*(8*a - 3*b)*cos(4*d*x + 4*c)^2 + b*sin(8*d*x + 8
*c)*sin(4*d*x + 4*c) - 4*b*sin(6*d*x + 6*c)*sin(4*d*x + 4*c) - 2*(8*a - 3*
b)*sin(4*d*x + 4*c)^2 - 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (4*b*cos(2
*d*x + 2*c) - b)*cos(4*d*x + 4*c))/(b^3*cos(8*d*x + 8*c)^2 + 16*b^3*cos(6*
d*x + 6*c)^2 + 16*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(8*d*x + 8*c)^2 + 16*b^3
*sin(6*d*x + 6*c)^2 + 16*b^3*sin(2*d*x + 2*c)^2 - 8*b^3*cos(2*d*x + 2*c) +
 b^3 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b -
48*a*b^2 + 9*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a*b^2 - 3*b^3)*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) - 2*(4*b^3*cos(6*d*x + 6*c) + 4*b^3*cos(2*d*x + 2*c) -
 b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*b^3*c
os(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(6*d*x +
6*c) - 4*(8*a*b^2 - 3*b^3 - 4*(8*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))*cos(4*d*
x + 4*c) - 4*(2*b^3*sin(6*d*x + 6*c) + 2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 -
 3*b^3)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*b^3*sin(2*d*x + 2*c) +
(8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c)), x) + x)/b
```

3.205.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(91) = 182$.

Time = 0.72 (sec) , antiderivative size = 912, normalized size of antiderivative = 7.18

$$\int \frac{\sin^4(c+dx)}{a-b\sin^4(c+dx)}\,dx =$$

$$-\frac{\frac{2\,(dx+c)}{b} + \frac{\left(\left(3\,\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}a^2-6\,\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}ab-\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}b^2\right)b^2|-a+b|-\left(3\,\sqrt{a^2-ab+\sqrt{ab}(a-\cdots\right.\right.}{\cdots}}{\cdots}$$

input
```
integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
-1/2*(2*(d*x + c)/b + ((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^
2 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b +
 sqrt(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) - (3*sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*a^3*b - 9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^2 +
5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^3 + sqrt(a^2 - a*b + sqrt(a*b)*(
a - b))*b^4)*abs(-a + b)*abs(b) - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*s
qrt(a*b)*a^2*b^2 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 -
 sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*abs(-a + b))*(pi*floor
((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b + sqrt(a^2*b^2 - (a*b
 - b^2)*a*b))/(a*b - b^2))))/((3*a^5*b^2 - 15*a^4*b^3 + 26*a^3*b^4 - 18*a^
2*b^5 + 3*a*b^6 + b^7)*abs(b)) + ((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*s
qrt(a*b)*a^2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(
a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) - (3*sqrt(a^
2 - a*b - sqrt(a*b)*(a - b))*a^3*b - 9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))
*a^2*b^2 + 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^3 + sqrt(a^2 - a*b -
sqrt(a*b)*(a - b))*b^4)*abs(-a + b)*abs(b) - (3*sqrt(a^2 - a*b - sqrt(a*b)
*(a - b))*sqrt(a*b)*a^2*b^2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a
*b)*a*b^3 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*abs(-a + b)
)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b - sqrt(a^2
*b^2 - (a*b - b^2)*a*b))/(a*b - b^2))))/((3*a^5*b^2 - 15*a^4*b^3 + 26*a...
```

3.205.9 Mupad [B] (verification not implemented)

Time = 16.32 (sec) , antiderivative size = 2991, normalized size of antiderivative = 23.55

$$\int \frac{\sin^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^4/(a - b*sin(c + d*x)^4),x)
```

output
```
- atan((18*a^5*tan(c + d*x))/(18*a^5 - 50*a^4*b + 32*a^3*b^2) - (50*a^4*ta
n(c + d*x))/(32*a^3*b - 50*a^4 + (18*a^5)/b) + (32*a^3*b*tan(c + d*x))/(32
*a^3*b - 50*a^4 + (18*a^5)/b))/(b*d) - (atan((((-(a*b^2 - (a*b^5)^(1/2))/(
16*(a*b^4 - b^5)))^(1/2)*(((-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(
1/2)*(320*a^3*b^5 - 64*a^2*b^6 - 448*a^4*b^4 + 192*a^5*b^3 + tan(c + d*x)*
(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(768*a^2*b^7 - 768*a^3
*b^6 - 768*a^4*b^5 + 768*a^5*b^4)) + tan(c + d*x)*(176*a^2*b^5 - 400*a^3*b
^4 + 80*a^4*b^3 + 144*a^5*b^2))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5
)))^(1/2) + 12*a^5*b - 16*a^2*b^4 + 28*a^3*b^3 - 24*a^4*b^2) + tan(c + d*x
)*(18*a^4*b + 6*a^5 - 4*a^2*b^3 - 20*a^3*b^2))*(-(a*b^2 - (a*b^5)^(1/2))/(
16*(a*b^4 - b^5)))^(1/2)*1i + ((-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)
))^(1/2)*(((-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(64*a^2*b^6
 - 320*a^3*b^5 + 448*a^4*b^4 - 192*a^5*b^3 + tan(c + d*x)*(-(a*b^2 - (a*b^
5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b
^5 + 768*a^5*b^4)) + tan(c + d*x)*(176*a^2*b^5 - 400*a^3*b^4 + 80*a^4*b^3
+ 144*a^5*b^2))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2) - 12*a
^5*b + 16*a^2*b^4 - 28*a^3*b^3 + 24*a^4*b^2) + tan(c + d*x)*(18*a^4*b + 6*
a^5 - 4*a^2*b^3 - 20*a^3*b^2))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)
))^(1/2)*1i)/(6*a^3*b - 6*a^4 + ((-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^
5)))^(1/2)*(((-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(320*a...
```

3.205. $\int \frac{\sin^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.206 $\int \frac{\sin^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.206.1 Optimal result

Integrand size = 24, antiderivative size = 125

$$\int \frac{\sin^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt{b}d} - \frac{\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt{b}d}$$

output
```
1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(1/4)/d/b^(1/2)/(
a^(1/2)-b^(1/2))^(1/2)-1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/
4))/a^(1/4)/d/b^(1/2)/(a^(1/2)+b^(1/2))^(1/2)
```

3.206.2 Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$\int \frac{\sin^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}d} - \frac{\text{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}d}$$

input
```
Integrate[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

output
```
-1/2*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]]/
(Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]*d) - ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c
 + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/(2*Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]
*d)
```

3.206.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.34, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3696, 1450, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{a-b\sin^4(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^2}{a-b\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\tan^2(c+dx)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1450}$$

$$\frac{\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{b}}\right)\int \frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tan(c+dx)+\frac{1}{2}\left(\frac{\sqrt{a}}{\sqrt{b}}+1\right)\int \frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\left(\frac{\sqrt{a}}{\sqrt{b}}+1\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\left(\sqrt{a}+\sqrt{b}\right)}+\frac{\left(1-\frac{\sqrt{a}}{\sqrt{b}}\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}}{d}$$

input
```
Int[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

output
```
(((1 + Sqrt[a]/Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1
/4)])/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*(Sqrt[a] + Sqrt[b])) + ((1 - Sqrt
[a]/Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^
(1/4)*(Sqrt[a] - Sqrt[b])*Sqrt[Sqrt[a] + Sqrt[b]]))/d
```

3.206.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

3.206. $\int \frac{\sin^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.206.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.69 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a^2b^2d^4-a\,b^3d^4\right)\_Z^4+2a\,d^2\_Z^2b\right)}\_R\ln\left(\mathrm{e}^{2i(dx+c)}+\left(2ia^2b\,d^3-2ia\,b^2d^3\right)\_R^3+\left(-2a^2d^2+2b\,d^2a\right)\ldots\right.\right.}{4}$
derivativedivides	$\frac{(a-b)\left(\frac{\left(\sqrt{ab}+a\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2(a-b)\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-a\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{d}$
default	$\frac{(a-b)\left(\frac{\left(\sqrt{ab}+a\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2(a-b)\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-a\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{d}$

input
```
int(sin(d*x+c)^2/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/4*sum(_R*ln(exp(2*I*(d*x+c))+(2*I*a^2*b*d^3-2*I*a*b^2*d^3)*_R^3+(-2*a^2
*d^2+2*a*b*d^2)*_R^2+4*I*a*d*_R-2/b*a-1),_R=RootOf(1+(a^2*b^2*d^4-a*b^3*d^
4)*_Z^4+2*a*d^2*_Z^2*b))
```

3.206.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1087 vs. 2(85) = 170.

Time = 0.41 (sec) , antiderivative size = 1087, normalized size of antiderivative = 8.70

$$\int \frac{\sin^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/8*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)
/((a*b - b^2)*d^2))*log(1/4*cos(d*x + c)^2 + 1/2*((a^2*b - a*b^2)*d^3*sqrt
(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4))*cos(d*x + c)*sin(d*x + c) - a*d*cos(
d*x + c)*sin(d*x + c))*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 +
 a*b^3)*d^4)) + 1)/((a*b - b^2)*d^2)) - 1/4*(2*(a^2 - a*b)*d^2*cos(d*x + c
)^2 - (a^2 - a*b)*d^2)*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1/4) +
1/8*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/
((a*b - b^2)*d^2))*log(1/4*cos(d*x + c)^2 - 1/2*((a^2*b - a*b^2)*d^3*sqrt(
1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4))*cos(d*x + c)*sin(d*x + c) - a*d*cos(d
*x + c)*sin(d*x + c))*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 +
a*b^3)*d^4)) + 1)/((a*b - b^2)*d^2)) - 1/4*(2*(a^2 - a*b)*d^2*cos(d*x + c)
^2 - (a^2 - a*b)*d^2)*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1/4) - 1
/8*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((
a*b - b^2)*d^2))*log(-1/4*cos(d*x + c)^2 + 1/2*((a^2*b - a*b^2)*d^3*sqrt(1
/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4))*cos(d*x + c)*sin(d*x + c) + a*d*cos(d*
x + c)*sin(d*x + c))*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*
b^3)*d^4)) - 1)/((a*b - b^2)*d^2)) - 1/4*(2*(a^2 - a*b)*d^2*cos(d*x + c)^2
 - (a^2 - a*b)*d^2)*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1/4) + 1/8
*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*
b - b^2)*d^2))*log(-1/4*cos(d*x + c)^2 - 1/2*((a^2*b - a*b^2)*d^3*sqrt(...
```

3.206.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**2/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.206.7 Maxima [F]

$$\int \frac{\sin^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\sin{(dx+c)}^2}{b\sin{(dx+c)}^4-a}\,dx$$

input
```
integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
-integrate(sin(d*x + c)^2/(b*sin(d*x + c)^4 - a), x)
```

3.206.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(85) = 170$.

Time = 0.79 (sec) , antiderivative size = 397, normalized size of antiderivative = 3.18

$$\int \frac{\sin^2(c+dx)}{a-b\sin^4(c+dx)}\,dx =$$

$$-\frac{\frac{\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}a^2-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}ab-\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}b^2\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(dx+c)}{\sqrt{\frac{4a+\sqrt{-16(a-b)a+16a}}{a-b}}}\right.\right.}{3a^5b-12a^4b^2+14a^3b^3-4a^2b^4-ab^5}}{}$$

input
```
integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
-1/2*((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2 - 6*sqrt(a^2 -
a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b + sqrt(a*b)*(a - b
))*sqrt(a*b)*b^2)*(pi*floor((d*x + c)/pi + 1/2) + arctan(2*tan(d*x + c)/sq
rt((4*a + sqrt(-16*(a - b)*a + 16*a^2))/(a - b))))*abs(a - b)/(3*a^5*b - 1
2*a^4*b^2 + 14*a^3*b^3 - 4*a^2*b^4 - a*b^5) + (3*sqrt(a^2 - a*b - sqrt(a*b
)*(a - b))*sqrt(a*b)*a^2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)
*a*b - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^2)*(pi*floor((d*x +
 c)/pi + 1/2) + arctan(2*tan(d*x + c)/sqrt((4*a - sqrt(-16*(a - b)*a + 16*
a^2))/(a - b))))*abs(a - b)/(3*a^5*b - 12*a^4*b^2 + 14*a^3*b^3 - 4*a^2*b^4
 - a*b^5))/d
```

3.206.9 Mupad [B] (verification not implemented)

Time = 16.78 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.54

$$\int \frac{\sin^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\ln\left(a\,b-a^2-\frac{a\tan(c+d\,x)\,(a-b)\,\sqrt{-\frac{1}{a\,b+\sqrt{a\,b^3}}}\,\left(2\,a\,b^2+a\,\sqrt{a\,b^3}+b\,\sqrt{a\,b^3}\right)}{a\,b+\sqrt{a\,b^3}}\right)\,\sqrt{-\frac{1}{a\,b+\sqrt{a\,b^3}}}}{4\,d} - \frac{\ln\left(a\,b-a^2-\frac{a\tan(c+d\,x)\,\sqrt{-\frac{1}{a\,b-\sqrt{a\,b^3}}}\,(a-b)\,\left(a\,\sqrt{a\,b^3}-2\,a\,b^2+b\,\sqrt{a\,b^3}\right)}{a\,b-\sqrt{a\,b^3}}\right)\,\sqrt{\frac{a\,b+\sqrt{a\,b^3}}{16\,(a\,b^3-a^2\,b^2)}}}{d} + \frac{\ln\left(a\,b-a^2+\frac{a\tan(c+d\,x)\,\sqrt{-\frac{1}{a\,b-\sqrt{a\,b^3}}}\,(a-b)\,\left(a\,\sqrt{a\,b^3}-2\,a\,b^2+b\,\sqrt{a\,b^3}\right)}{a\,b-\sqrt{a\,b^3}}\right)\,\sqrt{-\frac{1}{a\,b-\sqrt{a\,b^3}}}}{4\,d} - \frac{\ln\left(a\,b-a^2+\frac{a\tan(c+d\,x)\,(a-b)\,\sqrt{-\frac{1}{a\,b+\sqrt{a\,b^3}}}\,\left(2\,a\,b^2+a\,\sqrt{a\,b^3}+b\,\sqrt{a\,b^3}\right)}{a\,b+\sqrt{a\,b^3}}\right)\,\sqrt{\frac{a\,b-\sqrt{a\,b^3}}{16\,(a\,b^3-a^2\,b^2)}}}{d}$$

input
```
int(sin(c + d*x)^2/(a - b*sin(c + d*x)^4),x)
```

output
```
(log(a*b - a^2 - (a*tan(c + d*x)*(a - b)*(-1/(a*b + (a*b^3)^(1/2)))^(1/2)*
(2*a*b^2 + a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2)))/(a*b + (a*b^3)^(1/2)))*(-1/
(a*b + (a*b^3)^(1/2)))^(1/2))/(4*d) - (log(a*b - a^2 - (a*tan(c + d*x)*(-1
/(a*b - (a*b^3)^(1/2)))^(1/2)*(a - b)*(a*(a*b^3)^(1/2) - 2*a*b^2 + b*(a*b^
3)^(1/2)))/(a*b - (a*b^3)^(1/2)))*((a*b + (a*b^3)^(1/2))/(16*(a*b^3 - a^2*
b^2)))^(1/2))/d + (log(a*b - a^2 + (a*tan(c + d*x)*(-1/(a*b - (a*b^3)^(1/2
)))^(1/2)*(a - b)*(a*(a*b^3)^(1/2) - 2*a*b^2 + b*(a*b^3)^(1/2)))/(a*b - (a
*b^3)^(1/2)))*(-1/(a*b - (a*b^3)^(1/2)))^(1/2))/(4*d) - (log(a*b - a^2 + (
a*tan(c + d*x)*(a - b)*(-1/(a*b + (a*b^3)^(1/2)))^(1/2)*(2*a*b^2 + a*(a*b^
3)^(1/2) + b*(a*b^3)^(1/2)))/(a*b + (a*b^3)^(1/2)))*((a*b - (a*b^3)^(1/2))
/(16*(a*b^3 - a^2*b^2)))^(1/2))/d
```

3.207 $\int \frac{1}{a-b\sin^4(c+dx)}\,dx$

3.207.1 Optimal result

Integrand size = 15, antiderivative size = 115

$$\int \frac{1}{a-b\sin^4(c+dx)}\,dx=\frac{\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}d}+\frac{\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}d}$$

output
```
1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)-
b^(1/2))^(1/2)+1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(3
/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

3.207.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{1}{a-b\sin^4(c+dx)}\,dx=\frac{\frac{\arctan\left(\frac{(\sqrt{a}+\sqrt{b})\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}-\frac{\operatorname{arctanh}\left(\frac{(\sqrt{a}-\sqrt{b})\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}}{2\sqrt{a}d}$$

input
```
Integrate[(a - b*Sin[c + d*x]^4)^(-1),x]
```

output
```
(ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]]/Sqrt
[a + Sqrt[a]*Sqrt[b]] - ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a
 + Sqrt[a]*Sqrt[b]]]/Sqrt[-a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*d)
```

3.207.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.46, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3688, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\sin^4(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-b\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{3688}$$

$$\frac{\int \frac{\tan^2(c+dx)+1}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1480}$$

$$\frac{\frac{1}{2}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tan(c+dx)+\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\int \frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\left(\sqrt{a}+\sqrt{b}\right)}+\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}}{d}$$

input
```
Int[(a - b*Sin[c + d*x]^4)^(-1),x]
```

output
```
(((1 + Sqrt[b]/Sqrt[a])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1
/4)])/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*(Sqrt[a] + Sqrt[b])) + ((1 - Sqrt
[b]/Sqrt[a])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^
(1/4)*(Sqrt[a] - Sqrt[b])*Sqrt[Sqrt[a] + Sqrt[b]]))/d
```

3.207.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

3.207.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.57 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

method	result
risch	$\sum_{\_R=\operatorname{RootOf}\left(1+(256a^4d^4-256a^3b\,d^4)\_Z^4+32a^2d^2\_Z^2\right)} \_R\ln\left(e^{2i(dx+c)}+\left(\frac{128id^3a^4}{b}-128ia^3d^3\right)\_R^{3}$
derivativedivides	$\frac{(a-b)\left(\frac{\left(\sqrt{ab}+b\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{d}$
default	$\frac{(a-b)\left(\frac{\left(\sqrt{ab}+b\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{d}$

input
```
int(1/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(exp(2*I*(d*x+c))+(128*I/b*d^3*a^4-128*I*a^3*d^3)*_R^3+(-32/b*d^2
*a^3+32*a^2*d^2)*_R^2+(8*I/b*a^2*d+8*I*a*d)*_R-2/b*a-1),_R=RootOf(1+(256*a
^4*d^4-256*a^3*b*d^4)*_Z^4+32*a^2*d^2*_Z^2))
```

3.207.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(79) = 158$.

Time = 0.41 (sec) , antiderivative size = 1079, normalized size of antiderivative = 9.38

$$\int \frac{1}{a - b\sin^4(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/8*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((
a^2 - a*b)*d^2))*log(1/4*b*cos(d*x + c)^2 + 1/2*((a^4 - a^3*b)*d^3*sqrt(b/
((a^5 - 2*a^4*b + a^3*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) - a*b*d*cos(d*x
 + c)*sin(d*x + c))*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^
2)*d^4)) + 1)/((a^2 - a*b)*d^2)) - 1/4*(2*(a^3 - a^2*b)*d^2*cos(d*x + c)^2
 - (a^3 - a^2*b)*d^2)*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1/4*b) - 1
/8*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((a
^2 - a*b)*d^2))*log(1/4*b*cos(d*x + c)^2 - 1/2*((a^4 - a^3*b)*d^3*sqrt(b/(
(a^5 - 2*a^4*b + a^3*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) - a*b*d*cos(d*x
+ c)*sin(d*x + c))*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2
)*d^4)) + 1)/((a^2 - a*b)*d^2)) - 1/4*(2*(a^3 - a^2*b)*d^2*cos(d*x + c)^2
- (a^3 - a^2*b)*d^2)*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1/4*b) + 1/
8*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2
 - a*b)*d^2))*log(-1/4*b*cos(d*x + c)^2 + 1/2*((a^4 - a^3*b)*d^3*sqrt(b/((
a^5 - 2*a^4*b + a^3*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) + a*b*d*cos(d*x +
 c)*sin(d*x + c))*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*
d^4)) - 1)/((a^2 - a*b)*d^2)) - 1/4*(2*(a^3 - a^2*b)*d^2*cos(d*x + c)^2 -
(a^3 - a^2*b)*d^2)*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1/4*b) - 1/8*
sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 -
 a*b)*d^2))*log(-1/4*b*cos(d*x + c)^2 - 1/2*((a^4 - a^3*b)*d^3*sqrt(b/(...
```

3.207.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.207.7 Maxima [F]

$$\int \frac{1}{a-b\sin^4(c+dx)}\,dx = \int -\frac{1}{b\sin(dx+c)^4-a}\,dx$$

input
```
integrate(1/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
-integrate(1/(b*sin(d*x + c)^4 - a), x)
```

3.207.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(79) = 158.

Time = 0.45 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.14

$$\int \frac{1}{a-b\sin^4(c+dx)}\,dx = \frac{\frac{\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}a^2-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}ab-\sqrt{a^2-ab+\sqrt{ab}(a-b)}b^2\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(dx+c)}{\sqrt{\frac{4a+\sqrt{-16(a-b)a+16a^2}}{a-b}}}\right)\right)|a-b|}{3a^5-12a^4b+14a^3b^2-4a^2b^3-ab^4}+}{2d}$$

input
```
integrate(1/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
1/2*((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2 - 6*sqrt(a^2 - a*b + sqrt(
a*b)*(a - b))*a*b - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*b^2)*(pi*floor((d*
x + c)/pi + 1/2) + arctan(2*tan(d*x + c)/sqrt((4*a + sqrt(-16*(a - b)*a +
16*a^2))/(a - b))))*abs(a - b)/(3*a^5 - 12*a^4*b + 14*a^3*b^2 - 4*a^2*b^3
- a*b^4) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2 - 6*sqrt(a^2 - a*b -
 sqrt(a*b)*(a - b))*a*b - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^2)*(pi*flo
or((d*x + c)/pi + 1/2) + arctan(2*tan(d*x + c)/sqrt((4*a - sqrt(-16*(a - b
)*a + 16*a^2))/(a - b))))*abs(a - b)/(3*a^5 - 12*a^4*b + 14*a^3*b^2 - 4*a^
2*b^3 - a*b^4))/d
```

3.207.9 Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 671, normalized size of antiderivative = 5.83

$$\int \frac{1}{a-b\sin^4(c+dx)}\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{a^3\tan(c+dx)\sqrt{-\frac{1}{16\,a^2+16\,\sqrt{a^3\,b}}}\,4\mathrm{i}+a^5\tan(c+dx)\left(-\frac{1}{16\,a^2+16\,\sqrt{a^3\,b}}\right)^{3/2}64\mathrm{i}+a^3\tan(c+dx)\left(-\frac{1}{16\,a^2+16\,\sqrt{a^3\,b}}\right)^{3/2}\sqrt{a^3\,b}\,64\mathrm{i}+a^2\,b\tan \cdots}{\cdots}\right)}{\cdots} + \frac{\operatorname{atan}\left(\frac{a^3\tan(c+dx)\sqrt{-\frac{1}{16\,a^2-16\,\sqrt{a^3\,b}}}\,4\mathrm{i}+a^5\tan(c+dx)\left(-\frac{1}{16\,a^2-16\,\sqrt{a^3\,b}}\right)^{3/2}64\mathrm{i}-a^3\tan(c+dx)\left(-\frac{1}{16\,a^2-16\,\sqrt{a^3\,b}}\right)^{3/2}\sqrt{a^3\,b}\,64\mathrm{i}+a^2\,b \cdots}{\cdots}\right)}{\cdots}$$

input
```
int(1/(a - b*sin(c + d*x)^4),x)
```

output
```
(atan((a^3*tan(c + d*x)*(-1/(16*a^2 + 16*(a^3*b)^(1/2)))^(1/2)*4i + a^5*ta
n(c + d*x)*(-1/(16*a^2 + 16*(a^3*b)^(1/2)))^(3/2)*64i + a^3*tan(c + d*x)*(
-1/(16*a^2 + 16*(a^3*b)^(1/2)))^(3/2)*(a^3*b)^(1/2)*64i + a^2*b*tan(c + d*
x)*(-1/(16*a^2 + 16*(a^3*b)^(1/2)))^(1/2)*4i - a^4*b*tan(c + d*x)*(-1/(16*
a^2 + 16*(a^3*b)^(1/2)))^(3/2)*64i + a*tan(c + d*x)*(-1/(16*a^2 + 16*(a^3*
b)^(1/2)))^(1/2)*(a^3*b)^(1/2)*4i + b*tan(c + d*x)*(-1/(16*a^2 + 16*(a^3*b
)^(1/2)))^(1/2)*(a^3*b)^(1/2)*4i - a^2*b*tan(c + d*x)*(-1/(16*a^2 + 16*(a^
3*b)^(1/2)))^(3/2)*(a^3*b)^(1/2)*64i)/(a*b + (a^3*b)^(1/2)))*(-1/(16*a^2 +
 16*(a^3*b)^(1/2)))^(1/2)*2i)/d + (atan((a^3*tan(c + d*x)*(-1/(16*a^2 - 16
*(a^3*b)^(1/2)))^(1/2)*4i + a^5*tan(c + d*x)*(-1/(16*a^2 - 16*(a^3*b)^(1/2
)))^(3/2)*64i - a^3*tan(c + d*x)*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(3/2)*(a
^3*b)^(1/2)*64i + a^2*b*tan(c + d*x)*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(1/2
)*4i - a^4*b*tan(c + d*x)*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(3/2)*64i - a*t
an(c + d*x)*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(1/2)*(a^3*b)^(1/2)*4i - b*ta
n(c + d*x)*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(1/2)*(a^3*b)^(1/2)*4i + a^2*b
*tan(c + d*x)*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(3/2)*(a^3*b)^(1/2)*64i)/(a
*b - (a^3*b)^(1/2)))*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(1/2)*2i)/d
```

3.207. $\int \frac{1}{a-b\sin^4(c+dx)}\,dx$

3.208 $\int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.208.1 Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}-\sqrt{b}}d} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}+\sqrt{b}}d} - \frac{\cot(c+dx)}{ad}$$

output
```
-cot(d*x+c)/a/d+1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*b^(
1/2)/a^(5/4)/d/(a^(1/2)-b^(1/2))^(1/2)-1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*
tan(d*x+c)/a^(1/4))*b^(1/2)/a^(5/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

3.208.2 Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.03

$$\int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}} + \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}} + 2\cot(c+dx)}{2ad}$$

input
```
Integrate[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

output
```
-1/2*((Sqrt[b]*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*
Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + (Sqrt[b]*ArcTanh[((Sqrt[a] - Sqrt[b
])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] +
 2*Cot[c + d*x])/(a*d)
```

3.208.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3696, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^2\left(a-b\sin(c+dx)^4\right)}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\cot^2(c+dx)\left(\tan^2(c+dx)+1\right)^2}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1610}$$

$$\frac{\int \left(\frac{\cot^2(c+dx)}{a}+\frac{b\tan^2(c+dx)}{a\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cot(c+dx)}{a}}{d}$$

input
```
Int[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

output
```
((Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(5/
4)*Sqrt[Sqrt[a] - Sqrt[b]]) - (Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan
[c + d*x])/a^(1/4)])/(2*a^(5/4)*Sqrt[Sqrt[a] + Sqrt[b]]) - Cot[c + d*x]/a)
/d
```

3.208.3.1 Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

3.208.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.04 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{2i}{ad\left(e^{2i(dx+c)}-1\right)}-4\left(\sum_{\_R=\mathrm{RootOf}\left(\left(65536a^6d^4-65536a^5b\,d^4\right)\_Z^4+512a^3b\,d^2\_Z^2+b^2\right)}\_R\ln\left(e^{2i(dx+c)}+\right.\right.$
derivativedivides	$\frac{-\frac{1}{a\tan(dx+c)}+\frac{b(a-b)\left(\frac{\left(\sqrt{ab}+a\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2(a-b)\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-a\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{a}}{d}$
default	$\frac{-\frac{1}{a\tan(dx+c)}+\frac{b(a-b)\left(\frac{\left(\sqrt{ab}+a\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2(a-b)\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-a\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{a}}{d}$

input
```
int(csc(d*x+c)^2/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
-2*I/a/d/(exp(2*I*(d*x+c))-1)-4*sum(_R*ln(exp(2*I*(d*x+c))+(8192*I/b^2*d^3
*a^5-8192*I/b*d^3*a^4)*_R^3+(-512/b^2*d^2*a^4+512/b*d^2*a^3)*_R^2+64*I/b*a
^2*d*_R-2/b*a-1),_R=RootOf((65536*a^6*d^4-65536*a^5*b*d^4)*_Z^4+512*a^3*b*
d^2*_Z^2+b^2))
```

3.208.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. $2(99) = 198$.

Time = 0.41 (sec) , antiderivative size = 1229, normalized size of antiderivative = 8.84

$$\int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/8*(a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4
)) + b)/((a^3 - a^2*b)*d^2))*log(1/4*b^2*cos(d*x + c)^2 - 1/4*b^2 - 1/4*(2
*(a^4 - a^3*b)*d^2*cos(d*x + c)^2 - (a^4 - a^3*b)*d^2)*sqrt(b^3/((a^7 - 2*
a^6*b + a^5*b^2)*d^4)) + 1/2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b +
 a^5*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) - a^2*b*d*cos(d*x + c)*sin(d*x +
 c))*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) +
b)/((a^3 - a^2*b)*d^2)))*sin(d*x + c) - a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(
b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))*log(1/4*b^2
*cos(d*x + c)^2 - 1/4*b^2 - 1/4*(2*(a^4 - a^3*b)*d^2*cos(d*x + c)^2 - (a^4
 - a^3*b)*d^2)*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - 1/2*((a^5 - a^4
*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4))*cos(d*x + c)*sin(d*x + c
) - a^2*b*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/(
(a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2)))*sin(d*x + c) +
a*d*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)
/((a^3 - a^2*b)*d^2))*log(-1/4*b^2*cos(d*x + c)^2 + 1/4*b^2 - 1/4*(2*(a^4
- a^3*b)*d^2*cos(d*x + c)^2 - (a^4 - a^3*b)*d^2)*sqrt(b^3/((a^7 - 2*a^6*b
+ a^5*b^2)*d^4)) + 1/2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b
^2)*d^4))*cos(d*x + c)*sin(d*x + c) + a^2*b*d*cos(d*x + c)*sin(d*x + c))*s
qrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^
3 - a^2*b)*d^2)))*sin(d*x + c) - a*d*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/(...
```

3.208.6 Sympy [F]

$$\int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**2/(a-b*sin(d*x+c)**4),x)
```

output
```
Integral(csc(c + d*x)**2/(a - b*sin(c + d*x)**4), x)
```

3.208.7 Maxima [F]

$$\int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\csc(dx+c)^2}{b\sin(dx+c)^4-a}\,dx$$

input
```
integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
((a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 - 2*a*d*cos(2*d*x + 2*c)
 + a*d)*integrate(-4*(4*b^2*cos(6*d*x + 6*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2
+ 4*b^2*sin(6*d*x + 6*c)^2 + 4*b^2*sin(2*d*x + 2*c)^2 - 4*(8*a*b - 3*b^2)*
cos(4*d*x + 4*c)^2 - b^2*cos(2*d*x + 2*c) - 4*(8*a*b - 3*b^2)*sin(4*d*x +
4*c)^2 + 2*(8*a*b - 7*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (b^2*cos(6*
d*x + 6*c) - 2*b^2*cos(4*d*x + 4*c) + b^2*cos(2*d*x + 2*c))*cos(8*d*x + 8*
c) + (8*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 7*b^2)*cos(4*d*x + 4*c))*c
os(6*d*x + 6*c) + 2*(b^2 + (8*a*b - 7*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4
*c) - (b^2*sin(6*d*x + 6*c) - 2*b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c
))*sin(8*d*x + 8*c) + 2*(4*b^2*sin(2*d*x + 2*c) + (8*a*b - 7*b^2)*sin(4*d*
x + 4*c))*sin(6*d*x + 6*c))/(a*b^2*cos(8*d*x + 8*c)^2 + 16*a*b^2*cos(6*d*x
 + 6*c)^2 + 16*a*b^2*cos(2*d*x + 2*c)^2 + a*b^2*sin(8*d*x + 8*c)^2 + 16*a*
b^2*sin(6*d*x + 6*c)^2 + 16*a*b^2*sin(2*d*x + 2*c)^2 - 8*a*b^2*cos(2*d*x +
 2*c) + a*b^2 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*cos(4*d*x + 4*c)^2 + 4*(64
*a^3 - 48*a^2*b + 9*a*b^2)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b - 3*a*b^2)*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*a*b^2*cos(6*d*x + 6*c) + 4*a*b^2*cos
(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(8*d*x
+ 8*c) + 8*(4*a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4
*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b - 3*a*b^2 - 4*(8*a^2*b - 3*a*b^
2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*a*b^2*sin(6*d*x + 6*c) + 2...
```

3.208.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(99) = 198$.

Time = 0.75 (sec) , antiderivative size = 672, normalized size of antiderivative = 4.83

$$\int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx =$$

$$-\frac{\left(\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}a^2b-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}ab^2-\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}b^3\right)a^2|a-b|-\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}a^5-6\ \cdots\right.\right.}{(3a^8-15a^7b+26a^6b^2-18a^5b^3+3\cdots}$$

input
```
integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
-1/2*(((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^2
 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(
a - b))*sqrt(a*b)*b^3)*a^2*abs(a - b) - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a -
 b))*sqrt(a*b)*a^5 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b
 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2)*abs(a - b))*(pi*
floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^2 + sqrt(a^4 - (a^
2 - a*b)*a^2))/(a^2 - a*b))))/((3*a^8 - 15*a^7*b + 26*a^6*b^2 - 18*a^5*b^3
 + 3*a^4*b^4 + a^3*b^5)*abs(a)) - ((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*
sqrt(a*b)*a^2*b - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 -
sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*a^2*abs(a - b) - (3*sqr
t(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5 - 6*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*sqrt(a*b)*a^4*b - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*
b)*a^3*b^2)*abs(a - b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c
)/sqrt((a^2 - sqrt(a^4 - (a^2 - a*b)*a^2))/(a^2 - a*b))))/((3*a^8 - 15*a^7
*b + 26*a^6*b^2 - 18*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*abs(a)) + 2/(a*tan(d*x
 + c)))/d
```

3.208.9 Mupad [B] (verification not implemented)

Time = 14.81 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.67

$$\int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{2\left(\tan(c+dx)\,\left(4a^4b^4-4a^6b^2\right)-\frac{\tan(c+dx)\left(\sqrt{a^5b^3}+a^3b\right)\left(64a^9b-128a^8b^2+64a^7b^3\right)}{16\left(a^5b-a^6\right)}\right)\sqrt{\frac{\sqrt{a^5b^3}+a^3b}{16\left(a^5b-a^6\right)}}}{2a^3b^4-2a^4b^3}\right)\sqrt{\frac{\sqrt{a^5b^3}+a^3b}{16\left(a^5b-a^6\right)}}}{d} + \frac{2\,\mathrm{atanh}\left(\frac{2\left(\tan(c+dx)\,\left(4a^4b^4-4a^6b^2\right)+\frac{\tan(c+dx)\left(\sqrt{a^5b^3}-a^3b\right)\left(64a^9b-128a^8b^2+64a^7b^3\right)}{16\left(a^5b-a^6\right)}\right)\sqrt{-\frac{\sqrt{a^5b^3}-a^3b}{16\left(a^5b-a^6\right)}}}{2a^3b^4-2a^4b^3}\right)\sqrt{-\frac{\sqrt{a^5b^3}-a^3b}{16\left(a^5b-a^6\right)}}}{d} - \frac{\cot(c+dx)}{a\,d}$$

input
```
int(1/(sin(c + d*x)^2*(a - b*sin(c + d*x)^4)),x)
```

output
```
(2*atanh((2*(tan(c + d*x)*(4*a^4*b^4 - 4*a^6*b^2) - (tan(c + d*x)*((a^5*b^
3)^(1/2) + a^3*b)*(64*a^9*b + 64*a^7*b^3 - 128*a^8*b^2))/(16*(a^5*b - a^6)
))*(((a^5*b^3)^(1/2) + a^3*b)/(16*(a^5*b - a^6)))^(1/2))/(2*a^3*b^4 - 2*a^
4*b^3))*(((a^5*b^3)^(1/2) + a^3*b)/(16*(a^5*b - a^6)))^(1/2))/d + (2*atanh
((2*(tan(c + d*x)*(4*a^4*b^4 - 4*a^6*b^2) + (tan(c + d*x)*((a^5*b^3)^(1/2)
 - a^3*b)*(64*a^9*b + 64*a^7*b^3 - 128*a^8*b^2))/(16*(a^5*b - a^6)))*(-((a
^5*b^3)^(1/2) - a^3*b)/(16*(a^5*b - a^6)))^(1/2))/(2*a^3*b^4 - 2*a^4*b^3))
*(-((a^5*b^3)^(1/2) - a^3*b)/(16*(a^5*b - a^6)))^(1/2))/d - cot(c + d*x)/(
a*d)
```

3.208. $\int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.209 $\int \frac{\csc^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.209.1 Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{\csc^4(c+dx)}{a-b\sin^4(c+dx)}\,dx=\frac{b\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}-\sqrt{b}}d}+\frac{b\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}+\sqrt{b}}d}-\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}$$

output
```
-cot(d*x+c)/a/d-1/3*cot(d*x+c)^3/a/d+1/2*b*arctan((a^(1/2)-b^(1/2))^(1/2)*
tan(d*x+c)/a^(1/4))/a^(7/4)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*b*arctan((a^(1/2
)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(7/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

3.209.2 Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

$$\int \frac{\csc^4(c+dx)}{a-b\sin^4(c+dx)}\,dx=\frac{\frac{3b\arctan\left(\frac{(\sqrt{a}+\sqrt{b})\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}-\frac{3b\,\text{arctanh}\left(\frac{(\sqrt{a}-\sqrt{b})\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}-4\sqrt{a}\cot(c+dx)-2\sqrt{a}\cot(c+dx)\csc^2(c+dx)}{6a^{3/2}d}$$

input
```
Integrate[Csc[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

output
```
((3*b*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]]
)/Sqrt[a + Sqrt[a]*Sqrt[b]] - (3*b*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*
x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] - 4*Sqrt[a]*Co
t[c + d*x] - 2*Sqrt[a]*Cot[c + d*x]*Csc[c + d*x]^2)/(6*a^(3/2)*d)
```

3.209.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3696, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(c+dx)}{a-b\sin^4(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(c+dx)^4\left(a-b\sin(c+dx)^4\right)}dx$$

$$\Big\downarrow \text{ 3696}$$

$$\frac{\int \frac{\cot^4(c+dx)\left(\tan^2(c+dx)+1\right)^3}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 1610}$$

$$\frac{\int \left(\frac{\cot^4(c+dx)}{a}+\frac{\cot^2(c+dx)}{a}+\frac{b\left(\tan^2(c+dx)+1\right)}{a\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{b\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cot^3(c+dx)}{3a}-\frac{\cot(c+dx)}{a}}{d}$$

input
```
Int[Csc[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

output
```
((b*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(7/4)*Sqr
t[Sqrt[a] - Sqrt[b]]) + (b*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a
^(1/4)])/(2*a^(7/4)*Sqrt[Sqrt[a] + Sqrt[b]]) - Cot[c + d*x]/a - Cot[c + d*
x]^3/(3*a))/d
```

3.209.3.1 Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

3.209.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{\frac{b(a-b)\left(\frac{\left(\sqrt{ab}+b\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{a}-\frac{1}{3a\tan(dx+c)^{3}}-\frac{1}{a\tan(dx+c)}}{d}$
default	$\frac{\frac{b(a-b)\left(\frac{\left(\sqrt{ab}+b\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-b\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{a}-\frac{1}{3a\tan(dx+c)^{3}}-\frac{1}{a\tan(dx+c)}}{d}$
risch	$\frac{4i\left(3\,\mathrm{e}^{2i(dx+c)}-1\right)}{3da\left(\mathrm{e}^{2i(dx+c)}-1\right)^{3}}+16\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16777216a^{8}d^{4}-16777216a^{7}b\,d^{4}\right)\_Z^{4}+8192a^{4}b^{2}d^{2}\_Z^{2}+b^{4}\right)}\_R\ln\left(\mathrm{e}^{2i}\right.\right.$

input
```
int(csc(d*x+c)^4/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a*b*(a-b)*(1/2*((a*b)^(1/2)+b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(
a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*((a
*b)^(1/2)-b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b
)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2)))-1/3/a/tan(d*x+c)^3-1/a/tan(d*
x+c))
```

3.209.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1365 vs. $2(111) = 222$.

Time = 0.41 (sec) , antiderivative size = 1365, normalized size of antiderivative = 9.16

$$\int \frac{\csc^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/24*(3*(a*d*cos(d*x + c)^2 - a*d)*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^
9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*log(1/4*b^4*cos(d
*x + c)^2 - 1/4*b^4 - 1/4*(2*(a^5*b - a^4*b^2)*d^2*cos(d*x + c)^2 - (a^5*b
 - a^4*b^2)*d^2)*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + 1/2*(a^2*b^3*
d*cos(d*x + c)*sin(d*x + c) - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b +
 a^7*b^2)*d^4))*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b
^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2)))*sin(d*x +
 c) - 3*(a*d*cos(d*x + c)^2 - a*d)*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9
 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*log(1/4*b^4*cos(d*
x + c)^2 - 1/4*b^4 - 1/4*(2*(a^5*b - a^4*b^2)*d^2*cos(d*x + c)^2 - (a^5*b
- a^4*b^2)*d^2)*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - 1/2*(a^2*b^3*d
*cos(d*x + c)*sin(d*x + c) - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b +
a^7*b^2)*d^4))*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^
5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2)))*sin(d*x +
c) - 3*(a*d*cos(d*x + c)^2 - a*d)*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 -
 2*a^8*b + a^7*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))*log(-1/4*b^4*cos(d*x
 + c)^2 + 1/4*b^4 - 1/4*(2*(a^5*b - a^4*b^2)*d^2*cos(d*x + c)^2 - (a^5*b -
 a^4*b^2)*d^2)*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + 1/2*(a^2*b^3*d*
cos(d*x + c)*sin(d*x + c) + (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a
^7*b^2)*d^4))*cos(d*x + c)*sin(d*x + c))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b...
```

3.209.6 Sympy [F]

$$\int \frac{\csc^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int \frac{\csc^4{(c+dx)}}{a-b\sin^4{(c+dx)}}\,dx$$

input
```
integrate(csc(d*x+c)**4/(a-b*sin(d*x+c)**4),x)
```

output
```
Integral(csc(c + d*x)**4/(a - b*sin(c + d*x)**4), x)
```

3.209.7 Maxima [F]

$$\int \frac{\csc^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\csc\left(dx+c\right)^4}{b\sin\left(dx+c\right)^4-a}\,dx$$

input
```
integrate(csc(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
-4/3*(12*(a*b*d*cos(6*d*x + 6*c)^2 + 9*a*b*d*cos(4*d*x + 4*c)^2 + 9*a*b*d*
cos(2*d*x + 2*c)^2 + a*b*d*sin(6*d*x + 6*c)^2 + 9*a*b*d*sin(4*d*x + 4*c)^2
 - 18*a*b*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*b*d*sin(2*d*x + 2*c)^2
 - 6*a*b*d*cos(2*d*x + 2*c) + a*b*d - 2*(3*a*b*d*cos(4*d*x + 4*c) - 3*a*b*
d*cos(2*d*x + 2*c) + a*b*d)*cos(6*d*x + 6*c) - 6*(3*a*b*d*cos(2*d*x + 2*c)
 - a*b*d)*cos(4*d*x + 4*c) - 6*(a*b*d*sin(4*d*x + 4*c) - a*b*d*sin(2*d*x +
 2*c))*sin(6*d*x + 6*c))*integrate((b*cos(8*d*x + 8*c)*cos(4*d*x + 4*c) -
4*b*cos(6*d*x + 6*c)*cos(4*d*x + 4*c) - 2*(8*a - 3*b)*cos(4*d*x + 4*c)^2 +
 b*sin(8*d*x + 8*c)*sin(4*d*x + 4*c) - 4*b*sin(6*d*x + 6*c)*sin(4*d*x + 4*
c) - 2*(8*a - 3*b)*sin(4*d*x + 4*c)^2 - 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) - (4*b*cos(2*d*x + 2*c) - b)*cos(4*d*x + 4*c))/(a*b^2*cos(8*d*x + 8*c)
^2 + 16*a*b^2*cos(6*d*x + 6*c)^2 + 16*a*b^2*cos(2*d*x + 2*c)^2 + a*b^2*sin
(8*d*x + 8*c)^2 + 16*a*b^2*sin(6*d*x + 6*c)^2 + 16*a*b^2*sin(2*d*x + 2*c)^
2 - 8*a*b^2*cos(2*d*x + 2*c) + a*b^2 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*cos
(4*d*x + 4*c)^2 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*sin(4*d*x + 4*c)^2 + 16*
(8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*a*b^2*cos(6*d
*x + 6*c) + 4*a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4
*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8
*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b - 3*a*b^
2 - 4*(8*a^2*b - 3*a*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*a*b...
```

3.209.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(111) = 222$.

Time = 0.70 (sec) , antiderivative size = 937, normalized size of antiderivative = 6.29

$$\int \frac{\csc^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$= \frac{3\left(\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}a^2b-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}ab^2-\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}b^3\right)a^2|a-b|+\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}a^4b-9\sqrt{a^2-\right.\right.}{}$$

input
```
integrate(csc(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
1/6*(3*((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^
2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*sqrt(a*b)*b^3)*a^2*abs(a - b) + (3*sqrt(a^2 - a*b + sqrt(a*b)*(a
- b))*a^4*b - 9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^2 + 5*sqrt(a^2 -
 a*b + sqrt(a*b)*(a - b))*a^2*b^3 + sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*
b^4)*abs(a - b)*abs(a) - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*
a^4*b - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 - sqrt(a^2
 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^3)*abs(a - b))*(pi*floor((d*x
+ c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^2 + sqrt(a^4 - (a^2 - a*b)*a^
2))/(a^2 - a*b))))/((3*a^8 - 15*a^7*b + 26*a^6*b^2 - 18*a^5*b^3 + 3*a^4*b^
4 + a^3*b^5)*abs(a)) + 3*((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)
*a^2*b - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2
- a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*a^2*abs(a - b) + (3*sqrt(a^2 - a
*b - sqrt(a*b)*(a - b))*a^4*b - 9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*
b^2 + 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^3 + sqrt(a^2 - a*b - sqr
t(a*b)*(a - b))*a*b^4)*abs(a - b)*abs(a) - (3*sqrt(a^2 - a*b - sqrt(a*b)*(
a - b))*sqrt(a*b)*a^4*b - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*
a^3*b^2 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^3)*abs(a - b
))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^2 - sqrt(a^
4 - (a^2 - a*b)*a^2))/(a^2 - a*b))))/((3*a^8 - 15*a^7*b + 26*a^6*b^2 - ...
```

3.209.9 Mupad [B] (verification not implemented)

Time = 15.91 (sec) , antiderivative size = 1670, normalized size of antiderivative = 11.21

$$\int \frac{\csc^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(c + d*x)^4*(a - b*sin(c + d*x)^4)),x)
```

output
```
(atan((((((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(16*a^5*b^4
 - 32*a^6*b^3 + 16*a^7*b^2 + tan(c + d*x)*(((a^7*b^5)^(1/2) + a^4*b^2)/(16
*(a^7*b - a^8)))^(1/2)*(64*a^9*b + 64*a^7*b^3 - 128*a^8*b^2)) - tan(c + d*
x)*(4*a^3*b^5 - 4*a^5*b^3))*(((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^8)
))^(1/2)*1i - ((((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(16*
a^5*b^4 - 32*a^6*b^3 + 16*a^7*b^2 - tan(c + d*x)*(((a^7*b^5)^(1/2) + a^4*b
^2)/(16*(a^7*b - a^8)))^(1/2)*(64*a^9*b + 64*a^7*b^3 - 128*a^8*b^2)) + tan
(c + d*x)*(4*a^3*b^5 - 4*a^5*b^3))*(((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b
 - a^8)))^(1/2)*1i)/(((((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^8)))^(1/
2)*(16*a^5*b^4 - 32*a^6*b^3 + 16*a^7*b^2 + tan(c + d*x)*(((a^7*b^5)^(1/2)
+ a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(64*a^9*b + 64*a^7*b^3 - 128*a^8*b^2)
) - tan(c + d*x)*(4*a^3*b^5 - 4*a^5*b^3))*(((a^7*b^5)^(1/2) + a^4*b^2)/(16
*(a^7*b - a^8)))^(1/2) + ((((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^8)))
^(1/2)*(16*a^5*b^4 - 32*a^6*b^3 + 16*a^7*b^2 - tan(c + d*x)*(((a^7*b^5)^(1
/2) + a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(64*a^9*b + 64*a^7*b^3 - 128*a^8*
b^2)) + tan(c + d*x)*(4*a^3*b^5 - 4*a^5*b^3))*(((a^7*b^5)^(1/2) + a^4*b^2)
/(16*(a^7*b - a^8)))^(1/2) - 2*a^2*b^5 + 2*a^3*b^4))*(((a^7*b^5)^(1/2) + a
^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*2i)/d + (atan((((-((a^7*b^5)^(1/2) - a^4
*b^2)/(16*(a^7*b - a^8)))^(1/2)*(16*a^5*b^4 - 32*a^6*b^3 + 16*a^7*b^2 + ta
n(c + d*x)*(-((a^7*b^5)^(1/2) - a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(64*...
```

3.209. $\int \frac{\csc^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.210 $\int \frac{\csc^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.210.1 Optimal result

Integrand size = 24, antiderivative size = 178

$$\int \frac{\csc^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{b^{3/2}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}\sqrt{\sqrt{a}-\sqrt{b}}d} - \frac{b^{3/2}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}\sqrt{\sqrt{a}+\sqrt{b}}d} - \frac{(a+b)\cot(c+dx)}{a^2d} - \frac{2\cot^3(c+dx)}{3ad} - \frac{\cot^5(c+dx)}{5ad}$$

output
```
-(a+b)*cot(d*x+c)/a^2/d-2/3*cot(d*x+c)^3/a/d-1/5*cot(d*x+c)^5/a/d+1/2*b^(3
/2)*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(9/4)/d/(a^(1/2)-
b^(1/2))^(1/2)-1/2*b^(3/2)*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/
4))/a^(9/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

3.210.2 Mathematica [A] (verified)

Time = 3.62 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.98

$$\int \frac{\csc^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\frac{15b^{3/2}\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}} + \frac{15b^{3/2}\text{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}} + 2\cot(c+dx)\left(8a+15b+4a\csc^2(c+dx\right.}{30a^2d}$$

input
```
Integrate[Csc[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

output
```
-1/30*((15*b^(3/2)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt
[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + (15*b^(3/2)*ArcTanh[((Sqrt[a] -
 Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqr
t[b]] + 2*Cot[c + d*x]*(8*a + 15*b + 4*a*Csc[c + d*x]^2 + 3*a*Csc[c + d*x]
^4))/(a^2*d)
```

3.210.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3696, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(c+dx)}{a-b\sin^4(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(c+dx)^6\left(a-b\sin(c+dx)^4\right)}dx$$

$$\Big\downarrow \text{ 3696}$$

$$\frac{\int \frac{\cot^6(c+dx)\left(\tan^2(c+dx)+1\right)^4}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 1610}$$

$$\frac{\int \left(\frac{\cot^6(c+dx)}{a}+\frac{2\cot^4(c+dx)}{a}+\frac{(a+b)\cot^2(c+dx)}{a^2}+\frac{b^2\tan^2(c+dx)}{a^2\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{b^{3/2}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(a+b)\cot(c+dx)}{a^2}-\frac{\cot^5(c+dx)}{5a}-\frac{2\cot^3(c+dx)}{3a}}{d}$$

input
```
Int[Csc[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

output
```
((b^(3/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(9/
4)*Sqrt[Sqrt[a] - Sqrt[b]]) - (b^(3/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan
[c + d*x])/a^(1/4)])/(2*a^(9/4)*Sqrt[Sqrt[a] + Sqrt[b]]) - ((a + b)*Cot[c
+ d*x])/a^2 - (2*Cot[c + d*x]^3)/(3*a) - Cot[c + d*x]^5/(5*a))/d
```

3.210.3.1 Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

3.210.4 Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{b^2(a-b)\left(\frac{\left(\sqrt{ab}+a\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2(a-b)\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-a\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{a^2}-\frac{1}{5a\tan(dx+c)^5}-\frac{a+b}{a^2\tan(dx+c)}-\frac{\cdots}{3a\cdots}}{d}$
default	$\frac{\frac{b^2(a-b)\left(\frac{\left(\sqrt{ab}+a\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2(a-b)\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-a\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{a^2}-\frac{1}{5a\tan(dx+c)^5}-\frac{a+b}{a^2\tan(dx+c)}-\frac{\cdots}{3a\cdots}}{d}$
risch	$-\frac{2i\left(15\,e^{8i(dx+c)}b-60b\,e^{6i(dx+c)}+80a\,e^{4i(dx+c)}+90b\,e^{4i(dx+c)}-40a\,e^{2i(dx+c)}-60b\,e^{2i(dx+c)}+8a+15b\right)}{15d\,a^2\left(e^{2i(dx+c)}-1\right)^5}-64\Bigg(\sum_{\_R=\cdots}$

input
```
int(csc(d*x+c)^6/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2/a^2*(a-b)*(1/2*((a*b)^(1/2)+a)/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)+a)
*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*(
(a*b)^(1/2)-a)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a
+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2)))-1/5/a/tan(d*x+c)^5-(a+b)/a^
2/tan(d*x+c)-2/3/a/tan(d*x+c)^3)
```

3.210.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1477 vs. $2(134) = 268$.

Time = 0.46 (sec) , antiderivative size = 1477, normalized size of antiderivative = 8.30

$$\int \frac{\csc^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/120*(8*(8*a + 15*b)*cos(d*x + c)^5 - 80*(2*a + 3*b)*cos(d*x + c)^3 - 15
*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(-((a^5 - a^4
*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^2 + b^3)/((a^5 - a^4*b)*
d^2))*log(1/4*b^5*cos(d*x + c)^2 - 1/4*b^5 - 1/4*(2*(a^6*b - a^5*b^2)*d^2*
cos(d*x + c)^2 - (a^6*b - a^5*b^2)*d^2)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b
^2)*d^4)) + 1/2*(a^3*b^3*d*cos(d*x + c)*sin(d*x + c) - (a^8 - a^7*b)*sqrt(
b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c))*sqrt
(-((a^5 - a^4*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^2 + b^3)/((
a^5 - a^4*b)*d^2)))*sin(d*x + c) + 15*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(
d*x + c)^2 + a^2*d)*sqrt(-((a^5 - a^4*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*
b^2)*d^4))*d^2 + b^3)/((a^5 - a^4*b)*d^2))*log(1/4*b^5*cos(d*x + c)^2 - 1/
4*b^5 - 1/4*(2*(a^6*b - a^5*b^2)*d^2*cos(d*x + c)^2 - (a^6*b - a^5*b^2)*d^
2)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4)) - 1/2*(a^3*b^3*d*cos(d*x +
c)*sin(d*x + c) - (a^8 - a^7*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4)
)*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^5 - a^4*b)*sqrt(b^7/((a^11 - 2*
a^10*b + a^9*b^2)*d^4))*d^2 + b^3)/((a^5 - a^4*b)*d^2)))*sin(d*x + c) + 15
*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(((a^5 - a^4*
b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^2 - b^3)/((a^5 - a^4*b)*d
^2))*log(-1/4*b^5*cos(d*x + c)^2 + 1/4*b^5 - 1/4*(2*(a^6*b - a^5*b^2)*d^2*
cos(d*x + c)^2 - (a^6*b - a^5*b^2)*d^2)*sqrt(b^7/((a^11 - 2*a^10*b + a^...
```

3.210.6 Sympy [F]

$$\int \frac{\csc^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int \frac{\csc^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**6/(a-b*sin(d*x+c)**4),x)
```

output
```
Integral(csc(c + d*x)**6/(a - b*sin(c + d*x)**4), x)
```

3.210.7 Maxima [F]

$$\int \frac{\csc^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\csc\left(dx+c\right)^6}{b\sin\left(dx+c\right)^4-a}\,dx$$

input
```
integrate(csc(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/15*(300*b*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + 10*(3*b*sin(8*d*x + 8*c) -
 12*b*sin(6*d*x + 6*c) + 2*(8*a + 9*b)*sin(4*d*x + 4*c) - 4*(2*a + 3*b)*si
n(2*d*x + 2*c))*cos(10*d*x + 10*c) + 50*(6*b*sin(6*d*x + 6*c) - 4*(4*a + 3
*b)*sin(4*d*x + 4*c) + (8*a + 9*b)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 20
0*((8*a + 3*b)*sin(4*d*x + 4*c) - (4*a + 3*b)*sin(2*d*x + 2*c))*cos(6*d*x
+ 6*c) + 15*(a^2*d*cos(10*d*x + 10*c)^2 + 25*a^2*d*cos(8*d*x + 8*c)^2 + 10
0*a^2*d*cos(6*d*x + 6*c)^2 + 100*a^2*d*cos(4*d*x + 4*c)^2 + 25*a^2*d*cos(2
*d*x + 2*c)^2 + a^2*d*sin(10*d*x + 10*c)^2 + 25*a^2*d*sin(8*d*x + 8*c)^2 +
 100*a^2*d*sin(6*d*x + 6*c)^2 + 100*a^2*d*sin(4*d*x + 4*c)^2 - 100*a^2*d*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*a^2*d*sin(2*d*x + 2*c)^2 - 10*a^2*d*
cos(2*d*x + 2*c) + a^2*d - 2*(5*a^2*d*cos(8*d*x + 8*c) - 10*a^2*d*cos(6*d*
x + 6*c) + 10*a^2*d*cos(4*d*x + 4*c) - 5*a^2*d*cos(2*d*x + 2*c) + a^2*d)*c
os(10*d*x + 10*c) - 10*(10*a^2*d*cos(6*d*x + 6*c) - 10*a^2*d*cos(4*d*x + 4
*c) + 5*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(8*d*x + 8*c) - 20*(10*a^2*d*co
s(4*d*x + 4*c) - 5*a^2*d*cos(2*d*x + 2*c) + a^2*d)*cos(6*d*x + 6*c) - 20*(
5*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(4*d*x + 4*c) - 10*(a^2*d*sin(8*d*x +
 8*c) - 2*a^2*d*sin(6*d*x + 6*c) + 2*a^2*d*sin(4*d*x + 4*c) - a^2*d*sin(2*
d*x + 2*c))*sin(10*d*x + 10*c) - 50*(2*a^2*d*sin(6*d*x + 6*c) - 2*a^2*d*si
n(4*d*x + 4*c) + a^2*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 100*(2*a^2*d*s
in(4*d*x + 4*c) - a^2*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-...
```

3.210.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(134) = 268$.

Time = 0.74 (sec), antiderivative size = 471, normalized size of antiderivative = 2.65

$$\int \frac{\csc^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$= \frac{\frac{15\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}a^2b-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}ab^2-\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(dx+c)}{\sqrt{\frac{a^3+\sqrt{a^6-\left(a^3-a^2b\right)\cdots}}{a^3-a^2b}}}\cdots\right.\right.}{3a^7-12a^6b+14a^5b^2-4a^4b^3-a^3b^4}\cdots}{\cdots}$$

input
```
integrate(csc(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
1/30*(15*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a
^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b + sqrt(a*b)
*(a - b))*sqrt(a*b)*b^3)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x +
c)/sqrt((a^3 + sqrt(a^6 - (a^3 - a^2*b)*a^3))/(a^3 - a^2*b))))*abs(a - b)/
(3*a^7 - 12*a^6*b + 14*a^5*b^2 - 4*a^4*b^3 - a^3*b^4) - 15*(3*sqrt(a^2 - a
*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a
- b))*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)
*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^3 - sqrt(a^6
- (a^3 - a^2*b)*a^3))/(a^3 - a^2*b))))*abs(a - b)/(3*a^7 - 12*a^6*b + 14*a
^5*b^2 - 4*a^4*b^3 - a^3*b^4) - 2*(15*a*tan(d*x + c)^4 + 15*b*tan(d*x + c)
^4 + 10*a*tan(d*x + c)^2 + 3*a)/(a^2*tan(d*x + c)^5))/d
```

3.210.9 Mupad [B] (verification not implemented)

Time = 14.36 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.34

$$\int \frac{\csc^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{2\left(\tan(c+d\,x)\left(4\,a^7\,b^6-4\,a^9\,b^4\right)-\frac{\tan(c+d\,x)\left(\sqrt{a^9\,b^7}+a^5\,b^3\right)\left(64\,a^{14}\,b-128\,a^{13}\,b^2+64\,a^{12}\,b^3\right)}{16\left(a^9\,b-a^{10}\right)}\right)\sqrt{\frac{\sqrt{a^9\,b^7}+a^5\,b^3}{16\left(a^9\,b-a^{10}\right)}}}{2\,a^5\,b^7-2\,a^6\,b^6}\right)\sqrt{\frac{\sqrt{a^9\,b^7}+a^5\,b^3}{16\,(a^9\,b-a^{10})}}}{d} + \frac{2\,\mathrm{atanh}\left(\frac{2\left(\tan(c+d\,x)\left(4\,a^7\,b^6-4\,a^9\,b^4\right)+\frac{\tan(c+d\,x)\left(\sqrt{a^9\,b^7}-a^5\,b^3\right)\left(64\,a^{14}\,b-128\,a^{13}\,b^2+64\,a^{12}\,b^3\right)}{16\left(a^9\,b-a^{10}\right)}\right)\sqrt{-\frac{\sqrt{a^9\,b^7}-a^5\,b^3}{16\left(a^9\,b-a^{10}\right)}}}{2\,a^5\,b^7-2\,a^6\,b^6}\right)\sqrt{-\frac{\sqrt{a^9\,b^7-}}{16\,(a^9\,b}}}{d} - \frac{\frac{1}{5\,a}+\frac{2\tan(c+d\,x)^2}{3\,a}+\frac{\tan(c+d\,x)^4\,(a+b)}{a^2}}{d\tan\left(c+d\,x\right)^5}$$

input
```
int(1/(sin(c + d*x)^6*(a - b*sin(c + d*x)^4)),x)
```

output
```
(2*atanh((2*(tan(c + d*x)*(4*a^7*b^6 - 4*a^9*b^4) - (tan(c + d*x)*((a^9*b^
7)^(1/2) + a^5*b^3)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2))/(16*(a^9*b -
 a^10)))*(((a^9*b^7)^(1/2) + a^5*b^3)/(16*(a^9*b - a^10)))^(1/2))/(2*a^5*b
^7 - 2*a^6*b^6))*(((a^9*b^7)^(1/2) + a^5*b^3)/(16*(a^9*b - a^10)))^(1/2))/
d + (2*atanh((2*(tan(c + d*x)*(4*a^7*b^6 - 4*a^9*b^4) + (tan(c + d*x)*((a^
9*b^7)^(1/2) - a^5*b^3)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2))/(16*(a^9
*b - a^10)))*(-((a^9*b^7)^(1/2) - a^5*b^3)/(16*(a^9*b - a^10)))^(1/2))/(2*
a^5*b^7 - 2*a^6*b^6))*(-((a^9*b^7)^(1/2) - a^5*b^3)/(16*(a^9*b - a^10)))^(
1/2))/d - (1/(5*a) + (2*tan(c + d*x)^2)/(3*a) + (tan(c + d*x)^4*(a + b))/a
^2)/(d*tan(c + d*x)^5)
```

3.210. $\int \frac{\csc^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.211 $\int \frac{\csc^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.211.1 Optimal result

Integrand size = 24, antiderivative size = 197

$$\int \frac{\csc^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{b^2\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}\sqrt{\sqrt{a}-\sqrt{b}}d}+\frac{b^2\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}\sqrt{\sqrt{a}+\sqrt{b}}d} - \frac{(a+b)\cot(c+dx)}{a^2d}-\frac{(3a+b)\cot^3(c+dx)}{3a^2d} - \frac{3\cot^5(c+dx)}{5ad}-\frac{\cot^7(c+dx)}{7ad}$$

output
```
-(a+b)*cot(d*x+c)/a^2/d-1/3*(3*a+b)*cot(d*x+c)^3/a^2/d-3/5*cot(d*x+c)^5/a/
d-1/7*cot(d*x+c)^7/a/d+1/2*b^2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a
^(1/4))/a^(11/4)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*b^2*arctan((a^(1/2)+b^(1/2)
)^(1/2)*tan(d*x+c)/a^(1/4))/a^(11/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

3.211.2 Mathematica [A] (verified)

Time = 6.33 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.98

$$\int \frac{\csc^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\frac{105b^2\arctan\left(\frac{(\sqrt{a}+\sqrt{b})\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}-\frac{105b^2\text{arctanh}\left(\frac{(\sqrt{a}-\sqrt{b})\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}-2\sqrt{a}\cot(c+dx)\left(48a+70b+(24a+35b)\,\mathrm{c}\right.}{210a^{5/2}d}$$

input
```
Integrate[Csc[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

output
```
((105*b^2*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[
b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - (105*b^2*ArcTanh[((Sqrt[a] - Sqrt[b])*Ta
n[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] - 2*Sq
rt[a]*Cot[c + d*x]*(48*a + 70*b + (24*a + 35*b)*Csc[c + d*x]^2 + 18*a*Csc[
c + d*x]^4 + 15*a*Csc[c + d*x]^6))/(210*a^(5/2)*d)
```

3.211.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3696, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^8(c+dx)}{a-b\sin^4(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^8\left(a-b\sin(c+dx)^4\right)}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\cot^8(c+dx)\left(\tan^2(c+dx)+1\right)^5}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1610}$$

$$\frac{\int \left(\frac{\cot^8(c+dx)}{a}+\frac{3\cot^6(c+dx)}{a}+\frac{(3a+b)\cot^4(c+dx)}{a^2}+\frac{(a+b)\cot^2(c+dx)}{a^2}+\frac{b^2\left(\tan^2(c+dx)+1\right)}{a^2\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b^2\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(3a+b)\cot^3(c+dx)}{3a^2}-\frac{(a+b)\cot(c+dx)}{a^2}-\frac{\cot^7(c+dx)}{7a}-\frac{3\cot^5(c+dx)}{5a}}{d}$$

input
```
Int[Csc[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

output
```
((b^2*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(11/4)*
Sqrt[Sqrt[a] - Sqrt[b]]) + (b^2*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*
x])/a^(1/4)])/(2*a^(11/4)*Sqrt[Sqrt[a] + Sqrt[b]]) - ((a + b)*Cot[c + d*x]
)/a^2 - ((3*a + b)*Cot[c + d*x]^3)/(3*a^2) - (3*Cot[c + d*x]^5)/(5*a) - Co
t[c + d*x]^7/(7*a))/d
```

3.211.3.1 Defintions of rubi rules used

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

3.211. $\int \frac{\csc^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.211.4 Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{-\frac{1}{7a\tan(dx+c)^7}-\frac{a+b}{a^2\tan(dx+c)}-\frac{3a+b}{3a^2\tan(dx+c)^3}-\frac{3}{5a\tan(dx+c)^5}+\frac{b^2(a-b)\left(\frac{\left(\sqrt{ab}+b\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-b\right)a}{2\sqrt{ab}}\right.}{a^2}}{d}$
default	$\frac{-\frac{1}{7a\tan(dx+c)^7}-\frac{a+b}{a^2\tan(dx+c)}-\frac{3a+b}{3a^2\tan(dx+c)^3}-\frac{3}{5a\tan(dx+c)^5}+\frac{b^2(a-b)\left(\frac{\left(\sqrt{ab}+b\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(\sqrt{ab}-b\right)a}{2\sqrt{ab}}\right.}{a^2}}{d}$
risch	$\frac{4i\left(105b\,e^{10i(dx+c)}-455\,e^{8i(dx+c)}b+840a\,e^{6i(dx+c)}+770b\,e^{6i(dx+c)}-504a\,e^{4i(dx+c)}-630b\,e^{4i(dx+c)}+168a\,e^{2i(dx+c)}+24\right.}{105d\,a^2\left(e^{2i(dx+c)}-1\right)^7}$

input
```
int(csc(d*x+c)^8/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/7/a/tan(d*x+c)^7-(a+b)/a^2/tan(d*x+c)-1/3*(3*a+b)/a^2/tan(d*x+c)^3
-3/5/a/tan(d*x+c)^5+b^2/a^2*(a-b)*(1/2*((a*b)^(1/2)+b)/(a*b)^(1/2)/(a-b)/(
((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/((((a*b)^(1/2)+a)*(a-b
))^(1/2))+1/2*((a*b)^(1/2)-b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1
/2)*arctanh((-a+b)*tan(d*x+c)/((((a*b)^(1/2)-a)*(a-b))^(1/2))))
```

3.211.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1585 vs. 2(155) = 310.

Time = 0.44 (sec) , antiderivative size = 1585, normalized size of antiderivative = 8.05

$$\int \frac{\csc^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/840*(16*(24*a + 35*b)*cos(d*x + c)^7 - 56*(24*a + 35*b)*cos(d*x + c)^5
+ 560*(3*a + 4*b)*cos(d*x + c)^3 + 105*(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos
(d*x + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)*sqrt(-(b^4 + (a^6 - a^5*b)*s
qrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2))*log(
1/4*b^7*cos(d*x + c)^2 - 1/4*b^7 - 1/4*(2*(a^7*b^2 - a^6*b^3)*d^2*cos(d*x
+ c)^2 - (a^7*b^2 - a^6*b^3)*d^2)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d
^4)) + 1/2*(a^3*b^5*d*cos(d*x + c)*sin(d*x + c) - (a^10 - a^9*b)*sqrt(b^9/
((a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(-(
b^4 + (a^6 - a^5*b)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^
6 - a^5*b)*d^2)))*sin(d*x + c) - 105*(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d
*x + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)*sqrt(-(b^4 + (a^6 - a^5*b)*sqr
t(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2))*log(1/
4*b^7*cos(d*x + c)^2 - 1/4*b^7 - 1/4*(2*(a^7*b^2 - a^6*b^3)*d^2*cos(d*x +
c)^2 - (a^7*b^2 - a^6*b^3)*d^2)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4
)) - 1/2*(a^3*b^5*d*cos(d*x + c)*sin(d*x + c) - (a^10 - a^9*b)*sqrt(b^9/((
a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(-(b^
4 + (a^6 - a^5*b)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6
- a^5*b)*d^2)))*sin(d*x + c) - 105*(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x
 + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)*sqrt(-(b^4 - (a^6 - a^5*b)*sqrt(
b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2))*log(-...
```

3.211.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^8(c + dx)}{a - b\sin^4(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**8/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.211.7 Maxima [F]

$$\int \frac{\csc^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\csc(dx+c)^8}{b\sin(dx+c)^4-a}\,dx$$

input
```
integrate(csc(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
4/105*(735*b*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - 7*(15*b*sin(10*d*x + 10*c
) - 65*b*sin(8*d*x + 8*c) + 10*(12*a + 11*b)*sin(6*d*x + 6*c) - 18*(4*a +
5*b)*sin(4*d*x + 4*c) + (24*a + 35*b)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c)
 + 49*(15*b*sin(10*d*x + 10*c) - 65*b*sin(8*d*x + 8*c) + 10*(12*a + 11*b)*
sin(6*d*x + 6*c) - 18*(4*a + 5*b)*sin(4*d*x + 4*c) + (24*a + 35*b)*sin(2*d
*x + 2*c))*cos(12*d*x + 12*c) + 147*(40*b*sin(8*d*x + 8*c) - 5*(24*a + 17*
b)*sin(6*d*x + 6*c) + 3*(24*a + 25*b)*sin(4*d*x + 4*c) - 6*(4*a + 5*b)*sin
(2*d*x + 2*c))*cos(10*d*x + 10*c) + 245*(15*(8*a + 3*b)*sin(6*d*x + 6*c) -
 3*(24*a + 17*b)*sin(4*d*x + 4*c) + 2*(12*a + 11*b)*sin(2*d*x + 2*c))*cos(
8*d*x + 8*c) + 245*(24*b*sin(4*d*x + 4*c) - 13*b*sin(2*d*x + 2*c))*cos(6*d
*x + 6*c) - 420*(a^2*b^2*d*cos(14*d*x + 14*c)^2 + 49*a^2*b^2*d*cos(12*d*x
+ 12*c)^2 + 441*a^2*b^2*d*cos(10*d*x + 10*c)^2 + 1225*a^2*b^2*d*cos(8*d*x
+ 8*c)^2 + 1225*a^2*b^2*d*cos(6*d*x + 6*c)^2 + 441*a^2*b^2*d*cos(4*d*x + 4
*c)^2 + 49*a^2*b^2*d*cos(2*d*x + 2*c)^2 + a^2*b^2*d*sin(14*d*x + 14*c)^2 +
 49*a^2*b^2*d*sin(12*d*x + 12*c)^2 + 441*a^2*b^2*d*sin(10*d*x + 10*c)^2 +
1225*a^2*b^2*d*sin(8*d*x + 8*c)^2 + 1225*a^2*b^2*d*sin(6*d*x + 6*c)^2 + 44
1*a^2*b^2*d*sin(4*d*x + 4*c)^2 - 294*a^2*b^2*d*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 49*a^2*b^2*d*sin(2*d*x + 2*c)^2 - 14*a^2*b^2*d*cos(2*d*x + 2*c) +
 a^2*b^2*d - 2*(7*a^2*b^2*d*cos(12*d*x + 12*c) - 21*a^2*b^2*d*cos(10*d*x +
 10*c) + 35*a^2*b^2*d*cos(8*d*x + 8*c) - 35*a^2*b^2*d*cos(6*d*x + 6*c) ...
```

3.211.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(155) = 310$.

Time = 0.81 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.37

$$\int \frac{\csc^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$$
$$= \frac{105\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}a^2b^2-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}ab^3-\sqrt{a^2-ab+\sqrt{ab}(a-b)}b^4\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(dx+c)}{\sqrt{\frac{a^3+\sqrt{a^6-(a^3-a^2b)a^3}}{a^3-a^2b}}}\right)\right)|a-}{3a^7-12a^6b+14a^5b^2-4a^4b^3-a^3b^4}$$

input
```
integrate(csc(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
1/210*(105*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^2 - 6*sqrt(a^2 - a
*b + sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*b^4)*(
pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^3 + sqrt(a^6 -
(a^3 - a^2*b)*a^3))/(a^3 - a^2*b))))*abs(a - b)/(3*a^7 - 12*a^6*b + 14*a^5
*b^2 - 4*a^4*b^3 - a^3*b^4) + 105*(3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a
^2*b^2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b - sq
rt(a*b)*(a - b))*b^4)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/
sqrt((a^3 - sqrt(a^6 - (a^3 - a^2*b)*a^3))/(a^3 - a^2*b))))*abs(a - b)/(3*
a^7 - 12*a^6*b + 14*a^5*b^2 - 4*a^4*b^3 - a^3*b^4) - 2*(105*a*tan(d*x + c)
^6 + 105*b*tan(d*x + c)^6 + 105*a*tan(d*x + c)^4 + 35*b*tan(d*x + c)^4 + 6
3*a*tan(d*x + c)^2 + 15*a)/(a^2*tan(d*x + c)^7))/d
```

3.211.9 Mupad [B] (verification not implemented)

Time = 16.82 (sec) , antiderivative size = 1704, normalized size of antiderivative = 8.65

$$\int \frac{\csc^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(c + d*x)^8*(a - b*sin(c + d*x)^4)),x)
```

output
```
(atan((((((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(16*a^9*
b^5 - 32*a^10*b^4 + 16*a^11*b^3 + tan(c + d*x)*(((a^11*b^9)^(1/2) + a^6*b^
4)/(16*(a^11*b - a^12)))^(1/2)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2)) -
 tan(c + d*x)*(4*a^6*b^7 - 4*a^8*b^5))*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(
a^11*b - a^12)))^(1/2)*1i - ((((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a
^12)))^(1/2)*(16*a^9*b^5 - 32*a^10*b^4 + 16*a^11*b^3 - tan(c + d*x)*(((a^1
1*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(64*a^14*b + 64*a^12*b
^3 - 128*a^13*b^2)) + tan(c + d*x)*(4*a^6*b^7 - 4*a^8*b^5))*(((a^11*b^9)^(
1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*1i)/(((((a^11*b^9)^(1/2) + a^6
*b^4)/(16*(a^11*b - a^12)))^(1/2)*(16*a^9*b^5 - 32*a^10*b^4 + 16*a^11*b^3
+ tan(c + d*x)*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(
64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2)) - tan(c + d*x)*(4*a^6*b^7 - 4*a^8
*b^5))*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2) + ((((a^1
1*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(16*a^9*b^5 - 32*a^10*
b^4 + 16*a^11*b^3 - tan(c + d*x)*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b
 - a^12)))^(1/2)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2)) + tan(c + d*x)*
(4*a^6*b^7 - 4*a^8*b^5))*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)
))^(1/2) - 2*a^4*b^8 + 2*a^5*b^7))*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11
*b - a^12)))^(1/2)*2i)/d + (atan((((-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(a^1
1*b - a^12)))^(1/2)*(16*a^9*b^5 - 32*a^10*b^4 + 16*a^11*b^3 + tan(c + d...
```

3.211. $\int \frac{\csc^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.212 $\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

3.212.1 Optimal result

Integrand size = 24, antiderivative size = 236

$$\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{9/4}d} + \frac{\sqrt{a}\left(5\sqrt{a}+6\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{9/4}d} - \frac{\cos(c+dx)}{b^2d} - \frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)}$$

output
```
-cos(d*x+c)/b^2/d-1/4*a*cos(d*x+c)*(a+b-b*cos(d*x+c)^2)/(a-b)/b^2/d/(a-b+2
*b*cos(d*x+c)^2-b*cos(d*x+c)^4)+1/8*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(
1/2))^(1/2))*a^(1/2)*(5*a^(1/2)-6*b^(1/2))/b^(9/4)/d/(a^(1/2)-b^(1/2))^(3/
2)+1/8*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*a^(1/2)*(5*a^(1
/2)+6*b^(1/2))/b^(9/4)/d/(a^(1/2)+b^(1/2))^(3/2)
```

3.212.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.37 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.06

$$\int \frac{\sin^9(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx =$$

$$-\frac{32\cos(c+dx)+\frac{32a\cos(c+dx)(2a+b-b\cos(2(c+dx)))}{(a-b)(8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx)))}+\frac{ia\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1\right.}{}}{}$$

input
```
Integrate[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4)^2,x]
```

output
```
-1/32*(32*Cos[c + d*x] + (32*a*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)])
)/((a - b)*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (I*a
*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*b
*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*b*Log[1 - 2*Cos[c + d*x]*#1
+ #1^2] - 40*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 54*b*ArcTan
[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (20*I)*a*Log[1 - 2*Cos[c + d*x]*
#1 + #1^2]*#1^2 - (27*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 40*a*A
rcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 54*b*ArcTan[Sin[c + d*x]/(C
os[c + d*x] - #1)]*#1^4 - (20*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4
+ (27*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 2*b*ArcTan[Sin[c + d*x
]/(Cos[c + d*x] - #1)]*#1^6 - I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/
(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a - b))/(b^2*d)
```

3.212.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3694, 1517, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^9(c+dx)}{(a-b\sin^4(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^9}{(a-b\sin(c+dx)^4)^2}dx$$

$$\Big\downarrow \text{3694}$$

$$-\frac{\int \frac{(1-\cos^2(c+dx))^4}{(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b)^2}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{1517}$$

$$-\frac{\frac{a\cos(c+dx)(a-b\cos^2(c+dx)+b)}{4b^2(a-b)(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b)}-\frac{\int \frac{2\left(4a(a-b)\cos^4(c+dx)-a(7a-8b)\cos^2(c+dx)+a\left(\frac{a^2}{b}+a-4b\right)\right)}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{8ab(a-b)}}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{a\cos(c+dx)(a-b\cos^2(c+dx)+b)}{4b^2(a-b)(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b)}-\frac{\int \frac{4a(a-b)\cos^4(c+dx)-a(7a-8b)\cos^2(c+dx)+a\left(\frac{a^2}{b}+a-4b\right)}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{4ab(a-b)}}{d}$$

$$\Big\downarrow \text{2205}$$

$$-\frac{\frac{a\cos(c+dx)(a-b\cos^2(c+dx)+b)}{4b^2(a-b)(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b)}-\frac{\int \left(\frac{b\cos^2(c+dx)a^2+(5a-7b)a^2}{b\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)}-\frac{4a(a-b)}{b}\right)d\cos(c+dx)}{4ab(a-b)}}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{a\cos(c+dx)(a-b\cos^2(c+dx)+b)}{4b^2(a-b)(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b)}-\frac{\frac{a^{3/2}\left(-\sqrt{a}\sqrt{b}+5a-6b\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{5/4}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a^{3/2}\left(\sqrt{a}\sqrt{b}+5a-6b\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{5/4}\sqrt{\sqrt{a}+\sqrt{b}}}}{4ab(a-b)}}{d}$$

input
```
Int[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4)^2,x]
```

output
```
-((-1/4*((a^(3/2)*(5*a - Sqrt[a]*Sqrt[b] - 6*b)*ArcTan[(b^(1/4)*Cos[c + d*
x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(5/4)) + (a^(3/
2)*(5*a + Sqrt[a]*Sqrt[b] - 6*b)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[
a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(5/4)) - (4*a*(a - b)*Cos[c +
 d*x])/b)/(a*(a - b)*b) + (a*Cos[c + d*x]*(a + b - b*Cos[c + d*x]^2))/(4*(
a - b)*b^2*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)))/d)
```

3.212. $\int \frac{\sin^9(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

3.212.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.212. $\int \frac{\sin^9(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

3.212.4 Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{\cos(dx+c)}{b^2}+\frac{a\left(\frac{\frac{b\left(\cos^3(dx+c)\right)}{4a-4b}-\frac{(a+b)\cos(dx+c)}{4(a-b)}}{a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)}+\frac{b\left(\frac{\left(-\sqrt{ab}-6b+5a\right)\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-b\right)b}}-\frac{\left(-\sqrt{ab}+6b-5a\right)\operatorname{arctanh}\left(\right.}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+\right.}}\right.}{4a-4b}}{b^2}}{d}$
default	$\frac{-\frac{\cos(dx+c)}{b^2}+\frac{a\left(\frac{\frac{b\left(\cos^3(dx+c)\right)}{4a-4b}-\frac{(a+b)\cos(dx+c)}{4(a-b)}}{a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)}+\frac{b\left(\frac{\left(-\sqrt{ab}-6b+5a\right)\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-b\right)b}}-\frac{\left(-\sqrt{ab}+6b-5a\right)\operatorname{arctanh}\left(\right.}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+\right.}}\right.}{4a-4b}}{b^2}}{d}$
risch	$-\frac{e^{i(dx+c)}}{2b^2d}-\frac{e^{-i(dx+c)}}{2b^2d}-\frac{a\left(b\,e^{7i(dx+c)}-4a\,e^{5i(dx+c)}-b\,e^{5i(dx+c)}-4a\,e^{3i(dx+c)}-b\,e^{3i(dx+c)}+b\,e^{i(dx+c)}\right)}{2b^2(a-b)d\left(e^{8i(dx+c)}b-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}-\frac{i\left(\right.}{}$

input
```
int(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-cos(d*x+c)/b^2+a/b^2*((1/4*b/(a-b)*cos(d*x+c)^3-1/4*(a+b)/(a-b)*cos(
d*x+c))/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)+1/4/(a-b)*b*(1/2*(-(a*b)^(1/
2)-6*b+5*a)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a
*b)^(1/2)-b)*b)^(1/2))-1/2*(-(a*b)^(1/2)+6*b-5*a)/(a*b)^(1/2)/(((a*b)^(1/2
)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2)))))
```

3.212.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2649 vs. $2(188) = 376$.

Time = 0.60 (sec) , antiderivative size = 2649, normalized size of antiderivative = 11.22

$$\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
-1/16*(16*(a*b - b^2)*cos(d*x + c)^5 - 4*(7*a*b - 8*b^2)*cos(d*x + c)^3 +
((a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*
b^2 - 2*a*b^3 + b^4)*d)*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*s
qrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((
a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14
+ b^15)*d^4)) + 15*a^3 - 47*a^2*b + 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*
b^6 - b^7)*d^2))*log((625*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*
cos(d*x + c) + (2*(2*a^4*b^7 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11
)*d^3*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*
b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*
a*b^14 + b^15)*d^4)) - (125*a^5*b^2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*
b^5)*d)*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3
450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*
b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) +
15*a^3 - 47*a^2*b + 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))
) - ((a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (
a^2*b^2 - 2*a*b^3 + b^4)*d)*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^
2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)
/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^
14 + b^15)*d^4)) - 15*a^3 + 47*a^2*b - 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 ...
```

3.212.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**9/(a-b*sin(d*x+c)**4)**2,x)
```

output
```
Timed out
```

3.212.7 Maxima [F]

$$\int \frac{\sin^9(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \int \frac{\sin(dx+c)^9}{\left(b\sin(dx+c)^4-a\right)^2}\,dx$$

input
```
integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/2*((2*a*b^2 - 3*b^3)*cos(2*d*x + 2*c)*cos(d*x + c) - 4*(2*a*b^2 - 3*b^3)
*sin(3*d*x + 3*c)*sin(2*d*x + 2*c) + (2*a*b^2 - 3*b^3)*sin(2*d*x + 2*c)*si
n(d*x + c) - ((a*b^2 - b^3)*cos(9*d*x + 9*c) - 4*(a*b^2 - b^3)*cos(7*d*x +
 7*c) - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(5*d*x + 5*c) - 4*(a*b^2 - b^3)*
cos(3*d*x + 3*c) + (a*b^2 - b^3)*cos(d*x + c))*cos(10*d*x + 10*c) - (a*b^2
 - b^3 - (2*a*b^2 - 3*b^3)*cos(8*d*x + 8*c) - (20*a^2*b - 17*a*b^2 + 2*b^3
)*cos(6*d*x + 6*c) - (20*a^2*b - 17*a*b^2 + 2*b^3)*cos(4*d*x + 4*c) - (2*a
*b^2 - 3*b^3)*cos(2*d*x + 2*c))*cos(9*d*x + 9*c) - (4*(2*a*b^2 - 3*b^3)*co
s(7*d*x + 7*c) + 2*(16*a^2*b - 30*a*b^2 + 9*b^3)*cos(5*d*x + 5*c) + 4*(2*a
*b^2 - 3*b^3)*cos(3*d*x + 3*c) - (2*a*b^2 - 3*b^3)*cos(d*x + c))*cos(8*d*x
 + 8*c) + 4*(a*b^2 - b^3 - (20*a^2*b - 17*a*b^2 + 2*b^3)*cos(6*d*x + 6*c)
- (20*a^2*b - 17*a*b^2 + 2*b^3)*cos(4*d*x + 4*c) - (2*a*b^2 - 3*b^3)*cos(2
*d*x + 2*c))*cos(7*d*x + 7*c) - (2*(160*a^3 - 196*a^2*b + 67*a*b^2 - 6*b^3
)*cos(5*d*x + 5*c) + 4*(20*a^2*b - 17*a*b^2 + 2*b^3)*cos(3*d*x + 3*c) - (2
0*a^2*b - 17*a*b^2 + 2*b^3)*cos(d*x + c))*cos(6*d*x + 6*c) + 2*(8*a^2*b -
11*a*b^2 + 3*b^3 - (160*a^3 - 196*a^2*b + 67*a*b^2 - 6*b^3)*cos(4*d*x + 4*
c) - (16*a^2*b - 30*a*b^2 + 9*b^3)*cos(2*d*x + 2*c))*cos(5*d*x + 5*c) - (4
*(20*a^2*b - 17*a*b^2 + 2*b^3)*cos(3*d*x + 3*c) - (20*a^2*b - 17*a*b^2 + 2
*b^3)*cos(d*x + c))*cos(4*d*x + 4*c) + 4*(a*b^2 - b^3 - (2*a*b^2 - 3*b^3)*
cos(2*d*x + 2*c))*cos(3*d*x + 3*c) - (a*b^2 - b^3)*cos(d*x + c) + 2*((a...
```

3.212.8 Giac [F]

$$\int \frac{\sin^9(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \int \frac{\sin(dx+c)^9}{\left(b\sin(dx+c)^4-a\right)^2}\,dx$$

input
```
integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.212.9 Mupad [B] (verification not implemented)

Time = 16.00 (sec) , antiderivative size = 3941, normalized size of antiderivative = 16.70

$$\int \frac{\sin^9(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^9/(a - b*sin(c + d*x)^4)^2,x)
```

output
```
- cos(c + d*x)/(b^2*d) - ((cos(c + d*x)*(a*b + a^2))/(4*(a - b)) - (a*b*co
s(c + d*x)^3)/(4*(a - b)))/(d*(a*b^2 - b^3 + 2*b^3*cos(c + d*x)^2 - b^3*co
s(c + d*x)^4)) - (atan(((((1792*a^2*b^6 - 3072*a^3*b^5 + 1280*a^4*b^4)/(64
*(b^5 - 2*a*b^4 + a^2*b^3)) - (cos(c + d*x)*((25*a^2*(a^3*b^9)^(1/2) + 48*
b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a^3*b^9
)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2)*(256*a*b^7
- 512*a^2*b^6 + 256*a^3*b^5))/(4*(a^2*b - 2*a*b^2 + b^3)))*((25*a^2*(a^3*b
^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 -
69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1
/2) + (cos(c + d*x)*(25*a^4 - 59*a^3*b + 36*a^2*b^2))/(4*(a^2*b - 2*a*b^2
+ b^3)))*((25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47
*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*
a^2*b^10 + a^3*b^9)))^(1/2)*1i - (((1792*a^2*b^6 - 3072*a^3*b^5 + 1280*a^4
*b^4)/(64*(b^5 - 2*a*b^4 + a^2*b^3)) + (cos(c + d*x)*((25*a^2*(a^3*b^9)^(1
/2) + 48*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b
*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2)*(2
56*a*b^7 - 512*a^2*b^6 + 256*a^3*b^5))/(4*(a^2*b - 2*a*b^2 + b^3)))*((25*a
^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a
^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*
b^9)))^(1/2) - (cos(c + d*x)*(25*a^4 - 59*a^3*b + 36*a^2*b^2))/(4*(a^2*...
```

3.213 $\int \frac{\sin^7(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

3.213.1 Optimal result

Integrand size = 24, antiderivative size = 210

$$\int \frac{\sin^7(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \frac{\left(3\sqrt{a}-4\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{7/4}d} - \frac{\left(3\sqrt{a}+4\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{7/4}d} - \frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)}$$

output
```
-1/4*a*cos(d*x+c)*(2-cos(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d
*x+c)^4)+1/8*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(3*a^(1/2)
-4*b^(1/2))/b^(7/4)/d/(a^(1/2)-b^(1/2))^(3/2)-1/8*arctanh(b^(1/4)*cos(d*x+
c)/(a^(1/2)+b^(1/2))^(1/2))*(3*a^(1/2)+4*b^(1/2))/b^(7/4)/d/(a^(1/2)+b^(1/
2))^(3/2)
```

3.213.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.67 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.69

$$\int \frac{\sin^7(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$$

$$= \frac{\frac{16a(-5\cos(c+dx)+\cos(3(c+dx)))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))} - i\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&, \frac{6a\arctan\left(\frac{-}{co}\right.}{}\right.}{}$$

input
```
Integrate[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4)^2,x]
```

output
```
((16*a*(-5*Cos[c + d*x] + Cos[3*(c + d*x)]))/(8*a - 3*b + 4*b*Cos[2*(c + d
*x)] - b*Cos[4*(c + d*x)]) - I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4
 - 4*b*#1^6 + b*#1^8 & , (6*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - 8
*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - (3*I)*a*Log[1 - 2*Cos[c + d*
x]*#1 + #1^2] + (4*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 10*a*ArcTan[Si
n[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 24*b*ArcTan[Sin[c + d*x]/(Cos[c + d
*x] - #1)]*#1^2 + (5*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (12*I)*
b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 10*a*ArcTan[Sin[c + d*x]/(Cos[c
 + d*x] - #1)]*#1^4 - 24*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 -
 (5*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + (12*I)*b*Log[1 - 2*Cos[c
 + d*x]*#1 + #1^2]*#1^4 - 6*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^
6 + 8*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + (3*I)*a*Log[1 - 2*
Cos[c + d*x]*#1 + #1^2]*#1^6 - (4*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#
1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(32*(a - b)*
b*d)
```

3.213.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 3694, 1517, 27, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(c+dx)^7}{\left(a-b\sin(c+dx)^4\right)^2}dx$$

$$\Big\downarrow \text{ 3694}$$

$$-\frac{\int \frac{\left(1-\cos^2(c+dx)\right)^3}{\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)^2}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{ 1517}$$

$$-\frac{\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\int \frac{2a\left(2(a-2b)-(3a-4b)\cos^2(c+dx)\right)}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{8ab(a-b)}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\int \frac{2(a-2b)-(3a-4b)\cos^2(c+dx)}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{4b(a-b)}}{d}$$

$$\Big\downarrow \text{ 1480}$$

$$-\frac{\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{-\frac{1}{2}\left(-\sqrt{a}\sqrt{b}+3a-4b\right)\int \frac{1}{-b\cos^2(c+dx)-\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}d\cos(c+dx)-\frac{1}{2}\left(\sqrt{a}\sqrt{b}+3a-4b\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos(c+dx)}{4b(a-b)}}{d}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\frac{\left(-\sqrt{a}\sqrt{b}+3a-4b\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{1}{2}\left(\sqrt{a}\sqrt{b}+3a-4b\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos(c+dx)}{4b(a-b)}}{d}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\frac{\left(-\sqrt{a}\sqrt{b}+3a-4b\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\left(\sqrt{a}\sqrt{b}+3a-4b\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}}{4b(a-b)}}{d}$$

input
```
Int[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4)^2,x]
```

3.213. $\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

output
```
-((-1/4*(((3*a - Sqrt[a]*Sqrt[b] - 4*b)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt
[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(3/4)) - ((3*a + Sqrt[a
]*Sqrt[b] - 4*b)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/
(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/4))))/((a - b)*b) + (a*Cos[c + d*x]*(2 - Co
s[c + d*x]^2))/(4*(a - b)*b*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4
)))/d)
```

3.213.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

3.213. $\int \frac{\sin^7(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.213.4 Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\cos^3(dx+c)\right)}{4b(a-b)}-\frac{a\cos(dx+c)}{2b(a-b)}}{a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)}+\frac{\frac{\left(3a\sqrt{ab}-4\sqrt{ab}\,b-ab\right)\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\,b\sqrt{\left(\sqrt{ab}-b\right)b}}-\frac{\left(3a\sqrt{ab}-4\sqrt{ab}\,b+ab\right)\operatorname{arctanh}\left(\frac{\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)}}\right)}{2\sqrt{ab}\,b\sqrt{\left(\sqrt{ab}+b\right)b}}}{4a-4b}}{d}$
default	$\frac{\frac{\frac{a\left(\cos^3(dx+c)\right)}{4b(a-b)}-\frac{a\cos(dx+c)}{2b(a-b)}}{a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)}+\frac{\frac{\left(3a\sqrt{ab}-4\sqrt{ab}\,b-ab\right)\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\,b\sqrt{\left(\sqrt{ab}-b\right)b}}-\frac{\left(3a\sqrt{ab}-4\sqrt{ab}\,b+ab\right)\operatorname{arctanh}\left(\frac{\cos(dx+c)}{\sqrt{\left(\sqrt{ab}+b\right)}}\right)}{2\sqrt{ab}\,b\sqrt{\left(\sqrt{ab}+b\right)b}}}{4a-4b}}{d}$
risch	$-\frac{a\left(e^{7i(dx+c)}-5\,e^{5i(dx+c)}-5\,e^{3i(dx+c)}+e^{i(dx+c)}\right)}{2b(a-b)d\left(e^{8i(dx+c)}b-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}+\frac{i\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a^3b^7d^4-3a^2b^8 \right.\right.}\right.}{}$

input
```
int(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*((1/4*a/b/(a-b)*cos(d*x+c)^3-1/2*a/b/(a-b)*cos(d*x+c))/(a-b+2*b*cos(d*
x+c)^2-b*cos(d*x+c)^4)+1/4/(a-b)*(1/2*(3*a*(a*b)^(1/2)-4*(a*b)^(1/2)*b-a*b
)/(a*b)^(1/2)/b/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2
)-b)*b)^(1/2))-1/2*(3*a*(a*b)^(1/2)-4*(a*b)^(1/2)*b+a*b)/(a*b)^(1/2)/b/(((
a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2)))
```

3.213.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2507 vs. 2(161) = 322.

Time = 0.54 (sec) , antiderivative size = 2507, normalized size of antiderivative = 11.94

$$\int \frac{\sin^7(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
-1/16*(4*a*cos(d*x + c)^3 - ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b
^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(-((a^3*b^3 - 3*a^2*
b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^
2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a
^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b + 16*b^2)/((a^3*b^3 - 3*
a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log((81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b
^3)*cos(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^
9)*d^3*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)
/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12
 + b^13)*d^4)) - 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d)*sqrt(-(
(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*
a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 2
0*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b + 16*b^
2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) + ((a*b^2 - b^3)*d*cos(d*
x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*s
qrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b +
1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^
9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 3*a^2 + 15*a*b -
16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log((81*a^3 - 405*a^2
*b + 680*a*b^2 - 384*b^3)*cos(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*...
```

3.213.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**7/(a-b*sin(d*x+c)**4)**2,x)
```

output
```
Timed out
```

3.213.7 Maxima [F]

$$\int \frac{\sin^7(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \int \frac{\sin(dx+c)^7}{\left(b\sin(dx+c)^4-a\right)^2}\,dx$$

input
```
integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/2*(4*a*b*cos(2*d*x + 2*c)*cos(d*x + c) - 20*a*b*sin(3*d*x + 3*c)*sin(2*d
*x + 2*c) + 4*a*b*sin(2*d*x + 2*c)*sin(d*x + c) - a*b*cos(d*x + c) - (a*b*
cos(7*d*x + 7*c) - 5*a*b*cos(5*d*x + 5*c) - 5*a*b*cos(3*d*x + 3*c) + a*b*c
os(d*x + c))*cos(8*d*x + 8*c) + (4*a*b*cos(6*d*x + 6*c) + 4*a*b*cos(2*d*x
+ 2*c) - a*b + 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c))*cos(7*d*x + 7*c) - 4*(5
*a*b*cos(5*d*x + 5*c) + 5*a*b*cos(3*d*x + 3*c) - a*b*cos(d*x + c))*cos(6*d
*x + 6*c) - 5*(4*a*b*cos(2*d*x + 2*c) - a*b + 2*(8*a^2 - 3*a*b)*cos(4*d*x
+ 4*c))*cos(5*d*x + 5*c) - 2*(5*(8*a^2 - 3*a*b)*cos(3*d*x + 3*c) - (8*a^2
- 3*a*b)*cos(d*x + c))*cos(4*d*x + 4*c) - 5*(4*a*b*cos(2*d*x + 2*c) - a*b)
*cos(3*d*x + 3*c) + 2*((a*b^3 - b^4)*d*cos(8*d*x + 8*c)^2 + 16*(a*b^3 - b^
4)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*
cos(4*d*x + 4*c)^2 + 16*(a*b^3 - b^4)*d*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)
*d*sin(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^3*
b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 -
 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*
d*sin(2*d*x + 2*c)^2 - 8*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) + (a*b^3 - b^4)*
d - 2*(4*(a*b^3 - b^4)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^
4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)
*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4
*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(6*d*x +...
```

3.213.8 Giac [F]

$$\int \frac{\sin^7(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \int \frac{\sin(dx+c)^7}{\left(b\sin(dx+c)^4-a\right)^2}\,dx$$

input
```
integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.213.9 Mupad [B] (verification not implemented)

Time = 16.13 (sec) , antiderivative size = 3612, normalized size of antiderivative = 17.20

$$\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^7/(a - b*sin(c + d*x)^4)^2,x)
```

output
```
((a*cos(c + d*x)^3)/(4*b*(a - b)) - (a*cos(c + d*x))/(2*b*(a - b)))/(d*(a
- b + 2*b*cos(c + d*x)^2 - b*cos(c + d*x)^4)) - (atan(((((1024*a*b^6 - 153
6*a^2*b^5 + 512*a^3*b^4)/(64*(b^4 - 2*a*b^3 + a^2*b^2)) - (cos(c + d*x)*(-
(9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^
4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1
/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*(-(9
*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4
- 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2
) + (cos(c + d*x)*(16*a*b^2 - 23*a^2*b + 9*a^3))/(4*(a^2 - 2*a*b + b^2)))*
(-(9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*
b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^
(1/2)*1i - (((1024*a*b^6 - 1536*a^2*b^5 + 512*a^3*b^4)/(64*(b^4 - 2*a*b^3
+ a^2*b^2)) + (cos(c + d*x)*(-(9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2)
- 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^
10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))
/(4*(a^2 - 2*a*b + b^2)))*(-(9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) -
15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10
 - 3*a^2*b^8 + a^3*b^7)))^(1/2) - (cos(c + d*x)*(16*a*b^2 - 23*a^2*b + 9*a
^3))/(4*(a^2 - 2*a*b + b^2)))*(-(9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2
) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9...
```

3.214 $\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

3.214.1 Optimal result

Integrand size = 24, antiderivative size = 217

$$\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \frac{\left(\sqrt{a}-2\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{5/4}d} + \frac{\left(\sqrt{a}+2\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{5/4}d} - \frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)}$$

output
```
-1/4*cos(d*x+c)*(a+b-b*cos(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cos(d*x+c)^2-b*cos
(d*x+c)^4)+1/8*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(a^(1/2)
-2*b^(1/2))/b^(5/4)/d/a^(1/2)/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctanh(b^(1/4)*
cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(a^(1/2)+2*b^(1/2))/b^(5/4)/d/a^(1/2)/
(a^(1/2)+b^(1/2))^(3/2)
```

3.214.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.58 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.16

$$\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx =$$

$$-\frac{\frac{32\cos(c+dx)(2a+b-b\cos(2(c+dx)))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))}+i\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-2b\,\text{arcta}}{}\right.}{}$$

input
```
Integrate[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4)^2,x]
```

output
```
-1/32*((32*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(8*a - 3*b + 4*b*C
os[2*(c + d*x)] - b*Cos[4*(c + d*x)]) + I*RootSum[b - 4*b*#1^2 - 16*a*#1^4
 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x
] - #1)] + I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 8*a*ArcTan[Sin[c + d*x]
/(Cos[c + d*x] - #1)]*#1^2 + 22*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^2 + (4*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (11*I)*b*Log[1 -
2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 8*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #
1)]*#1^4 - 22*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (4*I)*a*Lo
g[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + (11*I)*b*Log[1 - 2*Cos[c + d*x]*#1
+ #1^2]*#1^4 + 2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*b*Log
[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*
#1^5 + b*#1^7) & ])/((a - b)*b*d)
```

3.214.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 3694, 1517, 27, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^5}{(a-b\sin(c+dx)^4)^2}dx$$

$\big\downarrow$ 3694

$$-\frac{\int \frac{(1-\cos^2(c+dx))^2}{(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b)^2}d\cos(c+dx)}{d}$$

$\big\downarrow$ 1517

$$-\frac{\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4b(a-b)(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b)}-\frac{\int \frac{2a\left(b\cos^2(c+dx)+a-3b\right)}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{8ab(a-b)}}{d}$$

$\big\downarrow$ 27

$$-\frac{\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4b(a-b)(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b)}-\frac{\int \frac{b\cos^2(c+dx)+a-3b}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{4b(a-b)}}{d}$$

$\big\downarrow$ 1480

$$-\frac{\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4b(a-b)(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b)}-\frac{\frac{1}{2}\sqrt{b}\left(\frac{a-2b}{\sqrt{a}}+\sqrt{b}\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos(c+dx)-\frac{1}{2}\sqrt{b}\left(\frac{a-2b}{\sqrt{a}}-\sqrt{b}\right)\int \frac{1}{-b\cos^2(c+dx)-}}{4b(a-b)}}{d}$$

$\big\downarrow$ 218

$$-\frac{\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4b(a-b)(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b)}-\frac{\frac{1}{2}\sqrt{b}\left(\frac{a-2b}{\sqrt{a}}+\sqrt{b}\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos(c+dx)+\frac{\left(\frac{a-2b}{\sqrt{a}}-\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}}{4b(a-b)}}{d}$$

$\big\downarrow$ 221

$$-\frac{\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4b(a-b)(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b)}-\frac{\frac{\left(\frac{a-2b}{\sqrt{a}}-\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\left(\frac{a-2b}{\sqrt{a}}+\sqrt{b}\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{b}\sqrt{\sqrt{a}+\sqrt{b}}}}{4b(a-b)}}{d}$$

input
```
Int[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4)^2,x]
```

3.214. $\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

output
```
-((-1/4*((((a - 2*b)/Sqrt[a] - Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt
[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(1/4)) + (((a - 2*b)/Sq
rt[a] + Sqrt[b])*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/
(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4)))/((a - b)*b) + (Cos[c + d*x]*(a + b -
b*Cos[c + d*x]^2))/(4*(a - b)*b*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*
x]^4)))/d)
```

3.214.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

3.214. $\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.214.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{-\frac{-\frac{\cos^3(dx+c)}{4(a-b)}+\frac{(a+b)\cos(dx+c)}{4b(a-b)}}{a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)}-\frac{\frac{\left(\sqrt{ab}+2b-a\right)\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-b\right)b}}-\frac{\left(\sqrt{ab}-2b+a\right)\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+b\right)b}}}{4(a-b)}}{d}$
default	$\frac{-\frac{-\frac{\cos^3(dx+c)}{4(a-b)}+\frac{(a+b)\cos(dx+c)}{4b(a-b)}}{a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)}-\frac{\frac{\left(\sqrt{ab}+2b-a\right)\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-b\right)b}}-\frac{\left(\sqrt{ab}-2b+a\right)\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+b\right)b}}}{4(a-b)}}{d}$
risch	$-\frac{b\,e^{7i(dx+c)}-4a\,e^{5i(dx+c)}-b\,e^{5i(dx+c)}-4a\,e^{3i(dx+c)}-b\,e^{3i(dx+c)}+b\,e^{i(dx+c)}}{2b(a-b)d\left(e^{8i(dx+c)}b-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}-\frac{i\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^5b^5d^4-3a^4b^6 \ldots\right.\right.}\right.}{\ldots}$

input
```
int(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-(-1/4/(a-b)*cos(d*x+c)^3+1/4*(a+b)/b/(a-b)*cos(d*x+c))/(a-b+2*b*cos(
d*x+c)^2-b*cos(d*x+c)^4)-1/4/(a-b)*(1/2*((a*b)^(1/2)+2*b-a)/(a*b)^(1/2)/((
(a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/2
*((a*b)^(1/2)-2*b+a)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x
+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))))
```

3.214.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2507 vs. $2(169) = 338$.

Time = 0.60 (sec) , antiderivative size = 2507, normalized size of antiderivative = 11.55

$$\int \frac{\sin^5(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
-1/16*(4*b*cos(d*x + c)^3 - ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b
^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(((a^4*b^2 - 3*a^3*b
^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 +
 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*
a^2*b^10 + a*b^11)*d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^
2*b^4 - a*b^5)*d^2))*log((a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*cos(d*x + c)
- (2*(a^4*b^5 - 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*sqrt((a^4 - 10*a^3*b +
41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^
4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) - (a^4*b - 8*a^3*b^2 + 23*
a^2*b^3 - 24*a*b^4)*d)*sqrt(((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2
*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*
b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) +
a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))) + ((a
*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b -
 2*a*b^2 + b^3)*d)*sqrt(-((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*sq
rt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6
 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) - a^2
 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))*log((a^3
- 9*a^2*b + 28*a*b^2 - 32*b^3)*cos(d*x + c) - (2*(a^4*b^5 - 3*a^3*b^6 + 3*
a^2*b^7 - a*b^8)*d^3*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*...
```

3.214.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**5/(a-b*sin(d*x+c)**4)**2,x)
```

output
```
Timed out
```

3.214.7 Maxima [F]

$$\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \int \frac{\sin(dx+c)^5}{\left(b\sin(dx+c)^4-a\right)^2}\,dx$$

input
```
integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/2*(4*b^2*cos(2*d*x + 2*c)*cos(d*x + c) + 4*b^2*sin(2*d*x + 2*c)*sin(d*x
+ c) - b^2*cos(d*x + c) - 4*(4*a*b + b^2)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c
) - (b^2*cos(7*d*x + 7*c) + b^2*cos(d*x + c) - (4*a*b + b^2)*cos(5*d*x + 5
*c) - (4*a*b + b^2)*cos(3*d*x + 3*c))*cos(8*d*x + 8*c) + (4*b^2*cos(6*d*x
+ 6*c) + 4*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c)
)*cos(7*d*x + 7*c) + 4*(b^2*cos(d*x + c) - (4*a*b + b^2)*cos(5*d*x + 5*c)
- (4*a*b + b^2)*cos(3*d*x + 3*c))*cos(6*d*x + 6*c) + (4*a*b + b^2 - 2*(32*
a^2 - 4*a*b - 3*b^2)*cos(4*d*x + 4*c) - 4*(4*a*b + b^2)*cos(2*d*x + 2*c))*
cos(5*d*x + 5*c) - 2*((32*a^2 - 4*a*b - 3*b^2)*cos(3*d*x + 3*c) - (8*a*b -
 3*b^2)*cos(d*x + c))*cos(4*d*x + 4*c) + (4*a*b + b^2 - 4*(4*a*b + b^2)*co
s(2*d*x + 2*c))*cos(3*d*x + 3*c) + 2*((a*b^3 - b^4)*d*cos(8*d*x + 8*c)^2 +
 16*(a*b^3 - b^4)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*
b^3 - 9*b^4)*d*cos(4*d*x + 4*c)^2 + 16*(a*b^3 - b^4)*d*cos(2*d*x + 2*c)^2
+ (a*b^3 - b^4)*d*sin(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*sin(6*d*x + 6*c)
^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*sin(4*d*x + 4*c)^2 +
16*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16
*(a*b^3 - b^4)*d*sin(2*d*x + 2*c)^2 - 8*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) +
 (a*b^3 - b^4)*d - 2*(4*(a*b^3 - b^4)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b^2 -
11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c)
- (a*b^3 - b^4)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^2*b^2 - 11*a*b^3 + 3*b^...
```

3.214.8 Giac [F]

$$\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \int \frac{\sin(dx+c)^5}{\left(b\sin(dx+c)^4-a\right)^2}\,dx$$

input
```
integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.214.9 Mupad [B] (verification not implemented)

Time = 16.69 (sec) , antiderivative size = 3839, normalized size of antiderivative = 17.69

$$\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^5/(a - b*sin(c + d*x)^4)^2,x)
```

output
```
(cos(c + d*x)^3/(4*(a - b)) - (cos(c + d*x)*(a + b))/(4*b*(a - b)))/(d*(a
- b + 2*b*cos(c + d*x)^2 - b*cos(c + d*x)^4)) - (atan(((((768*a*b^4 - 1024
*a^2*b^3 + 256*a^3*b^2)/(64*(a^2 - 2*a*b + b^2)) - (cos(c + d*x)*(-(a^2*(a
^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*
b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2
)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*(-(a^2
*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*b^5 - a^2*b^4 + a^3*b^3 - 5
*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(
1/2) + (cos(c + d*x)*(a^2*b - 3*a*b^2 + 4*b^3))/(4*(a^2 - 2*a*b + b^2)))*(
-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*b^5 - a^2*b^4 + a^3*b^
3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5
)))^(1/2)*1i - (((768*a*b^4 - 1024*a^2*b^3 + 256*a^3*b^2)/(64*(a^2 - 2*a*b
 + b^2)) + (cos(c + d*x)*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) -
4*a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3
*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4
))/(4*(a^2 - 2*a*b + b^2)))*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2)
 - 4*a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*
a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2) - (cos(c + d*x)*(a^2*b - 3*a*b^2 +
4*b^3))/(4*(a^2 - 2*a*b + b^2)))*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^
(1/2) - 4*a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*...
```

3.215 $\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

3.215.1 Optimal result

Integrand size = 24, antiderivative size = 186

$$\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = -\frac{\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/4}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/4}d}-\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)}$$

output
```
-1/4*cos(d*x+c)*(2-cos(d*x+c)^2)/(a-b)/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c
)^4)-1/8*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/b^(3/4)/d/a^(1
/2)/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2
))^(1/2))/b^(3/4)/d/a^(1/2)/(a^(1/2)+b^(1/2))^(3/2)
```

3.215.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.48 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.85

$$\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \frac{\frac{16(-5\cos(c+dx)+\cos(3(c+dx)))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))}-i\,\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{-2\arctan\left(\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4)^2,x]
```

output
```
((16*(-5*Cos[c + d*x] + Cos[3*(c + d*x)]))/(8*a - 3*b + 4*b*Cos[2*(c + d*x
)] - b*Cos[4*(c + d*x)]) - I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 -
 4*b*#1^6 + b*#1^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Lo
g[1 - 2*Cos[c + d*x]*#1 + #1^2] + 14*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #
1)]*#1^2 - (7*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 14*ArcTan[Sin[c
+ d*x]/(Cos[c + d*x] - #1)]*#1^4 + (7*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]
*#1^4 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*Log[1 - 2*Cos[
c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1
^7) & ])/(32*(a - b)*d)
```

3.215.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 3694, 1492, 27, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^3}{\left(a-b\sin(c+dx)^4\right)^2}dx$$

$$\Big\downarrow \text{3694}$$

$$-\frac{\int \frac{1-\cos^2(c+dx)}{\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)^2}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{1492}$$

$$-\frac{\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\int -\frac{2ab\left(2-\cos^2(c+dx)\right)}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{8ab(a-b)}}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{2-\cos^2(c+dx)}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{4(a-b)}+\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}}{d}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{\frac{-\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\int \frac{1}{-b\cos^2(c+dx)-\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}d\cos(c+dx)-\frac{1}{2}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos(c+dx)}{4(a-b)}+\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}}{d}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{\frac{\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{1}{2}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos(c+dx)}{4(a-b)}+\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}}{d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{\frac{\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}}{4(a-b)}+\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}}{d}$$

input
```
Int[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4)^2,x]
```

output
```
-(((((1 + Sqrt[b]/Sqrt[a])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sq
rt[b]]])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(3/4)) - ((1 - Sqrt[b]/Sqrt[a])*ArcT
anh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqr
t[b]]*b^(3/4)))/(4*(a - b)) + (Cos[c + d*x]*(2 - Cos[c + d*x]^2))/(4*(a -
b)*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)))/d)
```

3.215.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.215. $\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.215.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.10

3.215. $\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

method	result
derivativedivides	$\frac{b^2\left(\frac{\sqrt{ab}\left(\frac{\cos(dx+c)}{2(\sqrt{ab}+b)\left(-b(\cos^2(dx+c))+\sqrt{ab}+b\right)}+\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{(\sqrt{ab}+b)b}}\right)}{2(\sqrt{ab}+b)\sqrt{(\sqrt{ab}+b)b}}\right)}{4ab^2}-\frac{\sqrt{ab}\left(\frac{\cos(dx+c)}{2(\sqrt{ab}-b)\left(b(\cos^2(dx+c))+\sqrt{ab}-b\right)}+\ldots\right)}{4ab^2}\ldots\right)}{d}$
default	$\frac{b^2\left(\frac{\sqrt{ab}\left(\frac{\cos(dx+c)}{2(\sqrt{ab}+b)\left(-b(\cos^2(dx+c))+\sqrt{ab}+b\right)}+\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{(\sqrt{ab}+b)b}}\right)}{2(\sqrt{ab}+b)\sqrt{(\sqrt{ab}+b)b}}\right)}{4ab^2}-\frac{\sqrt{ab}\left(\frac{\cos(dx+c)}{2(\sqrt{ab}-b)\left(b(\cos^2(dx+c))+\sqrt{ab}-b\right)}+\ldots\right)}{4ab^2}\ldots\right)}{d}$
risch	$-\frac{e^{7i(dx+c)}-5e^{5i(dx+c)}-5e^{3i(dx+c)}+e^{i(dx+c)}}{2(a-b)d\left(e^{8i(dx+c)}b-4be^{6i(dx+c)}-16ae^{4i(dx+c)}+6be^{4i(dx+c)}-4be^{2i(dx+c)}+b\right)}+\frac{i\left(\sum_{\_R=\operatorname{RootOf}\left(-1+\left(16a^5b^3d^4-4\ldots\right.\right.}\ldots\right)}{\ldots}$

input
```
int(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*b^2*(1/4*(a*b)^(1/2)/a/b^2*(1/2*cos(d*x+c)/((a*b)^(1/2)+b)/(-b*cos(d*x
+c)^2+(a*b)^(1/2)+b)+1/2/((a*b)^(1/2)+b)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh
(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2)))-1/4*(a*b)^(1/2)/a/b^2*(1/2*cos(d
*x+c)/((a*b)^(1/2)-b)/(b*cos(d*x+c)^2+(a*b)^(1/2)-b)+1/2/((a*b)^(1/2)-b)/(
((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))))
```

3.215.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2049 vs. $2(141) = 282$.

Time = 0.42 (sec) , antiderivative size = 2049, normalized size of antiderivative = 11.02

$$\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
-1/16*(4*cos(d*x + c)^3 - ((a*b - b^2)*d*cos(d*x + c)^4 - 2*(a*b - b^2)*d*
cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*
b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5
*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4
*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log((a + 3*b)*cos(d*x + c) - ((a
^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sqrt((a^2 + 6*a*b + 9*b^2)/((a
^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*
b^9)*d^4)) - 2*(a^2*b + 3*a*b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3
- a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5
 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b -
 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))) + ((a*b - b^2)*d*cos(d*x + c)^4 - 2
*(a*b - b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(((a^4*b - 3*a^
3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^
6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) -
3*a - b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log((a + 3*b)*cos(
d*x + c) - ((a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sqrt((a^2 + 6*a*
b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 -
6*a^2*b^8 + a*b^9)*d^4)) + 2*(a^2*b + 3*a*b^2)*d)*sqrt(((a^4*b - 3*a^3*b^2
 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4
 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*...
```

3.215.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**3/(a-b*sin(d*x+c)**4)**2,x)
```

output
```
Timed out
```

3.215.7 Maxima [F]

$$\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx=\int \frac{\sin\left(dx+c\right)^3}{\left(b\sin\left(dx+c\right)^4-a\right)^2}\,dx$$

input
```
integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/2*(4*b*cos(2*d*x + 2*c)*cos(d*x + c) - 20*b*sin(3*d*x + 3*c)*sin(2*d*x +
 2*c) + 4*b*sin(2*d*x + 2*c)*sin(d*x + c) - (b*cos(7*d*x + 7*c) - 5*b*cos(
5*d*x + 5*c) - 5*b*cos(3*d*x + 3*c) + b*cos(d*x + c))*cos(8*d*x + 8*c) + (
4*b*cos(6*d*x + 6*c) + 2*(8*a - 3*b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*
c) - b)*cos(7*d*x + 7*c) - 4*(5*b*cos(5*d*x + 5*c) + 5*b*cos(3*d*x + 3*c)
- b*cos(d*x + c))*cos(6*d*x + 6*c) - 5*(2*(8*a - 3*b)*cos(4*d*x + 4*c) + 4
*b*cos(2*d*x + 2*c) - b)*cos(5*d*x + 5*c) - 2*(5*(8*a - 3*b)*cos(3*d*x + 3
*c) - (8*a - 3*b)*cos(d*x + c))*cos(4*d*x + 4*c) - 5*(4*b*cos(2*d*x + 2*c)
 - b)*cos(3*d*x + 3*c) - b*cos(d*x + c) + 2*((a*b^2 - b^3)*d*cos(8*d*x + 8
*c)^2 + 16*(a*b^2 - b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 57
*a*b^2 - 9*b^3)*d*cos(4*d*x + 4*c)^2 + 16*(a*b^2 - b^3)*d*cos(2*d*x + 2*c)
^2 + (a*b^2 - b^3)*d*sin(8*d*x + 8*c)^2 + 16*(a*b^2 - b^3)*d*sin(6*d*x + 6
*c)^2 + 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*d*sin(4*d*x + 4*c)^2 + 1
6*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a
*b^2 - b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a*b^2 - b^3)*d*cos(2*d*x + 2*c) + (a
*b^2 - b^3)*d - 2*(4*(a*b^2 - b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b - 11*a*
b^2 + 3*b^3)*d*cos(4*d*x + 4*c) + 4*(a*b^2 - b^3)*d*cos(2*d*x + 2*c) - (a*
b^2 - b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cos(4
*d*x + 4*c) + 4*(a*b^2 - b^3)*d*cos(2*d*x + 2*c) - (a*b^2 - b^3)*d)*cos(6*
d*x + 6*c) + 4*(4*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cos(2*d*x + 2*c) - (8*...
```

3.215.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(141) = 282$.

Time = 1.00 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.10

$$\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = -\frac{\frac{\cos(dx+c)^3}{d}-\frac{2\cos(dx+c)}{d}}{4\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2-a+b\right)(a-b)}$$

$$+\frac{\left(\left(a^2b-2\,ab^2+b^3\right)\sqrt{-b^2+\sqrt{ab}b}d^4-2\,\sqrt{ab}\sqrt{-b^2+\sqrt{ab}b}(a-b)d^2|-ad^2+bd^2|+\left(ad^2-bd^2\right)^2\sqrt{-b^2}\right.}{8\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\sqrt{ab}d^3|-ad^2+b}$$

$$-\frac{\left(\left(a^2b-2\,ab^2+b^3\right)\sqrt{-b^2-\sqrt{ab}b}d^4+2\,\sqrt{ab}\sqrt{-b^2-\sqrt{ab}b}(a-b)d^2|-ad^2+bd^2|+\left(ad^2-bd^2\right)^2\sqrt{-b^2}\right.}{8\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\sqrt{ab}d^3|-ad^2+b}$$

input
```
integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
-1/4*(cos(d*x + c)^3/d - 2*cos(d*x + c)/d)/((b*cos(d*x + c)^4 - 2*b*cos(d*
x + c)^2 - a + b)*(a - b)) + 1/8*((a^2*b - 2*a*b^2 + b^3)*sqrt(-b^2 + sqrt
(a*b)*b)*d^4 - 2*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*(a - b)*d^2*abs(-a*d^2
 + b*d^2) + (a*d^2 - b*d^2)^2*sqrt(-b^2 + sqrt(a*b)*b)*a)*arctan(cos(d*x +
 c)/(d*sqrt(-(a*b*d^2 - b^2*d^2 + sqrt((a*b*d^2 - b^2*d^2)^2 + (a*b*d^4 -
b^2*d^4)*(a^2 - 2*a*b + b^2)))/(a*b*d^4 - b^2*d^4))))/((a^3 - 3*a^2*b + 3*
a*b^2 - b^3)*sqrt(a*b)*d^3*abs(-a*d^2 + b*d^2)*abs(b)) - 1/8*((a^2*b - 2*a
*b^2 + b^3)*sqrt(-b^2 - sqrt(a*b)*b)*d^4 + 2*sqrt(a*b)*sqrt(-b^2 - sqrt(a*
b)*b)*(a - b)*d^2*abs(-a*d^2 + b*d^2) + (a*d^2 - b*d^2)^2*sqrt(-b^2 - sqrt
(a*b)*b)*a)*arctan(cos(d*x + c)/(d*sqrt(-(a*b*d^2 - b^2*d^2 - sqrt((a*b*d^
2 - b^2*d^2)^2 + (a*b*d^4 - b^2*d^4)*(a^2 - 2*a*b + b^2)))/(a*b*d^4 - b^2*
d^4))))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b)*d^3*abs(-a*d^2 + b*d^2)
*abs(b))
```

3.215.9 Mupad [B] (verification not implemented)

Time = 15.73 (sec) , antiderivative size = 3060, normalized size of antiderivative = 16.45

$$\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^3/(a - b*sin(c + d*x)^4)^2,x)
```

output
```
(cos(c + d*x)^3/(4*(a - b)) - cos(c + d*x)/(2*(a - b)))/(d*(a - b + 2*b*co
s(c + d*x)^2 - b*cos(c + d*x)^4)) - (atan(((((512*a*b^4 - 512*a^2*b^3)/(64
*(a^2 - 2*a*b + b^2)) - (cos(c + d*x)*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^
(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3
)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2))
)*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2
*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2) + (cos(c + d*x)*(a*b^2 + b
^3))/(4*(a^2 - 2*a*b + b^2)))*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) +
a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2
)*1i - (((512*a*b^4 - 512*a^2*b^3)/(64*(a^2 - 2*a*b + b^2)) + (cos(c + d*x
)*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2
*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 +
 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3
)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b
^3)))^(1/2) - (cos(c + d*x)*(a*b^2 + b^3))/(4*(a^2 - 2*a*b + b^2)))*((a*(a
^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3
*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2)*1i)/(b/(32*(a^2 - 2*a*b + b^2)) +
(((512*a*b^4 - 512*a^2*b^3)/(64*(a^2 - 2*a*b + b^2)) - (cos(c + d*x)*((a*(
a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 -
3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256...
```

3.216 $\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

3.216.1 Optimal result

Integrand size = 22, antiderivative size = 221

$$\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = -\frac{\left(3\sqrt{a}-2\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt[4]{b}d} - \frac{\left(3\sqrt{a}+2\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt[4]{b}d} - \frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)}$$

output
```
-1/4*cos(d*x+c)*(a+b-b*cos(d*x+c)^2)/a/(a-b)/d/(a-b+2*b*cos(d*x+c)^2-b*cos
(d*x+c)^4)-1/8*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(3*a^(1/
2)-2*b^(1/2))/a^(3/2)/b^(1/4)/d/(a^(1/2)-b^(1/2))^(3/2)-1/8*arctanh(b^(1/4
)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(3*a^(1/2)+2*b^(1/2))/a^(3/2)/b^(1/4
)/d/(a^(1/2)+b^(1/2))^(3/2)
```

3.216.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.48 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.12

$$\int \frac{\sin(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx =$$

$$-\frac{\frac{32\cos(c+dx)(2a+b-b\cos(2(c+dx)))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))} + i\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#1^4-4b\#1^6+b\#1^8\&, \frac{-2b\,\text{arcta}}{}\right.}{}$$

input
```
Integrate[Sin[c + d*x]/(a - b*Sin[c + d*x]^4)^2,x]
```

output
```
-1/32*((32*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(8*a - 3*b + 4*b*C
os[2*(c + d*x)] - b*Cos[4*(c + d*x)]) + I*RootSum[b - 4*b*#1^2 - 16*a*#1^4
 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x
] - #1)] + I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 24*a*ArcTan[Sin[c + d*x
]/(Cos[c + d*x] - #1)]*#1^2 - 10*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
]*#1^2 - (12*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (5*I)*b*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 24*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
 #1)]*#1^4 + 10*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (12*I)*a
*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - (5*I)*b*Log[1 - 2*Cos[c + d*x]*#
1 + #1^2]*#1^4 + 2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*b*L
og[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*
b*#1^5 + b*#1^7) & ])/(a*(a - b)*d)
```

3.216.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3042, 3694, 1405, 27, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(c+dx)}{\left(a-b\sin(c+dx)^4\right)^2}dx$$

$$\Big\downarrow \text{3694}$$

$$-\frac{\int \frac{1}{\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)^2}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{1405}$$

$$-\frac{\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4a(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\int -\frac{2b\left(-b\cos^2(c+dx)+3a-b\right)}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{8ab(a-b)}}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{-b\cos^2(c+dx)+3a-b}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{4a(a-b)}+\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4a(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}}{d}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{\frac{\frac{1}{2}\sqrt{b}\left(\frac{3a-2b}{\sqrt{a}}-\sqrt{b}\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos(c+dx)-\frac{1}{2}\sqrt{b}\left(\frac{3a-2b}{\sqrt{a}}+\sqrt{b}\right)\int \frac{1}{-b\cos^2(c+dx)-\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}d\cos(c+dx)}{4a(a-b)}+\frac{\cos(c+dx)(\ldots}{4a(a-b)(a-b\cos^4(\ldots}}{d}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{\frac{1}{2}\sqrt{b}\left(\frac{3a-2b}{\sqrt{a}}-\sqrt{b}\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos(c+dx)+\frac{\left(\frac{3a-2b}{\sqrt{a}}+\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}}{4a(a-b)}+\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4a(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)\ldots\right.}}{d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{\frac{\left(\frac{3a-2b}{\sqrt{a}}+\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\left(\frac{3a-2b}{\sqrt{a}}-\sqrt{b}\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{b}\sqrt{\sqrt{a}+\sqrt{b}}}}{4a(a-b)}+\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4a(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}}{d}$$

input
```
Int[Sin[c + d*x]/(a - b*Sin[c + d*x]^4)^2,x]
```

3.216. $\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

output
```
-((((((3*a - 2*b)/Sqrt[a] + Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sq
rt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(1/4)) + (((3*a - 2*b)/Sqr
t[a] - Sqrt[b])*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(
2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4)))/(4*a*(a - b)) + (Cos[c + d*x]*(a + b -
 b*Cos[c + d*x]^2))/(4*a*(a - b)*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d
*x]^4)))/d)
```

3.216.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.216. $\int \frac{\sin(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.216.4 Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.09

method	result
derivativedivides	$-\frac{b^{2}\left(\frac{\frac{\left(\sqrt{ab}+a\right)\cos(dx+c)}{2b(a-b)\left(\cos^{2}(dx+c)+\frac{\sqrt{ab}}{b}-1\right)}+\frac{\left(\sqrt{ab}+3a-2b\right)\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2(a-b)\sqrt{\left(\sqrt{ab}-b\right)b}}}{4\sqrt{ab}\,ab}+\frac{-\frac{\left(-\sqrt{ab}+a\right)\cos(dx+c)}{2b(a-b)\left(\cos^{2}(dx+c)-1-\frac{\sqrt{ab}}{b}\right)}-\frac{\left(\sqrt{ab}-3a+\cdots\right.}{2\cdots}}{4\sqrt{ab}\,ab}\cdots\right)}{d}$
default	$-\frac{b^{2}\left(\frac{\frac{\left(\sqrt{ab}+a\right)\cos(dx+c)}{2b(a-b)\left(\cos^{2}(dx+c)+\frac{\sqrt{ab}}{b}-1\right)}+\frac{\left(\sqrt{ab}+3a-2b\right)\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2(a-b)\sqrt{\left(\sqrt{ab}-b\right)b}}}{4\sqrt{ab}\,ab}+\frac{-\frac{\left(-\sqrt{ab}+a\right)\cos(dx+c)}{2b(a-b)\left(\cos^{2}(dx+c)-1-\frac{\sqrt{ab}}{b}\right)}-\frac{\left(\sqrt{ab}-3a+\cdots\right.}{2\cdots}}{4\sqrt{ab}\,ab}\cdots\right)}{d}$
risch	$-\frac{b\,e^{7i(dx+c)}-4a\,e^{5i(dx+c)}-b\,e^{5i(dx+c)}-4a\,e^{3i(dx+c)}-b\,e^{3i(dx+c)}+b\,e^{i(dx+c)}}{2a(a-b)d\left(e^{8i(dx+c)}b-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}-\frac{i\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4096a^{9}b\,d^{4}-122\cdots\right.\right.}\cdots\right)}{\cdots}$

input
```
int(sin(d*x+c)/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d*b^2*(1/4/(a*b)^(1/2)/a/b*(1/2*((a*b)^(1/2)+a)/b/(a-b)*cos(d*x+c)/(cos
(d*x+c)^2+(a*b)^(1/2)/b-1)+1/2*((a*b)^(1/2)+3*a-2*b)/(a-b)/(((a*b)^(1/2)-b
)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2)))+1/4/(a*b)^(1/2)
/a/b*(-1/2*(-(a*b)^(1/2)+a)/b/(a-b)*cos(d*x+c)/(cos(d*x+c)^2-1-(a*b)^(1/2)
/b)-1/2*((a*b)^(1/2)-3*a+2*b)/(a-b)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(
d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))))
```

3.216.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2269 vs. $2(173) = 346$.

Time = 0.56 (sec) , antiderivative size = 2269, normalized size of antiderivative = 10.27

$$\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
-1/16*(4*b*cos(d*x + c)^3 - ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b -
 a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^6 - 3*a^5*
b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a
^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4))
 + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*l
og((81*a^2 - 81*a*b + 20*b^2)*cos(d*x + c) + (2*(2*a^7*b - 7*a^6*b^2 + 9*a
^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d^3*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b
- 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*
d^4)) - (27*a^4 - 24*a^3*b + 5*a^2*b^2)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b
^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15
*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 -
 15*a*b + 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))) + ((a^2*b -
 a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a
^2*b + a*b^2)*d)*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*
a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*
a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - 15*a^2 + 15*a*b - 4*b^2)/((a^6 - 3*
a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log((81*a^2 - 81*a*b + 20*b^2)*cos(d*x
+ c) + (2*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d^3*sqrt
((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4
+ 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + (27*a^4 - 24*a^3*b + 5*a^2*...
```

3.216.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)/(a-b*sin(d*x+c)**4)**2,x)
```

output
```
Timed out
```

3.216.7 Maxima [F]

$$\int \frac{\sin(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \int \frac{\sin(dx+c)}{(b\sin(dx+c)^4-a)^2}\,dx$$

input
```
integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/2*(4*b^2*cos(2*d*x + 2*c)*cos(d*x + c) + 4*b^2*sin(2*d*x + 2*c)*sin(d*x
+ c) - b^2*cos(d*x + c) - 4*(4*a*b + b^2)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c
) - (b^2*cos(7*d*x + 7*c) + b^2*cos(d*x + c) - (4*a*b + b^2)*cos(5*d*x + 5
*c) - (4*a*b + b^2)*cos(3*d*x + 3*c))*cos(8*d*x + 8*c) + (4*b^2*cos(6*d*x
+ 6*c) + 4*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c)
)*cos(7*d*x + 7*c) + 4*(b^2*cos(d*x + c) - (4*a*b + b^2)*cos(5*d*x + 5*c)
- (4*a*b + b^2)*cos(3*d*x + 3*c))*cos(6*d*x + 6*c) + (4*a*b + b^2 - 2*(32*
a^2 - 4*a*b - 3*b^2)*cos(4*d*x + 4*c) - 4*(4*a*b + b^2)*cos(2*d*x + 2*c))*
cos(5*d*x + 5*c) - 2*((32*a^2 - 4*a*b - 3*b^2)*cos(3*d*x + 3*c) - (8*a*b -
 3*b^2)*cos(d*x + c))*cos(4*d*x + 4*c) + (4*a*b + b^2 - 4*(4*a*b + b^2)*co
s(2*d*x + 2*c))*cos(3*d*x + 3*c) + 2*((a^2*b^2 - a*b^3)*d*cos(8*d*x + 8*c)
^2 + 16*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 5
7*a^2*b^2 - 9*a*b^3)*d*cos(4*d*x + 4*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(2*d
*x + 2*c)^2 + (a^2*b^2 - a*b^3)*d*sin(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3
)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*s
in(4*d*x + 4*c)^2 + 16*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + 16*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a^2*b^2
 - a*b^3)*d*cos(2*d*x + 2*c) + (a^2*b^2 - a*b^3)*d - 2*(4*(a^2*b^2 - a*b^3
)*d*cos(6*d*x + 6*c) + 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(4*d*x + 4*
c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2 - a*b^3)*d)*cos(...
```

3.216.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 693 vs. $2(173) = 346$.

Time = 0.87 (sec) , antiderivative size = 693, normalized size of antiderivative = 3.14

$$\int \frac{\sin(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = -\frac{\frac{b\cos(dx+c)^3}{d} - \frac{a\cos(dx+c)}{d} - \frac{b\cos(dx+c)}{d}}{4\left(b\cos(dx+c)^4 - 2b\cos(dx+c)^2 - a + b\right)(a^2-ab)}$$

$$+\frac{\left((3a^4b - 8a^3b^2 + 7a^2b^3 - 2ab^4)\sqrt{-b^2+\sqrt{ab}b}d^4 - (3a^2-4ab+b^2)\sqrt{ab}\sqrt{-b^2+\sqrt{ab}b}d^2|-a^2d^2+ab\right.}{8(a^4-3a^3b+3a^2b^2-ab^3}$$

$$-\frac{\left((3a^4b - 8a^3b^2 + 7a^2b^3 - 2ab^4)\sqrt{-b^2-\sqrt{ab}b}d^4 + (3a^2-4ab+b^2)\sqrt{ab}\sqrt{-b^2-\sqrt{ab}b}d^2|-a^2d^2+ab\right.}{8(a^4-3a^3b+3a^2b^2-ab^3}$$

input
```
integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
-1/4*(b*cos(d*x + c)^3/d - a*cos(d*x + c)/d - b*cos(d*x + c)/d)/((b*cos(d*
x + c)^4 - 2*b*cos(d*x + c)^2 - a + b)*(a^2 - a*b)) + 1/8*((3*a^4*b - 8*a^
3*b^2 + 7*a^2*b^3 - 2*a*b^4)*sqrt(-b^2 + sqrt(a*b)*b)*d^4 - (3*a^2 - 4*a*b
 + b^2)*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*d^2*abs(-a^2*d^2 + a*b*d^2) + (
a^2*d^2 - a*b*d^2)^2*sqrt(-b^2 + sqrt(a*b)*b)*b)*arctan(cos(d*x + c)/(d*sq
rt(-(a^2*b*d^2 - a*b^2*d^2 + sqrt((a^2*b*d^2 - a*b^2*d^2)^2 + (a^2*b*d^4 -
 a*b^2*d^4)*(a^3 - 2*a^2*b + a*b^2)))/(a^2*b*d^4 - a*b^2*d^4))))/((a^4 - 3
*a^3*b + 3*a^2*b^2 - a*b^3)*sqrt(a*b)*d^3*abs(-a^2*d^2 + a*b*d^2)*abs(b))
- 1/8*((3*a^4*b - 8*a^3*b^2 + 7*a^2*b^3 - 2*a*b^4)*sqrt(-b^2 - sqrt(a*b)*b
)*d^4 + (3*a^2 - 4*a*b + b^2)*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*d^2*abs(-
a^2*d^2 + a*b*d^2) + (a^2*d^2 - a*b*d^2)^2*sqrt(-b^2 - sqrt(a*b)*b)*b)*arc
tan(cos(d*x + c)/(d*sqrt(-(a^2*b*d^2 - a*b^2*d^2 - sqrt((a^2*b*d^2 - a*b^2
*d^2)^2 + (a^2*b*d^4 - a*b^2*d^4)*(a^3 - 2*a^2*b + a*b^2)))/(a^2*b*d^4 - a
*b^2*d^4))))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sqrt(a*b)*d^3*abs(-a^2*d
^2 + a*b*d^2)*abs(b))
```

3.216.9 Mupad [B] (verification not implemented)

Time = 16.52 (sec) , antiderivative size = 3507, normalized size of antiderivative = 15.87

$$\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)/(a - b*sin(c + d*x)^4)^2,x)
```

output
```
((b*cos(c + d*x)^3)/(4*a*(a - b)) - (cos(c + d*x)*(a + b))/(4*a*(a - b)))/
(d*(a - b + 2*b*cos(c + d*x)^2 - b*cos(c + d*x)^4)) + (atan(((((256*a^3*b^
5 - 1024*a^4*b^4 + 768*a^5*b^3)/(64*(a^5 - 2*a^4*b + a^3*b^2)) - (cos(c +
d*x)*(256*a^3*b^6 - 512*a^4*b^5 + 256*a^5*b^4)*(-(15*a^5*b - 9*a*(a^9*b)^(
1/2) + 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 +
 3*a^7*b^3 - 3*a^8*b^2)))^(1/2))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*
b - 9*a*(a^9*b)^(1/2) + 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(
a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2) + (cos(c + d*x)*(4*b^5 -
11*a*b^4 + 9*a^2*b^3))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b - 9*a*(a
^9*b)^(1/2) + 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^
6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2)*1i - (((256*a^3*b^5 - 1024*a^4*b^4
+ 768*a^5*b^3)/(64*(a^5 - 2*a^4*b + a^3*b^2)) + (cos(c + d*x)*(256*a^3*b^6
 - 512*a^4*b^5 + 256*a^5*b^4)*(-(15*a^5*b - 9*a*(a^9*b)^(1/2) + 5*b*(a^9*b
)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^
8*b^2)))^(1/2))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b - 9*a*(a^9*b)^(
1/2) + 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 +
 3*a^7*b^3 - 3*a^8*b^2)))^(1/2) - (cos(c + d*x)*(4*b^5 - 11*a*b^4 + 9*a^2*
b^3))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b - 9*a*(a^9*b)^(1/2) + 5*b
*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3
 - 3*a^8*b^2)))^(1/2)*1i)/((9*a*b^3 - 4*b^4)/(32*(a^5 - 2*a^4*b + a^3*b...
```

3.217 $\int \frac{\csc(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

3.217.1 Optimal result

Integrand size = 22, antiderivative size = 325

$$\int \frac{\csc(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = -\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} - \frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}d} - \frac{\operatorname{arctanh}(\cos(c+dx))}{a^2d} + \frac{\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d} + \frac{\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}+\sqrt{b}}d} - \frac{b\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)}$$

output
```
-arctanh(cos(d*x+c))/a^2/d-1/4*b*cos(d*x+c)*(2-cos(d*x+c)^2)/a/(a-b)/d/(a-
b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)-1/8*b^(1/4)*arctan(b^(1/4)*cos(d*x+c)/(
a^(1/2)-b^(1/2))^(1/2))/a^(3/2)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*b^(1/4)*arct
anh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^(3/2)/d/(a^(1/2)+b^(1/2)
)^(3/2)-1/2*b^(1/4)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^2
/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*b^(1/4)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)
+b^(1/2))^(1/2))/a^2/d/(a^(1/2)+b^(1/2))^(1/2)
```

3.217.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.22 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.85

$$\int \frac{\csc(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$$

$$= \frac{16ab(-5\cos(c+dx)+\cos(3(c+dx)))}{(a-b)(8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx)))} - 32\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right) + 32\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right) - \frac{ib\text{RootSum}\Big[b}{}$$

input
```
Integrate[Csc[c + d*x]/(a - b*Sin[c + d*x]^4)^2,x]
```

output
```
((16*a*b*(-5*Cos[c + d*x] + Cos[3*(c + d*x)]))/((a - b)*(8*a - 3*b + 4*b*C
os[2*(c + d*x)] - b*Cos[4*(c + d*x)])) - 32*Log[Cos[(c + d*x)/2]] + 32*Log
[Sin[(c + d*x)/2]] - (I*b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*
b*#1^6 + b*#1^8 & , (-10*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 8*b*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + (5*I)*a*Log[1 - 2*Cos[c + d*x]*
#1 + #1^2] - (4*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 38*a*ArcTan[Sin[c
 + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 24*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
 - #1)]*#1^2 - (19*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (12*I)*b*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 38*a*ArcTan[Sin[c + d*x]/(Cos[c +
 d*x] - #1)]*#1^4 + 24*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (
19*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - (12*I)*b*Log[1 - 2*Cos[c
+ d*x]*#1 + #1^2]*#1^4 + 10*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^
6 - 8*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - (5*I)*a*Log[1 - 2*
Cos[c + d*x]*#1 + #1^2]*#1^6 + (4*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#
1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a - b))/(32
*a^2*d)
```

3.217.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3694, 1567, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.217. $\int \frac{\csc(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

$$\int \frac{\csc(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{\sin(c+dx)\,(a-b\sin(c+dx)^4)^2}dx$$

$\Big\downarrow$ 3694

$$-\frac{\int \frac{1}{(1-\cos^2(c+dx))(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b)^2}d\cos(c+dx)}{d}$$

$\Big\downarrow$ 1567

$$-\frac{\int \left(\frac{b-b\cos^2(c+dx)}{a^2(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b)}+\frac{b-b\cos^2(c+dx)}{a(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b)^2}-\frac{1}{a^2(\cos^2(c+dx)-1)}\right)d\cos(c+dx)}{d}$$

$\Big\downarrow$ 2009

$$-\frac{\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}+\sqrt{b}}}+\text{ar}}{d}$$

input
```
Int[Csc[c + d*x]/(a - b*Sin[c + d*x]^4)^2,x]
```

output
```
-(((b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*a^(
3/2)*(Sqrt[a] - Sqrt[b])^(3/2)) + (b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/S
qrt[Sqrt[a] - Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] - Sqrt[b]]) + ArcTanh[Cos[c +
 d*x]]/a^2 - (b^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b
]]])/(8*a^(3/2)*(Sqrt[a] + Sqrt[b])^(3/2)) - (b^(1/4)*ArcTanh[(b^(1/4)*Cos
[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] + Sqrt[b]]) + (b*
Cos[c + d*x]*(2 - Cos[c + d*x]^2))/(4*a*(a - b)*(a - b + 2*b*Cos[c + d*x]^
2 - b*Cos[c + d*x]^4)))/d)
```

3.217. $\int \frac{\csc(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

3.217.3.1 Defintions of rubi rules used

rule 1567
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.217.4 Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{-\frac{\ln(1+\cos(dx+c))}{2a^2}+\frac{\ln(\cos(dx+c)-1)}{2a^2}+\frac{b\left(\frac{\frac{a\left(\cos^3(dx+c)\right)}{4a-4b}-\frac{a\cos(dx+c)}{2(a-b)}}{a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)}+\frac{b\left(-\frac{\left(-5a\sqrt{ab}+4\sqrt{ab}\,b+ab\right)\operatorname{arctanh}\left(\frac{\cos(dx+\cdots}{\sqrt{\left(\sqrt{ab}+\cdots\right.}}\right.}{2\sqrt{ab}\,b\sqrt{\left(\sqrt{ab}+b\right)b}}\cdots\right.}{\cdots}\right.}{a^2}\cdots}{d}$
default	$\frac{-\frac{\ln(1+\cos(dx+c))}{2a^2}+\frac{\ln(\cos(dx+c)-1)}{2a^2}+\frac{b\left(\frac{\frac{a\left(\cos^3(dx+c)\right)}{4a-4b}-\frac{a\cos(dx+c)}{2(a-b)}}{a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)}+\frac{b\left(-\frac{\left(-5a\sqrt{ab}+4\sqrt{ab}\,b+ab\right)\operatorname{arctanh}\left(\frac{\cos(dx+\cdots}{\sqrt{\left(\sqrt{ab}+\cdots\right.}}\right.}{2\sqrt{ab}\,b\sqrt{\left(\sqrt{ab}+b\right)b}}\cdots\right.}{\cdots}\right.}{a^2}\cdots}{d}$
risch	$\frac{b\left(e^{7i(dx+c)}-5\,e^{5i(dx+c)}-5\,e^{3i(dx+c)}+e^{i(dx+c)}\right)}{2a(-a+b)d\left(e^{8i(dx+c)}b-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{d\,a^2}-\frac{\ln\left(e^{i(dx+c)}\cdots\right.}{d\,a^2}\cdots$

input
```
int(csc(d*x+c)/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a^2*ln(1+cos(d*x+c))+1/2/a^2*ln(cos(d*x+c)-1)+1/a^2*b*((1/4*a/(a
-b)*cos(d*x+c)^3-1/2*a/(a-b)*cos(d*x+c))/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c
)^4)+1/4/(a-b)*b*(-1/2*(-5*a*(a*b)^(1/2)+4*(a*b)^(1/2)*b+a*b)/(a*b)^(1/2)/
b/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2)
)+1/2*(-5*a*(a*b)^(1/2)+4*(a*b)^(1/2)*b-a*b)/(a*b)^(1/2)/b/(((a*b)^(1/2)-b
)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2)))))
```

3.217. $\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

3.217.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2711 vs. $2(244) = 488$.

Time = 0.86 (sec) , antiderivative size = 2711, normalized size of antiderivative = 8.34

$$\int \frac{\csc(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
-1/16*(4*a*b*cos(d*x + c)^3 - 8*a*b*cos(d*x + c) + ((a^3*b - a^2*b^2)*d*co
s(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*b + a^2
*b^2)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b
- 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15
*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 35*a^2
*b - 47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log((
625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)*cos(d*x + c) + ((5*a^10 -
18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)*d^3*sqrt((625*a^4*b - 1450
*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*
b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 2*(75*a^5*b
- 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*
b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b
^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 -
6*a^8*b^5 + a^7*b^6)*d^4)) + 35*a^2*b - 47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b
 + 3*a^5*b^2 - a^4*b^3)*d^2))) - ((a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(
a^3*b - a^2*b^2)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(((a^
7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 12
41*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10
*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*b + 47*a*b^2 - 16*
b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log((625*a^3*b - 1125...
```

3.217.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)/(a-b*sin(d*x+c)**4)**2,x)
```

output
```
Timed out
```

3.217.7 Maxima [F]

$$\int \frac{\csc(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \int \frac{\csc(dx+c)}{(b\sin(dx+c)^4-a)^2}\,dx$$

input
```
integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/2*(4*a*b^2*cos(2*d*x + 2*c)*cos(d*x + c) - 20*a*b^2*sin(3*d*x + 3*c)*sin
(2*d*x + 2*c) + 4*a*b^2*sin(2*d*x + 2*c)*sin(d*x + c) - a*b^2*cos(d*x + c)
 - (a*b^2*cos(7*d*x + 7*c) - 5*a*b^2*cos(5*d*x + 5*c) - 5*a*b^2*cos(3*d*x
+ 3*c) + a*b^2*cos(d*x + c))*cos(8*d*x + 8*c) + (4*a*b^2*cos(6*d*x + 6*c)
+ 4*a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c
))*cos(7*d*x + 7*c) - 4*(5*a*b^2*cos(5*d*x + 5*c) + 5*a*b^2*cos(3*d*x + 3*
c) - a*b^2*cos(d*x + c))*cos(6*d*x + 6*c) - 5*(4*a*b^2*cos(2*d*x + 2*c) -
a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(5*d*x + 5*c) - 2*(5*(8
*a^2*b - 3*a*b^2)*cos(3*d*x + 3*c) - (8*a^2*b - 3*a*b^2)*cos(d*x + c))*cos
(4*d*x + 4*c) - 5*(4*a*b^2*cos(2*d*x + 2*c) - a*b^2)*cos(3*d*x + 3*c) - 2*
((a^3*b^2 - a^2*b^3)*d*cos(8*d*x + 8*c)^2 + 16*(a^3*b^2 - a^2*b^3)*d*cos(6
*d*x + 6*c)^2 + 4*(64*a^5 - 112*a^4*b + 57*a^3*b^2 - 9*a^2*b^3)*d*cos(4*d*
x + 4*c)^2 + 16*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c)^2 + (a^3*b^2 - a^2*
b^3)*d*sin(8*d*x + 8*c)^2 + 16*(a^3*b^2 - a^2*b^3)*d*sin(6*d*x + 6*c)^2 +
4*(64*a^5 - 112*a^4*b + 57*a^3*b^2 - 9*a^2*b^3)*d*sin(4*d*x + 4*c)^2 + 16*
(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 1
6*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a^3*b^2 - a^2*b^3)*d*cos(2
*d*x + 2*c) + (a^3*b^2 - a^2*b^3)*d - 2*(4*(a^3*b^2 - a^2*b^3)*d*cos(6*d*x
 + 6*c) + 2*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*d*cos(4*d*x + 4*c) + 4*(a^3
*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) - (a^3*b^2 - a^2*b^3)*d)*cos(8*d*x +...
```

3.217.8 Giac [F]

$$\int \frac{\csc(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \int \frac{\csc(dx+c)}{(b\sin(dx+c)^4-a)^2}\,dx$$

input
```
integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.217.9 Mupad [B] (verification not implemented)

Time = 17.44 (sec) , antiderivative size = 7491, normalized size of antiderivative = 23.05

$$\int \frac{\csc(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(c + d*x)*(a - b*sin(c + d*x)^4)^2),x)
```

output
```
((b*cos(c + d*x)^3)/(4*a*(a - b)) - (b*cos(c + d*x))/(2*a*(a - b)))/(d*(a
- b + 2*b*cos(c + d*x)^2 - b*cos(c + d*x)^4)) - (atan(((((3072*a^3*b^7 - 1
0944*a^4*b^6 + 9776*a^5*b^5)/(256*(a^7 - 2*a^6*b + a^5*b^2)) - (((49152*a^
7*b^7 - 155648*a^8*b^6 + 172032*a^9*b^5 - 65536*a^10*b^4)/(256*(a^7 - 2*a^
6*b + a^5*b^2)) - (cos(c + d*x)*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/
2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^
10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2)*(98304*a^8*b^7 - 262144*a^9*b^6
 + 229376*a^10*b^5 - 65536*a^11*b^4))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*((2
5*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5
*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2))
)^(1/2) + (cos(c + d*x)*(18432*a^4*b^7 - 45440*a^5*b^6 + 29312*a^6*b^5))/(
128*(a^6 - 2*a^5*b + a^4*b^2)))*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/
2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^
10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2))*((25*a^2*(a^9*b)^(1/2) + 8*b^2
*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2)
)/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2) + (cos(c + d*x)*(76
8*b^7 - 2048*a*b^6 + 1425*a^2*b^5))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*((25*
a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b
^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^
(1/2)*1i - (((3072*a^3*b^7 - 10944*a^4*b^6 + 9776*a^5*b^5)/(256*(a^7 - ...
```

3.218 $\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

3.218.1 Optimal result

Integrand size = 24, antiderivative size = 320

$$\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \frac{x}{b^2}-\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}b^2d}+\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/2}d}-\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}b^2d}-\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/2}d}-\frac{\tan(c+dx)}{4(a-b)bd}+\frac{\tan^5(c+dx)}{4bd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)}$$

output
```
x/b^2+1/8*a^(1/4)*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/b^(3/
2)/d/(a^(1/2)-b^(1/2))^(3/2)-1/8*a^(1/4)*arctan((a^(1/2)+b^(1/2))^(1/2)*ta
n(d*x+c)/a^(1/4))/b^(3/2)/d/(a^(1/2)+b^(1/2))^(3/2)-1/2*a^(1/4)*arctan((a^
(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/b^2/d/(a^(1/2)-b^(1/2))^(1/2)-1/2
*a^(1/4)*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/b^2/d/(a^(1/2)
+b^(1/2))^(1/2)-1/4*tan(d*x+c)/(a-b)/b/d+1/4*tan(d*x+c)^5/b/d/(a+2*a*tan(d
*x+c)^2+(a-b)*tan(d*x+c)^4)
```

3.218.2 Mathematica [A] (verified)

Time = 6.69 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.82

$$\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$$
$$= \frac{8(c+dx) - \frac{\sqrt{a}\left(4\sqrt{a}+5\sqrt{b}\right)\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{a+\sqrt{a}\sqrt{b}}} + \frac{\sqrt{a}\left(4\sqrt{a}-5\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\sqrt{-a+\sqrt{a}\sqrt{b}}} + \frac{2ab(-6\sin(2(c}{(a-b)(8a-3b+4b\cos}}{8b^2d}$$

input
```
Integrate[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4)^2,x]
```

output
```
(8*(c + d*x) - (Sqrt[a]*(4*Sqrt[a] + 5*Sqrt[b])*ArcTan[((Sqrt[a] + Sqrt[b]
)*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])*Sqrt[a +
Sqrt[a]*Sqrt[b]]) + (Sqrt[a]*(4*Sqrt[a] - 5*Sqrt[b])*ArcTanh[((Sqrt[a] - S
qrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])*Sq
rt[-a + Sqrt[a]*Sqrt[b]]) + (2*a*b*(-6*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]
))/((a - b)*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])))/(8*b
^2*d)
```

3.218.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3696, 1650, 27, 1598, 27, 1442, 27, 1480, 218, 1610, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(c+dx)^8}{\left(a-b\sin(c+dx)^4\right)^2}dx$$

$$\Big\downarrow \; 3696$$

$$\frac{\int \frac{\tan^8(c+dx)}{(\tan^2(c+dx)+1)((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \; 1650$$

$$\frac{\frac{\int \frac{a\tan^4(c+dx)\left(\tan^2(c+dx)+1\right)}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{b}-\frac{\int \frac{\tan^4(c+dx)}{\left(\tan^2(c+dx)+1\right)\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}d\tan(c+dx)}{b}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{a\int \frac{\tan^4(c+dx)\left(\tan^2(c+dx)+1\right)}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{b}-\frac{\int \frac{\tan^4(c+dx)}{\left(\tan^2(c+dx)+1\right)\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}d\tan(c+dx)}{b}}{d}$$

$$\Big\downarrow \; 1598$$

$$\frac{\frac{a\left(\frac{\int -\frac{2b\tan^4(c+dx)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{8ab}+\frac{\tan^5(c+dx)}{4a\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}\right)}{b}-\frac{\int \frac{\tan^4(c+dx)}{\left(\tan^2(c+dx)+1\right)\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx\right.}}{b}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{a\left(\frac{\tan^5(c+dx)}{4a\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\int \frac{\tan^4(c+dx)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{4a}\right)}{b}-\frac{\int \frac{\tan^4(c+dx)}{\left(\tan^2(c+dx)+1\right)\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)\right.}}{b}}{d}$$

$$\Big\downarrow \; 1442$$

$$\frac{\frac{a\left(\frac{\tan^5(c+dx)}{4a\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\frac{\tan(c+dx)}{a-b}-\frac{\int \frac{a\left(2\tan^2(c+dx)+1\right)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{a-b}}{4a}\right)}{b}-\frac{\int \frac{\tan^4(c+dx)}{\left(\tan^2(c+dx)+1\right)\left((a-b)\tan^4(c+dx)+2\right.}}{b}}{d}$$

$$\Big\downarrow \; 27$$

3.218. $\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

$$\frac{\frac{a\left(\frac{\tan^5(c+dx)}{4a\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\frac{\tan(c+dx)}{a-b}-\frac{a\int\frac{2\tan^2(c+dx)+1}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{a-b}}{4a}\right)}{b}-\frac{\int\frac{\tan^4(c+dx)}{\left(\tan^2(c+dx)+1\right)\left((a-b)\tan^4(c+dx)+\cdots\right.}}{b}}{d}$$

$\downarrow$ 1480

$$\frac{\frac{a\left(\frac{\tan^5(c+dx)}{4a\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\frac{\tan(c+dx)}{a-b}-\frac{a\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\int\frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\tan(c+dx)}{2\sqrt{a}\sqrt{b}}+\frac{1}{2}\left(2-\frac{a+b}{\sqrt{a}\sqrt{b}}\right)\int\frac{1}{(a-b)\tan^2(c+dx)\cdots}\right.}{a-b}}{4a}\cdots\right.}{b}\cdots}{d}$$

$\downarrow$ 218

$$\frac{\frac{a\left(\frac{\tan^5(c+dx)}{4a\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\frac{\tan(c+dx)}{a-b}-\frac{a\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\left(2-\frac{a+b}{\sqrt{a}\sqrt{b}}\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{a-b}}{4a}\right)}{b}-\cdots}{d}$$

$\downarrow$ 1610

$$\frac{\frac{a\left(\frac{\tan^5(c+dx)}{4a\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\frac{\tan(c+dx)}{a-b}-\frac{a\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\left(2-\frac{a+b}{\sqrt{a}\sqrt{b}}\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{a-b}}{4a}\right)}{b}-\cdots}{d}$$

$\downarrow$ 2009

3.218. $\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

$$\frac{a\left(\frac{\tan^5(c+dx)}{4a\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\frac{\tan(c+dx)}{a-b}-\frac{a\left(\frac{(\sqrt{a}+\sqrt{b})\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\left(2-\frac{a+b}{\sqrt{a}\sqrt{b}}\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}(\sqrt{a}-\sqrt{b})\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{a-b}}{4a}\right)}{b} - \ldots}{d}$$

input
```
Int[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4)^2,x]
```

output
```
(-((-(ArcTan[Tan[c + d*x]]/b) + (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*T
an[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b) + (a^(1/4)*ArcTan[(Sq
rt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b
))/b) + (a*(-1/4*(-((a*(((Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b
]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*Sqrt[Sqrt[a] - Sqrt[b]]*Sqrt[b]) + (
(2 - (a + b)/(Sqrt[a]*Sqrt[b]))*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*
x])/a^(1/4)])/(2*a^(1/4)*(Sqrt[a] - Sqrt[b])*Sqrt[Sqrt[a] + Sqrt[b]])))/(a
 - b)) + Tan[c + d*x]/(a - b))/a + Tan[c + d*x]^5/(4*a*(a + 2*a*Tan[c + d*
x]^2 + (a - b)*Tan[c + d*x]^4))))/b)/d
```

3.218.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

3.218. $\int \frac{\sin^8(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1610
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

rule 1650
```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_) +
 (e_.)*(x_)^2), x_Symbol] :> Simp[-f^4/(c*d^2 - b*d*e + a*e^2)   Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Simp[d^2*(f
^4/(c*d^2 - b*d*e + a*e^2))   Int[(f*x)^(m - 4)*((a + b*x^2 + c*x^4)^(p + 1
)/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] && LtQ[p, -1] && GtQ[m, 2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

3.218. $\int \frac{\sin^8(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

3.218.4 Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(dx+c))}{b^2}-\frac{a\left(\frac{\frac{(\tan^3(dx+c))b}{2a-2b}+\frac{\tan(dx+c)b}{4a-4b}}{(\tan^4(dx+c))a-b(\tan^4(dx+c))+2a(\tan^2(dx+c))+a}+\frac{(4a\sqrt{ab}-6\sqrt{ab}\,b-3ab+5b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan(d}{\sqrt{(\sqrt{ab}-a)(}}\right.}{8\sqrt{ab}\,(a-b)\sqrt{(\sqrt{ab}-a)(a-b)}}\right.}{b^2}}{d}$
default	$\frac{\frac{\arctan(\tan(dx+c))}{b^2}-\frac{a\left(\frac{\frac{(\tan^3(dx+c))b}{2a-2b}+\frac{\tan(dx+c)b}{4a-4b}}{(\tan^4(dx+c))a-b(\tan^4(dx+c))+2a(\tan^2(dx+c))+a}+\frac{(4a\sqrt{ab}-6\sqrt{ab}\,b-3ab+5b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan(d}{\sqrt{(\sqrt{ab}-a)(}}\right.}{8\sqrt{ab}\,(a-b)\sqrt{(\sqrt{ab}-a)(a-b)}}\right.}{b^2}}{d}$
risch	$\frac{x}{b^2}-\frac{ia\left(b\,e^{6i(dx+c)}-8a\,e^{4i(dx+c)}+3b\,e^{4i(dx+c)}-5b\,e^{2i(dx+c)}+b\right)}{2b^2(a-b)d\left(e^{8i(dx+c)}b-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a^3b^8d^4-3\right.}\right.}{}$

input
```
int(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^2*arctan(tan(d*x+c))-a/b^2*((1/2*b/(a-b)*tan(d*x+c)^3+1/4*b/(a-b)
*tan(d*x+c))/(tan(d*x+c)^4*a-b*tan(d*x+c)^4+2*a*tan(d*x+c)^2+a)+1/8*(4*a*(
a*b)^(1/2)-6*(a*b)^(1/2)*b-3*a*b+5*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)
*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/8
*(4*a*(a*b)^(1/2)-6*(a*b)^(1/2)*b+3*a*b-5*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(
1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2)
)))
```

3.218.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3544 vs. 2(240) = 480.

Time = 0.94 (sec) , antiderivative size = 3544, normalized size of antiderivative = 11.08

$$\int \frac{\sin^8(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
1/32*(32*(a*b - b^2)*d*x*cos(d*x + c)^4 - 64*(a*b - b^2)*d*x*cos(d*x + c)^
2 - 32*(a^2 - 2*a*b + b^2)*d*x + ((a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^
3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*sqrt(-((a^3*b^4 -
 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 -
1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10
 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((
a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(32*a^3 - 166*a^2*b + 1125/4
*a*b^2 - 625/4*b^3 - 1/4*(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cos(
d*x + c)^2 + 1/2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9
)*d^3*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/
((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12
+ b^13)*d^4))*cos(d*x + c)*sin(d*x + c) - (24*a^3*b^2 - 127*a^2*b^3 + 220*
a*b^4 - 125*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^3*b^4 - 3*a^2*b^5
+ 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^
3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b
^11 - 6*a*b^12 + b^13)*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((a^3*b^4 - 3
*a^2*b^5 + 3*a*b^6 - b^7)*d^2)) + 1/4*(2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^
2*b^5 - 91*a*b^6 + 25*b^7)*d^2*cos(d*x + c)^2 - (16*a^4*b^3 - 73*a^3*b^4 +
 123*a^2*b^5 - 91*a*b^6 + 25*b^7)*d^2)*sqrt((64*a^5 - 464*a^4*b + 1241*a^3
*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 2...
```

3.218.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^8(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**8/(a-b*sin(d*x+c)**4)**2,x)
```

output
```
Timed out
```

3.218.7 Maxima [F]

$$\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \int \frac{\sin\left(dx+c\right)^8}{\left(b\sin\left(dx+c\right)^4-a\right)^2}\,dx$$

input
```
integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/2*(2*(a*b^2 - b^3)*d*x*cos(8*d*x + 8*c)^2 + 32*(a*b^2 - b^3)*d*x*cos(6*d
*x + 6*c)^2 + 8*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*d*x*cos(4*d*x + 4*
c)^2 + 32*(a*b^2 - b^3)*d*x*cos(2*d*x + 2*c)^2 + 2*(a*b^2 - b^3)*d*x*sin(8
*d*x + 8*c)^2 + 32*(a*b^2 - b^3)*d*x*sin(6*d*x + 6*c)^2 + 8*(64*a^3 - 112*
a^2*b + 57*a*b^2 - 9*b^3)*d*x*sin(4*d*x + 4*c)^2 + 32*(a*b^2 - b^3)*d*x*si
n(2*d*x + 2*c)^2 - 16*(a*b^2 - b^3)*d*x*cos(2*d*x + 2*c) - a*b^2*sin(2*d*x
 + 2*c) + 2*(a*b^2 - b^3)*d*x - (16*(a*b^2 - b^3)*d*x*cos(6*d*x + 6*c) + 8
*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*x*cos(4*d*x + 4*c) + 16*(a*b^2 - b^3)*d*x*
cos(2*d*x + 2*c) - a*b^2*sin(6*d*x + 6*c) + 5*a*b^2*sin(2*d*x + 2*c) - 4*(
a*b^2 - b^3)*d*x + (8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c))*cos(8*d*x + 8*c)
+ 2*(16*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*x*cos(4*d*x + 4*c) + 32*(a*b^2 - b^
3)*d*x*cos(2*d*x + 2*c) + 12*a*b^2*sin(2*d*x + 2*c) - 8*(a*b^2 - b^3)*d*x
+ 3*(8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c))*cos(6*d*x + 6*c) + 2*(16*(8*a^2*
b - 11*a*b^2 + 3*b^3)*d*x*cos(2*d*x + 2*c) - 4*(8*a^2*b - 11*a*b^2 + 3*b^3
)*d*x + 3*(8*a^2*b - 3*a*b^2)*sin(2*d*x + 2*c))*cos(4*d*x + 4*c) - 2*((a*b
^4 - b^5)*d*cos(8*d*x + 8*c)^2 + 16*(a*b^4 - b^5)*d*cos(6*d*x + 6*c)^2 + 4
*(64*a^3*b^2 - 112*a^2*b^3 + 57*a*b^4 - 9*b^5)*d*cos(4*d*x + 4*c)^2 + 16*(
a*b^4 - b^5)*d*cos(2*d*x + 2*c)^2 + (a*b^4 - b^5)*d*sin(8*d*x + 8*c)^2 + 1
6*(a*b^4 - b^5)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b^2 - 112*a^2*b^3 + 57*a*
b^4 - 9*b^5)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d...
```

3.218.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1563 vs. $2(240) = 480$.

Time = 0.93 (sec) , antiderivative size = 1563, normalized size of antiderivative = 4.88

$$\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

output

```
1/8*((2*(6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 21*sqrt(a^2
 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 16*sqrt(a^2 - a*b - sqrt(a*b
)*(a - b))*sqrt(a*b)*a*b^2 + 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*
b)*b^3)*(a*b^2 - b^3)^2*abs(-a + b) - (12*sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*a^5*b^2 - 63*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b^3 + 116*sqrt(a^
2 - a*b - sqrt(a*b)*(a - b))*a^3*b^4 - 86*sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*a^2*b^5 + 16*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^6 + 5*sqrt(a^2 -
a*b - sqrt(a*b)*(a - b))*b^7)*abs(-a*b^2 + b^3)*abs(-a + b) - (9*sqrt(a^2
- a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^4 - 51*sqrt(a^2 - a*b - sqrt(a*
b)*(a - b))*sqrt(a*b)*a^4*b^5 + 102*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sq
rt(a*b)*a^3*b^6 - 82*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^7
 + 17*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^8 + 5*sqrt(a^2 - a
*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^9)*abs(-a + b))*(pi*floor((d*x + c)/pi
 + 1/2) + arctan(tan(d*x + c)/sqrt((a^2*b^2 - a*b^3 + sqrt((a^2*b^2 - a*b^
3)^2 - (a^2*b^2 - a*b^3)*(a^2*b^2 - 2*a*b^3 + b^4)))/(a^2*b^2 - 2*a*b^3 +
b^4))))/((3*a^7*b^4 - 21*a^6*b^5 + 59*a^5*b^6 - 85*a^4*b^7 + 65*a^3*b^8 -
23*a^2*b^9 + a*b^10 + b^11)*abs(-a*b^2 + b^3)) + (2*(6*sqrt(a^2 - a*b + sq
rt(a*b)*(a - b))*sqrt(a*b)*a^3 - 21*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sq
rt(a*b)*a^2*b + 16*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 + 3
*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(a*b^2 - b^3)^2*abs...
```

3.218.9 Mupad [B] (verification not implemented)

Time = 17.82 (sec) , antiderivative size = 7640, normalized size of antiderivative = 23.88

$$\int \frac{\sin^8(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Too large to display}$$

input

```
int(sin(c + d*x)^8/(a - b*sin(c + d*x)^4)^2,x)
```

output

```
(atan(((((5120*a^2*b^7 - 17664*a^3*b^6 + 26688*a^4*b^5 - 16320*a^5*b^4 + 3
072*a^6*b^3)/(256*(a*b^5 - b^6)) + (((20480*a^2*b^11 - 110592*a^3*b^10 + 2
08896*a^4*b^9 - 167936*a^5*b^8 + 49152*a^6*b^7)/(256*(a*b^5 - b^6)) - (tan
(c + d*x)*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2
*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^
9 + a^3*b^8)))^(1/2)*(98304*a^2*b^12 - 196608*a^3*b^11 + 196608*a^5*b^9 -
98304*a^6*b^8))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9
)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*
(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) - (tan(c + d*x)*(21376*a^2
*b^8 - 84864*a^3*b^7 + 54912*a^4*b^6 + 20864*a^5*b^5 - 18432*a^6*b^4))/(12
8*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6
+ 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 -
3*a^2*b^9 + a^3*b^8)))^(1/2))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2)
 - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^
10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) + (tan(c + d*x)*(768*a^6 + 800*a^
2*b^4 + 4832*a^3*b^3 - 5295*a^4*b^2))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)
^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*
b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2)*1i -
 (((5120*a^2*b^7 - 17664*a^3*b^6 + 26688*a^4*b^5 - 16320*a^5*b^4 + 3072*a^
6*b^3)/(256*(a*b^5 - b^6)) + (((20480*a^2*b^11 - 110592*a^3*b^10 + 2088...
```

3.218. $\int \frac{\sin^8(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

3.219 $\int \frac{\sin^6(c+dx)}{(a-b\sin^4(c+dx))^2} dx$

3.219.1 Optimal result

Integrand size = 24, antiderivative size = 233

$$\int \frac{\sin^6(c+dx)}{(a-b\sin^4(c+dx))^2} dx = -\frac{\left(2\sqrt{a}-3\sqrt{b}\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/2}d} + \frac{\left(2\sqrt{a}+3\sqrt{b}\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/2}d} - \frac{\tan(c+dx)}{4(a-b)bd} + \frac{\sec^2(c+dx)\tan^3(c+dx)}{4bd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)}$$

output
```
-1/8*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)-3*b^(1/
2))/a^(1/4)/b^(3/2)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctan((a^(1/2)+b^(1/2))
^(1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)+3*b^(1/2))/a^(1/4)/b^(3/2)/d/(a^(1/2
)+b^(1/2))^(3/2)-1/4*tan(d*x+c)/(a-b)/b/d+1/4*sec(d*x+c)^2*tan(d*x+c)^3/b/
d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)
```

3.219.2 Mathematica [A] (verified)

Time = 2.95 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.02

$$\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$$
$$=\frac{\frac{\left(2a+\sqrt{a}\sqrt{b}-3b\right)\sqrt{b}\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}-\frac{\sqrt{b}\left(-2a+\sqrt{a}\sqrt{b}+3b\right)\operatorname{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{4b(-2a-b+b\cos(2(c+dx)))\,\mathrm{s}}{8a-3b+4b\cos(2(c+dx))-b\,\mathrm{c}}}{8(a-b)b^2d}$$

input
```
Integrate[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4)^2,x]
```

output
```
(((2*a + Sqrt[a]*Sqrt[b] - 3*b)*Sqrt[b]*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c
+ d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - (Sqrt[b]*(
-2*a + Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/S
qrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (4*b*(-2*a - b +
b*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] -
b*Cos[4*(c + d*x)]))/(8*(a - b)*b^2*d)
```

3.219.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 3696, 1440, 27, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(c+dx)^6}{\left(a-b\sin(c+dx)^4\right)^2}dx$$

$$\Big\downarrow \text{ 3696}$$

$$\frac{\int \frac{\tan^6(c+dx)}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 1440}$$

$$\frac{\frac{\tan^3(c+dx)(\tan^2(c+dx)+1)}{4b((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}-\frac{\int\frac{2a\tan^2(c+dx)(\tan^2(c+dx)+3)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{8ab}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\tan^3(c+dx)(\tan^2(c+dx)+1)}{4b((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}-\frac{\int\frac{\tan^2(c+dx)(\tan^2(c+dx)+3)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{4b}}{d}$$

$$\Big\downarrow \ 1602$$

$$\frac{\frac{\tan^3(c+dx)(\tan^2(c+dx)+1)}{4b((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}-\frac{\frac{\tan(c+dx)}{a-b}-\frac{\int\frac{a-(a-3b)\tan^2(c+dx)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{a-b}}{4b}}{d}$$

$$\Big\downarrow \ 1480$$

$$\frac{\frac{\tan^3(c+dx)(\tan^2(c+dx)+1)}{4b((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}-\frac{\frac{\tan(c+dx)}{a-b}-\frac{-\frac{1}{2}\left(-\frac{2\sqrt{a}(a-2b)}{\sqrt{b}}+a-3b\right)\int\frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tan(c+dx)-\frac{1}{2}\left(\frac{2\sqrt{a}(a-2b)}{\sqrt{b}}+a-3 \cdots\right.}{a-b}}{4b}}{d}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{\tan^3(c+dx)(\tan^2(c+dx)+1)}{4b((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}-\frac{\frac{\tan(c+dx)}{a-b}-\frac{-\frac{\left(\frac{2\sqrt{a}(a-2b)}{\sqrt{b}}+a-3b\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\left(\sqrt{a}+\sqrt{b}\right)}-\frac{\left(-\frac{2\sqrt{a}(a-2b)}{\sqrt{b}}+a-3b\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\cdots}}{\cdots}\right.}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\cdots}}}{a-b}}{4b}}{d}$$

input
```
Int[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4)^2,x]
```

output
```
(-1/4*(-((-1/2*((a + (2*Sqrt[a]*(a - 2*b))/Sqrt[b] - 3*b)*ArcTan[(Sqrt[Sqr
t[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*(
Sqrt[a] + Sqrt[b])) - ((a - (2*Sqrt[a]*(a - 2*b))/Sqrt[b] - 3*b)*ArcTan[(S
qrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(1/4)*(Sqrt[a] - Sqrt[
b])*Sqrt[Sqrt[a] + Sqrt[b]]))/(a - b)) + Tan[c + d*x]/(a - b))/b + (Tan[c
+ d*x]^3*(1 + Tan[c + d*x]^2))/(4*b*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[
c + d*x]^4)))/d
```

3.219. $\int \frac{\sin^6(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

3.219.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

3.219. $\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

3.219.4 Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{-\frac{(a+b)\left(\tan^3(dx+c)\right)}{4b(a-b)}-\frac{a\tan(dx+c)}{4b(a-b)}}{\left(\tan^4(dx+c)\right)a-b\left(\tan^4(dx+c)\right)+2a\left(\tan^2(dx+c)\right)+a}+\frac{\frac{\left(-a\sqrt{ab}+3\sqrt{ab}\,b-2a^2+4ab\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(-a\sqrt{ab}+3\sqrt{ab}\cdots\right.}{\cdots}}{4b}}{d}$
default	$\frac{\frac{-\frac{(a+b)\left(\tan^3(dx+c)\right)}{4b(a-b)}-\frac{a\tan(dx+c)}{4b(a-b)}}{\left(\tan^4(dx+c)\right)a-b\left(\tan^4(dx+c)\right)+2a\left(\tan^2(dx+c)\right)+a}+\frac{\frac{\left(-a\sqrt{ab}+3\sqrt{ab}\,b-2a^2+4ab\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(-a\sqrt{ab}+3\sqrt{ab}\cdots\right.}{\cdots}}{4b}}{d}$
risch	$-\frac{i\left(2a\,e^{6i(dx+c)}-b\,e^{6i(dx+c)}-8a\,e^{4i(dx+c)}+3b\,e^{4i(dx+c)}-2a\,e^{2i(dx+c)}-3b\,e^{2i(dx+c)}+b\right)}{2b(a-b)d\left(e^{8i(dx+c)}b-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}-\frac{\left(\_R=\text{RootOf}\left(\left(a^4b^6d^4-3a\cdots\right.\right.\right.}{\cdots}$

input
```
int(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*((-1/4*(a+b)/b/(a-b)*tan(d*x+c)^3-1/4*a/b/(a-b)*tan(d*x+c))/(tan(d*x+c
)^4*a-b*tan(d*x+c)^4+2*a*tan(d*x+c)^2+a)+1/4/b*(1/2*(-a*(a*b)^(1/2)+3*(a*b
)^(1/2)*b-2*a^2+4*a*b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arc
tan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*(-a*(a*b)^(1/2)+3*
(a*b)^(1/2)*b+2*a^2-4*a*b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)
*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))))
```

3.219.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3135 vs. 2(181) = 362.

Time = 0.86 (sec) , antiderivative size = 3135, normalized size of antiderivative = 13.45

$$\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
-1/32*(((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2
- (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*
d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 2
0*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15*a*b + 15*b^
2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(1/4*(20*a^2 - 81*a*b +
 81*b^2)*cos(d*x + c)^2 - 5*a^2 + 81/4*a*b - 81/4*b^2 + 1/2*((a^5*b^3 - 6*
a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7)*d^3*sqrt((25*a^2 - 90*a*b + 8
1*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^
2*b^8 + a*b^9)*d^4))*cos(d*x + c)*sin(d*x + c) + 2*(5*a^3*b - 19*a^2*b^2 +
 18*a*b^3)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*
b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*
a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15*
a*b + 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2)) + 1/4*(2*(4*a^5
*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2*cos(d*x + c)^2 -
(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2)*sqrt((25*a
^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15
*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))) - ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2
*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(-((a^3*b
^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*
b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*...
```

3.219.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx=\text{Timed out}$$

input
```
integrate(sin(d*x+c)**6/(a-b*sin(d*x+c)**4)**2,x)
```

output
```
Timed out
```

3.219.7 Maxima [F]

$$\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx=\int \frac{\sin\left(dx+c\right)^6}{\left(b\sin\left(dx+c\right)^4-a\right)^2}\,dx$$

input
```
integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/2*(2*(16*a^2 + 2*a*b - 3*b^2)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + ((2*a*
b - b^2)*sin(6*d*x + 6*c) - (8*a*b - 3*b^2)*sin(4*d*x + 4*c) - (2*a*b + 3*
b^2)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 2*((16*a^2 + 2*a*b - 3*b^2)*sin(
4*d*x + 4*c) + 4*(2*a*b + b^2)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - 2*((a*
b^3 - b^4)*d*cos(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*cos(6*d*x + 6*c)^2 +
4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*cos(4*d*x + 4*c)^2 + 16*(a
*b^3 - b^4)*d*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*d*sin(8*d*x + 8*c)^2 + 16
*(a*b^3 - b^4)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3
 - 9*b^4)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*d*sin(2*d*x + 2*c)^2 - 8*(
a*b^3 - b^4)*d*cos(2*d*x + 2*c) + (a*b^3 - b^4)*d - 2*(4*(a*b^3 - b^4)*d*c
os(6*d*x + 6*c) + 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*
(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(8*d*x + 8*c) + 8*(
2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*co
s(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^2*b^2 - 11*
a*b^3 + 3*b^4)*d*cos(2*d*x + 2*c) - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*cos(
4*d*x + 4*c) - 4*(2*(a*b^3 - b^4)*d*sin(6*d*x + 6*c) + (8*a^2*b^2 - 11*a*b
^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(8
*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a
*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-(4*(2*a*b ...
```

3.219.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1481 vs. $2(181) = 362$.

Time = 0.81 (sec) , antiderivative size = 1481, normalized size of antiderivative = 6.36

$$\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
1/8*(((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 15*sqrt(a^2 -
 a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 17*sqrt(a^2 - a*b - sqrt(a*b)*
(a - b))*sqrt(a*b)*a*b^2 + 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)
*b^3)*(a*b - b^2)^2*abs(-a + b) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a
^5*b - 12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b^2 + 14*sqrt(a^2 - a*b
- sqrt(a*b)*(a - b))*a^3*b^3 - 4*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b
^4 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^5)*abs(-a*b + b^2)*abs(-a + b
) - 2*(3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b - 18*sqrt(a^2
 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^2 + 38*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*sqrt(a*b)*a^4*b^3 - 32*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sq
rt(a*b)*a^3*b^4 + 7*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^5
+ 2*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^6)*abs(-a + b))*(pi*
floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^2*b - a*b^2 + sqrt
((a^2*b - a*b^2)^2 - (a^2*b - a*b^2)*(a^2*b - 2*a*b^2 + b^3)))/(a^2*b - 2*
a*b^2 + b^3))))/((3*a^8*b^2 - 21*a^7*b^3 + 59*a^6*b^4 - 85*a^5*b^5 + 65*a^
4*b^6 - 23*a^3*b^7 + a^2*b^8 + a*b^9)*abs(-a*b + b^2)) - ((3*sqrt(a^2 - a*
b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 15*sqrt(a^2 - a*b + sqrt(a*b)*(a -
b))*sqrt(a*b)*a^2*b + 17*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b
^2 + 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(a*b - b^2)^2*ab
s(-a + b) - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^5*b - 12*sqrt(a^2 ...
```

3.219.9 Mupad [B] (verification not implemented)

Time = 16.34 (sec) , antiderivative size = 3400, normalized size of antiderivative = 14.59

$$\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^6/(a - b*sin(c + d*x)^4)^2,x)
```

output
```
(atan(((((256*a^2*b^5 - 512*a^3*b^4 + 256*a^4*b^3)/(64*(a*b^3 - b^4)) - (t
an(c + d*x)*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^
4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*(256
*a^2*b^6 - 768*a^3*b^5 + 768*a^4*b^4 - 256*a^5*b^3))/(4*(a*b^2 - b^3)))*((
15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)
/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2) + (tan(c + d*x)*(9
*a*b^3 - 15*a^3*b + 4*a^4 + 10*a^2*b^2))/(4*(a*b^2 - b^3)))*((15*a*b^5 - 5
*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9
 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*1i - (((256*a^2*b^5 - 512*a^3*
b^4 + 256*a^4*b^3)/(64*(a*b^3 - b^4)) + (tan(c + d*x)*((15*a*b^5 - 5*a*(a*
b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a
^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*(256*a^2*b^6 - 768*a^3*b^5 + 768*a^4
*b^4 - 256*a^5*b^3))/(4*(a*b^2 - b^3)))*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9
*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3
*b^7 - a^4*b^6)))^(1/2) - (tan(c + d*x)*(9*a*b^3 - 15*a^3*b + 4*a^4 + 10*a
^2*b^2))/(4*(a*b^2 - b^3)))*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(
1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b
^6)))^(1/2)*1i)/((27*a*b^2 - 21*a^2*b + 4*a^3)/(32*(a*b^3 - b^4)) + (((256
*a^2*b^5 - 512*a^3*b^4 + 256*a^4*b^3)/(64*(a*b^3 - b^4)) - (tan(c + d*x)*(
(15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*...
```

3.219. $\int \frac{\sin^6(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

3.220 $\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

3.220.1 Optimal result

Integrand size = 24, antiderivative size = 195

$$\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \frac{\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}d} - \frac{\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}d} - \frac{\tan(c+dx)}{4a(a-b)d} + \frac{\tan^5(c+dx)}{4ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)}$$

output
```
1/8*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)-
b^(1/2))^(3/2)/b^(1/2)-1/8*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/
4))/a^(3/4)/d/b^(1/2)/(a^(1/2)+b^(1/2))^(3/2)-1/4*tan(d*x+c)/a/(a-b)/d+1/4
*tan(d*x+c)^5/a/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)
```

3.220.2 Mathematica [A] (verified)

Time = 6.53 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.15

$$\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = -\frac{\frac{(\sqrt{a}-\sqrt{b})\arctan\left(\frac{(\sqrt{a}+\sqrt{b})\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{(\sqrt{a}+\sqrt{b})\operatorname{arctanh}\left(\frac{(\sqrt{a}-\sqrt{b})\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{2(-6\sin(2(c+dx))+\sin(4(c+dx)))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))}}{8(a-b)d}$$

input

```
Integrate[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4)^2,x]
```

output

```
-1/8*(((Sqrt[a] - Sqrt[b])*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[
a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + ((Sqr
t[a] + Sqrt[b])*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[
a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) - (2*(-6*Sin[2*
(c + d*x)] + Sin[4*(c + d*x)]))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[
4*(c + d*x)]))/((a - b)*d)
```

3.220.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3696, 1598, 27, 1442, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^4}{(a-b\sin(c+dx)^4)^2}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\tan^4(c+dx)(\tan^2(c+dx)+1)}{((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1598}$$

$$\frac{\frac{\int -\frac{2b\tan^4(c+dx)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{8ab}+\frac{\tan^5(c+dx)}{4a((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\tan^5(c+dx)}{4a((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}-\frac{\int \frac{\tan^4(c+dx)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{4a}}{d}$$

$\Big\downarrow$ 1442

$$\frac{\frac{\tan^5(c+dx)}{4a((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}-\frac{\frac{\tan(c+dx)}{a-b}-\frac{\int \frac{a\left(2\tan^2(c+dx)+1\right)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{a-b}}{4a}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\tan^5(c+dx)}{4a((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}-\frac{\frac{\tan(c+dx)}{a-b}-\frac{a\int \frac{2\tan^2(c+dx)+1}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{a-b}}{4a}}{d}$$

$\Big\downarrow$ 1480

$$\frac{\frac{\tan^5(c+dx)}{4a((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}-\frac{\frac{\tan(c+dx)}{a-b}-\frac{a\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\int \frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\tan(c+dx)}{2\sqrt{a}\sqrt{b}}+\frac{1}{2}\left(2-\frac{a+b}{\sqrt{a}\sqrt{b}}\right)\int \frac{}{(a-b)\tan^2(c+d}\right.}{a-b}}{4a}}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{\tan^5(c+dx)}{4a((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}-\frac{\frac{\tan(c+dx)}{a-b}-\frac{a\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\left(2-\frac{a+b}{\sqrt{a}\sqrt{b}}\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{a-b}}{4a}}{d}$$

input
```
Int[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4)^2,x]
```

3.220. $\int \frac{\sin^4(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

output
```
(-1/4*(-((a*(((Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c +
d*x])/a^(1/4)])/(2*a^(3/4)*Sqrt[Sqrt[a] - Sqrt[b]]*Sqrt[b]) + ((2 - (a + b
)/(Sqrt[a]*Sqrt[b]))*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)
])/(2*a^(1/4)*(Sqrt[a] - Sqrt[b])*Sqrt[Sqrt[a] + Sqrt[b]])))/(a - b)) + Ta
n[c + d*x]/(a - b))/a + Tan[c + d*x]^5/(4*a*(a + 2*a*Tan[c + d*x]^2 + (a -
 b)*Tan[c + d*x]^4)))/d
```

3.220.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

3.220. $\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

3.220.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{-\frac{\tan^3(dx+c)}{2(a-b)}-\frac{\tan(dx+c)}{4(a-b)}}{(\tan^4(dx+c))a-b(\tan^4(dx+c))+2a(\tan^2(dx+c))+a}+\frac{(a+b+2\sqrt{ab})\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{(\sqrt{ab}+a)(a-b)}}\right)}{8\sqrt{ab}\,(a-b)\sqrt{(\sqrt{ab}+a)(a-b)}}+\frac{(-a-b+2\sqrt{ab})\,\mathrm{arctanh}\left(\frac{(-a+}{\sqrt{(\sqrt{}}}\right.}{8\sqrt{ab}\,(a-b)\sqrt{(\sqrt{ab}-a}}}{d}$
default	$\frac{\frac{-\frac{\tan^3(dx+c)}{2(a-b)}-\frac{\tan(dx+c)}{4(a-b)}}{(\tan^4(dx+c))a-b(\tan^4(dx+c))+2a(\tan^2(dx+c))+a}+\frac{(a+b+2\sqrt{ab})\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{(\sqrt{ab}+a)(a-b)}}\right)}{8\sqrt{ab}\,(a-b)\sqrt{(\sqrt{ab}+a)(a-b)}}+\frac{(-a-b+2\sqrt{ab})\,\mathrm{arctanh}\left(\frac{(-a+}{\sqrt{(\sqrt{}}}\right.}{8\sqrt{ab}\,(a-b)\sqrt{(\sqrt{ab}-a}}}{d}$
risch	$-\frac{i\left(b\,\mathrm{e}^{6i(dx+c)}-8a\,\mathrm{e}^{4i(dx+c)}+3b\,\mathrm{e}^{4i(dx+c)}-5b\,\mathrm{e}^{2i(dx+c)}+b\right)}{2b(a-b)d\left(\mathrm{e}^{8i(dx+c)}b-4b\,\mathrm{e}^{6i(dx+c)}-16a\,\mathrm{e}^{4i(dx+c)}+6b\,\mathrm{e}^{4i(dx+c)}-4b\,\mathrm{e}^{2i(dx+c)}+b\right)}+\left(\_R=\mathrm{RootOf}\left(1+\left(a^6b^2d^4-3a^5b\right.\right.\right.$

input
```
int(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*((-1/2/(a-b)*tan(d*x+c)^3-1/4/(a-b)*tan(d*x+c))/(tan(d*x+c)^4*a-b*tan(
d*x+c)^4+2*a*tan(d*x+c)^2+a)+1/8*(a+b+2*(a*b)^(1/2))/(a*b)^(1/2)/(a-b)/(((
a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))
^(1/2))+1/8*(-a-b+2*(a*b)^(1/2))/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))
^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2)))
```

3.220.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2796 vs. 2(151) = 302.

Time = 0.62 (sec) , antiderivative size = 2796, normalized size of antiderivative = 14.34

$$\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
-1/32*(((a*b - b^2)*d*cos(d*x + c)^4 - 2*(a*b - b^2)*d*cos(d*x + c)^2 - (a
^2 - 2*a*b + b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sq
rt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 1
5*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*
a^2*b^3 - a*b^4)*d^2))*log(1/4*(3*a + b)*cos(d*x + c)^2 + 1/2*(2*(a^6*b -
3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b -
6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^
4))*cos(d*x + c)*sin(d*x + c) - (3*a^3 + 4*a^2*b + a*b^2)*d*cos(d*x + c)*s
in(d*x + c))*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^
2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^
5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3
- a*b^4)*d^2)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*cos(d*x
+ c)^2 - (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sqrt((9*a^2 + 6*a*b +
b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^
6 + a^3*b^7)*d^4)) - 3/4*a - 1/4*b) - ((a*b - b^2)*d*cos(d*x + c)^4 - 2*(a
*b - b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(-((a^4*b - 3*a^3*
b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^
2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a
+ 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(1/4*(3*a + b)*co
s(d*x + c)^2 - 1/2*(2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*sqr...
```

3.220.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**4/(a-b*sin(d*x+c)**4)**2,x)
```

output
```
Timed out
```

3.220.7 Maxima [F]

$$\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \int \frac{\sin(dx+c)^4}{(b\sin(dx+c)^4-a)^2}\,dx$$

input
```
integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/2*(6*(8*a - 3*b)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + (b*sin(6*d*x + 6*c)
 - (8*a - 3*b)*sin(4*d*x + 4*c) - 5*b*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) +
 6*((8*a - 3*b)*sin(4*d*x + 4*c) + 4*b*sin(2*d*x + 2*c))*cos(6*d*x + 6*c)
- 2*((a*b^2 - b^3)*d*cos(8*d*x + 8*c)^2 + 16*(a*b^2 - b^3)*d*cos(6*d*x + 6
*c)^2 + 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*d*cos(4*d*x + 4*c)^2 + 1
6*(a*b^2 - b^3)*d*cos(2*d*x + 2*c)^2 + (a*b^2 - b^3)*d*sin(8*d*x + 8*c)^2
+ 16*(a*b^2 - b^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 57*a*b^2
 - 9*b^3)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^2 - b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a*
b^2 - b^3)*d*cos(2*d*x + 2*c) + (a*b^2 - b^3)*d - 2*(4*(a*b^2 - b^3)*d*cos
(6*d*x + 6*c) + 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cos(4*d*x + 4*c) + 4*(a*b
^2 - b^3)*d*cos(2*d*x + 2*c) - (a*b^2 - b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8
*a^2*b - 11*a*b^2 + 3*b^3)*d*cos(4*d*x + 4*c) + 4*(a*b^2 - b^3)*d*cos(2*d*
x + 2*c) - (a*b^2 - b^3)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^2*b - 11*a*b^2 +
3*b^3)*d*cos(2*d*x + 2*c) - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*cos(4*d*x + 4*
c) - 4*(2*(a*b^2 - b^3)*d*sin(6*d*x + 6*c) + (8*a^2*b - 11*a*b^2 + 3*b^3)*
d*sin(4*d*x + 4*c) + 2*(a*b^2 - b^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c)
+ 16*((8*a^2*b - 11*a*b^2 + 3*b^3)*d*sin(4*d*x + 4*c) + 2*(a*b^2 - b^3)*d*
sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate((4*b*cos(6*d*x + 6*c)^2 - 12
*(8*a - 3*b)*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + 4*b*sin(6*d*...
```

3.220.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1264 vs. $2(151) = 302$.

Time = 0.82 (sec) , antiderivative size = 1264, normalized size of antiderivative = 6.48

$$\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
1/8*((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5 - 9*sqrt(a^2 - a
*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b + 2*sqrt(a^2 - a*b - sqrt(a*b)*(a
- b))*sqrt(a*b)*a^3*b^2 + 10*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)
*a^2*b^3 - 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^4 - sqrt(a^
2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^5 - 2*(3*sqrt(a^2 - a*b - sqrt(a*
b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a
*b)*a*b^2 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(a - b)^2 +
 (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b - 12*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*a^3*b^2 + 14*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^3 - 4*
sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^4 - sqrt(a^2 - a*b - sqrt(a*b)*(a
- b))*b^5)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c
)/sqrt((a^2 - a*b + sqrt((a^2 - a*b)^2 - (a^2 - a*b)*(a^2 - 2*a*b + b^2)))
/(a^2 - 2*a*b + b^2))))/(3*a^8*b - 21*a^7*b^2 + 59*a^6*b^3 - 85*a^5*b^4 +
65*a^4*b^5 - 23*a^3*b^6 + a^2*b^7 + a*b^8) - (3*sqrt(a^2 - a*b + sqrt(a*b)
*(a - b))*sqrt(a*b)*a^5 - 9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*
a^4*b + 2*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 + 10*sqrt(
a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^3 - 5*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*sqrt(a*b)*a*b^4 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(
a*b)*b^5 - 2*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sq
rt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b + sq...
```

3.220.9 Mupad [B] (verification not implemented)

Time = 16.00 (sec) , antiderivative size = 2980, normalized size of antiderivative = 15.28

$$\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^4/(a - b*sin(c + d*x)^4)^2,x)
```

output
```
- (atan(((((128*a*b^3 + 128*a^3*b - 256*a^2*b^2)/(32*(a - b)) - (tan(c + d
*x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a
^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*(256*a^5*b - 256*a^2*b^4
 + 768*a^3*b^3 - 768*a^4*b^2))/(4*(a - b)))*((3*a*(a^3*b^3)^(1/2) + b*(a^3
*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a
^6*b^2)))^(1/2) - (tan(c + d*x)*(6*a*b + a^2 + b^2))/(4*(a - b)))*((3*a*(a
^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a
^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*1i - (((128*a*b^3 + 128*a^3*b - 256*
a^2*b^2)/(32*(a - b)) + (tan(c + d*x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^
(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2
)))^(1/2)*(256*a^5*b - 256*a^2*b^4 + 768*a^3*b^3 - 768*a^4*b^2))/(4*(a - b
)))*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a
^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2) + (tan(c + d*x)*(6*a*b +
 a^2 + b^2))/(4*(a - b)))*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*
b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*1i
)/((((128*a*b^3 + 128*a^3*b - 256*a^2*b^2)/(32*(a - b)) - (tan(c + d*x)*((
3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5
 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*(256*a^5*b - 256*a^2*b^4 + 768
*a^3*b^3 - 768*a^4*b^2))/(4*(a - b)))*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^
(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*...
```

3.220. $\int \frac{\sin^4(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

3.221 $\int \frac{\sin^2(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

3.221.1 Optimal result

Integrand size = 24, antiderivative size = 219

$$\int \frac{\sin^2(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \frac{\left(2\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}d} - \frac{\left(2\sqrt{a}+\sqrt{b}\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}d} - \frac{\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)}$$

output
```
1/8*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)-b^(1/2))
/a^(5/4)/d/(a^(1/2)-b^(1/2))^(3/2)/b^(1/2)-1/8*arctan((a^(1/2)+b^(1/2))^(1
/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)+b^(1/2))/a^(5/4)/d/b^(1/2)/(a^(1/2)+b^(
1/2))^(3/2)-1/4*tan(d*x+c)*(a+(a+b)*tan(d*x+c)^2)/a/(a-b)/d/(a+2*a*tan(d*x
+c)^2+(a-b)*tan(d*x+c)^4)
```

3.221.2 Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.16

$$\int \frac{\sin^2(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$$
$$= \frac{-\frac{\sqrt{a}\left(2a-\sqrt{a}\sqrt{b}-b\right)\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{a}\left(2a+\sqrt{a}\sqrt{b}-b\right)\operatorname{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{4\sqrt{a}(2a+b-b\cos(2(c+dx)))\,\mathrm{s}}{8a-3b+4b\cos(2(c+dx))-b\,\mathrm{co}}}{8a^{3/2}(a-b)d}$$

input
```
Integrate[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4)^2,x]
```

output
```
(-((Sqrt[a]*(2*a - Sqrt[a]*Sqrt[b] - b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c
+ d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b])) -
 (Sqrt[a]*(2*a + Sqrt[a]*Sqrt[b] - b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c +
 d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) -
 (4*Sqrt[a]*(2*a + b - b*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(8*a - 3*b +
4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]))/(8*a^(3/2)*(a - b)*d)
```

3.221.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3696, 1672, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(c+dx)^2}{(a-b\sin(c+dx)^4)^2}dx$$
$$\Big\downarrow \text{3696}$$
$$\frac{\int \frac{\tan^2(c+dx)\left(\tan^2(c+dx)+1\right)^2}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{d}$$
$$\Big\downarrow \text{1672}$$

$$-\frac{\frac{\int -\frac{2ab\left((3a-b)\tan^2(c+dx)+a\right)}{(a-b)\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}d\tan(c+dx)}{8a^2b}-\frac{\tan(c+dx)((a+b)\tan^2(c+dx)+a)}{4a(a-b)((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{(3a-b)\tan^2(c+dx)+a}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{4a(a-b)}-\frac{\tan(c+dx)((a+b)\tan^2(c+dx)+a)}{4a(a-b)((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}}{d}$$

$$\Big\downarrow \text{1480}$$

$$\frac{\frac{\frac{1}{2}\left(-\frac{2a^{3/2}}{\sqrt{b}}+3a-b\right)\int \frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tan(c+dx)+\frac{1}{2}\left(\frac{2a^{3/2}}{\sqrt{b}}+3a-b\right)\int \frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\tan(c+dx)}{4a(a-b)}-\frac{\tan(c+dx)((a+b)\tan^2(c+dx)+a)}{4a(a-b)((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\left(\frac{2a^{3/2}}{\sqrt{b}}+3a-b\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\left(\sqrt{a}+\sqrt{b}\right)}+\frac{\left(-\frac{2a^{3/2}}{\sqrt{b}}+3a-b\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}}{4a(a-b)}-\frac{\tan(c+dx)((a+b)\tan^2(c+dx)+a)}{4a(a-b)((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}}{d}$$

input
```
Int[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4)^2,x]
```

output
```
((((3*a + (2*a^(3/2))/Sqrt[b] - b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c +
 d*x])/a^(1/4)])/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*(Sqrt[a] + Sqrt[b])) +
 ((3*a - (2*a^(3/2))/Sqrt[b] - b)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c +
d*x])/a^(1/4)])/(2*a^(1/4)*(Sqrt[a] - Sqrt[b])*Sqrt[Sqrt[a] + Sqrt[b]]))/(
4*a*(a - b)) - (Tan[c + d*x]*(a + (a + b)*Tan[c + d*x]^2))/(4*a*(a - b)*(a
 + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)))/d
```

3.221.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.221. $\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1672
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*
Simp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
 c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

3.221.4 Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{\frac{-\frac{(a+b)\left(\tan^3(dx+c)\right)}{4(a-b)a}-\frac{\tan(dx+c)}{4(a-b)}}{\left(\tan^4(dx+c)\right)a-b\left(\tan^4(dx+c)\right)+2a\left(\tan^2(dx+c)\right)+a}+\frac{\frac{\left(3a\sqrt{ab}-\sqrt{ab}\,b+2a^2\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(3a\sqrt{ab}-\sqrt{ab}\,b-2a^2\right)\cdots}{2\sqrt{ab}\,(a-b\cdots}}{4a}}{d}$
default	$\frac{\frac{-\frac{(a+b)\left(\tan^3(dx+c)\right)}{4(a-b)a}-\frac{\tan(dx+c)}{4(a-b)}}{\left(\tan^4(dx+c)\right)a-b\left(\tan^4(dx+c)\right)+2a\left(\tan^2(dx+c)\right)+a}+\frac{\frac{\left(3a\sqrt{ab}-\sqrt{ab}\,b+2a^2\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(3a\sqrt{ab}-\sqrt{ab}\,b-2a^2\right)\cdots}{2\sqrt{ab}\,(a-b\cdots}}{4a}}{d}$
risch	$-\frac{i\left(2a\,e^{6i(dx+c)}-b\,e^{6i(dx+c)}-8a\,e^{4i(dx+c)}+3b\,e^{4i(dx+c)}-2a\,e^{2i(dx+c)}-3b\,e^{2i(dx+c)}+b\right)}{2a(a-b)d\left(e^{8i(dx+c)}b-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(256a^8b^2d^4\cdots\right.\right.}\right.}{\cdots}$

input
```
int(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*((-1/4*(a+b)/(a-b)/a*tan(d*x+c)^3-1/4/(a-b)*tan(d*x+c))/(tan(d*x+c)^4*
a-b*tan(d*x+c)^4+2*a*tan(d*x+c)^2+a)+1/4/a*(1/2*(3*a*(a*b)^(1/2)-(a*b)^(1/
2)*b+2*a^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*t
an(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*(3*a*(a*b)^(1/2)-(a*b)^(1/2)*
b-2*a^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*ta
n(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))))
```

3.221.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3445 vs. $2(169) = 338$.

Time = 0.92 (sec) , antiderivative size = 3445, normalized size of antiderivative = 15.73

$$\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
1/32*(((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)
^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^
2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b
 - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7
)*d^4)) + 4*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^
2))*log(8*a^3 - 7*a^2*b + 9/4*a*b^2 - 1/4*b^3 - 1/4*(32*a^3 - 28*a^2*b + 9
*a*b^2 - b^3)*cos(d*x + c)^2 + 1/2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6
*a^5*b^4 + a^4*b^5)*d^3*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 +
b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*
b^6 + a^5*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) - 2*(8*a^5 - 5*a^4*b + a^3*
b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 -
a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11
*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b
^7)*d^4)) + 4*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*
d^2)) + 1/4*(2*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2*c
os(d*x + c)^2 - (4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2)
*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*
b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))) -
 ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 -
(a^3 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*...
```

3.221.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**2/(a-b*sin(d*x+c)**4)**2,x)
```

output
```
Timed out
```

3.221.7 Maxima [F]

$$\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx=\int \frac{\sin\left(dx+c\right)^2}{\left(b\sin\left(dx+c\right)^4-a\right)^2}\,dx$$

input
```
integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/2*(2*(16*a^2 + 2*a*b - 3*b^2)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + ((2*a*
b - b^2)*sin(6*d*x + 6*c) - (8*a*b - 3*b^2)*sin(4*d*x + 4*c) - (2*a*b + 3*
b^2)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 2*((16*a^2 + 2*a*b - 3*b^2)*sin(
4*d*x + 4*c) + 4*(2*a*b + b^2)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*((a^
2*b^2 - a*b^3)*d*cos(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6
*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*cos(4*d*x + 4*c)^2
 + 16*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c)^2 + (a^2*b^2 - a*b^3)*d*sin(8*d
*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*
a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b - 11*a^2*
b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^2*b^2 - a*b^3)*
d*sin(2*d*x + 2*c)^2 - 8*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) + (a^2*b^2 -
 a*b^3)*d - 2*(4*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^3*b - 11*a^
2*b^2 + 3*a*b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*
c) - (a^2*b^2 - a*b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^3*b - 11*a^2*b^2 +
3*a*b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^
2*b^2 - a*b^3)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)
*d*cos(2*d*x + 2*c) - (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d)*cos(4*d*x + 4*c)
 - 4*(2*(a^2*b^2 - a*b^3)*d*sin(6*d*x + 6*c) + (8*a^3*b - 11*a^2*b^2 + 3*a
*b^3)*d*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c))*sin(8*d
*x + 8*c) + 16*((8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c) + 2...
```

3.221.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1407 vs. $2(169) = 338$.

Time = 0.81 (sec) , antiderivative size = 1407, normalized size of antiderivative = 6.42

$$\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
-1/8*(((9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 21*sqrt(a^
2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + 3*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*sqrt(a*b)*a*b^3 + sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*
b)*b^4)*(a^2 - a*b)^2*abs(-a + b) - (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))
*a^6*b - 12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^5*b^2 + 14*sqrt(a^2 - a*
b - sqrt(a*b)*(a - b))*a^4*b^3 - 4*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3
*b^4 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^5)*abs(-a^2 + a*b)*abs(-a
 + b) - 2*(3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^8 - 12*sqrt(a
^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b + 14*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*sqrt(a*b)*a^6*b^2 - 4*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqr
t(a*b)*a^5*b^3 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^4)*ab
s(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^3 -
 a^2*b + sqrt((a^3 - a^2*b)^2 - (a^3 - a^2*b)*(a^3 - 2*a^2*b + a*b^2)))/(a
^3 - 2*a^2*b + a*b^2))))/((3*a^10*b - 21*a^9*b^2 + 59*a^8*b^3 - 85*a^7*b^4
 + 65*a^6*b^5 - 23*a^5*b^6 + a^4*b^7 + a^3*b^8)*abs(-a^2 + a*b)) - ((9*sqr
t(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 21*sqrt(a^2 - a*b + sqr
t(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
sqrt(a*b)*a*b^3 + sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*(a^2
- a*b)^2*abs(-a + b) + (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^6*b - 12*s
qrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^5*b^2 + 14*sqrt(a^2 - a*b + sqrt(a...
```

3.221.9 Mupad [B] (verification not implemented)

Time = 17.17 (sec) , antiderivative size = 3842, normalized size of antiderivative = 17.54

$$\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^2/(a - b*sin(c + d*x)^4)^2,x)
```

output
```
- (tan(c + d*x)/(4*(a - b)) + (tan(c + d*x)^3*(a + b))/(4*a*(a - b)))/(d*(
a + 2*a*tan(c + d*x)^2 + tan(c + d*x)^4*(a - b))) - (atan(((((256*a^5*b +
256*a^3*b^3 - 512*a^4*b^2)/(64*(a^2*b - a^3)) - (tan(c + d*x)*(-(8*a^2*(a^
5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(
a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*(
256*a^6*b - 256*a^3*b^4 + 768*a^4*b^3 - 768*a^5*b^2))/(4*(a*b - a^2)))*(-(
8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2
- 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2))
)^(1/2) - (tan(c + d*x)*(9*a^2*b - 6*a*b^2 + 4*a^3 + b^3))/(4*(a*b - a^2))
)*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4
*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*
b^2)))^(1/2)*1i - (((256*a^5*b + 256*a^3*b^3 - 512*a^4*b^2)/(64*(a^2*b - a
^3)) + (tan(c + d*x)*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^
5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4
 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*(256*a^6*b - 256*a^3*b^4 + 768*a^4*b^3 - 7
68*a^5*b^2))/(4*(a*b - a^2)))*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/
2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 -
3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2) + (tan(c + d*x)*(9*a^2*b - 6*a*b^
2 + 4*a^3 + b^3))/(4*(a*b - a^2)))*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3
)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^...
```

3.221. $\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

3.222 $\int \frac{1}{(a-b\sin^4(c+dx))^2}\,dx$

3.222.1 Optimal result

Integrand size = 15, antiderivative size = 210

$$\int \frac{1}{(a-b\sin^4(c+dx))^2}\,dx = \frac{\left(4\sqrt{a}-3\sqrt{b}\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} + \frac{\left(4\sqrt{a}+3\sqrt{b}\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d} - \frac{b\tan(c+dx)\left(1+2\tan^2(c+dx)\right)}{4a(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)}$$

output
```
1/8*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(4*a^(1/2)-3*b^(1/2
))/a^(7/4)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctan((a^(1/2)+b^(1/2))^(1/2)*ta
n(d*x+c)/a^(1/4))*(4*a^(1/2)+3*b^(1/2))/a^(7/4)/d/(a^(1/2)+b^(1/2))^(3/2)-
1/4*b*tan(d*x+c)*(1+2*tan(d*x+c)^2)/a/(a-b)/d/(a+2*a*tan(d*x+c)^2+(a-b)*ta
n(d*x+c)^4)
```

3.222.2 Mathematica [A] (verified)

Time = 4.16 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \frac{\frac{\left(4a-\sqrt{a}\sqrt{b}-3b\right)\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}-\frac{\left(4a+\sqrt{a}\sqrt{b}-3b\right)\operatorname{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{a}b(-6\sin(2(c+dx))+\sin(4(c+dx)))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))}}{8a^{3/2}(a-b)d}$$

input
```
Integrate[(a - b*Sin[c + d*x]^4)^(-2),x]
```

output
```
(((4*a - Sqrt[a]*Sqrt[b] - 3*b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/
Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - ((4*a + Sqrt[a]*Sq
rt[b] - 3*b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*
Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (2*Sqrt[a]*b*(-6*Sin[2*(c + d*x)]
+ Sin[4*(c + d*x)]))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)
]))/(8*a^(3/2)*(a - b)*d)
```

3.222.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3688, 1517, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-b\sin(c+dx)^4\right)^2}dx$$

$$\Big\downarrow \text{3688}$$

$$\frac{\int \frac{\left(\tan^2(c+dx)+1\right)^3}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1517}$$

$$-\frac{\frac{\int -\frac{2ab\left(2(2a-b)\tan^2(c+dx)+4a-3b\right)}{(a-b)\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}d\tan(c+dx)}{8a^2b}-\frac{b\tan(c+dx)(2\tan^2(c+dx)+1)}{4a(a-b)((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{2(2a-b)\tan^2(c+dx)+4a-3b}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{4a(a-b)}-\frac{b\tan(c+dx)(2\tan^2(c+dx)+1)}{4a(a-b)((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}}{d}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{\frac{\frac{\left(4\sqrt{a}-3\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)^2\int \frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\tan(c+dx)}{2\sqrt{a}}+\frac{\left(4\sqrt{a}+3\sqrt{b}\right)\left(-2\sqrt{a}\sqrt{b}+a+b\right)\int \frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tan(c+dx)}{2\sqrt{a}}}{4a(a-b)}-\frac{\cdots}{4a(a\cdots}}{d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\frac{\left(4\sqrt{a}-3\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\left(4\sqrt{a}+3\sqrt{b}\right)\left(-2\sqrt{a}\sqrt{b}+a+b\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}}{4a(a-b)}-\frac{b\tan(c+dx)(2\tan^2(c+dx\cdots}{4a(a-b)((a-b)\tan^4(c+dx)+2a\tan\cdots}}{d}$$

input
```
Int[(a - b*Sin[c + d*x]^4)^(-2),x]
```

output
```
((((4*Sqrt[a] - 3*Sqrt[b])*(Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt
[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*Sqrt[Sqrt[a] - Sqrt[b]]) + ((4*Sqr
t[a] + 3*Sqrt[b])*(a - 2*Sqrt[a]*Sqrt[b] + b)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[
b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] - Sqrt[b])*Sqrt[Sqrt[a] +
Sqrt[b]]))/(4*a*(a - b)) - (b*Tan[c + d*x]*(1 + 2*Tan[c + d*x]^2))/(4*a*(a
 - b)*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)))/d
```

3.222.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.222. $\int \frac{1}{(a-b\sin^4(c+dx))^2}\,dx$

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

3.222.4 Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{\frac{-\frac{b\left(\tan^3(dx+c)\right)}{2a(a-b)}-\frac{b\tan(dx+c)}{4a(a-b)}}{\left(\tan^4(dx+c)\right)a-b\left(\tan^4(dx+c)\right)+2a\left(\tan^2(dx+c)\right)+a}+\frac{\frac{\left(4a\sqrt{ab}-2\sqrt{ab}\,b+5ab-3b^2\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(4a\sqrt{ab}-2\sqrt{ab}\,b\right.}{2}}{4a}}{d}$
default	$\frac{\frac{-\frac{b\left(\tan^3(dx+c)\right)}{2a(a-b)}-\frac{b\tan(dx+c)}{4a(a-b)}}{\left(\tan^4(dx+c)\right)a-b\left(\tan^4(dx+c)\right)+2a\left(\tan^2(dx+c)\right)+a}+\frac{\frac{\left(4a\sqrt{ab}-2\sqrt{ab}\,b+5ab-3b^2\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\left(4a\sqrt{ab}-2\sqrt{ab}\,b\right.}{2}}{4a}}{d}$
risch	$-\frac{i\left(b\,e^{6i(dx+c)}-8a\,e^{4i(dx+c)}+3b\,e^{4i(dx+c)}-5b\,e^{2i(dx+c)}+b\right)}{2a(a-b)d\left(e^{8i(dx+c)}b-4b\,e^{6i(dx+c)}-16a\,e^{4i(dx+c)}+6b\,e^{4i(dx+c)}-4b\,e^{2i(dx+c)}+b\right)}+\left(\_R=\mathrm{RootOf}\left((65536a^{10}d^4-\right.\right.$

input
```
int(1/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*((-1/2*b/a/(a-b)*tan(d*x+c)^3-1/4*b/a/(a-b)*tan(d*x+c))/(tan(d*x+c)^4*
a-b*tan(d*x+c)^4+2*a*tan(d*x+c)^2+a)+1/4/a*(1/2*(4*a*(a*b)^(1/2)-2*(a*b)^(
1/2)*b+5*a*b-3*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan
((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*(4*a*(a*b)^(1/2)-2*(a
*b)^(1/2)*b-5*a*b+3*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*a
rctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))))
```

3.222.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3477 vs. 2(164) = 328.

Time = 0.93 (sec) , antiderivative size = 3477, normalized size of antiderivative = 16.56

$$\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
-1/32*(((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c
)^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b
^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5
)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 +
 a^7*b^6)*d^4)) + 16*a^2 - 15*a*b + 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a
^3*b^3)*d^2))*log(96*a^3*b - 170*a^2*b^2 + 405/4*a*b^3 - 81/4*b^4 - 1/4*(3
84*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*cos(d*x + c)^2 + 1/2*(2*(2*a^
10 - 7*a^9*b + 9*a^8*b^2 - 5*a^7*b^3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1392
*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*
b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))*cos(d*x + c)*s
in(d*x + c) - (120*a^5*b - 217*a^4*b^2 + 132*a^3*b^3 - 27*a^2*b^4)*d*cos(d
*x + c)*sin(d*x + c))*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqr
t((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 -
6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d
^4)) + 16*a^2 - 15*a*b + 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2
)) + 1/4*(2*(16*a^8 - 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2*
cos(d*x + c)^2 - (16*a^8 - 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)
*d^2)*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/
((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a
^7*b^6)*d^4))) - ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*...
```

3.222.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a - b\sin^4(c + dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-b*sin(d*x+c)**4)**2,x)
```

output
```
Timed out
```

3.222.7 Maxima [F]

$$\int \frac{1}{\left(a - b\sin^4(c + dx)\right)^2}\,dx = \int \frac{1}{\left(b\sin\left(dx + c\right)^4 - a\right)^2}\,dx$$

input
```
integrate(1/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^2*sin(2*d*x + 2*c) - 6*(8*a*b - 3*b^2)*cos(4*d*x + 4*c)*sin(2*d*x
+ 2*c) - (b^2*sin(6*d*x + 6*c) - 5*b^2*sin(2*d*x + 2*c) - (8*a*b - 3*b^2)*
sin(4*d*x + 4*c))*cos(8*d*x + 8*c) - 6*(4*b^2*sin(2*d*x + 2*c) + (8*a*b -
3*b^2)*sin(4*d*x + 4*c))*cos(6*d*x + 6*c) + 2*((a^2*b^2 - a*b^3)*d*cos(8*d
*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*
a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*cos(4*d*x + 4*c)^2 + 16*(a^2*b^2 - a*b^3)*
d*cos(2*d*x + 2*c)^2 + (a^2*b^2 - a*b^3)*d*sin(8*d*x + 8*c)^2 + 16*(a^2*b^
2 - a*b^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a
*b^3)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c)^2 - 8
*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) + (a^2*b^2 - a*b^3)*d - 2*(4*(a^2*b^
2 - a*b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(4
*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2 - a*b^3)*d
)*cos(8*d*x + 8*c) + 8*(2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(4*d*x + 4
*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2 - a*b^3)*d)*cos(6*
d*x + 6*c) + 4*(4*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(2*d*x + 2*c) - (8
*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d)*cos(4*d*x + 4*c) - 4*(2*(a^2*b^2 - a*b^3
)*d*sin(6*d*x + 6*c) + (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c)
 + 2*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^3*b
 - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*d*sin...
```

3.222.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. $2(164) = 328$.

Time = 0.51 (sec) , antiderivative size = 1506, normalized size of antiderivative = 7.17

$$\int \frac{1}{\left(a - b\sin^4(c + dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
-1/8*((2*(6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 15*sqrt(a^
2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 4*sqrt(a^2 - a*b - sqrt(a*b
)*(a - b))*sqrt(a*b)*a*b^2 + sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)
*b^3)*(a^2 - a*b)^2*abs(-a + b) - (12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*
a^6 - 57*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^5*b + 92*sqrt(a^2 - a*b - s
qrt(a*b)*(a - b))*a^4*b^2 - 58*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b^3
 + 8*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^4 + 3*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*a*b^5)*abs(-a^2 + a*b)*abs(-a + b) - (15*sqrt(a^2 - a*b - s
qrt(a*b)*(a - b))*sqrt(a*b)*a^7 - 69*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*s
qrt(a*b)*a^6*b + 106*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^2
 - 62*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^3 + 7*sqrt(a^2 -
 a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^4 + 3*sqrt(a^2 - a*b - sqrt(a*b)
*(a - b))*sqrt(a*b)*a^2*b^5)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) +
arctan(tan(d*x + c)/sqrt((a^3 - a^2*b + sqrt((a^3 - a^2*b)^2 - (a^3 - a^2*
b)*(a^3 - 2*a^2*b + a*b^2)))/(a^3 - 2*a^2*b + a*b^2))))/((3*a^10 - 21*a^9*
b + 59*a^8*b^2 - 85*a^7*b^3 + 65*a^6*b^4 - 23*a^5*b^5 + a^4*b^6 + a^3*b^7)
*abs(-a^2 + a*b)) - (2*(6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^
3 - 15*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 4*sqrt(a^2 -
a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 + sqrt(a^2 - a*b + sqrt(a*b)*(a -
 b))*sqrt(a*b)*b^3)*(a^2 - a*b)^2*abs(-a + b) + (12*sqrt(a^2 - a*b + sq...
```

3.222.9 Mupad [B] (verification not implemented)

Time = 17.09 (sec) , antiderivative size = 3675, normalized size of antiderivative = 17.50

$$\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a - b*sin(c + d*x)^4)^2,x)
```

output
```
- (atan(((((512*a^6*b - 384*a^3*b^4 + 1280*a^4*b^3 - 1408*a^5*b^2)/(32*(a^
3*b - a^4)) - (tan(c + d*x)*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) -
 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^1
0 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*(256*a^7*b - 256*a^4*b^4 + 768*a^5*b^3 -
768*a^6*b^2))/(4*(a^2*b - a^3)))*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1
/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b
- a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2) - (tan(c + d*x)*(16*a^3*b - 26*a*b^3
 + 9*b^4 + 9*a^2*b^2))/(4*(a^2*b - a^3)))*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(
a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*
(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*1i - (((512*a^6*b - 384*a^3
*b^4 + 1280*a^4*b^3 - 1408*a^5*b^2)/(32*(a^3*b - a^4)) + (tan(c + d*x)*((2
4*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2
- 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2
)*(256*a^7*b - 256*a^4*b^4 + 768*a^5*b^3 - 768*a^6*b^2))/(4*(a^2*b - a^3))
)*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4
*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))
^(1/2) + (tan(c + d*x)*(16*a^3*b - 26*a*b^3 + 9*b^4 + 9*a^2*b^2))/(4*(a^2*
b - a^3)))*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^
6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a
^8*b^2)))^(1/2)*1i)/((32*a^2*b - 34*a*b^2 + 9*b^3)/(16*(a^3*b - a^4)) +...
```

3.222. $\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

3.223 $\int \frac{\csc^2(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

3.223.1 Optimal result

Integrand size = 24, antiderivative size = 236

$$\int \frac{\csc^2(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \frac{\left(6\sqrt{a}-5\sqrt{b}\right)\sqrt{b}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} - \frac{\left(6\sqrt{a}+5\sqrt{b}\right)\sqrt{b}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d} - \frac{\cot(c+dx)}{a^2d} - \frac{b\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a^2(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)}$$

output
```
-cot(d*x+c)/a^2/d+1/8*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(
6*a^(1/2)-5*b^(1/2))*b^(1/2)/a^(9/4)/d/(a^(1/2)-b^(1/2))^(3/2)-1/8*arctan(
(a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*b^(1/2)*(6*a^(1/2)+5*b^(1/2))/
a^(9/4)/d/(a^(1/2)+b^(1/2))^(3/2)-1/4*b*tan(d*x+c)*(a+(a+b)*tan(d*x+c)^2)/
a^2/(a-b)/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)
```

3.223.2 Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.16

$$\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \frac{-\frac{\left(6a\sqrt{b}+5\sqrt{a}b\right)\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{a+\sqrt{a}\sqrt{b}}}-\frac{\left(6a\sqrt{b}-5\sqrt{a}b\right)\text{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\sqrt{-a+\sqrt{a}\sqrt{b}}}-8\sqrt{a}\cot(c+dx)-\frac{4\sqrt{a}b(2a\ldots}{(a-b)(8a-\ldots}}{8a^{5/2}d}$$

input
```
Integrate[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4)^2,x]
```

output
```
(-(((6*a*Sqrt[b] + 5*Sqrt[a]*b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/
Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])*Sqrt[a + Sqrt[a]*Sqrt[b]]
)) - ((6*a*Sqrt[b] - 5*Sqrt[a]*b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x
])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])*Sqrt[-a + Sqrt[a]*Sqr
t[b]]) - 8*Sqrt[a]*Cot[c + d*x] - (4*Sqrt[a]*b*(2*a + b - b*Cos[2*(c + d*x
)])*Sin[2*(c + d*x)])/((a - b)*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4
*(c + d*x)])))/(8*a^(5/2)*d)
```

3.223.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3696, 1673, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^2\left(a-b\sin(c+dx)^4\right)^2}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\cot^2(c+dx)\left(\tan^2(c+dx)+1\right)^4}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \; 1673$$

$$\frac{-\frac{\int -\frac{2\cot^2(c+dx)\left(\frac{b\left(4a^2-ba-b^2\right)\tan^4(c+dx)}{a-b}+\frac{a(8a-7b)b\tan^2(c+dx)}{a-b}+4ab\right)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{8a^2b}-\frac{b\tan(c+dx)\left((a+b)\tan^2(c+dx)+a\right)}{4a^2(a-b)\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\cot^2(c+dx)\left(\frac{b\left(4a^2-ba-b^2\right)\tan^4(c+dx)}{a-b}+\frac{a(8a-7b)b\tan^2(c+dx)}{a-b}+4ab\right)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{4a^2b}-\frac{b\tan(c+dx)\left((a+b)\tan^2(c+dx)+a\right)}{4a^2(a-b)\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}}{d}$$

$$\Big\downarrow \; 2195$$

$$\frac{\frac{\int \left(\frac{\left((7a-5b)\tan^2(c+dx)+a\right)b^2}{(a-b)\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}+4\cot^2(c+dx)b\right)d\tan(c+dx)}{4a^2b}-\frac{b\tan(c+dx)\left((a+b)\tan^2(c+dx)+a\right)}{4a^2(a-b)\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{b^{3/2}\left(6\sqrt{a}-5\sqrt{b}\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{b^{3/2}\left(6\sqrt{a}+5\sqrt{b}\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-4b\cot(c+dx)}{4a^2b}-\frac{b\tan(c+dx)\left((a+b)\tan^2(c+d\right.}{4a^2(a-b)\left((a-b)\tan^4(c+dx)+2a\tan\right.}}{d}$$

input
```
Int[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4)^2,x]
```

output
```
((((6*Sqrt[a] - 5*Sqrt[b])*b^(3/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c +
 d*x])/a^(1/4)])/(2*a^(1/4)*(Sqrt[a] - Sqrt[b])^(3/2)) - ((6*Sqrt[a] + 5*S
qrt[b])*b^(3/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2
*a^(1/4)*(Sqrt[a] + Sqrt[b])^(3/2)) - 4*b*Cot[c + d*x])/(4*a^2*b) - (b*Tan
[c + d*x]*(a + (a + b)*Tan[c + d*x]^2))/(4*a^2*(a - b)*(a + 2*a*Tan[c + d*
x]^2 + (a - b)*Tan[c + d*x]^4)))/d
```

3.223. $\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

3.223.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1673
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 + c*x^4)^(p +
 1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
 e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5
) - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1]
&& ILtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

3.223. $\int \frac{\csc^2(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx$

3.223.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{-\frac{1}{a^2\tan(dx+c)}+\frac{b\left(\frac{-\frac{(a+b)\left(\tan^3(dx+c)\right)}{4(a-b)}-\frac{a\tan(dx+c)}{4(a-b)}}{\left(\tan^4(dx+c)\right)a-b\left(\tan^4(dx+c)\right)+2a\left(\tan^2(dx+c)\right)+a}+\frac{\left(7a\sqrt{ab}-5\sqrt{ab}\,b-6a^2+4ab\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+\cdots}{\sqrt{\left(\sqrt{ab}-a\right)(a-\cdots}}\right.}{8\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right.}{a^2}}{d}$
default	$\frac{-\frac{1}{a^2\tan(dx+c)}+\frac{b\left(\frac{-\frac{(a+b)\left(\tan^3(dx+c)\right)}{4(a-b)}-\frac{a\tan(dx+c)}{4(a-b)}}{\left(\tan^4(dx+c)\right)a-b\left(\tan^4(dx+c)\right)+2a\left(\tan^2(dx+c)\right)+a}+\frac{\left(7a\sqrt{ab}-5\sqrt{ab}\,b-6a^2+4ab\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+\cdots}{\sqrt{\left(\sqrt{ab}-a\right)(a-\cdots}}\right.}{8\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right.}{a^2}}{d}$
risch	Expression too large to display

input
```
int(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^2/tan(d*x+c)+1/a^2*b*((-1/4*(a+b)/(a-b)*tan(d*x+c)^3-1/4*a/(a-b)
*tan(d*x+c))/(tan(d*x+c)^4*a-b*tan(d*x+c)^4+2*a*tan(d*x+c)^2+a)+1/8*(7*a*(
a*b)^(1/2)-5*(a*b)^(1/2)*b-6*a^2+4*a*b)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)
*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/8
*(7*a*(a*b)^(1/2)-5*(a*b)^(1/2)*b+6*a^2-4*a*b)/(a*b)^(1/2)/(a-b)/(((a*b)^(
1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2)
)))
```

3.223.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3648 vs. $2(186) = 372$.

Time = 1.10 (sec) , antiderivative size = 3648, normalized size of antiderivative = 15.46

$$\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
-1/32*(8*(4*a*b - 5*b^2)*cos(d*x + c)^5 - 8*(7*a*b - 10*b^2)*cos(d*x + c)^
3 - ((a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x +
c)^2 - (a^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^
4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 +
 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*
a^10*b^5 + a^9*b^6)*d^4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 - 3*a^6*b
+ 3*a^5*b^2 - a^4*b^3)*d^2))*log(432*a^3*b^2 - 921*a^2*b^3 + 2625/4*a*b^4
- 625/4*b^5 - 1/4*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*cos
(d*x + c)^2 + 1/2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - 22*a^8*b^3 + 5*a^7*b
^4)*d^3*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 62
5*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^1
0*b^5 + a^9*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) - 2*(144*a^6*b - 303*a^5*
b^2 + 213*a^4*b^3 - 50*a^3*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^7 -
 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 71
61*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^
12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) + 36*a^2*b - 47*a*b^2 +
 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2)) + 1/4*(2*(36*a^9 - 1
33*a^8*b + 183*a^7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2*cos(d*x + c)^2 - (3
6*a^9 - 133*a^8*b + 183*a^7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2)*sqrt((230
4*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 ...
```

3.223.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**2/(a-b*sin(d*x+c)**4)**2,x)
```

output
```
Timed out
```

3.223.7 Maxima [F]

$$\int \frac{\csc^2(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \int \frac{\csc(dx+c)^2}{(b\sin(dx+c)^4-a)^2}\,dx$$

input
```
integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/2*(2*(48*a^2*b - 5*a*b^2 - 25*b^3)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + (
(6*a*b^2 - 5*b^3)*sin(8*d*x + 8*c) - 2*(13*a*b^2 - 10*b^3)*sin(6*d*x + 6*c
) - 2*(32*a^2*b - 47*a*b^2 + 15*b^3)*sin(4*d*x + 4*c) - 2*(7*a*b^2 - 10*b^
3)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c) + (2*(48*a^2*b - 5*a*b^2 - 25*b^3)
*sin(6*d*x + 6*c) + 2*(112*a^2*b - 165*a*b^2 + 50*b^3)*sin(4*d*x + 4*c) +
5*(8*a*b^2 - 15*b^3)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 2*(2*(256*a^3 -
432*a^2*b + 210*a*b^2 - 25*b^3)*sin(4*d*x + 4*c) + (112*a^2*b - 165*a*b^2
+ 50*b^3)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*((a^3*b^2 - a^2*b^3)*d*co
s(10*d*x + 10*c)^2 + 25*(a^3*b^2 - a^2*b^3)*d*cos(8*d*x + 8*c)^2 + 4*(64*a
^5 - 144*a^4*b + 105*a^3*b^2 - 25*a^2*b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^
5 - 144*a^4*b + 105*a^3*b^2 - 25*a^2*b^3)*d*cos(4*d*x + 4*c)^2 + 25*(a^3*b
^2 - a^2*b^3)*d*cos(2*d*x + 2*c)^2 + (a^3*b^2 - a^2*b^3)*d*sin(10*d*x + 10
*c)^2 + 25*(a^3*b^2 - a^2*b^3)*d*sin(8*d*x + 8*c)^2 + 4*(64*a^5 - 144*a^4*
b + 105*a^3*b^2 - 25*a^2*b^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 144*a^4*b
 + 105*a^3*b^2 - 25*a^2*b^3)*d*sin(4*d*x + 4*c)^2 + 20*(8*a^4*b - 13*a^3*b
^2 + 5*a^2*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*(a^3*b^2 - a^2*b^
3)*d*sin(2*d*x + 2*c)^2 - 10*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^3
*b^2 - a^2*b^3)*d - 2*(5*(a^3*b^2 - a^2*b^3)*d*cos(8*d*x + 8*c) + 2*(8*a^4
*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(6*d*x + 6*c) - 2*(8*a^4*b - 13*a^3*b^2
+ 5*a^2*b^3)*d*cos(4*d*x + 4*c) - 5*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2...
```

3.223.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1545 vs. 2(186) = 372.

Time = 0.89 (sec) , antiderivative size = 1545, normalized size of antiderivative = 6.55

$$\int \frac{\csc^2(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
-1/8*(((21*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 57*sqrt(a
^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + 23*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*sqrt(a*b)*a*b^3 + 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqr
t(a*b)*b^4)*(a^3 - a^2*b)^2*abs(-a + b) - (3*sqrt(a^2 - a*b - sqrt(a*b)*(a
 - b))*a^7*b - 12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^6*b^2 + 14*sqrt(a^
2 - a*b - sqrt(a*b)*(a - b))*a^5*b^3 - 4*sqrt(a^2 - a*b - sqrt(a*b)*(a - b
))*a^4*b^4 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b^5)*abs(-a^3 + a^2*b
)*abs(-a + b) - 2*(9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^10 -
42*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b + 66*sqrt(a^2 - a*b
 - sqrt(a*b)*(a - b))*sqrt(a*b)*a^8*b^2 - 40*sqrt(a^2 - a*b - sqrt(a*b)*(a
 - b))*sqrt(a*b)*a^7*b^3 + 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)
*a^6*b^4 + 2*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^5)*abs(-a
 + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^4 - a^3
*b + sqrt((a^4 - a^3*b)^2 - (a^4 - a^3*b)*(a^4 - 2*a^3*b + a^2*b^2)))/(a^4
 - 2*a^3*b + a^2*b^2))))/((3*a^12 - 21*a^11*b + 59*a^10*b^2 - 85*a^9*b^3 +
 65*a^8*b^4 - 23*a^7*b^5 + a^6*b^6 + a^5*b^7)*abs(-a^3 + a^2*b)) - ((21*sq
rt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 57*sqrt(a^2 - a*b + sq
rt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + 23*sqrt(a^2 - a*b + sqrt(a*b)*(a - b)
)*sqrt(a*b)*a*b^3 + 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*(
a^3 - a^2*b)^2*abs(-a + b) + (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^7...
```

3.223.9 Mupad [B] (verification not implemented)

Time = 18.23 (sec) , antiderivative size = 4411, normalized size of antiderivative = 18.69

$$\int \frac{\csc^2(c+dx)}{(a-b\sin^4(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(c + d*x)^2*(a - b*sin(c + d*x)^4)^2),x)
```

output
```
(atan((((-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15
*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^
9*b^3 - 3*a^10*b^2)))^(1/2)*(4096*a^10*b^8 - 24576*a^11*b^7 + 61440*a^12*b
^6 - 81920*a^13*b^5 + 61440*a^14*b^4 - 24576*a^15*b^3 + 4096*a^16*b^2 + ta
n(c + d*x)*(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b -
 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 +
 a^9*b^3 - 3*a^10*b^2)))^(1/2)*(65536*a^19*b - 65536*a^12*b^8 + 458752*a^1
3*b^7 - 1376256*a^14*b^6 + 2293760*a^15*b^5 - 2293760*a^16*b^4 + 1376256*a
^17*b^3 - 458752*a^18*b^2)) + tan(c + d*x)*(6400*a^7*b^9 - 39424*a^8*b^8 +
 93952*a^9*b^7 - 100352*a^10*b^6 + 26368*a^11*b^5 + 40448*a^12*b^4 - 36608
*a^13*b^3 + 9216*a^14*b^2))*(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(
1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(
3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*1i - ((-(48*a^2*(a^9*b^3)^
(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a
*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*
(4096*a^10*b^8 - 24576*a^11*b^7 + 61440*a^12*b^6 - 81920*a^13*b^5 + 61440*
a^14*b^4 - 24576*a^15*b^3 + 4096*a^16*b^2 - tan(c + d*x)*(-(48*a^2*(a^9*b^
3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 6
9*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/
2)*(65536*a^19*b - 65536*a^12*b^8 + 458752*a^13*b^7 - 1376256*a^14*b^6 ...
```

3.223. $\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

3.224 $\int \frac{\sin^9(c+dx)}{(a-b\sin^4(c+dx))^3} \, dx$

3.224.1 Optimal result

Integrand size = 24, antiderivative size = 315

$$\int \frac{\sin^9(c+dx)}{(a-b\sin^4(c+dx))^3} \, dx = -\frac{\left(5a-14\sqrt{a}\sqrt{b}+12b\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{9/4}d} - \frac{\left(5a+14\sqrt{a}\sqrt{b}+12b\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{9/4}d} - \frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(9a^2-11ab-10b^2-2(2a-5b)b\cos^2(c+dx)\right)}{32(a-b)^2b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)}$$

output
```
-1/8*a*cos(d*x+c)*(a+b-b*cos(d*x+c)^2)/(a-b)/b^2/d/(a-b+2*b*cos(d*x+c)^2-b
*cos(d*x+c)^4)^2+1/32*cos(d*x+c)*(9*a^2-11*a*b-10*b^2-2*(2*a-5*b)*b*cos(d*
x+c)^2)/(a-b)^2/b^2/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)-1/64*arctan(b^
(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(5*a+12*b-14*a^(1/2)*b^(1/2))/b^
(9/4)/d/a^(1/2)/(a^(1/2)-b^(1/2))^(5/2)-1/64*arctanh(b^(1/4)*cos(d*x+c)/(a
^(1/2)+b^(1/2))^(1/2))*(5*a+12*b+14*a^(1/2)*b^(1/2))/b^(9/4)/d/a^(1/2)/(a^
(1/2)+b^(1/2))^(5/2)
```

3.224.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 7.07 (sec) , antiderivative size = 785, normalized size of antiderivative = 2.49

$$\int \frac{\sin^9(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$$

$$= \frac{-\frac{32\cos(c+dx)(-9a^2+13ab+5b^2+(2a-5b)b\cos(2(c+dx)))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))}-\frac{512a(a-b)\cos(c+dx)(2a+b-b\cos(2(c+dx)))}{(-8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))^2}+i\text{RootSum}\left[b-4b\#1^2\right.}{}$$

input
```
Integrate[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
((-32*Cos[c + d*x]*(-9*a^2 + 13*a*b + 5*b^2 + (2*a - 5*b)*b*Cos[2*(c + d*x
)]))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]) - (512*a*(a -
 b)*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(
c + d*x)] + b*Cos[4*(c + d*x)])^2 + I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6
*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
- #1)] + 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + (2*I)*a*b*Log[1
 - 2*Cos[c + d*x]*#1 + #1^2] - (5*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]
 - 20*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 56*a*b*ArcTan[Si
n[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 78*b^2*ArcTan[Sin[c + d*x]/(Cos[c +
 d*x] - #1)]*#1^2 + (10*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (2
8*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (39*I)*b^2*Log[1 - 2*Cos
[c + d*x]*#1 + #1^2]*#1^2 + 20*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
]*#1^4 - 56*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 78*b^2*Arc
Tan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (10*I)*a^2*Log[1 - 2*Cos[c +
d*x]*#1 + #1^2]*#1^4 + (28*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 -
 (39*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 4*a*b*ArcTan[Sin[c +
d*x]/(Cos[c + d*x] - #1)]*#1^6 - 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
- #1)]*#1^6 - (2*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6 + (5*I)*b^2
*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 -
3*b*#1^5 + b*#1^7) & ])/(128*(a - b)^2*b^2*d)
```

3.224.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3694, 1517, 27, 2206, 27, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(c+dx)^9}{\left(a-b\sin(c+dx)^4\right)^3}dx$$

$$\Big\downarrow \ 3694$$

$$-\frac{\int \frac{\left(1-\cos^2(c+dx)\right)^4}{\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)^3}d\cos(c+dx)}{d}$$

$$\Big\downarrow \ 1517$$

$$-\frac{\frac{a\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8b^2(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\int \frac{2\left(8a(a-b)\cos^4(c+dx)-a(11a-16b)\cos^2(c+dx)+\frac{a\left(a^2+ba-8b^2\right)}{b}\right)}{\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)^2}d\cos(c+dx)}{16ab(a-b)}}{d}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{a\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8b^2(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\int \frac{8a(a-b)\cos^4(c+dx)-a(11a-16b)\cos^2(c+dx)+a\left(\frac{a^2}{b}+a-8b\right)}{\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)^2}d\cos(c+dx)}{8ab(a-b)}}{d}$$

$$\Big\downarrow \ 2206$$

$$-\frac{\frac{a\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8b^2(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\frac{a\cos(c+dx)\left(9a^2-2b(2a-5b)\cos^2(c+dx)-11ab-10b^2\right)}{4b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\int \frac{2a^2\left(5a^2-15ba+22b^2+2(2a-5b)b\cos^2(c+d\ldots\right.}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}}{8ab(a-b)}}{8ab(a-b)}}{d}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{a\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8b^2(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\frac{a\cos(c+dx)\left(9a^2-2b(2a-5b)\cos^2(c+dx)-11ab-10b^2\right)}{4b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{a\int\frac{5a^2-15ba+22b^2+2(2a-5b)b\cos^2(c+dx)}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}}{4b(a-b)}}{8ab(a-b)}}{d}$$

$\downarrow$ 1480

$$-\frac{\frac{a\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8b^2(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\frac{a\cos(c+dx)\left(9a^2-2b(2a-5b)\cos^2(c+dx)-11ab-10b^2\right)}{4b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{a\left(\frac{\sqrt{b}\left(-2\sqrt{a}\sqrt{b}+a+b\right)\left(14\sqrt{a}\sqrt{b}+5a+12b\right)\int}{2\sqrt{}}\right.}{}}{}}{d}$$

$\downarrow$ 218

$$-\frac{\frac{a\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8b^2(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\frac{a\cos(c+dx)\left(9a^2-2b(2a-5b)\cos^2(c+dx)-11ab-10b^2\right)}{4b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{a\left(\frac{\sqrt{b}\left(-2\sqrt{a}\sqrt{b}+a+b\right)\left(14\sqrt{a}\sqrt{b}+5a+12b\right)\int}{2\sqrt{}}\right.}{}}{8ab}}{d}$$

$\downarrow$ 221

$$-\frac{\frac{a\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8b^2(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\frac{a\cos(c+dx)\left(9a^2-2b(2a-5b)\cos^2(c+dx)-11ab-10b^2\right)}{4b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{a\left(\frac{\left(-14\sqrt{a}\sqrt{b}+5a+12b\right)\left(\sqrt{a}+\sqrt{b}\right)^2\arctan\left(\right.}{2\sqrt{a}\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}\right.}{}}{8ab(a-b)}}{d}$$

input
```
Int[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
-(((a*Cos[c + d*x]*(a + b - b*Cos[c + d*x]^2))/(8*(a - b)*b^2*(a - b + 2*b
*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)^2) - (-1/4*(a*(((Sqrt[a] + Sqrt[b])^2*
(5*a - 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[
a] - Sqrt[b]]])/(2*Sqrt[a]*Sqrt[Sqrt[a] - Sqrt[b]]*b^(1/4)) + ((a - 2*Sqrt
[a]*Sqrt[b] + b)*(5*a + 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTanh[(b^(1/4)*Cos[c
+ d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/
4))))/((a - b)*b) + (a*Cos[c + d*x]*(9*a^2 - 11*a*b - 10*b^2 - 2*(2*a - 5*
b)*b*Cos[c + d*x]^2))/(4*(a - b)*b*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c +
 d*x]^4)))/(8*a*(a - b)*b))/d)
```

3.224. $\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

3.224.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.224. $\int \frac{\sin^9(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.224.4 Maple [A] (verified)

Time = 5.59 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{-\frac{-\frac{(2a-5b)\left(\cos^7(dx+c)\right)}{16\left(a^2-2ab+b^2\right)}+\frac{3\left(3a^2-ab-10b^2\right)\left(\cos^5(dx+c)\right)}{32b\left(a^2-2ab+b^2\right)}-\frac{3\left(3a^2-2ab-5b^2\right)\left(\cos^3(dx+c)\right)}{16b\left(a^2-2ab+b^2\right)}-\frac{5\left(a^2-3ab-2b^2\right)\cos(dx+c)}{32b^2(a-b)}}{\left(a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)\right)^2}-\cdots}{d}$
default	$\frac{-\frac{-\frac{(2a-5b)\left(\cos^7(dx+c)\right)}{16\left(a^2-2ab+b^2\right)}+\frac{3\left(3a^2-ab-10b^2\right)\left(\cos^5(dx+c)\right)}{32b\left(a^2-2ab+b^2\right)}-\frac{3\left(3a^2-2ab-5b^2\right)\left(\cos^3(dx+c)\right)}{16b\left(a^2-2ab+b^2\right)}-\frac{5\left(a^2-3ab-2b^2\right)\cos(dx+c)}{32b^2(a-b)}}{\left(a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)\right)^2}-\cdots}{d}$
risch	Expression too large to display

input
```
int(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-(-1/16*(2*a-5*b)/(a^2-2*a*b+b^2)*cos(d*x+c)^7+3/32*(3*a^2-a*b-10*b^2
)/b/(a^2-2*a*b+b^2)*cos(d*x+c)^5-3/16*(3*a^2-2*a*b-5*b^2)/b/(a^2-2*a*b+b^2
)*cos(d*x+c)^3-5/32*(a^2-3*a*b-2*b^2)/b^2/(a-b)*cos(d*x+c))/(a-b+2*b*cos(d
*x+c)^2-b*cos(d*x+c)^4)^2-1/32/(a^2-2*a*b+b^2)/b*(-1/2*(-4*a*(a*b)^(1/2)+1
0*(a*b)^(1/2)*b-5*a^2+11*a*b-12*b^2)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)
*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))+1/2*(-4*a*(a*b)^(1/2)+10*
(a*b)^(1/2)*b+5*a^2-11*a*b+12*b^2)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*a
rctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))))
```

3.224.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4640 vs. 2(264) = 528.

Time = 1.07 (sec) , antiderivative size = 4640, normalized size of antiderivative = 14.73

$$\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
1/128*(8*(2*a*b^2 - 5*b^3)*cos(d*x + c)^7 - 12*(3*a^2*b - a*b^2 - 10*b^3)*
cos(d*x + c)^5 + 24*(3*a^2*b - 2*a*b^2 - 5*b^3)*cos(d*x + c)^3 + ((a^2*b^4
 - 2*a*b^5 + b^6)*d*cos(d*x + c)^8 - 4*(a^2*b^4 - 2*a*b^5 + b^6)*d*cos(d*x
 + c)^6 - 2*(a^3*b^3 - 5*a^2*b^4 + 7*a*b^5 - 3*b^6)*d*cos(d*x + c)^4 + 4*(
a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cos(d*x + c)^2 + (a^4*b^2 - 4*a^3*b
^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*d)*sqrt((15*a^4 - 94*a^3*b + 155*a^2*b^2 -
 76*a*b^3 - 144*b^4 + (a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a
^2*b^8 - a*b^9)*d^2*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^
5*b^3 + 269641*a^4*b^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a*b^7 +
147456*b^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a
^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^
2*b^18 + a*b^19)*d^4)))/((a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 +
5*a^2*b^8 - a*b^9)*d^2))*log((625*a^6 - 5250*a^5*b + 22509*a^4*b^2 - 57820
*a^3*b^3 + 96336*a^2*b^4 - 98304*a*b^5 + 55296*b^6)*cos(d*x + c) - ((a^8*b
^7 - 6*a^7*b^8 + 27*a^6*b^9 - 80*a^5*b^10 + 135*a^4*b^11 - 126*a^3*b^12 +
61*a^2*b^13 - 12*a*b^14)*d^3*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 -
117532*a^5*b^3 + 269641*a^4*b^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640
*a*b^7 + 147456*b^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^1
2 + 210*a^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^1
7 - 10*a^2*b^18 + a*b^19)*d^4)) + (125*a^7*b^2 - 1045*a^6*b^3 + 4305*a^...
```

3.224.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**9/(a-b*sin(d*x+c)**4)**3,x)
```

output
```
Timed out
```

3.224.7 Maxima [F]

$$\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx=\int -\frac{\sin\left(dx+c\right)^9}{\left(b\sin\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/8*(8*(2*a*b^4 - 5*b^5)*cos(2*d*x + 2*c)*cos(d*x + c) - 8*(18*a^2*b^3 -
20*a*b^4 - 25*b^5)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c) + 8*(2*a*b^4 - 5*b^5)
*sin(2*d*x + 2*c)*sin(d*x + c) - ((2*a*b^4 - 5*b^5)*cos(15*d*x + 15*c) - (
18*a^2*b^3 - 20*a*b^4 - 25*b^5)*cos(13*d*x + 13*c) + 3*(18*a^2*b^3 - 8*a*b
^4 - 15*b^5)*cos(11*d*x + 11*c) + (160*a^3*b^2 - 388*a^2*b^3 + 2*a*b^4 + 2
5*b^5)*cos(9*d*x + 9*c) + (160*a^3*b^2 - 388*a^2*b^3 + 2*a*b^4 + 25*b^5)*c
os(7*d*x + 7*c) + 3*(18*a^2*b^3 - 8*a*b^4 - 15*b^5)*cos(5*d*x + 5*c) - (18
*a^2*b^3 - 20*a*b^4 - 25*b^5)*cos(3*d*x + 3*c) + (2*a*b^4 - 5*b^5)*cos(d*x
 + c))*cos(16*d*x + 16*c) - (2*a*b^4 - 5*b^5 - 8*(2*a*b^4 - 5*b^5)*cos(14*
d*x + 14*c) - 4*(16*a^2*b^3 - 54*a*b^4 + 35*b^5)*cos(12*d*x + 12*c) + 8*(3
2*a^2*b^3 - 94*a*b^4 + 35*b^5)*cos(10*d*x + 10*c) + 2*(256*a^3*b^2 - 832*a
^2*b^3 + 550*a*b^4 - 175*b^5)*cos(8*d*x + 8*c) + 8*(32*a^2*b^3 - 94*a*b^4
+ 35*b^5)*cos(6*d*x + 6*c) - 4*(16*a^2*b^3 - 54*a*b^4 + 35*b^5)*cos(4*d*x
+ 4*c) - 8*(2*a*b^4 - 5*b^5)*cos(2*d*x + 2*c))*cos(15*d*x + 15*c) - 8*((18
*a^2*b^3 - 20*a*b^4 - 25*b^5)*cos(13*d*x + 13*c) - 3*(18*a^2*b^3 - 8*a*b^4
 - 15*b^5)*cos(11*d*x + 11*c) - (160*a^3*b^2 - 388*a^2*b^3 + 2*a*b^4 + 25*
b^5)*cos(9*d*x + 9*c) - (160*a^3*b^2 - 388*a^2*b^3 + 2*a*b^4 + 25*b^5)*cos
(7*d*x + 7*c) - 3*(18*a^2*b^3 - 8*a*b^4 - 15*b^5)*cos(5*d*x + 5*c) + (18*a
^2*b^3 - 20*a*b^4 - 25*b^5)*cos(3*d*x + 3*c) - (2*a*b^4 - 5*b^5)*cos(d*x +
 c))*cos(14*d*x + 14*c) + (18*a^2*b^3 - 20*a*b^4 - 25*b^5 - 4*(144*a^3*...
```

3.224.8 Giac [F]

$$\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx=\int -\frac{\sin\left(dx+c\right)^9}{\left(b\sin\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.224.9 Mupad [B] (verification not implemented)

Time = 19.41 (sec) , antiderivative size = 6675, normalized size of antiderivative = 21.19

$$\int \frac{\sin^9(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^9/(a - b*sin(c + d*x)^4)^3,x)
```

output
```
((cos(c + d*x)^7*(2*a - 5*b))/(16*(a^2 - 2*a*b + b^2)) + (3*cos(c + d*x)^5
*(a*b - 3*a^2 + 10*b^2))/(32*b*(a^2 - 2*a*b + b^2)) - (5*cos(c + d*x)*(3*a
*b - a^2 + 2*b^2))/(32*b^2*(a - b)) - (3*cos(c + d*x)^3*(2*a*b - 3*a^2 + 5
*b^2))/(16*b*(a - b)^2))/(d*(a^2 - 2*a*b + b^2 + cos(c + d*x)^2*(4*a*b - 4
*b^2) - cos(c + d*x)^4*(2*a*b - 6*b^2) - 4*b^2*cos(c + d*x)^6 + b^2*cos(c
+ d*x)^8)) + (atan((((180224*a*b^8 - 483328*a^2*b^7 + 466944*a^3*b^6 - 20
4800*a^4*b^5 + 40960*a^5*b^4)/(16384*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^
4 + a^4*b^3)) - (cos(c + d*x)*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9
)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 +
 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*
b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*
a^6*b^10 - a^7*b^9)))^(1/2)*(16384*a*b^9 - 65536*a^2*b^8 + 98304*a^3*b^7 -
 65536*a^4*b^6 + 16384*a^5*b^5))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 -
 4*a^3*b^2)))*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*
b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*(a^
3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(163
84*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b
^9)))^(1/2) + (cos(c + d*x)*(25*a^4 - 94*a^3*b - 164*a*b^3 + 144*b^4 + 161
*a^2*b^2))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*(-(25*a^
4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + ...
```

3.225 $\int \frac{\sin^7(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

3.225.1 Optimal result

Integrand size = 24, antiderivative size = 290

$$\int \frac{\sin^7(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \frac{3\left(\sqrt{a}-2\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{7/4}d} - \frac{3\left(\sqrt{a}+2\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{7/4}d} - \frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(5a-17b-3(a-3b)\cos^2(c+dx)\right)}{32(a-b)^2bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)}$$

output
```
-1/8*a*cos(d*x+c)*(2-cos(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d
*x+c)^4)^2+1/32*cos(d*x+c)*(5*a-17*b-3*(a-3*b)*cos(d*x+c)^2)/(a-b)^2/b/d/(
a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)+3/64*arctan(b^(1/4)*cos(d*x+c)/(a^(1/
2)-b^(1/2))^(1/2))*(a^(1/2)-2*b^(1/2))/b^(7/4)/d/a^(1/2)/(a^(1/2)-b^(1/2))
^(5/2)-3/64*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(a^(1/2)+2
*b^(1/2))/b^(7/4)/d/a^(1/2)/(a^(1/2)+b^(1/2))^(5/2)
```

3.225.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.27 (sec) , antiderivative size = 630, normalized size of antiderivative = 2.17

$$\int \frac{\sin^7(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$$

$$= \frac{-\frac{32\cos(c+dx)(-7a+25b+3(a-3b)\cos(2(c+dx)))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))} + \frac{512a(a-b)(-5\cos(c+dx)+\cos(3(c+dx)))}{(-8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))^2} - 3i\mathrm{RootSum}\left[b-4b\#1^2-16\right.}{}$$

input
```
Integrate[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
((-32*Cos[c + d*x]*(-7*a + 25*b + 3*(a - 3*b)*Cos[2*(c + d*x)]))/(8*a - 3*
b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]) + (512*a*(a - b)*(-5*Cos[c
+ d*x] + Cos[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(
c + d*x)])^2 - (3*I)*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^
6 + b*#1^8 & , (2*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - 6*b*ArcTan[
Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]
+ (3*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 6*a*ArcTan[Sin[c + d*x]/(Cos
[c + d*x] - #1)]*#1^2 + 34*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2
 + (3*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (17*I)*b*Log[1 - 2*Cos
[c + d*x]*#1 + #1^2]*#1^2 + 6*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#
1^4 - 34*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (3*I)*a*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + (17*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^
2]*#1^4 - 2*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + 6*b*ArcTan[S
in[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + I*a*Log[1 - 2*Cos[c + d*x]*#1 + #1
^2]*#1^6 - (3*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*
#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(256*(a - b)^2*b*d)
```

3.225.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3694, 1517, 27, 1492, 27, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(c+dx)^7}{\left(a-b\sin(c+dx)^4\right)^3}dx$$

$$\Big\downarrow \; 3694$$

$$-\frac{\int \frac{\left(1-\cos^2(c+dx)\right)^3}{\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)^3}d\cos(c+dx)}{d}$$

$$\Big\downarrow \; 1517$$

$$-\frac{\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\int \frac{2a\left(2(a-4b)-(3a-8b)\cos^2(c+dx)\right)}{\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)^2}d\cos(c+dx)}{16ab(a-b)}}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\int \frac{2(a-4b)-(3a-8b)\cos^2(c+dx)}{\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)^2}d\cos(c+dx)}{8b(a-b)}}{d}$$

$$\Big\downarrow \; 1492$$

$$-\frac{\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\frac{\cos(c+dx)\left(-3(a-3b)\cos^2(c+dx)+5a-17b\right)}{4(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\int -\frac{6ab\left(-\left((a-3b)\cos^2(c+dx)\right)+a-5b\right)}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{8ab(a-b)}}{8b(a-b)}}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\frac{3\int \frac{-\left((a-3b)\cos^2(c+dx)\right)+a-5b}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{4(a-b)}+\frac{\cos(c+dx)\left(-3(a-3b)\cos^2(c+dx)+5a-17b\right)}{4(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}}{8b(a-b)}}{d}$$

$$\Big\downarrow \; 1480$$

$$-\frac{\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\frac{3\left(-\frac{1}{2}\left(-\frac{2b^{3/2}}{\sqrt{a}}+a-3b\right)\int \frac{1}{-b\cos^2(c+dx)-\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}d\cos(c+dx)-\frac{1}{2}\left(\frac{2b^{3/2}}{\sqrt{a}}+a-3b\right)\int \frac{\cdots}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{\cdots}}\cdots}{4(a-b)}\cdots}{8b(a-b)}}{d}$$

$$\Big\downarrow \; 218$$

3.225. $\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

$$-\frac{\frac{a\cos(c+dx)(2-\cos^2(c+dx))}{8b(a-b)(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b)^2}-\frac{\frac{3\left(\frac{\left(-\frac{2b^{3/2}}{\sqrt{a}}+a-3b\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{1}{2}\left(\frac{2b^{3/2}}{\sqrt{a}}+a-3b\right)\int\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos}{4(a-b)}}{8b(a-b)}}{d}$$

$$\downarrow 221$$

$$-\frac{\frac{a\cos(c+dx)(2-\cos^2(c+dx))}{8b(a-b)(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b)^2}-\frac{\frac{3\left(\frac{\left(-\frac{2b^{3/2}}{\sqrt{a}}+a-3b\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\left(\frac{2b^{3/2}}{\sqrt{a}}+a-3b\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{4(a-b)}+\frac{}{}}{8b(a-b)}}{d}$$

input
```
Int[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
-(((a*Cos[c + d*x]*(2 - Cos[c + d*x]^2))/(8*(a - b)*b*(a - b + 2*b*Cos[c +
 d*x]^2 - b*Cos[c + d*x]^4)^2) - ((3*(((a - 3*b - (2*b^(3/2))/Sqrt[a])*Arc
Tan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a] - Sqr
t[b]]*b^(3/4)) - ((a - 3*b + (2*b^(3/2))/Sqrt[a])*ArcTanh[(b^(1/4)*Cos[c +
 d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/4))))/(4*
(a - b)) + (Cos[c + d*x]*(5*a - 17*b - 3*(a - 3*b)*Cos[c + d*x]^2))/(4*(a
- b)*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)))/(8*(a - b)*b))/d)
```

3.225.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.225. $\int \frac{\sin^7(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.225. $\int \frac{\sin^7(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

3.225.4 Maple [A] (verified)

Time = 5.26 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\frac{\frac{3(a-3b)\left(\cos^7(dx+c)\right)}{32\left(a^2-2ab+b^2\right)}-\frac{(11a-35b)\left(\cos^5(dx+c)\right)}{32\left(a^2-2ab+b^2\right)}+\frac{\left(a^2+18ab-43b^2\right)\left(\cos^3(dx+c)\right)}{32b\left(a^2-2ab+b^2\right)}-\frac{(3a+17b)\cos(dx+c)}{32(a-b)b}}{\left(a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)\right)^2}+\frac{-\frac{3\left(a\sqrt{ab}-3\sqrt{ab}\,b+2b^2\right.}{64\sqrt{ab}\,b}}{}}{d}$
default	$\frac{\frac{\frac{3(a-3b)\left(\cos^7(dx+c)\right)}{32\left(a^2-2ab+b^2\right)}-\frac{(11a-35b)\left(\cos^5(dx+c)\right)}{32\left(a^2-2ab+b^2\right)}+\frac{\left(a^2+18ab-43b^2\right)\left(\cos^3(dx+c)\right)}{32b\left(a^2-2ab+b^2\right)}-\frac{(3a+17b)\cos(dx+c)}{32(a-b)b}}{\left(a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)\right)^2}+\frac{-\frac{3\left(a\sqrt{ab}-3\sqrt{ab}\,b+2b^2\right.}{64\sqrt{ab}\,b}}{}}{d}$
risch	Expression too large to display

input
```
int(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*((3/32*(a-3*b)/(a^2-2*a*b+b^2)*cos(d*x+c)^7-1/32*(11*a-35*b)/(a^2-2*a*
b+b^2)*cos(d*x+c)^5+1/32*(a^2+18*a*b-43*b^2)/b/(a^2-2*a*b+b^2)*cos(d*x+c)^
3-1/32*(3*a+17*b)/(a-b)/b*cos(d*x+c))/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4
)^2+3/32/(a^2-2*a*b+b^2)*(-1/2*(a*(a*b)^(1/2)-3*(a*b)^(1/2)*b+2*b^2)/(a*b)
^(1/2)/b/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b
)^(1/2))+1/2*(a*(a*b)^(1/2)-3*(a*b)^(1/2)*b-2*b^2)/(a*b)^(1/2)/b/(((a*b)^(
1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))))
```

3.225.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4185 vs. 2(234) = 468.

Time = 0.75 (sec) , antiderivative size = 4185, normalized size of antiderivative = 14.43

$$\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
1/128*(12*(a*b - 3*b^2)*cos(d*x + c)^7 - 4*(11*a*b - 35*b^2)*cos(d*x + c)^
5 + 4*(a^2 + 18*a*b - 43*b^2)*cos(d*x + c)^3 + 3*((a^2*b^3 - 2*a*b^4 + b^5
)*d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3
*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*
b^3 + 3*a*b^4 - b^5)*d*cos(d*x + c)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4
*a*b^4 + b^5)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5
*a^2*b^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167
*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 12
0*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 4
5*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) + a^3 - 10*a^2*b + 21*a*b^2 + 4*b
^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d
^2))*log(27*(a^4 - 10*a^3*b + 29*a^2*b^2 - 4*a*b^3 - 64*b^4)*cos(d*x + c)
+ 27*((a^8*b^5 - 8*a^7*b^6 + 23*a^6*b^7 - 30*a^5*b^8 + 15*a^4*b^9 + 4*a^3*
b^10 - 7*a^2*b^11 + 2*a*b^12)*d^3*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a
^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*
a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*
a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) - (a^5*b^2 - 11*a^4*b
^3 + 35*a^3*b^4 - 9*a^2*b^5 - 80*a*b^6)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 1
0*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b + 46*
a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - ...
```

3.225.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**7/(a-b*sin(d*x+c)**4)**3,x)
```

output
```
Timed out
```

3.225.7 Maxima [F]

$$\int \frac{\sin^7(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \int -\frac{\sin(dx+c)^7}{(b\sin(dx+c)^4-a)^3}\,dx$$

input
```
integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/16*(24*(a*b^3 - 3*b^4)*cos(2*d*x + 2*c)*cos(d*x + c) - 8*(23*a*b^3 - 77
*b^4)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c) + 24*(a*b^3 - 3*b^4)*sin(2*d*x + 2
*c)*sin(d*x + c) - (3*(a*b^3 - 3*b^4)*cos(15*d*x + 15*c) - (23*a*b^3 - 77*
b^4)*cos(13*d*x + 13*c) + (16*a^2*b^2 + 131*a*b^3 - 177*b^4)*cos(11*d*x +
11*c) - (144*a^2*b^2 + 367*a*b^3 - 109*b^4)*cos(9*d*x + 9*c) - (144*a^2*b^
2 + 367*a*b^3 - 109*b^4)*cos(7*d*x + 7*c) + (16*a^2*b^2 + 131*a*b^3 - 177*
b^4)*cos(5*d*x + 5*c) - (23*a*b^3 - 77*b^4)*cos(3*d*x + 3*c) + 3*(a*b^3 -
3*b^4)*cos(d*x + c))*cos(16*d*x + 16*c) - 3*(a*b^3 - 3*b^4 - 8*(a*b^3 - 3*
b^4)*cos(14*d*x + 14*c) - 4*(8*a^2*b^2 - 31*a*b^3 + 21*b^4)*cos(12*d*x + 1
2*c) + 8*(16*a^2*b^2 - 55*a*b^3 + 21*b^4)*cos(10*d*x + 10*c) + 2*(128*a^3*
b - 480*a^2*b^2 + 323*a*b^3 - 105*b^4)*cos(8*d*x + 8*c) + 8*(16*a^2*b^2 -
55*a*b^3 + 21*b^4)*cos(6*d*x + 6*c) - 4*(8*a^2*b^2 - 31*a*b^3 + 21*b^4)*co
s(4*d*x + 4*c) - 8*(a*b^3 - 3*b^4)*cos(2*d*x + 2*c))*cos(15*d*x + 15*c) -
8*((23*a*b^3 - 77*b^4)*cos(13*d*x + 13*c) - (16*a^2*b^2 + 131*a*b^3 - 177*
b^4)*cos(11*d*x + 11*c) + (144*a^2*b^2 + 367*a*b^3 - 109*b^4)*cos(9*d*x +
9*c) + (144*a^2*b^2 + 367*a*b^3 - 109*b^4)*cos(7*d*x + 7*c) - (16*a^2*b^2
+ 131*a*b^3 - 177*b^4)*cos(5*d*x + 5*c) + (23*a*b^3 - 77*b^4)*cos(3*d*x +
3*c) - 3*(a*b^3 - 3*b^4)*cos(d*x + c))*cos(14*d*x + 14*c) + (23*a*b^3 - 77
*b^4 - 4*(184*a^2*b^2 - 777*a*b^3 + 539*b^4)*cos(12*d*x + 12*c) + 8*(368*a
^2*b^2 - 1393*a*b^3 + 539*b^4)*cos(10*d*x + 10*c) + 2*(2944*a^3*b - 120...
```

3.225.8 Giac [F]

$$\int \frac{\sin^7(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \int -\frac{\sin(dx+c)^7}{(b\sin(dx+c)^4-a)^3}\,dx$$

input
```
integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.225.9 Mupad [B] (verification not implemented)

Time = 18.68 (sec) , antiderivative size = 5824, normalized size of antiderivative = 20.08

$$\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^7/(a - b*sin(c + d*x)^4)^3,x)
```

output
```
((3*cos(c + d*x)^7*(a - 3*b))/(32*(a^2 - 2*a*b + b^2)) - (cos(c + d*x)^5*(
11*a - 35*b))/(32*(a^2 - 2*a*b + b^2)) + (cos(c + d*x)^3*(18*a*b + a^2 - 4
3*b^2))/(32*b*(a - b)^2) - (cos(c + d*x)*(3*a + 17*b))/(32*b*(a - b)))/(d*
(a^2 - 2*a*b + b^2 + cos(c + d*x)^2*(4*a*b - 4*b^2) - cos(c + d*x)^4*(2*a*
b - 6*b^2) - 4*b^2*cos(c + d*x)^6 + b^2*cos(c + d*x)^8)) + (atan(((((3*(81
920*a*b^7 - 180224*a^2*b^6 + 114688*a^3*b^5 - 16384*a^4*b^4))/(32768*(b^6
- 4*a*b^5 + 6*a^2*b^4 - 4*a^3*b^3 + a^4*b^2)) - (cos(c + d*x)*((9*(a^3*(a^
3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5
+ a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^
2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1
/2)*(16384*a*b^8 - 65536*a^2*b^7 + 98304*a^3*b^6 - 65536*a^4*b^5 + 16384*a
^5*b^4))/(256*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*((9*(a^3*(a^3*
b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 +
a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*
b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2
) + (cos(c + d*x)*(81*a*b^2 - 54*a^2*b + 9*a^3 + 36*b^3))/(256*(a^4 - 4*a^
3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*((9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*
b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^
7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^
4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2)*1i - (((3*(81920*a*b...
```

3.226 $\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

3.226.1 Optimal result

Integrand size = 24, antiderivative size = 313

$$\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \frac{\left(3a-10\sqrt{a}\sqrt{b}+4b\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{5/4}d} + \frac{\left(3a+10\sqrt{a}\sqrt{b}+4b\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{5/4}d} - \frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(a^2-11ab-2b^2+2b(2a+b)\cos^2(c+dx)\right)}{32a(a-b)^2bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)}$$

output
```
-1/8*cos(d*x+c)*(a+b-b*cos(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cos(d*x+c)^2-b*cos
(d*x+c)^4)^2+1/32*cos(d*x+c)*(a^2-11*a*b-2*b^2+2*b*(2*a+b)*cos(d*x+c)^2)/a
/(a-b)^2/b/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)+1/64*arctan(b^(1/4)*cos
(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(3*a+4*b-10*a^(1/2)*b^(1/2))/a^(3/2)/b^(5
/4)/d/(a^(1/2)-b^(1/2))^(5/2)+1/64*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(
1/2))^(1/2))*(3*a+4*b+10*a^(1/2)*b^(1/2))/a^(3/2)/b^(5/4)/d/(a^(1/2)+b^(1/
2))^(5/2)
```

3.226.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.24 (sec) , antiderivative size = 786, normalized size of antiderivative = 2.51

$$\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$$

$$= \frac{\frac{32\cos(c+dx)(a^2-9ab-b^2+b(2a+b)\cos(2(c+dx)))}{a(8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx)))} - \frac{512(a-b)\cos(c+dx)(2a+b-b\cos(2(c+dx)))}{(-8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))^2} + \frac{i\text{RootSum}\left[b-4b\#1^2-16a\#1^4+\right.}{}}{}$$

input
```
Integrate[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
((32*Cos[c + d*x]*(a^2 - 9*a*b - b^2 + b*(2*a + b)*Cos[2*(c + d*x)]))/(a*(
8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) - (512*(a - b)*Cos
[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c + d*x
)] + b*Cos[4*(c + d*x)])^2 + (I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^
4 - 4*b*#1^6 + b*#1^8 & , (4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
+ 2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - (2*I)*a*b*Log[1 - 2*Cos
[c + d*x]*#1 + #1^2] - I*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 12*a^2*Ar
cTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 64*a*b*ArcTan[Sin[c + d*x]/(
Cos[c + d*x] - #1)]*#1^2 + 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^2 - (6*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (32*I)*a*b*Log[
1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (5*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 +
 #1^2]*#1^2 - 12*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 64*a*
b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 10*b^2*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)]*#1^4 + (6*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]
*#1^4 - (32*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + (5*I)*b^2*Log[
1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - 4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*
x] - #1)]*#1^6 - 2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + (2*
I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6 + I*b^2*Log[1 - 2*Cos[c + d*
x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) &
])/a)/(128*(a - b)^2*b*d)
```

3.226.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3694, 1517, 27, 1492, 27, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(c+dx)^5}{\left(a-b\sin(c+dx)^4\right)^3}dx$$

$$\Big\downarrow \; 3694$$

$$-\frac{\int \frac{\left(1-\cos^2(c+dx)\right)^2}{\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)^3}d\cos(c+dx)}{d}$$

$$\Big\downarrow \; 1517$$

$$-\frac{\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\int \frac{2a\left(5b\cos^2(c+dx)+a-7b\right)}{\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)^2}d\cos(c+dx)}{16ab(a-b)}}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\int \frac{5b\cos^2(c+dx)+a-7b}{\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)^2}d\cos(c+dx)}{8b(a-b)}}{d}$$

$$\Big\downarrow \; 1492$$

$$-\frac{\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\frac{\cos(c+dx)\left(a^2+2b(2a+b)\cos^2(c+dx)-11ab-2b^2\right)}{4a(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\int -\frac{2b\left(3a^2-17ba+2b^2+2b(2a+b)\cos^2(c+dx)\right)}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos}{8ab(a-b)}}{8b(a-b)}}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\frac{\int \frac{3a^2-17ba+2b^2+2b(2a+b)\cos^2(c+dx)}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{4a(a-b)}+\frac{\cos(c+dx)\left(a^2+2b(2a+b)\cos^2(c+dx)-11ab-2b^2\right)}{4a(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}}{8b(a-b)}}{d}$$

$\downarrow$ 1480

$$-\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\frac{\frac{\frac{\sqrt{b}\left(-2\sqrt{a}\sqrt{b}+a+b\right)\left(10\sqrt{a}\sqrt{b}+3a+4b\right)\int\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos(c+dx)}{2\sqrt{a}}-\frac{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)^2\left(-10\cdots\right.}{\cdots}}{4a(a-b)}\cdots}{8\cdots}}{d}$$

$\downarrow$ 218

$$-\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\frac{\frac{\frac{\sqrt{b}\left(-2\sqrt{a}\sqrt{b}+a+b\right)\left(10\sqrt{a}\sqrt{b}+3a+4b\right)\int\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos(c+dx)}{2\sqrt{a}}+\frac{\left(-10\sqrt{a}\sqrt{b}+3a+4b\cdots\right.}{\cdots}}{4a(a-b)}\cdots}{8b(a-b)}}{d}$$

$\downarrow$ 221

$$-\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8b(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\frac{\cos(c+dx)\left(a^2+2b(2a+b)\cos^2(c+dx)-11ab-2b^2\right)}{4a(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}+\frac{\frac{\left(-10\sqrt{a}\sqrt{b}+3a+4b\right)\left(\sqrt{a}+\sqrt{b}\right)^2\arctan\left(\frac{\sqrt[4]{b}\cos(c\cdots}{\sqrt{\sqrt{a}-\cdots}}\right.}{2\sqrt{a}\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}\cdots}{8b(a-b)}}{d}$$

input
```
Int[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
-(((Cos[c + d*x]*(a + b - b*Cos[c + d*x]^2))/(8*(a - b)*b*(a - b + 2*b*Cos
[c + d*x]^2 - b*Cos[c + d*x]^4)^2) - ((((Sqrt[a] + Sqrt[b])^2*(3*a - 10*Sq
rt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]
])/(2*Sqrt[a]*Sqrt[Sqrt[a] - Sqrt[b]]*b^(1/4)) + ((a - 2*Sqrt[a]*Sqrt[b] +
 b)*(3*a + 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[S
qrt[a] + Sqrt[b]]])/(2*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4)))/(4*a*(a -
 b)) + (Cos[c + d*x]*(a^2 - 11*a*b - 2*b^2 + 2*b*(2*a + b)*Cos[c + d*x]^2)
)/(4*a*(a - b)*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)))/(8*(a - b
)*b))/d)
```

3.226. $\int \frac{\sin^5(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

3.226.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

3.226. $\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.226.4 Maple [A] (verified)

Time = 5.97 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{-\frac{-\frac{b(2a+b)\left(\cos^7(dx+c)\right)}{16a\left(a^2-2ab+b^2\right)}+\frac{\left(a^2-19ab-6b^2\right)\left(\cos^5(dx+c)\right)}{32a\left(a^2-2ab+b^2\right)}-\frac{\left(5a^2-14ab-3b^2\right)\left(\cos^3(dx+c)\right)}{16a\left(a^2-2ab+b^2\right)}+\frac{\left(3a^2+15ab+2b^2\right)\cos(dx+c)}{32(a-b)ab}}{\left(a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)\right)^2}-\frac{\left(4a\sqrt{a}\right.}{}}{d}$
default	$\frac{-\frac{-\frac{b(2a+b)\left(\cos^7(dx+c)\right)}{16a\left(a^2-2ab+b^2\right)}+\frac{\left(a^2-19ab-6b^2\right)\left(\cos^5(dx+c)\right)}{32a\left(a^2-2ab+b^2\right)}-\frac{\left(5a^2-14ab-3b^2\right)\left(\cos^3(dx+c)\right)}{16a\left(a^2-2ab+b^2\right)}+\frac{\left(3a^2+15ab+2b^2\right)\cos(dx+c)}{32(a-b)ab}}{\left(a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)\right)^2}-\frac{\left(4a\sqrt{a}\right.}{}}{d}$
risch	Expression too large to display

input
```
int(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-(1/16*b*(2*a+b)/a/(a^2-2*a*b+b^2)*cos(d*x+c)^7+1/32*(a^2-19*a*b-6*b^
2)/a/(a^2-2*a*b+b^2)*cos(d*x+c)^5-1/16*(5*a^2-14*a*b-3*b^2)/a/(a^2-2*a*b+b
^2)*cos(d*x+c)^3+1/32*(3*a^2+15*a*b+2*b^2)/(a-b)/a/b*cos(d*x+c))/(a-b+2*b*
cos(d*x+c)^2-b*cos(d*x+c)^4)^2-1/32/a/(a^2-2*a*b+b^2)*(1/2*(4*a*(a*b)^(1/2
)+2*(a*b)^(1/2)*b-3*a^2+13*a*b-4*b^2)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2
)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/2*(4*a*(a*b)^(1/2)+2*(a
*b)^(1/2)*b+3*a^2-13*a*b+4*b^2)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arct
anh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))))
```

3.226.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4524 vs. $2(262) = 524$.

Time = 0.97 (sec) , antiderivative size = 4524, normalized size of antiderivative = 14.45

$$\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
-1/128*(8*(2*a*b^2 + b^3)*cos(d*x + c)^7 + 4*(a^2*b - 19*a*b^2 - 6*b^3)*co
s(d*x + c)^5 - 8*(5*a^2*b - 14*a*b^2 - 3*b^3)*cos(d*x + c)^3 + ((a^3*b^3 -
 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*c
os(d*x + c)^6 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*d*cos(d*x +
c)^4 + 4*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 + (a^5
*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)*sqrt((15*a^4 - 30*a^3*b
 - 229*a^2*b^2 + 116*a*b^3 - 16*b^4 + (a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 -
10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2*sqrt((81*a^6 - 1548*a^5*b + 12814*a^
4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^
5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10
+ 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)
))/((a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*
d^2))*log((81*a^5 - 1458*a^4*b + 9389*a^3*b^2 - 24972*a^2*b^3 + 10896*a*b^
4 - 1280*b^5)*cos(d*x + c) + ((a^10*b^4 + 10*a^9*b^5 - 69*a^8*b^6 + 160*a^
7*b^7 - 185*a^6*b^8 + 114*a^5*b^9 - 35*a^4*b^10 + 4*a^3*b^11)*d^3*sqrt((81
*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^4 - 48160
*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 +
 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 -
10*a^4*b^14 + a^3*b^15)*d^4)) - (27*a^7*b - 411*a^6*b^2 + 2383*a^5*b^3 - 5
529*a^4*b^4 + 1962*a^3*b^5 - 160*a^2*b^6)*d)*sqrt((15*a^4 - 30*a^3*b - ...
```

3.226.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**5/(a-b*sin(d*x+c)**4)**3,x)
```

output
```
Timed out
```

3.226.7 Maxima [F]

$$\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \int -\frac{\sin(dx+c)^5}{\left(b\sin(dx+c)^4-a\right)^3}\,dx$$

input
```
integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
1/8*(8*(2*a*b^4 + b^5)*cos(2*d*x + 2*c)*cos(d*x + c) + 8*(2*a^2*b^3 - 24*a
*b^4 - 5*b^5)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c) + 8*(2*a*b^4 + b^5)*sin(2*
d*x + 2*c)*sin(d*x + c) - ((2*a*b^4 + b^5)*cos(15*d*x + 15*c) + (2*a^2*b^3
 - 24*a*b^4 - 5*b^5)*cos(13*d*x + 13*c) - (70*a^2*b^3 - 76*a*b^4 - 9*b^5)*
cos(11*d*x + 11*c) + (96*a^3*b^2 + 164*a^2*b^3 - 54*a*b^4 - 5*b^5)*cos(9*d
*x + 9*c) + (96*a^3*b^2 + 164*a^2*b^3 - 54*a*b^4 - 5*b^5)*cos(7*d*x + 7*c)
 - (70*a^2*b^3 - 76*a*b^4 - 9*b^5)*cos(5*d*x + 5*c) + (2*a^2*b^3 - 24*a*b^
4 - 5*b^5)*cos(3*d*x + 3*c) + (2*a*b^4 + b^5)*cos(d*x + c))*cos(16*d*x + 1
6*c) - (2*a*b^4 + b^5 - 8*(2*a*b^4 + b^5)*cos(14*d*x + 14*c) - 4*(16*a^2*b
^3 - 6*a*b^4 - 7*b^5)*cos(12*d*x + 12*c) + 8*(32*a^2*b^3 + 2*a*b^4 - 7*b^5
)*cos(10*d*x + 10*c) + 2*(256*a^3*b^2 - 64*a^2*b^3 - 26*a*b^4 + 35*b^5)*co
s(8*d*x + 8*c) + 8*(32*a^2*b^3 + 2*a*b^4 - 7*b^5)*cos(6*d*x + 6*c) - 4*(16
*a^2*b^3 - 6*a*b^4 - 7*b^5)*cos(4*d*x + 4*c) - 8*(2*a*b^4 + b^5)*cos(2*d*x
 + 2*c))*cos(15*d*x + 15*c) + 8*((2*a^2*b^3 - 24*a*b^4 - 5*b^5)*cos(13*d*x
 + 13*c) - (70*a^2*b^3 - 76*a*b^4 - 9*b^5)*cos(11*d*x + 11*c) + (96*a^3*b^
2 + 164*a^2*b^3 - 54*a*b^4 - 5*b^5)*cos(9*d*x + 9*c) + (96*a^3*b^2 + 164*a
^2*b^3 - 54*a*b^4 - 5*b^5)*cos(7*d*x + 7*c) - (70*a^2*b^3 - 76*a*b^4 - 9*b
^5)*cos(5*d*x + 5*c) + (2*a^2*b^3 - 24*a*b^4 - 5*b^5)*cos(3*d*x + 3*c) + (
2*a*b^4 + b^5)*cos(d*x + c))*cos(14*d*x + 14*c) - (2*a^2*b^3 - 24*a*b^4 -
5*b^5 - 4*(16*a^3*b^2 - 206*a^2*b^3 + 128*a*b^4 + 35*b^5)*cos(12*d*x + ...
```

3.226.8 Giac [F]

$$\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \int -\frac{\sin(dx+c)^5}{\left(b\sin(dx+c)^4-a\right)^3}\,dx$$

input
```
integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.226.9 Mupad [B] (verification not implemented)

Time = 19.51 (sec) , antiderivative size = 6362, normalized size of antiderivative = 20.33

$$\int \frac{\sin^5(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^5/(a - b*sin(c + d*x)^4)^3,x)
```

output
```
- ((cos(c + d*x)^3*(14*a*b - 5*a^2 + 3*b^2))/(16*a*(a - b)^2) - (cos(c + d
*x)^5*(19*a*b - a^2 + 6*b^2))/(32*a*(a^2 - 2*a*b + b^2)) + (b*cos(c + d*x)
^7*(2*a + b))/(16*a*(a^2 - 2*a*b + b^2)) + (cos(c + d*x)*(15*a*b + 3*a^2 +
 2*b^2))/(32*a*b*(a - b)))/(d*(a^2 - 2*a*b + b^2 + cos(c + d*x)^2*(4*a*b -
 4*b^2) - cos(c + d*x)^4*(2*a*b - 6*b^2) - 4*b^2*cos(c + d*x)^6 + b^2*cos(
c + d*x)^8)) - (atan(((((16384*a^3*b^6 - 172032*a^4*b^5 + 319488*a^5*b^4 -
 188416*a^6*b^3 + 24576*a^7*b^2)/(16384*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b
^3 + 6*a^5*b^2)) - (cos(c + d*x)*((80*b^3*(a^9*b^5)^(1/2) - 9*a^3*(a^9*b^5
)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3
 - 301*a*b^2*(a^9*b^5)^(1/2) + 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^10
- 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2)*(16
384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^5 + 16384*a^7*b^
4))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((80*b^3*(a^9
*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b
^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^9*b^5)^(1/2) + 86*a^2*b*(a^9*b
^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10
*b^6 - a^11*b^5)))^(1/2) + (cos(c + d*x)*(9*a^4*b - 100*a*b^4 + 16*b^5 + 2
09*a^2*b^3 - 62*a^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^
4*b^2)))*((80*b^3*(a^9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 1
16*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^9*b^5...
```

3.227 $\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

3.227.1 Optimal result

Integrand size = 24, antiderivative size = 288

$$\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = -\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/4}d} + \frac{\left(5\sqrt{a}+2\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/4}d} - \frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} - \frac{\cos(c+dx)\left(11a+b-(5a+b)\cos^2(c+dx)\right)}{32a(a-b)^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)}$$

output

```
-1/8*cos(d*x+c)*(2-cos(d*x+c)^2)/(a-b)/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c
)^4)^2-1/32*cos(d*x+c)*(11*a+b-(5*a+b)*cos(d*x+c)^2)/a/(a-b)^2/d/(a-b+2*b*
cos(d*x+c)^2-b*cos(d*x+c)^4)-1/64*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/
2))^(1/2))*(5*a^(1/2)-2*b^(1/2))/a^(3/2)/b^(3/4)/d/(a^(1/2)-b^(1/2))^(5/2)
+1/64*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(5*a^(1/2)+2*b^(
1/2))/a^(3/2)/b^(3/4)/d/(a^(1/2)+b^(1/2))^(5/2)
```

3.227.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.14 (sec) , antiderivative size = 631, normalized size of antiderivative = 2.19

$$\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$$

$$= \frac{\frac{32\cos(c+dx)(-17a-b+(5a+b)\cos(2(c+dx)))}{a(8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx)))}+\frac{512(a-b)(-5\cos(c+dx)+\cos(3(c+dx)))}{(-8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))^2}+\frac{i\,\text{RootSum}\left[b-4b\#1^2-16a\#1^4+6b\#\right.}{}}{}$$

input
```
Integrate[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
((32*Cos[c + d*x]*(-17*a - b + (5*a + b)*Cos[2*(c + d*x)]))/(a*(8*a - 3*b
+ 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (512*(a - b)*(-5*Cos[c + d
*x] + Cos[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c +
 d*x)])^2 + (I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*
#1^8 & , (10*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 2*b*ArcTan[Sin[c
 + d*x]/(Cos[c + d*x] - #1)] - (5*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] -
 I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 94*a*ArcTan[Sin[c + d*x]/(Cos[c +
 d*x] - #1)]*#1^2 + 10*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (
47*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (5*I)*b*Log[1 - 2*Cos[c +
 d*x]*#1 + #1^2]*#1^2 + 94*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4
 - 10*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (47*I)*a*Log[1 - 2
*Cos[c + d*x]*#1 + #1^2]*#1^4 + (5*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*
#1^4 - 10*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - 2*b*ArcTan[Sin
[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + (5*I)*a*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2]*#1^6 + I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1
^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/a)/(256*(a - b)^2*d)
```

3.227.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3694, 1492, 27, 1492, 27, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.227. $\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

$$\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(c+dx)^3}{(a-b\sin(c+dx)^4)^3}dx$$

$$\Big\downarrow \; 3694$$

$$-\frac{\int \frac{1-\cos^2(c+dx)}{(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b)^3}d\cos(c+dx)}{d}$$

$$\Big\downarrow \; 1492$$

$$-\frac{\frac{\cos(c+dx)(2-\cos^2(c+dx))}{8(a-b)(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b)^2}-\frac{\int -\frac{2ab\left(6-5\cos^2(c+dx)\right)}{\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)^2}d\cos(c+dx)}{16ab(a-b)}}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{6-5\cos^2(c+dx)}{\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)^2}d\cos(c+dx)}{8(a-b)}+\frac{\cos(c+dx)(2-\cos^2(c+dx))}{8(a-b)(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b)^2}}{d}$$

$$\Big\downarrow \; 1492$$

$$-\frac{\frac{\frac{\cos(c+dx)\left(-\left((5a+b)\cos^2(c+dx)\right)+11a+b\right)}{4a(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\int -\frac{2b\left(-\left((5a+b)\cos^2(c+dx)\right)+13a-b\right)}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{8ab(a-b)}}{8(a-b)}+\frac{\cos(c+dx)(2-\cos^2(c+dx))}{8(a-b)(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b)^2}}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{\int \frac{-\left((5a+b)\cos^2(c+dx)\right)+13a-b}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{4a(a-b)}+\frac{\cos(c+dx)\left(-\left((5a+b)\cos^2(c+dx)\right)+11a+b\right)}{4a(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}}{8(a-b)}+\frac{\cos(c+dx)(2-\cos^2(c+dx))}{8(a-b)(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b)^2}}{d}$$

$$\Big\downarrow \; 1480$$

$$-\frac{\frac{\frac{-\frac{1}{2}\left(\frac{2\sqrt{b}(4a-b)}{\sqrt{a}}+5a+b\right)\int \frac{1}{-b\cos^2(c+dx)-\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}d\cos(c+dx)-\frac{1}{2}\left(-\frac{2\sqrt{b}(4a-b)}{\sqrt{a}}+5a+b\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos(c+dx)}{4a(a-b)}+\frac{\cos(c+dx)\left(-\left((5a \cdots\right.\right.}{4a(a-b)\left(a-b\cos^4 \cdots\right.}}{8(a-b)}}{d}$$

$$\Big\downarrow \; 218$$

3.227. $\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}dx$

$$-\frac{\frac{\frac{\left(\frac{2\sqrt{b}(4a-b)}{\sqrt{a}}+5a+b\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{1}{2}\left(-\frac{2\sqrt{b}(4a-b)}{\sqrt{a}}+5a+b\right)\int\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos(c+dx)}{4a(a-b)}+\frac{\cos(c+dx)\left(-\left((5a+b)\cos^2(c+dx)\right)+11\right.}{4a(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+d\right.}}{8(a-b)}}{d}$$

$\downarrow$ 221

$$-\frac{\frac{\frac{\frac{\left(\frac{2\sqrt{b}(4a-b)}{\sqrt{a}}+5a+b\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\left(-\frac{2\sqrt{b}(4a-b)}{\sqrt{a}}+5a+b\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}}{4a(a-b)}+\frac{\cos(c+dx)\left(-\left((5a+b)\cos^2(c+dx)\right)+11a+b\right)}{4a(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}}{8(a-b)}+\frac{}{8}}{d}$$

input
```
Int[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
-(((Cos[c + d*x]*(2 - Cos[c + d*x]^2))/(8*(a - b)*(a - b + 2*b*Cos[c + d*x
]^2 - b*Cos[c + d*x]^4)^2) + ((((5*a + (2*(4*a - b)*Sqrt[b])/Sqrt[a] + b)*
ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a] -
Sqrt[b]]*b^(3/4)) - ((5*a - (2*(4*a - b)*Sqrt[b])/Sqrt[a] + b)*ArcTanh[(b^
(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b
^(3/4)))/(4*a*(a - b)) + (Cos[c + d*x]*(11*a + b - (5*a + b)*Cos[c + d*x]^
2))/(4*a*(a - b)*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)))/(8*(a -
 b)))/d)
```

3.227.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.227. $\int\frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.227.4 Maple [A] (verified)

Time = 5.86 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{b^3\left(\frac{\frac{\left(-5a\sqrt{ab}-\sqrt{ab}\,b+8ab-2b^2\right)\left(\cos^3(dx+c)\right)}{4b^2\left(a^2-2ab+b^2\right)}+\frac{\left(7a-5\sqrt{ab}-2b\right)\cos(dx+c)}{4b^2(a-b)}}{\left(\cos^2(dx+c)-1-\frac{\sqrt{ab}}{b}\right)^2}+\frac{\left(5a\sqrt{ab}+\sqrt{ab}\,b-8ab+2b^2\right)\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{4b\left(a^2-2ab+b^2\right)\sqrt{\left(\sqrt{ab}+b\right)b}}}{16\sqrt{ab}\,a\,b^2}\right.}{d}$
default	$\frac{b^3\left(\frac{\frac{\left(-5a\sqrt{ab}-\sqrt{ab}\,b+8ab-2b^2\right)\left(\cos^3(dx+c)\right)}{4b^2\left(a^2-2ab+b^2\right)}+\frac{\left(7a-5\sqrt{ab}-2b\right)\cos(dx+c)}{4b^2(a-b)}}{\left(\cos^2(dx+c)-1-\frac{\sqrt{ab}}{b}\right)^2}+\frac{\left(5a\sqrt{ab}+\sqrt{ab}\,b-8ab+2b^2\right)\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{4b\left(a^2-2ab+b^2\right)\sqrt{\left(\sqrt{ab}+b\right)b}}}{16\sqrt{ab}\,a\,b^2}\right.}{d}$
risch	$-\frac{5ab\,e^{15i(dx+c)}+b^2e^{15i(dx+c)}-49ab\,e^{13i(dx+c)}-5b^2e^{13i(dx+c)}-144a^2e^{11i(dx+c)}+165ab\,e^{11i(dx+c)}+9b^2e^{11i(dx+c)}+78}{1}$

input
```
int(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*b^3*(1/16/(a*b)^(1/2)/a/b^2*((1/4*(-5*a*(a*b)^(1/2)-(a*b)^(1/2)*b+8*a*
b-2*b^2)/b^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3+1/4*(7*a-5*(a*b)^(1/2)-2*b)/b^2/
(a-b)*cos(d*x+c))/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2+1/4*(5*a*(a*b)^(1/2)+(a
*b)^(1/2)*b-8*a*b+2*b^2)/b/(a^2-2*a*b+b^2)/(((a*b)^(1/2)+b)*b)^(1/2)*arcta
nh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2)))-1/16/(a*b)^(1/2)/a/b^2*((1/4*(
5*a*(a*b)^(1/2)+(a*b)^(1/2)*b+8*a*b-2*b^2)/b^2/(a^2-2*a*b+b^2)*cos(d*x+c)^
3+1/4*(7*a+5*(a*b)^(1/2)-2*b)/b^2/(a-b)*cos(d*x+c))/(cos(d*x+c)^2+(a*b)^(1
/2)/b-1)^2+1/4*(5*a*(a*b)^(1/2)+(a*b)^(1/2)*b+8*a*b-2*b^2)/b/(a^2-2*a*b+b^
2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2)
)))
```

3.227. $\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

3.227.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4050 vs. $2(233) = 466$.

Time = 0.75 (sec) , antiderivative size = 4050, normalized size of antiderivative = 14.06

$$\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
-1/128*(4*(5*a*b + b^2)*cos(d*x + c)^7 - 12*(7*a*b + b^2)*cos(d*x + c)^5 -
 12*(3*a^2 - 10*a*b - b^2)*cos(d*x + c)^3 + ((a^3*b^2 - 2*a^2*b^3 + a*b^4)
*d*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(
a^4*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a
^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2
- 4*a^2*b^3 + a*b^4)*d)*sqrt(-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^
4 + 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 -
10780*a*b^3 + 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*
b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11
 - 10*a^4*b^12 + a^3*b^13)*d^4)) + 105*a^3 + 70*a^2*b - 35*a*b^2 + 4*b^3)/
((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))
*log((625*a^3 + 3750*a^2*b - 1491*a*b^2 + 140*b^3)*cos(d*x + c) + ((5*a^10
*b^2 - 16*a^9*b^3 + 3*a^8*b^4 + 50*a^7*b^5 - 85*a^6*b^6 + 60*a^5*b^7 - 19*
a^4*b^8 + 2*a^3*b^9)*d^3*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 1078
0*a*b^3 + 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6
+ 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 1
0*a^4*b^12 + a^3*b^13)*d^4)) - (325*a^5*b + 1977*a^4*b^2 - 609*a^3*b^3 + 3
5*a^2*b^4)*d)*sqrt(-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*
b^5 - a^3*b^6)*d^2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^
3 + 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 2...
```

3.227.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**3/(a-b*sin(d*x+c)**4)**3,x)
```

output
```
Timed out
```

3.227.7 Maxima [F]

$$\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \int -\frac{\sin(dx+c)^3}{(b\sin(dx+c)^4-a)^3}\,dx$$

input
```
integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
1/16*(8*(5*a*b^3 + b^4)*cos(2*d*x + 2*c)*cos(d*x + c) - 8*(49*a*b^3 + 5*b^
4)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c) + 8*(5*a*b^3 + b^4)*sin(2*d*x + 2*c)*
sin(d*x + c) - ((5*a*b^3 + b^4)*cos(15*d*x + 15*c) - (49*a*b^3 + 5*b^4)*co
s(13*d*x + 13*c) - 3*(48*a^2*b^2 - 55*a*b^3 - 3*b^4)*cos(11*d*x + 11*c) +
(784*a^2*b^2 - 377*a*b^3 - 5*b^4)*cos(9*d*x + 9*c) + (784*a^2*b^2 - 377*a*
b^3 - 5*b^4)*cos(7*d*x + 7*c) - 3*(48*a^2*b^2 - 55*a*b^3 - 3*b^4)*cos(5*d*
x + 5*c) - (49*a*b^3 + 5*b^4)*cos(3*d*x + 3*c) + (5*a*b^3 + b^4)*cos(d*x +
 c))*cos(16*d*x + 16*c) - (5*a*b^3 + b^4 - 8*(5*a*b^3 + b^4)*cos(14*d*x +
14*c) - 4*(40*a^2*b^2 - 27*a*b^3 - 7*b^4)*cos(12*d*x + 12*c) + 8*(80*a^2*b
^2 - 19*a*b^3 - 7*b^4)*cos(10*d*x + 10*c) + 2*(640*a^3*b - 352*a^2*b^2 + 7
9*a*b^3 + 35*b^4)*cos(8*d*x + 8*c) + 8*(80*a^2*b^2 - 19*a*b^3 - 7*b^4)*cos
(6*d*x + 6*c) - 4*(40*a^2*b^2 - 27*a*b^3 - 7*b^4)*cos(4*d*x + 4*c) - 8*(5*
a*b^3 + b^4)*cos(2*d*x + 2*c))*cos(15*d*x + 15*c) - 8*((49*a*b^3 + 5*b^4)*
cos(13*d*x + 13*c) + 3*(48*a^2*b^2 - 55*a*b^3 - 3*b^4)*cos(11*d*x + 11*c)
- (784*a^2*b^2 - 377*a*b^3 - 5*b^4)*cos(9*d*x + 9*c) - (784*a^2*b^2 - 377*
a*b^3 - 5*b^4)*cos(7*d*x + 7*c) + 3*(48*a^2*b^2 - 55*a*b^3 - 3*b^4)*cos(5*
d*x + 5*c) + (49*a*b^3 + 5*b^4)*cos(3*d*x + 3*c) - (5*a*b^3 + b^4)*cos(d*x
 + c))*cos(14*d*x + 14*c) + (49*a*b^3 + 5*b^4 - 4*(392*a^2*b^2 - 303*a*b^3
 - 35*b^4)*cos(12*d*x + 12*c) + 8*(784*a^2*b^2 - 263*a*b^3 - 35*b^4)*cos(1
0*d*x + 10*c) + 2*(6272*a^3*b - 4064*a^2*b^2 + 1235*a*b^3 + 175*b^4)*co...
```

3.227.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. 2(233) = 466.

Time = 1.65 (sec) , antiderivative size = 1076, normalized size of antiderivative = 3.74

$$\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx =$$

$$-\frac{\frac{5\,ab\cos(dx+c)^7}{d}+\frac{b^2\cos(dx+c)^7}{d}-\frac{21\,ab\cos(dx+c)^5}{d}-\frac{3\,b^2\cos(dx+c)^5}{d}-\frac{9\,a^2\cos(dx+c)^3}{d}+\frac{30\,ab\cos(dx+c)^3}{d}+\frac{3\,b^2\cos(dx+c)^3}{d}-}{32\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2-a+b\right)^2(a^3-2\,a^2b+ab}$$

$$-\frac{\left(2\left(4\,a^6b-17\,a^5b^2+28\,a^4b^3-22\,a^3b^4+8\,a^2b^5-ab^6\right)\sqrt{ab}\sqrt{-b^2+\sqrt{ab}b}d^4+(13\,a^4b-27\,a^3b^2+15\,a^2b\right.}{}$$

$$+\frac{\left(2\left(4\,a^6b-17\,a^5b^2+28\,a^4b^3-22\,a^3b^4+8\,a^2b^5-ab^6\right)\sqrt{-b^2-\sqrt{ab}b}d^4-(13\,a^3-27\,a^2b+15\,ab^2-b^3)\right.}{}$$

input
```
integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
-1/32*(5*a*b*cos(d*x + c)^7/d + b^2*cos(d*x + c)^7/d - 21*a*b*cos(d*x + c)
^5/d - 3*b^2*cos(d*x + c)^5/d - 9*a^2*cos(d*x + c)^3/d + 30*a*b*cos(d*x +
c)^3/d + 3*b^2*cos(d*x + c)^3/d + 19*a^2*cos(d*x + c)/d - 18*a*b*cos(d*x +
 c)/d - b^2*cos(d*x + c)/d)/((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 - a +
b)^2*(a^3 - 2*a^2*b + a*b^2)) - 1/64*(2*(4*a^6*b - 17*a^5*b^2 + 28*a^4*b^3
 - 22*a^3*b^4 + 8*a^2*b^5 - a*b^6)*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*d^4
+ (13*a^4*b - 27*a^3*b^2 + 15*a^2*b^3 - a*b^4)*sqrt(-b^2 + sqrt(a*b)*b)*d^
2*abs(a^3*d^2 - 2*a^2*b*d^2 + a*b^2*d^2) + (a^3*d^2 - 2*a^2*b*d^2 + a*b^2*
d^2)^2*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*(5*a + b))*arctan(cos(d*x + c)/(
d*sqrt(-(a^3*b*d^2 - 2*a^2*b^2*d^2 + a*b^3*d^2 - sqrt((a^3*b*d^2 - 2*a^2*b
^2*d^2 + a*b^3*d^2)^2 + (a^3*b*d^4 - 2*a^2*b^2*d^4 + a*b^3*d^4)*(a^4 - 3*a
^3*b + 3*a^2*b^2 - a*b^3)))/(a^3*b*d^4 - 2*a^2*b^2*d^4 + a*b^3*d^4))))/((a
^7*b - 5*a^6*b^2 + 10*a^5*b^3 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^3*abs(
a^3*d^2 - 2*a^2*b*d^2 + a*b^2*d^2)*abs(b)) + 1/64*(2*(4*a^6*b - 17*a^5*b^2
 + 28*a^4*b^3 - 22*a^3*b^4 + 8*a^2*b^5 - a*b^6)*sqrt(-b^2 - sqrt(a*b)*b)*d
^4 - (13*a^3 - 27*a^2*b + 15*a*b^2 - b^3)*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*
b)*d^2*abs(a^3*d^2 - 2*a^2*b*d^2 + a*b^2*d^2) + (a^3*d^2 - 2*a^2*b*d^2 + a
*b^2*d^2)^2*sqrt(-b^2 - sqrt(a*b)*b)*(5*a + b))*arctan(cos(d*x + c)/(d*sqr
t(-(a^3*b*d^2 - 2*a^2*b^2*d^2 + a*b^3*d^2 + sqrt((a^3*b*d^2 - 2*a^2*b^2*d^
2 + a*b^3*d^2)^2 + (a^3*b*d^4 - 2*a^2*b^2*d^4 + a*b^3*d^4)*(a^4 - 3*a^3...
```

3.227.9 Mupad [B] (verification not implemented)

Time = 18.40 (sec) , antiderivative size = 5566, normalized size of antiderivative = 19.33

$$\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^3/(a - b*sin(c + d*x)^4)^3,x)
```

output
```
- (atan(((((16384*a^3*b^6 - 245760*a^4*b^5 + 442368*a^5*b^4 - 212992*a^6*b
^3)/(32768*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)) - (cos(c + d
*x)*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4
*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8
 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2)*(1
6384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^5 + 16384*a^7*b
^4))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((35*b^2*(a^
9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^
3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 1
0*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2) + (cos(c + d*x)*(4
*b^5 - 31*a*b^4 + 74*a^2*b^3 + 25*a^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b^4
- 4*a^3*b^3 + 6*a^4*b^2)))*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/
2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)
^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4
 - a^11*b^3)))^(1/2)*1i - (((16384*a^3*b^6 - 245760*a^4*b^5 + 442368*a^5*b
^4 - 212992*a^6*b^3)/(32768*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b
^2)) + (cos(c + d*x)*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4
*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2)
)/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^1
1*b^3)))^(1/2)*(16384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a...
```

3.227. $\int \frac{\sin^3(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

3.228 $\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

3.228.1 Optimal result

Integrand size = 22, antiderivative size = 313

$$\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = -\frac{3\left(7a-10\sqrt{a}\sqrt{b}+4b\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt[4]{b}d} - \frac{3\left(7a+10\sqrt{a}\sqrt{b}+4b\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt[4]{b}d} - \frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} - \frac{\cos(c+dx)\left((7a-3b)(a+2b)-6(2a-b)b\cos^2(c+dx)\right)}{32a^2(a-b)^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)}$$

output
```
-1/8*cos(d*x+c)*(a+b-b*cos(d*x+c)^2)/a/(a-b)/d/(a-b+2*b*cos(d*x+c)^2-b*cos
(d*x+c)^4)^2-1/32*cos(d*x+c)*((7*a-3*b)*(a+2*b)-6*(2*a-b)*b*cos(d*x+c)^2)/
a^2/(a-b)^2/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)-3/64*arctan(b^(1/4)*co
s(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(7*a+4*b-10*a^(1/2)*b^(1/2))/a^(5/2)/b^(
1/4)/d/(a^(1/2)-b^(1/2))^(5/2)-3/64*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^
(1/2))^(1/2))*(7*a+4*b+10*a^(1/2)*b^(1/2))/a^(5/2)/b^(1/4)/d/(a^(1/2)+b^(1
/2))^(5/2)
```

3.228.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.65 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.50

$$\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$$

$$= \frac{-\frac{32\cos(c+dx)(7a^2+5ab-3b^2+3b(-2a+b)\cos(2(c+dx)))}{8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx))} - \frac{512a(a-b)\cos(c+dx)(2a+b-b\cos(2(c+dx)))}{(-8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))^2} + 3i\text{RootSum}\left[b-4b\#1\right.}{}$$

input
```
Integrate[Sin[c + d*x]/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
((-32*Cos[c + d*x]*(7*a^2 + 5*a*b - 3*b^2 + 3*b*(-2*a + b)*Cos[2*(c + d*x)
]))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]) - (512*a*(a -
b)*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c
 + d*x)] + b*Cos[4*(c + d*x)])^2 + (3*I)*RootSum[b - 4*b*#1^2 - 16*a*#1^4
+ 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x
] - #1)] - 2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - (2*I)*a*b*Log[
1 - 2*Cos[c + d*x]*#1 + #1^2] + I*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] -
28*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 24*a*b*ArcTan[Sin[c
 + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*
x] - #1)]*#1^2 + (14*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (12*I
)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (5*I)*b^2*Log[1 - 2*Cos[c +
 d*x]*#1 + #1^2]*#1^2 + 28*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1
^4 - 24*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 10*b^2*ArcTan[
Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (14*I)*a^2*Log[1 - 2*Cos[c + d*x]
*#1 + #1^2]*#1^4 + (12*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - (5*
I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - 4*a*b*ArcTan[Sin[c + d*x]/
(Cos[c + d*x] - #1)]*#1^6 + 2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^6 + (2*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6 - I*b^2*Log[1 - 2
*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 +
 b*#1^7) & ])/(128*a^2*(a - b)^2*d)
```

3.228.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {3042, 3694, 1405, 27, 1492, 27, 1480, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(c+dx)}{\left(a-b\sin(c+dx)^4\right)^3}dx$$

$$\Big\downarrow \ 3694$$

$$-\frac{\int \frac{1}{\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)^3}d\cos(c+dx)}{d}$$

$$\Big\downarrow \ 1405$$

$$-\frac{\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8a(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}-\frac{\int -\frac{2b\left(-5b\cos^2(c+dx)+7a-b\right)}{\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)^2}d\cos(c+dx)}{16ab(a-b)}}{d}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\int \frac{-5b\cos^2(c+dx)+7a-b}{\left(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b\right)^2}d\cos(c+dx)}{8a(a-b)}+\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8a(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}}{d}$$

$$\Big\downarrow \ 1492$$

$$-\frac{\frac{\frac{\cos(c+dx)\left((7a-3b)(a+2b)-6b(2a-b)\cos^2(c+dx)\right)}{4a(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\int -\frac{6b\left(7a^2-5ba+2b^2-2(2a-b)b\cos^2(c+dx)\right)}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{8ab(a-b)}}{8a(a-b)}+\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8a(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+d\right.}}{d}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{3\int \frac{7a^2-5ba+2b^2-2(2a-b)b\cos^2(c+dx)}{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}d\cos(c+dx)}{4a(a-b)}+\frac{\cos(c+dx)\left((7a-3b)(a+2b)-6b(2a-b)\cos^2(c+dx)\right)}{4a(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}}{8a(a-b)}+\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8a(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right.}}{d}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{\frac{3\left(\frac{\sqrt{b}\left(-2\sqrt{a}\sqrt{b}+a+b\right)\left(10\sqrt{a}\sqrt{b}+7a+4b\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos(c+dx)}{2\sqrt{a}}-\frac{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)^2\left(-10\sqrt{a}\sqrt{b}+7a+4b\right)\int \frac{1}{-b\cos^2(c+dx)-\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}d\cos\cdots}{2\sqrt{a}}\cdots\right.}{4a(a-b)}\cdots}{8a(a-b)}\cdots}{d}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{3\left(\frac{\sqrt{b}\left(-2\sqrt{a}\sqrt{b}+a+b\right)\left(10\sqrt{a}\sqrt{b}+7a+4b\right)\int \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}-b\cos^2(c+dx)}d\cos(c+dx)}{2\sqrt{a}}+\frac{\left(-10\sqrt{a}\sqrt{b}+7a+4b\right)\left(\sqrt{a}+\sqrt{b}\right)^2\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{4a(a-b)}+\frac{\cos(c+dx\cdots}{4a(a-\cdots}}{8a(a-b)}\cdots}{d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{3\left(\frac{\left(-10\sqrt{a}\sqrt{b}+7a+4b\right)\left(\sqrt{a}+\sqrt{b}\right)^2\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\left(-2\sqrt{a}\sqrt{b}+a+b\right)\left(10\sqrt{a}\sqrt{b}+7a+4b\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{4a(a-b)}+\frac{\cos(c+dx)\left((7a-3b)(a+\cdots\right.}{4a(a-b)\left(a-b\cos^4(c\cdots\right.}}{8a(a-b)}\cdots}{d}$$

input
```
Int[Sin[c + d*x]/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
-(((Cos[c + d*x]*(a + b - b*Cos[c + d*x]^2))/(8*a*(a - b)*(a - b + 2*b*Cos
[c + d*x]^2 - b*Cos[c + d*x]^4)^2) + ((3*(((Sqrt[a] + Sqrt[b])^2*(7*a - 10
*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[
b]]])/(2*Sqrt[a]*Sqrt[Sqrt[a] - Sqrt[b]]*b^(1/4)) + ((a - 2*Sqrt[a]*Sqrt[b
] + b)*(7*a + 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqr
t[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4))))/(4*a*
(a - b)) + (Cos[c + d*x]*((7*a - 3*b)*(a + 2*b) - 6*(2*a - b)*b*Cos[c + d*
x]^2))/(4*a*(a - b)*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)))/(8*a
*(a - b)))/d)
```

3.228. $\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

3.228.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.228. $\int \frac{\sin(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.228.4 Maple [A] (verified)

Time = 6.15 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.37

method	result
derivativedivides	$b^3\left(\frac{\frac{\frac{3\left(-4a\sqrt{ab}+2\sqrt{ab}\,b+3a^2-ab\right)\left(\cos^3(dx+c)\right)}{4b^2\left(a^2-2ab+b^2\right)}+\frac{\left(-11a\sqrt{ab}+6\sqrt{ab}\,b+5ab\right)\cos(dx+c)}{4b^3(a-b)}}{\left(\cos^2(dx+c)-1-\frac{\sqrt{ab}}{b}\right)^2}+\frac{3\left(4a\sqrt{ab}-2\sqrt{ab}\,b-7a^2+9ab-4b^2\right)\operatorname{arctanh}\ldots}{4b\left(a^2-2ab+b^2\right)\sqrt{\left(\sqrt{ab}+b\ldots\right.}}}{16b\,a^2\sqrt{ab}}\ldots\right.$
default	$b^3\left(\frac{\frac{\frac{3\left(-4a\sqrt{ab}+2\sqrt{ab}\,b+3a^2-ab\right)\left(\cos^3(dx+c)\right)}{4b^2\left(a^2-2ab+b^2\right)}+\frac{\left(-11a\sqrt{ab}+6\sqrt{ab}\,b+5ab\right)\cos(dx+c)}{4b^3(a-b)}}{\left(\cos^2(dx+c)-1-\frac{\sqrt{ab}}{b}\right)^2}+\frac{3\left(4a\sqrt{ab}-2\sqrt{ab}\,b-7a^2+9ab-4b^2\right)\operatorname{arctanh}\ldots}{4b\left(a^2-2ab+b^2\right)\sqrt{\left(\sqrt{ab}+b\ldots\right.}}}{16b\,a^2\sqrt{ab}}\ldots\right.$
risch	Expression too large to display

input
```
int(sin(d*x+c)/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*b^3*(1/16/b/a^2/(a*b)^(1/2)*((3/4*(-4*a*(a*b)^(1/2)+2*(a*b)^(1/2)*b+3*
a^2-a*b)/b^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3+1/4*(-11*a*(a*b)^(1/2)+6*(a*b)^(
1/2)*b+5*a*b)/b^3/(a-b)*cos(d*x+c))/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2+3/4*(
4*a*(a*b)^(1/2)-2*(a*b)^(1/2)*b-7*a^2+9*a*b-4*b^2)/b/(a^2-2*a*b+b^2)/(((a*
b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2)))-1/16
/b/a^2/(a*b)^(1/2)*((3/4*(4*a*(a*b)^(1/2)-2*(a*b)^(1/2)*b+3*a^2-a*b)/b^2/(
a^2-2*a*b+b^2)*cos(d*x+c)^3+1/4*(11*a*(a*b)^(1/2)-6*(a*b)^(1/2)*b+5*a*b)/b
^3/(a-b)*cos(d*x+c))/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2+3/4*(4*a*(a*b)^(1/2)
-2*(a*b)^(1/2)*b+7*a^2-9*a*b+4*b^2)/b/(a^2-2*a*b+b^2)/(((a*b)^(1/2)-b)*b)^
(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))))
```

3.228.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4160 vs. $2(263) = 526$.

Time = 0.89 (sec) , antiderivative size = 4160, normalized size of antiderivative = 13.29

$$\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
-1/128*(24*(2*a*b^2 - b^3)*cos(d*x + c)^7 - 4*(7*a^2*b + 35*a*b^2 - 18*b^3
)*cos(d*x + c)^5 - 8*(a^2*b - 22*a*b^2 + 9*b^3)*cos(d*x + c)^3 + 3*((a^4*b
^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*
b^4)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*co
s(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^
2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(105*a^4 -
210*a^3*b + 189*a^2*b^2 - 84*a*b^3 + 16*b^4 + (a^10 - 5*a^9*b + 10*a^8*b^2
 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^3*b + 497
4*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 1
20*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45
*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 -
10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))*log(27*(2401*a^4 - 4802*a^3*b + 41
89*a^2*b^2 - 1788*a*b^3 + 336*b^4)*cos(d*x + c) - 27*((11*a^12*b - 66*a^11
*b^2 + 169*a^10*b^3 - 240*a^9*b^4 + 205*a^8*b^5 - 106*a^7*b^6 + 31*a^6*b^7
 - 4*a^5*b^8)*d^3*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3
+ 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*
b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10
+ a^5*b^11)*d^4)) - (343*a^7 - 623*a^6*b + 515*a^5*b^2 - 213*a^4*b^3 + 42*
a^3*b^4)*d)*sqrt(-(105*a^4 - 210*a^3*b + 189*a^2*b^2 - 84*a*b^3 + 16*b^4 +
 (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*s...
```

3.228.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)/(a-b*sin(d*x+c)**4)**3,x)
```

output
```
Timed out
```

3.228.7 Maxima [F]

$$\int \frac{\sin(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \int -\frac{\sin(dx+c)}{(b\sin(dx+c)^4-a)^3}\,dx$$

input
```
integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
1/8*(24*(2*a*b^4 - b^5)*cos(2*d*x + 2*c)*cos(d*x + c) - 8*(14*a^2*b^3 + 28
*a*b^4 - 15*b^5)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c) + 24*(2*a*b^4 - b^5)*si
n(2*d*x + 2*c)*sin(d*x + c) - (3*(2*a*b^4 - b^5)*cos(15*d*x + 15*c) - (14*
a^2*b^3 + 28*a*b^4 - 15*b^5)*cos(13*d*x + 13*c) - (86*a^2*b^3 - 128*a*b^4
+ 27*b^5)*cos(11*d*x + 11*c) + (352*a^3*b^2 - 60*a^2*b^3 - 106*a*b^4 + 15*
b^5)*cos(9*d*x + 9*c) + (352*a^3*b^2 - 60*a^2*b^3 - 106*a*b^4 + 15*b^5)*co
s(7*d*x + 7*c) - (86*a^2*b^3 - 128*a*b^4 + 27*b^5)*cos(5*d*x + 5*c) - (14*
a^2*b^3 + 28*a*b^4 - 15*b^5)*cos(3*d*x + 3*c) + 3*(2*a*b^4 - b^5)*cos(d*x
+ c))*cos(16*d*x + 16*c) - 3*(2*a*b^4 - b^5 - 8*(2*a*b^4 - b^5)*cos(14*d*x
 + 14*c) - 4*(16*a^2*b^3 - 22*a*b^4 + 7*b^5)*cos(12*d*x + 12*c) + 8*(32*a^
2*b^3 - 30*a*b^4 + 7*b^5)*cos(10*d*x + 10*c) + 2*(256*a^3*b^2 - 320*a^2*b^
3 + 166*a*b^4 - 35*b^5)*cos(8*d*x + 8*c) + 8*(32*a^2*b^3 - 30*a*b^4 + 7*b^
5)*cos(6*d*x + 6*c) - 4*(16*a^2*b^3 - 22*a*b^4 + 7*b^5)*cos(4*d*x + 4*c) -
 8*(2*a*b^4 - b^5)*cos(2*d*x + 2*c))*cos(15*d*x + 15*c) - 8*((14*a^2*b^3 +
 28*a*b^4 - 15*b^5)*cos(13*d*x + 13*c) + (86*a^2*b^3 - 128*a*b^4 + 27*b^5)
*cos(11*d*x + 11*c) - (352*a^3*b^2 - 60*a^2*b^3 - 106*a*b^4 + 15*b^5)*cos(
9*d*x + 9*c) - (352*a^3*b^2 - 60*a^2*b^3 - 106*a*b^4 + 15*b^5)*cos(7*d*x +
 7*c) + (86*a^2*b^3 - 128*a*b^4 + 27*b^5)*cos(5*d*x + 5*c) + (14*a^2*b^3 +
 28*a*b^4 - 15*b^5)*cos(3*d*x + 3*c) - 3*(2*a*b^4 - b^5)*cos(d*x + c))*cos
(14*d*x + 14*c) + (14*a^2*b^3 + 28*a*b^4 - 15*b^5 - 4*(112*a^3*b^2 + 12...
```

3.228.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(263) = 526$.

Time = 1.96 (sec) , antiderivative size = 793, normalized size of antiderivative = 2.53

$$\int \frac{\sin(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$$

$$= \frac{3\left(4a^2b-2ab^2-(7a^2-9ab+4b^2)\sqrt{ab}\right)\sqrt{-b^2-\sqrt{ab}b}\arctan\left(\frac{\cos}{d\sqrt{-\frac{a^4bd^2-2a^3b^2d^2+a^2b^3d^2+\sqrt{\left(a^4bd^2-2a^3b^2d^2+a^2}}{a^4bd^4}}}\right.}{64\left(a^5b-2a^4b^2+a^3b^3+(a^5-2a^4b+a^3b^2)\sqrt{ab}\right)d|b|}$$

$$+ \frac{3\left(4a^2b-2ab^2+(7a^2-9ab+4b^2)\sqrt{ab}\right)\sqrt{-b^2+\sqrt{ab}b}\arctan\left(\frac{}{d\sqrt{-\frac{a^4bd^2-2a^3b^2d^2+a^2b^3d^2-\sqrt{\left(a^4bd^2-2a^3b^2d^2+}}{a^4b}}}\right.}{64\left(a^5b-2a^4b^2+a^3b^3-(a^5-2a^4b+a^3b^2)\sqrt{ab}\right)d|b|}$$

$$- \frac{\frac{12ab^2\cos(dx+c)^7}{d}-\frac{6b^3\cos(dx+c)^7}{d}-\frac{7a^2b\cos(dx+c)^5}{d}-\frac{35ab^2\cos(dx+c)^5}{d}+\frac{18b^3\cos(dx+c)^5}{d}-\frac{2a^2b\cos(dx+c)^3}{d}+\frac{44ab^2\cos(}{d}}{32\left(b\cos(dx+c)^4-2b\cos(dx+c)^2-a+\right.}$$

input
```
integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
3/64*(4*a^2*b - 2*a*b^2 - (7*a^2 - 9*a*b + 4*b^2)*sqrt(a*b))*sqrt(-b^2 - s
qrt(a*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(a^4*b*d^2 - 2*a^3*b^2*d^2 + a^2*
b^3*d^2 + sqrt((a^4*b*d^2 - 2*a^3*b^2*d^2 + a^2*b^3*d^2)^2 + (a^4*b*d^4 -
2*a^3*b^2*d^4 + a^2*b^3*d^4)*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)))/(a^4*
b*d^4 - 2*a^3*b^2*d^4 + a^2*b^3*d^4))))/((a^5*b - 2*a^4*b^2 + a^3*b^3 + (a
^5 - 2*a^4*b + a^3*b^2)*sqrt(a*b))*d*abs(b)) + 3/64*(4*a^2*b - 2*a*b^2 + (
7*a^2 - 9*a*b + 4*b^2)*sqrt(a*b))*sqrt(-b^2 + sqrt(a*b)*b)*arctan(cos(d*x
+ c)/(d*sqrt(-(a^4*b*d^2 - 2*a^3*b^2*d^2 + a^2*b^3*d^2 - sqrt((a^4*b*d^2 -
 2*a^3*b^2*d^2 + a^2*b^3*d^2)^2 + (a^4*b*d^4 - 2*a^3*b^2*d^4 + a^2*b^3*d^4
)*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)))/(a^4*b*d^4 - 2*a^3*b^2*d^4 + a^2
*b^3*d^4))))/((a^5*b - 2*a^4*b^2 + a^3*b^3 - (a^5 - 2*a^4*b + a^3*b^2)*sqr
t(a*b))*d*abs(b)) - 1/32*(12*a*b^2*cos(d*x + c)^7/d - 6*b^3*cos(d*x + c)^7
/d - 7*a^2*b*cos(d*x + c)^5/d - 35*a*b^2*cos(d*x + c)^5/d + 18*b^3*cos(d*x
 + c)^5/d - 2*a^2*b*cos(d*x + c)^3/d + 44*a*b^2*cos(d*x + c)^3/d - 18*b^3*
cos(d*x + c)^3/d + 11*a^3*cos(d*x + c)/d + 4*a^2*b*cos(d*x + c)/d - 21*a*b
^2*cos(d*x + c)/d + 6*b^3*cos(d*x + c)/d)/((b*cos(d*x + c)^4 - 2*b*cos(d*x
 + c)^2 - a + b)^2*(a^4 - 2*a^3*b + a^2*b^2))
```

3.228.9 Mupad [B] (verification not implemented)

Time = 18.28 (sec) , antiderivative size = 5753, normalized size of antiderivative = 18.38

$$\int \frac{\sin(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)/(a - b*sin(c + d*x)^4)^3,x)
```

output
```
(atan(((((3*(16384*a^5*b^7 - 73728*a^6*b^6 + 155648*a^7*b^5 - 155648*a^8*b
^4 + 57344*a^9*b^3))/(16384*(a^10 - 4*a^9*b + a^6*b^4 - 4*a^7*b^3 + 6*a^8*
b^2)) - (cos(c + d*x)*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) -
 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*
(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10
*a^13*b^3 - 5*a^14*b^2)))^(1/2)*(16384*a^5*b^8 - 65536*a^6*b^7 + 98304*a^7
*b^6 - 65536*a^8*b^5 + 16384*a^9*b^4))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*a
^5*b^3 + 6*a^6*b^2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) -
 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*
(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10
*a^13*b^3 - 5*a^14*b^2)))^(1/2) + (cos(c + d*x)*(144*b^7 - 612*a*b^6 + 108
9*a^2*b^5 - 990*a^3*b^4 + 441*a^4*b^3))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*
a^5*b^3 + 6*a^6*b^2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2)
- 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b
*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 1
0*a^13*b^3 - 5*a^14*b^2)))^(1/2)*1i - (((3*(16384*a^5*b^7 - 73728*a^6*b^6
+ 155648*a^7*b^5 - 155648*a^8*b^4 + 57344*a^9*b^3))/(16384*(a^10 - 4*a^9*b
 + a^6*b^4 - 4*a^7*b^3 + 6*a^8*b^2)) + (cos(c + d*x)*((9*(49*a^2*(a^15*b)^
(1/2) + 21*b^2*(a^15*b)^(1/2) - 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*
a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b...
```

3.229 $\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

3.229.1 Optimal result

Integrand size = 22, antiderivative size = 617

$$\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = -\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d} - \frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} - \frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}-\sqrt{b}}d} - \frac{\operatorname{arctanh}(\cos(c+dx))}{a^3d} + \frac{\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{5/2}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d} + \frac{\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}+\sqrt{b}}d} + \frac{\left(5\sqrt{a}+2\sqrt{b}\right)\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}d} - \frac{b\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} - \frac{b\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4a^2(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} - \frac{b\cos(c+dx)\left(11a+b-(5a+b)\cos^2(c+dx)\right)}{32a^2(a-b)^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)}$$

output
```
-arctanh(cos(d*x+c))/a^3/d-1/8*b*cos(d*x+c)*(2-cos(d*x+c)^2)/a/(a-b)/d/(a-
b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)^2-1/4*b*cos(d*x+c)*(2-cos(d*x+c)^2)/a^2
/(a-b)/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)-1/32*b*cos(d*x+c)*(11*a+b-(
5*a+b)*cos(d*x+c)^2)/a^2/(a-b)^2/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)-1
/64*b^(1/4)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(5*a^(1/2)-
2*b^(1/2))/a^(5/2)/d/(a^(1/2)-b^(1/2))^(5/2)-1/8*b^(1/4)*arctan(b^(1/4)*co
s(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^(5/2)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*b^
(1/4)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^(5/2)/d/(a^(1/
2)+b^(1/2))^(3/2)+1/64*b^(1/4)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2)
)^(1/2))*(5*a^(1/2)+2*b^(1/2))/a^(5/2)/d/(a^(1/2)+b^(1/2))^(5/2)-1/2*b^(1/
4)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^3/d/(a^(1/2)-b^(1/
2))^(1/2)+1/2*b^(1/4)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/
a^3/d/(a^(1/2)+b^(1/2))^(1/2)
```

3.229.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 10.95 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.49

$$\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$$

$$= \frac{\frac{32ab\cos(c+dx)(-41a+23b+(13a-7b)\cos(2(c+dx)))}{(a-b)^2(8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx)))}+\frac{512a^2b(-5\cos(c+dx)+\cos(3(c+dx)))}{(a-b)(-8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))^2}-256\log\left(\cos\left(\frac{1}{2}(c+dx\right)\right.}{}$$

input
```
Integrate[Csc[c + d*x]/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
((32*a*b*Cos[c + d*x]*(-41*a + 23*b + (13*a - 7*b)*Cos[2*(c + d*x)]))/((a
- b)^2*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (512*a^2
*b*(-5*Cos[c + d*x] + Cos[3*(c + d*x)]))/((a - b)*(-8*a + 3*b - 4*b*Cos[2*
(c + d*x)] + b*Cos[4*(c + d*x)])^2) - 256*Log[Cos[(c + d*x)/2]] + 256*Log[
Sin[(c + d*x)/2]] - (I*b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b
*#1^6 + b*#1^8 & , (-90*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 142
*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - 64*b^2*ArcTan[Sin[c + d*x]
/(Cos[c + d*x] - #1)] + (45*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - (71
*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (32*I)*b^2*Log[1 - 2*Cos[c + d
*x]*#1 + #1^2] + 398*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 5
06*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 192*b^2*ArcTan[Sin[
c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (199*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2]*#1^2 + (253*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (96*I
)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 398*a^2*ArcTan[Sin[c + d*x]
/(Cos[c + d*x] - #1)]*#1^4 + 506*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #
1)]*#1^4 - 192*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (199*I)
*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - (253*I)*a*b*Log[1 - 2*Cos[c
+ d*x]*#1 + #1^2]*#1^4 + (96*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4
 + 90*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - 142*a*b*ArcTan[S
in[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + 64*b^2*ArcTan[Sin[c + d*x]/(Cos...
```

3.229.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 591, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 3694, 1567, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(c+dx)\left(a-b\sin(c+dx)^4\right)^3}dx$$

$$\Big\downarrow \; 3694$$

$$-\frac{\int \frac{1}{(1-\cos^2(c+dx))(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b)^3}d\cos(c+dx)}{d}$$

$$\Big\downarrow \; 1567$$

$$-\frac{\int \left(\frac{b-b\cos^2(c+dx)}{a^3(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b)}+\frac{b-b\cos^2(c+dx)}{a^2(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b)^2}+\frac{b-b\cos^2(c+dx)}{a(-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b)^3}-\frac{}{a^3}\right.}{d}$$

$\downarrow$ 2009

$$-\frac{\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{b}\left(5\sqrt{a}-2\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{\sqrt[4]{b}\left(5\sqrt{a}+2\sqrt{b}\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\sqrt[4]{b}\text{arctanh}}{8a^{5/2}(}}{}$$

input
```
Int[Csc[c + d*x]/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
-((((5*Sqrt[a] - 2*Sqrt[b])*b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqr
t[a] - Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)) + (b^(1/4)*ArcTan
[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*a^(5/2)*(Sqrt[a] - Sq
rt[b])^(3/2)) + (b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(2*a^3*Sqrt[Sqrt[a] - Sqrt[b]]) + ArcTanh[Cos[c + d*x]]/a^3 - (b^(1
/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*a^(5/2)*(S
qrt[a] + Sqrt[b])^(3/2)) - (b^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sq
rt[a] + Sqrt[b]]])/(2*a^3*Sqrt[Sqrt[a] + Sqrt[b]]) - ((5*Sqrt[a] + 2*Sqrt[
b])*b^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*a
^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)) + (b*Cos[c + d*x]*(2 - Cos[c + d*x]^2))/
(8*a*(a - b)*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)^2) + (b*Cos[c
 + d*x]*(2 - Cos[c + d*x]^2))/(4*a^2*(a - b)*(a - b + 2*b*Cos[c + d*x]^2 -
 b*Cos[c + d*x]^4)) + (b*Cos[c + d*x]*(11*a + b - (5*a + b)*Cos[c + d*x]^2
))/(32*a^2*(a - b)^2*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)))/d)
```

3.229.3.1 Defintions of rubi rules used

rule 1567
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.229.4 Maple [A] (verified)

Time = 7.50 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.62

method	result
derivativedivides	$b\left(\frac{-\frac{ab(13a-7b)\left(\cos^7(dx+c)\right)}{32\left(a^2-2ab+b^2\right)}+\frac{(53a-29b)ab\left(\cos^5(dx+c)\right)}{32a^2-64ab+32b^2}+\frac{a\left(17a^2-78ab+37b^2\right)\left(\cos^3(dx+c)\right)}{32a^2-64ab+32b^2}-\frac{5(7a-3b)a\cos(dx+c)}{32(a-b)}}{\left(a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)\right)^2}+b\left(\frac{\left(-45\right.}{}\right.\right.$
default	$b\left(\frac{-\frac{ab(13a-7b)\left(\cos^7(dx+c)\right)}{32\left(a^2-2ab+b^2\right)}+\frac{(53a-29b)ab\left(\cos^5(dx+c)\right)}{32a^2-64ab+32b^2}+\frac{a\left(17a^2-78ab+37b^2\right)\left(\cos^3(dx+c)\right)}{32a^2-64ab+32b^2}-\frac{5(7a-3b)a\cos(dx+c)}{32(a-b)}}{\left(a-b+2b\left(\cos^2(dx+c)\right)-b\left(\cos^4(dx+c)\right)\right)^2}+b\left(\frac{\left(-45\right.}{}\right.\right.$
risch	Expression too large to display

input
```
int(csc(d*x+c)/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b/a^3*((-1/32*a*b*(13*a-7*b)/(a^2-2*a*b+b^2)*cos(d*x+c)^7+1/32*(53*a-
29*b)*a*b/(a^2-2*a*b+b^2)*cos(d*x+c)^5+1/32*a*(17*a^2-78*a*b+37*b^2)/(a^2-
2*a*b+b^2)*cos(d*x+c)^3-5/32*(7*a-3*b)*a/(a-b)*cos(d*x+c))/(a-b+2*b*cos(d*
x+c)^2-b*cos(d*x+c)^4)^2+1/32/(a^2-2*a*b+b^2)*b*(1/2*(-45*a^2*(a*b)^(1/2)+
71*a*b*(a*b)^(1/2)-32*b^2*(a*b)^(1/2)-16*a^2*b+10*a*b^2)/(a*b)^(1/2)/b/(((
a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/2*
(-45*a^2*(a*b)^(1/2)+71*a*b*(a*b)^(1/2)-32*b^2*(a*b)^(1/2)+16*a^2*b-10*a*b
^2)/(a*b)^(1/2)/b/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(
1/2)+b)*b)^(1/2))))-1/2/a^3*ln(1+cos(d*x+c))+1/2/a^3*ln(cos(d*x+c)-1))
```

3.229.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5020 vs. $2(481) = 962$.

Time = 2.20 (sec) , antiderivative size = 5020, normalized size of antiderivative = 8.14

$$\int \frac{\csc(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.229.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)/(a-b*sin(d*x+c)**4)**3,x)
```

output
```
Timed out
```

3.229.7 Maxima [F]

$$\int \frac{\csc(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \int -\frac{\csc(dx+c)}{(b\sin(dx+c)^4-a)^3}\,dx$$

input
```
integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
1/16*(8*(13*a^2*b^4 - 7*a*b^5)*cos(2*d*x + 2*c)*cos(d*x + c) - 8*(121*a^2*
b^4 - 67*a*b^5)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c) + 8*(13*a^2*b^4 - 7*a*b^
5)*sin(2*d*x + 2*c)*sin(d*x + c) - ((13*a^2*b^4 - 7*a*b^5)*cos(15*d*x + 15
*c) - (121*a^2*b^4 - 67*a*b^5)*cos(13*d*x + 13*c) - (272*a^3*b^3 - 461*a^2
*b^4 + 159*a*b^5)*cos(11*d*x + 11*c) + (1424*a^3*b^3 - 1121*a^2*b^4 + 99*a
*b^5)*cos(9*d*x + 9*c) + (1424*a^3*b^3 - 1121*a^2*b^4 + 99*a*b^5)*cos(7*d*
x + 7*c) - (272*a^3*b^3 - 461*a^2*b^4 + 159*a*b^5)*cos(5*d*x + 5*c) - (121
*a^2*b^4 - 67*a*b^5)*cos(3*d*x + 3*c) + (13*a^2*b^4 - 7*a*b^5)*cos(d*x + c
))*cos(16*d*x + 16*c) - (13*a^2*b^4 - 7*a*b^5 - 8*(13*a^2*b^4 - 7*a*b^5)*c
os(14*d*x + 14*c) - 4*(104*a^3*b^3 - 147*a^2*b^4 + 49*a*b^5)*cos(12*d*x +
12*c) + 8*(208*a^3*b^3 - 203*a^2*b^4 + 49*a*b^5)*cos(10*d*x + 10*c) + 2*(1
664*a^4*b^2 - 2144*a^3*b^3 + 1127*a^2*b^4 - 245*a*b^5)*cos(8*d*x + 8*c) +
8*(208*a^3*b^3 - 203*a^2*b^4 + 49*a*b^5)*cos(6*d*x + 6*c) - 4*(104*a^3*b^3
 - 147*a^2*b^4 + 49*a*b^5)*cos(4*d*x + 4*c) - 8*(13*a^2*b^4 - 7*a*b^5)*cos
(2*d*x + 2*c))*cos(15*d*x + 15*c) - 8*((121*a^2*b^4 - 67*a*b^5)*cos(13*d*x
 + 13*c) + (272*a^3*b^3 - 461*a^2*b^4 + 159*a*b^5)*cos(11*d*x + 11*c) - (1
424*a^3*b^3 - 1121*a^2*b^4 + 99*a*b^5)*cos(9*d*x + 9*c) - (1424*a^3*b^3 -
1121*a^2*b^4 + 99*a*b^5)*cos(7*d*x + 7*c) + (272*a^3*b^3 - 461*a^2*b^4 + 1
59*a*b^5)*cos(5*d*x + 5*c) + (121*a^2*b^4 - 67*a*b^5)*cos(3*d*x + 3*c) - (
13*a^2*b^4 - 7*a*b^5)*cos(d*x + c))*cos(14*d*x + 14*c) + (121*a^2*b^4 -...
```

3.229.8 Giac [F]

$$\int \frac{\csc(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \int -\frac{\csc(dx+c)}{(b\sin(dx+c)^4-a)^3}\,dx$$

input
```
integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.229.9 Mupad [B] (verification not implemented)

Time = 20.97 (sec) , antiderivative size = 12247, normalized size of antiderivative = 19.85

$$\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input

```
int(1/(sin(c + d*x)*(a - b*sin(c + d*x)^4)^3),x)
```

output

```
- ((5*cos(c + d*x)*(7*a*b - 3*b^2))/(32*a^2*(a - b)) - (cos(c + d*x)^3*(17
*a^2*b - 78*a*b^2 + 37*b^3))/(32*a^2*(a - b)^2) - (cos(c + d*x)^5*(53*a*b^
2 - 29*b^3))/(32*a^2*(a - b)^2) + (b*cos(c + d*x)^7*(13*a*b - 7*b^2))/(32*
a^2*(a - b)^2))/(d*(a^2 - 2*a*b + b^2 + cos(c + d*x)^2*(4*a*b - 4*b^2) - c
os(c + d*x)^4*(2*a*b - 6*b^2) - 4*b^2*cos(c + d*x)^6 + b^2*cos(c + d*x)^8)
) - (atan(((((((((192*a^11*b^9 - 990*a^12*b^8 + 2050*a^13*b^7 - 2154*a^14*b
^6 + 1158*a^15*b^5 - 256*a^16*b^4)/(2*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11
*b^3 + 6*a^12*b^2)) - (cos(c + d*x)*(402653184*a^12*b^9 - 1879048192*a^13*
b^8 + 3489660928*a^14*b^7 - 3221225472*a^15*b^6 + 1476395008*a^16*b^5 - 26
8435456*a^17*b^4))/(2097152*a^3*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6
*a^10*b^2)))/(2*a^3) + (cos(c + d*x)*(75497472*a^6*b^9 - 337215488*a^7*b^8
 + 592748544*a^8*b^7 - 489406464*a^9*b^6 + 163684352*a^10*b^5))/(1048576*(
a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))/(2*a^3) - (12*a^5*b^
9 - (4311*a^6*b^8)/64 + (307961*a^7*b^7)/2048 - (1290253*a^8*b^6)/8192 + (
546059*a^9*b^5)/8192)/(2*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12
*b^2)))*1i)/(2*a^3) - (cos(c + d*x)*(3145728*b^9 - 14417920*a*b^8 + 264532
64*a^2*b^7 - 23076232*a^3*b^6 + 8247825*a^4*b^5)*1i)/(1048576*(a^12 - 4*a^
11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))/a^3 - ((((((192*a^11*b^9 - 990*
a^12*b^8 + 2050*a^13*b^7 - 2154*a^14*b^6 + 1158*a^15*b^5 - 256*a^16*b^4)/(
2*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)) + (cos(c + d*...
```

3.230 $\int \frac{\sin^8(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

3.230.1 Optimal result

Integrand size = 24, antiderivative size = 319

$$\begin{aligned}\int \frac{\sin^8(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = &-\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d}\\&+\frac{\left(2\sqrt{a}+5\sqrt{b}\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/2}d}\\&-\frac{(a+5b)\tan(c+dx)}{32a(a-b)^2bd}+\frac{\tan^3(c+dx)}{32a(a-b)bd}\\&+\frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2}\\&-\frac{\sec^2(c+dx)\tan^5(c+dx)}{32abd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)}\end{aligned}$$

output
```
-1/64*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)-5*b^(1
/2))/a^(3/4)/b^(3/2)/d/(a^(1/2)-b^(1/2))^(5/2)+1/64*arctan((a^(1/2)+b^(1/2
))^(1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)+5*b^(1/2))/a^(3/4)/b^(3/2)/d/(a^(1
/2)+b^(1/2))^(5/2)-1/32*(a+5*b)*tan(d*x+c)/a/(a-b)^2/b/d+1/32*tan(d*x+c)^3
/a/(a-b)/b/d+1/8*tan(d*x+c)^9/a/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)^
2-1/32*sec(d*x+c)^2*tan(d*x+c)^5/a/b/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c
)^4)
```

3.230.2 Mathematica [A] (verified)

Time = 10.98 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.04

$$\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \frac{\frac{\left(2a^{3/2}\sqrt{b}+ab-8\sqrt{a}b^{3/2}+5b^2\right)\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)^2\sqrt{b}\text{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{8b(5a-14b-}{8a-3b}}{64(a-b)^2b^2d}$$

input
```
Integrate[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
(((2*a^(3/2)*Sqrt[b] + a*b - 8*Sqrt[a]*b^(3/2) + 5*b^2)*ArcTan[((Sqrt[a] +
 Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt
[a]*Sqrt[b]]) + ((2*Sqrt[a] - 5*Sqrt[b])*(Sqrt[a] + Sqrt[b])^2*Sqrt[b]*Arc
Tanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt
[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + (8*b*(5*a - 14*b + (-2*a + 5*b)*Cos[2*(c
 + d*x)])*Sin[2*(c + d*x)])/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c
 + d*x)]) + (64*a*(a - b)*b*(-6*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/(-8*
a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^2)/(64*(a - b)^2*b^2*
d)
```

3.230.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3696, 1598, 27, 1440, 27, 1602, 27, 1602, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^8}{\left(a-b\sin(c+dx)^4\right)^3}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\tan^8(c+dx)(\tan^2(c+dx)+1)}{((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)^3}d\tan(c+dx)}{d}$$

$\downarrow$ 1598

$$\frac{\frac{\int -\frac{2b\tan^8(c+dx)}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{16ab}+\frac{\tan^9(c+dx)}{8a((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)^2}}{d}$$

$\downarrow$ 27

$$\frac{\frac{\tan^9(c+dx)}{8a((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)^2}-\frac{\int \frac{\tan^8(c+dx)}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{8a}}{d}$$

$\downarrow$ 1440

$$\frac{\frac{\tan^9(c+dx)}{8a((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)^2}-\frac{\frac{\tan^5(c+dx)\left(\tan^2(c+dx)+1\right)}{4b\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\int \frac{2a\tan^4(c+dx)\left(3\tan^2(c+dx)+5\right)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{8ab}}{8a}}{d}$$

$\downarrow$ 27

$$\frac{\frac{\tan^9(c+dx)}{8a((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)^2}-\frac{\frac{\tan^5(c+dx)\left(\tan^2(c+dx)+1\right)}{4b\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\int \frac{\tan^4(c+dx)\left(3\tan^2(c+dx)+5\right)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{4b}}{8a}}{d}$$

$\downarrow$ 1602

$$\frac{\frac{\tan^9(c+dx)}{8a((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)^2}-\frac{\frac{\tan^5(c+dx)\left(\tan^2(c+dx)+1\right)}{4b\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\frac{\tan^3(c+dx)}{a-b}-\frac{\int \frac{3\tan^2(c+dx)\left((a+5b)\tan^2(c+dx)+3a\right)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+}{3(a-b)}}{4b}}{8a}}{d}$$

$\downarrow$ 27

$$\frac{\frac{\tan^9(c+dx)}{8a((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)^2}-\frac{\frac{\tan^5(c+dx)\left(\tan^2(c+dx)+1\right)}{4b\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\frac{\tan^3(c+dx)}{a-b}-\frac{\int \frac{\tan^2(c+dx)\left((a+5b)\tan^2(c+dx)+3a\right)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx}{a-b}}{4b}}{8a}}{d}$$

$\downarrow$ 1602

3.230. $\int \frac{\sin^8(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

$$\frac{\frac{\tan^9(c+dx)}{8a((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)^2}-\frac{\frac{\tan^5(c+dx)\left(\tan^2(c+dx)+1\right)}{4b\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\frac{\tan^3(c+dx)}{a-b}-\frac{\frac{(a+5b)\tan(c+dx)}{a-b}-\frac{\int\frac{a\left(-\left((a-13b)\tan^2(c+dx)\right)\right.}{(a-b)\tan^4(c+dx)+2a\tan^2(\ldots}}{a-b}}{a-b}}{4b}}{8a}}{d}$$

$\downarrow$ 27

$$\frac{\frac{\tan^9(c+dx)}{8a((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)^2}-\frac{\frac{\tan^5(c+dx)\left(\tan^2(c+dx)+1\right)}{4b\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\frac{\tan^3(c+dx)}{a-b}-\frac{\frac{(a+5b)\tan(c+dx)}{a-b}-\frac{a\int\frac{-\left((a-13b)\tan^2(c+dx)\right.}{(a-b)\tan^4(c+dx)+2a\tan^2\ldots}}{a-b}}{a-b}}{4b}}{8a}}{d}$$

$\downarrow$ 1480

$$\frac{\frac{\tan^9(c+dx)}{8a((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)^2}-\frac{\frac{\tan^5(c+dx)\left(\tan^2(c+dx)+1\right)}{4b\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\tan^3(c+dx)}{a-b}-\frac{(a+5b)\tan(c+dx)}{a-b}-\frac{a\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)^3\left(2\sqrt{a}+5\sqrt{b}\right)\int\ldots}{\ldots}\right.}{\ldots}}{\ldots}}{d}$$

$\downarrow$ 218

$$\frac{\frac{\tan^9(c+dx)}{8a((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)^2}-\frac{\frac{\tan^5(c+dx)\left(\tan^2(c+dx)+1\right)}{4b\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\tan^3(c+dx)}{a-b}-\frac{(a+5b)\tan(c+dx)}{a-b}-\frac{a\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\left(2\sqrt{a}+5\sqrt{b}\right)\text{ar}\ldots}{2a^{3/4}\sqrt{\ldots}}\right.}{\ldots}}{8a}}{d}$$

input
```
Int[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
(Tan[c + d*x]^9/(8*a*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)^2)
- ((Tan[c + d*x]^5*(1 + Tan[c + d*x]^2))/(4*b*(a + 2*a*Tan[c + d*x]^2 + (a
 - b)*Tan[c + d*x]^4)) - (Tan[c + d*x]^3/(a - b) - (-((a*(-1/2*((2*Sqrt[a]
 - 5*Sqrt[b])*(Sqrt[a] + Sqrt[b])^2*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c
+ d*x])/a^(1/4)])/(a^(3/4)*Sqrt[Sqrt[a] - Sqrt[b]]*Sqrt[b]) + ((Sqrt[a] -
Sqrt[b])^2*(2*Sqrt[a] + 5*Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c +
 d*x])/a^(1/4)])/(2*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[b]]*Sqrt[b])))/(a - b)) +
((a + 5*b)*Tan[c + d*x])/(a - b))/(a - b))/(4*b))/(8*a))/d
```

3.230. $\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

3.230.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

3.230. $\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

3.230.4 Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{\frac{-\frac{(a+19b)\left(\tan^7(dx+c)\right)}{32(a-b)b}-\frac{3\left(a^2+10ab-3b^2\right)\left(\tan^5(dx+c)\right)}{32b\left(a^2-2ab+b^2\right)}-\frac{3(a+7b)a\left(\tan^3(dx+c)\right)}{32b\left(a^2-2ab+b^2\right)}-\frac{a(a+5b)\tan(dx+c)}{32b\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^4(dx+c)\right)a-b\left(\tan^4(dx+c)\right)+2a\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{(a-b)\left(\frac{\left(-a\sqrt{ab}+13\sqrt{c\cdots}\right.}{\cdots}\right.}{\cdots}}{d}$
default	$\frac{\frac{-\frac{(a+19b)\left(\tan^7(dx+c)\right)}{32(a-b)b}-\frac{3\left(a^2+10ab-3b^2\right)\left(\tan^5(dx+c)\right)}{32b\left(a^2-2ab+b^2\right)}-\frac{3(a+7b)a\left(\tan^3(dx+c)\right)}{32b\left(a^2-2ab+b^2\right)}-\frac{a(a+5b)\tan(dx+c)}{32b\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^4(dx+c)\right)a-b\left(\tan^4(dx+c)\right)+2a\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{(a-b)\left(\frac{\left(-a\sqrt{ab}+13\sqrt{c\cdots}\right.}{\cdots}\right.}{\cdots}}{d}$
risch	Expression too large to display

input
```
int(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*((-1/32*(a+19*b)/(a-b)/b*tan(d*x+c)^7-3/32*(a^2+10*a*b-3*b^2)/b/(a^2-2
*a*b+b^2)*tan(d*x+c)^5-3/32*(a+7*b)*a/b/(a^2-2*a*b+b^2)*tan(d*x+c)^3-1/32*
a*(a+5*b)/b/(a^2-2*a*b+b^2)*tan(d*x+c))/(tan(d*x+c)^4*a-b*tan(d*x+c)^4+2*a
*tan(d*x+c)^2+a)^2+1/32/b/(a^2-2*a*b+b^2)*(a-b)*(1/2*(-a*(a*b)^(1/2)+13*(a
*b)^(1/2)*b-2*a^2+9*a*b+5*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(
1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*(-a*(a*b)^
(1/2)+13*(a*b)^(1/2)*b+2*a^2-9*a*b-5*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-
a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))))
```

3.230.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5219 vs. 2(263) = 526.

Time = 1.33 (sec) , antiderivative size = 5219, normalized size of antiderivative = 16.36

$$\int \frac{\sin^8(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.230.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^8(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**8/(a-b*sin(d*x+c)**4)**3,x)
```

output
```
Timed out
```

3.230.7 Maxima [F]

$$\int \frac{\sin^8(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \int -\frac{\sin(dx+c)^8}{(b\sin(dx+c)^4-a)^3}\,dx$$

input
```
integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/8*(4*(72*a^2*b^2 - 155*a*b^3 + 26*b^4)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c
) + ((a*b^3 - 4*b^4)*sin(14*d*x + 14*c) - (32*a^2*b^2 - 58*a*b^3 - b^4)*si
n(12*d*x + 12*c) + 3*(48*a^2*b^2 - 73*a*b^3 + 20*b^4)*sin(10*d*x + 10*c) +
 (256*a^3*b - 832*a^2*b^2 + 550*a*b^3 - 175*b^4)*sin(8*d*x + 8*c) + (112*a
^2*b^2 - 533*a*b^3 + 220*b^4)*sin(6*d*x + 6*c) - (32*a^2*b^2 - 158*a*b^3 +
 141*b^4)*sin(4*d*x + 4*c) - (17*a*b^3 - 44*b^4)*sin(2*d*x + 2*c))*cos(16*
d*x + 16*c) + 2*(2*(72*a^2*b^2 - 155*a*b^3 + 26*b^4)*sin(12*d*x + 12*c) -
8*(80*a^2*b^2 - 145*a*b^3 + 44*b^4)*sin(10*d*x + 10*c) - 3*(384*a^3*b - 13
12*a^2*b^2 + 873*a*b^3 - 280*b^4)*sin(8*d*x + 8*c) - 16*(32*a^2*b^2 - 151*
a*b^3 + 62*b^4)*sin(6*d*x + 6*c) + 2*(72*a^2*b^2 - 355*a*b^3 + 310*b^4)*si
n(4*d*x + 4*c) + 24*(3*a*b^3 - 8*b^4)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c)
 - 2*(2*(128*a^3*b - 456*a^2*b^2 + 1233*a*b^3 - 434*b^4)*sin(10*d*x + 10*c
) - (6400*a^3*b - 13888*a^2*b^2 + 8566*a*b^3 - 2485*b^4)*sin(8*d*x + 8*c)
- 2*(128*a^3*b + 2744*a^2*b^2 - 4711*a*b^3 + 1554*b^4)*sin(6*d*x + 6*c) +
4*(400*a^2*b^2 - 918*a*b^3 + 497*b^4)*sin(4*d*x + 4*c) - 2*(72*a^2*b^2 - 3
55*a*b^3 + 310*b^4)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) - 2*((2048*a^4 +
18560*a^3*b - 24752*a^2*b^2 + 13175*a*b^3 - 2800*b^4)*sin(8*d*x + 8*c) + 8
*(256*a^3*b + 2400*a^2*b^2 - 2379*a*b^3 + 560*b^4)*sin(6*d*x + 6*c) - 2*(1
28*a^3*b + 2744*a^2*b^2 - 4711*a*b^3 + 1554*b^4)*sin(4*d*x + 4*c) + 16*(32
*a^2*b^2 - 151*a*b^3 + 62*b^4)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c) - 2...
```

3.230.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1987 vs. $2(263) = 526$.

Time = 1.32 (sec) , antiderivative size = 1987, normalized size of antiderivative = 6.23

$$\int \frac{\sin^8(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/64*(((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 45*sqrt(a^2
- a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 77*sqrt(a^2 - a*b + sqrt(a*b)
*(a - b))*sqrt(a*b)*a*b^2 + 13*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*
b)*b^3)*(a^2*b - 2*a*b^2 + b^3)^2*abs(-a + b) + (3*sqrt(a^2 - a*b + sqrt(a
*b)*(a - b))*a^6*b - 49*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b^3 + 112*
sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^4 - 87*sqrt(a^2 - a*b + sqrt(a*b
)*(a - b))*a^2*b^5 + 16*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^6 + 5*sqrt
(a^2 - a*b + sqrt(a*b)*(a - b))*b^7)*abs(a^2*b - 2*a*b^2 + b^3)*abs(-a + b
) - (6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^8*b - 63*sqrt(a^2 -
 a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^2 + 229*sqrt(a^2 - a*b + sqrt(a*
b)*(a - b))*sqrt(a*b)*a^6*b^3 - 367*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sq
rt(a*b)*a^5*b^4 + 233*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^
5 + 27*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^6 - 89*sqrt(a^2
 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^7 + 19*sqrt(a^2 - a*b + sqrt(a
*b)*(a - b))*sqrt(a*b)*a*b^8 + 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(
a*b)*b^9)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)
/sqrt((a^3*b - 2*a^2*b^2 + a*b^3 + sqrt((a^3*b - 2*a^2*b^2 + a*b^3)^2 - (a
^3*b - 2*a^2*b^2 + a*b^3)*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)))/(a^3*b - 3
*a^2*b^2 + 3*a*b^3 - b^4))))/((3*a^10*b^2 - 27*a^9*b^3 + 104*a^8*b^4 - 224
*a^7*b^5 + 294*a^6*b^6 - 238*a^5*b^7 + 112*a^4*b^8 - 24*a^3*b^9 - a^2*b...
```

3.230.9 Mupad [B] (verification not implemented)

Time = 19.35 (sec) , antiderivative size = 5508, normalized size of antiderivative = 17.27

$$\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^8/(a - b*sin(c + d*x)^4)^3,x)
```

output
```
(atan(((((229376*a^2*b^6 - 81920*a*b^7 - 196608*a^3*b^5 + 32768*a^4*b^4 +
16384*a^5*b^3)/(32768*(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3)) - (tan(c + d*
x)*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^
3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11
 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2)*(16
384*a^2*b^8 - 81920*a^3*b^7 + 163840*a^4*b^6 - 163840*a^5*b^5 + 81920*a^6*
b^4 - 16384*a^7*b^3))/(256*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))*((25*b^
2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35
*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b
^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2) + (tan(c + d*
x)*(259*a*b^3 - 35*a^3*b + 4*a^4 + 25*b^4 + 35*a^2*b^2))/(256*(3*a*b^4 - b
^5 - 3*a^2*b^3 + a^3*b^2)))*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1
/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9
)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b
^7 - a^8*b^6)))^(1/2)*1i - (((229376*a^2*b^6 - 81920*a*b^7 - 196608*a^3*b^
5 + 32768*a^4*b^4 + 16384*a^5*b^3)/(32768*(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3
*b^3)) + (tan(c + d*x)*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) +
 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/
2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 -
a^8*b^6)))^(1/2)*(16384*a^2*b^8 - 81920*a^3*b^7 + 163840*a^4*b^6 - 1638...
```

3.230. $\int \frac{\sin^8(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

3.231 $\int \frac{\sin^6(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

3.231.1 Optimal result

Integrand size = 24, antiderivative size = 343

$$\int \frac{\sin^6(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = -\frac{\left(4a-10\sqrt{a}\sqrt{b}+3b\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d} + \frac{\left(4a+10\sqrt{a}\sqrt{b}+3b\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/2}d} - \frac{\tan(c+dx)\left(a(a+3b)+(a^2+6ab+b^2)\tan^2(c+dx)\right)}{8(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\frac{2a(a^2-ab-8b^2)}{(a-b)^3}+\frac{(2a^2+15ab+3b^2)\tan^2(c+dx)}{(a-b)^2}\right)}{32abd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)}$$

output
```
-1/64*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(4*a+3*b-10*a^(1/
2)*b^(1/2))/a^(5/4)/b^(3/2)/d/(a^(1/2)-b^(1/2))^(5/2)+1/64*arctan((a^(1/2)
+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(4*a+3*b+10*a^(1/2)*b^(1/2))/a^(5/4)/b
^(3/2)/d/(a^(1/2)+b^(1/2))^(5/2)-1/8*tan(d*x+c)*(a*(a+3*b)+(a^2+6*a*b+b^2)
*tan(d*x+c)^2)/(a-b)^3/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)^2-1/32*ta
n(d*x+c)*(2*a*(a^2-a*b-8*b^2)/(a-b)^3+(2*a^2+15*a*b+3*b^2)*tan(d*x+c)^2/(a
-b)^2)/a/b/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)
```

3.231.2 Mathematica [A] (verified)

Time = 6.23 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.02

$$\int \frac{\sin^6(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$$
$$= \frac{\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\sqrt{b}\left(4a+10\sqrt{a}\sqrt{b}+3b\right)\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{a\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\sqrt{b}\left(4a-10\sqrt{a}\sqrt{b}+3b\right)\operatorname{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{a\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{4b}{\cdots}}{64(a-b)^2b^2d}$$

input
```
Integrate[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
(((Sqrt[a] - Sqrt[b])^2*Sqrt[b]*(4*a + 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[((
Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(a*Sqrt[a + S
qrt[a]*Sqrt[b]]) + ((Sqrt[a] + Sqrt[b])^2*Sqrt[b]*(4*a - 10*Sqrt[a]*Sqrt[b
] + 3*b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt
[b]]])/(a*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + (4*b*(4*a^2 - 19*a*b - 3*b^2 + 3*b
*(a + b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(a*(8*a - 3*b + 4*b*Cos[2*(c
+ d*x)] - b*Cos[4*(c + d*x)])) - (128*(a - b)*b*(2*a + b - b*Cos[2*(c + d*
x)])*Sin[2*(c + d*x)])/(-8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d
*x)])^2)/(64*(a - b)^2*b^2*d)
```

3.231.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3696, 1672, 27, 2206, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(c+dx)^6}{(a-b\sin(c+dx)^4)^3}dx$$

$$\Big\downarrow \; 3696$$

$$\frac{\int \frac{\tan^6(c+dx)\left(\tan^2(c+dx)+1\right)^2}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^3}d\tan(c+dx)}{d}$$

$$\Big\downarrow \; 1672$$

$$\frac{-\frac{\int -\frac{2\left(\frac{8a^2b\tan^6(c+dx)}{a-b}-\frac{16a^2b^2\tan^4(c+dx)}{(a-b)^2}-\frac{a^2b\left(5a^2+6ba-3b^2\right)\tan^2(c+dx)}{(a-b)^3}+\frac{a^3b(a+3b)}{(a-b)^3}\right)}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{16a^2b}-\frac{\tan(c+dx)\left(\left(a^2+6ab+b^2\right)\tan^2(c+dx)+a\right.}{8(a-b)^3\left((a-b)\tan^4(c+dx)+2a\tan^2(c+\right.}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\frac{8a^2b\tan^6(c+dx)}{a-b}-\frac{16a^2b^2\tan^4(c+dx)}{(a-b)^2}-\frac{a^2b\left(5a^2+6ba-3b^2\right)\tan^2(c+dx)}{(a-b)^3}+\frac{a^3b(a+3b)}{(a-b)^3}}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{8a^2b}-\frac{\tan(c+dx)\left(\left(a^2+6ab+b^2\right)\tan^2(c+dx)+a(a+3b)\right)}{8(a-b)^3\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}}{d}$$

$$\Big\downarrow \; 2206$$

$$\frac{\frac{-\frac{\int -\frac{2a^3b\left(2a(a+2b)-\left(2a^2-17ba+3b^2\right)\tan^2(c+dx)\right)}{(a-b)^2\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}d\tan(c+dx)}{8a^2b}-\frac{a\tan(c+dx)\left(\frac{\left(2a^2+15ab+3b^2\right)\tan^2(c+dx)}{(a-b)^2}+\frac{2a\left(a^2-ab-8b^2\right)}{(a-b)^3}\right)}{4\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}}{8a^2b}-\frac{\tan(c+dx)\left(\left(a^2+6ab\right.\right.}{8(a-b)^3\left((a-b)\tan^4\right.}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{a\int \frac{2a(a+2b)-\left(2a^2-17ba+3b^2\right)\tan^2(c+dx)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{4(a-b)^2}-\frac{a\tan(c+dx)\left(\frac{\left(2a^2+15ab+3b^2\right)\tan^2(c+dx)}{(a-b)^2}+\frac{2a\left(a^2-ab-8b^2\right)}{(a-b)^3}\right)}{4\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}}{8a^2b}-\frac{\tan(c+dx)\left(\left(a^2+6ab+b^2\right)\tan\right.}{8(a-b)^3\left((a-b)\tan^4(c+dx)+\right.}}{d}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{\frac{a\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)^3\left(10\sqrt{a}\sqrt{b}+4a+3b\right)\int \frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tan(c+dx)}{2\sqrt{b}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^3\left(-10\sqrt{a}\sqrt{b}+4a+3b\right)\int \frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\tan(c+d\right.}{2\sqrt{b}}}{4(a-b)^2}}{8a^2b}}{d}$$

$$\Big\downarrow \; 218$$

3.231. $\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}dx$

$$\frac{\frac{\frac{a\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\left(10\sqrt{a}\sqrt{b}+4a+3b\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(-10\sqrt{a}\sqrt{b}+4a+3b\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{4(a-b)^2}-\frac{a\tan(c+dx)\left(\frac{\left(2a^2+15ab\right.}{}\right.}{4\left((a-b)\tan^4\right.}}{8a^2b}}{d}$$

input
```
Int[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
(-1/8*(Tan[c + d*x]*(a*(a + 3*b) + (a^2 + 6*a*b + b^2)*Tan[c + d*x]^2))/((
a - b)^3*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)^2) + ((a*(-1/2*
((Sqrt[a] + Sqrt[b])^2*(4*a - 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[(Sqrt[Sqrt[
a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*Sqr
t[b]) + ((Sqrt[a] - Sqrt[b])^2*(4*a + 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[(Sq
rt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(1/4)*Sqrt[Sqrt[a] + Sq
rt[b]]*Sqrt[b])))/(4*(a - b)^2) - (a*Tan[c + d*x]*((2*a*(a^2 - a*b - 8*b^2
))/(a - b)^3 + ((2*a^2 + 15*a*b + 3*b^2)*Tan[c + d*x]^2)/(a - b)^2))/(4*(a
 + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)))/(8*a^2*b))/d
```

3.231.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.231. $\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

rule 1672
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*
Simp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
 c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

3.231. $\int \frac{\sin^6(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

3.231.4 Maple [A] (verified)

Time = 4.79 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{-\frac{(2a^2+15ab+3b^2)(\tan^7(dx+c))}{32(a-b)ab}-\frac{(3a^2+14ab-5b^2)(\tan^5(dx+c))}{16b(a^2-2ab+b^2)}-\frac{(6a^2+19ab-b^2)(\tan^3(dx+c))}{32b(a^2-2ab+b^2)}-\frac{a(a+2b)\tan(dx+c)}{16b(a^2-2ab+b^2)}}{((\tan^4(dx+c))a-b(\tan^4(dx+c))+2a(\tan^2(dx+c))+a)^2}+\frac{(a-b)}{}$
default	$\frac{-\frac{(2a^2+15ab+3b^2)(\tan^7(dx+c))}{32(a-b)ab}-\frac{(3a^2+14ab-5b^2)(\tan^5(dx+c))}{16b(a^2-2ab+b^2)}-\frac{(6a^2+19ab-b^2)(\tan^3(dx+c))}{32b(a^2-2ab+b^2)}-\frac{a(a+2b)\tan(dx+c)}{16b(a^2-2ab+b^2)}}{((\tan^4(dx+c))a-b(\tan^4(dx+c))+2a(\tan^2(dx+c))+a)^2}+\frac{(a-b)}{}$
risch	Expression too large to display

input
```
int(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*((-1/32*(2*a^2+15*a*b+3*b^2)/(a-b)/a/b*tan(d*x+c)^7-1/16*(3*a^2+14*a*b
-5*b^2)/b/(a^2-2*a*b+b^2)*tan(d*x+c)^5-1/32*(6*a^2+19*a*b-b^2)/b/(a^2-2*a*
b+b^2)*tan(d*x+c)^3-1/16*a*(a+2*b)/b/(a^2-2*a*b+b^2)*tan(d*x+c))/(tan(d*x+
c)^4*a-b*tan(d*x+c)^4+2*a*tan(d*x+c)^2+a)^2+1/32/a/b/(a^2-2*a*b+b^2)*(a-b)
*(1/2*(-2*a^2*(a*b)^(1/2)+17*a*b*(a*b)^(1/2)-3*b^2*(a*b)^(1/2)+4*a^3-15*a^
2*b-a*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*
tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2*(-2*a^2*(a*b)^(1/2)+17*a*b*(
a*b)^(1/2)-3*b^2*(a*b)^(1/2)-4*a^3+15*a^2*b+a*b^2)/(a*b)^(1/2)/(a-b)/(((a*
b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(
1/2))))
```

3.231.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5961 vs. 2(291) = 582.

Time = 2.03 (sec) , antiderivative size = 5961, normalized size of antiderivative = 17.38

$$\int \frac{\sin^6(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.231.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(sin(d*x+c)**6/(a-b*sin(d*x+c)**4)**3,x)
```

output
```
Timed out
```

3.231.7 Maxima [F]

$$\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx=\int -\frac{\sin\left(dx+c\right)^6}{\left(b\sin\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/16*(4*(32*a^3*b^2 - 84*a^2*b^3 - 83*a*b^4 + 21*b^5)*cos(4*d*x + 4*c)*si
n(2*d*x + 2*c) + ((4*a^2*b^3 - 13*a*b^4 + 3*b^5)*sin(14*d*x + 14*c) - 3*(8
*a^2*b^3 - 33*a*b^4 + 7*b^5)*sin(12*d*x + 12*c) + (64*a^3*b^2 + 68*a^2*b^3
 - 225*a*b^4 + 63*b^5)*sin(10*d*x + 10*c) - 3*(128*a^3*b^2 + 32*a^2*b^3 -
61*a*b^4 + 35*b^5)*sin(8*d*x + 8*c) - (64*a^3*b^2 + 452*a^2*b^3 - 9*a*b^4
- 105*b^5)*sin(6*d*x + 6*c) + 3*(40*a^2*b^3 - 29*a*b^4 - 21*b^5)*sin(4*d*x
 + 4*c) - (4*a^2*b^3 - 37*a*b^4 - 21*b^5)*sin(2*d*x + 2*c))*cos(16*d*x + 1
6*c) + 2*(2*(32*a^3*b^2 - 84*a^2*b^3 - 83*a*b^4 + 21*b^5)*sin(12*d*x + 12*
c) - 8*(64*a^3*b^2 - 84*a^2*b^3 - 43*a*b^4 + 21*b^5)*sin(10*d*x + 10*c) -
(512*a^4*b - 3584*a^3*b^2 + 1388*a^2*b^3 - 11*a*b^4 - 315*b^5)*sin(8*d*x +
 8*c) + 16*(172*a^2*b^3 - 37*a*b^4 - 21*b^5)*sin(6*d*x + 6*c) + 2*(32*a^3*
b^2 - 372*a^2*b^3 + 289*a*b^4 + 105*b^5)*sin(4*d*x + 4*c) + 8*(4*a^2*b^3 -
 25*a*b^4 - 9*b^5)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c) - 2*(2*(512*a^4*b
- 672*a^3*b^2 + 1228*a^2*b^3 + 21*a*b^4 - 147*b^5)*sin(10*d*x + 10*c) - 3*
(3072*a^4*b - 6272*a^3*b^2 + 2920*a^2*b^3 - 413*a*b^4 - 245*b^5)*sin(8*d*x
 + 8*c) - 2*(512*a^4*b + 3936*a^3*b^2 - 6740*a^2*b^3 + 1281*a*b^4 + 441*b^
5)*sin(6*d*x + 6*c) + 12*(192*a^3*b^2 - 416*a^2*b^3 + 161*a*b^4 + 49*b^5)*
sin(4*d*x + 4*c) - 2*(32*a^3*b^2 - 372*a^2*b^3 + 289*a*b^4 + 105*b^5)*sin(
2*d*x + 2*c))*cos(12*d*x + 12*c) - 2*((8192*a^5 + 27136*a^4*b - 37696*a^3*
b^2 + 17644*a^2*b^3 - 2079*a*b^4 - 735*b^5)*sin(8*d*x + 8*c) + 8*(1024*...
```

3.231.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2231 vs. $2(291) = 582$.

Time = 1.40 (sec) , antiderivative size = 2231, normalized size of antiderivative = 6.50

$$\int \frac{\sin^6(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/64*(((6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4 - 63*sqrt(a^2
- a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b + 109*sqrt(a^2 - a*b + sqrt(a*b
)*(a - b))*sqrt(a*b)*a^2*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*
b)*a*b^3 - 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*(a^3*b - 2
*a^2*b^2 + a*b^3)^2*abs(-a + b) + 2*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))
*a^8*b - 9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^7*b^2 - 4*sqrt(a^2 - a*b
+ sqrt(a*b)*(a - b))*a^6*b^3 + 34*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^5*
b^4 - 33*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b^5 + 7*sqrt(a^2 - a*b +
sqrt(a*b)*(a - b))*a^3*b^6 + 2*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^7
)*abs(a^3*b - 2*a^2*b^2 + a*b^3)*abs(-a + b) - (12*sqrt(a^2 - a*b + sqrt(a
*b)*(a - b))*sqrt(a*b)*a^11*b - 117*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sq
rt(a*b)*a^10*b^2 + 431*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b
^3 - 773*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^8*b^4 + 703*sqrt(
a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^5 - 279*sqrt(a^2 - a*b + sq
rt(a*b)*(a - b))*sqrt(a*b)*a^6*b^6 + 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))
*sqrt(a*b)*a^5*b^7 + 17*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*
b^8 + sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^9)*abs(-a + b))*
(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^4*b - 2*a^3*b^
2 + a^2*b^3 + sqrt((a^4*b - 2*a^3*b^2 + a^2*b^3)^2 - (a^4*b - 2*a^3*b^2 +
a^2*b^3)*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)))/(a^4*b - 3*a^3*b^2 +...
```

3.231.9 Mupad [B] (verification not implemented)

Time = 20.09 (sec) , antiderivative size = 6391, normalized size of antiderivative = 18.63

$$\int \frac{\sin^6(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^6/(a - b*sin(c + d*x)^4)^3,x)
```

output
```
- ((tan(c + d*x)^3*(19*a*b + 6*a^2 - b^2))/(32*(a^2*b - 2*a*b^2 + b^3)) +
(a*tan(c + d*x)*(a + 2*b))/(16*(a^2*b - 2*a*b^2 + b^3)) + (tan(c + d*x)^7*
(15*a*b + 2*a^2 + 3*b^2))/(32*a*(a*b - b^2)) + (tan(c + d*x)^5*(14*a*b + 3
*a^2 - 5*b^2))/(16*(a - b)*(a*b - b^2)))/(d*(tan(c + d*x)^8*(a^2 - 2*a*b +
 b^2) + a^2 - tan(c + d*x)^4*(2*a*b - 6*a^2) - tan(c + d*x)^6*(4*a*b - 4*a
^2) + 4*a^2*tan(c + d*x)^2)) - (atan(((((65536*a^3*b^7 - 163840*a^4*b^6 +
98304*a^5*b^5 + 32768*a^6*b^4 - 32768*a^7*b^3)/(32768*(a^2*b^6 - 3*a^3*b^5
 + 3*a^4*b^4 - a^5*b^3)) - (tan(c + d*x)*((9*b^3*(a^5*b^9)^(1/2) - 80*a^3*
(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16
*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(1/2) + 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a
^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(
1/2)*(16384*a^3*b^8 - 81920*a^4*b^7 + 163840*a^5*b^6 - 163840*a^6*b^5 + 81
920*a^7*b^4 - 16384*a^8*b^3))/(256*(a*b^5 - 3*a^2*b^4 + 3*a^3*b^3 - a^4*b^
2)))*((9*b^3*(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^
4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(1/2)
+ 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 -
10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2) + (tan(c + d*x)*(16*a^5 - 116*a
^4*b - 101*a*b^4 + 9*b^5 + 331*a^2*b^3 + 149*a^3*b^2))/(256*(a*b^5 - 3*a^2
*b^4 + 3*a^3*b^3 - a^4*b^2)))*((9*b^3*(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9)^(
1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3...
```

3.231. $\int \frac{\sin^6(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

3.232 $\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

3.232.1 Optimal result

Integrand size = 24, antiderivative size = 313

$$\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \frac{3\left(2\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}d} - \frac{3\left(2\sqrt{a}+\sqrt{b}\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt{b}d} - \frac{b\tan(c+dx)\left(3a+b+4(a+b)\tan^2(c+dx)\right)}{8(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\frac{9a^2-24ab-b^2}{(a-b)^3}+\frac{(17a+3b)\tan^2(c+dx)}{(a-b)^2}\right)}{32ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)}$$

output
```
3/64*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)-b^(1/2)
)/a^(7/4)/d/(a^(1/2)-b^(1/2))^(5/2)/b^(1/2)-3/64*arctan((a^(1/2)+b^(1/2))^
(1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)+b^(1/2))/a^(7/4)/d/b^(1/2)/(a^(1/2)+b
^(1/2))^(5/2)-1/8*b*tan(d*x+c)*(3*a+b+4*(a+b)*tan(d*x+c)^2)/(a-b)^3/d/(a+2
*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)^2-1/32*tan(d*x+c)*((9*a^2-24*a*b-b^2)/
(a-b)^3+(17*a+3*b)*tan(d*x+c)^2/(a-b)^2)/a/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan
(d*x+c)^4)
```

3.232.2 Mathematica [A] (verified)

Time = 11.44 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.01

$$\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$$
$$= \frac{-\frac{3\left(2a^{3/2}-3a\sqrt{b}+b^{3/2}\right)\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{a^{3/2}\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{3\left(2a^{3/2}+3a\sqrt{b}-b^{3/2}\right)\text{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{a^{3/2}\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{8(-7a-2b+(2a+b)\cos\ldots}{a(8a-3b+4b\cos(2(c\ldots}}{64(a-b)^2d}$$

input
```
Integrate[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
((-3*(2*a^(3/2) - 3*a*Sqrt[b] + b^(3/2))*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c
 + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(a^(3/2)*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sq
rt[b]) - (3*(2*a^(3/2) + 3*a*Sqrt[b] - b^(3/2))*ArcTanh[((Sqrt[a] - Sqrt[b
])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(a^(3/2)*Sqrt[-a + Sqrt[a]*S
qrt[b]]*Sqrt[b]) + (8*(-7*a - 2*b + (2*a + b)*Cos[2*(c + d*x)])*Sin[2*(c +
 d*x)])/(a*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (64*
(a - b)*(-6*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*
(c + d*x)] + b*Cos[4*(c + d*x)])^2)/(64*(a - b)^2*d)
```

3.232.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3696, 1672, 27, 2206, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin(c+dx)^4}{(a-b\sin(c+dx)^4)^3}dx$$
$$\Big\downarrow \; 3696$$

$$\frac{\int \frac{\tan^4(c+dx)\left(\tan^2(c+dx)+1\right)^3}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^3}d\tan(c+dx)}{d}$$

$\big\downarrow$ 1672

$$\frac{-\frac{\int -\frac{2\left(\frac{8a^2b\tan^6(c+dx)}{a-b}+\frac{8a^2(a-3b)b\tan^4(c+dx)}{(a-b)^2}-\frac{4a^2(3a-b)b^2\tan^2(c+dx)}{(a-b)^3}+\frac{a^2b^2(3a+b)}{(a-b)^3}\right)}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{16a^2b}-\frac{b\tan(c+dx)\left(4(a+b)\tan^2(c+dx)+3a+\right.}{8(a-b)^3((a-b)\tan^4(c+dx)+2a\tan^2(c+dx}}{d}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{\frac{8a^2b\tan^6(c+dx)}{a-b}+\frac{8a^2(a-3b)b\tan^4(c+dx)}{(a-b)^2}-\frac{4a^2(3a-b)b^2\tan^2(c+dx)}{(a-b)^3}+\frac{a^2b^2(3a+b)}{(a-b)^3}}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{8a^2b}-\frac{b\tan(c+dx)\left(4(a+b)\tan^2(c+dx)+3a+b\right)}{8(a-b)^3((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)^2}}{d}$$

$\big\downarrow$ 2206

$$\frac{\frac{-\frac{\int -\frac{6a^3b^2\left((5a-b)\tan^2(c+dx)+3a-b\right)}{(a-b)^2\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}d\tan(c+dx)}{8a^2b}-\frac{ab\tan(c+dx)\left(\frac{9a^2-24ab-b^2}{(a-b)^3}+\frac{(17a+3b)\tan^2(c+dx)}{(a-b)^2}\right)}{4\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}}{8a^2b}-\frac{b\tan(c+dx)\left(4(a+b)\tan^2(\right.}{8(a-b)^3((a-b)\tan^4(c+dx)+2a}}{d}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{3ab\int \frac{(5a-b)\tan^2(c+dx)+3a-b}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{4(a-b)^2}-\frac{ab\tan(c+dx)\left(\frac{9a^2-24ab-b^2}{(a-b)^3}+\frac{(17a+3b)\tan^2(c+dx)}{(a-b)^2}\right)}{4\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}}{8a^2b}-\frac{b\tan(c+dx)\left(4(a+b)\tan^2(c+dx)+3\right.}{8(a-b)^3((a-b)\tan^4(c+dx)+2a\tan^2(c+}}{d}$$

$\big\downarrow$ 1480

$$\frac{\frac{\frac{3ab\left(\frac{\left(2\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)^3\int \frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\tan(c+dx)}{2\sqrt{a}\sqrt{b}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^3\left(2\sqrt{a}+\sqrt{b}\right)\int \frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tan(c+dx)}{2\sqrt{a}\sqrt{b}}\right)}{4(a-b)^2}-\frac{ab\tan(}{}}{8a^2b}}{d}$$

$\big\downarrow$ 218

3.232. $\int \frac{\sin^4(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}dx$

$$\frac{\frac{\frac{3ab\left(\frac{(2\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})^2\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{(\sqrt{a}-\sqrt{b})^2(2\sqrt{a}+\sqrt{b})\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{4(a-b)^2}-\frac{ab\tan(c+dx)\left(\frac{9a^2-24ab-b^2}{(a-b)^3}+\frac{(17a+3b}{(}\right.}{4\left((a-b)\tan^4(c+dx)+2a\tan^2(c+\right.}}{8a^2b}}{d}$$

input
```
Int[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
(-1/8*(b*Tan[c + d*x]*(3*a + b + 4*(a + b)*Tan[c + d*x]^2))/((a - b)^3*(a
+ 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)^2) + ((3*a*b*(((2*Sqrt[a] -
 Sqrt[b])*(Sqrt[a] + Sqrt[b])^2*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*
x])/a^(1/4)])/(2*a^(3/4)*Sqrt[Sqrt[a] - Sqrt[b]]*Sqrt[b]) - ((Sqrt[a] - Sq
rt[b])^2*(2*Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x
])/a^(1/4)])/(2*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[b]]*Sqrt[b])))/(4*(a - b)^2) -
 (a*b*Tan[c + d*x]*((9*a^2 - 24*a*b - b^2)/(a - b)^3 + ((17*a + 3*b)*Tan[c
 + d*x]^2)/(a - b)^2))/(4*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4
)))/(8*a^2*b))/d
```

3.232.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1672
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*
Simp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
 c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

3.232. $\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

3.232.4 Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{\frac{-\frac{(17a+3b)\left(\tan^7(dx+c)\right)}{32a(a-b)}-\frac{\left(43a^2-18ab-b^2\right)\left(\tan^5(dx+c)\right)}{32a\left(a^2-2ab+b^2\right)}-\frac{(35a-11b)\left(\tan^3(dx+c)\right)}{32\left(a^2-2ab+b^2\right)}-\frac{3(3a-b)\tan(dx+c)}{32\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^4(dx+c)\right)a-b\left(\tan^4(dx+c)\right)+2a\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{3(a-b)\left(\frac{\left(5a\sqrt{ab}-\sqrt{\right.}}{}\right.}{}}{d}$
default	$\frac{\frac{-\frac{(17a+3b)\left(\tan^7(dx+c)\right)}{32a(a-b)}-\frac{\left(43a^2-18ab-b^2\right)\left(\tan^5(dx+c)\right)}{32a\left(a^2-2ab+b^2\right)}-\frac{(35a-11b)\left(\tan^3(dx+c)\right)}{32\left(a^2-2ab+b^2\right)}-\frac{3(3a-b)\tan(dx+c)}{32\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^4(dx+c)\right)a-b\left(\tan^4(dx+c)\right)+2a\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{3(a-b)\left(\frac{\left(5a\sqrt{ab}-\sqrt{\right.}}{}\right.}{}}{d}$
risch	Expression too large to display

input
```
int(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*((-1/32*(17*a+3*b)/a/(a-b)*tan(d*x+c)^7-1/32*(43*a^2-18*a*b-b^2)/a/(a^
2-2*a*b+b^2)*tan(d*x+c)^5-1/32*(35*a-11*b)/(a^2-2*a*b+b^2)*tan(d*x+c)^3-3/
32*(3*a-b)/(a^2-2*a*b+b^2)*tan(d*x+c))/(tan(d*x+c)^4*a-b*tan(d*x+c)^4+2*a*
tan(d*x+c)^2+a)^2+3/32/a/(a^2-2*a*b+b^2)*(a-b)*(1/2*(5*a*(a*b)^(1/2)-(a*b)
^(1/2)*b-2*a^2-3*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*
arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2*(5*a*(a*b)^(1
/2)-(a*b)^(1/2)*b+2*a^2+3*a*b-b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b
))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))))
```

3.232.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5510 vs. 2(261) = 522.

Time = 1.44 (sec) , antiderivative size = 5510, normalized size of antiderivative = 17.60

$$\int \frac{\sin^4(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.232.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**4/(a-b*sin(d*x+c)**4)**3,x)
```

output
```
Timed out
```

3.232.7 Maxima [F]

$$\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \int -\frac{\sin(dx+c)^4}{\left(b\sin(dx+c)^4-a\right)^3}\,dx$$

input
```
integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*a*b^3*sin(2*d*x + 2*c) - 12*(8*a^2*b^2 + 13*a*b^3 - 2*b^4)*cos(4*d
*x + 4*c)*sin(2*d*x + 2*c) - (3*a*b^3*sin(14*d*x + 14*c) - 3*(10*a*b^3 - b
^4)*sin(12*d*x + 12*c) - (80*a^2*b^2 - 111*a*b^3 + 16*b^4)*sin(10*d*x + 10
*c) + (256*a^3*b - 64*a^2*b^2 - 26*a*b^3 + 35*b^4)*sin(8*d*x + 8*c) + (336
*a^2*b^2 - 95*a*b^3 - 40*b^4)*sin(6*d*x + 6*c) - (64*a^2*b^2 - 54*a*b^3 -
25*b^4)*sin(4*d*x + 4*c) - (19*a*b^3 + 8*b^4)*sin(2*d*x + 2*c))*cos(16*d*x
 + 16*c) - 2*(6*(8*a^2*b^2 + 13*a*b^3 - 2*b^4)*sin(12*d*x + 12*c) + 8*(16*
a^2*b^2 - 45*a*b^3 + 8*b^4)*sin(10*d*x + 10*c) - (1408*a^3*b - 544*a^2*b^2
 + a*b^3 + 140*b^4)*sin(8*d*x + 8*c) - 16*(96*a^2*b^2 - 29*a*b^3 - 10*b^4)
*sin(6*d*x + 6*c) + 2*(152*a^2*b^2 - 129*a*b^3 - 50*b^4)*sin(4*d*x + 4*c)
+ 8*(11*a*b^3 + 4*b^4)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c) - 2*(2*(640*a^
3*b - 488*a^2*b^2 + 389*a*b^3 - 70*b^4)*sin(10*d*x + 10*c) - (4096*a^4 - 8
448*a^3*b + 3744*a^2*b^2 - 414*a*b^3 - 385*b^4)*sin(8*d*x + 8*c) - 2*(2688
*a^3*b - 4072*a^2*b^2 + 861*a*b^3 + 238*b^4)*sin(6*d*x + 6*c) + 4*(256*a^3
*b - 560*a^2*b^2 + 206*a*b^3 + 77*b^4)*sin(4*d*x + 4*c) + 2*(152*a^2*b^2 -
 129*a*b^3 - 50*b^4)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) - 2*((26624*a^4
- 33152*a^3*b + 15632*a^2*b^2 - 2453*a*b^3 - 420*b^4)*sin(8*d*x + 8*c) + 8
*(3328*a^3*b - 3104*a^2*b^2 + 529*a*b^3 + 84*b^4)*sin(6*d*x + 6*c) - 2*(26
88*a^3*b - 4072*a^2*b^2 + 861*a*b^3 + 238*b^4)*sin(4*d*x + 4*c) - 16*(96*a
^2*b^2 - 29*a*b^3 - 10*b^4)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c) - 2*((...
```

3.232.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1986 vs. $2(261) = 522$.

Time = 1.41 (sec) , antiderivative size = 1986, normalized size of antiderivative = 6.35

$$\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
-1/64*(3*((15*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 33*sqr
t(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*sqrt(a*b)*a*b^3 + sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(
a*b)*b^4)*(a^3 - 2*a^2*b + a*b^2)^2*abs(-a + b) - (9*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*a^7*b - 48*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^6*b^2 + 93
*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^5*b^3 - 80*sqrt(a^2 - a*b + sqrt(a*
b)*(a - b))*a^4*b^4 + 27*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^5 - sqr
t(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^7)*abs(a^3 - 2*a^2*b + a*b^2)*abs(-a
+ b) - (6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^10 - 27*sqrt(a^2
 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b + 25*sqrt(a^2 - a*b + sqrt(a*b
)*(a - b))*sqrt(a*b)*a^8*b^2 + 53*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt
(a*b)*a^7*b^3 - 131*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^4
+ 103*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^5 - 29*sqrt(a^2
- a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^6 - sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*sqrt(a*b)*a^3*b^7 + sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)
*a^2*b^8)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)
/sqrt((a^4 - 2*a^3*b + a^2*b^2 + sqrt((a^4 - 2*a^3*b + a^2*b^2)^2 - (a^4 -
 2*a^3*b + a^2*b^2)*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)))/(a^4 - 3*a^3*b +
 3*a^2*b^2 - a*b^3))))/((3*a^12*b - 27*a^11*b^2 + 104*a^10*b^3 - 224*a^9*b
^4 + 294*a^8*b^5 - 238*a^7*b^6 + 112*a^6*b^7 - 24*a^5*b^8 - a^4*b^9 + a...
```

3.232.9 Mupad [B] (verification not implemented)

Time = 18.73 (sec) , antiderivative size = 5892, normalized size of antiderivative = 18.82

$$\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^4/(a - b*sin(c + d*x)^4)^3,x)
```

output
```
- (atan(((((3*(49152*a^7*b + 16384*a^3*b^5 - 98304*a^4*b^4 + 196608*a^5*b^
3 - 163840*a^6*b^2))/(32768*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)) - (tan(
c + d*x)*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4
*b^4 - 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^
3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11
*b^3 - a^12*b^2)))^(1/2)*(16384*a^9*b - 16384*a^4*b^6 + 81920*a^5*b^5 - 16
3840*a^6*b^4 + 163840*a^7*b^3 - 81920*a^8*b^2))/(256*(3*a^4*b - a^5 + a^2*
b^3 - 3*a^3*b^2)))*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a
^7*b + a^4*b^4 - 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2
*b*(a^7*b^3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^
4 + 5*a^11*b^3 - a^12*b^2)))^(1/2) - (tan(c + d*x)*(333*a^3*b - 45*a*b^3 +
 36*a^4 + 9*b^4 - 45*a^2*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2))
)*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^4 -
10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/2
)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 -
a^12*b^2)))^(1/2)*1i - (((3*(49152*a^7*b + 16384*a^3*b^5 - 98304*a^4*b^4 +
 196608*a^5*b^3 - 163840*a^6*b^2))/(32768*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4
*b^2)) + (tan(c + d*x)*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) +
 4*a^7*b + a^4*b^4 - 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5
*a^2*b*(a^7*b^3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*...
```

3.232. $\int \frac{\sin^4(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

3.233 $\int \frac{\sin^2(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

3.233.1 Optimal result

Integrand size = 24, antiderivative size = 347

$$\int \frac{\sin^2(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \frac{\left(12a-14\sqrt{a}\sqrt{b}+5b\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}d} - \frac{\left(12a+14\sqrt{a}\sqrt{b}+5b\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt{b}d} - \frac{b\tan(c+dx)\left(a(a+3b)+(a^2+6ab+b^2)\tan^2(c+dx)\right)}{8a(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\frac{2a(5a^2-9ab-4b^2)}{(a-b)^3}+\frac{5(2a^2+3ab-b^2)\tan^2(c+dx)}{(a-b)^2}\right)}{32a^2d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)}$$

output
```
1/64*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(12*a+5*b-14*a^(1/
2)*b^(1/2))/a^(9/4)/d/(a^(1/2)-b^(1/2))^(5/2)/b^(1/2)-1/64*arctan((a^(1/2)
+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(12*a+5*b+14*a^(1/2)*b^(1/2))/a^(9/4)/
d/b^(1/2)/(a^(1/2)+b^(1/2))^(5/2)-1/8*b*tan(d*x+c)*(a*(a+3*b)+(a^2+6*a*b+b
^2)*tan(d*x+c)^2)/a/(a-b)^3/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)^2-1/
32*tan(d*x+c)*(2*a*(5*a^2-9*a*b-4*b^2)/(a-b)^3+5*(2*a^2+3*a*b-b^2)*tan(d*x
+c)^2/(a-b)^2)/a^2/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)
```

3.233.2 Mathematica [A] (verified)

Time = 7.17 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.32

$$\int \frac{\sin^2(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$$
$$= \frac{\left(-12a^{5/2}\sqrt{b}+10a^2b+11a^{3/2}b^{3/2}-4ab^2-5\sqrt{a}b^{5/2}\right)\arctan\left(\frac{\left(\sqrt{a}\sqrt{b}+b\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}}\right)}{64a^{5/2}\sqrt{a+\sqrt{a}\sqrt{b}}(a-b)^2bd}$$
$$-\frac{\left(12a^{5/2}\sqrt{b}+10a^2b-11a^{3/2}b^{3/2}-4ab^2+5\sqrt{a}b^{5/2}\right)\operatorname{arctanh}\left(\frac{\left(\sqrt{a}\sqrt{b}-b\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}\sqrt{b}}\right)}{64a^{5/2}\sqrt{-a+\sqrt{a}\sqrt{b}}(a-b)^2bd}$$
$$+\frac{-4a\sin(2(c+dx))-2b\sin(2(c+dx))+b\sin(4(c+dx))}{a(a-b)d(-8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))^2}$$
$$+\frac{24a^2\sin(2(c+dx))+22ab\sin(2(c+dx))-10b^2\sin(2(c+dx))-11ab\sin(4(c+dx))+5b^2\sin(4(c+d}{32a^2(a-b)^2d(-8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))}$$

input
```
Integrate[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
((-12*a^(5/2)*Sqrt[b] + 10*a^2*b + 11*a^(3/2)*b^(3/2) - 4*a*b^2 - 5*Sqrt[a
]*b^(5/2))*ArcTan[((Sqrt[a]*Sqrt[b] + b)*Tan[c + d*x])/(Sqrt[a + Sqrt[a]*S
qrt[b]]*Sqrt[b])])/(64*a^(5/2)*Sqrt[a + Sqrt[a]*Sqrt[b]]*(a - b)^2*b*d) -
((12*a^(5/2)*Sqrt[b] + 10*a^2*b - 11*a^(3/2)*b^(3/2) - 4*a*b^2 + 5*Sqrt[a]
*b^(5/2))*ArcTanh[((Sqrt[a]*Sqrt[b] - b)*Tan[c + d*x])/(Sqrt[-a + Sqrt[a]*
Sqrt[b]]*Sqrt[b])])/(64*a^(5/2)*Sqrt[-a + Sqrt[a]*Sqrt[b]]*(a - b)^2*b*d)
+ (-4*a*Sin[2*(c + d*x)] - 2*b*Sin[2*(c + d*x)] + b*Sin[4*(c + d*x)])/(a*(
a - b)*d*(-8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^2) + (24
*a^2*Sin[2*(c + d*x)] + 22*a*b*Sin[2*(c + d*x)] - 10*b^2*Sin[2*(c + d*x)]
- 11*a*b*Sin[4*(c + d*x)] + 5*b^2*Sin[4*(c + d*x)])/(32*a^2*(a - b)^2*d*(-
8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]))
```

3.233.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3696, 1672, 27, 2206, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(c+dx)^2}{\left(a-b\sin(c+dx)^4\right)^3}dx$$

$$\Big\downarrow \ 3696$$

$$\frac{\int \frac{\tan^2(c+dx)\left(\tan^2(c+dx)+1\right)^4}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^3}d\tan(c+dx)}{d}$$

$$\Big\downarrow \ 1672$$

$$\frac{-\frac{\int -\frac{2\left(\frac{8a^2b\tan^6(c+dx)}{a-b}+\frac{16a^2(a-2b)b\tan^4(c+dx)}{(a-b)^2}+\frac{ab\left(8a^3-29ba^2+18b^2a-5b^3\right)\tan^2(c+dx)}{(a-b)^3}+\frac{a^2b^2(a+3b)}{(a-b)^3}\right)}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{16a^2b}-\frac{b\tan(c+dx)\left(\left(a^2+6ab+b^2\right)\cdots}{8a(a-b)^3((a-b)\tan^4(c+d\cdots}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{\frac{8a^2b\tan^6(c+dx)}{a-b}+\frac{16a^2(a-2b)b\tan^4(c+dx)}{(a-b)^2}+\frac{ab\left(8a^3-29ba^2+18b^2a-5b^3\right)\tan^2(c+dx)}{(a-b)^3}+\frac{a^2b^2(a+3b)}{(a-b)^3}}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{8a^2b}-\frac{b\tan(c+dx)\left(\left(a^2+6ab+b^2\right)\tan^2(c+\cdots}{8a(a-b)^3((a-b)\tan^4(c+dx)+2a\,\mathrm{ta}\cdots}}{d}$$

$$\Big\downarrow \ 2206$$

$$\frac{\frac{-\frac{\int -\frac{2a^2b^2\left(\left(22a^2-15ba+5b^2\right)\tan^2(c+dx)+2a(5a-2b)\right)}{(a-b)^2\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}d\tan(c+dx)}{8a^2b}-\frac{b\tan(c+dx)\left(\frac{5\left(2a^2+3ab-b^2\right)\tan^2(c+dx)}{(a-b)^2}+\frac{2a\left(5a^2-9ab-4b^2\right)}{(a-b)^3}\right)}{4\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}}{8a^2b}-\frac{b\tan(c+dx)\left(\left(a^2\cdots\right.\right.}{8a(a-b)^3((a-b)\cdots}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{b\int \frac{(22a^2-15ba+5b^2)\tan^2(c+dx)+2a(5a-2b)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{4(a-b)^2}-\frac{b\tan(c+dx)\left(\frac{5(2a^2+3ab-b^2)\tan^2(c+dx)}{(a-b)^2}+\frac{2a(5a^2-9ab-4b^2)}{(a-b)^3}\right)}{4\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}}{8a^2b}-\frac{b\tan(c+dx)\left((a^2+6ab+b^2\cdots\right.}{8a(a-b)^3\left((a-b)\tan^4(c+a\cdots\right.}}{d}$$

$\downarrow$ 1480

$$\frac{\frac{\frac{b\left(\frac{(\sqrt{a}+\sqrt{b})^3(-14\sqrt{a}\sqrt{b}+12a+5b)\int\frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}(\sqrt{a}+\sqrt{b})}d\tan(c+dx)}{2\sqrt{b}}-\frac{(\sqrt{a}-\sqrt{b})^3(14\sqrt{a}\sqrt{b}+12a+5b)\int\frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}(\sqrt{a}-\sqrt{b})}d\tan(c+\cdots}{2\sqrt{b}}\cdots\right.}{4(a-b)^2}\cdots}{8a^2b}\cdots}{d}$$

$\downarrow$ 218

$$\frac{\frac{\frac{b\left(\frac{(\sqrt{a}+\sqrt{b})^2(-14\sqrt{a}\sqrt{b}+12a+5b)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{(\sqrt{a}-\sqrt{b})^2(14\sqrt{a}\sqrt{b}+12a+5b)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{4(a-b)^2}-\frac{b\tan(c+dx)\left(\frac{5(2a^2+3\cdots}{\cdots}\right.}{4\left((a-b)\,\mathrm{ta}\cdots\right.}}{8a^2b}\cdots}{d}$$

input
```
Int[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
(-1/8*(b*Tan[c + d*x]*(a*(a + 3*b) + (a^2 + 6*a*b + b^2)*Tan[c + d*x]^2))/
(a*(a - b)^3*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)^2) + ((b*((
(Sqrt[a] + Sqrt[b])^2*(12*a - 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTan[(Sqrt[Sqrt[
a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*S
qrt[b]) - ((Sqrt[a] - Sqrt[b])^2*(12*a + 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTan[
(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(1/4)*Sqrt[Sqrt[a] +
 Sqrt[b]]*Sqrt[b])))/(4*(a - b)^2) - (b*Tan[c + d*x]*((2*a*(5*a^2 - 9*a*b
- 4*b^2))/(a - b)^3 + (5*(2*a^2 + 3*a*b - b^2)*Tan[c + d*x]^2)/(a - b)^2))
/(4*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)))/(8*a^2*b))/d
```

3.233. $\int \frac{\sin^2(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

3.233.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1672
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*
Simp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
 c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.233. $\int \frac{\sin^2(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

3.233.4 Maple [A] (verified)

Time = 5.49 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{-\frac{5(2a^2+3ab-b^2)(\tan^7(dx+c))}{32a^2(a-b)}-\frac{3(5a^2+2ab-3b^2)(\tan^5(dx+c))}{16a(a^2-2ab+b^2)}-\frac{3(10a^2+ab-3b^2)(\tan^3(dx+c))}{32a(a^2-2ab+b^2)}-\frac{(5a-2b)\tan(dx+c)}{16(a^2-2ab+b^2)}}{\left((\tan^4(dx+c))a-b(\tan^4(dx+c))+2a(\tan^2(dx+c))+a\right)^2}+ \frac{(a-b)}{}$
default	$\frac{-\frac{5(2a^2+3ab-b^2)(\tan^7(dx+c))}{32a^2(a-b)}-\frac{3(5a^2+2ab-3b^2)(\tan^5(dx+c))}{16a(a^2-2ab+b^2)}-\frac{3(10a^2+ab-3b^2)(\tan^3(dx+c))}{32a(a^2-2ab+b^2)}-\frac{(5a-2b)\tan(dx+c)}{16(a^2-2ab+b^2)}}{\left((\tan^4(dx+c))a-b(\tan^4(dx+c))+2a(\tan^2(dx+c))+a\right)^2}+ \frac{(a-b)}{}$
risch	Expression too large to display

input
```
int(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*((-5/32*(2*a^2+3*a*b-b^2)/a^2/(a-b)*tan(d*x+c)^7-3/16*(5*a^2+2*a*b-3*b
^2)/a/(a^2-2*a*b+b^2)*tan(d*x+c)^5-3/32*(10*a^2+a*b-3*b^2)/a/(a^2-2*a*b+b^
2)*tan(d*x+c)^3-1/16*(5*a-2*b)/(a^2-2*a*b+b^2)*tan(d*x+c))/(tan(d*x+c)^4*a
-b*tan(d*x+c)^4+2*a*tan(d*x+c)^2+a)^2+1/32/a^2/(a^2-2*a*b+b^2)*(a-b)*(1/2*
(22*a^2*(a*b)^(1/2)-15*a*b*(a*b)^(1/2)+5*b^2*(a*b)^(1/2)-12*a^3+a^2*b-a*b^
2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+
c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2*(22*a^2*(a*b)^(1/2)-15*a*b*(a*b)^(1/
2)+5*b^2*(a*b)^(1/2)+12*a^3-a^2*b+a*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a
)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))))
```

3.233.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6215 vs. 2(295) = 590.

Time = 2.37 (sec) , antiderivative size = 6215, normalized size of antiderivative = 17.91

$$\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.233.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**2/(a-b*sin(d*x+c)**4)**3,x)
```

output
```
Timed out
```

3.233.7 Maxima [F]

$$\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \int -\frac{\sin\left(dx+c\right)^2}{\left(b\sin\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
1/16*(4*(96*a^3*b^2 + 36*a^2*b^3 - 53*a*b^4 + 35*b^5)*cos(4*d*x + 4*c)*sin
(2*d*x + 2*c) + ((12*a^2*b^3 - 11*a*b^4 + 5*b^5)*sin(14*d*x + 14*c) - (104
*a^2*b^3 - 85*a*b^4 + 35*b^5)*sin(12*d*x + 12*c) - (320*a^3*b^2 - 652*a^2*
b^3 + 407*a*b^4 - 105*b^5)*sin(10*d*x + 10*c) + (1408*a^3*b^2 - 1696*a^2*b
^3 + 865*a*b^4 - 175*b^5)*sin(8*d*x + 8*c) + (320*a^3*b^2 + 756*a^2*b^3 -
849*a*b^4 + 175*b^5)*sin(6*d*x + 6*c) - (248*a^2*b^3 - 383*a*b^4 + 105*b^5
)*sin(4*d*x + 4*c) - (12*a^2*b^3 + 77*a*b^4 - 35*b^5)*sin(2*d*x + 2*c))*co
s(16*d*x + 16*c) + 2*(2*(96*a^3*b^2 + 36*a^2*b^3 - 53*a*b^4 + 35*b^5)*sin(
12*d*x + 12*c) + 8*(64*a^3*b^2 - 196*a^2*b^3 + 125*a*b^4 - 35*b^5)*sin(10*
d*x + 10*c) - 3*(512*a^4*b + 1024*a^3*b^2 - 1556*a^2*b^3 + 865*a*b^4 - 175
*b^5)*sin(8*d*x + 8*c) - 16*(128*a^3*b^2 + 124*a^2*b^3 - 173*a*b^4 + 35*b^
5)*sin(6*d*x + 6*c) + 2*(96*a^3*b^2 + 324*a^2*b^3 - 649*a*b^4 + 175*b^5)*s
in(4*d*x + 4*c) + 24*(4*a^2*b^3 + 11*a*b^4 - 5*b^5)*sin(2*d*x + 2*c))*cos(
14*d*x + 14*c) + 2*(2*(2560*a^4*b - 4128*a^3*b^2 + 3644*a^2*b^3 - 1379*a*b
^4 + 245*b^5)*sin(10*d*x + 10*c) - (9216*a^4*b - 25984*a^3*b^2 + 21304*a^2
*b^3 - 8575*a*b^4 + 1225*b^5)*sin(8*d*x + 8*c) - 2*(2560*a^4*b + 480*a^3*b
^2 - 7908*a^2*b^3 + 5033*a*b^4 - 735*b^5)*sin(6*d*x + 6*c) + 4*(576*a^3*b^
2 - 1696*a^2*b^3 + 1323*a*b^4 - 245*b^5)*sin(4*d*x + 4*c) + 2*(96*a^3*b^2
+ 324*a^2*b^3 - 649*a*b^4 + 175*b^5)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c)
+ 2*((40960*a^5 - 24064*a^4*b - 22080*a^3*b^2 + 27516*a^2*b^3 - 11095*a...
```

3.233.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1184 vs. 2(295) = 590.

Time = 1.98 (sec) , antiderivative size = 1184, normalized size of antiderivative = 3.41

$$\int \frac{\sin^2(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/64*((30*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b - 72*sqrt(a^2 - a*b +
sqrt(a*b)*(a - b))*a^3*b^2 + 14*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^
3 + 4*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^4 + 36*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*sqrt(a*b)*a^4 - 105*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqr
t(a*b)*a^3*b + 69*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 -
19*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - 5*sqrt(a^2 - a*b
+ sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*(pi*floor((d*x + c)/pi + 1/2) + arctan
(tan(d*x + c)/sqrt((a^5 - 2*a^4*b + a^3*b^2 + sqrt((a^5 - 2*a^4*b + a^3*b^
2)^2 - (a^5 - 2*a^4*b + a^3*b^2)*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)))/(
a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3))))*abs(-a + b)/(3*a^9*b - 18*a^8*b^2
+ 41*a^7*b^3 - 44*a^6*b^4 + 21*a^5*b^5 - 2*a^4*b^6 - a^3*b^7) + (30*sqrt(a
^2 - a*b - sqrt(a*b)*(a - b))*a^4*b - 72*sqrt(a^2 - a*b - sqrt(a*b)*(a - b
))*a^3*b^2 + 14*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^3 + 4*sqrt(a^2 -
 a*b - sqrt(a*b)*(a - b))*a*b^4 - 36*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*s
qrt(a*b)*a^4 + 105*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 6
9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + 19*sqrt(a^2 - a*
b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 + 5*sqrt(a^2 - a*b - sqrt(a*b)*(a -
 b))*sqrt(a*b)*b^4)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sq
rt((a^5 - 2*a^4*b + a^3*b^2 - sqrt((a^5 - 2*a^4*b + a^3*b^2)^2 - (a^5 - 2*
a^4*b + a^3*b^2)*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)))/(a^5 - 3*a^4*b...
```

3.233.9 Mupad [B] (verification not implemented)

Time = 19.03 (sec) , antiderivative size = 6646, normalized size of antiderivative = 19.15

$$\int \frac{\sin^2(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(c + d*x)^2/(a - b*sin(c + d*x)^4)^3,x)
```

output
```
- ((tan(c + d*x)*(5*a - 2*b))/(16*(a^2 - 2*a*b + b^2)) + (3*tan(c + d*x)^3
*(a*b + 10*a^2 - 3*b^2))/(32*a*(a^2 - 2*a*b + b^2)) + (5*tan(c + d*x)^7*(3
*a*b + 2*a^2 - b^2))/(32*a^2*(a - b)) + (3*tan(c + d*x)^5*(2*a*b + 5*a^2 -
 3*b^2))/(16*a*(a - b)^2))/(d*(tan(c + d*x)^8*(a^2 - 2*a*b + b^2) + a^2 -
tan(c + d*x)^4*(2*a*b - 6*a^2) - tan(c + d*x)^6*(4*a*b - 4*a^2) + 4*a^2*ta
n(c + d*x)^2)) - (atan(((((163840*a^9*b + 65536*a^5*b^5 - 360448*a^6*b^4 +
 688128*a^7*b^3 - 557056*a^8*b^2)/(32768*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*
b^2)) - (tan(c + d*x)*(-(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2)
- 144*a^9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b^2 + 349*a^2
*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/
2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3
- a^14*b^2)))^(1/2)*(16384*a^10*b - 16384*a^5*b^6 + 81920*a^6*b^5 - 163840
*a^7*b^4 + 163840*a^8*b^3 - 81920*a^9*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3
- 3*a^4*b^2)))*(-(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a
^9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a
^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16
384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*
b^2)))^(1/2) - (tan(c + d*x)*(460*a^4*b - 149*a*b^4 + 144*a^5 + 25*b^5 + 4
43*a^2*b^3 - 635*a^3*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*(-
(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 15*a^5*...
```

3.233. $\int \frac{\sin^2(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

3.234 $\int \frac{1}{(a-b\sin^4(c+dx))^3}\,dx$

3.234.1 Optimal result

Integrand size = 15, antiderivative size = 319

$$\begin{aligned}\int \frac{1}{(a-b\sin^4(c+dx))^3}\,dx &= \frac{\left(32a-50\sqrt{a}\sqrt{b}+21b\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d} \\ &+ \frac{\left(32a+50\sqrt{a}\sqrt{b}+21b\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}d} \\ &- \frac{b^2\tan(c+dx)\left(3a+b+4(a+b)\tan^2(c+dx)\right)}{8a(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} \\ &- \frac{b\tan(c+dx)\left(\frac{17a^2-40ab+7b^2}{(a-b)^3}+\frac{(33a-13b)\tan^2(c+dx)}{(a-b)^2}\right)}{32a^2d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)}\end{aligned}$$

output
```
1/64*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(32*a+21*b-50*a^(1
/2)*b^(1/2))/a^(11/4)/d/(a^(1/2)-b^(1/2))^(5/2)+1/64*arctan((a^(1/2)+b^(1/
2))^(1/2)*tan(d*x+c)/a^(1/4))*(32*a+21*b+50*a^(1/2)*b^(1/2))/a^(11/4)/d/(a
^(1/2)+b^(1/2))^(5/2)-1/8*b^2*tan(d*x+c)*(3*a+b+4*(a+b)*tan(d*x+c)^2)/a/(a
-b)^3/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)^2-1/32*b*tan(d*x+c)*((17*a
^2-40*a*b+7*b^2)/(a-b)^3+(33*a-13*b)*tan(d*x+c)^2/(a-b)^2)/a^2/d/(a+2*a*ta
n(d*x+c)^2+(a-b)*tan(d*x+c)^4)
```

3.234.2 Mathematica [A] (verified)

Time = 8.21 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$$
$$= \frac{\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\left(32a+50\sqrt{a}\sqrt{b}+21b\right)\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(32a-50\sqrt{a}\sqrt{b}+21b\right)\text{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{8\sqrt{}}{}}{64a^{5/2}(a-b)^2d}$$

input
```
Integrate[(a - b*Sin[c + d*x]^4)^(-3),x]
```

output
```
(((Sqrt[a] - Sqrt[b])^2*(32*a + 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTan[((Sqrt[a
] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sq
rt[b]] - ((Sqrt[a] + Sqrt[b])^2*(32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTanh
[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a +
 Sqrt[a]*Sqrt[b]] + (8*Sqrt[a]*b*(-19*a + 10*b + (6*a - 3*b)*Cos[2*(c + d*
x)])*Sin[2*(c + d*x)])/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*
x)]) + (64*a^(3/2)*(a - b)*b*(-6*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/(-8
*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^2)/(64*a^(5/2)*(a -
b)^2*d)
```

3.234.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3688, 1517, 27, 2206, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\left(a-b\sin(c+dx)^4\right)^3}dx$$
$$\Big\downarrow \text{ 3688}$$

$$\frac{\int \frac{\left(\tan^2(c+dx)+1\right)^5}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^3}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1517}$$

$$\frac{-\frac{\int -\frac{2\left(\frac{8a^2b\tan^6(c+dx)}{a-b}+\frac{8a^2(3a-5b)b\tan^4(c+dx)}{(a-b)^2}+\frac{4ab\left(6a^3-18ba^2+15b^2a-5b^3\right)\tan^2(c+dx)}{(a-b)^3}+\frac{ab\left(8a^3-24ba^2+27b^2a-7b^3\right)}{(a-b)^3}\right)}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{16a^2b}-\frac{b^2\tan}{8a(a-b)^3(}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\frac{8a^2b\tan^6(c+dx)}{a-b}+\frac{8a^2(3a-5b)b\tan^4(c+dx)}{(a-b)^2}+\frac{4ab\left(6a^3-18ba^2+15b^2a-5b^3\right)\tan^2(c+dx)}{(a-b)^3}+\frac{ab\left(8a^3-24ba^2+27b^2a-7b^3\right)}{(a-b)^3}}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{8a^2b}-\frac{b^2\tan(c+dx)(}{8a(a-b)^3((a-b)\tan}}{d}$$

$$\Big\downarrow \text{2206}$$

$$\frac{\frac{-\frac{\int -\frac{2a^2b^2\left(32a^2-47ba+21b^2+\left(32a^2-33ba+13b^2\right)\tan^2(c+dx)\right)}{(a-b)^2\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}d\tan(c+dx)}{8a^2b}-\frac{b^2\tan(c+dx)\left(\frac{17a^2-40ab+7b^2}{(a-b)^3}+\frac{(33a-13b)\tan^2(c+dx)}{(a-b)^2}\right)}{4\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}}{8a^2b}-\frac{b^2\tan(c+dx}{8a(a-b)^3((a-b)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{b\int \frac{32a^2-47ba+21b^2+\left(32a^2-33ba+13b^2\right)\tan^2(c+dx)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{4(a-b)^2}-\frac{b^2\tan(c+dx)\left(\frac{17a^2-40ab+7b^2}{(a-b)^3}+\frac{(33a-13b)\tan^2(c+dx)}{(a-b)^2}\right)}{4\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}}{8a^2b}-\frac{b^2\tan(c+dx)(4(a+b)}{8a(a-b)^3((a-b)\tan^4(c+d}}{d}$$

$$\Big\downarrow \text{1480}$$

$$\frac{\frac{b\left(\frac{\left(50\sqrt{a}\sqrt{b}+32a+21b\right)\left(\sqrt{a}-\sqrt{b}\right)^3\int \frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tan(c+dx)}{2\sqrt{a}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^3\left(-50\sqrt{a}\sqrt{b}+32a+21b\right)\int \frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\tan(}{2\sqrt{a}}\right.}{4(a-b)^2}}{8a^2b}}{d}$$

$$\Big\downarrow \text{218}$$

3.234. $\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

$$\frac{\frac{\frac{b\left(\frac{\left(50\sqrt{a}\sqrt{b}+32a+21b\right)\left(\sqrt{a}-\sqrt{b}\right)^2\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(-50\sqrt{a}\sqrt{b}+32a+21b\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{4(a-b)^2}-\frac{b^2\tan(c+dx)\left(\frac{17a^2-}{(}\right.}{4\left((a-b)\tan\right.}}{8a^2b}}{d}$$

input
```
Int[(a - b*Sin[c + d*x]^4)^(-3),x]
```

output
```
(-1/8*(b^2*Tan[c + d*x]*(3*a + b + 4*(a + b)*Tan[c + d*x]^2))/(a*(a - b)^3
*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)^2) + ((b*(((Sqrt[a] + S
qrt[b])^2*(32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b
]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*Sqrt[Sqrt[a] - Sqrt[b]]) + ((Sqrt[a]
 - Sqrt[b])^2*(32*a + 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTan[(Sqrt[Sqrt[a] + Sq
rt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[b]])))/(4*(a
 - b)^2) - (b^2*Tan[c + d*x]*((17*a^2 - 40*a*b + 7*b^2)/(a - b)^3 + ((33*a
 - 13*b)*Tan[c + d*x]^2)/(a - b)^2))/(4*(a + 2*a*Tan[c + d*x]^2 + (a - b)*
Tan[c + d*x]^4)))/(8*a^2*b))/d
```

3.234.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

3.234. $\int \frac{1}{(a-b\sin^4(c+dx))^3}\,dx$

3.234.4 Maple [A] (verified)

Time = 5.66 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{\frac{-\frac{(33a-13b)b\left(\tan^7(dx+c)\right)}{32a^2(a-b)}-\frac{b\left(83a^2-66ab+7b^2\right)\left(\tan^5(dx+c)\right)}{32a^2\left(a^2-2ab+b^2\right)}-\frac{(67a-43b)b\left(\tan^3(dx+c)\right)}{32a\left(a^2-2ab+b^2\right)}-\frac{b(17a-11b)\tan(dx+c)}{32a\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^4(dx+c)\right)a-b\left(\tan^4(dx+c)\right)+2a\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{(a-b)\left(\frac{(32a}{}\right.}{}}{}$
default	$\frac{\frac{-\frac{(33a-13b)b\left(\tan^7(dx+c)\right)}{32a^2(a-b)}-\frac{b\left(83a^2-66ab+7b^2\right)\left(\tan^5(dx+c)\right)}{32a^2\left(a^2-2ab+b^2\right)}-\frac{(67a-43b)b\left(\tan^3(dx+c)\right)}{32a\left(a^2-2ab+b^2\right)}-\frac{b(17a-11b)\tan(dx+c)}{32a\left(a^2-2ab+b^2\right)}}{\left(\left(\tan^4(dx+c)\right)a-b\left(\tan^4(dx+c)\right)+2a\left(\tan^2(dx+c)\right)+a\right)^2}+\frac{(a-b)\left(\frac{(32a}{}\right.}{}}{}$
risch	Expression too large to display

input
```
int(1/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*((-1/32*(33*a-13*b)/a^2*b/(a-b)*tan(d*x+c)^7-1/32/a^2*b*(83*a^2-66*a*b
+7*b^2)/(a^2-2*a*b+b^2)*tan(d*x+c)^5-1/32*(67*a-43*b)/a*b/(a^2-2*a*b+b^2)*
tan(d*x+c)^3-1/32*b*(17*a-11*b)/a/(a^2-2*a*b+b^2)*tan(d*x+c))/(tan(d*x+c)^
4*a-b*tan(d*x+c)^4+2*a*tan(d*x+c)^2+a)^2+1/32/a^2/(a^2-2*a*b+b^2)*(a-b)*(1
/2*(32*a^2*(a*b)^(1/2)-33*a*b*(a*b)^(1/2)+13*b^2*(a*b)^(1/2)-46*a^2*b+55*a
*b^2-21*b^3)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b
)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2*(32*a^2*(a*b)^(1/2)-33*a*b
*(a*b)^(1/2)+13*b^2*(a*b)^(1/2)+46*a^2*b-55*a*b^2+21*b^3)/(a*b)^(1/2)/(a-b
)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(
a-b))^(1/2))))
```

3.234.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6152 vs. $2(271) = 542$.

Time = 2.51 (sec) , antiderivative size = 6152, normalized size of antiderivative = 19.29

$$\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.234.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a - b\sin^4(c + dx)\right)^3}\, dx = \text{Timed out}$$

input
```
integrate(1/(a-b*sin(d*x+c)**4)**3,x)
```

output
```
Timed out
```

3.234.7 Maxima [F]

$$\int \frac{1}{\left(a - b\sin^4(c + dx)\right)^3}\, dx = \int -\frac{1}{\left(b\sin\left(dx + c\right)^4 - a\right)^3}\, dx$$

input
```
integrate(1/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
1/8*(4*(120*a^2*b^3 - 77*a*b^4 + 14*b^5)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + ((7*a*b^4 - 4*b^5)*sin(14*d*x + 14*c) - (32*a^2*b^3 + 2*a*b^4 - 7*b^5)*
sin(12*d*x + 12*c) - (16*a^2*b^3 - 3*a*b^4 - 28*b^5)*sin(10*d*x + 10*c) +
3*(256*a^3*b^2 - 320*a^2*b^3 + 166*a*b^4 - 35*b^5)*sin(8*d*x + 8*c) + (784
*a^2*b^3 - 723*a*b^4 + 140*b^5)*sin(6*d*x + 6*c) - (160*a^2*b^3 - 266*a*b^
4 + 91*b^5)*sin(4*d*x + 4*c) - (55*a*b^4 - 28*b^5)*sin(2*d*x + 2*c))*cos(1
6*d*x + 16*c) + 2*(2*(120*a^2*b^3 - 77*a*b^4 + 14*b^5)*sin(12*d*x + 12*c)
- 8*(48*a^2*b^3 - 55*a*b^4 + 28*b^5)*sin(10*d*x + 10*c) - (3968*a^3*b^2 -
5024*a^2*b^3 + 2621*a*b^4 - 560*b^5)*sin(8*d*x + 8*c) - 16*(224*a^2*b^3 -
209*a*b^4 + 42*b^5)*sin(6*d*x + 6*c) + 2*(376*a^2*b^3 - 613*a*b^4 + 210*b^
5)*sin(4*d*x + 4*c) + 8*(31*a*b^4 - 16*b^5)*sin(2*d*x + 2*c))*cos(14*d*x +
 14*c) + 2*(2*(1152*a^3*b^2 - 520*a^2*b^3 - 455*a*b^4 + 294*b^5)*sin(10*d*
x + 10*c) - (8192*a^4*b - 23296*a^3*b^2 + 21376*a^2*b^3 - 9394*a*b^4 + 171
5*b^5)*sin(8*d*x + 8*c) - 2*(5248*a^3*b^2 - 10888*a^2*b^3 + 6433*a*b^4 - 1
078*b^5)*sin(6*d*x + 6*c) + 4*(512*a^3*b^2 - 1520*a^2*b^3 + 1330*a*b^4 - 3
43*b^5)*sin(4*d*x + 4*c) + 2*(376*a^2*b^3 - 613*a*b^4 + 210*b^5)*sin(2*d*x
 + 2*c))*cos(12*d*x + 12*c) + 2*((51200*a^4*b - 84864*a^3*b^2 + 56016*a^2*
b^3 - 18081*a*b^4 + 1960*b^5)*sin(8*d*x + 8*c) + 8*(6400*a^3*b^2 - 8608*a^
2*b^3 + 3437*a*b^4 - 392*b^5)*sin(6*d*x + 6*c) - 2*(5248*a^3*b^2 - 10888*a
^2*b^3 + 6433*a*b^4 - 1078*b^5)*sin(4*d*x + 4*c) - 16*(224*a^2*b^3 - 20...
```

3.234.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1131 vs. $2(271) = 542$.

Time = 0.71 (sec) , antiderivative size = 1131, normalized size of antiderivative = 3.55

$$\int \frac{1}{(a - b\sin^4(c + dx))^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/64*((96*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4 - 333*sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*a^3*b + 313*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^2
- 79*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^3 - 21*sqrt(a^2 - a*b + sqrt(
a*b)*(a - b))*b^4 + 42*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 -
 108*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 34*sqrt(a^2 - a
*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 + 8*sqrt(a^2 - a*b + sqrt(a*b)*(a
- b))*sqrt(a*b)*b^3)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/s
qrt((a^5 - 2*a^4*b + a^3*b^2 + sqrt((a^5 - 2*a^4*b + a^3*b^2)^2 - (a^5 - 2
*a^4*b + a^3*b^2)*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)))/(a^5 - 3*a^4*b +
 3*a^3*b^2 - a^2*b^3))))*abs(-a + b)/(3*a^9 - 18*a^8*b + 41*a^7*b^2 - 44*a
^6*b^3 + 21*a^5*b^4 - 2*a^4*b^5 - a^3*b^6) + (96*sqrt(a^2 - a*b - sqrt(a*b
)*(a - b))*a^4 - 333*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b + 313*sqrt(
a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^2 - 79*sqrt(a^2 - a*b - sqrt(a*b)*(a
- b))*a*b^3 - 21*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^4 - 42*sqrt(a^2 - a
*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 + 108*sqrt(a^2 - a*b - sqrt(a*b)*(a
- b))*sqrt(a*b)*a^2*b - 34*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a
*b^2 - 8*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(pi*floor((d*x
 + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^5 - 2*a^4*b + a^3*b^2 - sqrt
((a^5 - 2*a^4*b + a^3*b^2)^2 - (a^5 - 2*a^4*b + a^3*b^2)*(a^5 - 3*a^4*b +
3*a^3*b^2 - a^2*b^3)))/(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3))))*abs(-a ...
```

3.234.9 Mupad [B] (verification not implemented)

Time = 19.24 (sec) , antiderivative size = 6267, normalized size of antiderivative = 19.65

$$\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a - b*sin(c + d*x)^4)^3,x)
```

output
```
- ((tan(c + d*x)^5*(83*a^2*b - 66*a*b^2 + 7*b^3))/(32*a^2*(a - b)^2) + (ta
n(c + d*x)^7*(33*a*b - 13*b^2))/(32*a^2*(a - b)) + (tan(c + d*x)*(17*a*b -
 11*b^2))/(32*a*(a^2 - 2*a*b + b^2)) + (tan(c + d*x)^3*(67*a*b - 43*b^2))/
(32*a*(a^2 - 2*a*b + b^2)))/(d*(tan(c + d*x)^8*(a^2 - 2*a*b + b^2) + a^2 -
 tan(c + d*x)^4*(2*a*b - 6*a^2) - tan(c + d*x)^6*(4*a*b - 4*a^2) + 4*a^2*t
an(c + d*x)^2)) - (atan((((524288*a^10*b - 344064*a^5*b^6 + 1802240*a^6*b
^5 - 3866624*a^7*b^4 + 4227072*a^8*b^3 - 2342912*a^9*b^2)/(32768*(3*a^8*b
- a^9 + a^6*b^3 - 3*a^7*b^2)) - (tan(c + d*x)*((1920*a^4*(a^11*b)^(1/2) +
441*b^4*(a^11*b)^(1/2) - 1916*a^9*b + 1024*a^10 + 105*a^6*b^4 - 570*a^7*b^
3 + 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2) +
 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12*
b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2)*(16384*a^11*b - 16384*a^6*b^6 + 8
1920*a^7*b^5 - 163840*a^8*b^4 + 163840*a^9*b^3 - 81920*a^10*b^2))/(256*(3*
a^6*b - a^7 + a^4*b^3 - 3*a^5*b^2)))*((1920*a^4*(a^11*b)^(1/2) + 441*b^4*(
a^11*b)^(1/2) - 1916*a^9*b + 1024*a^10 + 105*a^6*b^4 - 570*a^7*b^3 + 1501*
a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2
*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*
a^13*b^3 - 10*a^14*b^2)))^(1/2) - (tan(c + d*x)*(1024*a^5*b - 2141*a*b^5 +
 441*b^6 + 4099*a^2*b^4 - 3139*a^3*b^3 + 4*a^4*b^2))/(256*(3*a^6*b - a^7 +
 a^4*b^3 - 3*a^5*b^2)))*((1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a^11*b)^(1...
```

3.234. $\int \frac{1}{(a-b\sin^4(c+dx))^3}\,dx$

3.235 $\int \frac{\csc^2(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

3.235.1 Optimal result

Integrand size = 24, antiderivative size = 357

$$\int \frac{\csc^2(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \frac{3\sqrt{b}\left(20a-34\sqrt{a}\sqrt{b}+15b\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d} - \frac{3\sqrt{b}\left(20a+34\sqrt{a}\sqrt{b}+15b\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}d} - \frac{\cot(c+dx)}{a^3d} - \frac{b^2\tan(c+dx)\left(a(a+3b)+(a^2+6ab+b^2)\tan^2(c+dx)\right)}{8a^2(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{b\tan(c+dx)\left(\frac{2a^2(9a-17b)}{(a-b)^3}+\frac{(18a^2+15ab-13b^2)\tan^2(c+dx)}{(a-b)^2}\right)}{32a^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)}$$

output
```
-cot(d*x+c)/a^3/d+3/64*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*
b^(1/2)*(20*a+15*b-34*a^(1/2)*b^(1/2))/a^(13/4)/d/(a^(1/2)-b^(1/2))^(5/2)-
3/64*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*b^(1/2)*(20*a+15*b
+34*a^(1/2)*b^(1/2))/a^(13/4)/d/(a^(1/2)+b^(1/2))^(5/2)-1/8*b^2*tan(d*x+c)
*(a*(a+3*b)+(a^2+6*a*b+b^2)*tan(d*x+c)^2)/a^2/(a-b)^3/d/(a+2*a*tan(d*x+c)^
2+(a-b)*tan(d*x+c)^4)^2-1/32*b*tan(d*x+c)*(2*a^2*(9*a-17*b)/(a-b)^3+(18*a^
2+15*a*b-13*b^2)*tan(d*x+c)^2/(a-b)^2)/a^3/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan
(d*x+c)^4)
```

3.235.2 Mathematica [A] (verified)

Time = 11.40 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = -\frac{\frac{3\sqrt{b}(20a+34\sqrt{a}\sqrt{b}+15b)\arctan\left(\frac{(\sqrt{a}+\sqrt{b})\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{(\sqrt{a}+\sqrt{b})^2\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{3\sqrt{b}(20a-34\sqrt{a}\sqrt{b}+15b)\operatorname{arctanh}\left(\frac{(\sqrt{a}-\sqrt{b})\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{(\sqrt{a}-\sqrt{b})^2\sqrt{-a+\sqrt{a}\sqrt{b}}}+64\cot(c+}{64a^3d}$$

input
```
Integrate[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
-1/64*((3*Sqrt[b]*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTan[((Sqrt[a] + Sq
rt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])^2*Sq
rt[a + Sqrt[a]*Sqrt[b]]) + (3*Sqrt[b]*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*A
rcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((S
qrt[a] - Sqrt[b])^2*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + 64*Cot[c + d*x] + (4*b*(
28*a^2 + 3*a*b - 13*b^2 + b*(-19*a + 13*b)*Cos[2*(c + d*x)])*Sin[2*(c + d*
x)])/((a - b)^2*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) +
 (128*a*b*(2*a + b - b*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/((a - b)*(-8*a
+ 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^2))/(a^3*d)
```

3.235.3 Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3696, 1673, 27, 2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^2\,(a-b\sin(c+dx)^4)^3}dx$$

$$\Big\downarrow \text{3696}$$

$$\frac{\int \frac{\cot^2(c+dx)\left(\tan^2(c+dx)+1\right)^6}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^3}d\tan(c+dx)}{d}$$

$\Big\downarrow$ 1673

$$-\frac{\frac{\int -\frac{2\cot^2(c+dx)\left(\frac{8a^2b\tan^8(c+dx)}{a-b}+\frac{16a^2(2a-3b)b\tan^6(c+dx)}{(a-b)^2}+\frac{b\left(48a^4-136ba^3+115b^2a^2-30b^3a-5b^4\right)\tan^4(c+dx)}{(a-b)^3}+\frac{ab\left(32a^3-96ba^2+97b^2a-29b^3\right)\tan^2(c+}{(a-b)^3}\right.}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}}{16a^2b}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\cot^2(c+dx)\left(\frac{8a^2b\tan^8(c+dx)}{a-b}+\frac{16a^2(2a-3b)b\tan^6(c+dx)}{(a-b)^2}+\frac{b\left(48a^4-136ba^3+115b^2a^2-30b^3a-5b^4\right)\tan^4(c+dx)}{(a-b)^3}+\frac{ab\left(32a^3-96ba^2+97b^2a-29b^3\right)\tan^2(c+dx)}{(a-b)^3}+\right.}{\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}}{8a^2b}}{d}$$

$\Big\downarrow$ 2198

$$-\frac{\frac{\frac{\int -\frac{2\cot^2(c+dx)\left(\frac{ab^2\left(32a^3-18ba^2-15b^2a+13b^3\right)\tan^4(c+dx)}{(a-b)^2}+\frac{2a^2b^2\left(32a^2-55ba+26b^2\right)\tan^2(c+dx)}{(a-b)^2}+32a^2b^2\right)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{8a^2b}-\frac{b^2\tan(c+dx)\left(\frac{\left(18a^2+15ab-1\right.}{(a\ldots}\right.}{4a\left((a-b)\tan^4(c+\ldots\right.}}{8a^2b}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\cot^2(c+dx)\left(\frac{ab^2\left(32a^3-18ba^2-15b^2a+13b^3\right)\tan^4(c+dx)}{(a-b)^2}+\frac{2a^2b^2\left(32a^2-55ba+26b^2\right)\tan^2(c+dx)}{(a-b)^2}+32a^2b^2\right)}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{4a^2b}-\frac{b^2\tan(c+dx)\left(\frac{\left(18a^2+15ab-13b^2\right)}{(a-b)^2}\right.}{4a\left((a-b)\tan^4(c+dx)+2\right.}}{8a^2b}}{d}$$

$\Big\downarrow$ 2195

$$\frac{\frac{\frac{\int \left(\frac{3a\left(\left(26a^2-37ba+15b^2\right)\tan^2(c+dx)+2a(3a-2b)\right)b^3}{(a-b)^2\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}+32a\cot^2(c+dx)b^2\right)d\tan(c+dx)}{4a^2b}-\frac{b^2\tan(c+dx)\left(\frac{\left(18a^2+15ab-13b^2\right)\tan^2(c+dx)}{(a-b)^2}+\frac{2a^2(9a-17b)}{(a-b)^3}\right)}{4a\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}}{8a^2b}-\ldots}{d}$$

$\Big\downarrow$ 2009

3.235. $\displaystyle\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

$$\frac{\frac{\frac{3a^{3/4}b^{5/2}\left(-34\sqrt{a}\sqrt{b}+20a+15b\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3a^{3/4}b^{5/2}\left(34\sqrt{a}\sqrt{b}+20a+15b\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-32ab^2\cot(c+dx)}{4a^2b}-\frac{b^2\tan(c+}{}}{8a^2b}}{d}$$

input
```
Int[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4)^3,x]
```

output
```
(-1/8*(b^2*Tan[c + d*x]*((a*(a + 3*b))/(a - b)^3 + ((a^2 + 6*a*b + b^2)*Ta
n[c + d*x]^2)/(a - b)^3))/(a^2*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d
*x]^4)^2) + (((3*a^(3/4)*b^(5/2)*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTan
[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*(Sqrt[a] - Sqrt[b])^(
5/2)) - (3*a^(3/4)*b^(5/2)*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTan[(Sqrt
[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*(Sqrt[a] + Sqrt[b])^(5/2))
- 32*a*b^2*Cot[c + d*x])/(4*a^2*b) - (b^2*Tan[c + d*x]*((2*a^2*(9*a - 17*b
))/(a - b)^3 + ((18*a^2 + 15*a*b - 13*b^2)*Tan[c + d*x]^2)/(a - b)^2))/(4*
a*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)))/(8*a^2*b))/d
```

3.235.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1673
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 + c*x^4)^(p +
 1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
 e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5
) - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1]
&& ILtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.235. $\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3696
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)
^(m/2 + 2*p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &
& IntegerQ[m/2] && IntegerQ[p]
```

3.235.4 Maple [A] (verified)

Time = 6.47 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{b\left(\frac{-\frac{(18a^2+15ab-13b^2)(\tan^7(dx+c))}{32(a-b)}-\frac{a(27a^2-2ab-13b^2)(\tan^5(dx+c))}{16(a^2-2ab+b^2)}-\frac{(54a^2-13ab-17b^2)a(\tan^3(dx+c))}{32(a^2-2ab+b^2)}-\frac{3a^2(3a-2b)\tan}{16(a^2-2ab+}}{((\tan^4(dx+c))a-b(\tan^4(dx+c))+2a(\tan^2(dx+c))+a)^2}\right.}{}$
default	$\frac{b\left(\frac{-\frac{(18a^2+15ab-13b^2)(\tan^7(dx+c))}{32(a-b)}-\frac{a(27a^2-2ab-13b^2)(\tan^5(dx+c))}{16(a^2-2ab+b^2)}-\frac{(54a^2-13ab-17b^2)a(\tan^3(dx+c))}{32(a^2-2ab+b^2)}-\frac{3a^2(3a-2b)\tan}{16(a^2-2ab+}}{((\tan^4(dx+c))a-b(\tan^4(dx+c))+2a(\tan^2(dx+c))+a)^2}\right.}{}$
risch	Expression too large to display

input
```
int(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b/a^3*((-1/32*(18*a^2+15*a*b-13*b^2)/(a-b)*tan(d*x+c)^7-1/16*a*(27*a^
2-2*a*b-13*b^2)/(a^2-2*a*b+b^2)*tan(d*x+c)^5-1/32*(54*a^2-13*a*b-17*b^2)*a
/(a^2-2*a*b+b^2)*tan(d*x+c)^3-3/16*a^2*(3*a-2*b)/(a^2-2*a*b+b^2)*tan(d*x+c
))/(tan(d*x+c)^4*a-b*tan(d*x+c)^4+2*a*tan(d*x+c)^2+a)^2+3/32/(a^2-2*a*b+b^
2)*(a-b)*(1/2*(26*a^2*(a*b)^(1/2)-37*a*b*(a*b)^(1/2)+15*b^2*(a*b)^(1/2)+20
*a^3-27*a^2*b+11*a*b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*ar
ctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*(26*a^2*(a*b)^(1/
2)-37*a*b*(a*b)^(1/2)+15*b^2*(a*b)^(1/2)-20*a^3+27*a^2*b-11*a*b^2)/(a*b)^(
1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)
^(1/2)-a)*(a-b))^(1/2))))-1/a^3/tan(d*x+c))
```

3.235. $\int \frac{\csc^2(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx$

3.235.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6323 vs. 2(305) = 610.

Time = 2.65 (sec) , antiderivative size = 6323, normalized size of antiderivative = 17.71

$$\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.235.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**2/(a-b*sin(d*x+c)**4)**3,x)
```

output
```
Timed out
```

3.235.7 Maxima [F]

$$\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \int -\frac{\csc\left(dx+c\right)^2}{\left(b\sin\left(dx+c\right)^4-a\right)^3}\,dx$$

input
```
integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
1/16*(12*(160*a^3*b^3 - 57*a^2*b^4 - 195*a*b^5 + 135*b^6)*cos(4*d*x + 4*c)
*sin(2*d*x + 2*c) + (3*(20*a^2*b^4 - 33*a*b^5 + 15*b^6)*sin(16*d*x + 16*c)
 - 12*(43*a^2*b^4 - 68*a*b^5 + 30*b^6)*sin(14*d*x + 14*c) - 4*(400*a^3*b^3
 - 1137*a^2*b^4 + 1031*a*b^5 - 315*b^6)*sin(12*d*x + 12*c) + 12*(592*a^3*b
^3 - 1237*a^2*b^4 + 886*a*b^5 - 210*b^6)*sin(10*d*x + 10*c) + 2*(4096*a^4*
b^2 - 12192*a^3*b^3 + 13634*a^2*b^4 - 7113*a*b^5 + 1575*b^6)*sin(8*d*x + 8
*c) + 4*(880*a^3*b^3 - 2855*a^2*b^4 + 2512*a*b^5 - 630*b^6)*sin(6*d*x + 6*
c) - 4*(256*a^3*b^3 - 823*a^2*b^4 + 903*a*b^5 - 315*b^6)*sin(4*d*x + 4*c)
- 12*(19*a^2*b^4 - 54*a*b^5 + 30*b^6)*sin(2*d*x + 2*c))*cos(18*d*x + 18*c)
 + 3*(4*(160*a^3*b^3 - 57*a^2*b^4 - 195*a*b^5 + 135*b^6)*sin(14*d*x + 14*c
) + 4*(400*a^3*b^3 - 1671*a^2*b^4 + 1800*a*b^5 - 630*b^6)*sin(12*d*x + 12*
c) - 2*(2560*a^4*b^2 + 3232*a^3*b^3 - 13806*a^2*b^4 + 11469*a*b^5 - 2835*b
^6)*sin(10*d*x + 10*c) - 4*(4864*a^4*b^2 - 14576*a^3*b^3 + 16221*a^2*b^4 -
 8430*a*b^5 + 1890*b^6)*sin(8*d*x + 8*c) - 4*(1840*a^3*b^3 - 6825*a^2*b^4
+ 6243*a*b^5 - 1575*b^6)*sin(6*d*x + 6*c) + 4*(608*a^3*b^3 - 2025*a^2*b^4
+ 2292*a*b^5 - 810*b^6)*sin(4*d*x + 4*c) + 9*(56*a^2*b^4 - 183*a*b^5 + 105
*b^6)*sin(2*d*x + 2*c))*cos(16*d*x + 16*c) + 4*(4*(3200*a^4*b^2 - 7536*a^3
*b^3 + 7612*a^2*b^4 - 3915*a*b^5 + 945*b^6)*sin(12*d*x + 12*c) - 6*(3968*a
^4*b^2 - 14864*a^3*b^3 + 19013*a^2*b^4 - 10224*a*b^5 + 1890*b^6)*sin(10*d*
x + 10*c) - 2*(32768*a^5*b - 117888*a^4*b^2 + 172048*a^3*b^3 - 127323*a...
```

3.235.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2203 vs. $2(305) = 610$.

Time = 1.67 (sec) , antiderivative size = 2203, normalized size of antiderivative = 6.17

$$\int \frac{\csc^2(c+dx)}{(a-b\sin^4(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/64*(3*((78*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b - 267*sqr
t(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 + 241*sqrt(a^2 - a*b +
sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^3 - 53*sqrt(a^2 - a*b + sqrt(a*b)*(a -
b))*sqrt(a*b)*a*b^4 - 15*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^5
)*(a^5 - 2*a^4*b + a^3*b^2)^2*abs(-a + b) + 2*(9*sqrt(a^2 - a*b + sqrt(a*b
)*(a - b))*a^10*b - 51*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^9*b^2 + 108*s
qrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^8*b^3 - 106*sqrt(a^2 - a*b + sqrt(a*b
)*(a - b))*a^7*b^4 + 45*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^6*b^5 - 3*sq
rt(a^2 - a*b + sqrt(a*b)*(a - b))*a^5*b^6 - 2*sqrt(a^2 - a*b + sqrt(a*b)*(
a - b))*a^4*b^7)*abs(a^5 - 2*a^4*b + a^3*b^2)*abs(-a + b) - (60*sqrt(a^2 -
 a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^15 - 441*sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*sqrt(a*b)*a^14*b + 1339*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(
a*b)*a^13*b^2 - 2185*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^12*b^
3 + 2059*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^11*b^4 - 1091*sqr
t(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^10*b^5 + 265*sqrt(a^2 - a*b +
 sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b^6 + 5*sqrt(a^2 - a*b + sqrt(a*b)*(a -
b))*sqrt(a*b)*a^8*b^7 - 11*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a
^7*b^8)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/s
qrt((a^6 - 2*a^5*b + a^4*b^2 + sqrt((a^6 - 2*a^5*b + a^4*b^2)^2 - (a^6 - 2
*a^5*b + a^4*b^2)*(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)))/(a^6 - 3*a^5*...
```

3.235.9 Mupad [B] (verification not implemented)

Time = 19.94 (sec) , antiderivative size = 7364, normalized size of antiderivative = 20.63

$$\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(c + d*x)^2*(a - b*sin(c + d*x)^4)^3),x)
```

output

```
(atan((((-(9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) - 400*a^
11*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 + 1044*a^10*b^2 + 2085*a^2
*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3
)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 -
10*a^16*b^2)))^(1/2)*(2315255808*a^15*b^12 - 201326592*a^14*b^13 - 1207959
5520*a^16*b^11 + 37748736000*a^17*b^10 - 78517370880*a^18*b^9 + 1141521776
64*a^19*b^8 - 118380036096*a^20*b^7 + 87577067520*a^21*b^6 - 45298483200*a
^22*b^5 + 15602810880*a^23*b^4 - 3221225472*a^24*b^3 + 301989888*a^25*b^2
+ tan(c + d*x)*(-(9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) -
 400*a^11*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 + 1044*a^10*b^2 + 2
085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a
^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15
*b^3 - 10*a^16*b^2)))^(1/2)*(2147483648*a^29*b + 2147483648*a^17*b^13 - 25
769803776*a^18*b^12 + 141733920768*a^19*b^11 - 472446402560*a^20*b^10 + 10
63004405760*a^21*b^9 - 1700807049216*a^22*b^8 + 1984274890752*a^23*b^7 - 1
700807049216*a^24*b^6 + 1063004405760*a^25*b^5 - 472446402560*a^26*b^4 + 1
41733920768*a^27*b^3 - 25769803776*a^28*b^2)) + tan(c + d*x)*(3024617472*a
^11*b^13 - 265420800*a^10*b^14 - 15574892544*a^12*b^12 + 47520940032*a^13*
b^11 - 94402510848*a^14*b^10 + 125505110016*a^15*b^9 - 108421447680*a^16*b
^8 + 51536461824*a^17*b^7 + 484835328*a^18*b^6 - 18454413312*a^19*b^5 +...
```

3.235. $\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

3.236 $\int \frac{1}{1-\sin^4(x)}\,dx$

3.236.1 Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \frac{1}{1-\sin^4(x)}\,dx = \frac{\arctan\left(\sqrt{2}\tan(x)\right)}{2\sqrt{2}} + \frac{\tan(x)}{2}$$

output
```
1/4*arctan(2^(1/2)*tan(x))*2^(1/2)+1/2*tan(x)
```

3.236.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{1-\sin^4(x)}\,dx = \frac{1}{4}\left(\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right) + 2\tan(x)\right)$$

input
```
Integrate[(1 - Sin[x]^4)^(-1),x]
```

output
```
(Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]] + 2*Tan[x])/4
```

3.236.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3688, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\sin^4(x)} \, dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{1-\sin(x)^4}dx$$
$$\Big\downarrow \text{ 3688}$$
$$\int \frac{\tan^2(x)+1}{2\tan^2(x)+1}d\tan(x)$$
$$\Big\downarrow \text{ 299}$$
$$\frac{1}{2}\int \frac{1}{2\tan^2(x)+1}d\tan(x)+\frac{\tan(x)}{2}$$
$$\Big\downarrow \text{ 216}$$
$$\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{2\sqrt{2}}+\frac{\tan(x)}{2}$$

input
```
Int[(1 - Sin[x]^4)^(-1),x]
```

output
```
ArcTan[Sqrt[2]*Tan[x]]/(2*Sqrt[2]) + Tan[x]/2
```

3.236.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

3.236.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{\arctan\left(\sqrt{2}\,\tan(x)\right)\sqrt{2}}{4}+\frac{\tan(x)}{2}$	18
risch	$\frac{i}{e^{2ix}+1}+\frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}-3\right)}{8}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}-3\right)}{8}$	52

input
```
int(1/(1-sin(x)^4),x,method=_RETURNVERBOSE)
```

output
```
1/4*arctan(2^(1/2)*tan(x))*2^(1/2)+1/2*tan(x)
```

3.236.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \frac{1}{1-\sin^4(x)}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right)\cos(x)-4\sin(x)}{8\cos(x)}$$

input
```
integrate(1/(1-sin(x)^4),x, algorithm="fricas")
```

output
```
-1/8*(sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x)))
*cos(x) - 4*sin(x))/cos(x)
```

3.236.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 724 vs. $2(20) = 40$.

Time = 21.73 (sec) , antiderivative size = 724, normalized size of antiderivative = 28.96

$$\int \frac{1}{1-\sin^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-sin(x)**4),x)
```

output
```
54608393*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) +
 pi*floor((x/2 - pi/2)/pi))*tan(x/2)**2/(63977712*sqrt(2)*tan(x/2)**2 + 90
478148*tan(x/2)**2 - 90478148 - 63977712*sqrt(2)) + 77227930*sqrt(3 - 2*sq
rt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))*ta
n(x/2)**2/(63977712*sqrt(2)*tan(x/2)**2 + 90478148*tan(x/2)**2 - 90478148
- 63977712*sqrt(2)) - 77227930*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 -
 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(63977712*sqrt(2)*tan(x/2)**2 +
90478148*tan(x/2)**2 - 90478148 - 63977712*sqrt(2)) - 54608393*sqrt(2)*sqr
t(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/
2)/pi))/(63977712*sqrt(2)*tan(x/2)**2 + 90478148*tan(x/2)**2 - 90478148 -
63977712*sqrt(2)) + 9369319*sqrt(2)*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqr
t(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)**2/(63977712*sqrt(
2)*tan(x/2)**2 + 90478148*tan(x/2)**2 - 90478148 - 63977712*sqrt(2)) + 132
50218*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((
x/2 - pi/2)/pi))*tan(x/2)**2/(63977712*sqrt(2)*tan(x/2)**2 + 90478148*tan(
x/2)**2 - 90478148 - 63977712*sqrt(2)) - 13250218*sqrt(2*sqrt(2) + 3)*(ata
n(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(63977712*sqr
t(2)*tan(x/2)**2 + 90478148*tan(x/2)**2 - 90478148 - 63977712*sqrt(2)) - 9
369319*sqrt(2)*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + p
i*floor((x/2 - pi/2)/pi))/(63977712*sqrt(2)*tan(x/2)**2 + 90478148*tan(...
```

3.236.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1}{1-\sin^4(x)}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right) + \frac{1}{2}\tan(x)$$

input
```
integrate(1/(1-sin(x)^4),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(sqrt(2)*tan(x)) + 1/2*tan(x)
```

3.236.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(17) = 34.

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{1}{1-\sin^4(x)}\,dx = \frac{1}{4}\sqrt{2}\left(x + \arctan\left(-\frac{\sqrt{2}\sin(2x) - 2\sin(2x)}{\sqrt{2}\cos(2x) + \sqrt{2} - 2\cos(2x) + 2}\right)\right) + \frac{1}{2}\tan(x)$$

input
```
integrate(1/(1-sin(x)^4),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x)
 + sqrt(2) - 2*cos(2*x) + 2))) + 1/2*tan(x)
```

3.236.9 Mupad [B] (verification not implemented)

Time = 13.77 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1}{1-\sin^4(x)}\,dx = \frac{\tan(x)}{2} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan(x)\right)}{4}$$

input
```
int(-1/(sin(x)^4 - 1),x)
```

output
```
tan(x)/2 + (2^(1/2)*atan(2^(1/2)*tan(x)))/4
```

3.237 $\int \frac{1}{a+b\sin^4(x)}\,dx$

3.237.1 Optimal result

Integrand size = 10, antiderivative size = 487

$$\int \frac{1}{a+b\sin^4(x)}\,dx = -\frac{\left(\sqrt{a}+\sqrt{a+b}\right)\arctan\left(\frac{\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}-\sqrt{2}(a+b)^{3/4}\tan(x)}{\sqrt[4]{a}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}$$
$$+\frac{\left(\sqrt{a}+\sqrt{a+b}\right)\arctan\left(\frac{\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}+\sqrt{2}(a+b)^{3/4}\tan(x)}{\sqrt[4]{a}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}$$
$$+\frac{\left(\sqrt{a}-\sqrt{a+b}\right)\log\left(\sqrt{a}\sqrt[4]{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\tan(x)+(a+b)^{3/4}\tan^2(x)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}$$
$$-\frac{\left(\sqrt{a}-\sqrt{a+b}\right)\log\left(\sqrt{a}\sqrt[4]{a+b}+\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\tan(x)+(a+b)^{3/4}\tan^2(x)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}$$

output
```
1/8*ln((a+b)^(1/4)*a^(1/2)-a^(1/4)*2^(1/2)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)
*tan(x)+(a+b)^(3/4)*tan(x)^2)*(a^(1/2)-(a+b)^(1/2))/a^(3/4)/(a+b)^(1/4)*2^
(1/2)/(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)-1/8*ln((a+b)^(1/4)*a^(1/2)+a^(1/4)*2
^(1/2)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)*tan(x)+(a+b)^(3/4)*tan(x)^2)*(a^(1/
2)-(a+b)^(1/2))/a^(3/4)/(a+b)^(1/4)*2^(1/2)/(a+b-a^(1/2)*(a+b)^(1/2))^(1/2
)-1/4*arctan((a^(1/4)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)-(a+b)^(3/4)*2^(1/2)*
tan(x))/a^(1/4)/(a+b+a^(1/2)*(a+b)^(1/2))^(1/2))*(a^(1/2)+(a+b)^(1/2))/a^(
3/4)/(a+b)^(1/4)*2^(1/2)/(a+b+a^(1/2)*(a+b)^(1/2))^(1/2)+1/4*arctan((a^(1/
4)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)+(a+b)^(3/4)*2^(1/2)*tan(x))/a^(1/4)/(a+
b+a^(1/2)*(a+b)^(1/2))^(1/2))*(a^(1/2)+(a+b)^(1/2))/a^(3/4)/(a+b)^(1/4)*2^
(1/2)/(a+b+a^(1/2)*(a+b)^(1/2))^(1/2)
```

3.237.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.01 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.30

$$\int \frac{1}{a+b\sin^4(x)}\,dx = \frac{\left(\sqrt{a}-i\sqrt{b}\right)\sqrt{a+i\sqrt{a}\sqrt{b}}\arctan\left(\frac{\sqrt{a+i\sqrt{a}\sqrt{b}}\tan(x)}{\sqrt{a}}\right)-\left(\sqrt{a}+i\sqrt{b}\right)\sqrt{-a+i\sqrt{a}\sqrt{b}}\text{arctanh}\left(\frac{\sqrt{-a+i\sqrt{a}\sqrt{b}}\,\text{t}}{\sqrt{a}}\right.}{2a(a+b)}$$

input
```
Integrate[(a + b*Sin[x]^4)^(-1),x]
```

output
```
((Sqrt[a] - I*Sqrt[b])*Sqrt[a + I*Sqrt[a]*Sqrt[b]]*ArcTan[(Sqrt[a + I*Sqrt
[a]*Sqrt[b]]*Tan[x])/Sqrt[a]] - (Sqrt[a] + I*Sqrt[b])*Sqrt[-a + I*Sqrt[a]*
Sqrt[b]]*ArcTanh[(Sqrt[-a + I*Sqrt[a]*Sqrt[b]]*Tan[x])/Sqrt[a]])/(2*a*(a +
 b))
```

3.237.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3688, 1483, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin^4(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{a+b\sin(x)^4}dx$$

$$\Big\downarrow \text{ 3688}$$

$$\int \frac{\tan^2(x)+1}{(a+b)\tan^4(x)+2a\tan^2(x)+a}d\tan(x)$$

$$\Big\downarrow \text{ 1483}$$

$$\frac{\sqrt[4]{a+b}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}-\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\tan(x)}{\tan^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}+\frac{\sqrt[4]{a+b}\int \frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\tan(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}}{\tan^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\tan^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}(a+b)^{5/4}}-\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int -\frac{\sqrt{2}\left(\sqrt[4]{a} \cdots\right.}{\tan^2(x)-\frac{\sqrt{2} \cdots}{\cdots}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}+\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\tan^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\left(\sqrt{2}\tan \cdots\right.}{\tan^2(x)+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\Big\downarrow \text{ 25}$$

3.237. $\int \frac{1}{a+b\sin^4(x)}\,dx$

$$\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\tan^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\left(\frac{\sqrt[4]{a}\sqrt{\cdots}}{(\cdots}\right.}{\tan^2(x)-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}}\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\tan^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\left(\sqrt{2}\tan\cdots\right.}{\tan^2(x)+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}}\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\tan^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\cdots}}{(a+b)^{3/4}}}{\tan^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\cdots}}{(a+\cdots}}}{\sqrt{2}\cdots}\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\tan^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a+b}}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\tan(x)+\frac{\sqrt[4]{a}\cdots}{\cdots}}{\tan^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\cdots}}{(a+\cdots}}}{\sqrt{2}\cdots}\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\Big\downarrow \text{ 1083}$$

3.237. $\int \frac{1}{a+b\sin^4(x)}\,dx$

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}-\sqrt{2}\tan(x)}{\tan^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}}-\frac{\sqrt{2}\sqrt[4]{a}\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{\cdots}{-\left(2\tan(x)-\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\cdots}\right.\cdots}}{\cdots}\right.\cdots}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\tan(x)+\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}}{\tan^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}}-\frac{\sqrt{2}\sqrt[4]{a}\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{\cdots}{-\left(2\tan(x)+\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\cdots}\right.\cdots}}{\cdots}\right.\cdots}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$\downarrow$ 217

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}-\sqrt{2}\tan(x)}{\tan^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}}+\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(2\tan(x)-\cdots\right.}{\sqrt{2}\sqrt[4]{a}\cdots}\right.\cdots}{\sqrt{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\cdots}\right.\cdots}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\tan(x)+\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}}{\tan^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\tan(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\tan(x)}{\sqrt{2}}+\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(\frac{\sqrt{2}\sqrt[4]{a}\cdots}{(a\cdots}\right.\cdots}{\sqrt{2}\sqrt[4]{a}\cdots}\right.\cdots}{\sqrt{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\cdots}\right.\cdots}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$\downarrow$ 1103

3.237. $\int \frac{1}{a+b\sin^4(x)}\,dx$

$$\frac{\sqrt[4]{a+b}\left(\frac{(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(2\tan(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}{(a+b)^{3/4}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right)}{\sqrt{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}-\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\log\left((a+b)^{3/4}\text{ta}\right.\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}{(a+b)^{3/4}}+2\tan(x)\right)}{\sqrt{2}\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right)}{\sqrt{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}+\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\log\left((a+b)^{3/4}\text{ta}\right.\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

input
```
Int[(a + b*Sin[x]^4)^(-1),x]
```

output
```
((a + b)^(1/4)*(((Sqrt[a] + Sqrt[a + b])*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]
*ArcTan[((a + b)^(3/4)*(-((Sqrt[2]*a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b
]])/(a + b)^(3/4)) + 2*Tan[x]))/(Sqrt[2]*a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt
[a + b]])])/(Sqrt[a + b]*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]) - ((1 - Sqrt[a
]/Sqrt[a + b])*Log[Sqrt[a]*(a + b)^(1/4) - Sqrt[2]*a^(1/4)*Sqrt[a + b - Sq
rt[a]*Sqrt[a + b]]*Tan[x] + (a + b)^(3/4)*Tan[x]^2])/2))/(2*Sqrt[2]*a^(3/4
)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]) + ((a + b)^(1/4)*(((Sqrt[a] + Sqrt[a
+ b])*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]*ArcTan[((a + b)^(3/4)*((Sqrt[2]*a^
(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]])/(a + b)^(3/4) + 2*Tan[x]))/(Sqrt[
2]*a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]])])/(Sqrt[a + b]*Sqrt[a + b +
Sqrt[a]*Sqrt[a + b]]) + ((1 - Sqrt[a]/Sqrt[a + b])*Log[Sqrt[a]*(a + b)^(1/
4) + Sqrt[2]*a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]*Tan[x] + (a + b)^(3
/4)*Tan[x]^2])/2))/(2*Sqrt[2]*a^(3/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]])
```

3.237. $\int \frac{1}{a+b\sin^4(x)}\,dx$

3.237.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

3.237.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.21

method	result
risch	$\sum_{\_R=\text{RootOf}\left(1+\left(256a^4+256a^3b\right)\_Z^4+32a^2\_Z^2\right)} \_R \ln\left(\mathrm{e}^{2ix}+\left(-\frac{128ia^4}{b}-128ia^3\right)\_R^3+\left(\frac{32a^3}{b}+32a^2\right)\_R$
default	$-\frac{\left(-a^{\frac{5}{2}}\sqrt{2\sqrt{a^2+ab}-2a}-a^{\frac{3}{2}}\sqrt{a^2+ab}\,\sqrt{2\sqrt{a^2+ab}-2a}+\sqrt{a+b}\,\sqrt{a^2+ab}\,\sqrt{2\sqrt{a^2+ab}-2a}\,a+\sqrt{a+b}\,\sqrt{2\sqrt{a^2+ab}-2a}\,a^2\right)\ln\left(-\sqrt{a+b}\left(\tan^2(x)\right)+\tan(x\right.}{2\sqrt{a+b}}$

input
```
int(1/(a+b*sin(x)^4),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(exp(2*I*x)+(-128*I/b*a^4-128*I*a^3)*_R^3+(32/b*a^3+32*a^2)*_R^2+
(-8*I/b*a^2+8*I*a)*_R+2/b*a-1),_R=RootOf(1+(256*a^4+256*a^3*b)*_Z^4+32*a^2
*_Z^2))
```

3.237.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 823 vs. 2(344) = 688.

Time = 0.38 (sec) , antiderivative size = 823, normalized size of antiderivative = 1.69

$$\int \frac{1}{a+b\sin^4(x)}\,dx = -\frac{1}{8}\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+1}{a^2+ab}}\log\Bigg(\frac{1}{4}b\cos(x)^2$$

$$+\frac{1}{2}\left(ab\cos(x)\sin(x)+(a^4+a^3b)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+}{a^2+ab}}$$

$$-\frac{1}{4}\left(a^3+a^2b-2\left(a^3+a^2b\right)\cos(x)^2\right)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-\frac{1}{4}b\Bigg)$$

$$+\frac{1}{8}\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+1}{a^2+ab}}\log\Bigg(\frac{1}{4}b\cos(x)^2$$

$$-\frac{1}{2}\left(ab\cos(x)\sin(x)+(a^4+a^3b)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-}{a^2+ab}}$$

$$-\frac{1}{4}\left(a^3+a^2b-2\left(a^3+a^2b\right)\cos(x)^2\right)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-\frac{1}{4}b\Bigg)$$

$$+\frac{1}{8}\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-1}{a^2+ab}}\log\Bigg(-\frac{1}{4}b\cos(x)^2$$

$$+\frac{1}{2}\left(ab\cos(x)\sin(x)-(a^4+a^3b)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-}{a^2+ab}}$$

$$-\frac{1}{4}\left(a^3+a^2b-2\left(a^3+a^2b\right)\cos(x)^2\right)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+\frac{1}{4}b\Bigg)$$

$$-\frac{1}{8}\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-1}{a^2+ab}}\log\Bigg(-\frac{1}{4}b\cos(x)^2$$

$$-\frac{1}{2}\left(ab\cos(x)\sin(x)-(a^4+a^3b)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-}{a^2+ab}}$$

$$-\frac{1}{4}\left(a^3+a^2b-2\left(a^3+a^2b\right)\cos(x)^2\right)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+\frac{1}{4}b\Bigg)$$

input
```
integrate(1/(a+b*sin(x)^4),x, algorithm="fricas")
```

output
```
-1/8*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b
))*log(1/4*b*cos(x)^2 + 1/2*(a*b*cos(x)*sin(x) + (a^4 + a^3*b)*sqrt(-b/(a^
5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2
*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b)) - 1/4*(a^3 + a^2*b - 2*(a^3 + a^2*b)*
cos(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1/4*b) + 1/8*sqrt(-((a^2 +
a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b))*log(1/4*b*cos(x)
^2 - 1/2*(a*b*cos(x)*sin(x) + (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b
^2))*cos(x)*sin(x))*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))
+ 1)/(a^2 + a*b)) - 1/4*(a^3 + a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(
a^5 + 2*a^4*b + a^3*b^2)) - 1/4*b) + 1/8*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 +
2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b))*log(-1/4*b*cos(x)^2 + 1/2*(a*b*cos(x
)*sin(x) - (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))
*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b)) -
1/4*(a^3 + a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*
b^2)) + 1/4*b) - 1/8*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))
- 1)/(a^2 + a*b))*log(-1/4*b*cos(x)^2 - 1/2*(a*b*cos(x)*sin(x) - (a^4 + a^
3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(((a^2 + a*b)*s
qrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b)) - 1/4*(a^3 + a^2*b - 2
*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1/4*b)
```

3.237.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a + b\sin^4(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sin(x)**4),x)
```

output
```
Timed out
```

3.237.7 Maxima [F]

$$\int \frac{1}{a+b\sin^4(x)}\,dx = \int \frac{1}{b\sin{(x)}^4+a}\,dx$$

input
```
integrate(1/(a+b*sin(x)^4),x, algorithm="maxima")
```

output
```
integrate(1/(b*sin(x)^4 + a), x)
```

3.237.8 Giac [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.65

$$\int \frac{1}{a+b\sin^4(x)}\,dx = \frac{\left(3\sqrt{a^2+ab+\sqrt{-ab}(a+b)}a^2+6\sqrt{a^2+ab+\sqrt{-ab}(a+b)}ab-\sqrt{a^2+ab+\sqrt{-ab}(a+b)}b^2\right)\left(\pi\left\lfloor\frac{x}{\pi}+\right.\right.}{2\left(3a^5+12a^4b+14a^3b^2+4a^2b^3-ab^4\right)} + \frac{\left(3\sqrt{a^2+ab-\sqrt{-ab}(a+b)}a^2+6\sqrt{a^2+ab-\sqrt{-ab}(a+b)}ab-\sqrt{a^2+ab-\sqrt{-ab}(a+b)}b^2\right)\left(\pi\left\lfloor\frac{x}{\pi}\right.\right.}{2\left(3a^5+12a^4b+14a^3b^2+4a^2b^3-ab^4\right)}$$

input
```
integrate(1/(a+b*sin(x)^4),x, algorithm="giac")
```

output
```
1/2*(3*sqrt(a^2 + a*b + sqrt(-a*b)*(a + b))*a^2 + 6*sqrt(a^2 + a*b + sqrt(
-a*b)*(a + b))*a*b - sqrt(a^2 + a*b + sqrt(-a*b)*(a + b))*b^2)*(pi*floor(x
/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a + sqrt(-16*(a + b)*a + 16*a^2))/(a
+ b))))*abs(a + b)/(3*a^5 + 12*a^4*b + 14*a^3*b^2 + 4*a^2*b^3 - a*b^4) + 1
/2*(3*sqrt(a^2 + a*b - sqrt(-a*b)*(a + b))*a^2 + 6*sqrt(a^2 + a*b - sqrt(-
a*b)*(a + b))*a*b - sqrt(a^2 + a*b - sqrt(-a*b)*(a + b))*b^2)*(pi*floor(x/
pi + 1/2) + arctan(2*tan(x)/sqrt((4*a - sqrt(-16*(a + b)*a + 16*a^2))/(a +
 b))))*abs(a + b)/(3*a^5 + 12*a^4*b + 14*a^3*b^2 + 4*a^2*b^3 - a*b^4)
```

3.237.9 Mupad [B] (verification not implemented)

Time = 14.97 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.84

$$\int \frac{1}{a+b\sin^4(x)}\,dx$$
$$=\operatorname{atan}\left(\frac{a^3\tan(x)\,\sqrt{-\frac{a^2-\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}}\,4\mathrm{i}+a^5\tan(x)\,\left(-\frac{a^2-\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}\right)^{3/2}64\mathrm{i}-a^2\,b\tan(x)\,\sqrt{-\frac{a^2-\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}}\,4\mathrm{i}+a^4\,b}{\sqrt{-a^3\,b}}\right.$$
$$-\operatorname{atan}\left(\frac{a^3\tan(x)\,\sqrt{-\frac{a^2+\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}}\,4\mathrm{i}+a^5\tan(x)\,\left(-\frac{a^2+\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}\right)^{3/2}64\mathrm{i}-a^2\,b\tan(x)\,\sqrt{-\frac{a^2+\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}}\,4\mathrm{i}+a}{\sqrt{-a^3\,b}}\right.$$

input
```
int(1/(a + b*sin(x)^4),x)
```

output
```
atan((a^3*tan(x)*(-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*4i +
a^5*tan(x)*(-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(3/2)*64i - a^2*b
*tan(x)*(-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*4i + a^4*b*tan
(x)*(-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(3/2)*64i)/(-a^3*b)^(1/2
))*(-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*2i - atan((a^3*tan(
x)*(-(a^2 + (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*4i + a^5*tan(x)*(-(
a^2 + (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(3/2)*64i - a^2*b*tan(x)*(-(a^2
 + (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*4i + a^4*b*tan(x)*(-(a^2 + (
-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(3/2)*64i)/(-a^3*b)^(1/2))*(-(a^2 + (-
a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*2i
```

3.238 $\int \frac{1}{1+\sin^4(x)}\,dx$

3.238.1 Optimal result

Integrand size = 8, antiderivative size = 309

$$\int \frac{1}{1+\sin^4(x)}\,dx = \frac{x}{2\sqrt{-1+\sqrt{2}}} + \frac{\arctan\left(\frac{\sqrt{-1+\sqrt{2}}-2\sqrt{-1+\sqrt{2}}\cos^2(x)-\left(-2+\sqrt{2}\right)\cos(x)\sin(x)}{2+\sqrt{1+\sqrt{2}}+\left(-2+\sqrt{2}\right)\cos^2(x)-2\sqrt{-1+\sqrt{2}}\cos(x)\sin(x)}\right)}{4\sqrt{-1+\sqrt{2}}} - \frac{\arctan\left(\frac{\sqrt{-1+\sqrt{2}}-2\sqrt{-1+\sqrt{2}}\cos^2(x)+\left(-2+\sqrt{2}\right)\cos(x)\sin(x)}{2+\sqrt{1+\sqrt{2}}+\left(-2+\sqrt{2}\right)\cos^2(x)+2\sqrt{-1+\sqrt{2}}\cos(x)\sin(x)}\right)}{4\sqrt{-1+\sqrt{2}}} - \frac{1}{8}\sqrt{-1+\sqrt{2}}\log\left(\sqrt{2}-2\sqrt{-1+\sqrt{2}}\tan(x)+2\tan^2(x)\right) + \frac{1}{8}\sqrt{-1+\sqrt{2}}\log\left(1+\sqrt{2\left(-1+\sqrt{2}\right)}\tan(x)+\sqrt{2}\tan^2(x)\right)$$

output
```
1/2*x/(2^(1/2)-1)^(1/2)+1/4*arctan((-cos(x)*sin(x)*(-2+2^(1/2))+(2^(1/2)-1
)^(1/2)-2*cos(x)^2*(2^(1/2)-1)^(1/2))/(2+cos(x)^2*(-2+2^(1/2))-2*cos(x)*si
n(x)*(2^(1/2)-1)^(1/2)+(1+2^(1/2))^(1/2)))/(2^(1/2)-1)^(1/2)-1/4*arctan((c
os(x)*sin(x)*(-2+2^(1/2))+(2^(1/2)-1)^(1/2)-2*cos(x)^2*(2^(1/2)-1)^(1/2))/
(2+cos(x)^2*(-2+2^(1/2))+2*cos(x)*sin(x)*(2^(1/2)-1)^(1/2)+(1+2^(1/2))^(1/
2)))/(2^(1/2)-1)^(1/2)-1/8*ln(2^(1/2)-2*(2^(1/2)-1)^(1/2)*tan(x)+2*tan(x)^
2)*(2^(1/2)-1)^(1/2)+1/8*ln(1+(-2+2*2^(1/2))^(1/2)*tan(x)+2^(1/2)*tan(x)^2
)*(2^(1/2)-1)^(1/2)
```

3.238.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.15

$$\int \frac{1}{1+\sin^4(x)}\,dx = \frac{\arctan\left(\sqrt{1-i}\tan(x)\right)}{2\sqrt{1-i}} + \frac{\arctan\left(\sqrt{1+i}\tan(x)\right)}{2\sqrt{1+i}}$$

input
```
Integrate[(1 + Sin[x]^4)^(-1),x]
```

output
```
ArcTan[Sqrt[1 - I]*Tan[x]]/(2*Sqrt[1 - I]) + ArcTan[Sqrt[1 + I]*Tan[x]]/(2
*Sqrt[1 + I])
```

3.238.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.65, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3688, 1483, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^4(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(x)^4+1}dx$$

$$\Big\downarrow \text{3688}$$

$$\int \frac{\tan^2(x)+1}{2\tan^4(x)+2\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{1483}$$

$$\frac{\int \frac{2\sqrt{-1+\sqrt{2}}-\left(2-\sqrt{2}\right)\tan(x)}{2\tan^2(x)-2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}} + \frac{\int \frac{\left(2-\sqrt{2}\right)\tan(x)+2\sqrt{-1+\sqrt{2}}}{2\tan^2(x)+2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{2\tan^2(x)-2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)-\frac{1}{4}\left(2-\sqrt{2}\right)\int -\frac{2\left(\sqrt{-1+\sqrt{2}}-2\tan(x)\right)}{2\tan^2(x)-2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{2\tan^2(x)+2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)+\frac{1}{4}\left(2-\sqrt{2}\right)\int \frac{2\left(2\tan(x)+\sqrt{-1+\sqrt{2}}\right)}{2\tan^2(x)+2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{2\tan^2(x)-2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)+\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{\sqrt{-1+\sqrt{2}}-2\tan(x)}{2\tan^2(x)-2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{2\tan^2(x)+2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)+\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{2\tan(x)+\sqrt{-1+\sqrt{2}}}{2\tan^2(x)+2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

$$\Big\downarrow \ 1083$$

$$\frac{\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{\sqrt{-1+\sqrt{2}}-2\tan(x)}{2\tan^2(x)-2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)-\sqrt{2\left(1+\sqrt{2}\right)}\int \frac{1}{-\left(4\tan(x)-2\sqrt{-1+\sqrt{2}}\right)^2-4\left(1+\sqrt{2}\right)}d\left(4\tan(x)-2\right.}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

$$\frac{\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{2\tan(x)+\sqrt{-1+\sqrt{2}}}{2\tan^2(x)+2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)-\sqrt{2\left(1+\sqrt{2}\right)}\int \frac{1}{-\left(4\tan(x)+2\sqrt{-1+\sqrt{2}}\right)^2-4\left(1+\sqrt{2}\right)}d\left(4\tan(x)+2\right.}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{\sqrt{-1+\sqrt{2}}-2\tan(x)}{2\tan^2(x)-2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)+\frac{\arctan\left(\frac{4\tan(x)-2\sqrt{\sqrt{2}-1}}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2}}}{2\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{2\tan(x)+\sqrt{-1+\sqrt{2}}}{2\tan^2(x)+2\sqrt{-1+\sqrt{2}}\tan(x)+\sqrt{2}}d\tan(x)+\frac{\arctan\left(\frac{4\tan(x)+2\sqrt{\sqrt{2}-1}}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2}}}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

$$\Big\downarrow \ 1103$$

3.238. $\int \frac{1}{1+\sin^4(x)}\,dx$

$$\frac{\frac{\arctan\left(\frac{4\tan(x)-2\sqrt{\sqrt{2}-1}}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2}}-\frac{1}{4}(2-\sqrt{2})\log\left(2\tan^2(x)-2\sqrt{\sqrt{2}-1}\tan(x)+\sqrt{2}\right)}{2\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\frac{\arctan\left(\frac{4\tan(x)+2\sqrt{\sqrt{2}-1}}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2}}+\frac{1}{4}(2-\sqrt{2})\log\left(\sqrt{2}\tan^2(x)+\sqrt{2\left(\sqrt{2}-1\right)}\tan(x)+1\right)}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

input
```
Int[(1 + Sin[x]^4)^(-1),x]
```

output
```
(ArcTan[(-2*Sqrt[-1 + Sqrt[2]] + 4*Tan[x])/(2*Sqrt[1 + Sqrt[2]])]/Sqrt[2]
- ((2 - Sqrt[2])*Log[Sqrt[2] - 2*Sqrt[-1 + Sqrt[2]]*Tan[x] + 2*Tan[x]^2])/
4)/(2*Sqrt[2*(-1 + Sqrt[2])]) + (ArcTan[(2*Sqrt[-1 + Sqrt[2]] + 4*Tan[x])/
(2*Sqrt[1 + Sqrt[2]])]/Sqrt[2] + ((2 - Sqrt[2])*Log[1 + Sqrt[2*(-1 + Sqrt[
2])]*Tan[x] + Sqrt[2]*Tan[x]^2])/4)/(2*Sqrt[2*(-1 + Sqrt[2])])
```

3.238.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

3.238.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.41

method	result
risch	$\frac{\sqrt{-2+2i}\,\ln\left(\mathrm{e}^{2ix}+i\sqrt{-2+2i}+\sqrt{-2+2i}-1+2i\right)}{8}-\frac{\sqrt{-2+2i}\,\ln\left(\mathrm{e}^{2ix}-i\sqrt{-2+2i}-\sqrt{-2+2i}-1+2i\right)}{8}+\frac{\sqrt{-2-2i}\,\ln\left(\mathrm{e}^{2ix}+i\sqrt{-2-2i}-\right.}{8}$
default	$\frac{\sqrt{2}\left(-\frac{\sqrt{-2+2\sqrt{2}}\,\ln\left(-\sqrt{-2+2\sqrt{2}}\,\sqrt{2}\,\tan(x)+2\left(\tan^2(x)\right)+\sqrt{2}\right)}{4}+\frac{\left(-\frac{\left(-2+2\sqrt{2}\right)\sqrt{2}}{4}+2\right)\arctan\left(\frac{-\sqrt{2}\,\sqrt{-2+2\sqrt{2}}+4\tan(x)}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{1+\sqrt{2}}}\right)}{4}+\frac{\sqrt{2}\left(\frac{\sqrt{-}}{}\right.}{}$

input
```
int(1/(1+sin(x)^4),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-2+2*I)^(1/2)*ln(exp(2*I*x)+I*(-2+2*I)^(1/2)+(-2+2*I)^(1/2)-1+2*I)-1/
8*(-2+2*I)^(1/2)*ln(exp(2*I*x)-I*(-2+2*I)^(1/2)-(-2+2*I)^(1/2)-1+2*I)+1/8*
(-2-2*I)^(1/2)*ln(exp(2*I*x)+I*(-2-2*I)^(1/2)-(-2-2*I)^(1/2)-1-2*I)-1/8*(-
2-2*I)^(1/2)*ln(exp(2*I*x)-I*(-2-2*I)^(1/2)+(-2-2*I)^(1/2)-1-2*I)
```

3.238.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.38

$$\int \frac{1}{1+\sin^4(x)}\,dx = \frac{1}{16}\sqrt{2}\sqrt{i-1}\log\left(-(i-1)\sqrt{2}\sqrt{i-1}\cos(x)\sin(x)+(2i-1)\cos(x)^2-i+1\right) - \frac{1}{16}\sqrt{2}\sqrt{i-1}\log\left((i-1)\sqrt{2}\sqrt{i-1}\cos(x)\sin(x)+(2i-1)\cos(x)^2-i+1\right) - \frac{1}{16}\sqrt{2}\sqrt{-i-1}\log\left((i+1)\sqrt{2}\sqrt{-i-1}\cos(x)\sin(x)+(2i+1)\cos(x)^2-i-1\right) + \frac{1}{16}\sqrt{2}\sqrt{-i-1}\log\left(-(i+1)\sqrt{2}\sqrt{-i-1}\cos(x)\sin(x)+(2i+1)\cos(x)^2-i-1\right)$$

input
```
integrate(1/(1+sin(x)^4),x, algorithm="fricas")
```

output
```
1/16*sqrt(2)*sqrt(I - 1)*log(-(I - 1)*sqrt(2)*sqrt(I - 1)*cos(x)*sin(x) +
(2*I - 1)*cos(x)^2 - I + 1) - 1/16*sqrt(2)*sqrt(I - 1)*log((I - 1)*sqrt(2)
*sqrt(I - 1)*cos(x)*sin(x) + (2*I - 1)*cos(x)^2 - I + 1) - 1/16*sqrt(2)*sq
rt(-I - 1)*log((I + 1)*sqrt(2)*sqrt(-I - 1)*cos(x)*sin(x) + (2*I + 1)*cos(
x)^2 - I - 1) + 1/16*sqrt(2)*sqrt(-I - 1)*log(-(I + 1)*sqrt(2)*sqrt(-I - 1
)*cos(x)*sin(x) + (2*I + 1)*cos(x)^2 - I - 1)
```

3.238.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1+\sin^4(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1+sin(x)**4),x)
```

output
```
Timed out
```

3.238.7 Maxima [F]

$$\int \frac{1}{1+\sin^4(x)}\,dx = \int \frac{1}{\sin(x)^4+1}\,dx$$

input
```
integrate(1/(1+sin(x)^4),x, algorithm="maxima")
```

output
```
integrate(1/(sin(x)^4 + 1), x)
```

3.238.8 Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.55

$$\begin{aligned}\int \frac{1}{1+\sin^4(x)}\,dx &= \frac{1}{4}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}+2\tan(x)\right)}{\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}\\&+\frac{1}{4}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}-2\tan(x)\right)}{\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}\\&+\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\tan(x)^2+\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{\frac{1}{2}}\right)\\&-\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\tan(x)^2-\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{\frac{1}{2}}\right)\end{aligned}$$

input
```
integrate(1/(1+sin(x)^4),x, algorithm="giac")
```

output
```
1/4*(pi*floor(x/pi + 1/2) + arctan(2*(1/2)^(3/4)*((1/2)^(1/4)*sqrt(-sqrt(2
) + 2) + 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) + 1) + 1/4*(pi*floor(x
/pi + 1/2) + arctan(-2*(1/2)^(3/4)*((1/2)^(1/4)*sqrt(-sqrt(2) + 2) - 2*tan
(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) + 1) + 1/8*sqrt(sqrt(2) - 1)*log(tan
(x)^2 + (1/2)^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(1/2)) - 1/8*sqrt(sqrt
(2) - 1)*log(tan(x)^2 - (1/2)^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(1/2))
```

3.238.9 Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+\sin^4(x)}\,dx = \operatorname{atanh}\left(\frac{\tan(x)}{8\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}}-\frac{\tan(x)}{8\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}}+\frac{\sqrt{2}\tan(x)}{16\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}}+\frac{\sqrt{2}\tan(x)}{16\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}}\right)\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right)+\operatorname{atanh}\left(\frac{\tan(x)}{8\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}}+\frac{\tan(x)}{8\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}}+\frac{\sqrt{2}\tan(x)}{16\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}}-\frac{\sqrt{2}\tan(x)}{16\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}}\right)\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right)-\frac{(x-\operatorname{atan}(\tan(x)))\left(\pi\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right)1\mathrm{i}+\pi\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right)1\mathrm{i}\right)}{\pi}$$

input
```
int(1/(sin(x)^4 + 1),x)
```

output
```
atanh(tan(x)/(8*(- 2^(1/2)/64 - 1/64)^(1/2)) - tan(x)/(8*(2^(1/2)/64 - 1/6
4)^(1/2)) + (2^(1/2)*tan(x))/(16*(- 2^(1/2)/64 - 1/64)^(1/2)) + (2^(1/2)*t
an(x))/(16*(2^(1/2)/64 - 1/64)^(1/2)))*(2*(- 2^(1/2)/64 - 1/64)^(1/2) - 2*
(2^(1/2)/64 - 1/64)^(1/2)) + atanh(tan(x)/(8*(- 2^(1/2)/64 - 1/64)^(1/2))
+ tan(x)/(8*(2^(1/2)/64 - 1/64)^(1/2)) + (2^(1/2)*tan(x))/(16*(- 2^(1/2)/6
4 - 1/64)^(1/2)) - (2^(1/2)*tan(x))/(16*(2^(1/2)/64 - 1/64)^(1/2)))*(2*(-
2^(1/2)/64 - 1/64)^(1/2) + 2*(2^(1/2)/64 - 1/64)^(1/2)) - ((x - atan(tan(x
)))*(pi*(2*(- 2^(1/2)/64 - 1/64)^(1/2) - 2*(2^(1/2)/64 - 1/64)^(1/2))*1i +
 pi*(2*(- 2^(1/2)/64 - 1/64)^(1/2) + 2*(2^(1/2)/64 - 1/64)^(1/2))*1i))/pi
```

3.239 $\int \sin(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$

3.239.1 Optimal result

Integrand size = 23, antiderivative size = 477

$$\int \sin(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3d} + \frac{2\sqrt{b}\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3\sqrt{a+b}d\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)} - \frac{2\sqrt[4]{b}(a+b)^{3/4}\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}{(a+b)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right.}{3d\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}} + \frac{(a+b)^{3/4}\left(\sqrt{b}-\sqrt{a+b}\right)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}{(a+b)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\cos(c}{\sqrt[4]{a+}}\right.\right.}{3\sqrt[4]{b}d\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}$$

output
```
-1/3*cos(d*x+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/d+2/3*cos(d*x+
c)*b^(1/2)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/d/(1+cos(d*x+c)^2*b
^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-2/3*b^(1/4)*(a+b)^(3/4)*(cos(2*arctan(b^(1
/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b
)^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/2*(2+2
*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*((a+b-2*
b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^
2)^(1/2)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)+1/3*(a+b)^(3/4)*(co
s(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*
cos(d*x+c)/(a+b)^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(
1/4))),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^
(1/2))*(b^(1/2)-(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/
(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b^(1/4)/d/(a+b-2*b*cos(d*x+c
)^2+b*cos(d*x+c)^4)^(1/2)
```

3.239.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 26.94 (sec) , antiderivative size = 3120, normalized size of antiderivative = 6.54

$$\int \sin(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
-1/6*(Cos[c + d*x]*Sqrt[8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*
x)]])/(Sqrt[2]*d) + (Sqrt[2]*a*b^(3/2)*Sqrt[((-I)*(Sqrt[a] + I*Sqrt[b])*Se
c[c + d*x]^2)/Sqrt[b]]*(4*a*Sin[c + d*x] + 3*b*Sin[c + d*x] - b*Sin[3*(c +
 d*x)])*Sqrt[((-I)*(Sqrt[a] + I*Sqrt[b])*(1 + Tan[c + d*x]^2))/Sqrt[b]]*Sq
rt[((-I)*a + Sqrt[a]*Sqrt[b] - I*a*Tan[c + d*x]^2 - I*b*Tan[c + d*x]^2)/(S
qrt[a]*Sqrt[b])]*Sqrt[(I*a + Sqrt[a]*Sqrt[b] + I*a*Tan[c + d*x]^2 + I*b*Ta
n[c + d*x]^2)/(Sqrt[a]*Sqrt[b])]*Sqrt[((a + b)*(a + 2*a*Tan[c + d*x]^2 + a
*Tan[c + d*x]^4 + b*Tan[c + d*x]^4))/(a*b)]*Sqrt[Cos[c + d*x]^4*(a + 2*a*T
an[c + d*x]^2 + a*Tan[c + d*x]^4 + b*Tan[c + d*x]^4)]*(a + 2*a*Tan[c + d*x
]^2 + a*Tan[c + d*x]^4 + b*Tan[c + d*x]^4 + ((-(Sqrt[b]*EllipticE[ArcSin[S
qrt[1 + (I*Sqrt[a])/Sqrt[b] + (I*(a + b)*Tan[c + d*x]^2)/(Sqrt[a]*Sqrt[b])
]/Sqrt[2]], (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b])]) + ((-I)*Sqrt[a] + Sqrt[b])
*EllipticF[ArcSin[Sqrt[1 + (I*Sqrt[a])/Sqrt[b] + (I*(a + b)*Tan[c + d*x]^2
)/(Sqrt[a]*Sqrt[b])]/Sqrt[2]], (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b])])*Sqrt[(1
 - (I*Sqrt[a])/Sqrt[b])*Sec[c + d*x]^2]*(Sqrt[a] + (Sqrt[a] - I*Sqrt[b])*T
an[c + d*x]^2)*(a - I*Sqrt[a]*Sqrt[b] + (a + b)*Tan[c + d*x]^2))/(Sqrt[a]*
Sqrt[b]*Sqrt[((a + b)*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4))/(
a*b)])))/(3*d*Sqrt[8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]]*
Sqrt[Cos[c + d*x]^4*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)]*(I*
a^(7/2)*Sec[c + d*x]^4*Tan[c + d*x]^3 - a^3*Sqrt[b]*Sec[c + d*x]^4*Tan[...
```

3.239.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3694, 1404, 27, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)\sqrt{a+b\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{3694}$$

$$-\frac{\int \sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{1404}$$

$$-\frac{\frac{1}{3}\int \frac{2(-b\cos^2(c+dx)+a+b)}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)+\frac{1}{3}\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{d}$$

$\downarrow$ 27

$$-\frac{\frac{2}{3}\int \frac{-b\cos^2(c+dx)+a+b}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)+\frac{1}{3}\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{d}$$

$\downarrow$ 1511

$$-\frac{\frac{2}{3}\left(\sqrt{b}\sqrt{a+b}\int \frac{1-\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)-\sqrt{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)\int \frac{1}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+}}\right.}{d}$$

$\downarrow$ 1416

$$-\frac{\frac{2}{3}\left(\sqrt{b}\sqrt{a+b}\int \frac{1-\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)-\frac{(a+b)^{3/4}\left(\sqrt{b}-\sqrt{a+b}\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\,c}{(a+b)\left(\frac{\sqrt{b}\cos^2(c-}{\sqrt{a+b}}\right.}}}{2\sqrt[4]{b}\sqrt{a+b\cos^4(c+c}}\right.}{c}$$

$\downarrow$ 1509

$$-\frac{\frac{2}{3}\left(\sqrt{b}\sqrt{a+b}\left(\frac{\sqrt[4]{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{\sqrt[4]{b}\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}-\frac{\cos(c+dx}{}\right.\right.}{}$$

input

```
Int[Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

3.239. $\int \sin(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$

output
```
-(((Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/3 +
(2*(Sqrt[b]*Sqrt[a + b]*(-((Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 +
 b*Cos[c + d*x]^4])/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])))
+ ((a + b)^(1/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b -
2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]
^2)/Sqrt[a + b])^2)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/
4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(b^(1/4)*Sqrt[a + b - 2*b*Cos[c + d*x]^
2 + b*Cos[c + d*x]^4])) - ((a + b)^(3/4)*(Sqrt[b] - Sqrt[a + b])*(1 + (Sqr
t[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos
[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*Ellip
ticF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a +
 b])/2])/(2*b^(1/4)*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])))
/3)/d)
```

3.239.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.239.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.26 (sec) , antiderivative size = 439, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\frac{4\cos(dx+c)\sqrt{a+b-2b\left(\cos^2(dx+c)\right)+b\left(\cos^4(dx+c)\right)}}{3}+\frac{4\left(\frac{2a}{3}+\frac{2b}{3}\right)\sqrt{1-\frac{\left(i\sqrt{a}\,\sqrt{b}+b\right)\left(\cos^2(dx+c)\right)}{a+b}}\,\sqrt{1+\frac{\left(i\sqrt{a}\,\sqrt{b}-b\right)\left(\cos^2(dx+c)\right)}{a+b}}\,F\left(\cos(dx+c\right.}{\sqrt{\frac{i\sqrt{a}\,\sqrt{b}+b}{a+b}}\,\sqrt{a+b-2b\left(\cos^2(dx+c)\right)+b\left(\cos^4(dx+c)\right.}}}{}$

input
```
int(sin(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*(4/3*cos(d*x+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)+4*(2/3*
a+2/3*b)/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b
)*cos(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(
a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*EllipticF(cos(d*x+c)*((I*a^(1/2
)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))+16/3*b
*(a+b)/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*
cos(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+
b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/(-2*b+2*I*a^(1/2)*b^(1/2))*(Ellip
ticF(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/
2)-b)/(a+b))^(1/2))-EllipticE(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/
2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))))
```

3.239.5 Fricas [F]

$$\int \sin(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx=\int \sqrt{b\sin\left(dx+c\right)^4+a}\sin\left(dx+c\right)\,dx$$

input
```
integrate(sin(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*sin(d*x + c),
 x)
```

3.239.6 Sympy [F]

$$\int \sin(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx=\int \sqrt{a+b\sin^4\left(c+dx\right)}\sin\left(c+dx\right)dx$$

input
```
integrate(sin(d*x+c)*(a+b*sin(d*x+c)**4)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(c + d*x)**4)*sin(c + d*x), x)
```

3.239.7 Maxima [F]

$$\int \sin(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = \int \sqrt{b\sin\left(dx+c\right)^4+a}\sin\left(dx+c\right)\,dx$$

input
```
integrate(sin(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(d*x + c)^4 + a)*sin(d*x + c), x)
```

3.239.8 Giac [F]

$$\int \sin(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = \int \sqrt{b\sin\left(dx+c\right)^4+a}\sin\left(dx+c\right)\,dx$$

input
```
integrate(sin(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(d*x + c)^4 + a)*sin(d*x + c), x)
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \sin(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = \int \sin\left(c+d\,x\right)\,\sqrt{b\sin\left(c+d\,x\right)^4+a}\,dx$$

input
```
int(sin(c + d*x)*(a + b*sin(c + d*x)^4)^(1/2),x)
```

output
```
int(sin(c + d*x)*(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.240 $\int \csc(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$

3.240.1 Optimal result

Integrand size = 23, antiderivative size = 521

$$\int \csc(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}\right)}{2d}$$
$$+\frac{\sqrt{b}\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{\sqrt{a+b}d\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)}$$
$$-\frac{\sqrt[4]{b}(a+b)^{3/4}\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}{(a+b)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right)}{d\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}$$
$$-\frac{\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)^2\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}{(a+b)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}+\sqrt{a+b}\right)^2}{4\sqrt{b}\sqrt{a+b}},2\,\text{arcta}\right.}{4\sqrt[4]{b}d\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}$$

output
```
1/2*arctan(cos(d*x+c)*(-a)^(1/2)/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/
2))*(-a)^(1/2)/d+cos(d*x+c)*b^(1/2)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^
(1/2)/d/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-b^(1/4)*(a+b)^(3/
4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*cos(d*x+c)/(a+b)^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*cos(d*x+c)/(
a+b)^(1/4))),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/
(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*
b^(1/2)/(a+b)^(1/2))^2)^(1/2)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2
)-1/4*(a+b)^(1/4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/
cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*EllipticPi(sin(2*arctan(b^(1
/4)*cos(d*x+c)/(a+b)^(1/4))),1/4*(b^(1/2)+(a+b)^(1/2))^2/b^(1/2)/(a+b)^(1/
2),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2
))*(b^(1/2)-(a+b)^(1/2))^2*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1
+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b^(1/4)/d/(a+b-2*b*cos(d*x+c)^
2+b*cos(d*x+c)^4)^(1/2)
```

3.240.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 24.44 (sec) , antiderivative size = 2045, normalized size of antiderivative = 3.93

$$\int \csc(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
(b*Cos[c + d*x]*Sqrt[8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]
]*Cot[c + d*x]^2*(1 + Cot[c + d*x]^2)^2*Sqrt[(a + b + 2*a*Cot[c + d*x]^2 +
 a*Cot[c + d*x]^4)/(1 + Cot[c + d*x]^2)^2]*Sqrt[(I*(I*Sqrt[a] + Sqrt[b])*(
(-I)*Sqrt[a] + Sqrt[b] - I*Sqrt[a]*Cot[c + d*x]^2)*Tan[c + d*x]^2)/(Sqrt[a
]*Sqrt[b])]*Sqrt[((-I)*((-I)*Sqrt[a] + Sqrt[b])*(I*Sqrt[a] + Sqrt[b] + I*S
qrt[a]*Cot[c + d*x]^2)*Tan[c + d*x]^2)/(Sqrt[a]*Sqrt[b])]*Sqrt[((a + b)*(a
 + b + 2*a*Cot[c + d*x]^2 + a*Cot[c + d*x]^4)*Tan[c + d*x]^4)/(a*b)]*(1 +
Tan[c + d*x]^2)*Sqrt[(b*Tan[c + d*x]^4 + a*(1 + Tan[c + d*x]^2)^2)/(1 + Ta
n[c + d*x]^2)^2]*(1 + (b*(Sqrt[b]*EllipticE[ArcSin[Sqrt[(I*(a - I*Sqrt[a]*
Sqrt[b] + a*Tan[c + d*x]^2 + b*Tan[c + d*x]^2))/(Sqrt[a]*Sqrt[b])]/Sqrt[2]
], (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b])] + I*(Sqrt[a] + I*Sqrt[b])*EllipticF[
ArcSin[Sqrt[(I*(a - I*Sqrt[a]*Sqrt[b] + a*Tan[c + d*x]^2 + b*Tan[c + d*x]^
2))/(Sqrt[a]*Sqrt[b])]/Sqrt[2]], (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b])])*Sqrt[
(((-I)*Sqrt[a] + Sqrt[b])*(1 + Tan[c + d*x]^2))/Sqrt[b]]*Sqrt[(I*(a - I*Sq
rt[a]*Sqrt[b] + a*Tan[c + d*x]^2 + b*Tan[c + d*x]^2))/(Sqrt[a]*Sqrt[b])])/
((a + b)*Sqrt[((-I)*(a + I*Sqrt[a]*Sqrt[b] + a*Tan[c + d*x]^2 + b*Tan[c +
d*x]^2))/(Sqrt[a]*Sqrt[b])]*(Sqrt[b]*Tan[c + d*x]^2 - I*Sqrt[a]*(1 + Tan[c
 + d*x]^2))) - (a*((-I)*Sqrt[b]*EllipticE[ArcSin[Sqrt[(I*(a - I*Sqrt[a]*Sq
rt[b] + a*Tan[c + d*x]^2 + b*Tan[c + d*x]^2))/(Sqrt[a]*Sqrt[b])]/Sqrt[2]],
 (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b])] + (Sqrt[a] + I*Sqrt[b])*EllipticF[A...
```

3.240.3 Rubi [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3694, 1523, 25, 27, 1509, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin(c+dx)^4}}{\sin(c+dx)}dx$$

$$\Big\downarrow \text{3694}$$

$$-\frac{\int \frac{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}{1-\cos^2(c+dx)}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{1523}$$

$$-\frac{\frac{a\int \frac{\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1}{\left(1-\cos^2(c+dx)\right)\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{\frac{\sqrt{b}}{\sqrt{a+b}}+1}-\frac{\int -\frac{\sqrt{b}\left(-\sqrt{b}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\cos^2(c+dx)+\sqrt{b}+\sqrt{a+b}\right)}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{\frac{\sqrt{b}}{\sqrt{a+b}}+1}}{d}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{a\int \frac{\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1}{\left(1-\cos^2(c+dx)\right)\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{\frac{\sqrt{b}}{\sqrt{a+b}}+1}+\frac{\int \frac{\sqrt{b}\left(-\sqrt{b}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\cos^2(c+dx)+\sqrt{b}+\sqrt{a+b}\right)}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{\frac{\sqrt{b}}{\sqrt{a+b}}+1}}{d}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{a\int \frac{\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1}{\left(1-\cos^2(c+dx)\right)\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{\frac{\sqrt{b}}{\sqrt{a+b}}+1}+\frac{\sqrt{b}\int \frac{-\sqrt{b}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\cos^2(c+dx)+\sqrt{b}+\sqrt{a+b}}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{\frac{\sqrt{b}}{\sqrt{a+b}}+1}}{d}$$

$$\Big\downarrow \ 1509$$

$$-\frac{\frac{a\int \frac{\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1}{\left(1-\cos^2(c+dx)\right)\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{\frac{\sqrt{b}}{\sqrt{a+b}}+1}+\frac{\sqrt{b}\left(\frac{\sqrt[4]{a+b}\left(\sqrt{a+b}+\sqrt{b}\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)}}}{\sqrt[4]{b}\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c}}\right.}{}}{d}$$

$$\Big\downarrow \ 2222$$

$$-\frac{a\left(\frac{\sqrt[4]{a+b}\left(1-\frac{\sqrt{b}}{\sqrt{a+b}}\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}+\sqrt{a+b}\right)^2}{4\sqrt{b}\sqrt{a+b}},2\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right),\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{4\sqrt[4]{b}\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}+\right.}{\frac{\sqrt{b}}{\sqrt{a+b}}+1}$$

input
```
Int[Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

3.240. $\int \csc(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$

output
```
-(((Sqrt[b]*(-(((Sqrt[b] + Sqrt[a + b])*Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[
c + d*x]^2 + b*Cos[c + d*x]^4])/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqr
t[a + b]))) + ((a + b)^(1/4)*(Sqrt[b] + Sqrt[a + b])*(1 + (Sqrt[b]*Cos[c +
 d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)
/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticE[2*ArcTa
n[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(b^
(1/4)*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])))/(1 + Sqrt[b]/
Sqrt[a + b]) + (a*(((1 + Sqrt[b]/Sqrt[a + b])*ArcTanh[(Sqrt[a]*Cos[c + d*x
])/Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4]])/(2*Sqrt[a]) + ((a
 + b)^(1/4)*(1 - Sqrt[b]/Sqrt[a + b])*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a
 + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 +
(Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticPi[(Sqrt[b] + Sqrt[a + b
])^2/(4*Sqrt[b]*Sqrt[a + b]), 2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4
)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(4*b^(1/4)*Sqrt[a + b - 2*b*Cos[c + d*x]
^2 + b*Cos[c + d*x]^4])))/(1 + Sqrt[b]/Sqrt[a + b]))/d)
```

3.240.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1523
```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(c*d^2 - b*d*e + a*e^2)/(e*(e - d*q))   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] - Simp[1/(e*(e
 - d*q))   Int[(c*d - b*e + a*e*q - (c*e - a*d*q^3)*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.240. $\int \csc(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.240.4 Maple [F]

$$\int \csc(dx+c)\sqrt{a+b\left(\sin^4(dx+c)\right)}dx$$

input
```
int(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x)
```

output
```
int(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x)
```

3.240.5 Fricas [F]

$$\int \csc(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = \int \sqrt{b\sin(dx+c)^4+a}\csc(dx+c)\,dx$$

input
```
integrate(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*csc(d*x + c),
 x)
```

3.240.6 Sympy [F]

$$\int \csc(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = \int \sqrt{a+b\sin^4{(c+dx)}}\csc{(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)*(a+b*sin(d*x+c)**4)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(c + d*x)**4)*csc(c + d*x), x)
```

3.240.7 Maxima [F]

$$\int \csc(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = \int \sqrt{b\sin{(dx+c)}^4+a}\csc{(dx+c)}\;dx$$

input
```
integrate(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(d*x + c)^4 + a)*csc(d*x + c), x)
```

3.240.8 Giac [F]

$$\int \csc(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = \int \sqrt{b\sin{(dx+c)}^4+a}\csc{(dx+c)}\;dx$$

input
```
integrate(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(d*x + c)^4 + a)*csc(d*x + c), x)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \csc(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx=\int \frac{\sqrt{b\sin\left(c+d\,x\right)^4+a}}{\sin\left(c+d\,x\right)}\,dx$$

input
```
int((a + b*sin(c + d*x)^4)^(1/2)/sin(c + d*x),x)
```

output
```
int((a + b*sin(c + d*x)^4)^(1/2)/sin(c + d*x), x)
```

3.241 $\int \frac{\sin^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.241.1 Optimal result

Integrand size = 25, antiderivative size = 484

$$\int \frac{\sin^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3bd} + \frac{2\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3\sqrt{b}\sqrt{a+b}d\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)} - \frac{2(a+b)^{3/4}\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}{(a+b)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right)}{3b^{3/4}d\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}} + \frac{\sqrt[4]{a+b}\left(a-2b+2\sqrt{b}\sqrt{a+b}\right)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}{(a+b)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\cdots}{\cdots}\right)\cdots\right)}{6b^{5/4}d\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}$$

output
```
-1/3*cos(d*x+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/b/d+2/3*cos(d*
x+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/d/b^(1/2)/(1+cos(d*x+c)^2
*b^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-2/3*(a+b)^(3/4)*(cos(2*arctan(b^(1/4)*co
s(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4
)))*EllipticE(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/2*(2+2*b^(1/
2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*((a+b-2*b*cos(
d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/
2)/b^(3/4)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)+1/6*(a+b)^(1/4)*(
cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4
)*cos(d*x+c)/(a+b)^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)
^(1/4))),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b
)^(1/2))*(a-2*b+2*b^(1/2)*(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)
^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b^(5/4)/d/(a+b-2*b
*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)
```

3.241.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 27.46 (sec) , antiderivative size = 2854, normalized size of antiderivative = 5.90

$$\int \frac{\sin^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sin[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
-1/6*(Cos[c + d*x]*Sqrt[8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*
x)]])/(Sqrt[2]*b*d) + (2*Sqrt[2]*Sqrt[a]*((Sqrt[2]*Sin[c + d*x])/Sqrt[8*a
+ 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]] - (2*Sqrt[2]*a*Sin[c +
d*x])/(3*b*Sqrt[8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]]) -
(Sqrt[2]*Sin[3*(c + d*x)])/(3*Sqrt[8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Co
s[4*(c + d*x)]]))*Sqrt[((a + b)*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c +
d*x]^4))/(a*b)]*(-2*(Sqrt[a] - I*Sqrt[b])*Sqrt[b]*EllipticE[ArcSin[Sqrt[1
+ (I*Sqrt[a])/Sqrt[b] + (I*(a + b)*Tan[c + d*x]^2)/(Sqrt[a]*Sqrt[b])]/Sqrt
[2]], (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b])]*Sqrt[(1 - (I*Sqrt[a])/Sqrt[b])*Se
c[c + d*x]^2] + I*(a - 2*b)*EllipticF[ArcSin[Sqrt[1 + (I*Sqrt[a])/Sqrt[b]
+ (I*(a + b)*Tan[c + d*x]^2)/(Sqrt[a]*Sqrt[b])]/Sqrt[2]], (2*Sqrt[a])/(Sqr
t[a] - I*Sqrt[b])]*Sqrt[(1 - (I*Sqrt[a])/Sqrt[b])*Sec[c + d*x]^2] + 2*Sqrt
[a]*Sqrt[b]*Sqrt[((a + b)*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4
))/(a*b)]))/(3*Sqrt[b]*(a + b)*d*Sqrt[8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b
*Cos[4*(c + d*x)]]*(Sec[c + d*x]^2)^(3/2)*((Sqrt[2]*(4*a*Sec[c + d*x]^2*Ta
n[c + d*x] + 4*(a + b)*Sec[c + d*x]^2*Tan[c + d*x]^3)*(-2*(Sqrt[a] - I*Sqr
t[b])*Sqrt[b]*EllipticE[ArcSin[Sqrt[1 + (I*Sqrt[a])/Sqrt[b] + (I*(a + b)*T
an[c + d*x]^2)/(Sqrt[a]*Sqrt[b])]/Sqrt[2]], (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[
b])]*Sqrt[(1 - (I*Sqrt[a])/Sqrt[b])*Sec[c + d*x]^2] + I*(a - 2*b)*Elliptic
F[ArcSin[Sqrt[1 + (I*Sqrt[a])/Sqrt[b] + (I*(a + b)*Tan[c + d*x]^2)/(Sqr...
```

3.241.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3694, 1518, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^5}{\sqrt{a+b\sin(c+dx)^4}}dx$$

$$\Big\downarrow \text{3694}$$

$$-\frac{\int \frac{\left(1-\cos^2(c+dx)\right)^2}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{d}$$

$\downarrow$ 1518

$$-\frac{\frac{\int -\frac{2b\cos^2(c+dx)+a-2b}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{3b}+\frac{\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{3b}}{d}$$

$\downarrow$ 25

$$-\frac{\frac{\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{3b}-\frac{\int \frac{2b\cos^2(c+dx)+a-2b}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{3b}}{d}$$

$\downarrow$ 1511

$$-\frac{\frac{\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{3b}-\frac{\left(2\sqrt{b}\sqrt{a+b}+a-2b\right)\int \frac{1}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)-2\sqrt{b}\sqrt{a+b}\int \frac{1}{\sqrt{b\cos^4(c\cdots}}}{3b}}{d}$$

$\downarrow$ 1416

$$-\frac{\frac{\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{3b}-\frac{\frac{\sqrt[4]{a+b}\left(2\sqrt{b}\sqrt{a+b}+a-2b\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}\text{EllipticF}\left(2\,\cdots\right.}{2\sqrt[4]{b}\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\cdots}{\cdots}}{d}$$

$\downarrow$ 1509

$$-\frac{\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{3b}-\frac{\frac{\sqrt[4]{a+b}\left(2\sqrt{b}\sqrt{a+b}+a-2b\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}\text{EllipticF}\left(2\,\cdots\right.}{2\sqrt[4]{b}\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\cdots}{\cdots}$$

input

```
Int[Sin[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]^4],x]
```

3.241. $\int \frac{\sin^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

output
```
-(((Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/(3*b
) - (-2*Sqrt[b]*Sqrt[a + b]*(-((Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]
^2 + b*Cos[c + d*x]^4])/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b]
))) + ((a + b)^(1/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a +
b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c +
d*x]^2)/Sqrt[a + b])^2)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)
^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(b^(1/4)*Sqrt[a + b - 2*b*Cos[c + d
*x]^2 + b*Cos[c + d*x]^4])) + ((a + b)^(1/4)*(a - 2*b + 2*Sqrt[b]*Sqrt[a +
 b])*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c +
d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a +
 b])^2)]*EllipticF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sq
rt[b]/Sqrt[a + b])/2])/(2*b^(1/4)*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[
c + d*x]^4]))/(3*b))/d)
```

3.241.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.241. $\int \frac{\sin^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.241.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.87 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.73

method	result
default	$-\frac{\sqrt{1-\frac{\left(i\sqrt{a}\,\sqrt{b}+b\right)\left(\cos^2(dx+c)\right)}{a+b}}\,\sqrt{1+\frac{\left(i\sqrt{a}\,\sqrt{b}-b\right)\left(\cos^2(dx+c)\right)}{a+b}}\,F\left(\cos(dx+c)\sqrt{\frac{i\sqrt{a}\,\sqrt{b}+b}{a+b}},\sqrt{-1-\frac{2\left(i\sqrt{a}\,\sqrt{b}-b\right)}{a+b}}\right)}{d\sqrt{\frac{i\sqrt{a}\,\sqrt{b}+b}{a+b}}\,\sqrt{a+b-2b(\cos^2(dx+c))+b(\cos^4(dx+c))}}-\frac{4(a+b)\sqrt{1-\frac{(i\sqrt{}}{}}}{}$

input
```
int(sin(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*co
s(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-
2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*EllipticF(cos(d*x+c)*((I*a^(1/2)*b^
(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))-4/d*(a+b)/
((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos(d*x
+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*b*c
os(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/(-2*b+2*I*a^(1/2)*b^(1/2))*(EllipticF(co
s(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(
a+b))^(1/2))-EllipticE(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-
2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2)))-4/d*(1/12/b*cos(d*x+c)*(a+b-2*b*cos
(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)-1/12*(a+b)/b/((I*a^(1/2)*b^(1/2)+b)/(a+b))
^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^
(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(
1/2)*EllipticF(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(
1/2)*b^(1/2)-b)/(a+b))^(1/2))-2/3*(a+b)/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2
)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)
-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/
(-2*b+2*I*a^(1/2)*b^(1/2))*(EllipticF(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a
+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))-EllipticE(cos(d*x+c)*
((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^...
```

3.241.5 Fricas [F]

$$\int \frac{\sin^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\sin(dx+c)^5}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

input
```
integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c)/sqrt(b*cos(d
*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b), x)
```

3.241.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**5/(a+b*sin(d*x+c)**4)**(1/2),x)
```

output
```
Timed out
```

3.241.7 Maxima [F]

$$\int \frac{\sin^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\sin{(dx+c)}^5}{\sqrt{b\sin{(dx+c)}^4+a}}\,dx$$

input
```
integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)^5/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.241.8 Giac [F]

$$\int \frac{\sin^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\sin{(dx+c)}^5}{\sqrt{b\sin{(dx+c)}^4+a}}\,dx$$

input
```
integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\sin\left(c+d\,x\right)^5}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

input
```
int(sin(c + d*x)^5/(a + b*sin(c + d*x)^4)^(1/2),x)
```

output
```
int(sin(c + d*x)^5/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.242 $\int \frac{\sin^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.242.1 Optimal result

Integrand size = 25, antiderivative size = 431

$$\int \frac{\sin^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{\sqrt{b}\sqrt{a+b}\,d\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)}$$
$$-\frac{(a+b)^{3/4}\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}{(a+b)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)\middle|\frac{1}{2}\left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right)}{b^{3/4}d\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}$$
$$-\frac{\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}{(a+b)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)\ldots\right.}{2b^{3/4}d\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}$$

output
```
cos(d*x+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/d/b^(1/2)/(1+cos(d*
x+c)^2*b^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-(a+b)^(3/4)*(cos(2*arctan(b^(1/4)*
cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1
/4)))*EllipticE(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/2*(2+2*b^(
1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*((a+b-2*b*co
s(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(
1/2)/b^(3/4)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)-1/2*(a+b)^(1/4)
*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*cos(d*x+c)/(a+b)^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+
b)^(1/4))),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a
+b)^(1/2))*(b^(1/2)-(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a
+b)/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b^(3/4)/d/(a+b-2*b*cos(d
*x+c)^2+b*cos(d*x+c)^4)^(1/2)
```

3.242.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 23.80 (sec) , antiderivative size = 2663, normalized size of antiderivative = 6.18

$$\int \frac{\sin^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sin[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
(Sqrt[a]*((3*Sin[c + d*x])/(Sqrt[2]*Sqrt[8*a + 3*b - 4*b*Cos[2*(c + d*x)]
+ b*Cos[4*(c + d*x)]]) - Sin[3*(c + d*x)]/(Sqrt[2]*Sqrt[8*a + 3*b - 4*b*Co
s[2*(c + d*x)] + b*Cos[4*(c + d*x)]]))*Sqrt[((a + b)*(a + 2*a*Tan[c + d*x]
^2 + (a + b)*Tan[c + d*x]^4))/(a*b)]*(-((Sqrt[a] - I*Sqrt[b])*EllipticE[Ar
cSin[Sqrt[1 + (I*Sqrt[a])/Sqrt[b] + (I*(a + b)*Tan[c + d*x]^2)/(Sqrt[a]*Sq
rt[b])]/Sqrt[2]], (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b])]*Sqrt[(1 - (I*Sqrt[a])
/Sqrt[b])*Sec[c + d*x]^2]) - I*Sqrt[b]*EllipticF[ArcSin[Sqrt[1 + (I*Sqrt[a
])/Sqrt[b] + (I*(a + b)*Tan[c + d*x]^2)/(Sqrt[a]*Sqrt[b])]/Sqrt[2]], (2*Sq
rt[a])/(Sqrt[a] - I*Sqrt[b])]*Sqrt[(1 - (I*Sqrt[a])/Sqrt[b])*Sec[c + d*x]^
2] + Sqrt[a]*Sqrt[((a + b)*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^
4))/(a*b)]))/((a + b)*d*((1 + Cos[2*(c + d*x)])^(-1))^(3/2)*Sqrt[8*a + 3*b
 - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]]*(((4*a*Sec[c + d*x]^2*Tan[c
+ d*x] + 4*(a + b)*Sec[c + d*x]^2*Tan[c + d*x]^3)*(-((Sqrt[a] - I*Sqrt[b])
*EllipticE[ArcSin[Sqrt[1 + (I*Sqrt[a])/Sqrt[b] + (I*(a + b)*Tan[c + d*x]^2
)/(Sqrt[a]*Sqrt[b])]/Sqrt[2]], (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b])]*Sqrt[(1
- (I*Sqrt[a])/Sqrt[b])*Sec[c + d*x]^2]) - I*Sqrt[b]*EllipticF[ArcSin[Sqrt[
1 + (I*Sqrt[a])/Sqrt[b] + (I*(a + b)*Tan[c + d*x]^2)/(Sqrt[a]*Sqrt[b])]/Sq
rt[2]], (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b])]*Sqrt[(1 - (I*Sqrt[a])/Sqrt[b])*
Sec[c + d*x]^2] + Sqrt[a]*Sqrt[((a + b)*(a + 2*a*Tan[c + d*x]^2 + (a + b)*
Tan[c + d*x]^4))/(a*b)]))/(2*Sqrt[a]*b*((1 + Cos[2*(c + d*x)])^(-1))^(3...
```

3.242.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3694, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(c+dx)^3}{\sqrt{a+b\sin(c+dx)^4}}dx$$

$$\Big\downarrow \; 3694$$

$$-\frac{\int \frac{1-\cos^2(c+dx)}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{d}$$

$$\Big\downarrow \; 1511$$

$$-\frac{\left(1-\frac{\sqrt{a+b}}{\sqrt{b}}\right)\int \frac{1}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)+\frac{\sqrt{a+b}\int \frac{1-\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{\sqrt{b}}}{d}$$

$$\Big\downarrow \; 1416$$

$$-\frac{\frac{\sqrt{a+b}\int \frac{1-\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{\sqrt{b}}+\frac{\sqrt[4]{a+b}\left(1-\frac{\sqrt{a+b}}{\sqrt{b}}\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}\text{EllipticF}\Big(}{2\sqrt[4]{b}\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}}{d}$$

$$\Big\downarrow \; 1509$$

$$-\frac{\frac{\sqrt[4]{a+b}\left(1-\frac{\sqrt{a+b}}{\sqrt{b}}\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right),\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{2\sqrt[4]{b}\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}+\frac{\sqrt{a+b}\Bigg(}{}}{}$$

input
```
Int[Sin[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

3.242. $\int \frac{\sin^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

output
```
-(((Sqrt[a + b]*(-((Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c
 + d*x]^4])/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b]))) + ((a +
b)^(1/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[
c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt
[a + b])^2)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1
+ Sqrt[b]/Sqrt[a + b])/2])/(b^(1/4)*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Co
s[c + d*x]^4])))/Sqrt[b] + ((a + b)^(1/4)*(1 - Sqrt[a + b]/Sqrt[b])*(1 + (
Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*
Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*El
lipticF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[
a + b])/2])/(2*b^(1/4)*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4]
))/d)
```

3.242.3.1 Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.242.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.85 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\sqrt{1-\frac{\left(i\sqrt{a}\,\sqrt{b}+b\right)\left(\cos^{2}(dx+c)\right)}{a+b}}\,\sqrt{1+\frac{\left(i\sqrt{a}\,\sqrt{b}-b\right)\left(\cos^{2}(dx+c)\right)}{a+b}}\,F\left(\cos(dx+c)\sqrt{\frac{i\sqrt{a}\,\sqrt{b}+b}{a+b}},\sqrt{-1-\frac{2\left(i\sqrt{a}\,\sqrt{b}-b\right)}{a+b}}\right)}{d\sqrt{\frac{i\sqrt{a}\,\sqrt{b}+b}{a+b}}\,\sqrt{a+b-2b(\cos^{2}(dx+c))+b(\cos^{4}(dx+c))}}-\frac{2(a+b)\sqrt{1-}\cdots}{\cdots}$

input
```
int(sin(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*co
s(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-
2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*EllipticF(cos(d*x+c)*((I*a^(1/2)*b^
(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))-2/d*(a+b)/
((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos(d*x
+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*b*c
os(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/(-2*b+2*I*a^(1/2)*b^(1/2))*(EllipticF(co
s(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(
a+b))^(1/2))-EllipticE(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-
2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2)))
```

3.242.5 Fricas [F]

$$\int \frac{\sin^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx=\int \frac{\sin\left(dx+c\right)^3}{\sqrt{b\sin\left(dx+c\right)^4+a}}\,dx$$

input
```
integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
integral(-(cos(d*x + c)^2 - 1)*sin(d*x + c)/sqrt(b*cos(d*x + c)^4 - 2*b*co
s(d*x + c)^2 + a + b), x)
```

3.242.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)**4)**(1/2),x)
```

output
```
Timed out
```

3.242.7 Maxima [F]

$$\int \frac{\sin^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\sin\left(dx+c\right)^3}{\sqrt{b\sin\left(dx+c\right)^4+a}}\,dx$$

input
```
integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)^3/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.242.8 Giac [F]

$$\int \frac{\sin^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\sin\left(dx+c\right)^3}{\sqrt{b\sin\left(dx+c\right)^4+a}}\,dx$$

input
```
integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx=\int \frac{\sin\left(c+d\,x\right)^3}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

input
```
int(sin(c + d*x)^3/(a + b*sin(c + d*x)^4)^(1/2),x)
```

output
```
int(sin(c + d*x)^3/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.243 $\int \frac{\sin(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.243.1 Optimal result

Integrand size = 23, antiderivative size = 171

$$\int \frac{\sin(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\sqrt[4]{a+b}\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}{(a+b)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right),\frac{1}{2}\left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\cdots\right)\right)}{2\sqrt[4]{b}d\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}$$

output
```
-1/2*(a+b)^(1/4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*EllipticF(sin(2*arctan(b^(1/4
)*cos(d*x+c)/(a+b)^(1/4))),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x
+c)^2*b^(1/2)/(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1
+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b^(1/4)/d/(a+b-2*b*cos(d*x+c)^
2+b*cos(d*x+c)^4)^(1/2)
```

3.243.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.81 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.72

$$\int \frac{\sin(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$
$$=\frac{2\sqrt{2}\cos^3(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{a}}{\sqrt{b}}+\frac{i(a+b)\tan^2(c+dx)}{\sqrt{a}\sqrt{b}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}}\right)\sqrt{\left(1-\frac{i\sqrt{a}}{\sqrt{b}}\right)\sec^2(c+dx)\left(\sqrt{a}+\right.}}{\left(\sqrt{a}+i\sqrt{b}\right)d\sqrt{8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx))}\sqrt{1+}}$$

input
```
Integrate[Sin[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
(2*Sqrt[2]*Cos[c + d*x]^3*EllipticF[ArcSin[Sqrt[1 + (I*Sqrt[a])/Sqrt[b] +
(I*(a + b)*Tan[c + d*x]^2)/(Sqrt[a]*Sqrt[b])]/Sqrt[2]], (2*Sqrt[a])/(Sqrt[
a] - I*Sqrt[b])]*Sqrt[(1 - (I*Sqrt[a])/Sqrt[b])*Sec[c + d*x]^2]*(Sqrt[a] +
 (Sqrt[a] + I*Sqrt[b])*Tan[c + d*x]^2)*Sqrt[((-I)*(a + I*Sqrt[a]*Sqrt[b] +
 (a + b)*Tan[c + d*x]^2))/(Sqrt[a]*Sqrt[b])])/((Sqrt[a] + I*Sqrt[b])*d*Sqr
t[8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]]*Sqrt[1 + (I*Sqrt[
a])/Sqrt[b] + (I*(a + b)*Tan[c + d*x]^2)/(Sqrt[a]*Sqrt[b])])
```

3.243.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3694, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(c+dx)}{\sqrt{a+b\sin(c+dx)^4}}dx$$
$$\Big\downarrow \text{3694}$$
$$-\frac{\int \frac{1}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{d}$$
$$\Big\downarrow \text{1416}$$

$$-\frac{\sqrt[4]{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right),\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{2\sqrt[4]{b}d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}$$

input
```
Int[Sin[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
-1/2*((a + b)^(1/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b
 - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d
*x]^2)/Sqrt[a + b])^2)]*EllipticF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^
(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(b^(1/4)*d*Sqrt[a + b - 2*b*Cos[c +
d*x]^2 + b*Cos[c + d*x]^4])
```

3.243.3.1 Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.243. $\int \frac{\sin(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.243.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.05 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{\sqrt{1-\frac{\left(i\sqrt{a}\,\sqrt{b}+b\right)\left(\cos^2(dx+c)\right)}{a+b}}\,\sqrt{1+\frac{\left(i\sqrt{a}\,\sqrt{b}-b\right)\left(\cos^2(dx+c)\right)}{a+b}}\,F\left(\cos(dx+c)\sqrt{\frac{i\sqrt{a}\,\sqrt{b}+b}{a+b}},\sqrt{-1-\frac{2\left(i\sqrt{a}\,\sqrt{b}-b\right)}{a+b}}\right)}{d\sqrt{\frac{i\sqrt{a}\,\sqrt{b}+b}{a+b}}\,\sqrt{a+b-2b(\cos^2(dx+c))+b(\cos^4(dx+c))}}$	163

input
```
int(sin(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*co
s(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-
2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*EllipticF(cos(d*x+c)*((I*a^(1/2)*b^
(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))
```

3.243.5 Fricas [F]

$$\int \frac{\sin(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\sin(dx+c)}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
integral(sin(d*x + c)/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b),
 x)
```

3.243.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)**4)**(1/2),x)
```

output
```
Timed out
```

3.243.7 Maxima [F]

$$\int \frac{\sin(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\sin(dx+c)}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.243.8 Giac [F]

$$\int \frac{\sin(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\sin(dx+c)}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

input
```
integrate(sin(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\sin(c+d\,x)}{\sqrt{b\sin(c+d\,x)^4+a}}\,dx$$

input
```
int(sin(c + d*x)/(a + b*sin(c + d*x)^4)^(1/2),x)
```

output
```
int(sin(c + d*x)/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.244 $\int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.244.1 Optimal result

Integrand size = 23, antiderivative size = 469

$$\int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}\right)}{2\sqrt{-a}d}$$

$$+\frac{\sqrt[4]{b}\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}{(a+b)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\cos(c}{\sqrt[4]{a+}\right.\right.}{2ad\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}$$

$$-\frac{\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)^2\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}{(a+b)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}+\sqrt{a+b}\right)^2}{4\sqrt{b}\sqrt{a+b}},2\,\text{arcta}\right.}{4a\sqrt[4]{b}d\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}$$

output
```
-1/2*arctan(cos(d*x+c)*(-a)^(1/2)/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1
/2))/d/(-a)^(1/2)+1/2*b^(1/4)*(a+b)^(1/4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)
/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*Elli
pticF(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/2*(2+2*b^(1/2)/(a+b)
^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*(b^(1/2)-(a+b)^(1/2))*
((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)/(a+b)
^(1/2))^2)^(1/2)/a/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)-1/4*(a+b)
^(1/4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arcta
n(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*cos(d*x
+c)/(a+b)^(1/4))),1/4*(b^(1/2)+(a+b)^(1/2))^2/b^(1/2)/(a+b)^(1/2),1/2*(2+2
*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*(b^(1/2)
-(a+b)^(1/2))^2*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)
^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/a/b^(1/4)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d
*x+c)^4)^(1/2)
```

3.244.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.57 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.04

$$\int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

$$= \frac{\cos^3(c+dx)\sqrt{\left(1-\frac{i\sqrt{a}}{\sqrt{b}}\right)\sec^2(c+dx)}\left(\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{i\sqrt{a}}{\sqrt{b}}+\frac{i(a+b)\tan^2(c+dx)}{\sqrt{a}\sqrt{b}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}}\right)\left(a-i\sqrt{a}\sqrt{\cdots}\right.\right.}{\cdots}$$

input
```
Integrate[Csc[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
(Cos[c + d*x]^3*Sqrt[(1 - (I*Sqrt[a])/Sqrt[b])*Sec[c + d*x]^2]*(EllipticF[
ArcSin[Sqrt[1 + (I*Sqrt[a])/Sqrt[b] + (I*(a + b)*Tan[c + d*x]^2)/(Sqrt[a]*
Sqrt[b])]/Sqrt[2]], (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b])]*(a - I*Sqrt[a]*Sqrt
[b] + (a + b)*Tan[c + d*x]^2)*Sqrt[((-I)*(a + I*Sqrt[a]*Sqrt[b] + (a + b)*
Tan[c + d*x]^2))/(Sqrt[a]*Sqrt[b])] + I*(Sqrt[a] + I*Sqrt[b])*Sqrt[b]*Elli
pticPi[((-2*I)*Sqrt[b])/(Sqrt[a] - I*Sqrt[b]), ArcSin[Sqrt[1 + (I*Sqrt[a])
/Sqrt[b] + (I*(a + b)*Tan[c + d*x]^2)/(Sqrt[a]*Sqrt[b])]/Sqrt[2]], (2*Sqrt
[a])/(Sqrt[a] - I*Sqrt[b])]*Sqrt[1 + (I*Sqrt[a])/Sqrt[b] + (I*(a + b)*Tan[
c + d*x]^2)/(Sqrt[a]*Sqrt[b])]*Sqrt[((a + b)*(a*Sec[c + d*x]^4 + b*Tan[c +
 d*x]^4))/(a*b)]))/((a + b)*d*Sqrt[1 + (I*Sqrt[a])/Sqrt[b] + (I*(a + b)*Ta
n[c + d*x]^2)/(Sqrt[a]*Sqrt[b])]*Sqrt[Cos[c + d*x]^4*(a + 2*a*Tan[c + d*x]
^2 + (a + b)*Tan[c + d*x]^4)])
```

3.244.3 Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3694, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(c+dx)\sqrt{a+b\sin(c+dx)^4}}dx$$

$$\Big\downarrow \ 3694$$

$$-\frac{\int \frac{1}{\left(1-\cos^2(c+dx)\right)\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{d}$$

$$\Big\downarrow \ 1540$$

$$-\frac{\frac{(a+b)\left(1-\frac{\sqrt{b}}{\sqrt{a+b}}\right)\int \frac{\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1}{\left(1-\cos^2(c+dx)\right)\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{a}-\frac{\sqrt{b}\left(\sqrt{b}-\sqrt{a+b}\right)\int \frac{1}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c}{a}}{d}$$

$$\Big\downarrow \ 1416$$

$$-\frac{\frac{(a+b)\left(1-\frac{\sqrt{b}}{\sqrt{a+b}}\right)\int \frac{\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1}{\left(1-\cos^2(c+dx)\right)\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{a}-\frac{\sqrt[4]{b}\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(\cdots}{(a+b)(\cdots}}\cdots}{2a\sqrt{a+b\cos\cdots}}}{d}$$

$$\Big\downarrow \text{2222}$$

$$-\frac{(a+b)\left(1-\frac{\sqrt{b}}{\sqrt{a+b}}\right)\left(\frac{\sqrt[4]{a+b}\left(1-\frac{\sqrt{b}}{\sqrt{a+b}}\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}+\sqrt{a+b}\right)^2}{4\sqrt{b}\sqrt{a+b}},2\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right),\cdots\right.}{4\sqrt[4]{b}\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\cdots\right.}{a}\cdots$$

input
```
Int[Csc[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
-((-1/2*(b^(1/4)*(a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])*(1 + (Sqrt[b]*Cos[c
 + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^
4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticF[2*Arc
Tan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(
a*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4]) + ((a + b)*(1 - Sqr
t[b]/Sqrt[a + b])*(((1 + Sqrt[b]/Sqrt[a + b])*ArcTanh[(Sqrt[a]*Cos[c + d*x
])/Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4]])/(2*Sqrt[a]) + ((a
 + b)^(1/4)*(1 - Sqrt[b]/Sqrt[a + b])*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a
 + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 +
(Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticPi[(Sqrt[b] + Sqrt[a + b
])^2/(4*Sqrt[b]*Sqrt[a + b]), 2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4
)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(4*b^(1/4)*Sqrt[a + b - 2*b*Cos[c + d*x]
^2 + b*Cos[c + d*x]^4])))/a)/d)
```

3.244.3.1 Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.244.4 Maple [F]

$$\int \frac{\csc (dx+c)}{\sqrt{a+b\left(\sin ^4(dx+c)\right)}}dx$$

input
```
int(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x)
```

output
```
int(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x)
```

3.244.5 Fricas [F]

$$\int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\csc(dx+c)}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b),
 x)
```

3.244.6 Sympy [F]

$$\int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)**4)**(1/2),x)
```

output
```
Integral(csc(c + d*x)/sqrt(a + b*sin(c + d*x)**4), x)
```

3.244.7 Maxima [F]

$$\int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\csc(dx+c)}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.244.8 Giac [F]

$$\int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\csc(dx+c)}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

input
```
integrate(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{1}{\sin(c+d\,x)\,\sqrt{b\sin(c+d\,x)^4+a}}\,dx$$

input
```
int(1/(sin(c + d*x)*(a + b*sin(c + d*x)^4)^(1/2)),x)
```

output
```
int(1/(sin(c + d*x)*(a + b*sin(c + d*x)^4)^(1/2)), x)
```

3.245 $\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.245.1 Optimal result

Integrand size = 25, antiderivative size = 776

$$
\begin{aligned}
\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx &= -\frac{\arctan\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}\right)}{4\sqrt{-a}d} \\
&- \frac{\sqrt{b}\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{2a\sqrt{a+b}d\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)} \\
&- \frac{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}\cot(c+dx)\csc(c+dx)}{2ad} \\
&+ \frac{\sqrt[4]{b}(a+b)^{3/4}\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}{(a+b)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right)}{2ad\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}} \\
&- \frac{\sqrt[4]{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}{(a+b)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\cos(c+\cdots}{\sqrt[4]{a+b}\cdots}\right.\right.}{2a\sqrt[4]{a+b}d\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}} \\
&- \frac{\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)^2\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}{(a+b)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}+\sqrt{a+b}\right)^2}{4\sqrt{b}\sqrt{a+b}},2\operatorname{arcta}\cdots\right.}{8a\sqrt[4]{b}d\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}
\end{aligned}
$$

output
```
-1/4*arctan(cos(d*x+c)*(-a)^(1/2)/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1
/2))/d/(-a)^(1/2)-1/2*cot(d*x+c)*csc(d*x+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*
x+c)^4)^(1/2)/a/d-1/2*cos(d*x+c)*b^(1/2)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c
)^4)^(1/2)/a/d/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)+1/2*b^(1/4
)*(a+b)^(3/4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(
2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*c
os(d*x+c)/(a+b)^(1/4))),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)
^2*b^(1/2)/(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+co
s(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/a/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d
*x+c)^4)^(1/2)-1/8*(a+b)^(1/4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4
)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*EllipticPi(sin(
2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/4*(b^(1/2)+(a+b)^(1/2))^2/b^(1
/2)/(a+b)^(1/2),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/
2)/(a+b)^(1/2))*(b^(1/2)-(a+b)^(1/2))^2*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c
)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/a/b^(1/4)/d/(a+b-
2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)-1/2*b^(1/4)*(cos(2*arctan(b^(1/4)*c
os(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/
4)))*EllipticF(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/2*(2+2*b^(1
/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*(a+b-b^(1/2)*
(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)...
```

3.245.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 25.16 (sec) , antiderivative size = 1442, normalized size of antiderivative = 1.86

$$\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[Csc[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
-1/4*(Sqrt[8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]]*Cot[c +
d*x]*Csc[c + d*x])/(Sqrt[2]*a*d) + (Sec[c + d*x]*(-a - 2*a*Tan[c + d*x]^2
- a*Tan[c + d*x]^4 - b*Tan[c + d*x]^4 - (b*(Sqrt[b]*EllipticE[ArcSin[Sqrt[
(I*(a - I*Sqrt[a]*Sqrt[b] + a*Tan[c + d*x]^2 + b*Tan[c + d*x]^2))/(Sqrt[a]
*Sqrt[b])]/Sqrt[2]], (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b])] + I*(Sqrt[a] + I*S
qrt[b])*EllipticF[ArcSin[Sqrt[(I*(a - I*Sqrt[a]*Sqrt[b] + a*Tan[c + d*x]^2
 + b*Tan[c + d*x]^2))/(Sqrt[a]*Sqrt[b])]/Sqrt[2]], (2*Sqrt[a])/(Sqrt[a] -
I*Sqrt[b])])*Sqrt[(((-I)*Sqrt[a] + Sqrt[b])*(1 + Tan[c + d*x]^2))/Sqrt[b]]
*Sqrt[(I*(a - I*Sqrt[a]*Sqrt[b] + a*Tan[c + d*x]^2 + b*Tan[c + d*x]^2))/(S
qrt[a]*Sqrt[b])]*(Sqrt[b]*Tan[c + d*x]^2 + I*Sqrt[a]*(1 + Tan[c + d*x]^2))
)/((a + b)*Sqrt[((-I)*(a + I*Sqrt[a]*Sqrt[b] + a*Tan[c + d*x]^2 + b*Tan[c
+ d*x]^2))/(Sqrt[a]*Sqrt[b])]) + (a*((-I)*Sqrt[b]*EllipticE[ArcSin[Sqrt[(I
*(a - I*Sqrt[a]*Sqrt[b] + a*Tan[c + d*x]^2 + b*Tan[c + d*x]^2))/(Sqrt[a]*S
qrt[b])]/Sqrt[2]], (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b])] + (Sqrt[a] + I*Sqrt[
b])*EllipticF[ArcSin[Sqrt[(I*(a - I*Sqrt[a]*Sqrt[b] + a*Tan[c + d*x]^2 + b
*Tan[c + d*x]^2))/(Sqrt[a]*Sqrt[b])]/Sqrt[2]], (2*Sqrt[a])/(Sqrt[a] - I*Sq
rt[b])])*Sqrt[(((-I)*Sqrt[a] + Sqrt[b])*(1 + Tan[c + d*x]^2))/Sqrt[b]]*Sqr
t[(I*(a - I*Sqrt[a]*Sqrt[b] + a*Tan[c + d*x]^2 + b*Tan[c + d*x]^2))/(Sqrt[
a]*Sqrt[b])]*((-I)*Sqrt[b]*Tan[c + d*x]^2 + Sqrt[a]*(1 + Tan[c + d*x]^2)))
/((a + b)*Sqrt[((-I)*(a + I*Sqrt[a]*Sqrt[b] + a*Tan[c + d*x]^2 + b*Tan[...
```

3.245.3 Rubi [A] (warning: unable to verify)

Time = 1.30 (sec) , antiderivative size = 802, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3694, 1551, 25, 2232, 25, 27, 1509, 2226, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(c+dx)^3\sqrt{a+b\sin(c+dx)^4}}dx$$

$$\Big\downarrow \text{ 3694}$$

$$-\frac{\int \frac{1}{(1-\cos^2(c+dx))^2\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{ 1551}$$

$$-\frac{\frac{\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{2a(1-\cos^2(c+dx))}-\frac{\int -\frac{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}{\left(1-\cos^2(c+dx)\right)\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{2a}}{d}$$

$\downarrow$ 25

$$-\frac{\frac{\int \frac{-b\cos^4(c+dx)+2b\cos^2(c+dx)+a-b}{\left(1-\cos^2(c+dx)\right)\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{2a}+\frac{\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{2a(1-\cos^2(c+dx))}}{d}$$

$\downarrow$ 2232

$$-\frac{\frac{-\sqrt{b}\sqrt{a+b}\int \frac{1-\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)-\frac{\int -\frac{b\left(\sqrt{b}\left(\sqrt{b}-\sqrt{a+b}\right)\cos^2(c+dx)+a-b+\sqrt{b}\sqrt{a+b}\right)}{\left(1-\cos^2(c+dx)\right)\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{b}}{2a}+\frac{\cos(c+dx)\sqrt{a+\dots}}{2\dots}}{d}$$

$\downarrow$ 25

$$-\frac{\frac{\frac{\int \frac{b\left(\sqrt{b}\left(\sqrt{b}-\sqrt{a+b}\right)\cos^2(c+dx)+a-b+\sqrt{b}\sqrt{a+b}\right)}{\left(1-\cos^2(c+dx)\right)\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{b}-\sqrt{b}\sqrt{a+b}\int \frac{1-\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{2a}+\frac{\cos(c+dx)\sqrt{a+b\cos\dots}}{2a(1-\dots}}{d}$$

$\downarrow$ 27

$$-\frac{\frac{\int \frac{\sqrt{b}\left(\sqrt{b}-\sqrt{a+b}\right)\cos^2(c+dx)+a-b+\sqrt{b}\sqrt{a+b}}{\left(1-\cos^2(c+dx)\right)\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)-\sqrt{b}\sqrt{a+b}\int \frac{1-\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{2a}+\frac{\cos(c+dx)\sqrt{a+b\cos\dots}}{2a(1\dots}}{d}$$

$\downarrow$ 1509

$$-\frac{\frac{\int \frac{\sqrt{b}\left(\sqrt{b}-\sqrt{a+b}\right)\cos^2(c+dx)+a-b+\sqrt{b}\sqrt{a+b}}{\left(1-\cos^2(c+dx)\right)\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)-\sqrt{b}\sqrt{a+b}\left(\frac{\sqrt[4]{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}E\left(2\,a\dots\right.}{\sqrt[4]{b}\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx\dots}}\right.}{2a}}{d}$$

$\downarrow$ 2226

3.245. $\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

$$-\frac{\frac{2\sqrt{b}\left(-\sqrt{b}\sqrt{a+b}+a+b\right)\int \frac{1}{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)}{\sqrt{a+b}}-\sqrt{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)\int \frac{\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1}{\left(1-\cos^2(c+dx)\right)\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos}{}$$

$$\downarrow 1416$$

$$-\frac{-\sqrt{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)\int \frac{\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1}{\left(1-\cos^2(c+dx)\right)\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}d\cos(c+dx)+\frac{\sqrt[4]{b}\left(-\sqrt{b}\sqrt{a+b}+a+b\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)}{(a+b)\left(\frac{\sqrt{b}\cos}{\sqrt{}}\right.}}}{\sqrt[4]{a+b}\sqrt{a+b\,c}}}{}$$

$$\downarrow 2222$$

$$-\frac{\frac{\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}\cos(c+dx)}{2a(1-\cos^2(c+dx))}+\frac{-\sqrt{b}\sqrt{a+b}\left(\frac{\sqrt[4]{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{}}\right.\right.}{\sqrt[4]{b}\sqrt{b\cos^4(c+dx)-2b\cos^2(c+dx)+a+b}}\right.}{}}{}$$

input
```
Int[Csc[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

3.245. $\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

output
```
-(((Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/(2*a
*(1 - Cos[c + d*x]^2)) + (-(Sqrt[b]*Sqrt[a + b]*(-((Cos[c + d*x]*Sqrt[a +
b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/((a + b)*(1 + (Sqrt[b]*Cos[c +
 d*x]^2)/Sqrt[a + b]))) + ((a + b)^(1/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqr
t[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1
 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticE[2*ArcTan[(b^(1/4)*C
os[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(b^(1/4)*Sqrt[a
 + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4]))) + (b^(1/4)*(a + b - Sqrt[
b]*Sqrt[a + b])*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2
*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^
2)/Sqrt[a + b])^2)]*EllipticF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4
)], (1 + Sqrt[b]/Sqrt[a + b])/2])/((a + b)^(1/4)*Sqrt[a + b - 2*b*Cos[c +
d*x]^2 + b*Cos[c + d*x]^4]) - Sqrt[a + b]*(Sqrt[b] - Sqrt[a + b])*(((1 + S
qrt[b]/Sqrt[a + b])*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + b - 2*b*Cos[c
+ d*x]^2 + b*Cos[c + d*x]^4]])/(2*Sqrt[a]) + ((a + b)^(1/4)*(1 - Sqrt[b]/S
qrt[a + b])*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*C
os[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/S
qrt[a + b])^2)]*EllipticPi[(Sqrt[b] + Sqrt[a + b])^2/(4*Sqrt[b]*Sqrt[a + b
]), 2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a +
b])/2])/(4*b^(1/4)*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])...
```

3.245.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.245. $\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1551
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2
*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c
*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*
a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2226
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Simp[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

rule 2232
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + b*x^2
 + c*x^4], x], x] + Simp[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d -
 a*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
&&  !GtQ[b^2 - 4*a*c, 0]
```

3.245. $\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3694
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
 x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
 1)/2]
```

3.245.4 Maple [F]

$$\int \frac{\csc^3(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}dx$$

input
```
int(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

output
```
int(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

3.245.5 Fricas [F]

$$\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\csc{(dx+c)}^3}{\sqrt{b\sin{(dx+c)}^4+a}}\,dx$$

input
```
integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)^3/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b
), x)
```

3.245.6 Sympy [F]

$$\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

input
```
integrate(csc(d*x+c)**3/(a+b*sin(d*x+c)**4)**(1/2),x)
```

output
```
Integral(csc(c + d*x)**3/sqrt(a + b*sin(c + d*x)**4), x)
```

3.245.7 Maxima [F]

$$\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\csc(dx+c)^3}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

input
```
integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(d*x + c)^3/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.245.8 Giac [F]

$$\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\csc(dx+c)^3}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

input
```
integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx=\int \frac{1}{\sin\left(c+d\,x\right)^3\,\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

input
```
int(1/(sin(c + d*x)^3*(a + b*sin(c + d*x)^4)^(1/2)),x)
```

output
```
int(1/(sin(c + d*x)^3*(a + b*sin(c + d*x)^4)^(1/2)), x)
```

3.246 $\int \frac{\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.246.1 Optimal result

Integrand size = 25, antiderivative size = 499

$$\int \frac{\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}}\right)\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}}{2\sqrt{b}d\sqrt{a+b\sin^4(c+dx)}}$$

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{a+b}\right)\cos^2(c+dx)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)\left(\sqrt{a}+\sqrt{a+b}\right.}{2b\sqrt[4]{a+b}d\sqrt{a+b\sin^4(c+dx)}}$$

$$+\frac{\left(\sqrt{a}+\sqrt{a+b}\right)^2\cos^2(c+dx)\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{a+b}\right)^2}{4\sqrt{a}\sqrt{a+b}},2\arctan\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)\left(\sqrt{\right.}{4\sqrt[4]{a}b\sqrt[4]{a+b}d\sqrt{a+b\sin^4(c+dx)}}$$

output
```
-1/2*arctan(b^(1/2)*tan(d*x+c)/(a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)^(1/
2))*cos(d*x+c)^2*(a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)^(1/2)/d/b^(1/2)/(
a+b*sin(d*x+c)^4)^(1/2)-1/2*a^(1/4)*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)
*tan(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)
))*EllipticF(sin(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2
)/(a+b)^(1/2))^(1/2))*(a^(1/2)+(a+b)^(1/2))*((a+2*a*tan(d*x+c)^2+(a+b)*tan
(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^(1/2)*(a^(1/2)+(a+b)^(1/2
)*tan(d*x+c)^2)/b/(a+b)^(1/4)/d/(a+b*sin(d*x+c)^4)^(1/2)+1/4*cos(d*x+c)^2*
(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arctan((a+b)
^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticPi(sin(2*arctan((a+b)^(1/4)*tan(d*x+c)
/a^(1/4))),-1/4*(a^(1/2)-(a+b)^(1/2))^2/a^(1/2)/(a+b)^(1/2),1/2*(2-2*a^(1/
2)/(a+b)^(1/2))^(1/2))*(a^(1/2)+(a+b)^(1/2))^2*((a+2*a*tan(d*x+c)^2+(a+b)*
tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^(1/2)*(a^(1/2)+(a+b)^(
1/2)*tan(d*x+c)^2)/a^(1/4)/b/(a+b)^(1/4)/d/(a+b*sin(d*x+c)^4)^(1/2)
```

3.246.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.32 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.58

$$\int \frac{\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx =$$

$$-\frac{2i\cos^2(c+dx)\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right),\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}\right)-\text{EllipticPi}\left(\frac{\sqrt{a}}{\sqrt{a}-i\sqrt{b}},i\text{arcsinh}\left(\right.\right.\right.}{\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}d\sqrt{8a+3b-4b\cos(2(c}$$

input
```
Integrate[Sin[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
((-2*I)*Cos[c + d*x]^2*(EllipticF[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*
Tan[c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])] - EllipticPi[S
qrt[a]/(Sqrt[a] - I*Sqrt[b]), I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[
c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])])*Sqrt[1 + (1 + (I*
Sqrt[b])/Sqrt[a])*Tan[c + d*x]^2]*Sqrt[2 + (2 - ((2*I)*Sqrt[b])/Sqrt[a])*T
an[c + d*x]^2])/(Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*d*Sqrt[8*a + 3*b - 4*b*Cos[
2*(c + d*x)] + b*Cos[4*(c + d*x)]])
```

3.246.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3698, 1656, 27, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^2}{\sqrt{a+b\sin(c+dx)^4}}dx$$

$$\Big\downarrow \text{3698}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\int \frac{\tan^2(c+dx)}{(\tan^2(c+dx)+1)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}d\tan(c+}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$\Big\downarrow \text{1656}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\left(\frac{a\left(\frac{\sqrt{a+b}}{\sqrt{a}}+1\right)\int \frac{\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}}{\sqrt{a}\left(\tan^2(c+dx)+1\right)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}}{b}\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\left(\frac{\sqrt{a}\left(\frac{\sqrt{a+b}}{\sqrt{a}}+1\right)\int \frac{\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}}{\left(\tan^2(c+dx)+1\right)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}d}{b}\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\left(\frac{\sqrt{a}\left(\frac{\sqrt{a+b}}{\sqrt{a}}+1\right)\int \frac{\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}}{\left(\tan^2(c+dx)+1\right)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}d}{b}\right.}{d\sqrt{a+}}$$

$$\Big\downarrow \text{2220}$$

$$\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\left(\frac{\sqrt{a}\left(\frac{\sqrt{a+b}}{\sqrt{a}}+1\right)\left(\frac{(\sqrt{a}-\sqrt{a+b})\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)}}\right.}{2\sqrt{b}}\right.}{}\right.$$

input
```
Int[Sin[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
(Cos[c + d*x]^2*Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4]*(-1/
2*(a^(3/4)*(1 + Sqrt[a + b]/Sqrt[a])*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan
[c + d*x])/a^(1/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*T
an[c + d*x]^2)*Sqrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqr
t[a] + Sqrt[a + b]*Tan[c + d*x]^2)^2])/(b*(a + b)^(1/4)*Sqrt[a + 2*a*Tan[c
 + d*x]^2 + (a + b)*Tan[c + d*x]^4]) + (Sqrt[a]*(1 + Sqrt[a + b]/Sqrt[a])*
(((Sqrt[a] - Sqrt[a + b])*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a + 2*a*Tan[c
 + d*x]^2 + (a + b)*Tan[c + d*x]^4]])/(2*Sqrt[b]) + ((Sqrt[a] + Sqrt[a + b
])*EllipticPi[-1/4*(Sqrt[a] - Sqrt[a + b])^2/(Sqrt[a]*Sqrt[a + b]), 2*ArcT
an[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sq
rt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)
*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)^2])/(4*a^(1/4)*(a
+ b)^(1/4)*Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4])))/b))/(d
*Sqrt[a + b*Sin[c + d*x]^4])
```

3.246.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.246. $\int \frac{\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

rule 1656
```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(-a)*((e + d*q)/(c*d^2 - a*e^2))
   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[a*d*((e + d*q)/(c*d^2 - a*e
^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a] && NeQ[c*d^2 -
a*e^2, 0]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3698
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1)
*(a + b*Sin[e + f*x]^4)^p*((Sec[e + f*x]^2)^(2*p)/(f*Apart[a*(1 + Tan[e + f
*x]^2)^2 + b*Tan[e + f*x]^4]^p))   Subst[Int[x^m*(ExpandToSum[a*(1 + ff^2*x
^2)^2 + b*ff^4*x^4, x]^p/(1 + ff^2*x^2)^(m/2 + 2*p + 1)), x], x, Tan[e + f*
x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[p - 1
/2]
```

3.246.4 Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 881, normalized size of antiderivative = 1.77

method	result
default	$-\frac{\sqrt{\left(\left(\cos^2(2dx+2c)\right)b+b-2b\cos(2dx+2c)+4a\right)\left(\sin^2(2dx+2c)\right)}\,\sqrt{-ab}\,\sqrt{\frac{\left(-b+\sqrt{-ab}\right)\left(-1+\cos(2dx+2c)\right)}{\sqrt{-ab}\,\left(1+\cos(2dx+2c)\right)}}\,\left(1+\cos(2dx+2c)\right)^2\sqrt{\frac{-b\cos(2d}{\sqrt{-ab}\,(1}}}{2\left(-b+\sqrt{-ab}\right)\sqrt{\frac{\left(-1+\cos(2dx+2c)\right)\left(1+\cos(2dx+2c)\right)\left(-b\cos(2dx+2c)-\right.}{b}}}$

input
```
int(sin(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c)+4*a)*sin(2*d*x+2*c)^2)^(1/2
)*(-a*b)^(1/2)*((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))/(-a*b)^(1/2)/(1+cos(
2*d*x+2*c)))^(1/2)*(1+cos(2*d*x+2*c))^2*((-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)
+b)/(-a*b)^(1/2)/(1+cos(2*d*x+2*c)))^(1/2)*((b*cos(2*d*x+2*c)+2*(-a*b)^(1/
2)-b)/(-a*b)^(1/2)/(1+cos(2*d*x+2*c)))^(1/2)*(EllipticF(((-b+(-a*b)^(1/2))
*(-1+cos(2*d*x+2*c))/(-a*b)^(1/2)/(1+cos(2*d*x+2*c)))^(1/2),((b+(-a*b)^(1/
2))/(-b+(-a*b)^(1/2)))^(1/2))-2*EllipticPi(((-b+(-a*b)^(1/2))*(-1+cos(2*d*
x+2*c))/(-a*b)^(1/2)/(1+cos(2*d*x+2*c)))^(1/2),(-a*b)^(1/2)/(-b+(-a*b)^(1/
2)),((b+(-a*b)^(1/2))/(-b+(-a*b)^(1/2)))^(1/2)))/(-b+(-a*b)^(1/2))/(1/b*(-
1+cos(2*d*x+2*c))*(1+cos(2*d*x+2*c))*(-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)*
(b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)-b))^(1/2)/sin(2*d*x+2*c)/(cos(2*d*x+2*c)^
2*b+b-2*b*cos(2*d*x+2*c)+4*a)^(1/2)/d-1/2*((cos(2*d*x+2*c)^2*b+b-2*b*cos(2
*d*x+2*c)+4*a)*sin(2*d*x+2*c)^2)^(1/2)*(-a*b)^(1/2)*((-b+(-a*b)^(1/2))*(-1
+cos(2*d*x+2*c))/(-a*b)^(1/2)/(1+cos(2*d*x+2*c)))^(1/2)*(1+cos(2*d*x+2*c))
^2*((-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)/(-a*b)^(1/2)/(1+cos(2*d*x+2*c)))^
(1/2)*((b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)-b)/(-a*b)^(1/2)/(1+cos(2*d*x+2*c))
)^(1/2)*EllipticF(((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))/(-a*b)^(1/2)/(1+c
os(2*d*x+2*c)))^(1/2),((b+(-a*b)^(1/2))/(-b+(-a*b)^(1/2)))^(1/2))/(-b+(-a*
b)^(1/2))/(1/b*(-1+cos(2*d*x+2*c))*(1+cos(2*d*x+2*c))*(-b*cos(2*d*x+2*c)+2
*(-a*b)^(1/2)+b)*(b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)-b))^(1/2)/sin(2*d*x+2...
```

3.246.5 Fricas [F]

$$\int \frac{\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\sin(dx+c)^2}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

input
```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
integral(-(cos(d*x + c)^2 - 1)/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2
+ a + b), x)
```

3.246.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**4)**(1/2),x)
```

output
```
Timed out
```

3.246.7 Maxima [F]

$$\int \frac{\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\sin\left(dx+c\right)^2}{\sqrt{b\sin\left(dx+c\right)^4+a}}\,dx$$

input
```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.246.8 Giac [F]

$$\int \frac{\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\sin\left(dx+c\right)^2}{\sqrt{b\sin\left(dx+c\right)^4+a}}\,dx$$

input
```
integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx=\int \frac{\sin\left(c+d\,x\right)^2}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

input
```
int(sin(c + d*x)^2/(a + b*sin(c + d*x)^4)^(1/2),x)
```

output
```
int(sin(c + d*x)^2/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.247 $\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.247.1 Optimal result

Integrand size = 16, antiderivative size = 162

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\cos^2(c+dx)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx)\right)\sqrt{\frac{a+2a\cdots}{(\cdots}}}{2\sqrt[4]{a}\sqrt[4]{a+b}\,d\sqrt{a+b\sin^4(c+dx)}}$$

output
```
1/2*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/c
os(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan((a+b)^
(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*((a+2*a*ta
n(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^(1/2)
*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/a^(1/4)/(a+b)^(1/4)/d/(a+b*sin(d*x+c)^
4)^(1/2)
```

3.247.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{2i\cos^2(c+dx)\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right),\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}\right)\sqrt{1+\left(1+\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}\sqrt{2\cdots}}{\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\,d\sqrt{8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx))}}$$

input
```
Integrate[1/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
((-2*I)*Cos[c + d*x]^2*EllipticF[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*T
an[c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])]*Sqrt[1 + (1 + (
I*Sqrt[b])/Sqrt[a])*Tan[c + d*x]^2]*Sqrt[2 + (2 - ((2*I)*Sqrt[b])/Sqrt[a])
*Tan[c + d*x]^2])/(Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*d*Sqrt[8*a + 3*b - 4*b*Co
s[2*(c + d*x)] + b*Cos[4*(c + d*x)]])
```

3.247.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 3689, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\sin(c+dx)^4}}dx$$

$$\Big\downarrow \text{3689}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\int \frac{1}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}d\tan(c+dx)}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)\right.}{2\sqrt[4]{a}d\sqrt[4]{a+b}\sqrt{a+b\sin^4(c+dx)}}$$

input
```
Int[1/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
(Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)],
(1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(
a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Ta
n[c + d*x]^2)^2])/(2*a^(1/4)*(a + b)^(1/4)*d*Sqrt[a + b*Sin[c + d*x]^4])
```

3.247.3.1 Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3689
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff*(a + b*Sin[e + f*x]^4)^p*((Sec[e + f
*x]^2)^(2*p)/(f*(a + 2*a*Tan[e + f*x]^2 + (a + b)*Tan[e + f*x]^4)^p))   Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x]
, x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[p - 1/2
]
```

3.247.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(181) = 362$.

Time = 2.88 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.44

method	result
default	$-\frac{\sqrt{\left(\left(\cos^{2}\left(2dx+2c\right)\right)b+b-2b\cos\left(2dx+2c\right)+4a\right)\left(\sin^{2}\left(2dx+2c\right)\right)}\,\sqrt{-ab}\,\sqrt{\frac{\left(-b+\sqrt{-ab}\right)\left(-1+\cos\left(2dx+2c\right)\right)}{\sqrt{-ab}\,\left(1+\cos\left(2dx+2c\right)\right)}}\,\left(1+\cos\left(2dx+2c\right)\right)^{2}\sqrt{\frac{-b\cos(2d}{\sqrt{-ab}\,(1}}}{\left(-b+\sqrt{-ab}\right)\sqrt{\frac{\left(-1+\cos\left(2dx+2c\right)\right)\left(1+\cos\left(2dx+2c\right)\right)\left(-b\cos\left(2dx+2c\right)+2\sqrt{-ab}+b\right)\left(b\cos\left(2dx+2c\right)+2\sqrt{-ab}-b}{b}}}$

input
```
int(1/(a+b*sin(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c)+4*a)*sin(2*d*x+2*c)^2)^(1/2)*(-
a*b)^(1/2)*((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))/(-a*b)^(1/2)/(1+cos(2*d*
x+2*c)))^(1/2)*(1+cos(2*d*x+2*c))^2*((-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)/
(-a*b)^(1/2)/(1+cos(2*d*x+2*c)))^(1/2)*((b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)-b
)/(-a*b)^(1/2)/(1+cos(2*d*x+2*c)))^(1/2)*EllipticF(((-b+(-a*b)^(1/2))*(-1+
cos(2*d*x+2*c))/(-a*b)^(1/2)/(1+cos(2*d*x+2*c)))^(1/2),((b+(-a*b)^(1/2))/(
-b+(-a*b)^(1/2)))^(1/2))/(-b+(-a*b)^(1/2))/(1/b*(-1+cos(2*d*x+2*c))*(1+cos
(2*d*x+2*c))*(-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)*(b*cos(2*d*x+2*c)+2*(-a*
b)^(1/2)-b))^(1/2)/sin(2*d*x+2*c)/(cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c)
+4*a)^(1/2)/d
```

3.247.5 Fricas [F]

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sin{(dx+c)}^4+a}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b), x)
```

3.247.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sin^4{(c+dx)}}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c)**4)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sin(c + d*x)**4), x)
```

3.247.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sin{(dx+c)}^4+a}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.247.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sin{(dx+c)}^4+a}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,{\sin\left(c+d\,x\right)}^4+a}}\,dx$$

input
```
int(1/(a + b*sin(c + d*x)^4)^(1/2),x)
```

output
```
int(1/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.248 $\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.248.1 Optimal result

Integrand size = 25, antiderivative size = 493

$$\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$
$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}}$$
$$+ \frac{\sqrt{a+b}\cos(c+dx)\sin(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx)\right)}$$
$$- \frac{\sqrt[4]{a+b}\cos^2(c+dx)E\left(2\arctan\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx)\right)\sqrt{\frac{a+2\cdots}{}}}{a^{3/4}d\sqrt{a+b\sin^4(c+dx)}}$$
$$+ \frac{\left(a+b+\sqrt{a}\sqrt{a+b}\right)\cos^2(c+dx)\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)\left(\sqrt{a}+\sqrt{a+\cdots}\right.}{2a^{3/4}(a+b)^{3/4}d\sqrt{a+b\sin^4(c+dx)}}$$

output
```
-(a+b)^(1/4)*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2
)^(1/2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticE(sin(2*arct
an((a+b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*(
(a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)
^2)^(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/a^(3/4)/d/(a+b*sin(d*x+c)^4)^
(1/2)+1/2*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(
1/2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan(
(a+b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*(a+b
+a^(1/2)*(a+b)^(1/2))*((a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a
+b)^(1/2)*tan(d*x+c)^2)^2)^(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/a^(3/4
)/(a+b)^(3/4)/d/(a+b*sin(d*x+c)^4)^(1/2)-cos(d*x+c)^2*cot(d*x+c)*(a+2*a*ta
n(d*x+c)^2+(a+b)*tan(d*x+c)^4)/a/d/(a+b*sin(d*x+c)^4)^(1/2)+cos(d*x+c)*sin
(d*x+c)*(a+b)^(1/2)*(a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/a/d/(a+b*sin(d
*x+c)^4)^(1/2)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)
```

3.248.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.20 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.94

$$\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\sqrt{8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx))}\cot(c+dx)}{2\sqrt{2}ad}$$
$$-\frac{2\sqrt{2}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}(a+b\sin^4(c+dx))\tan(c+dx)+\sqrt{a}\left(i\sqrt{a}+\sqrt{b}\right)\cos^2(c+dx)E\left(i\text{arcsinh}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\right.\right.\right.}{}$$

input
```
Integrate[Csc[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
-1/2*(Sqrt[8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]]*Cot[c +
d*x])/(Sqrt[2]*a*d) - (2*Sqrt[2]*(Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*(a + b*Sin
[c + d*x]^4)*Tan[c + d*x] + Sqrt[a]*(I*Sqrt[a] + Sqrt[b])*Cos[c + d*x]^2*E
llipticE[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a] +
 I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])]*Sqrt[1 + (1 - (I*Sqrt[b])/Sqrt[a])*Tan[
c + d*x]^2]*Sqrt[1 + (1 + (I*Sqrt[b])/Sqrt[a])*Tan[c + d*x]^2] - Sqrt[a]*S
qrt[b]*Cos[c + d*x]^2*EllipticF[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Ta
n[c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])]*Sqrt[1 + (1 - (I
*Sqrt[b])/Sqrt[a])*Tan[c + d*x]^2]*Sqrt[1 + (1 + (I*Sqrt[b])/Sqrt[a])*Tan[
c + d*x]^2]))/(a*Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*d*Sqrt[8*a + 3*b - 4*b*Cos[
2*(c + d*x)] + b*Cos[4*(c + d*x)]])
```

3.248.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3698, 1604, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(c+dx)^2\sqrt{a+b\sin(c+dx)^4}}dx$$

$$\Big\downarrow \text{ 3698}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\int \frac{\cot^2(c+dx)(\tan^2(c+dx)+1)}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}d\tan(c+dx)}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$\Big\downarrow \text{ 1604}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\Bigg(-\frac{\int -\frac{(a+b)\tan^2(c+dx)+a}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}d\tan(c+dx)}{a}-\frac{\cot(c+dx)\cdots}{\cdots}}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\Bigg(\frac{\int \frac{(a+b)\tan^2(c+dx)+a}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}d\tan(c+dx)}{a}-\frac{\cot(c+dx)\sqrt{(a\cdots}}{\cdots}}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$\Big\downarrow \text{ 1511}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\Bigg(\frac{\frac{\sqrt{a}(\sqrt{a}\sqrt{a+b}+a+b)\int \frac{1}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}d\tan(c+dx)}{\sqrt{a+b}}-\cdots}{a}\cdots}{d\sqrt{a+b\sin^4(c+dx)}\cdots}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\left(\frac{\frac{\sqrt{a}\left(\sqrt{a}\sqrt{a+b}+a+b\right)\int\frac{1}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}d\tan(c+dx)}{\sqrt{a+b}}-\cdots}{a}\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$\downarrow$ 1416

$$\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\left(\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{a+b}+a+b\right)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2\cdots}{\left(\sqrt{a+b}\tan^2(c+dx)+\cdots\right.}}}{2(a+b)^{3/4}\sqrt{(a+b)\tan^4(c+\cdots}}}{\cdots}\right.$$

$\downarrow$ 1509

$$\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\left(\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{a+b}+a+b\right)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2\cdots}{\left(\sqrt{a+b}\tan^2(c+dx)+\cdots\right.}}}{2(a+b)^{3/4}\sqrt{(a+b)\tan^4(c+\cdots}}}{\cdots}\right.$$

input
```
Int[Csc[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

3.248. $\int\frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

output
```
(Cos[c + d*x]^2*Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4]*(-((
Cot[c + d*x]*Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4])/a) + (
(a^(1/4)*(a + b + Sqrt[a]*Sqrt[a + b])*EllipticF[2*ArcTan[((a + b)^(1/4)*T
an[c + d*x])/a^(1/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]
*Tan[c + d*x]^2)*Sqrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(S
qrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)^2])/(2*(a + b)^(3/4)*Sqrt[a + 2*a*Tan
[c + d*x]^2 + (a + b)*Tan[c + d*x]^4]) - Sqrt[a + b]*(-((Tan[c + d*x]*Sqrt
[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4])/(Sqrt[a] + Sqrt[a + b]*
Tan[c + d*x]^2)) + (a^(1/4)*EllipticE[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x]
)/a^(1/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*
x]^2)*Sqrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sq
rt[a + b]*Tan[c + d*x]^2)^2])/((a + b)^(1/4)*Sqrt[a + 2*a*Tan[c + d*x]^2 +
 (a + b)*Tan[c + d*x]^4]))/a))/(d*Sqrt[a + b*Sin[c + d*x]^4])
```

3.248.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

3.248. $\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3698
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1)
*(a + b*Sin[e + f*x]^4)^p*((Sec[e + f*x]^2)^(2*p)/(f*Apart[a*(1 + Tan[e + f
*x]^2)^2 + b*Tan[e + f*x]^4]^p))   Subst[Int[x^m*(ExpandToSum[a*(1 + ff^2*x
^2)^2 + b*ff^4*x^4, x]^p/(1 + ff^2*x^2)^(m/2 + 2*p + 1)), x], x, Tan[e + f*
x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[p - 1
/2]
```

3.248.4 Maple [F]

$$\int \frac{\csc^2(dx+c)}{\sqrt{a+b(\sin^4(dx+c))}}dx$$

input
```
int(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

output
```
int(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

3.248.5 Fricas [F]

$$\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\csc\left(dx+c\right)^2}{\sqrt{b\sin\left(dx+c\right)^4+a}}\,dx$$

input
```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
integral(csc(d*x + c)^2/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b
), x)
```

3.248.6 Sympy [F]

$$\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\csc^2\left(c+dx\right)}{\sqrt{a+b\sin^4\left(c+dx\right)}}\,dx$$

input
```
integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**4)**(1/2),x)
```

output
```
Integral(csc(c + d*x)**2/sqrt(a + b*sin(c + d*x)**4), x)
```

3.248.7 Maxima [F]

$$\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\csc\left(dx+c\right)^2}{\sqrt{b\sin\left(dx+c\right)^4+a}}\,dx$$

input
```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.248.8 Giac [F]

$$\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\csc\left(dx+c\right)^2}{\sqrt{b\sin\left(dx+c\right)^4+a}}\,dx$$

input
```
integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{1}{\sin\left(c+d\,x\right)^2\,\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

input
```
int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x)^4)^(1/2)),x)
```

output
```
int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x)^4)^(1/2)), x)
```

3.249 $\int \frac{1}{a+b\sin^5(x)}\,dx$

3.249.1 Optimal result

Integrand size = 10, antiderivative size = 384

$$\int \frac{1}{a+b\sin^5(x)}\,dx = \frac{2\arctan\left(\frac{\sqrt[5]{b}+\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}+\frac{2\arctan\left(\frac{(-1)^{2/5}\sqrt[5]{b}+\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}+\frac{2\arctan\left(\frac{(-1)^{4/5}\sqrt[5]{b}+\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}-\frac{2\arctan\left(\frac{(-1)^{3/5}\left(\sqrt[5]{b}+(-1)^{2/5}\sqrt[5]{a}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}-\frac{2\arctan\left(\frac{\sqrt[5]{-1}\left(\sqrt[5]{b}+(-1)^{4/5}\sqrt[5]{a}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}$$

output
```
2/5*arctan((b^(1/5)+a^(1/5)*tan(1/2*x))/(a^(2/5)-b^(2/5))^(1/2))/a^(4/5)/(
a^(2/5)-b^(2/5))^(1/2)-2/5*arctan((-1)^(3/5)*(b^(1/5)+(-1)^(2/5)*a^(1/5)*t
an(1/2*x))/(a^(2/5)+(-1)^(1/5)*b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)+(-1)^(1/5)
*b^(2/5))^(1/2)-2/5*arctan((-1)^(1/5)*(b^(1/5)+(-1)^(4/5)*a^(1/5)*tan(1/2*
x))/(a^(2/5)-(-1)^(2/5)*b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)-(-1)^(2/5)*b^(2/5
))^(1/2)+2/5*arctan(((-1)^(4/5)*b^(1/5)+a^(1/5)*tan(1/2*x))/(a^(2/5)+(-1)^
(3/5)*b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)+(-1)^(3/5)*b^(2/5))^(1/2)+2/5*arcta
n(((-1)^(2/5)*b^(1/5)+a^(1/5)*tan(1/2*x))/(a^(2/5)-(-1)^(4/5)*b^(2/5))^(1/
2))/a^(4/5)/(a^(2/5)-(-1)^(4/5)*b^(2/5))^(1/2)
```

3.249.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.39

$$\int \frac{1}{a+b\sin^5(x)}\,dx = \frac{8}{5}i\text{RootSum}\left[ib-5ib\#1^2+10ib\#1^4+32a\#1^5-10ib\#1^6+5ib\#1^8 - ib\#1^{10}\&, \frac{2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^3-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^3}{b-4b\#1^2+16ia\#1^3+6b\#1^4-4b\#1^6+b\#1^8}\&\right]$$

input
```
Integrate[(a + b*Sin[x]^5)^(-1),x]
```

output
```
((8*I)/5)*RootSum[I*b - (5*I)*b*#1^2 + (10*I)*b*#1^4 + 32*a*#1^5 - (10*I)*
b*#1^6 + (5*I)*b*#1^8 - I*b*#1^10 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3
 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3)/(b - 4*b*#1^2 + (16*I)*a*#1^3 + 6*b
*#1^4 - 4*b*#1^6 + b*#1^8) & ]
```

3.249.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin^5(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{a+b\sin(x)^5}dx$$

$$\Big\downarrow \ 3692$$

$$\int \left(-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-\sqrt[5]{b}\sin(x)\right)}-\frac{1}{5a^{4/5}\left(\sqrt[5]{-1}\sqrt[5]{b}\sin(x)-\sqrt[5]{a}\right)}-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\sin(x)\right)}-\frac{}{5a^{4/5}\left(\right.}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{2\arctan\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{b}}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}+\frac{2\arctan\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+(-1)^{2/5}\sqrt[5]{b}}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}+\frac{2\arctan\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+(-1)^{4/5}\sqrt[5]{b}}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}-\frac{2\arctan\left(\frac{(-1)^{3/5}\left((-1)^{2/5}\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{b}\right)}{\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}-\frac{2\arctan\left(\frac{\sqrt[5]{-1}\left((-1)^{4/5}\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{b}\right)}{\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}$$

input
```
Int[(a + b*Sin[x]^5)^(-1),x]
```

output
```
(2*ArcTan[(b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) - b^(2/5)]])/(5*a^(4/5
)*Sqrt[a^(2/5) - b^(2/5)]) + (2*ArcTan[((-1)^(2/5)*b^(1/5) + a^(1/5)*Tan[x
/2])/Sqrt[a^(2/5) - (-1)^(4/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(
4/5)*b^(2/5)]) + (2*ArcTan[((-1)^(4/5)*b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^
(2/5) + (-1)^(3/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(3/5)*b^(2/5)
]) - (2*ArcTan[((-1)^(3/5)*(b^(1/5) + (-1)^(2/5)*a^(1/5)*Tan[x/2]))/Sqrt[a
^(2/5) + (-1)^(1/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(1/5)*b^(2/5
)]) - (2*ArcTan[((-1)^(1/5)*(b^(1/5) + (-1)^(4/5)*a^(1/5)*Tan[x/2]))/Sqrt[
a^(2/5) - (-1)^(2/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(2/5)*b^(2/
5)])
```

3.249.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.249.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.28

method	result
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a\_Z^{10}+5a\_Z^{8}+10a\_Z^{6}+32b\_Z^{5}+10a\_Z^{4}+5a\_Z^{2}+a\right)}\frac{\left(\_R^{8}+4\_R^{6}+6\_R^{4}+4\_R^{2}+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^{9}a+4\_R^{7}a+6\_R^{5}a+16\_R^{4}b+4\_R^{3}a+\_Ra}\right)}{5}$
risch	$\sum_{\_R=\operatorname{RootOf}\left(1+\left(9765625a^{10}-9765625a^{8}b^{2}\right)\_Z^{10}+1953125a^{8}\_Z^{8}+156250a^{6}\_Z^{6}+6250a^{4}\_Z^{4}+125a^{2}\_Z^{2}\right)}\_R\ln\left(\mathrm{e}^{ix}+\left(1\right.\right.$

input
```
int(1/(a+b*sin(x)^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*sum((_R^8+4*_R^6+6*_R^4+4*_R^2+1)/(_R^9*a+4*_R^7*a+6*_R^5*a+16*_R^4*b+
4*_R^3*a+_R*a)*ln(tan(1/2*x)-_R),_R=RootOf(_Z^10*a+5*_Z^8*a+10*_Z^6*a+32*_
Z^5*b+10*_Z^4*a+5*_Z^2*a+a))
```

3.249.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\sin^5(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+b*sin(x)^5),x, algorithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

3.249.6 Sympy [F]

$$\int \frac{1}{a+b\sin^5(x)}\,dx = \int \frac{1}{a+b\sin^5{(x)}}\,dx$$

input
```
integrate(1/(a+b*sin(x)**5),x)
```

output
```
Integral(1/(a + b*sin(x)**5), x)
```

3.249.7 Maxima [F]

$$\int \frac{1}{a+b\sin^5(x)}\,dx = \int \frac{1}{b\sin{(x)}^5+a}\,dx$$

input
```
integrate(1/(a+b*sin(x)^5),x, algorithm="maxima")
```

output
```
integrate(1/(b*sin(x)^5 + a), x)
```

3.249.8 Giac [F]

$$\int \frac{1}{a + b\sin^5(x)}\,dx = \int \frac{1}{b\sin{(x)}^5 + a}\,dx$$

input
```
integrate(1/(a+b*sin(x)^5),x, algorithm="giac")
```

output
```
integrate(1/(b*sin(x)^5 + a), x)
```

3.249.9 Mupad [B] (verification not implemented)

Time = 19.69 (sec) , antiderivative size = 1515, normalized size of antiderivative = 3.95

$$\int \frac{1}{a + b\sin^5(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*sin(x)^5),x)
```

output
```
symsum(log(-10995116277760*a*b^7*(16*tan(x/2) + 56*root(9765625*a^8*b^2*d^
10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 -
 125*a^2*d^2 - 1, d, k)*a + 5425*root(9765625*a^8*b^2*d^10 - 9765625*a^10*
d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1,
d, k)^3*a^3 + 196875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 19531
25*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^5
+ 3171875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8
- 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^7 + 19140625*
root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a
^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^9*a^9 + 1560*root(9765625*a
^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*
a^4*d^4 - 125*a^2*d^2 - 1, d, k)^2*a^2*tan(x/2) + 57000*root(9765625*a^8*b
^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*
d^4 - 125*a^2*d^2 - 1, d, k)^4*a^4*tan(x/2) + 925000*root(9765625*a^8*b^2*
d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4
 - 125*a^2*d^2 - 1, d, k)^6*a^6*tan(x/2) + 5625000*root(9765625*a^8*b^2*d^
10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 -
 125*a^2*d^2 - 1, d, k)^8*a^8*tan(x/2) + 14000*root(9765625*a^8*b^2*d^10 -
 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125
*a^2*d^2 - 1, d, k)^4*a^3*b + 175000*root(9765625*a^8*b^2*d^10 - 976562...
```

3.250 $\int \frac{1}{a+b\sin^6(x)}\,dx$

3.250.1 Optimal result

Integrand size = 10, antiderivative size = 171

$$\int \frac{1}{a+b\sin^6(x)}\,dx = \frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

output
```
1/3*arctan((a^(1/3)+b^(1/3))^(1/2)*tan(x)/a^(1/6))/a^(5/6)/(a^(1/3)+b^(1/3
))^(1/2)+1/3*arctan((a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)*tan(x)/a^(1/6))/a^(
5/6)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)+1/3*arctan((a^(1/3)+(-1)^(2/3)*b^(
1/3))^(1/2)*tan(x)/a^(1/6))/a^(5/6)/(a^(1/3)+(-1)^(2/3)*b^(1/3))^(1/2)
```

3.250.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.87

$$\int \frac{1}{a+b\sin^6(x)}\,dx = -\frac{8}{3}\text{RootSum}\left[b-6b\#1+15b\#1^2-64a\#1^3-20b\#1^3+15b\#1^4-6b\#1^5+b\#1^6\&,\frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^2-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^2}{-b+5b\#1-32a\#1^2-10b\#1^2+10b\#1^3-5b\#1^4+b\#1^5}\&\right]$$

input
```
Integrate[(a + b*Sin[x]^6)^(-1),x]
```

output
```
(-8*RootSum[b - 6*b*#1 + 15*b*#1^2 - 64*a*#1^3 - 20*b*#1^3 + 15*b*#1^4 - 6
*b*#1^5 + b*#1^6 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2 - I*Log[1 -
2*Cos[2*x]*#1 + #1^2]*#1^2)/(-b + 5*b*#1 - 32*a*#1^2 - 10*b*#1^2 + 10*b*#1
^3 - 5*b*#1^4 + b*#1^5) & ])/3
```

3.250.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin^6(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin(x)^6}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{\int \frac{1}{\frac{\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{(-1)^{2/3}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}+1}dx}{3a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{1}{\frac{\sqrt[3]{b}\sin(x)^2}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin(x)^2}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{(-1)^{2/3}\sqrt[3]{b}\sin(x)^2}{\sqrt[3]{a}}+1}dx}{3a}$$

$$\Big\downarrow \text{ 3660}$$

$$\frac{\int \frac{1}{\left(\frac{\sqrt[3]{b}}{\sqrt[3]{a}}+1\right)\tan^2(x)+1}d\tan(x)}{3a}+\frac{\int \frac{1}{\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)\tan^2(x)+1}d\tan(x)}{3a}+\frac{\int \frac{1}{\left(\frac{(-1)^{2/3}\sqrt[3]{b}}{\sqrt[3]{a}}+1\right)\tan^2(x)+1}d\tan(x)}{3a}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

input
```
Int[(a + b*Sin[x]^6)^(-1),x]
```

output
```
ArcTan[(Sqrt[a^(1/3) + b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) +
 b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(
3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) + (-1
)^(2/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(2/3)*b^(
1/3)])
```

3.250.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.250. $\int \frac{1}{a+b\sin^6(x)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

3.250.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.40

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^6+3a\_Z^4+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^5a+\_R^5b+2\_R^3a+\_Ra}\right)}{6}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(46656a^6+46656a^5b)\_Z^6+3888a^4\_Z^4+108a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(\frac{15552ia^6}{b}+15552ia^5\right)\_R^5+\left(\right.\right.$

input
```
int(1/(a+b*sin(x)^6),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum((_R^4+2*_R^2+1)/(_R^5*a+_R^5*b+2*_R^3*a+_R*a)*ln(tan(x)-_R),_R=Roo
tOf((a+b)*_Z^6+3*a*_Z^4+3*a*_Z^2+a))
```

3.250.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 15501, normalized size of antiderivative = 90.65

$$\int \frac{1}{a+b\sin^6(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(x)^6),x, algorithm="fricas")
```

output
```
Too large to include
```

3.250.6 Sympy [F]

$$\int \frac{1}{a+b\sin^6(x)}\,dx = \int \frac{1}{a+b\sin^6{(x)}}\,dx$$

input
```
integrate(1/(a+b*sin(x)**6),x)
```

output
```
Integral(1/(a + b*sin(x)**6), x)
```

3.250.7 Maxima [F]

$$\int \frac{1}{a+b\sin^6(x)}\,dx = \int \frac{1}{b\sin{(x)}^6+a}\,dx$$

input
```
integrate(1/(a+b*sin(x)^6),x, algorithm="maxima")
```

output
```
integrate(1/(b*sin(x)^6 + a), x)
```

3.250.8 Giac [F]

$$\int \frac{1}{a+b\sin^6(x)}\,dx=\int \frac{1}{b\sin{(x)}^6+a}\,dx$$

input
```
integrate(1/(a+b*sin(x)^6),x, algorithm="giac")
```

output
```
integrate(1/(b*sin(x)^6 + a), x)
```

3.250.9 Mupad [B] (verification not implemented)

Time = 15.28 (sec) , antiderivative size = 513, normalized size of antiderivative = 3.00

$$\int \frac{1}{a+b\sin^6(x)}\,dx$$
$$=\sum_{k=1}^{6}\ln\left(-\frac{b^3\,(a+b)\,\left(-\cot(x)+\mathrm{root}(46656\,a^5\,b\,d^6+46656\,a^6\,d^6+3888\,a^4\,d^4+108\,a^2\,d^2+1,d,k)\,a\,8+\right.}{}\right.$$
$$+46656\,a^6\,d^6+3888\,a^4\,d^4+108\,a^2\,d^2+1,d,k)$$

input
```
int(1/(a + b*sin(x)^6),x)
```

output
```
symsum(log(-(3*b^3*(a + b)*(8*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*
a^4*d^4 + 108*a^2*d^2 + 1, d, k)*a - cot(x) + 2*root(46656*a^5*b*d^6 + 466
56*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)*b + 504*root(46656*a^5*
b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^3*a^3 + 7776
*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d,
 k)^5*a^5 - 144*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*
a^2*d^2 + 1, d, k)^3*a^2*b + 7776*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3
888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^5*a^4*b - 60*root(46656*a^5*b*d^6 + 4
6656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^2*a^2*cot(x) - 864*ro
ot(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)
^4*a^4*cot(x) - 864*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 +
108*a^2*d^2 + 1, d, k)^4*a^3*b*cot(x) + 12*root(46656*a^5*b*d^6 + 46656*a^
6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^2*a*b*cot(x)))/cot(x))*root(
46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k), k
, 1, 6)
```

3.251 $\int \frac{1}{a+b\sin^8(x)}\,dx$

3.251.1 Optimal result

Integrand size = 10, antiderivative size = 245

$$\int \frac{1}{a+b\sin^8(x)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}} - \frac{\arctan\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}} - \frac{\arctan\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}} - \frac{\arctan\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

output
```
-1/4*arctan(((-a)^(1/4)-b^(1/4))^(1/2)*tan(x)/(-a)^(1/8))/(-a)^(7/8)/((-a)
^(1/4)-b^(1/4))^(1/2)-1/4*arctan(((-a)^(1/4)-I*b^(1/4))^(1/2)*tan(x)/(-a)^
(1/8))/(-a)^(7/8)/((-a)^(1/4)-I*b^(1/4))^(1/2)-1/4*arctan(((-a)^(1/4)+I*b^
(1/4))^(1/2)*tan(x)/(-a)^(1/8))/(-a)^(7/8)/((-a)^(1/4)+I*b^(1/4))^(1/2)-1/
4*arctan(((-a)^(1/4)+b^(1/4))^(1/2)*tan(x)/(-a)^(1/8))/(-a)^(7/8)/((-a)^(1
/4)+b^(1/4))^(1/2)
```

3.251.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.71

$$\int \frac{1}{a+b\sin^8(x)}\,dx$$
$$= 8\text{RootSum}\Big[b-8b\#1+28b\#1^2-56b\#1^3+256a\#1^4+70b\#1^4-56b\#1^5+28b\#1^6-8b\#1^7 +b\#1^8\&, \frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^3-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^3}{-b+7b\#1-21b\#1^2+128a\#1^3+35b\#1^3-35b\#1^4+21b\#1^5-7b\#1^6+b\#1^7}\&\Big]$$

input
```
Integrate[(a + b*Sin[x]^8)^(-1),x]
```

output
```
8*RootSum[b - 8*b*#1 + 28*b*#1^2 - 56*b*#1^3 + 256*a*#1^4 + 70*b*#1^4 - 56
*b*#1^5 + 28*b*#1^6 - 8*b*#1^7 + b*#1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] -
 #1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/(-b + 7*b*#1 - 21*b*#1^
2 + 128*a*#1^3 + 35*b*#1^3 - 35*b*#1^4 + 21*b*#1^5 - 7*b*#1^6 + b*#1^7) &
]
```

3.251.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin^8(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{a+b\sin(x)^8}dx$$
$$\Big\downarrow \; 3690$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}+1}dx}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin(x)^2}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin(x)^2}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\sin(x)^2}{\sqrt[4]{-a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\sin(x)^2}{\sqrt[4]{-a}}+1}dx}{4a}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\int \frac{1}{\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}\right)\tan^2(x)+1}d\tan(x)}{4a}+\frac{\int \frac{1}{\left(\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}+1\right)\tan^2(x)+1}d\tan(x)}{4a}+\frac{\int \frac{1}{\left(\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}+1\right)\tan^2(x)+1}d\tan(x)}{4a}+\frac{\int \frac{1}{\left(\frac{\sqrt[4]{b}a}{(-a)^{5/4}}+1\right)\tan^2(x)+1}d\tan(x)}{4a}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt[8]{-a}\arctan\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4a\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}+\frac{\sqrt[8]{-a}\arctan\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4a\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}+\frac{\sqrt[8]{-a}\arctan\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4a\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}+\frac{(-a)^{5/8}\arctan\left(\frac{\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}\tan(x)}{(-a)^{5/8}}\right)}{4a\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}}$$

input
```
Int[(a + b*Sin[x]^8)^(-1),x]
```

output
```
((-a)^(1/8)*ArcTan[(Sqrt[(-a)^(1/4) - I*b^(1/4)]*Tan[x])/(-a)^(1/8)])/(4*a
*Sqrt[(-a)^(1/4) - I*b^(1/4)]) + ((-a)^(1/8)*ArcTan[(Sqrt[(-a)^(1/4) + I*b
^(1/4)]*Tan[x])/(-a)^(1/8)])/(4*a*Sqrt[(-a)^(1/4) + I*b^(1/4)]) + ((-a)^(1
/8)*ArcTan[(Sqrt[(-a)^(1/4) + b^(1/4)]*Tan[x])/(-a)^(1/8)])/(4*a*Sqrt[(-a)
^(1/4) + b^(1/4)]) + ((-a)^(5/8)*ArcTan[(Sqrt[(-a)^(5/4) + a*b^(1/4)]*Tan[
x])/(-a)^(5/8)])/(4*a*Sqrt[(-a)^(5/4) + a*b^(1/4)])
```

3.251. $\int \frac{1}{a+b\sin^8(x)}\,dx$

3.251.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

3.251.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.69 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.35

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^8+4a\_Z^6+6a\_Z^4+4a\_Z^2+a\right)}\frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^7a+\_R^7b+3\_R^5a+3\_R^3a+\_Ra}\right)}{8}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(16777216a^8+16777216a^7b)\_Z^8+1048576a^6\_Z^6+24576a^4\_Z^4+256a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(-\frac{4194304ia^8}{b}\right.\right.$

input
```
int(1/(a+b*sin(x)^8),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7*a+_R^7*b+3*_R^5*a+3*_R^3*a+_R*a)*ln(t
an(x)-_R),_R=RootOf((a+b)*_Z^8+4*a*_Z^6+6*a*_Z^4+4*a*_Z^2+a))
```

3.251.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 665483 vs. 2(165) = 330.

Time = 6.40 (sec) , antiderivative size = 665483, normalized size of antiderivative = 2716.26

$$\int \frac{1}{a+b\sin^8(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(x)^8),x, algorithm="fricas")
```

output
```
Too large to include
```

3.251.6 Sympy [F]

$$\int \frac{1}{a+b\sin^8(x)}\,dx = \int \frac{1}{a+b\sin^8{(x)}}\,dx$$

input
```
integrate(1/(a+b*sin(x)**8),x)
```

output
```
Integral(1/(a + b*sin(x)**8), x)
```

3.251.7 Maxima [F]

$$\int \frac{1}{a+b\sin^8(x)}\,dx = \int \frac{1}{b\sin{(x)}^8+a}\,dx$$

input
```
integrate(1/(a+b*sin(x)^8),x, algorithm="maxima")
```

output
```
integrate(1/(b*sin(x)^8 + a), x)
```

3.251.8 Giac [F]

$$\int \frac{1}{a+b\sin^8(x)}\,dx = \int \frac{1}{b\sin\left(x\right)^8+a}\,dx$$

input
```
integrate(1/(a+b*sin(x)^8),x, algorithm="giac")
```

output
```
integrate(1/(b*sin(x)^8 + a), x)
```

3.251.9 Mupad [B] (verification not implemented)

Time = 16.28 (sec) , antiderivative size = 816, normalized size of antiderivative = 3.33

$$\begin{aligned}\int \frac{1}{a+b\sin^8(x)}\,dx = \sum_{k=1}^{8} \ln\Big(-b^5\,(a\\
&+b)\,\Big(\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\big)^2\,a^2\,800\\
&+\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\big)^4\,a^4\,43008\\
&+\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\big)^6\,a^6\,786432\\
&+\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\big)\,b\tan(x)\,4\\
&-\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\big)^4\,a^3\,b\,6144\\
&+\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\big)^6\,a^5\,b\,786432\\
&-\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\big)^3\,a^3\tan(x)\\
&-\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\big)^5\,a^5\tan(x)\\
&-\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\big)^7\,a^7\tan(x)\\
&+\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\big)^2\,a\,b\,32\\
&-\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\big)\,a\tan(x)\,60\\
&-\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\big)^3\,a^2\,b\tan(x)\\
&+\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\big)^5\,a^4\,b\tan(x)\\
&-\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\big)^7\,a^6\,b\tan(x)\\
&+5\Big)\,2\Big)\,\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2\\
&+1,d,k\big)\end{aligned}$$

input
```
int(1/(a + b*sin(x)^8),x)
```

output
```
symsum(log(-2*b^5*(a + b)*(800*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8
+ 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^2*a^2 + 43008*r
oot(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^
4 + 256*a^2*d^2 + 1, d, k)^4*a^4 + 786432*root(16777216*a^7*b*d^8 + 167772
16*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^6*a^
6 + 4*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576
*a^4*d^4 + 256*a^2*d^2 + 1, d, k)*b*tan(x) - 6144*root(16777216*a^7*b*d^8
+ 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d,
 k)^4*a^3*b + 786432*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*
a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^6*a^5*b - 9984*root(16777
216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a
^2*d^2 + 1, d, k)^3*a^3*tan(x) - 557056*root(16777216*a^7*b*d^8 + 16777216
*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^5*a^5*
tan(x) - 10485760*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6
*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^7*a^7*tan(x) + 32*root(16777
216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a
^2*d^2 + 1, d, k)^2*a*b - 60*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 +
1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)*a*tan(x) - 768*ro
ot(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4
 + 256*a^2*d^2 + 1, d, k)^3*a^2*b*tan(x) + 98304*root(16777216*a^7*b*d^...
```

3.251. $\int \frac{1}{a+b\sin^8(x)}\,dx$

3.252 $\int \frac{1}{a-b\sin^5(x)}\,dx$

3.252.1 Optimal result

Integrand size = 11, antiderivative size = 379

$$\int \frac{1}{a-b\sin^5(x)}\,dx = -\frac{2\arctan\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}-\frac{2\arctan\left(\frac{(-1)^{2/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}-\frac{2\arctan\left(\frac{(-1)^{4/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}+\frac{2\arctan\left(\frac{\sqrt[5]{-1}\sqrt[5]{b}+\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}+\frac{2\arctan\left(\frac{(-1)^{3/5}\sqrt[5]{b}+\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}$$

output
```
-2/5*arctan((b^(1/5)-a^(1/5)*tan(1/2*x))/(a^(2/5)-b^(2/5))^(1/2))/a^(4/5)/
(a^(2/5)-b^(2/5))^(1/2)+2/5*arctan(((-1)^(3/5)*b^(1/5)+a^(1/5)*tan(1/2*x))
/(a^(2/5)+(-1)^(1/5)*b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)+(-1)^(1/5)*b^(2/5))^
(1/2)+2/5*arctan(((-1)^(1/5)*b^(1/5)+a^(1/5)*tan(1/2*x))/(a^(2/5)-(-1)^(2/
5)*b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)-(-1)^(2/5)*b^(2/5))^(1/2)-2/5*arctan((
(-1)^(4/5)*b^(1/5)-a^(1/5)*tan(1/2*x))/(a^(2/5)+(-1)^(3/5)*b^(2/5))^(1/2))
/a^(4/5)/(a^(2/5)+(-1)^(3/5)*b^(2/5))^(1/2)-2/5*arctan(((-1)^(2/5)*b^(1/5)
-a^(1/5)*tan(1/2*x))/(a^(2/5)-(-1)^(4/5)*b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)-
(-1)^(4/5)*b^(2/5))^(1/2)
```

3.252.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.39

$$\int \frac{1}{a-b\sin^5(x)}\,dx$$
$$= -\frac{8}{5}i\text{RootSum}\Bigg[-ib+5ib\#1^2-10ib\#1^4+32a\#1^5+10ib\#1^6-5ib\#1^8 + ib\#1^{10}\&, \frac{2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^3-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^3}{b-4b\#1^2-16ia\#1^3+6b\#1^4-4b\#1^6+b\#1^8}\&\Bigg]$$

input
```
Integrate[(a - b*Sin[x]^5)^(-1),x]
```

output
```
((-8*I)/5)*RootSum[(-I)*b + (5*I)*b*#1^2 - (10*I)*b*#1^4 + 32*a*#1^5 + (10
*I)*b*#1^6 - (5*I)*b*#1^8 + I*b*#1^10 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*
#1^3 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3)/(b - 4*b*#1^2 - (16*I)*a*#1^3 +
 6*b*#1^4 - 4*b*#1^6 + b*#1^8) & ]
```

3.252.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\sin^5(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{a-b\sin(x)^5}dx$$
$$\Big\downarrow \text{ 3692}$$

$$\int \left(\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-\sqrt[5]{b}\sin(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\sin(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\sin(x)\right)}+\frac{}{5a^{4/5}\left(\sqrt[5]{a}+\right.}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\arctan\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}+\frac{2\arctan\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{-1}\sqrt[5]{b}}{\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}+\frac{2\arctan\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+(-1)^{3/5}\sqrt[5]{b}}{\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}-\frac{2\arctan\left(\frac{(-1)^{4/5}\left(\sqrt[5]{-1}\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{b}\right)}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}-\frac{2\arctan\left(\frac{(-1)^{2/5}\left((-1)^{3/5}\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{b}\right)}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}$$

input
```
Int[(a - b*Sin[x]^5)^(-1),x]
```

output
```
(-2*ArcTan[(b^(1/5) - a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) - b^(2/5)]])/(5*a^(4/
5)*Sqrt[a^(2/5) - b^(2/5)]) + (2*ArcTan[((-1)^(1/5)*b^(1/5) + a^(1/5)*Tan[
x/2])/Sqrt[a^(2/5) - (-1)^(2/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^
(2/5)*b^(2/5)]) + (2*ArcTan[((-1)^(3/5)*b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a
^(2/5) + (-1)^(1/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(1/5)*b^(2/5
)]) - (2*ArcTan[((-1)^(4/5)*(b^(1/5) + (-1)^(1/5)*a^(1/5)*Tan[x/2]))/Sqrt[
a^(2/5) + (-1)^(3/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(3/5)*b^(2/
5)]) - (2*ArcTan[((-1)^(2/5)*(b^(1/5) + (-1)^(3/5)*a^(1/5)*Tan[x/2]))/Sqrt
[a^(2/5) - (-1)^(4/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(4/5)*b^(2
/5)])
```

3.252.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.252. $\int \frac{1}{a-b\sin^5(x)}\,dx$

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.252.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.29

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{10}+5a\_Z^{8}+10a\_Z^{6}-32b\_Z^{5}+10a\_Z^{4}+5a\_Z^{2}+a\right)}\frac{\left(\_R^{8}+4\_R^{6}+6\_R^{4}+4\_R^{2}+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^{9}a+4\_R^{7}a+6\_R^{5}a-16\_R^{4}b+4\_R^{3}a+\_Ra}\right)}{5}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+\left(9765625a^{10}-9765625a^{8}b^{2}\right)\_Z^{10}+1953125a^{8}\_Z^{8}+156250a^{6}\_Z^{6}+6250a^{4}\_Z^{4}+125a^{2}\_Z^{2}\right)}\_R\ln\left(\mathrm{e}^{ix}+\left(-\right.\right.$

input
```
int(1/(a-b*sin(x)^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*sum((_R^8+4*_R^6+6*_R^4+4*_R^2+1)/(_R^9*a+4*_R^7*a+6*_R^5*a-16*_R^4*b+
4*_R^3*a+_R*a)*ln(tan(1/2*x)-_R),_R=RootOf(_Z^10*a+5*_Z^8*a+10*_Z^6*a-32*_
Z^5*b+10*_Z^4*a+5*_Z^2*a+a))
```

3.252.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{a-b\sin^5(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a-b*sin(x)^5),x, algorithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

3.252.6 Sympy [F]

$$\int \frac{1}{a - b\sin^5(x)}\, dx = \int \frac{1}{a - b\sin^5\left(x\right)}\, dx$$

input
```
integrate(1/(a-b*sin(x)**5),x)
```

output
```
Integral(1/(a - b*sin(x)**5), x)
```

3.252.7 Maxima [F]

$$\int \frac{1}{a - b\sin^5(x)}\, dx = \int -\frac{1}{b\sin\left(x\right)^5 - a}\, dx$$

input
```
integrate(1/(a-b*sin(x)^5),x, algorithm="maxima")
```

output
```
-integrate(1/(b*sin(x)^5 - a), x)
```

3.252.8 Giac [F]

$$\int \frac{1}{a - b\sin^5(x)}\, dx = \int -\frac{1}{b\sin\left(x\right)^5 - a}\, dx$$

input
```
integrate(1/(a-b*sin(x)^5),x, algorithm="giac")
```

output
```
integrate(-1/(b*sin(x)^5 - a), x)
```

3.252.9 Mupad [B] (verification not implemented)

Time = 18.73 (sec) , antiderivative size = 1515, normalized size of antiderivative = 4.00

$$\int \frac{1}{a - b\sin^5(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(a - b*sin(x)^5),x)
```

output
```
symsum(log(10995116277760*a*b^7*(16*tan(x/2) + 56*root(9765625*a^8*b^2*d^1
0 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 -
125*a^2*d^2 - 1, d, k)*a + 5425*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d
^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d
, k)^3*a^3 + 196875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 195312
5*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^5 +
 3171875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 -
 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^7 + 19140625*r
oot(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^
6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^9*a^9 + 1560*root(9765625*a^
8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a
^4*d^4 - 125*a^2*d^2 - 1, d, k)^2*a^2*tan(x/2) + 57000*root(9765625*a^8*b^
2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d
^4 - 125*a^2*d^2 - 1, d, k)^4*a^4*tan(x/2) + 925000*root(9765625*a^8*b^2*d
^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4
- 125*a^2*d^2 - 1, d, k)^6*a^6*tan(x/2) + 5625000*root(9765625*a^8*b^2*d^1
0 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 -
125*a^2*d^2 - 1, d, k)^8*a^8*tan(x/2) - 14000*root(9765625*a^8*b^2*d^10 -
9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*
a^2*d^2 - 1, d, k)^4*a^3*b - 175000*root(9765625*a^8*b^2*d^10 - 9765625...
```

3.253 $\int \frac{1}{a-b\sin^6(x)}\,dx$

3.253.1 Optimal result

Integrand size = 11, antiderivative size = 175

$$\int \frac{1}{a-b\sin^6(x)}\,dx = \frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}} + \frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}} + \frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

output
```
1/3*arctan((a^(1/3)-b^(1/3))^(1/2)*tan(x)/a^(1/6))/a^(5/6)/(a^(1/3)-b^(1/3
))^(1/2)+1/3*arctan((a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)*tan(x)/a^(1/6))/a^(
5/6)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)+1/3*arctan((a^(1/3)-(-1)^(2/3)*b^(
1/3))^(1/2)*tan(x)/a^(1/6))/a^(5/6)/(a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)
```

3.253.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.85

$$\int \frac{1}{a-b\sin^6(x)}\,dx = \frac{8}{3}\text{RootSum}\left[b-6b\#1+15b\#1^2+64a\#1^3-20b\#1^3+15b\#1^4-6b\#1^5 + b\#1^6\&, \frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^2-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^2}{-b+5b\#1+32a\#1^2-10b\#1^2+10b\#1^3-5b\#1^4+b\#1^5}\&\right]$$

input
```
Integrate[(a - b*Sin[x]^6)^(-1),x]
```

output
```
(8*RootSum[b - 6*b*#1 + 15*b*#1^2 + 64*a*#1^3 - 20*b*#1^3 + 15*b*#1^4 - 6*
b*#1^5 + b*#1^6 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2 - I*Log[1 - 2
*Cos[2*x]*#1 + #1^2]*#1^2)/(-b + 5*b*#1 + 32*a*#1^2 - 10*b*#1^2 + 10*b*#1^
3 - 5*b*#1^4 + b*#1^5) & ])/3
```

3.253.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\sin^6(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-b\sin(x)^6}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{(-1)^{2/3}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[3]{b}\sin(x)^2}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin(x)^2}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{(-1)^{2/3}\sqrt[3]{b}\sin(x)^2}{\sqrt[3]{a}}}dx}{3a}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\int \frac{1}{\left(1-\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)\tan^2(x)+1}d\tan(x)}{3a}+\frac{\int \frac{1}{\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}+1\right)\tan^2(x)+1}d\tan(x)}{3a}+\frac{\int \frac{1}{\left(1-\frac{(-1)^{2/3}\sqrt[3]{b}}{\sqrt[3]{a}}\right)\tan^2(x)+1}d\tan(x)}{3a}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

input
```
Int[(a - b*Sin[x]^6)^(-1),x]
```

output
```
ArcTan[(Sqrt[a^(1/3) - b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) -
 b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(
3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) - (-1
)^(2/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(2/3)*b^(
1/3)])
```

3.253.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.253. $\int \frac{1}{a-b\sin^6(x)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

3.253.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.41

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a-b)\_Z^{6}+3a\_Z^{4}+3a\_Z^{2}+a\right)}\frac{\left(\_R^{4}+2\_R^{2}+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^{5}a-\_R^{5}b+2\_R^{3}a+\_Ra}\right)}{6}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(46656a^{6}-46656a^{5}b)\_Z^{6}+3888a^{4}\_Z^{4}+108a^{2}\_Z^{2}\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(-\frac{15552ia^{6}}{b}+15552ia^{5}\right)\_R^{5}+\right.$

input
```
int(1/(a-b*sin(x)^6),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum((_R^4+2*_R^2+1)/(_R^5*a-_R^5*b+2*_R^3*a+_R*a)*ln(tan(x)-_R),_R=Roo
tOf((a-b)*_Z^6+3*a*_Z^4+3*a*_Z^2+a))
```

3.253.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 16697, normalized size of antiderivative = 95.41

$$\int \frac{1}{a-b\sin^6(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*sin(x)^6),x, algorithm="fricas")
```

output
```
Too large to include
```

3.253.6 Sympy [F]

$$\int \frac{1}{a-b\sin^6(x)}\,dx = \int \frac{1}{a-b\sin^6{(x)}}\,dx$$

input
```
integrate(1/(a-b*sin(x)**6),x)
```

output
```
Integral(1/(a - b*sin(x)**6), x)
```

3.253.7 Maxima [F]

$$\int \frac{1}{a-b\sin^6(x)}\,dx = \int -\frac{1}{b\sin{(x)}^6-a}\,dx$$

input
```
integrate(1/(a-b*sin(x)^6),x, algorithm="maxima")
```

output
```
-integrate(1/(b*sin(x)^6 - a), x)
```

3.253.8 Giac [F]

$$\int \frac{1}{a-b\sin^6(x)}\,dx=\int -\frac{1}{b\sin{(x)}^6-a}\,dx$$

input
```
integrate(1/(a-b*sin(x)^6),x, algorithm="giac")
```

output
```
integrate(-1/(b*sin(x)^6 - a), x)
```

3.253.9 Mupad [B] (verification not implemented)

Time = 14.75 (sec) , antiderivative size = 513, normalized size of antiderivative = 2.93

$$\int \frac{1}{a-b\sin^6(x)}\,dx$$
$$=\sum_{k=1}^{6}\ln\left(-\frac{b^3\,(a-b)\,\left(\cot(x)-\mathrm{root}(46656\,a^5\,b\,d^6-46656\,a^6\,d^6-3888\,a^4\,d^4-108\,a^2\,d^2-1,d,k)\;a\,8+\mathrm{ro}\right.}{}\right.$$
$$-46656\,a^6\,d^6-3888\,a^4\,d^4-108\,a^2\,d^2-1,d,k)$$

input
```
int(1/(a - b*sin(x)^6),x)
```

output
```
symsum(log(-(3*b^3*(a - b)*(cot(x) - 8*root(46656*a^5*b*d^6 - 46656*a^6*d^
6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)*a + 2*root(46656*a^5*b*d^6 - 466
56*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)*b - 504*root(46656*a^5*
b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^3*a^3 - 7776
*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d,
 k)^5*a^5 - 144*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*
a^2*d^2 - 1, d, k)^3*a^2*b + 7776*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3
888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^5*a^4*b + 60*root(46656*a^5*b*d^6 - 4
6656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^2*a^2*cot(x) + 864*ro
ot(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)
^4*a^4*cot(x) - 864*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 -
108*a^2*d^2 - 1, d, k)^4*a^3*b*cot(x) + 12*root(46656*a^5*b*d^6 - 46656*a^
6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^2*a*b*cot(x)))/cot(x))*root(
46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k), k
, 1, 6)
```

3.254 $\int \frac{1}{a-b\sin^8(x)}\,dx$

3.254.1 Optimal result

Integrand size = 11, antiderivative size = 213

$$\int \frac{1}{a-b\sin^8(x)}\,dx = \frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}} + \frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}} + \frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}} + \frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

output
```
1/4*arctan((a^(1/4)-b^(1/4))^(1/2)*tan(x)/a^(1/8))/a^(7/8)/(a^(1/4)-b^(1/4
))^(1/2)+1/4*arctan((a^(1/4)-I*b^(1/4))^(1/2)*tan(x)/a^(1/8))/a^(7/8)/(a^(
1/4)-I*b^(1/4))^(1/2)+1/4*arctan((a^(1/4)+I*b^(1/4))^(1/2)*tan(x)/a^(1/8))
/a^(7/8)/(a^(1/4)+I*b^(1/4))^(1/2)+1/4*arctan((a^(1/4)+b^(1/4))^(1/2)*tan(
x)/a^(1/8))/a^(7/8)/(a^(1/4)+b^(1/4))^(1/2)
```

3.254.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.02 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.82

$$\int \frac{1}{a-b\sin^8(x)}\,dx$$
$$=-8\text{RootSum}\Bigg[b-8b\#1+28b\#1^2-56b\#1^3-256a\#1^4+70b\#1^4-56b\#1^5+28b\#1^6-8b\#1^7$$
$$+b\#1^8\&,\frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^3-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^3}{-b+7b\#1-21b\#1^2-128a\#1^3+35b\#1^3-35b\#1^4+21b\#1^5-7b\#1^6+b\#1^7}\&\Bigg]$$

input
```
Integrate[(a - b*Sin[x]^8)^(-1),x]
```

output
```
-8*RootSum[b - 8*b*#1 + 28*b*#1^2 - 56*b*#1^3 - 256*a*#1^4 + 70*b*#1^4 - 5
6*b*#1^5 + 28*b*#1^6 - 8*b*#1^7 + b*#1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x]
- #1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/(-b + 7*b*#1 - 21*b*#1
^2 - 128*a*#1^3 + 35*b*#1^3 - 35*b*#1^4 + 21*b*#1^5 - 7*b*#1^6 + b*#1^7) &
 ]
```

3.254.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\sin^8(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{a-b\sin(x)^8}dx$$
$$\Big\downarrow \; 3690$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}+1}dx}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin(x)^2}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin(x)^2}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\sin(x)^2}{\sqrt[4]{a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\sin(x)^2}{\sqrt[4]{a}}+1}dx}{4a}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\int \frac{1}{\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right)\tan^2(x)+1}d\tan(x)}{4a}+\frac{\int \frac{1}{\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{a}}\right)\tan^2(x)+1}d\tan(x)}{4a}+\frac{\int \frac{1}{\left(\frac{i\sqrt[4]{b}}{\sqrt[4]{a}}+1\right)\tan^2(x)+1}d\tan(x)}{4a}+\frac{\int \frac{1}{\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}+1\right)\tan^2(x)+1}d\tan(x)}{4a}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

input
```
Int[(a - b*Sin[x]^8)^(-1),x]
```

output
```
ArcTan[(Sqrt[a^(1/4) - b^(1/4)]*Tan[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) -
 b^(1/4)]) + ArcTan[(Sqrt[a^(1/4) - I*b^(1/4)]*Tan[x])/a^(1/8)]/(4*a^(7/8)
*Sqrt[a^(1/4) - I*b^(1/4)]) + ArcTan[(Sqrt[a^(1/4) + I*b^(1/4)]*Tan[x])/a^
(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) + I*b^(1/4)]) + ArcTan[(Sqrt[a^(1/4) + b^(1
/4)]*Tan[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) + b^(1/4)])
```

3.254. $\int \frac{1}{a-b\sin^8(x)}\,dx$

3.254.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

3.254.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.73 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.41

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a-b)\_Z^8+4a\_Z^6+6a\_Z^4+4a\_Z^2+a\right)}\frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^7a-\_R^7b+3\_R^5a+3\_R^3a+\_Ra}\right)}{8}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(16777216a^8-16777216a^7b)\_Z^8+1048576a^6\_Z^6+24576a^4\_Z^4+256a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(\frac{4194304ia^8}{b}-\right.\right.$

input
```
int(1/(a-b*sin(x)^8),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7*a-_R^7*b+3*_R^5*a+3*_R^3*a+_R*a)*ln(t
an(x)-_R),_R=RootOf((a-b)*_Z^8+4*a*_Z^6+6*a*_Z^4+4*a*_Z^2+a))
```

3.254.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 643307 vs. 2(133) = 266.

Time = 6.89 (sec) , antiderivative size = 643307, normalized size of antiderivative = 3020.22

$$\int \frac{1}{a-b\sin^8(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*sin(x)^8),x, algorithm="fricas")
```

output
```
Too large to include
```

3.254.6 Sympy [F]

$$\int \frac{1}{a-b\sin^8(x)}\,dx = \int \frac{1}{a-b\sin^8{(x)}}\,dx$$

input
```
integrate(1/(a-b*sin(x)**8),x)
```

output
```
Integral(1/(a - b*sin(x)**8), x)
```

3.254.7 Maxima [F]

$$\int \frac{1}{a-b\sin^8(x)}\,dx = \int -\frac{1}{b\sin{(x)}^8-a}\,dx$$

input
```
integrate(1/(a-b*sin(x)^8),x, algorithm="maxima")
```

output
```
-integrate(1/(b*sin(x)^8 - a), x)
```

3.254.8 Giac [F]

$$\int \frac{1}{a-b\sin^8(x)}\,dx = \int -\frac{1}{b\sin{(x)}^8 - a}\,dx$$

input
```
integrate(1/(a-b*sin(x)^8),x, algorithm="giac")
```

output
```
integrate(-1/(b*sin(x)^8 - a), x)
```

3.254.9 Mupad [B] (verification not implemented)

Time = 16.36 (sec) , antiderivative size = 818, normalized size of antiderivative = 3.84

$$\int \frac{1}{a-b\sin^8(x)}\,dx = \sum_{k=1}^{8} \ln\Big(-b^5\,(a$$
$$-b)\,\Big(-\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\big)^2\,a^2\,8$$
$$-\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\big)^4\,a^4\,43008$$
$$-\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\big)^6\,a^6\,786432$$
$$+\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\big)\,b\tan(x)\,4$$
$$-\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\big)^4\,a^3\,b\,6144$$
$$+\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\big)^6\,a^5\,b\,78643$$
$$+\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\big)^3\,a^3\tan(x)$$
$$+\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\big)^5\,a^5\tan(x)$$
$$+\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\big)^7\,a^7\tan(x)$$
$$+\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\big)^2\,a\,b\,32$$
$$+\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\big)\,a\tan(x)\,60$$
$$-\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\big)^3\,a^2\,b\tan(x)$$
$$+\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\big)^5\,a^4\,b\tan(x)$$
$$-\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\big)^7\,a^6\,b\tan(x)$$
$$-5\Big)\,2\Big)\,\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2$$
$$-1,d,k\big)$$

input
```
int(1/(a - b*sin(x)^8),x)
```

output
```
symsum(log(-2*b^5*(a - b)*(4*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 -
1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)*b*tan(x) - 43008*
root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d
^4 - 256*a^2*d^2 - 1, d, k)^4*a^4 - 786432*root(16777216*a^7*b*d^8 - 16777
216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^6*a
^6 - 800*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24
576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^2*a^2 - 6144*root(16777216*a^7*b*d^8
- 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d,
 k)^4*a^3*b + 786432*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*
a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^6*a^5*b + 9984*root(16777
216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a
^2*d^2 - 1, d, k)^3*a^3*tan(x) + 557056*root(16777216*a^7*b*d^8 - 16777216
*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^5*a^5*
tan(x) + 10485760*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6
*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^7*a^7*tan(x) + 32*root(16777
216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a
^2*d^2 - 1, d, k)^2*a*b + 60*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 -
1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)*a*tan(x) - 768*ro
ot(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4
 - 256*a^2*d^2 - 1, d, k)^3*a^2*b*tan(x) + 98304*root(16777216*a^7*b*d^...
```

3.254. $\int \frac{1}{a-b\sin^8(x)}\,dx$

3.255 $\int \frac{1}{1+\sin^5(x)}\,dx$

3.255.1 Optimal result

Integrand size = 8, antiderivative size = 195

$$\int \frac{1}{1+\sin^5(x)}\,dx=\frac{2\arctan\left(\frac{(-1)^{2/5}+\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}}+\frac{2\arctan\left(\frac{(-1)^{4/5}+\tan\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}}-\frac{2\arctan\left(\frac{(-1)^{3/5}\left(1+(-1)^{2/5}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{2\arctan\left(\frac{\sqrt[5]{-1}\left(1+(-1)^{4/5}\tan\left(\frac{x}{2}\right)\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}}-\frac{\cos(x)}{5(1+\sin(x))}$$

output
```
-1/5*cos(x)/(1+sin(x))-2/5*arctan((-1)^(3/5)*(1+(-1)^(2/5)*tan(1/2*x))/(1+
(-1)^(1/5))^(1/2))/(1+(-1)^(1/5))^(1/2)-2/5*arctan((-1)^(1/5)*(1+(-1)^(4/5
)*tan(1/2*x))/(1-(-1)^(2/5))^(1/2))/(1-(-1)^(2/5))^(1/2)+2/5*arctan(((-1)^
(4/5)+tan(1/2*x))/(1+(-1)^(3/5))^(1/2))/(1+(-1)^(3/5))^(1/2)+2/5*arctan(((
-1)^(2/5)+tan(1/2*x))/(1-(-1)^(4/5))^(1/2))/(1-(-1)^(4/5))^(1/2)
```

3.255.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.09 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.11

$$\int \frac{1}{1+\sin^5(x)}\,dx$$
$$= -\frac{1}{10} i\text{RootSum}\Bigg[1+2i\#1-8\#1^2-14i\#1^3+30\#1^4+14i\#1^5-8\#1^6-2i\#1^7$$
$$+\#1^8\&,\frac{-2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)+i\log\left(1-2\cos(x)\#1+\#1^2\right)-8i\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1-4\log\left(1-\right.}{}$$
$$+\frac{2\sin\left(\frac{x}{2}\right)}{5\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}$$

input
```
Integrate[(1 + Sin[x]^5)^(-1),x]
```

output
```
(-1/10*I)*RootSum[1 + (2*I)*#1 - 8*#1^2 - (14*I)*#1^3 + 30*#1^4 + (14*I)*#
1^5 - 8*#1^6 - (2*I)*#1^7 + #1^8 & , (-2*ArcTan[Sin[x]/(Cos[x] - #1)] + I*
Log[1 - 2*Cos[x]*#1 + #1^2] - (8*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 - 4*Lo
g[1 - 2*Cos[x]*#1 + #1^2]*#1 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - (15*
I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^2 + (80*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*
#1^3 + 40*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3 - 30*ArcTan[Sin[x]/(Cos[x] - #1
)]*#1^4 + (15*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4 - (8*I)*ArcTan[Sin[x]/(C
os[x] - #1)]*#1^5 - 4*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^5 + 2*ArcTan[Sin[x]/(
Cos[x] - #1)]*#1^6 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/(I - 8*#1 - (21*I
)*#1^2 + 60*#1^3 + (35*I)*#1^4 - 24*#1^5 - (7*I)*#1^6 + 4*#1^7) & ] + (2*S
in[x/2])/(5*(Cos[x/2] + Sin[x/2]))
```

3.255.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^5(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(x)^5+1}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(-\frac{1}{5\left(\sqrt[5]{-1}\sin(x)-1\right)}-\frac{1}{5\left(-(-1)^{2/5}\sin(x)-1\right)}-\frac{1}{5\left((-1)^{3/5}\sin(x)-1\right)}-\frac{1}{5\left(-(-1)^{4/5}\sin(x)-1\right)}-\frac{1}{5(-\text{s}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{2/5}}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}}+\frac{2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{4/5}}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}}-\frac{2\arctan\left(\frac{(-1)^{3/5}\left((-1)^{2/5}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{2\arctan\left(\frac{\sqrt[5]{-1}\left((-1)^{4/5}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}}-\frac{\cos(x)}{5(\sin(x)+1)}$$

input
```
Int[(1 + Sin[x]^5)^(-1),x]
```

output
```
(2*ArcTan[((-1)^(2/5) + Tan[x/2])/Sqrt[1 - (-1)^(4/5)]])/(5*Sqrt[1 - (-1)^
(4/5)]) + (2*ArcTan[((-1)^(4/5) + Tan[x/2])/Sqrt[1 + (-1)^(3/5)]])/(5*Sqrt
[1 + (-1)^(3/5)]) - (2*ArcTan[((-1)^(3/5)*(1 + (-1)^(2/5)*Tan[x/2]))/Sqrt[
1 + (-1)^(1/5)]])/(5*Sqrt[1 + (-1)^(1/5)]) - (2*ArcTan[((-1)^(1/5)*(1 + (-
1)^(4/5)*Tan[x/2]))/Sqrt[1 - (-1)^(2/5)]])/(5*Sqrt[1 - (-1)^(2/5)]) - Cos[
x]/(5*(1 + Sin[x]))
```

3.255.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.255.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.68 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.45

method	result
risch	$-\frac{2}{5(e^{ix}+i)}+\left(\sum_{\_R=\mathrm{RootOf}(1953125\_Z^8+156250\_Z^6+6250\_Z^4+125\_Z^2+1)}\_R\ln\left(e^{ix}+2343750\_R^7+234375\right.\right.$
default	$\frac{2\left(\sum_{\_R=\mathrm{RootOf}(\_Z^8-2\_Z^7+8\_Z^6-14\_Z^5+30\_Z^4-14\_Z^3+8\_Z^2-2\_Z+1)}\frac{\left(2\_R^6-3\_R^5+10\_R^4-10\_R^3+10\_R^2-3\_R+2\right)}{4\_R^7-7\_R^6+24\_R^5-35\_R^4+60\_R^3-21\_}\right.}{5}$

input
```
int(1/(1+sin(x)^5),x,method=_RETURNVERBOSE)
```

output
```
-2/5/(exp(I*x)+I)+sum(_R*ln(exp(I*x)+2343750*_R^7+234375*I*_R^6+140625*_R^
5+15625*I*_R^4+4375*_R^3+500*I*_R^2+50*_R+6*I),_R=RootOf(1953125*_Z^8+1562
50*_Z^6+6250*_Z^4+125*_Z^2+1))
```

3.255.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. 2(133) = 266.

Time = 0.41 (sec) , antiderivative size = 848, normalized size of antiderivative = 4.35

$$\int \frac{1}{1+\sin^5(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1+sin(x)^5),x, algorithm="fricas")
```

output
```
-1/100*((sqrt(5)*cos(x) + sqrt(5)*sin(x) + sqrt(5))*sqrt(2*sqrt(5)*sqrt(2*
sqrt(5) - 5) - 10)*log(-sqrt(2*sqrt(5)*sqrt(2*sqrt(5) - 5) - 10)*(3*sqrt(5
) + 5)*sqrt(2*sqrt(5) - 5)*cos(x) + 5*sqrt(2*sqrt(5) - 5)*(sqrt(5) + 3)*si
n(x) + 5*(sqrt(5) - 1)*sin(x) + 20) - (sqrt(5)*cos(x) + sqrt(5)*sin(x) + s
qrt(5))*sqrt(2*sqrt(5)*sqrt(2*sqrt(5) - 5) - 10)*log(-sqrt(2*sqrt(5)*sqrt(
2*sqrt(5) - 5) - 10)*(3*sqrt(5) + 5)*sqrt(2*sqrt(5) - 5)*cos(x) - 5*sqrt(2
*sqrt(5) - 5)*(sqrt(5) + 3)*sin(x) - 5*(sqrt(5) - 1)*sin(x) - 20) + (sqrt(
5)*cos(x) + sqrt(5)*sin(x) + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(2*sqrt(5) - 5)
- 10)*log(-sqrt(-2*sqrt(5)*sqrt(2*sqrt(5) - 5) - 10)*(3*sqrt(5) + 5)*sqrt(
2*sqrt(5) - 5)*cos(x) + 5*sqrt(2*sqrt(5) - 5)*(sqrt(5) + 3)*sin(x) - 5*(sq
rt(5) - 1)*sin(x) - 20) - (sqrt(5)*cos(x) + sqrt(5)*sin(x) + sqrt(5))*sqrt
(-2*sqrt(5)*sqrt(2*sqrt(5) - 5) - 10)*log(-sqrt(-2*sqrt(5)*sqrt(2*sqrt(5)
- 5) - 10)*(3*sqrt(5) + 5)*sqrt(2*sqrt(5) - 5)*cos(x) - 5*sqrt(2*sqrt(5) -
 5)*(sqrt(5) + 3)*sin(x) + 5*(sqrt(5) - 1)*sin(x) + 20) - (sqrt(5)*cos(x)
+ sqrt(5)*sin(x) + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2*sqrt(5) - 5) - 10)*log(
-sqrt(2*sqrt(5)*sqrt(-2*sqrt(5) - 5) - 10)*(3*sqrt(5) - 5)*sqrt(-2*sqrt(5)
 - 5)*cos(x) + 5*(sqrt(5) - 3)*sqrt(-2*sqrt(5) - 5)*sin(x) - 5*(sqrt(5) +
1)*sin(x) + 20) + (sqrt(5)*cos(x) + sqrt(5)*sin(x) + sqrt(5))*sqrt(2*sqrt(
5)*sqrt(-2*sqrt(5) - 5) - 10)*log(-sqrt(2*sqrt(5)*sqrt(-2*sqrt(5) - 5) - 1
0)*(3*sqrt(5) - 5)*sqrt(-2*sqrt(5) - 5)*cos(x) - 5*(sqrt(5) - 3)*sqrt(-...
```

3.255.6 Sympy [F]

$$\int \frac{1}{1+\sin^5(x)}\,dx=\int \frac{1}{(\sin(x)+1)\left(\sin^4(x)-\sin^3(x)+\sin^2(x)-\sin(x)+1\right)}\,dx$$

input
```
integrate(1/(1+sin(x)**5),x)
```

output
```
Integral(1/((sin(x) + 1)*(sin(x)**4 - sin(x)**3 + sin(x)**2 - sin(x) + 1))
, x)
```

3.255.7 Maxima [F]

$$\int \frac{1}{1+\sin^5(x)}\,dx = \int \frac{1}{\sin(x)^5+1}\,dx$$

input
```
integrate(1/(1+sin(x)^5),x, algorithm="maxima")
```

output
```
-1/5*(5*(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)*integrate(-2/5*((4*cos(6*x) -
 40*cos(4*x) + 4*cos(2*x) - sin(7*x) + 15*sin(5*x) - 15*sin(3*x) + sin(x))
*cos(8*x) + 2*(22*cos(5*x) - 22*cos(3*x) + 2*cos(x) - 8*sin(6*x) + 55*sin(
4*x) - 8*sin(2*x))*cos(7*x) - 2*cos(7*x)^2 + 4*(110*cos(4*x) - 16*cos(2*x)
 - 44*sin(5*x) + 44*sin(3*x) - 4*sin(x) + 1)*cos(6*x) - 32*cos(6*x)^2 + 2*
(210*cos(3*x) - 22*cos(x) - 505*sin(4*x) + 88*sin(2*x))*cos(5*x) - 210*cos
(5*x)^2 + 10*(44*cos(2*x) - 101*sin(3*x) + 11*sin(x) - 4)*cos(4*x) - 1200*
cos(4*x)^2 + 44*(cos(x) - 4*sin(2*x))*cos(3*x) - 210*cos(3*x)^2 - 4*(4*sin
(x) - 1)*cos(2*x) - 32*cos(2*x)^2 - 2*cos(x)^2 + (cos(7*x) - 15*cos(5*x) +
 15*cos(3*x) - cos(x) + 4*sin(6*x) - 40*sin(4*x) + 4*sin(2*x))*sin(8*x) +
(16*cos(6*x) - 110*cos(4*x) + 16*cos(2*x) + 44*sin(5*x) - 44*sin(3*x) + 4*
sin(x) - 1)*sin(7*x) - 2*sin(7*x)^2 + 8*(22*cos(5*x) - 22*cos(3*x) + 2*cos
(x) + 55*sin(4*x) - 8*sin(2*x))*sin(6*x) - 32*sin(6*x)^2 + (1010*cos(4*x)
- 176*cos(2*x) + 420*sin(3*x) - 44*sin(x) + 15)*sin(5*x) - 210*sin(5*x)^2
+ 10*(101*cos(3*x) - 11*cos(x) + 44*sin(2*x))*sin(4*x) - 1200*sin(4*x)^2 +
 (176*cos(2*x) + 44*sin(x) - 15)*sin(3*x) - 210*sin(3*x)^2 + 16*cos(x)*sin
(2*x) - 32*sin(2*x)^2 - 2*sin(x)^2 + sin(x))/(2*(8*cos(6*x) - 30*cos(4*x)
+ 8*cos(2*x) - 2*sin(7*x) + 14*sin(5*x) - 14*sin(3*x) + 2*sin(x) - 1)*cos(
8*x) - cos(8*x)^2 + 8*(7*cos(5*x) - 7*cos(3*x) + cos(x) - 4*sin(6*x) + 15*
sin(4*x) - 4*sin(2*x))*cos(7*x) - 4*cos(7*x)^2 + 16*(30*cos(4*x) - 8*co...
```

3.255.8 Giac [F]

$$\int \frac{1}{1+\sin^5(x)}\,dx = \int \frac{1}{\sin(x)^5+1}\,dx$$

input
```
integrate(1/(1+sin(x)^5),x, algorithm="giac")
```

output
```
sage0*x
```

3.255.9 Mupad [B] (verification not implemented)

Time = 14.27 (sec) , antiderivative size = 3513, normalized size of antiderivative = 18.02

$$\int \frac{1}{1+\sin^5(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(x)^5 + 1),x)
```

output
```
2*atanh((989855744*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301
989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x
/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*(- (2*5^(1/2))/5
- 1)^(1/2))/25 + (16777216*5^(1/2))/5 - 16777216*(- (2*5^(1/2))/5 - 1)^(1/
2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 1845493
76/25)) - (2030043136*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/
2))/(5*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308
622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*(- (2
*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 - 16777216*(- (2*5^(1/2)
)/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/
25 + 184549376/25)) + (1627389952*5^(1/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50
- 1/50)^(1/2))/(25*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))
/125 + (1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5
^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 - 16777216*(
- (2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5
- 1)^(1/2))/25 + 184549376/25)) + (553648128*(- (2*5^(1/2))/5 - 1)^(1/2)*(
(- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 +
 (2382364672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2)*(- (2*5^(1/2))/5
 - 1)^(1/2))/25 - (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (16
777216*5^(1/2))/5 - 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (436207616*5...
```

3.256 $\int \frac{1}{1+\sin^6(x)} dx$

3.256.1 Optimal result

Integrand size = 8, antiderivative size = 103

$$\int \frac{1}{1+\sin^6(x)}\,dx=\frac{x}{3\sqrt{2}}+\frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{3\sqrt{2}}+\frac{\arctan\left(\sqrt{1-\sqrt[3]{-1}}\tan(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\arctan\left(\sqrt{1+(-1)^{2/3}}\tan(x)\right)}{3\sqrt{1+(-1)^{2/3}}}$$

output
```
1/6*x*2^(1/2)+1/6*arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)+1/3*a
rctan((1-(-1)^(1/3))^(1/2)*tan(x))/(1-(-1)^(1/3))^(1/2)+1/3*arctan((1+(-1)
^(2/3))^(1/2)*tan(x))/(1+(-1)^(2/3))^(1/2)
```

3.256.2 Mathematica [A] (verified)

Time = 5.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

$$\int \frac{1}{1+\sin^6(x)}\,dx=\frac{1}{12}\left(-2\sqrt{3}\arctan\left(\frac{1-2\tan(x)}{\sqrt{3}}\right)+2\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)+2\sqrt{3}\arctan\left(\frac{1+2\tan(x)}{\sqrt{3}}\right)-\log(2-\sin(2x))+\log(2+\sin(2x))\right)$$

input
```
Integrate[(1 + Sin[x]^6)^(-1),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(1 - 2*Tan[x])/Sqrt[3]] + 2*Sqrt[2]*ArcTan[Sqrt[2]*Tan[
x]] + 2*Sqrt[3]*ArcTan[(1 + 2*Tan[x])/Sqrt[3]] - Log[2 - Sin[2*x]] + Log[2
 + Sin[2*x]])/12
```

3.256.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^6(x)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(x)^6+1}dx$$
$$\Big\downarrow \text{3690}$$
$$\frac{1}{3}\int \frac{1}{\sin^2(x)+1}dx+\frac{1}{3}\int \frac{1}{1-\sqrt[3]{-1}\sin^2(x)}dx+\frac{1}{3}\int \frac{1}{(-1)^{2/3}\sin^2(x)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{3}\int \frac{1}{\sin(x)^2+1}dx+\frac{1}{3}\int \frac{1}{1-\sqrt[3]{-1}\sin(x)^2}dx+\frac{1}{3}\int \frac{1}{(-1)^{2/3}\sin(x)^2+1}dx$$
$$\Big\downarrow \text{3660}$$
$$\frac{1}{3}\int \frac{1}{2\tan^2(x)+1}d\tan(x)+\frac{1}{3}\int \frac{1}{\left(1-\sqrt[3]{-1}\right)\tan^2(x)+1}d\tan(x)+\frac{1}{3}\int \frac{1}{\left(1+(-1)^{2/3}\right)\tan^2(x)+1}d\tan(x)$$
$$\Big\downarrow \text{216}$$
$$\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{3\sqrt{2}}+\frac{\arctan\left(\sqrt{1-\sqrt[3]{-1}}\tan(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\arctan\left(\sqrt{1+(-1)^{2/3}}\tan(x)\right)}{3\sqrt{1+(-1)^{2/3}}}$$

input
```
Int[(1 + Sin[x]^6)^(-1),x]
```

output
```
ArcTan[Sqrt[2]*Tan[x]]/(3*Sqrt[2]) + ArcTan[Sqrt[1 - (-1)^(1/3)]*Tan[x]]/(
3*Sqrt[1 - (-1)^(1/3)]) + ArcTan[Sqrt[1 + (-1)^(2/3)]*Tan[x]]/(3*Sqrt[1 +
(-1)^(2/3)])
```

3.256.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

3.256.4 Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.70

method	result
default	$-\frac{\ln\left(\tan^2(x)-\tan(x)+1\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2\tan(x)-1)\sqrt{3}}{3}\right)}{6}+\frac{\arctan\left(\sqrt{2}\,\tan(x)\right)\sqrt{2}}{6}+\frac{\ln\left(\tan^2(x)+\tan(x)+1\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\right.}{}$
risch	$-\frac{\ln\left(e^{2ix}-2i-i\sqrt{3}\right)}{12}+\frac{i\ln\left(e^{2ix}-2i-i\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(e^{2ix}-2i+i\sqrt{3}\right)}{12}-\frac{i\ln\left(e^{2ix}-2i+i\sqrt{3}\right)\sqrt{3}}{12}+\frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}-3\right)}{12}-$

input
```
int(1/(1+sin(x)^6),x,method=_RETURNVERBOSE)
```

output
```
-1/12*ln(tan(x)^2-tan(x)+1)+1/6*3^(1/2)*arctan(1/3*(2*tan(x)-1)*3^(1/2))+1
/6*arctan(2^(1/2)*tan(x))*2^(1/2)+1/12*ln(tan(x)^2+tan(x)+1)+1/6*3^(1/2)*a
rctan(1/3*(2*tan(x)+1)*3^(1/2))
```

3.256.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.34

$$\int \frac{1}{1+\sin^6(x)}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)+\sqrt{3}}{3\left(2\cos(x)^2-1\right)}\right) + \frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)-\sqrt{3}}{3\left(2\cos(x)^2-1\right)}\right) - \frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right) + \frac{1}{24}\log\left(-\cos(x)^4+\cos(x)^2+2\cos(x)\sin(x)+1\right) - \frac{1}{24}\log\left(-\cos(x)^4+\cos(x)^2-2\cos(x)\sin(x)+1\right)$$

input
```
integrate(1/(1+sin(x)^6),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) + sqrt(3))/(2*cos(x)^2 -
1)) + 1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) - sqrt(3))/(2*cos(x
)^2 - 1)) - 1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(
x)*sin(x))) + 1/24*log(-cos(x)^4 + cos(x)^2 + 2*cos(x)*sin(x) + 1) - 1/24*
log(-cos(x)^4 + cos(x)^2 - 2*cos(x)*sin(x) + 1)
```

3.256.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1+\sin^6(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1+sin(x)**6),x)
```

output
```
Timed out
```

3.256.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.69

$$\int \frac{1}{1+\sin^6(x)}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\tan(x)+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\tan(x)-1)\right) + \frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right) + \frac{1}{12}\log\left(\tan(x)^2+\tan(x)+1\right) - \frac{1}{12}\log\left(\tan(x)^2-\tan(x)+1\right)$$

input
```
integrate(1/(1+sin(x)^6),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x) + 1)) + 1/6*sqrt(3)*arctan(1/3*sq
rt(3)*(2*tan(x) - 1)) + 1/6*sqrt(2)*arctan(sqrt(2)*tan(x)) + 1/12*log(tan(
x)^2 + tan(x) + 1) - 1/12*log(tan(x)^2 - tan(x) + 1)
```

3.256.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(73) = 146.

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.80

$$\int \frac{1}{1+\sin^6(x)}\,dx = \frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)+\cos(2x)-2\sin(2x)+1}{\sqrt{3}\cos(2x)+\sqrt{3}-2\cos(2x)-\sin(2x)+2}\right)\right) + \frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)-\cos(2x)-2\sin(2x)-1}{\sqrt{3}\cos(2x)+\sqrt{3}-2\cos(2x)+\sin(2x)+2}\right)\right) + \frac{1}{6}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-2\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-2\cos(2x)+2}\right)\right) + \frac{1}{12}\log\left(\tan(x)^2+\tan(x)+1\right) - \frac{1}{12}\log\left(\tan(x)^2-\tan(x)+1\right)$$

input
```
integrate(1/(1+sin(x)^6),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) + cos(2*x) - 2*sin(2*x) + 1)/(s
qrt(3)*cos(2*x) + sqrt(3) - 2*cos(2*x) - sin(2*x) + 2))) + 1/6*sqrt(3)*(x
+ arctan(-(sqrt(3)*sin(2*x) - cos(2*x) - 2*sin(2*x) - 1)/(sqrt(3)*cos(2*x)
 + sqrt(3) - 2*cos(2*x) + sin(2*x) + 2))) + 1/6*sqrt(2)*(x + arctan(-(sqrt
(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x) + sqrt(2) - 2*cos(2*x) + 2)))
 + 1/12*log(tan(x)^2 + tan(x) + 1) - 1/12*log(tan(x)^2 - tan(x) + 1)
```

3.256.9 Mupad [B] (verification not implemented)

Time = 13.96 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95

$$\int \frac{1}{1+\sin^6(x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan(x)\right)}{6} + \mathrm{atan}\left(\frac{\sqrt{3}\tan(x)}{2} + \frac{\tan(x)\ 1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{6} - \frac{1}{6}\mathrm{i}\right) - \mathrm{atan}\left(-\frac{\sqrt{3}\tan(x)}{2} + \frac{\tan(x)\ 1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{6} + \frac{1}{6}\mathrm{i}\right) + \frac{(x - \mathrm{atan}(\tan(x)))\left(\frac{\pi\sqrt{2}}{6} + \pi\left(\frac{\sqrt{3}}{6} - \frac{1}{6}\mathrm{i}\right) + \pi\left(\frac{\sqrt{3}}{6} + \frac{1}{6}\mathrm{i}\right)\right)}{\pi}$$

input
```
int(1/(sin(x)^6 + 1),x)
```

output
```
atan((tan(x)*1i)/2 + (3^(1/2)*tan(x))/2)*(3^(1/2)/6 - 1i/6) - atan((tan(x)
*1i)/2 - (3^(1/2)*tan(x))/2)*(3^(1/2)/6 + 1i/6) + (2^(1/2)*atan(2^(1/2)*ta
n(x)))/6 + ((x - atan(tan(x)))*((2^(1/2)*pi)/6 + pi*(3^(1/2)/6 - 1i/6) + p
i*(3^(1/2)/6 + 1i/6)))/pi
```

3.257 $\int \frac{1}{1+\sin^8(x)}\,dx$

3.257.1 Optimal result

Integrand size = 8, antiderivative size = 218

$$\int \frac{1}{1+\sin^8(x)}\,dx = \frac{1}{8}\left(\sqrt{1+\sqrt{4-2\sqrt{2}}}+\sqrt{2+2\sqrt[4]{2}+2\sqrt{1+\sqrt{2}}+2\sqrt{2+\sqrt{2}}}+\sqrt{1+\sqrt{4+2\sqrt{2}}}\right)(x-\arctan(\tan(x))) + \frac{\arctan\left(\sqrt{1-\sqrt[4]{-1}}\tan(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}} + \frac{\arctan\left(\sqrt{1+\sqrt[4]{-1}}\tan(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}} + \frac{\arctan\left(\sqrt{1-(-1)^{3/4}}\tan(x)\right)}{4\sqrt{1-(-1)^{3/4}}} + \frac{\arctan\left(\sqrt{1+(-1)^{3/4}}\tan(x)\right)}{4\sqrt{1+(-1)^{3/4}}}$$

output
```
1/4*arctan((1-(-1)^(1/4))^(1/2)*tan(x))/(1-(-1)^(1/4))^(1/2)+1/4*arctan((1
+(-1)^(1/4))^(1/2)*tan(x))/(1+(-1)^(1/4))^(1/2)+1/4*arctan((1-(-1)^(3/4))^
(1/2)*tan(x))/(1-(-1)^(3/4))^(1/2)+1/4*arctan((1+(-1)^(3/4))^(1/2)*tan(x))
/(1+(-1)^(3/4))^(1/2)+1/8*(x-arctan(tan(x)))*((1+(4-2*2^(1/2))^(1/2))^(1/2
)+(2+2*2^(1/4)+2*(1+2^(1/2))^(1/2)+2*(2+2^(1/2))^(1/2))^(1/2)+(1+(4+2*2^(1
/2))^(1/2))^(1/2))
```

3.257.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.65

$$\int \frac{1}{1+\sin^8(x)}\,dx = 8\text{RootSum}\left[1-8\#1+28\#1^2-56\#1^3+326\#1^4-56\#1^5+28\#1^6-8\#1^7 + \#1^8\&, \frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^3-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^3}{-1+7\#1-21\#1^2+163\#1^3-35\#1^4+21\#1^5-7\#1^6+\#1^7}\&\right]$$

input
```
Integrate[(1 + Sin[x]^8)^(-1),x]
```

output
```
8*RootSum[1 - 8*#1 + 28*#1^2 - 56*#1^3 + 326*#1^4 - 56*#1^5 + 28*#1^6 - 8*
#1^7 + #1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^3 - I*Log[1 - 2*Cos
[2*x]*#1 + #1^2]*#1^3)/(-1 + 7*#1 - 21*#1^2 + 163*#1^3 - 35*#1^4 + 21*#1^5
 - 7*#1^6 + #1^7) & ]
```

3.257.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.59, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^8(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(x)^8+1}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{1}{4}\int \frac{1}{1-\sqrt[4]{-1}\sin^2(x)}dx+\frac{1}{4}\int \frac{1}{\sqrt[4]{-1}\sin^2(x)+1}dx+\frac{1}{4}\int \frac{1}{1-(-1)^{3/4}\sin^2(x)}dx+\frac{1}{4}\int \frac{1}{(-1)^{3/4}\sin^2(x)+1}dx$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\int \frac{1}{1-\sqrt[4]{-1}\sin(x)^2}dx+\frac{1}{4}\int \frac{1}{\sqrt[4]{-1}\sin(x)^2+1}dx+\frac{1}{4}\int \frac{1}{1-(-1)^{3/4}\sin(x)^2}dx+\frac{1}{4}\int \frac{1}{(-1)^{3/4}\sin(x)^2+1}dx$$

$\Big\downarrow$ 3660

$$\frac{1}{4}\int \frac{1}{\left(1-\sqrt[4]{-1}\right)\tan^2(x)+1}d\tan(x)+\frac{1}{4}\int \frac{1}{\left(1+\sqrt[4]{-1}\right)\tan^2(x)+1}d\tan(x)+\frac{1}{4}\int \frac{1}{\left(1-(-1)^{3/4}\right)\tan^2(x)+1}d\tan(x)+\frac{1}{4}\int \frac{1}{\left(1+(-1)^{3/4}\right)\tan^2(x)+1}d\tan(x)$$

$\Big\downarrow$ 216

$$\frac{\arctan\left(\sqrt{1-\sqrt[4]{-1}}\tan(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}}+\frac{\arctan\left(\sqrt{1+\sqrt[4]{-1}}\tan(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}}+\frac{\arctan\left(\sqrt{1-(-1)^{3/4}}\tan(x)\right)}{4\sqrt{1-(-1)^{3/4}}}+\frac{\arctan\left(\sqrt{1+(-1)^{3/4}}\tan(x)\right)}{4\sqrt{1+(-1)^{3/4}}}$$

input
```
Int[(1 + Sin[x]^8)^(-1),x]
```

output
```
ArcTan[Sqrt[1 - (-1)^(1/4)]*Tan[x]]/(4*Sqrt[1 - (-1)^(1/4)]) + ArcTan[Sqrt
[1 + (-1)^(1/4)]*Tan[x]]/(4*Sqrt[1 + (-1)^(1/4)]) + ArcTan[Sqrt[1 - (-1)^(
3/4)]*Tan[x]]/(4*Sqrt[1 - (-1)^(3/4)]) + ArcTan[Sqrt[1 + (-1)^(3/4)]*Tan[x
]]/(4*Sqrt[1 + (-1)^(3/4)])
```

3.257.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

3.257.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.33

method	result
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(2\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+1\right)}\frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{2\_R^7+3\_R^5+3\_R^3+\_R}\right)}{8}$
risch	$\left(\sum_{\_R=\operatorname{RootOf}\left(8192\_Z^4+(128+128i)\_Z^2+1+i\right)}\_R\ln\left(\mathrm{e}^{2ix}+(1024-1024i)\_R^3+(128+128i)\_R^2+(1\right.\right.$

input
```
int(1/(1+sin(x)^8),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum((_R^6+3*_R^4+3*_R^2+1)/(2*_R^7+3*_R^5+3*_R^3+_R)*ln(tan(x)-_R),_R=
RootOf(2*_Z^8+4*_Z^6+6*_Z^4+4*_Z^2+1))
```

3.257.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(152) = 304$.

Time = 0.43 (sec) , antiderivative size = 893, normalized size of antiderivative = 4.10

$$\int \frac{1}{1+\sin^8(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1+sin(x)^8),x, algorithm="fricas")
```

output
```
-1/32*sqrt(2)*sqrt(-sqrt(2*sqrt(2) - 3) - 1)*log(2*(sqrt(2) + 1)*cos(x)^2
+ (2*(sqrt(2) + 2)*cos(x)^2 - sqrt(2) - 2)*sqrt(2*sqrt(2) - 3) + 2*(sqrt(2
*sqrt(2) - 3)*(sqrt(2) + 1)*cos(x)*sin(x) + (sqrt(2) + 1)*cos(x)*sin(x))*s
qrt(-sqrt(2*sqrt(2) - 3) - 1) - sqrt(2) - 2) + 1/32*sqrt(2)*sqrt(-sqrt(2*s
qrt(2) - 3) - 1)*log(2*(sqrt(2) + 1)*cos(x)^2 + (2*(sqrt(2) + 2)*cos(x)^2
- sqrt(2) - 2)*sqrt(2*sqrt(2) - 3) - 2*(sqrt(2*sqrt(2) - 3)*(sqrt(2) + 1)*
cos(x)*sin(x) + (sqrt(2) + 1)*cos(x)*sin(x))*sqrt(-sqrt(2*sqrt(2) - 3) - 1
) - sqrt(2) - 2) - 1/32*sqrt(2)*sqrt(sqrt(2*sqrt(2) - 3) - 1)*log(-2*(sqrt
(2) + 1)*cos(x)^2 + (2*(sqrt(2) + 2)*cos(x)^2 - sqrt(2) - 2)*sqrt(2*sqrt(2
) - 3) + 2*(sqrt(2*sqrt(2) - 3)*(sqrt(2) + 1)*cos(x)*sin(x) - (sqrt(2) + 1
)*cos(x)*sin(x))*sqrt(sqrt(2*sqrt(2) - 3) - 1) + sqrt(2) + 2) + 1/32*sqrt(
2)*sqrt(sqrt(2*sqrt(2) - 3) - 1)*log(-2*(sqrt(2) + 1)*cos(x)^2 + (2*(sqrt(
2) + 2)*cos(x)^2 - sqrt(2) - 2)*sqrt(2*sqrt(2) - 3) - 2*(sqrt(2*sqrt(2) -
3)*(sqrt(2) + 1)*cos(x)*sin(x) - (sqrt(2) + 1)*cos(x)*sin(x))*sqrt(sqrt(2*
sqrt(2) - 3) - 1) + sqrt(2) + 2) + 1/32*sqrt(2)*sqrt(-sqrt(-2*sqrt(2) - 3)
 - 1)*log(2*(sqrt(2) - 1)*cos(x)^2 + (2*(sqrt(2) - 2)*cos(x)^2 - sqrt(2) +
 2)*sqrt(-2*sqrt(2) - 3) + 2*((sqrt(2) - 1)*sqrt(-2*sqrt(2) - 3)*cos(x)*si
n(x) + (sqrt(2) - 1)*cos(x)*sin(x))*sqrt(-sqrt(-2*sqrt(2) - 3) - 1) - sqrt
(2) + 2) - 1/32*sqrt(2)*sqrt(-sqrt(-2*sqrt(2) - 3) - 1)*log(2*(sqrt(2) - 1
)*cos(x)^2 + (2*(sqrt(2) - 2)*cos(x)^2 - sqrt(2) + 2)*sqrt(-2*sqrt(2) -...
```

3.257.6 Sympy [F]

$$\int \frac{1}{1+\sin^8(x)}\,dx = \int \frac{1}{\sin^8(x)+1}\,dx$$

input
```
integrate(1/(1+sin(x)**8),x)
```

output
```
Integral(1/(sin(x)**8 + 1), x)
```

3.257.7 Maxima [F]

$$\int \frac{1}{1+\sin^8(x)}\,dx = \int \frac{1}{\sin(x)^8+1}\,dx$$

input
```
integrate(1/(1+sin(x)^8),x, algorithm="maxima")
```

output
```
integrate(1/(sin(x)^8 + 1), x)
```

3.257.8 Giac [F]

$$\int \frac{1}{1+\sin^8(x)}\,dx = \int \frac{1}{\sin(x)^8+1}\,dx$$

input
```
integrate(1/(1+sin(x)^8),x, algorithm="giac")
```

output
```
sage0*x
```

3.257.9 Mupad [B] (verification not implemented)

Time = 14.96 (sec) , antiderivative size = 945, normalized size of antiderivative = 4.33

$$\int \frac{1}{1+\sin^8(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(x)^8 + 1),x)
```

output
```
atan((tan(x)*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*1i)/(256*((3*2^(1
/2)*(- 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)^(1/2)/512)) - (2^(1/
2)*tan(x)*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*1i)/(256*((3*2^(1/2)
*(- 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)^(1/2)/512)) - (tan(x)*(
- 2*2^(1/2) - 3)^(1/2)*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*7i)/(25
6*((3*2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)^(1/2)/512)
) + (2^(1/2)*tan(x)*(- 2*2^(1/2) - 3)^(1/2)*((- 2*2^(1/2) - 3)^(1/2)/128 -
 1/128)^(1/2)*5i)/(256*((3*2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^
(1/2) - 3)^(1/2)/512)))*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*2i - a
tan((tan(x)*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*1i)/(256*((3*2^(
1/2)*(- 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)^(1/2)/512)) - (2^(1
/2)*tan(x)*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*1i)/(256*((3*2^(1
/2)*(- 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)^(1/2)/512)) + (tan(x
)*(- 2*2^(1/2) - 3)^(1/2)*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*7i
)/(256*((3*2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)^(1/2)
/512)) - (2^(1/2)*tan(x)*(- 2*2^(1/2) - 3)^(1/2)*(- (- 2*2^(1/2) - 3)^(1/2
)/128 - 1/128)^(1/2)*5i)/(256*((3*2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2048 -
(- 2*2^(1/2) - 3)^(1/2)/512)))*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/
2)*2i + atan((tan(x)*(- (2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*1i)/(256*
((3*2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2048 + (2*2^(1/2) - 3)^(1/2)/512)) +...
```

3.257. $\int \frac{1}{1+\sin^8(x)}\,dx$

3.258 $\int \frac{1}{1-\sin^5(x)}\,dx$

3.258.1 Optimal result

Integrand size = 10, antiderivative size = 187

$$\int \frac{1}{1-\sin^5(x)}\,dx = -\frac{2\arctan\left(\frac{(-1)^{2/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}} - \frac{2\arctan\left(\frac{(-1)^{4/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}} + \frac{2\arctan\left(\frac{\sqrt[5]{-1}+\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}} + \frac{2\arctan\left(\frac{(-1)^{3/5}+\tan\left(\frac{x}{2}\right)}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}} + \frac{\cos(x)}{5(1-\sin(x))}$$

output
```
1/5*cos(x)/(1-sin(x))+2/5*arctan(((-1)^(3/5)+tan(1/2*x))/(1+(-1)^(1/5))^(1
/2))/(1+(-1)^(1/5))^(1/2)+2/5*arctan(((-1)^(1/5)+tan(1/2*x))/(1-(-1)^(2/5)
)^(1/2))/(1-(-1)^(2/5))^(1/2)-2/5*arctan(((-1)^(4/5)-tan(1/2*x))/(1+(-1)^(
3/5))^(1/2))/(1+(-1)^(3/5))^(1/2)-2/5*arctan(((-1)^(2/5)-tan(1/2*x))/(1-(-
1)^(4/5))^(1/2))/(1-(-1)^(4/5))^(1/2)
```

3.258.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.09 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.21

$$\int \frac{1}{1-\sin^5(x)}\,dx$$
$$= \frac{1}{10} i \text{RootSum}\Bigg[1 - 2i\#1 - 8\#1^2 + 14i\#1^3 + 30\#1^4 - 14i\#1^5 - 8\#1^6 + 2i\#1^7$$
$$+\#1^8\&, \frac{-2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right) + i\log\left(1-2\cos(x)\#1+\#1^2\right) + 8i\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1 + 4\log\left(1-\right.}{}$$
$$+ \frac{2\sin\left(\frac{x}{2}\right)}{5\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}$$

input
```
Integrate[(1 - Sin[x]^5)^(-1),x]
```

output
```
(I/10)*RootSum[1 - (2*I)*#1 - 8*#1^2 + (14*I)*#1^3 + 30*#1^4 - (14*I)*#1^5
 - 8*#1^6 + (2*I)*#1^7 + #1^8 & , (-2*ArcTan[Sin[x]/(Cos[x] - #1)] + I*Log
[1 - 2*Cos[x]*#1 + #1^2] + (8*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 + 4*Log[1
 - 2*Cos[x]*#1 + #1^2]*#1 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - (15*I)*
Log[1 - 2*Cos[x]*#1 + #1^2]*#1^2 - (80*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^
3 - 40*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3 - 30*ArcTan[Sin[x]/(Cos[x] - #1)]*
#1^4 + (15*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4 + (8*I)*ArcTan[Sin[x]/(Cos[
x] - #1)]*#1^5 + 4*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^5 + 2*ArcTan[Sin[x]/(Cos
[x] - #1)]*#1^6 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/(-I - 8*#1 + (21*I)*
#1^2 + 60*#1^3 - (35*I)*#1^4 - 24*#1^5 + (7*I)*#1^6 + 4*#1^7) & ] + (2*Sin
[x/2])/(5*(Cos[x/2] - Sin[x/2]))
```

3.258.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\sin^5(x)}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-\sin(x)^5}dx$$

$\big\downarrow$ 3692

$$\int \left(\frac{1}{5\left(\sqrt[5]{-1}\sin(x)+1\right)}+\frac{1}{5\left(1-(-1)^{2/5}\sin(x)\right)}+\frac{1}{5\left((-1)^{3/5}\sin(x)+1\right)}+\frac{1}{5\left(1-(-1)^{4/5}\sin(x)\right)}+\frac{1}{5(1-\sin(}\right.$$

$\big\downarrow$ 2009

$$\frac{2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)+\sqrt[5]{-1}}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{3/5}}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{2\arctan\left(\frac{(-1)^{4/5}\left(\sqrt[5]{-1}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}}-\frac{2\arctan\left(\frac{(-1)^{2/5}\left((-1)^{3/5}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}}+\frac{\cos(x)}{5(1-\sin(x))}$$

input
```
Int[(1 - Sin[x]^5)^(-1),x]
```

output
```
(2*ArcTan[((-1)^(1/5) + Tan[x/2])/Sqrt[1 - (-1)^(2/5)]])/(5*Sqrt[1 - (-1)^
(2/5)]) + (2*ArcTan[((-1)^(3/5) + Tan[x/2])/Sqrt[1 + (-1)^(1/5)]])/(5*Sqrt
[1 + (-1)^(1/5)]) - (2*ArcTan[((-1)^(4/5)*(1 + (-1)^(1/5)*Tan[x/2]))/Sqrt[
1 + (-1)^(3/5)]])/(5*Sqrt[1 + (-1)^(3/5)]) - (2*ArcTan[((-1)^(2/5)*(1 + (-
1)^(3/5)*Tan[x/2]))/Sqrt[1 - (-1)^(4/5)]])/(5*Sqrt[1 - (-1)^(4/5)]) + Cos[
x]/(5*(1 - Sin[x]))
```

3.258.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.258. $\int \frac{1}{1-\sin^5(x)}\,dx$

3.258.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.65 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.47

method	result
risch	$\frac{2}{5(e^{ix}-i)}+\left(\sum_{\_R=\mathrm{RootOf}\left(1953125\_Z^{8}+156250\_Z^{6}+6250\_Z^{4}+125\_Z^{2}+1\right)}\_R\ln\left(e^{ix}-2343750\_R^{7}-234375i\_\right.\right.$
default	$\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+2\_Z^{7}+8\_Z^{6}+14\_Z^{5}+30\_Z^{4}+14\_Z^{3}+8\_Z^{2}+2\_Z+1\right)}\frac{\left(2\_R^{6}+3\_R^{5}+10\_R^{4}+10\_R^{3}+10\_R^{2}+3\_R+2\right)}{4\_R^{7}+7\_R^{6}+24\_R^{5}+35\_R^{4}+60\_R^{3}+21\_}\right.}{5}$

input
```
int(1/(1-sin(x)^5),x,method=_RETURNVERBOSE)
```

output
```
2/5/(exp(I*x)-I)+sum(_R*ln(exp(I*x)-2343750*_R^7-234375*I*_R^6-140625*_R^5
-15625*I*_R^4-4375*_R^3-500*I*_R^2-50*_R-6*I),_R=RootOf(1953125*_Z^8+15625
0*_Z^6+6250*_Z^4+125*_Z^2+1))
```

3.258.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 858 vs. 2(127) = 254.

Time = 0.39 (sec) , antiderivative size = 858, normalized size of antiderivative = 4.59

$$\int \frac{1}{1-\sin^5(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-sin(x)^5),x, algorithm="fricas")
```

output
```
-1/100*((sqrt(5)*cos(x) - sqrt(5)*sin(x) + sqrt(5))*sqrt(2*sqrt(5)*sqrt(2*
sqrt(5) - 5) - 10)*log(-sqrt(2*sqrt(5)*sqrt(2*sqrt(5) - 5) - 10)*(3*sqrt(5
) + 5)*sqrt(2*sqrt(5) - 5)*cos(x) + 5*sqrt(2*sqrt(5) - 5)*(sqrt(5) + 3)*si
n(x) + 5*(sqrt(5) - 1)*sin(x) - 20) - (sqrt(5)*cos(x) - sqrt(5)*sin(x) + s
qrt(5))*sqrt(2*sqrt(5)*sqrt(2*sqrt(5) - 5) - 10)*log(-sqrt(2*sqrt(5)*sqrt(
2*sqrt(5) - 5) - 10)*(3*sqrt(5) + 5)*sqrt(2*sqrt(5) - 5)*cos(x) - 5*sqrt(2
*sqrt(5) - 5)*(sqrt(5) + 3)*sin(x) - 5*(sqrt(5) - 1)*sin(x) + 20) + (sqrt(
5)*cos(x) - sqrt(5)*sin(x) + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(2*sqrt(5) - 5)
- 10)*log(-sqrt(-2*sqrt(5)*sqrt(2*sqrt(5) - 5) - 10)*(3*sqrt(5) + 5)*sqrt(
2*sqrt(5) - 5)*cos(x) + 5*sqrt(2*sqrt(5) - 5)*(sqrt(5) + 3)*sin(x) - 5*(sq
rt(5) - 1)*sin(x) + 20) - (sqrt(5)*cos(x) - sqrt(5)*sin(x) + sqrt(5))*sqrt
(-2*sqrt(5)*sqrt(2*sqrt(5) - 5) - 10)*log(-sqrt(-2*sqrt(5)*sqrt(2*sqrt(5)
- 5) - 10)*(3*sqrt(5) + 5)*sqrt(2*sqrt(5) - 5)*cos(x) - 5*sqrt(2*sqrt(5) -
 5)*(sqrt(5) + 3)*sin(x) + 5*(sqrt(5) - 1)*sin(x) - 20) - (sqrt(5)*cos(x)
- sqrt(5)*sin(x) + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2*sqrt(5) - 5) - 10)*log(
-sqrt(2*sqrt(5)*sqrt(-2*sqrt(5) - 5) - 10)*(3*sqrt(5) - 5)*sqrt(-2*sqrt(5)
 - 5)*cos(x) + 5*(sqrt(5) - 3)*sqrt(-2*sqrt(5) - 5)*sin(x) - 5*(sqrt(5) +
1)*sin(x) - 20) + (sqrt(5)*cos(x) - sqrt(5)*sin(x) + sqrt(5))*sqrt(2*sqrt(
5)*sqrt(-2*sqrt(5) - 5) - 10)*log(-sqrt(2*sqrt(5)*sqrt(-2*sqrt(5) - 5) - 1
0)*(3*sqrt(5) - 5)*sqrt(-2*sqrt(5) - 5)*cos(x) - 5*(sqrt(5) - 3)*sqrt(-...
```

3.258.6 Sympy [F]

$$\int \frac{1}{1-\sin^5(x)}\,dx = -\int \frac{1}{\sin^5(x)-1}\,dx$$

input
```
integrate(1/(1-sin(x)**5),x)
```

output
```
-Integral(1/(sin(x)**5 - 1), x)
```

3.258.7 Maxima [F]

$$\int \frac{1}{1-\sin^5(x)}\,dx = \int -\frac{1}{\sin{(x)}^5-1}\,dx$$

input
```
integrate(1/(1-sin(x)^5),x, algorithm="maxima")
```

output
```
1/5*(5*(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)*integrate(2/5*((4*cos(6*x) - 4
0*cos(4*x) + 4*cos(2*x) + sin(7*x) - 15*sin(5*x) + 15*sin(3*x) - sin(x))*c
os(8*x) + 2*(22*cos(5*x) - 22*cos(3*x) + 2*cos(x) + 8*sin(6*x) - 55*sin(4*
x) + 8*sin(2*x))*cos(7*x) - 2*cos(7*x)^2 + 4*(110*cos(4*x) - 16*cos(2*x) +
 44*sin(5*x) - 44*sin(3*x) + 4*sin(x) + 1)*cos(6*x) - 32*cos(6*x)^2 + 2*(2
10*cos(3*x) - 22*cos(x) + 505*sin(4*x) - 88*sin(2*x))*cos(5*x) - 210*cos(5
*x)^2 + 10*(44*cos(2*x) + 101*sin(3*x) - 11*sin(x) - 4)*cos(4*x) - 1200*co
s(4*x)^2 + 44*(cos(x) + 4*sin(2*x))*cos(3*x) - 210*cos(3*x)^2 + 4*(4*sin(x
) + 1)*cos(2*x) - 32*cos(2*x)^2 - 2*cos(x)^2 - (cos(7*x) - 15*cos(5*x) + 1
5*cos(3*x) - cos(x) - 4*sin(6*x) + 40*sin(4*x) - 4*sin(2*x))*sin(8*x) - (1
6*cos(6*x) - 110*cos(4*x) + 16*cos(2*x) - 44*sin(5*x) + 44*sin(3*x) - 4*si
n(x) - 1)*sin(7*x) - 2*sin(7*x)^2 - 8*(22*cos(5*x) - 22*cos(3*x) + 2*cos(x
) - 55*sin(4*x) + 8*sin(2*x))*sin(6*x) - 32*sin(6*x)^2 - (1010*cos(4*x) -
176*cos(2*x) - 420*sin(3*x) + 44*sin(x) + 15)*sin(5*x) - 210*sin(5*x)^2 -
10*(101*cos(3*x) - 11*cos(x) - 44*sin(2*x))*sin(4*x) - 1200*sin(4*x)^2 - (
176*cos(2*x) - 44*sin(x) - 15)*sin(3*x) - 210*sin(3*x)^2 - 16*cos(x)*sin(2
*x) - 32*sin(2*x)^2 - 2*sin(x)^2 - sin(x))/(2*(8*cos(6*x) - 30*cos(4*x) +
8*cos(2*x) + 2*sin(7*x) - 14*sin(5*x) + 14*sin(3*x) - 2*sin(x) - 1)*cos(8*
x) - cos(8*x)^2 + 8*(7*cos(5*x) - 7*cos(3*x) + cos(x) + 4*sin(6*x) - 15*si
n(4*x) + 4*sin(2*x))*cos(7*x) - 4*cos(7*x)^2 + 16*(30*cos(4*x) - 8*cos(...
```

3.258.8 Giac [F]

$$\int \frac{1}{1-\sin^5(x)}\,dx = \int -\frac{1}{\sin{(x)}^5-1}\,dx$$

input
```
integrate(1/(1-sin(x)^5),x, algorithm="giac")
```

output
```
sage0*x
```

3.258.9 Mupad [B] (verification not implemented)

Time = 13.67 (sec) , antiderivative size = 3513, normalized size of antiderivative = 18.79

$$\int \frac{1}{1-\sin^5(x)}\,dx = \text{Too large to display}$$

input
```
int(-1/(sin(x)^5 - 1),x)
```

output
```
2*atanh((989855744*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301
989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x
/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*5^(1/2))/5
- 1)^(1/2))/25 - (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/
2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - 1845493
76/25)) + (2030043136*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/
2))/(5*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308
622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2
*5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2)
)/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/
25 - 184549376/25)) + (1627389952*5^(1/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50
- 1/50)^(1/2))/(25*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))
/125 + (1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5
^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 + 16777216*(
- (2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5
- 1)^(1/2))/25 - 184549376/25)) + (553648128*(- (2*5^(1/2))/5 - 1)^(1/2)*(
(- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 +
 (2382364672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2)*(- (2*5^(1/2))/5
 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (16
777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (436207616*5...
```

3.259 $\int \frac{1}{1-\sin^6(x)}\,dx$

3.259.1 Optimal result

Integrand size = 10, antiderivative size = 71

$$\int \frac{1}{1-\sin^6(x)}\,dx=\frac{\arctan\left(\sqrt{1+\sqrt[3]{-1}}\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\arctan\left(\sqrt{1-(-1)^{2/3}}\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\tan(x)}{3}$$

output
```
1/3*arctan((1+(-1)^(1/3))^(1/2)*tan(x))/(1+(-1)^(1/3))^(1/2)+1/3*arctan((1
-(-1)^(2/3))^(1/2)*tan(x))/(1-(-1)^(2/3))^(1/2)+1/3*tan(x)
```

3.259.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.65

$$\int \frac{1}{1-\sin^6(x)}\,dx$$

$$=\frac{\cos(x)(15-8\cos(2x)+\cos(4x))\left(i\sqrt[4]{-3}(3i+\sqrt{3})\arctan\left(\frac{1}{2}\sqrt[4]{-\frac{1}{3}}(-3i+\sqrt{3})\tan(x)\right)\cos(x)+\sqrt[4]{-3}(\right.}{144\left(-1+\sin^6(x)\right)}$$

input
```
Integrate[(1 - Sin[x]^6)^(-1),x]
```

output
```
(Cos[x]*(15 - 8*Cos[2*x] + Cos[4*x])*(I*(-3)^(1/4)*(3*I + Sqrt[3])*ArcTan[
((-1/3)^(1/4)*(-3*I + Sqrt[3])*Tan[x])/2]*Cos[x] + (-3)^(1/4)*(-3*I + Sqrt
[3])*ArcTan[((-1)^(3/4)*(3*I + Sqrt[3])*Tan[x])/(2*3^(1/4))]*Cos[x] - 6*Si
n[x]))/(144*(-1 + Sin[x]^6))
```

3.259.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3690, 3042, 3654, 3042, 3660, 216, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\sin^6(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-\sin(x)^6}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{1}{3}\int \frac{1}{1-\sin^2(x)}dx+\frac{1}{3}\int \frac{1}{\sqrt[3]{-1}\sin^2(x)+1}dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\sin^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{1}{1-\sin(x)^2}dx+\frac{1}{3}\int \frac{1}{\sqrt[3]{-1}\sin(x)^2+1}dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\sin(x)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{1}{3}\int \frac{1}{\sqrt[3]{-1}\sin(x)^2+1}dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\sin(x)^2}dx+\frac{1}{3}\int \sec^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{1}{\sqrt[3]{-1}\sin(x)^2+1}dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\sin(x)^2}dx+\frac{1}{3}\int \csc\left(x+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3660}$$

$$\frac{1}{3}\int \frac{1}{\left(1+\sqrt[3]{-1}\right)\tan^2(x)+1}d\tan(x)+\frac{1}{3}\int \frac{1}{\left(1-(-1)^{2/3}\right)\tan^2(x)+1}d\tan(x)+\frac{1}{3}\int \csc\left(x+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\int \csc\left(x+\frac{\pi}{2}\right)^2dx+\frac{\arctan\left(\sqrt{1+\sqrt[3]{-1}}\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\arctan\left(\sqrt{1-(-1)^{2/3}}\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{1}{3}\int 1d(-\tan(x))+\frac{\arctan\left(\sqrt{1+\sqrt[3]{-1}}\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\arctan\left(\sqrt{1-(-1)^{2/3}}\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\arctan\left(\sqrt{1+\sqrt[3]{-1}}\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\arctan\left(\sqrt{1-(-1)^{2/3}}\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\tan(x)}{3}$$

input
```
Int[(1 - Sin[x]^6)^(-1),x]
```

output
```
ArcTan[Sqrt[1 + (-1)^(1/3)]*Tan[x]]/(3*Sqrt[1 + (-1)^(1/3)]) + ArcTan[Sqrt
[1 - (-1)^(2/3)]*Tan[x]]/(3*Sqrt[1 - (-1)^(2/3)]) + Tan[x]/3
```

3.259.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.259.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.72 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

method	result
risch	$\frac{2i}{3(e^{2ix}+1)}+\left(\sum_{\_R=\text{RootOf}\left(3888\_Z^4+108\_Z^2+1\right)}\_R\ln\left(e^{2ix}-1296i\_R^3+216\_R^2+1\right)\right)$
default	$\frac{\tan(x)}{3}+\frac{\sqrt{3}\left(\frac{\sqrt{2\sqrt{3}-3}\,\ln\left(\sqrt{3}+\sqrt{2\sqrt{3}-3}\,\sqrt{3}\,\tan(x)+3\left(\tan^2(x)\right)\right)}{6}+\frac{2\left(-\frac{\left(2\sqrt{3}-3\right)\sqrt{3}}{6}+2\right)\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{3}+6\tan(x)}{\sqrt{6\sqrt{3}+9}}\right)}{\sqrt{6\sqrt{3}+9}}\right)}{6}+\frac{\sqrt{3}\left(-\ldots\right.}{}$

input
```
int(1/(1-sin(x)^6),x,method=_RETURNVERBOSE)
```

output
```
2/3*I/(exp(2*I*x)+1)+sum(_R*ln(exp(2*I*x)-1296*I*_R^3+216*_R^2+1),_R=RootO
f(3888*_Z^4+108*_Z^2+1))
```

3.259.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.37

$$\int \frac{1}{1-\sin^6(x)}\,dx =$$
$$-\frac{\sqrt{6}\sqrt{i\sqrt{3}-3}\cos(x)\log\left(\sqrt{6}(i\sqrt{3}-3)^{\frac{3}{2}}\cos(x)\sin(x)-6\left(-i\sqrt{3}+2\right)\cos(x)^2-3i\sqrt{3}+9\right)-\sqrt{6}\sqrt{}}{}$$

input
```
integrate(1/(1-sin(x)^6),x, algorithm="fricas")
```

output
```
-1/72*(sqrt(6)*sqrt(I*sqrt(3) - 3)*cos(x)*log(sqrt(6)*(I*sqrt(3) - 3)^(3/2
)*cos(x)*sin(x) - 6*(-I*sqrt(3) + 2)*cos(x)^2 - 3*I*sqrt(3) + 9) - sqrt(6)
*sqrt(I*sqrt(3) - 3)*cos(x)*log(sqrt(6)*sqrt(I*sqrt(3) - 3)*(-I*sqrt(3) +
3)*cos(x)*sin(x) - 6*(-I*sqrt(3) + 2)*cos(x)^2 - 3*I*sqrt(3) + 9) + sqrt(6
)*sqrt(-I*sqrt(3) - 3)*cos(x)*log(sqrt(6)*(I*sqrt(3) + 3)*sqrt(-I*sqrt(3)
- 3)*cos(x)*sin(x) - 6*(-I*sqrt(3) - 2)*cos(x)^2 - 3*I*sqrt(3) - 9) - sqrt
(6)*sqrt(-I*sqrt(3) - 3)*cos(x)*log(sqrt(6)*(-I*sqrt(3) - 3)^(3/2)*cos(x)*
sin(x) - 6*(-I*sqrt(3) - 2)*cos(x)^2 - 3*I*sqrt(3) - 9) - 24*sin(x))/cos(x
)
```

3.259.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1-\sin^6(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1-sin(x)**6),x)
```

output
```
Timed out
```

3.259.7 Maxima [F]

$$\int \frac{1}{1-\sin^6(x)}\,dx = \int -\frac{1}{\sin(x)^6-1}\,dx$$

input
```
integrate(1/(1-sin(x)^6),x, algorithm="maxima")
```

output
```
-1/3*(3*(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*integrate(-4/3*((cos(6*
x) - 10*cos(4*x) + cos(2*x))*cos(8*x) + (110*cos(4*x) - 16*cos(2*x) + 1)*c
os(6*x) - 8*cos(6*x)^2 + 10*(11*cos(2*x) - 1)*cos(4*x) - 300*cos(4*x)^2 -
8*cos(2*x)^2 + (sin(6*x) - 10*sin(4*x) + sin(2*x))*sin(8*x) + 2*(55*sin(4*
x) - 8*sin(2*x))*sin(6*x) - 8*sin(6*x)^2 - 300*sin(4*x)^2 + 110*sin(4*x)*s
in(2*x) - 8*sin(2*x)^2 + cos(2*x))/(2*(8*cos(6*x) - 30*cos(4*x) + 8*cos(2*
x) - 1)*cos(8*x) - cos(8*x)^2 + 16*(30*cos(4*x) - 8*cos(2*x) + 1)*cos(6*x)
 - 64*cos(6*x)^2 + 60*(8*cos(2*x) - 1)*cos(4*x) - 900*cos(4*x)^2 - 64*cos(
2*x)^2 + 4*(4*sin(6*x) - 15*sin(4*x) + 4*sin(2*x))*sin(8*x) - sin(8*x)^2 +
 32*(15*sin(4*x) - 4*sin(2*x))*sin(6*x) - 64*sin(6*x)^2 - 900*sin(4*x)^2 +
 480*sin(4*x)*sin(2*x) - 64*sin(2*x)^2 + 16*cos(2*x) - 1), x) - 2*sin(2*x)
)/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

3.259.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(49) = 98$.

Time = 0.35 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.77

$$\begin{aligned}
\int \frac{1}{1-\sin^6(x)}\,dx
&= \frac{1}{18}\left(\pi\left\lfloor \frac{x}{\pi}+\frac{1}{2}\right\rfloor - \arctan\left(-\frac{3\left(\frac{1}{3}\right)^{\frac{3}{4}}\left(\left(\frac{1}{3}\right)^{\frac{1}{4}}\left(\sqrt{6}-\sqrt{2}\right)+4\tan(x)\right)}{\sqrt{6}+\sqrt{2}}\right)\right)\sqrt{6\sqrt{3}+9} \\
&\quad + \frac{1}{18}\left(\pi\left\lfloor \frac{x}{\pi}+\frac{1}{2}\right\rfloor + \arctan\left(-\frac{3\left(\frac{1}{3}\right)^{\frac{3}{4}}\left(\left(\frac{1}{3}\right)^{\frac{1}{4}}\left(\sqrt{6}-\sqrt{2}\right)-4\tan(x)\right)}{\sqrt{6}+\sqrt{2}}\right)\right)\sqrt{6\sqrt{3}+9} \\
&\quad + \frac{1}{36}\sqrt{6\sqrt{3}-9}\log\left(\frac{1}{2}\left(\sqrt{6}\left(\frac{1}{3}\right)^{\frac{1}{4}}-\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\right)\tan(x)+\tan(x)^2+\sqrt{\frac{1}{3}}\right) \\
&\quad - \frac{1}{36}\sqrt{6\sqrt{3}-9}\log\left(-\frac{1}{2}\left(\sqrt{6}\left(\frac{1}{3}\right)^{\frac{1}{4}}-\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\right)\tan(x)+\tan(x)^2+\sqrt{\frac{1}{3}}\right) \\
&\quad + \frac{1}{3}\tan(x)
\end{aligned}$$

input
```
integrate(1/(1-sin(x)^6),x, algorithm="giac")
```

output
```
1/18*(pi*floor(x/pi + 1/2) - arctan(-3*(1/3)^(3/4)*((1/3)^(1/4)*(sqrt(6) -
 sqrt(2)) + 4*tan(x))/(sqrt(6) + sqrt(2))))*sqrt(6*sqrt(3) + 9) + 1/18*(pi
*floor(x/pi + 1/2) + arctan(-3*(1/3)^(3/4)*((1/3)^(1/4)*(sqrt(6) - sqrt(2)
) - 4*tan(x))/(sqrt(6) + sqrt(2))))*sqrt(6*sqrt(3) + 9) + 1/36*sqrt(6*sqrt
(3) - 9)*log(1/2*(sqrt(6)*(1/3)^(1/4) - sqrt(2)*(1/3)^(1/4))*tan(x) + tan(
x)^2 + sqrt(1/3)) - 1/36*sqrt(6*sqrt(3) - 9)*log(-1/2*(sqrt(6)*(1/3)^(1/4)
 - sqrt(2)*(1/3)^(1/4))*tan(x) + tan(x)^2 + sqrt(1/3)) + 1/3*tan(x)
```

3.259.9 Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.39

$$\int \frac{1}{1-\sin^6(x)}\,dx = \frac{\tan(x)}{3} - \frac{\sqrt{6}\,\mathrm{atan}\left(3^{1/4}\sqrt{6}\tan(x)\,\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+3^{3/4}\sqrt{6}\tan(x)\,\left(\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)\right)\left(3^{1/4}\,(1+1\mathrm{i})+3^{3/4}\,(-1+1\mathrm{i})\right)1\mathrm{i}}{36} + \frac{\sqrt{6}\,\mathrm{atan}\left(3^{1/4}\sqrt{6}\tan(x)\,\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)+3^{3/4}\sqrt{6}\tan(x)\,\left(\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)\right)\left(3^{1/4}\,(1-\mathrm{i})+3^{3/4}\,(-1-\mathrm{i})\right)1\mathrm{i}}{36}$$

input
```
int(-1/(sin(x)^6 - 1),x)
```

output
```
tan(x)/3 - (6^(1/2)*atan(3^(1/4)*6^(1/2)*tan(x)*(1/4 - 1i/4) + 3^(3/4)*6^(
1/2)*tan(x)*(1/12 + 1i/12))*(3^(1/4)*(1 + 1i) - 3^(3/4)*(1 - 1i))*1i)/36 +
 (6^(1/2)*atan(3^(1/4)*6^(1/2)*tan(x)*(1/4 + 1i/4) + 3^(3/4)*6^(1/2)*tan(x
)*(1/12 - 1i/12))*(3^(1/4)*(1 - 1i) - 3^(3/4)*(1 + 1i))*1i)/36
```

3.260 $\int \frac{1}{1-\sin^8(x)}\,dx$

3.260.1 Optimal result

Integrand size = 10, antiderivative size = 89

$$\int \frac{1}{1-\sin^8(x)}\,dx = \frac{x}{4\sqrt{2}}+\frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{4\sqrt{2}}+\frac{\arctan\left(\sqrt{1-i}\tan(x)\right)}{4\sqrt{1-i}}+\frac{\arctan\left(\sqrt{1+i}\tan(x)\right)}{4\sqrt{1+i}}+\frac{\tan(x)}{4}$$

output
```
1/4*arctan((1-I)^(1/2)*tan(x))/(1-I)^(1/2)+1/4*arctan((1+I)^(1/2)*tan(x))/
(1+I)^(1/2)+1/8*x*2^(1/2)+1/8*arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2
^(1/2)+1/4*tan(x)
```

3.260.2 Mathematica [A] (verified)

Time = 3.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

$$\int \frac{1}{1-\sin^8(x)}\,dx = \frac{1}{8}\left(\frac{2\arctan\left(\sqrt{1-i}\tan(x)\right)}{\sqrt{1-i}}+\frac{2\arctan\left(\sqrt{1+i}\tan(x)\right)}{\sqrt{1+i}}+\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)+2\tan(x)\right)$$

input
```
Integrate[(1 - Sin[x]^8)^(-1),x]
```

output
```
((2*ArcTan[Sqrt[1 - I]*Tan[x]])/Sqrt[1 - I] + (2*ArcTan[Sqrt[1 + I]*Tan[x]
])/Sqrt[1 + I] + Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]] + 2*Tan[x])/8
```

3.260.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3690, 3042, 3654, 3042, 3660, 216, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\sin^8(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-\sin(x)^8}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{1}{4}\int \frac{1}{1-\sin^2(x)}dx+\frac{1}{4}\int \frac{1}{1-i\sin^2(x)}dx+\frac{1}{4}\int \frac{1}{i\sin^2(x)+1}dx+\frac{1}{4}\int \frac{1}{\sin^2(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{1}{1-\sin(x)^2}dx+\frac{1}{4}\int \frac{1}{1-i\sin(x)^2}dx+\frac{1}{4}\int \frac{1}{i\sin(x)^2+1}dx+\frac{1}{4}\int \frac{1}{\sin(x)^2+1}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{1}{4}\int \frac{1}{1-i\sin(x)^2}dx+\frac{1}{4}\int \frac{1}{i\sin(x)^2+1}dx+\frac{1}{4}\int \frac{1}{\sin(x)^2+1}dx+\frac{1}{4}\int \sec^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{1}{1-i\sin(x)^2}dx+\frac{1}{4}\int \frac{1}{i\sin(x)^2+1}dx+\frac{1}{4}\int \frac{1}{\sin(x)^2+1}dx+\frac{1}{4}\int \csc\left(x+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3660}$$

$$\frac{1}{4}\int \frac{1}{(1-i)\tan^2(x)+1}d\tan(x)+\frac{1}{4}\int \frac{1}{(1+i)\tan^2(x)+1}d\tan(x)+\frac{1}{4}\int \frac{1}{2\tan^2(x)+1}d\tan(x)+\frac{1}{4}\int \csc\left(x+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\int \csc\left(x+\frac{\pi}{2}\right)^2dx+\frac{\arctan\left(\sqrt{1-i}\tan(x)\right)}{4\sqrt{1-i}}+\frac{\arctan\left(\sqrt{1+i}\tan(x)\right)}{4\sqrt{1+i}}+\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{4\sqrt{2}}$$

$$\Big\downarrow \textbf{4254}$$

$$-\frac{1}{4}\int 1d(-\tan(x))+\frac{\arctan\left(\sqrt{1-i}\tan(x)\right)}{4\sqrt{1-i}}+\frac{\arctan\left(\sqrt{1+i}\tan(x)\right)}{4\sqrt{1+i}}+\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{4\sqrt{2}}$$

$$\Big\downarrow \textbf{24}$$

$$\frac{\arctan\left(\sqrt{1-i}\tan(x)\right)}{4\sqrt{1-i}}+\frac{\arctan\left(\sqrt{1+i}\tan(x)\right)}{4\sqrt{1+i}}+\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{4\sqrt{2}}+\frac{\tan(x)}{4}$$

input
```
Int[(1 - Sin[x]^8)^(-1),x]
```

output
```
ArcTan[Sqrt[1 - I]*Tan[x]]/(4*Sqrt[1 - I]) + ArcTan[Sqrt[1 + I]*Tan[x]]/(4
*Sqrt[1 + I]) + ArcTan[Sqrt[2]*Tan[x]]/(4*Sqrt[2]) + Tan[x]/4
```

3.260.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.260.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(65) = 130$.

Time = 5.77 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.98

method	result
risch	$\frac{i}{2\,e^{2ix}+2}+\frac{\sqrt{-2-2i}\,\ln\left(e^{2ix}+i\sqrt{-2-2i}-\sqrt{-2-2i}-1-2i\right)}{16}-\frac{\sqrt{-2-2i}\,\ln\left(e^{2ix}-i\sqrt{-2-2i}+\sqrt{-2-2i}-1-2i\right)}{16}+\frac{\sqrt{-2+2i}\,\ln\left(e^{2i\cdots\right.}{}$
default	$\frac{\tan(x)}{4}+\frac{\sqrt{2}\left(-\frac{\sqrt{-2+2\sqrt{2}}\,\ln\left(-\sqrt{-2+2\sqrt{2}}\,\sqrt{2}\,\tan(x)+2\left(\tan^{2}(x)\right)+\sqrt{2}\right)}{4}+\frac{\left(-\frac{\left(-2+2\sqrt{2}\right)\sqrt{2}}{4}+2\right)\arctan\left(\frac{-\sqrt{2}\,\sqrt{-2+2\sqrt{2}}+4\tan(x)}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{1+\sqrt{2}}}\right)}{8}+$

input
```
int(1/(1-sin(x)^8),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/(exp(2*I*x)+1)+1/16*(-2-2*I)^(1/2)*ln(exp(2*I*x)+I*(-2-2*I)^(1/2)-(-
2-2*I)^(1/2)-1-2*I)-1/16*(-2-2*I)^(1/2)*ln(exp(2*I*x)-I*(-2-2*I)^(1/2)+(-2
-2*I)^(1/2)-1-2*I)+1/16*(-2+2*I)^(1/2)*ln(exp(2*I*x)+I*(-2+2*I)^(1/2)+(-2+
2*I)^(1/2)-1+2*I)-1/16*(-2+2*I)^(1/2)*ln(exp(2*I*x)-I*(-2+2*I)^(1/2)-(-2+2
*I)^(1/2)-1+2*I)+1/16*I*2^(1/2)*ln(exp(2*I*x)-2*2^(1/2)-3)-1/16*I*2^(1/2)*
ln(exp(2*I*x)+2*2^(1/2)-3)
```

3.260.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(57) = 114.

Time = 0.32 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.87

$$\int \frac{1}{1-\sin^8(x)}\,dx$$
$$= \frac{\sqrt{2}\sqrt{i-1}\cos(x)\log\left(-(i-1)\sqrt{2}\sqrt{i-1}\cos(x)\sin(x)+(2i-1)\cos(x)^2-i+1\right)-\sqrt{2}\sqrt{i-1}\cos(x)}{}$$

input
```
integrate(1/(1-sin(x)^8),x, algorithm="fricas")
```

output
```
1/32*(sqrt(2)*sqrt(I - 1)*cos(x)*log(-(I - 1)*sqrt(2)*sqrt(I - 1)*cos(x)*s
in(x) + (2*I - 1)*cos(x)^2 - I + 1) - sqrt(2)*sqrt(I - 1)*cos(x)*log((I -
1)*sqrt(2)*sqrt(I - 1)*cos(x)*sin(x) + (2*I - 1)*cos(x)^2 - I + 1) - sqrt(
2)*sqrt(-I - 1)*cos(x)*log((I + 1)*sqrt(2)*sqrt(-I - 1)*cos(x)*sin(x) + (2
*I + 1)*cos(x)^2 - I - 1) + sqrt(2)*sqrt(-I - 1)*cos(x)*log(-(I + 1)*sqrt(
2)*sqrt(-I - 1)*cos(x)*sin(x) + (2*I + 1)*cos(x)^2 - I - 1) - 2*sqrt(2)*ar
ctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x)))*cos(x) + 8*sin(
x))/cos(x)
```

3.260.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1-\sin^8(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1-sin(x)**8),x)
```

output
```
Timed out
```

3.260.7 Maxima [F]

$$\int \frac{1}{1-\sin^8(x)}\,dx = \int -\frac{1}{\sin(x)^8-1}\,dx$$

input
```
integrate(1/(1-sin(x)^8),x, algorithm="maxima")
```

output
```
1/16*((sqrt(2)*cos(2*x)^2 + sqrt(2)*sin(2*x)^2 + 2*sqrt(2)*cos(2*x) + sqrt
(2))*arctan2(2*sqrt(2)*sin(x)/(2*(sqrt(2) + 1)*cos(x) + cos(x)^2 + sin(x)^
2 + 2*sqrt(2) + 3), (cos(x)^2 + sin(x)^2 + 2*cos(x) - 1)/(2*(sqrt(2) + 1)*
cos(x) + cos(x)^2 + sin(x)^2 + 2*sqrt(2) + 3)) - (sqrt(2)*cos(2*x)^2 + sqr
t(2)*sin(2*x)^2 + 2*sqrt(2)*cos(2*x) + sqrt(2))*arctan2(2*sqrt(2)*sin(x)/(
2*(sqrt(2) - 1)*cos(x) + cos(x)^2 + sin(x)^2 - 2*sqrt(2) + 3), (cos(x)^2 +
 sin(x)^2 - 2*cos(x) - 1)/(2*(sqrt(2) - 1)*cos(x) + cos(x)^2 + sin(x)^2 -
2*sqrt(2) + 3)) + 128*(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*integrate
(((4*cos(2*x) - 1)*cos(4*x) - cos(8*x)*cos(4*x) + 4*cos(6*x)*cos(4*x) - 22
*cos(4*x)^2 - sin(8*x)*sin(4*x) + 4*sin(6*x)*sin(4*x) - 22*sin(4*x)^2 + 4*
sin(4*x)*sin(2*x))/(2*(4*cos(6*x) - 22*cos(4*x) + 4*cos(2*x) - 1)*cos(8*x)
 - cos(8*x)^2 + 8*(22*cos(4*x) - 4*cos(2*x) + 1)*cos(6*x) - 16*cos(6*x)^2
+ 44*(4*cos(2*x) - 1)*cos(4*x) - 484*cos(4*x)^2 - 16*cos(2*x)^2 + 4*(2*sin
(6*x) - 11*sin(4*x) + 2*sin(2*x))*sin(8*x) - sin(8*x)^2 + 16*(11*sin(4*x)
- 2*sin(2*x))*sin(6*x) - 16*sin(6*x)^2 - 484*sin(4*x)^2 + 176*sin(4*x)*sin
(2*x) - 16*sin(2*x)^2 + 8*cos(2*x) - 1), x) + 8*sin(2*x))/(cos(2*x)^2 + si
n(2*x)^2 + 2*cos(2*x) + 1)
```

3.260.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(57) = 114$.

Time = 0.53 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.47

$$\int \frac{1}{1-\sin^8(x)}\,dx$$
$$= \frac{1}{8}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-2\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-2\cos(2x)+2}\right)\right)$$
$$+\frac{1}{8}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}+2\tan(x)\right)}{\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}$$
$$+\frac{1}{8}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}-2\tan(x)\right)}{\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}$$
$$+\frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(\tan(x)^2+\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{\frac{1}{2}}\right)$$
$$-\frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(\tan(x)^2-\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{\frac{1}{2}}\right)+\frac{1}{4}\tan(x)$$

input
```
integrate(1/(1-sin(x)^8),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x)
 + sqrt(2) - 2*cos(2*x) + 2))) + 1/8*(pi*floor(x/pi + 1/2) + arctan(2*(1/2
)^(3/4)*((1/2)^(1/4)*sqrt(-sqrt(2) + 2) + 2*tan(x))/sqrt(sqrt(2) + 2)))*sq
rt(sqrt(2) + 1) + 1/8*(pi*floor(x/pi + 1/2) + arctan(-2*(1/2)^(3/4)*((1/2)
^(1/4)*sqrt(-sqrt(2) + 2) - 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) + 1
) + 1/16*sqrt(sqrt(2) - 1)*log(tan(x)^2 + (1/2)^(1/4)*sqrt(-sqrt(2) + 2)*t
an(x) + sqrt(1/2)) - 1/16*sqrt(sqrt(2) - 1)*log(tan(x)^2 - (1/2)^(1/4)*sqr
t(-sqrt(2) + 2)*tan(x) + sqrt(1/2)) + 1/4*tan(x)
```

3.260. $\int \frac{1}{1-\sin^8(x)}\,dx$

3.260.9 Mupad [B] (verification not implemented)

Time = 13.79 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.58

$$\int \frac{1}{1-\sin^8(x)}\,dx = \frac{\tan(x)}{4} + \operatorname{atan}\left(\sqrt{2}\tan(x)\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,8i - \sqrt{2}\tan(x)\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,8i\right)\left(\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2i + \sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2i\right) + \operatorname{atan}\left(\sqrt{2}\tan(x)\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,8i + \sqrt{2}\tan(x)\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,8i\right)\left(\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2i - \sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2i\right) + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\tan(x)\right)}{8}$$

input
```
int(-1/(sin(x)^8 - 1),x)
```

output
```
tan(x)/4 + atan(2^(1/2)*tan(x)*(- 2^(1/2)/256 - 1/256)^(1/2)*8i - 2^(1/2)*
tan(x)*(2^(1/2)/256 - 1/256)^(1/2)*8i)*((- 2^(1/2)/256 - 1/256)^(1/2)*2i +
 (2^(1/2)/256 - 1/256)^(1/2)*2i) + atan(2^(1/2)*tan(x)*(- 2^(1/2)/256 - 1/
256)^(1/2)*8i + 2^(1/2)*tan(x)*(2^(1/2)/256 - 1/256)^(1/2)*8i)*((- 2^(1/2)
/256 - 1/256)^(1/2)*2i - (2^(1/2)/256 - 1/256)^(1/2)*2i) + (2^(1/2)*atan(2
^(1/2)*tan(x)))/8
```

3.261 $\int \frac{\cos^9(x)}{a-a\sin^2(x)}\,dx$

3.261.1 Optimal result

Integrand size = 16, antiderivative size = 38

$$\int \frac{\cos^9(x)}{a-a\sin^2(x)}\,dx=\frac{\sin(x)}{a}-\frac{\sin^3(x)}{a}+\frac{3\sin^5(x)}{5a}-\frac{\sin^7(x)}{7a}$$

output
```
sin(x)/a-sin(x)^3/a+3/5*sin(x)^5/a-1/7*sin(x)^7/a
```

3.261.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{\cos^9(x)}{a-a\sin^2(x)}\,dx=\frac{\sin(x)-\sin^3(x)+\frac{3\sin^5(x)}{5}-\frac{\sin^7(x)}{7}}{a}$$

input
```
Integrate[Cos[x]^9/(a - a*Sin[x]^2),x]
```

output
```
(Sin[x] - Sin[x]^3 + (3*Sin[x]^5)/5 - Sin[x]^7/7)/a
```

3.261.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3042, 3654, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^9(x)}{a-a\sin^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^9}{a-a\sin(x)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \cos^7(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(x+\frac{\pi}{2}\right)^7dx}{a}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\int \left(-\sin^6(x)+3\sin^4(x)-3\sin^2(x)+1\right)d(-\sin(x))}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\sin^7(x)}{7}-\frac{3\sin^5(x)}{5}+\sin^3(x)-\sin(x)}{a}$$

input
```
Int[Cos[x]^9/(a - a*Sin[x]^2),x]
```

output
```
-((-Sin[x] + Sin[x]^3 - (3*Sin[x]^5)/5 + Sin[x]^7/7)/a)
```

3.261.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.261.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$-\frac{\frac{\left(\sin^{7}(x)\right)}{7}-\frac{3\left(\sin^{5}(x)\right)}{5}+\sin^{3}(x)-\sin(x)}{a}$	27
default	$-\frac{\frac{\left(\sin^{7}(x)\right)}{7}-\frac{3\left(\sin^{5}(x)\right)}{5}+\sin^{3}(x)-\sin(x)}{a}$	27
parallelrisch	$\frac{1225\sin(x)+5\sin(7x)+49\sin(5x)+245\sin(3x)}{2240a}$	29
risch	$\frac{35\sin(x)}{64a}+\frac{\sin(7x)}{448a}+\frac{7\sin(5x)}{320a}+\frac{7\sin(3x)}{64a}$	36

input
```
int(cos(x)^9/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/a*(1/7*sin(x)^7-3/5*sin(x)^5+sin(x)^3-sin(x))
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{\cos^9(x)}{a - a\sin^2(x)}\,dx = \frac{\left(5\,\cos\left(x\right)^6 + 6\,\cos\left(x\right)^4 + 8\,\cos\left(x\right)^2 + 16\right)\sin\left(x\right)}{35\,a}$$

input
```
integrate(cos(x)^9/(a-a*sin(x)^2),x, algorithm="fricas")
```

output
```
1/35*(5*cos(x)^6 + 6*cos(x)^4 + 8*cos(x)^2 + 16)*sin(x)/a
```

3.261.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(29) = 58.

Time = 12.86 (sec) , antiderivative size = 580, normalized size of antiderivative = 15.26

$$\int \frac{\cos^9(x)}{a - a\sin^2(x)}\,dx$$
$$= \frac{70\tan^{13}\left(\frac{x}{2}\right)}{35a\tan^{14}\left(\frac{x}{2}\right) + 245a\tan^{12}\left(\frac{x}{2}\right) + 735a\tan^{10}\left(\frac{x}{2}\right) + 1225a\tan^{8}\left(\frac{x}{2}\right) + 1225a\tan^{6}\left(\frac{x}{2}\right) + 735a\tan^{4}\left(\frac{x}{2}\right) + 24}$$
$$+ \frac{140\tan^{11}\left(\frac{x}{2}\right)}{35a\tan^{14}\left(\frac{x}{2}\right) + 245a\tan^{12}\left(\frac{x}{2}\right) + 735a\tan^{10}\left(\frac{x}{2}\right) + 1225a\tan^{8}\left(\frac{x}{2}\right) + 1225a\tan^{6}\left(\frac{x}{2}\right) + 735a\tan^{4}\left(\frac{x}{2}\right) +}$$
$$+ \frac{602\tan^{9}\left(\frac{x}{2}\right)}{35a\tan^{14}\left(\frac{x}{2}\right) + 245a\tan^{12}\left(\frac{x}{2}\right) + 735a\tan^{10}\left(\frac{x}{2}\right) + 1225a\tan^{8}\left(\frac{x}{2}\right) + 1225a\tan^{6}\left(\frac{x}{2}\right) + 735a\tan^{4}\left(\frac{x}{2}\right) +}$$
$$+ \frac{424\tan^{7}\left(\frac{x}{2}\right)}{35a\tan^{14}\left(\frac{x}{2}\right) + 245a\tan^{12}\left(\frac{x}{2}\right) + 735a\tan^{10}\left(\frac{x}{2}\right) + 1225a\tan^{8}\left(\frac{x}{2}\right) + 1225a\tan^{6}\left(\frac{x}{2}\right) + 735a\tan^{4}\left(\frac{x}{2}\right) +}$$
$$+ \frac{602\tan^{5}\left(\frac{x}{2}\right)}{35a\tan^{14}\left(\frac{x}{2}\right) + 245a\tan^{12}\left(\frac{x}{2}\right) + 735a\tan^{10}\left(\frac{x}{2}\right) + 1225a\tan^{8}\left(\frac{x}{2}\right) + 1225a\tan^{6}\left(\frac{x}{2}\right) + 735a\tan^{4}\left(\frac{x}{2}\right) +}$$
$$+ \frac{140\tan^{3}\left(\frac{x}{2}\right)}{35a\tan^{14}\left(\frac{x}{2}\right) + 245a\tan^{12}\left(\frac{x}{2}\right) + 735a\tan^{10}\left(\frac{x}{2}\right) + 1225a\tan^{8}\left(\frac{x}{2}\right) + 1225a\tan^{6}\left(\frac{x}{2}\right) + 735a\tan^{4}\left(\frac{x}{2}\right) +}$$
$$+ \frac{70\tan\left(\frac{x}{2}\right)}{35a\tan^{14}\left(\frac{x}{2}\right) + 245a\tan^{12}\left(\frac{x}{2}\right) + 735a\tan^{10}\left(\frac{x}{2}\right) + 1225a\tan^{8}\left(\frac{x}{2}\right) + 1225a\tan^{6}\left(\frac{x}{2}\right) + 735a\tan^{4}\left(\frac{x}{2}\right) +}$$

input
```
integrate(cos(x)**9/(a-a*sin(x)**2),x)
```

output
```
70*tan(x/2)**13/(35*a*tan(x/2)**14 + 245*a*tan(x/2)**12 + 735*a*tan(x/2)**
10 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2)**6 + 735*a*tan(x/2)**4 + 245*a*t
an(x/2)**2 + 35*a) + 140*tan(x/2)**11/(35*a*tan(x/2)**14 + 245*a*tan(x/2)*
*12 + 735*a*tan(x/2)**10 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2)**6 + 735*a
*tan(x/2)**4 + 245*a*tan(x/2)**2 + 35*a) + 602*tan(x/2)**9/(35*a*tan(x/2)*
*14 + 245*a*tan(x/2)**12 + 735*a*tan(x/2)**10 + 1225*a*tan(x/2)**8 + 1225*
a*tan(x/2)**6 + 735*a*tan(x/2)**4 + 245*a*tan(x/2)**2 + 35*a) + 424*tan(x/
2)**7/(35*a*tan(x/2)**14 + 245*a*tan(x/2)**12 + 735*a*tan(x/2)**10 + 1225*
a*tan(x/2)**8 + 1225*a*tan(x/2)**6 + 735*a*tan(x/2)**4 + 245*a*tan(x/2)**2
 + 35*a) + 602*tan(x/2)**5/(35*a*tan(x/2)**14 + 245*a*tan(x/2)**12 + 735*a
*tan(x/2)**10 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2)**6 + 735*a*tan(x/2)**
4 + 245*a*tan(x/2)**2 + 35*a) + 140*tan(x/2)**3/(35*a*tan(x/2)**14 + 245*a
*tan(x/2)**12 + 735*a*tan(x/2)**10 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2)*
*6 + 735*a*tan(x/2)**4 + 245*a*tan(x/2)**2 + 35*a) + 70*tan(x/2)/(35*a*tan
(x/2)**14 + 245*a*tan(x/2)**12 + 735*a*tan(x/2)**10 + 1225*a*tan(x/2)**8 +
 1225*a*tan(x/2)**6 + 735*a*tan(x/2)**4 + 245*a*tan(x/2)**2 + 35*a)
```

3.261.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{\cos^9(x)}{a-a\sin^2(x)}\,dx = -\frac{5\,\sin{(x)}^7 - 21\,\sin{(x)}^5 + 35\,\sin{(x)}^3 - 35\,\sin{(x)}}{35\,a}$$

input
```
integrate(cos(x)^9/(a-a*sin(x)^2),x, algorithm="maxima")
```

output
```
-1/35*(5*sin(x)^7 - 21*sin(x)^5 + 35*sin(x)^3 - 35*sin(x))/a
```

3.261.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{\cos^9(x)}{a-a\sin^2(x)}\,dx = -\frac{5\,\sin{(x)}^7 - 21\,\sin{(x)}^5 + 35\,\sin{(x)}^3 - 35\,\sin{(x)}}{35\,a}$$

input
```
integrate(cos(x)^9/(a-a*sin(x)^2),x, algorithm="giac")
```

output
```
-1/35*(5*sin(x)^7 - 21*sin(x)^5 + 35*sin(x)^3 - 35*sin(x))/a
```

3.261.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{\cos^9(x)}{a - a\sin^2(x)}\,dx = \frac{\sin(x)}{a} - \frac{\sin(x)^3}{a} + \frac{3\sin(x)^5}{5a} - \frac{\sin(x)^7}{7a}$$

input
```
int(cos(x)^9/(a - a*sin(x)^2),x)
```

output
```
sin(x)/a - sin(x)^3/a + (3*sin(x)^5)/(5*a) - sin(x)^7/(7*a)
```

3.262 $\int \frac{\cos^7(x)}{a-a\sin^2(x)}\,dx$

3.262.1 Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{\cos^7(x)}{a-a\sin^2(x)}\,dx = \frac{\sin(x)}{a} - \frac{2\sin^3(x)}{3a} + \frac{\sin^5(x)}{5a}$$

output
```
sin(x)/a-2/3*sin(x)^3/a+1/5*sin(x)^5/a
```

3.262.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\cos^7(x)}{a-a\sin^2(x)}\,dx = \frac{\sin(x) - \frac{2\sin^3(x)}{3} + \frac{\sin^5(x)}{5}}{a}$$

input
```
Integrate[Cos[x]^7/(a - a*Sin[x]^2),x]
```

output
```
(Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5)/a
```

3.262.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3042, 3654, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^7(x)}{a-a\sin^2(x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(x)^7}{a-a\sin(x)^2}dx$$
$$\Big\downarrow \text{ 3654}$$
$$\frac{\int \cos^5(x)dx}{a}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \sin\left(x+\frac{\pi}{2}\right)^5dx}{a}$$
$$\Big\downarrow \text{ 3113}$$
$$-\frac{\int \left(\sin^4(x)-2\sin^2(x)+1\right)d(-\sin(x))}{a}$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)}{a}$$

input
```
Int[Cos[x]^7/(a - a*Sin[x]^2),x]
```

output
```
-((-Sin[x] + (2*Sin[x]^3)/3 - Sin[x]^5/5)/a)
```

3.262.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.262.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{\frac{\left(\sin^5(x)\right)}{5}-\frac{2\left(\sin^3(x)\right)}{3}+\sin(x)}{a}$
default	$\frac{\frac{\left(\sin^5(x)\right)}{5}-\frac{2\left(\sin^3(x)\right)}{3}+\sin(x)}{a}$
parallelrisch	$\frac{150\sin(x)+3\sin(5x)+25\sin(3x)}{240a}$
risch	$\frac{5\sin(x)}{8a}+\frac{\sin(5x)}{80a}+\frac{5\sin(3x)}{48a}$
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}-\frac{14\left(\tan^3\left(\frac{x}{2}\right)\right)}{3a}-\frac{42\left(\tan^5\left(\frac{x}{2}\right)\right)}{5a}-\frac{86\left(\tan^7\left(\frac{x}{2}\right)\right)}{15a}+\frac{86\left(\tan^9\left(\frac{x}{2}\right)\right)}{15a}+\frac{42\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{5a}+\frac{14\left(\tan^{13}\left(\frac{x}{2}\right)\right)}{3a}+\frac{2\left(\tan^{15}\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^7\left(\tan^2\left(\frac{x}{2}\right)-1\right)}$

input
```
int(cos(x)^7/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/5*sin(x)^5-2/3*sin(x)^3+sin(x))
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{\cos^7(x)}{a-a\sin^2(x)}\,dx=\frac{\left(3\,\cos\left(x\right)^4+4\,\cos\left(x\right)^2+8\right)\sin\left(x\right)}{15\,a}$$

input
```
integrate(cos(x)^7/(a-a*sin(x)^2),x, algorithm="fricas")
```

output
```
1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)/a
```

3.262.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(22) = 44.

Time = 5.83 (sec) , antiderivative size = 311, normalized size of antiderivative = 10.72

$$\begin{aligned}&\int \frac{\cos^7(x)}{a-a\sin^2(x)}\,dx\\&=\frac{30\tan^9\left(\frac{x}{2}\right)}{15a\tan^{10}\left(\frac{x}{2}\right)+75a\tan^8\left(\frac{x}{2}\right)+150a\tan^6\left(\frac{x}{2}\right)+150a\tan^4\left(\frac{x}{2}\right)+75a\tan^2\left(\frac{x}{2}\right)+15a}\\&\quad+\frac{40\tan^7\left(\frac{x}{2}\right)}{15a\tan^{10}\left(\frac{x}{2}\right)+75a\tan^8\left(\frac{x}{2}\right)+150a\tan^6\left(\frac{x}{2}\right)+150a\tan^4\left(\frac{x}{2}\right)+75a\tan^2\left(\frac{x}{2}\right)+15a}\\&\quad+\frac{116\tan^5\left(\frac{x}{2}\right)}{15a\tan^{10}\left(\frac{x}{2}\right)+75a\tan^8\left(\frac{x}{2}\right)+150a\tan^6\left(\frac{x}{2}\right)+150a\tan^4\left(\frac{x}{2}\right)+75a\tan^2\left(\frac{x}{2}\right)+15a}\\&\quad+\frac{40\tan^3\left(\frac{x}{2}\right)}{15a\tan^{10}\left(\frac{x}{2}\right)+75a\tan^8\left(\frac{x}{2}\right)+150a\tan^6\left(\frac{x}{2}\right)+150a\tan^4\left(\frac{x}{2}\right)+75a\tan^2\left(\frac{x}{2}\right)+15a}\\&\quad+\frac{30\tan\left(\frac{x}{2}\right)}{15a\tan^{10}\left(\frac{x}{2}\right)+75a\tan^8\left(\frac{x}{2}\right)+150a\tan^6\left(\frac{x}{2}\right)+150a\tan^4\left(\frac{x}{2}\right)+75a\tan^2\left(\frac{x}{2}\right)+15a}\end{aligned}$$

input
```
integrate(cos(x)**7/(a-a*sin(x)**2),x)
```

output
```
30*tan(x/2)**9/(15*a*tan(x/2)**10 + 75*a*tan(x/2)**8 + 150*a*tan(x/2)**6 +
 150*a*tan(x/2)**4 + 75*a*tan(x/2)**2 + 15*a) + 40*tan(x/2)**7/(15*a*tan(x
/2)**10 + 75*a*tan(x/2)**8 + 150*a*tan(x/2)**6 + 150*a*tan(x/2)**4 + 75*a*
tan(x/2)**2 + 15*a) + 116*tan(x/2)**5/(15*a*tan(x/2)**10 + 75*a*tan(x/2)**
8 + 150*a*tan(x/2)**6 + 150*a*tan(x/2)**4 + 75*a*tan(x/2)**2 + 15*a) + 40*
tan(x/2)**3/(15*a*tan(x/2)**10 + 75*a*tan(x/2)**8 + 150*a*tan(x/2)**6 + 15
0*a*tan(x/2)**4 + 75*a*tan(x/2)**2 + 15*a) + 30*tan(x/2)/(15*a*tan(x/2)**1
0 + 75*a*tan(x/2)**8 + 150*a*tan(x/2)**6 + 150*a*tan(x/2)**4 + 75*a*tan(x/
2)**2 + 15*a)
```

3.262.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{\cos^7(x)}{a - a\sin^2(x)}\,dx = \frac{3\,\sin(x)^5 - 10\,\sin(x)^3 + 15\,\sin(x)}{15\,a}$$

input
```
integrate(cos(x)^7/(a-a*sin(x)^2),x, algorithm="maxima")
```

output
```
1/15*(3*sin(x)^5 - 10*sin(x)^3 + 15*sin(x))/a
```

3.262.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{\cos^7(x)}{a - a\sin^2(x)}\,dx = \frac{3\,\sin(x)^5 - 10\,\sin(x)^3 + 15\,\sin(x)}{15\,a}$$

input
```
integrate(cos(x)^7/(a-a*sin(x)^2),x, algorithm="giac")
```

output
```
1/15*(3*sin(x)^5 - 10*sin(x)^3 + 15*sin(x))/a
```

3.262.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{\cos^7(x)}{a - a\sin^2(x)}\,dx = \frac{\frac{\sin(x)^5}{5} - \frac{2\sin(x)^3}{3} + \sin(x)}{a}$$

input
```
int(cos(x)^7/(a - a*sin(x)^2),x)
```

output
```
(sin(x) - (2*sin(x)^3)/3 + sin(x)^5/5)/a
```

3.263 $\int \frac{\cos^5(x)}{a-a\sin^2(x)}\,dx$

3.263.1 Optimal result

Integrand size = 16, antiderivative size = 18

$$\int \frac{\cos^5(x)}{a-a\sin^2(x)}\,dx=\frac{\sin(x)}{a}-\frac{\sin^3(x)}{3a}$$

output
```
sin(x)/a-1/3*sin(x)^3/a
```

3.263.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\cos^5(x)}{a-a\sin^2(x)}\,dx=\frac{\sin(x)-\frac{\sin^3(x)}{3}}{a}$$

input
```
Integrate[Cos[x]^5/(a - a*Sin[x]^2),x]
```

output
```
(Sin[x] - Sin[x]^3/3)/a
```

3.263.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3042, 3654, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(x)}{a-a\sin^2(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(x)^5}{a-a\sin(x)^2}dx$$
$$\Big\downarrow \text{3654}$$
$$\frac{\int \cos^3(x)dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \sin\left(x+\frac{\pi}{2}\right)^3dx}{a}$$
$$\Big\downarrow \text{3113}$$
$$-\frac{\int \left(1-\sin^2(x)\right)d(-\sin(x))}{a}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\frac{\sin^3(x)}{3}-\sin(x)}{a}$$

input
```
Int[Cos[x]^5/(a - a*Sin[x]^2),x]
```

output
```
-((-Sin[x] + Sin[x]^3/3)/a)
```

3.263.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.263.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{-\frac{\left(\sin^{3}(x)\right)}{3}+\sin(x)}{a}$	14
default	$\frac{-\frac{\left(\sin^{3}(x)\right)}{3}+\sin(x)}{a}$	14
parallelrisch	$\frac{9\sin(x)+\sin(3x)}{12a}$	15
risch	$\frac{3\sin(x)}{4a}+\frac{\sin(3x)}{12a}$	18
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}-\frac{10\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{3a}-\frac{4\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{3a}+\frac{4\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{3a}+\frac{10\left(\tan^{9}\left(\frac{x}{2}\right)\right)}{3a}+\frac{2\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{5}\left(\tan^{2}\left(\frac{x}{2}\right)-1\right)}$	87

input
```
int(cos(x)^5/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/3*sin(x)^3+sin(x))
```

3.263. $\int \frac{\cos^5(x)}{a-a\sin^2(x)}\,dx$

3.263.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{\cos^5(x)}{a - a\sin^2(x)}\,dx = \frac{\left(\cos{(x)}^2 + 2\right)\sin{(x)}}{3\,a}$$

input
```
integrate(cos(x)^5/(a-a*sin(x)^2),x, algorithm="fricas")
```

output
```
1/3*(cos(x)^2 + 2)*sin(x)/a
```

3.263.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(12) = 24.

Time = 2.59 (sec) , antiderivative size = 124, normalized size of antiderivative = 6.89

$$\int \frac{\cos^5(x)}{a - a\sin^2(x)}\,dx = \frac{6\tan^5\left(\frac{x}{2}\right)}{3a\tan^6\left(\frac{x}{2}\right) + 9a\tan^4\left(\frac{x}{2}\right) + 9a\tan^2\left(\frac{x}{2}\right) + 3a} + \frac{4\tan^3\left(\frac{x}{2}\right)}{3a\tan^6\left(\frac{x}{2}\right) + 9a\tan^4\left(\frac{x}{2}\right) + 9a\tan^2\left(\frac{x}{2}\right) + 3a} + \frac{6\tan\left(\frac{x}{2}\right)}{3a\tan^6\left(\frac{x}{2}\right) + 9a\tan^4\left(\frac{x}{2}\right) + 9a\tan^2\left(\frac{x}{2}\right) + 3a}$$

input
```
integrate(cos(x)**5/(a-a*sin(x)**2),x)
```

output
```
6*tan(x/2)**5/(3*a*tan(x/2)**6 + 9*a*tan(x/2)**4 + 9*a*tan(x/2)**2 + 3*a)
+ 4*tan(x/2)**3/(3*a*tan(x/2)**6 + 9*a*tan(x/2)**4 + 9*a*tan(x/2)**2 + 3*a
) + 6*tan(x/2)/(3*a*tan(x/2)**6 + 9*a*tan(x/2)**4 + 9*a*tan(x/2)**2 + 3*a)
```

3.263.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\cos^5(x)}{a - a\sin^2(x)}\,dx = -\frac{\sin(x)^3 - 3\sin(x)}{3a}$$

input
```
integrate(cos(x)^5/(a-a*sin(x)^2),x, algorithm="maxima")
```

output
```
-1/3*(sin(x)^3 - 3*sin(x))/a
```

3.263.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\cos^5(x)}{a - a\sin^2(x)}\,dx = -\frac{\sin(x)^3 - 3\sin(x)}{3a}$$

input
```
integrate(cos(x)^5/(a-a*sin(x)^2),x, algorithm="giac")
```

output
```
-1/3*(sin(x)^3 - 3*sin(x))/a
```

3.263.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\cos^5(x)}{a - a\sin^2(x)}\,dx = \frac{3\sin(x) - \sin(x)^3}{3a}$$

input
```
int(cos(x)^5/(a - a*sin(x)^2),x)
```

output
```
(3*sin(x) - sin(x)^3)/(3*a)
```

3.264 $\int \frac{\cos^3(x)}{a-a\sin^2(x)}\,dx$

3.264.1 Optimal result

Integrand size = 16, antiderivative size = 6

$$\int \frac{\cos^3(x)}{a-a\sin^2(x)}\,dx = \frac{\sin(x)}{a}$$

output
```
sin(x)/a
```

3.264.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(x)}{a-a\sin^2(x)}\,dx = \frac{\sin(x)}{a}$$

input
```
Integrate[Cos[x]^3/(a - a*Sin[x]^2),x]
```

output
```
Sin[x]/a
```

3.264.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3654, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(x)}{a-a\sin^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)^3}{a-a\sin(x)^2}dx$$

$$\Big\downarrow \text{ 3654}$$

$$\frac{\int \cos(x)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sin\left(x+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{\sin(x)}{a}$$

input
```
Int[Cos[x]^3/(a - a*Sin[x]^2),x]
```

output
```
Sin[x]/a
```

3.264.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.264.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{\sin(x)}{a}$	7
default	$\frac{\sin(x)}{a}$	7
risch	$\frac{\sin(x)}{a}$	7
parallelrisch	$\frac{\sin(x)}{a}$	7
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}-\frac{2\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}+\frac{2\left(\tan^5\left(\frac{x}{2}\right)\right)}{a}+\frac{2\left(\tan^7\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3\left(\tan^2\left(\frac{x}{2}\right)-1\right)}$	65

input
```
int(cos(x)^3/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
sin(x)/a
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(x)}{a-a\sin^2(x)}\,dx=\frac{\sin(x)}{a}$$

input
```
integrate(cos(x)^3/(a-a*sin(x)^2),x, algorithm="fricas")
```

output
```
sin(x)/a
```

3.264.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.

Time = 0.81 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{\cos^3(x)}{a - a\sin^2(x)}\,dx = \frac{2\tan\left(\frac{x}{2}\right)}{a\tan^2\left(\frac{x}{2}\right) + a}$$

input
```
integrate(cos(x)**3/(a-a*sin(x)**2),x)
```

output
```
2*tan(x/2)/(a*tan(x/2)**2 + a)
```

3.264.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(x)}{a - a\sin^2(x)}\,dx = \frac{\sin(x)}{a}$$

input
```
integrate(cos(x)^3/(a-a*sin(x)^2),x, algorithm="maxima")
```

output
```
sin(x)/a
```

3.264.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(x)}{a - a\sin^2(x)}\,dx = \frac{\sin(x)}{a}$$

input
```
integrate(cos(x)^3/(a-a*sin(x)^2),x, algorithm="giac")
```

output
```
sin(x)/a
```

3.264.9 Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(x)}{a-a\sin^2(x)}\,dx=\frac{\sin(x)}{a}$$

input
```
int(cos(x)^3/(a - a*sin(x)^2),x)
```

output
```
sin(x)/a
```

3.265 $\int \frac{\cos(x)}{a-a\sin^2(x)}\,dx$

3.265.1 Optimal result

Integrand size = 14, antiderivative size = 7

$$\int \frac{\cos(x)}{a-a\sin^2(x)}\,dx=\frac{\operatorname{arctanh}(\sin(x))}{a}$$

output
```
arctanh(sin(x))/a
```

3.265.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a-a\sin^2(x)}\,dx=\frac{\operatorname{arctanh}(\sin(x))}{a}$$

input
```
Integrate[Cos[x]/(a - a*Sin[x]^2),x]
```

output
```
ArcTanh[Sin[x]]/a
```

3.265.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3654, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{a-a\sin^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{a-a\sin(x)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sec(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\text{arctanh}(\sin(x))}{a}$$

input
```
Int[Cos[x]/(a - a*Sin[x]^2),x]
```

output
```
ArcTanh[Sin[x]]/a
```

3.265.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.265.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}(\sin(x))}{a}$	8
default	$\frac{\operatorname{arctanh}(\sin(x))}{a}$	8
parallelrisch	$\frac{-\ln(\tan(\frac{x}{2})-1)+\ln(\tan(\frac{x}{2})+1)}{a}$	22
norman	$\frac{\ln(\tan(\frac{x}{2})+1)}{a}-\frac{\ln(\tan(\frac{x}{2})-1)}{a}$	25
risch	$\frac{\ln(e^{ix}+i)}{a}-\frac{\ln(e^{ix}-i)}{a}$	29

input
```
int(cos(x)/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctanh(sin(x))/a
```

3.265.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(7) = 14.

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.86

$$\int \frac{\cos(x)}{a-a\sin^2(x)}\,dx = \frac{\log\left(\sin\left(x\right)+1\right)-\log\left(-\sin\left(x\right)+1\right)}{2\,a}$$

input
```
integrate(cos(x)/(a-a*sin(x)^2),x, algorithm="fricas")
```

output
```
1/2*(log(sin(x) + 1) - log(-sin(x) + 1))/a
```

3.265.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(5) = 10$.

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.71

$$\int \frac{\cos(x)}{a - a\sin^2(x)}\,dx = -\frac{\log(\sin(x) - 1)}{2a} + \frac{\log(\sin(x) + 1)}{2a}$$

input
```
integrate(cos(x)/(a-a*sin(x)**2),x)
```

output
```
-log(sin(x) - 1)/(2*a) + log(sin(x) + 1)/(2*a)
```

3.265.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(7) = 14$.

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 3.00

$$\int \frac{\cos(x)}{a - a\sin^2(x)}\,dx = \frac{\log(\sin(x) + 1)}{2\,a} - \frac{\log(\sin(x) - 1)}{2\,a}$$

input
```
integrate(cos(x)/(a-a*sin(x)^2),x, algorithm="maxima")
```

output
```
1/2*log(sin(x) + 1)/a - 1/2*log(sin(x) - 1)/a
```

3.265.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(7) = 14$.

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 3.29

$$\int \frac{\cos(x)}{a - a\sin^2(x)}\,dx = \frac{\log(\sin(x) + 1)}{2\,a} - \frac{\log(-\sin(x) + 1)}{2\,a}$$

input
```
integrate(cos(x)/(a-a*sin(x)^2),x, algorithm="giac")
```

output
```
1/2*log(sin(x) + 1)/a - 1/2*log(-sin(x) + 1)/a
```

3.265.9 Mupad [B] (verification not implemented)

Time = 13.74 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a - a\sin^2(x)}\,dx = \frac{\operatorname{atanh}(\sin(x))}{a}$$

input
```
int(cos(x)/(a - a*sin(x)^2),x)
```

output
```
atanh(sin(x))/a
```

3.266 $\int \frac{\sec^3(x)}{a-a\sin^2(x)}\,dx$

3.266.1 Optimal result

Integrand size = 16, antiderivative size = 35

$$\int \frac{\sec^3(x)}{a-a\sin^2(x)}\,dx=\frac{3\text{arctanh}(\sin(x))}{8a}+\frac{3\sec(x)\tan(x)}{8a}+\frac{\sec^3(x)\tan(x)}{4a}$$

output
```
3/8*arctanh(sin(x))/a+3/8*sec(x)*tan(x)/a+1/4*sec(x)^3*tan(x)/a
```

3.266.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{\sec^3(x)}{a-a\sin^2(x)}\,dx=\frac{\frac{3}{8}\text{arctanh}(\sin(x))+\frac{3}{8}\sec(x)\tan(x)+\frac{1}{4}\sec^3(x)\tan(x)}{a}$$

input
```
Integrate[Sec[x]^3/(a - a*Sin[x]^2),x]
```

output
```
((3*ArcTanh[Sin[x]])/8 + (3*Sec[x]*Tan[x])/8 + (Sec[x]^3*Tan[x])/4)/a
```

3.266.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3654, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(x)}{a-a\sin^2(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\cos(x)^3\left(a-a\sin(x)^2\right)}dx$$
$$\Big\downarrow \text{3654}$$
$$\frac{\int \sec^5(x)dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^5dx}{a}$$
$$\Big\downarrow \text{4255}$$
$$\frac{\frac{3}{4}\int \sec^3(x)dx+\frac{1}{4}\tan(x)\sec^3(x)}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\frac{3}{4}\int \csc\left(x+\frac{\pi}{2}\right)^3dx+\frac{1}{4}\tan(x)\sec^3(x)}{a}$$
$$\Big\downarrow \text{4255}$$
$$\frac{\frac{3}{4}\left(\frac{\int \sec(x)dx}{2}+\frac{1}{2}\tan(x)\sec(x)\right)+\frac{1}{4}\tan(x)\sec^3(x)}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\frac{3}{4}\left(\frac{1}{2}\int \csc\left(x+\frac{\pi}{2}\right)dx+\frac{1}{2}\tan(x)\sec(x)\right)+\frac{1}{4}\tan(x)\sec^3(x)}{a}$$
$$\Big\downarrow \text{4257}$$
$$\frac{\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\sin(x))+\frac{1}{2}\tan(x)\sec(x)\right)+\frac{1}{4}\tan(x)\sec^3(x)}{a}$$

input
```
Int[Sec[x]^3/(a - a*Sin[x]^2),x]
```

output
```
((Sec[x]^3*Tan[x])/4 + (3*(ArcTanh[Sin[x]]/2 + (Sec[x]*Tan[x])/2))/4)/a
```

3.266.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.266.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

method	result	size
parallelrisch	$\frac{3\ln(-\cot(x)+\csc(x)+1)-3\ln(-\cot(x)+\csc(x)-1)+3\sec(x)\tan(x)+2\tan(x)\left(\sec^{3}(x)\right)}{8a}$	43
default	$\frac{-\frac{1}{16(1+\sin(x))^{2}}-\frac{3}{16(1+\sin(x))}+\frac{3\ln(1+\sin(x))}{16}+\frac{1}{16(\sin(x)-1)^{2}}-\frac{3}{16(\sin(x)-1)}-\frac{3\ln(\sin(x)-1)}{16}}{a}$	52
risch	$-\frac{i\left(3\,e^{7ix}+11\,e^{5ix}-11\,e^{3ix}-3\,e^{ix}\right)}{4\left(e^{2ix}+1\right)^{4}a}-\frac{3\ln\left(e^{ix}-i\right)}{8a}+\frac{3\ln\left(e^{ix}+i\right)}{8a}$	74
norman	$\frac{\frac{5\tan\left(\frac{x}{2}\right)}{4a}+\frac{3\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{4a}+\frac{3\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{4a}+\frac{5\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{4a}}{\left(\tan^{2}\left(\frac{x}{2}\right)-1\right)^{4}}-\frac{3\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{8a}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{8a}$	80

input
```
int(sec(x)^3/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(3*ln(-cot(x)+csc(x)+1)-3*ln(-cot(x)+csc(x)-1)+3*sec(x)*tan(x)+2*tan(x
)*sec(x)^3)/a
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \frac{\sec^3(x)}{a-a\sin^2(x)}\,dx = \frac{3\,\cos(x)^4\log(\sin(x)+1)-3\,\cos(x)^4\log(-\sin(x)+1)+2\left(3\,\cos(x)^2+2\right)\sin(x)}{16\,a\cos(x)^4}$$

input
```
integrate(sec(x)^3/(a-a*sin(x)^2),x, algorithm="fricas")
```

output
```
1/16*(3*cos(x)^4*log(sin(x) + 1) - 3*cos(x)^4*log(-sin(x) + 1) + 2*(3*cos(
x)^2 + 2)*sin(x))/(a*cos(x)^4)
```

3.266.6 Sympy [F]

$$\int \frac{\sec^3(x)}{a-a\sin^2(x)}\,dx = -\frac{\int \frac{\sec^3(x)}{\sin^2(x)-1}\,dx}{a}$$

input
```
integrate(sec(x)**3/(a-a*sin(x)**2),x)
```

output
```
-Integral(sec(x)**3/(sin(x)**2 - 1), x)/a
```

3.266.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int \frac{\sec^3(x)}{a-a\sin^2(x)}\,dx = -\frac{3\,\sin(x)^3-5\,\sin(x)}{8\left(a\sin(x)^4-2\,a\sin(x)^2+a\right)} + \frac{3\,\log(\sin(x)+1)}{16\,a} - \frac{3\,\log(\sin(x)-1)}{16\,a}$$

input
```
integrate(sec(x)^3/(a-a*sin(x)^2),x, algorithm="maxima")
```

output
```
-1/8*(3*sin(x)^3 - 5*sin(x))/(a*sin(x)^4 - 2*a*sin(x)^2 + a) + 3/16*log(si
n(x) + 1)/a - 3/16*log(sin(x) - 1)/a
```

3.266.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int \frac{\sec^3(x)}{a-a\sin^2(x)}\,dx = \frac{3\,\log\left(\sin\left(x\right)+1\right)}{16\,a} - \frac{3\,\log\left(-\sin\left(x\right)+1\right)}{16\,a} - \frac{3\,\sin\left(x\right)^3-5\,\sin\left(x\right)}{8\,\left(\sin\left(x\right)^2-1\right)^2a}$$

input
```
integrate(sec(x)^3/(a-a*sin(x)^2),x, algorithm="giac")
```

output
```
3/16*log(sin(x) + 1)/a - 3/16*log(-sin(x) + 1)/a - 1/8*(3*sin(x)^3 - 5*sin
(x))/((sin(x)^2 - 1)^2*a)
```

3.266.9 Mupad [B] (verification not implemented)

Time = 13.60 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{\sec^3(x)}{a-a\sin^2(x)}\,dx = \frac{3\,\mathrm{atanh}(\sin\left(x\right))}{8\,a} + \frac{3\,\sin\left(x\right)}{8\,a\cos\left(x\right)^2} + \frac{\sin\left(x\right)}{4\,a\cos\left(x\right)^4}$$

input
```
int(1/(cos(x)^3*(a - a*sin(x)^2)),x)
```

output
```
(3*atanh(sin(x)))/(8*a) + (3*sin(x))/(8*a*cos(x)^2) + sin(x)/(4*a*cos(x)^4
)
```

3.267 $\int \frac{\cos^6(x)}{a-a\sin^2(x)}\,dx$

3.267.1 Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{\cos^6(x)}{a-a\sin^2(x)}\,dx=\frac{3x}{8a}+\frac{3\cos(x)\sin(x)}{8a}+\frac{\cos^3(x)\sin(x)}{4a}$$

output
```
3/8*x/a+3/8*cos(x)*sin(x)/a+1/4*cos(x)^3*sin(x)/a
```

3.267.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{\cos^6(x)}{a-a\sin^2(x)}\,dx=\frac{\frac{3x}{8}+\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)}{a}$$

input
```
Integrate[Cos[x]^6/(a - a*Sin[x]^2),x]
```

output
```
((3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32)/a
```

3.267.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 3654, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(x)}{a-a\sin^2(x)} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(x)^6}{a-a\sin(x)^2}dx$$
$$\Big\downarrow \text{3654}$$
$$\frac{\int \cos^4(x)dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \sin\left(x+\frac{\pi}{2}\right)^4dx}{a}$$
$$\Big\downarrow \text{3115}$$
$$\frac{\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)}{a}$$
$$\Big\downarrow \text{3115}$$
$$\frac{\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)}{a}$$
$$\Big\downarrow \text{24}$$
$$\frac{\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a}$$

input
```
Int[Cos[x]^6/(a - a*Sin[x]^2),x]
```

output
```
((Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[x]*Sin[x])/2))/4)/a
```

3.267.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.267.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

method	result
parallelrisch	$\frac{12x+\sin(4x)+8\sin(2x)}{32a}$
risch	$\frac{3x}{8a}+\frac{\sin(4x)}{32a}+\frac{\sin(2x)}{4a}$
default	$\frac{\frac{\tan(x)}{4\left(1+\tan^2(x)\right)^2}+\frac{3\tan(x)}{8\left(1+\tan^2(x)\right)}+\frac{3\arctan(\tan(x))}{8}}{a}$
norman	$\frac{\frac{\tan^7\left(\frac{x}{2}\right)}{a}-\frac{5\tan\left(\frac{x}{2}\right)}{4a}-\frac{\tan^3\left(\frac{x}{2}\right)}{2a}+\frac{5\left(\tan^5\left(\frac{x}{2}\right)\right)}{4a}+\frac{5\left(\tan^9\left(\frac{x}{2}\right)\right)}{4a}-\frac{\tan^{11}\left(\frac{x}{2}\right)}{2a}-\frac{5\left(\tan^{13}\left(\frac{x}{2}\right)\right)}{4a}-\frac{3x}{8a}-\frac{15x\left(\tan^2\left(\frac{x}{2}\right)\right)}{8a}-\frac{27x\left(\tan^4\left(\frac{x}{2}\right)\right)}{8a}-\cdots}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6\left(\tan^2\left(\frac{x}{2}\right)-1\right)}$

input
```
int(cos(x)^6/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/32*(12*x+sin(4*x)+8*sin(2*x))/a
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{\cos^6(x)}{a - a\sin^2(x)}\,dx = \frac{(2\,\cos{(x)}^3 + 3\,\cos{(x)})\sin{(x)} + 3\,x}{8\,a}$$

input
```
integrate(cos(x)^6/(a-a*sin(x)^2),x, algorithm="fricas")
```

output
```
1/8*((2*cos(x)^3 + 3*cos(x))*sin(x) + 3*x)/a
```

3.267.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(29) = 58.

Time = 3.75 (sec) , antiderivative size = 473, normalized size of antiderivative = 14.33

$$\begin{aligned}\int \frac{\cos^6(x)}{a - a\sin^2(x)}\,dx &= \frac{3x\tan^8\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right) + 32a\tan^6\left(\frac{x}{2}\right) + 48a\tan^4\left(\frac{x}{2}\right) + 32a\tan^2\left(\frac{x}{2}\right) + 8a} \\
&+ \frac{12x\tan^6\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right) + 32a\tan^6\left(\frac{x}{2}\right) + 48a\tan^4\left(\frac{x}{2}\right) + 32a\tan^2\left(\frac{x}{2}\right) + 8a} \\
&+ \frac{18x\tan^4\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right) + 32a\tan^6\left(\frac{x}{2}\right) + 48a\tan^4\left(\frac{x}{2}\right) + 32a\tan^2\left(\frac{x}{2}\right) + 8a} \\
&+ \frac{12x\tan^2\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right) + 32a\tan^6\left(\frac{x}{2}\right) + 48a\tan^4\left(\frac{x}{2}\right) + 32a\tan^2\left(\frac{x}{2}\right) + 8a} \\
&+ \frac{3x}{8a\tan^8\left(\frac{x}{2}\right) + 32a\tan^6\left(\frac{x}{2}\right) + 48a\tan^4\left(\frac{x}{2}\right) + 32a\tan^2\left(\frac{x}{2}\right) + 8a} \\
&- \frac{10\tan^7\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right) + 32a\tan^6\left(\frac{x}{2}\right) + 48a\tan^4\left(\frac{x}{2}\right) + 32a\tan^2\left(\frac{x}{2}\right) + 8a} \\
&+ \frac{6\tan^5\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right) + 32a\tan^6\left(\frac{x}{2}\right) + 48a\tan^4\left(\frac{x}{2}\right) + 32a\tan^2\left(\frac{x}{2}\right) + 8a} \\
&- \frac{6\tan^3\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right) + 32a\tan^6\left(\frac{x}{2}\right) + 48a\tan^4\left(\frac{x}{2}\right) + 32a\tan^2\left(\frac{x}{2}\right) + 8a} \\
&+ \frac{10\tan\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right) + 32a\tan^6\left(\frac{x}{2}\right) + 48a\tan^4\left(\frac{x}{2}\right) + 32a\tan^2\left(\frac{x}{2}\right) + 8a}\end{aligned}$$

input
```
integrate(cos(x)**6/(a-a*sin(x)**2),x)
```

output
```
3*x*tan(x/2)**8/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 3
2*a*tan(x/2)**2 + 8*a) + 12*x*tan(x/2)**6/(8*a*tan(x/2)**8 + 32*a*tan(x/2)
**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 18*x*tan(x/2)**4/(8*a*t
an(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a)
 + 12*x*tan(x/2)**2/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4
 + 32*a*tan(x/2)**2 + 8*a) + 3*x/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*
a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) - 10*tan(x/2)**7/(8*a*tan(x/2)**8
+ 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 6*tan(x/
2)**5/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/
2)**2 + 8*a) - 6*tan(x/2)**3/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*ta
n(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 10*tan(x/2)/(8*a*tan(x/2)**8 + 32*a*
tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a)
```

3.267.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{\cos^6(x)}{a-a\sin^2(x)}\,dx = \frac{3\,\tan(x)^3+5\,\tan(x)}{8\left(a\tan(x)^4+2\,a\tan(x)^2+a\right)}+\frac{3\,x}{8\,a}$$

input
```
integrate(cos(x)^6/(a-a*sin(x)^2),x, algorithm="maxima")
```

output
```
1/8*(3*tan(x)^3 + 5*tan(x))/(a*tan(x)^4 + 2*a*tan(x)^2 + a) + 3/8*x/a
```

3.267.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{\cos^6(x)}{a-a\sin^2(x)}\,dx = \frac{3\,\arctan(\tan(x))}{8\,a}+\frac{\frac{3\,\tan(x)^3}{a}+\frac{5\,\tan(x)}{a}}{8\left(\tan(x)^2+1\right)^2}$$

input
```
integrate(cos(x)^6/(a-a*sin(x)^2),x, algorithm="giac")
```

output
```
3/8*arctan(tan(x))/a + 1/8*(3*tan(x)^3/a + 5*tan(x)/a)/(tan(x)^2 + 1)^2
```

3.267.9 Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{\cos^6(x)}{a - a\sin^2(x)}\, dx = \frac{\sin(2x)}{4a} + \frac{\sin(4x)}{32a} + \frac{3x}{8a}$$

input
```
int(cos(x)^6/(a - a*sin(x)^2),x)
```

output
```
sin(2*x)/(4*a) + sin(4*x)/(32*a) + (3*x)/(8*a)
```

3.268 $\int \frac{\cos^4(x)}{a-a\sin^2(x)}\,dx$

3.268.1 Optimal result

Integrand size = 16, antiderivative size = 20

$$\int \frac{\cos^4(x)}{a-a\sin^2(x)}\,dx=\frac{x}{2a}+\frac{\cos(x)\sin(x)}{2a}$$

output
```
1/2*x/a+1/2*cos(x)*sin(x)/a
```

3.268.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos^4(x)}{a-a\sin^2(x)}\,dx=\frac{\frac{x}{2}+\frac{1}{4}\sin(2x)}{a}$$

input
```
Integrate[Cos[x]^4/(a - a*Sin[x]^2),x]
```

output
```
(x/2 + Sin[2*x]/4)/a
```

3.268.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 3654, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(x)}{a-a\sin^2(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos(x)^4}{a-a\sin(x)^2}dx$$

$$\Big\downarrow \ 3654$$

$$\frac{\int \cos^2(x)dx}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \sin\left(x+\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \ 3115$$

$$\frac{\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)}{a}$$

$$\Big\downarrow \ 24$$

$$\frac{\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)}{a}$$

input
```
Int[Cos[x]^4/(a - a*Sin[x]^2),x]
```

output
```
(x/2 + (Cos[x]*Sin[x])/2)/a
```

3.268.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.268.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

method	result	size
parallelrisch	$\frac{2x+\sin(2x)}{4a}$	14
risch	$\frac{x}{2a}+\frac{\sin(2x)}{4a}$	17
default	$\frac{\frac{\tan(x)}{2+2\left(\tan^2(x)\right)}+\frac{\arctan(\tan(x))}{2}}{a}$	23
norman	$\frac{\frac{x\left(\tan^6\left(\frac{x}{2}\right)\right)}{a}-\frac{\tan\left(\frac{x}{2}\right)}{a}+\frac{2\left(\tan^5\left(\frac{x}{2}\right)\right)}{a}-\frac{\tan^9\left(\frac{x}{2}\right)}{a}-\frac{x}{2a}-\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2a}-\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}+\frac{3x\left(\tan^8\left(\frac{x}{2}\right)\right)}{2a}+\frac{x\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{2a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4\left(\tan^2\left(\frac{x}{2}\right)-1\right)}$	119

input
```
int(cos(x)^4/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*x+sin(2*x))/a
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{\cos^4(x)}{a - a\sin^2(x)}\, dx = \frac{\cos(x)\sin(x) + x}{2\, a}$$

input
```
integrate(cos(x)^4/(a-a*sin(x)^2),x, algorithm="fricas")
```

output
```
1/2*(cos(x)*sin(x) + x)/a
```

3.268.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(14) = 28.

Time = 1.45 (sec) , antiderivative size = 153, normalized size of antiderivative = 7.65

$$\int \frac{\cos^4(x)}{a - a\sin^2(x)}\, dx = \frac{x\tan^4\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right) + 4a\tan^2\left(\frac{x}{2}\right) + 2a} + \frac{2x\tan^2\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right) + 4a\tan^2\left(\frac{x}{2}\right) + 2a} + \frac{x}{2a\tan^4\left(\frac{x}{2}\right) + 4a\tan^2\left(\frac{x}{2}\right) + 2a} - \frac{2\tan^3\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right) + 4a\tan^2\left(\frac{x}{2}\right) + 2a} + \frac{2\tan\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right) + 4a\tan^2\left(\frac{x}{2}\right) + 2a}$$

input
```
integrate(cos(x)**4/(a-a*sin(x)**2),x)
```

output
```
x*tan(x/2)**4/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) + 2*x*tan(x/2)**2/
(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) + x/(2*a*tan(x/2)**4 + 4*a*tan(x
/2)**2 + 2*a) - 2*tan(x/2)**3/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) +
2*tan(x/2)/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a)
```

3.268.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{\cos^4(x)}{a - a\sin^2(x)}\, dx = \frac{x}{2\,a} + \frac{\tan(x)}{2\left(a\tan(x)^2 + a\right)}$$

input
```
integrate(cos(x)^4/(a-a*sin(x)^2),x, algorithm="maxima")
```

output
```
1/2*x/a + 1/2*tan(x)/(a*tan(x)^2 + a)
```

3.268.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\cos^4(x)}{a - a\sin^2(x)}\, dx = \frac{\arctan(\tan(x))}{2\,a} + \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)a}$$

input
```
integrate(cos(x)^4/(a-a*sin(x)^2),x, algorithm="giac")
```

output
```
1/2*arctan(tan(x))/a + 1/2*tan(x)/((tan(x)^2 + 1)*a)
```

3.268.9 Mupad [B] (verification not implemented)

Time = 13.33 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \frac{\cos^4(x)}{a - a\sin^2(x)}\, dx = \frac{2\,x + \sin(2\,x)}{4\,a}$$

input
```
int(cos(x)^4/(a - a*sin(x)^2),x)
```

output
```
(2*x + sin(2*x))/(4*a)
```

3.269 $\int \frac{\cos^2(x)}{a-a\sin^2(x)}\,dx$

3.269.1 Optimal result

Integrand size = 16, antiderivative size = 5

$$\int \frac{\cos^2(x)}{a-a\sin^2(x)}\,dx=\frac{x}{a}$$

output
```
x/a
```

3.269.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(x)}{a-a\sin^2(x)}\,dx=\frac{x}{a}$$

input
```
Integrate[Cos[x]^2/(a - a*Sin[x]^2),x]
```

output
```
x/a
```

3.269.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 3654, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(x)}{a - a\sin^2(x)}\, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(x)^2}{a - a\sin(x)^2} dx$$
$$\Big\downarrow \text{3654}$$
$$\frac{\int 1 dx}{a}$$
$$\Big\downarrow \text{24}$$
$$\frac{x}{a}$$

input
```
Int[Cos[x]^2/(a - a*Sin[x]^2),x]
```

output
```
x/a
```

3.269.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.269.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
risch	$\frac{x}{a}$	6
default	$\frac{\arctan(\tan(x))}{a}$	8
norman	$\frac{\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}+\frac{x\left(\tan^6\left(\frac{x}{2}\right)\right)}{a}-\frac{x}{a}-\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2\left(\tan^2\left(\frac{x}{2}\right)-1\right)}$	63

input
```
int(cos(x)^2/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
x/a
```

3.269.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(x)}{a-a\sin^2(x)}\,dx=\frac{x}{a}$$

input
```
integrate(cos(x)^2/(a-a*sin(x)^2),x, algorithm="fricas")
```

output
```
x/a
```

3.269.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.40

$$\int \frac{\cos^2(x)}{a-a\sin^2(x)}\,dx=\frac{x}{a}$$

input
```
integrate(cos(x)**2/(a-a*sin(x)**2),x)
```

output
```
x/a
```

3.269.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(x)}{a - a\sin^2(x)}\,dx = \frac{x}{a}$$

input
```
integrate(cos(x)^2/(a-a*sin(x)^2),x, algorithm="maxima")
```

output
```
x/a
```

3.269.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(5) = 10.

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.80

$$\int \frac{\cos^2(x)}{a - a\sin^2(x)}\,dx = \frac{\arctan\left(\frac{|a|\tan(x)}{a}\right)}{|a|}$$

input
```
integrate(cos(x)^2/(a-a*sin(x)^2),x, algorithm="giac")
```

output
```
arctan(abs(a)*tan(x)/a)/abs(a)
```

3.269.9 Mupad [B] (verification not implemented)

Time = 12.92 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(x)}{a - a\sin^2(x)}\,dx = \frac{x}{a}$$

input
```
int(cos(x)^2/(a - a*sin(x)^2),x)
```

output
```
x/a
```

3.270 $\int \frac{\sec(x)}{a-a\sin^2(x)}\,dx$

3.270.1 Optimal result

Integrand size = 14, antiderivative size = 22

$$\int \frac{\sec(x)}{a-a\sin^2(x)}\,dx=\frac{\operatorname{arctanh}(\sin(x))}{2a}+\frac{\sec(x)\tan(x)}{2a}$$

output
```
1/2*arctanh(sin(x))/a+1/2*sec(x)*tan(x)/a
```

3.270.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sec(x)}{a-a\sin^2(x)}\,dx=\frac{\frac{1}{2}\operatorname{arctanh}(\sin(x))+\frac{1}{2}\sec(x)\tan(x)}{a}$$

input
```
Integrate[Sec[x]/(a - a*Sin[x]^2),x]
```

output
```
(ArcTanh[Sin[x]]/2 + (Sec[x]*Tan[x])/2)/a
```

3.270.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3654, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(x)}{a-a\sin^2(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\cos(x)\left(a-a\sin(x)^2\right)}dx$$
$$\Big\downarrow \text{ 3654}$$
$$\frac{\int \sec^3(x)dx}{a}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^3dx}{a}$$
$$\Big\downarrow \text{ 4255}$$
$$\frac{\frac{\int \sec(x)dx}{2}+\frac{1}{2}\tan(x)\sec(x)}{a}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\frac{1}{2}\int \csc\left(x+\frac{\pi}{2}\right)dx+\frac{1}{2}\tan(x)\sec(x)}{a}$$
$$\Big\downarrow \text{ 4257}$$
$$\frac{\frac{1}{2}\text{arctanh}(\sin(x))+\frac{1}{2}\tan(x)\sec(x)}{a}$$

input
```
Int[Sec[x]/(a - a*Sin[x]^2),x]
```

output
```
(ArcTanh[Sin[x]]/2 + (Sec[x]*Tan[x])/2)/a
```

3.270.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.270.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

method	result	size
parallelrisch	$\frac{\sec(x)\tan(x)-\ln(-\cot(x)+\csc(x)-1)+\ln(-\cot(x)+\csc(x)+1)}{2a}$	32
default	$\frac{-\frac{1}{4(1+\sin(x))}+\frac{\ln(1+\sin(x))}{4}-\frac{1}{4(\sin(x)-1)}-\frac{\ln(\sin(x)-1)}{4}}{a}$	36
norman	$\frac{\frac{\tan\left(\frac{x}{2}\right)}{a}+\frac{\tan^3\left(\frac{x}{2}\right)}{a}}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)^2}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{2a}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{2a}$	56
risch	$-\frac{i(e^{3ix}-e^{ix})}{(e^{2ix}+1)^2 a}-\frac{\ln(e^{ix}-i)}{2a}+\frac{\ln(e^{ix}+i)}{2a}$	58

input
```
int(sec(x)/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(sec(x)*tan(x)-ln(-cot(x)+csc(x)-1)+ln(-cot(x)+csc(x)+1))/a
```

3.270.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{\sec(x)}{a - a\sin^2(x)}\,dx = \frac{\cos(x)^2\log(\sin(x)+1) - \cos(x)^2\log(-\sin(x)+1) + 2\sin(x)}{4\,a\cos(x)^2}$$

input
```
integrate(sec(x)/(a-a*sin(x)^2),x, algorithm="fricas")
```

output
```
1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) + 2*sin(x))/(a*c
os(x)^2)
```

3.270.6 Sympy [F]

$$\int \frac{\sec(x)}{a - a\sin^2(x)}\,dx = -\frac{\int \frac{\sec(x)}{\sin^2(x)-1}\,dx}{a}$$

input
```
integrate(sec(x)/(a-a*sin(x)**2),x)
```

output
```
-Integral(sec(x)/(sin(x)**2 - 1), x)/a
```

3.270.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{\sec(x)}{a - a\sin^2(x)}\,dx = \frac{\log(\sin(x)+1)}{4\,a} - \frac{\log(\sin(x)-1)}{4\,a} - \frac{\sin(x)}{2\,(a\sin(x)^2 - a)}$$

input
```
integrate(sec(x)/(a-a*sin(x)^2),x, algorithm="maxima")
```

output
```
1/4*log(sin(x) + 1)/a - 1/4*log(sin(x) - 1)/a - 1/2*sin(x)/(a*sin(x)^2 - a
)
```

3.270.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(18) = 36.

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{\sec(x)}{a - a\sin^2(x)}\,dx = \frac{\log(\sin(x) + 1)}{4\,a} - \frac{\log(-\sin(x) + 1)}{4\,a} - \frac{\sin(x)}{2\left(\sin(x)^2 - 1\right)a}$$

input
```
integrate(sec(x)/(a-a*sin(x)^2),x, algorithm="giac")
```

output
```
1/4*log(sin(x) + 1)/a - 1/4*log(-sin(x) + 1)/a - 1/2*sin(x)/((sin(x)^2 - 1
)*a)
```

3.270.9 Mupad [B] (verification not implemented)

Time = 13.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{\sec(x)}{a - a\sin^2(x)}\,dx = \frac{\operatorname{atanh}(\sin(x))}{2\,a} + \frac{\sin(x)}{2\,\left(a - a\sin(x)^2\right)}$$

input
```
int(1/(cos(x)*(a - a*sin(x)^2)),x)
```

output
```
atanh(sin(x))/(2*a) + sin(x)/(2*(a - a*sin(x)^2))
```

3.271 $\int \frac{\sec^2(x)}{a-a\sin^2(x)}\,dx$

3.271.1 Optimal result

Integrand size = 16, antiderivative size = 18

$$\int \frac{\sec^2(x)}{a-a\sin^2(x)}\,dx=\frac{\tan(x)}{a}+\frac{\tan^3(x)}{3a}$$

output
```
tan(x)/a+1/3*tan(x)^3/a
```

3.271.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\sec^2(x)}{a-a\sin^2(x)}\,dx=\frac{\tan(x)+\frac{\tan^3(x)}{3}}{a}$$

input
```
Integrate[Sec[x]^2/(a - a*Sin[x]^2),x]
```

output
```
(Tan[x] + Tan[x]^3/3)/a
```

3.271.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3042, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{a-a\sin^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(x)^2\left(a-a\sin(x)^2\right)}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sec^4(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^4dx}{a}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int \left(\tan^2(x)+1\right)d(-\tan(x))}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{3}\tan^3(x)-\tan(x)}{a}$$

input
```
Int[Sec[x]^2/(a - a*Sin[x]^2),x]
```

output
```
-((-Tan[x] - Tan[x]^3/3)/a)
```

3.271.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.271.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\frac{\left(\tan^3(x)\right)}{3}+\tan(x)}{a}$	14
parallelrisch	$\frac{\tan(x)\left(2+\sec^2(x)\right)}{3a}$	14
risch	$\frac{4i\left(3\,\mathrm{e}^{2ix}+1\right)}{3\left(\mathrm{e}^{2ix}+1\right)^3 a}$	25
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{4\left(\tan^3\left(\frac{x}{2}\right)\right)}{3a}-\frac{2\left(\tan^5\left(\frac{x}{2}\right)\right)}{a}}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)^3}$	44

input
```
int(sec(x)^2/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/3*tan(x)^3+tan(x))
```

3.271.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\sec^2(x)}{a - a\sin^2(x)}\,dx = \frac{(2\cos(x)^2 + 1)\sin(x)}{3\,a\cos(x)^3}$$

input
```
integrate(sec(x)^2/(a-a*sin(x)^2),x, algorithm="fricas")
```

output
```
1/3*(2*cos(x)^2 + 1)*sin(x)/(a*cos(x)^3)
```

3.271.6 Sympy [F]

$$\int \frac{\sec^2(x)}{a - a\sin^2(x)}\,dx = -\frac{\int \frac{\sec^2(x)}{\sin^2(x)-1}\,dx}{a}$$

input
```
integrate(sec(x)**2/(a-a*sin(x)**2),x)
```

output
```
-Integral(sec(x)**2/(sin(x)**2 - 1), x)/a
```

3.271.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(x)}{a - a\sin^2(x)}\,dx = \frac{\tan(x)^3 + 3\tan(x)}{3\,a}$$

input
```
integrate(sec(x)^2/(a-a*sin(x)^2),x, algorithm="maxima")
```

output
```
1/3*(tan(x)^3 + 3*tan(x))/a
```

3.271.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(x)}{a-a\sin^2(x)}\,dx=\frac{\tan{(x)}^3+3\,\tan{(x)}}{3\,a}$$

input
```
integrate(sec(x)^2/(a-a*sin(x)^2),x, algorithm="giac")
```

output
```
1/3*(tan(x)^3 + 3*tan(x))/a
```

3.271.9 Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{\sec^2(x)}{a-a\sin^2(x)}\,dx=\frac{\tan(x)\,\left(\tan(x)^2+3\right)}{3\,a}$$

input
```
int(1/(cos(x)^2*(a - a*sin(x)^2)),x)
```

output
```
(tan(x)*(tan(x)^2 + 3))/(3*a)
```

3.272 $\int \frac{\sec^4(x)}{a-a\sin^2(x)}\,dx$

3.272.1 Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{\sec^4(x)}{a-a\sin^2(x)}\,dx = \frac{\tan(x)}{a} + \frac{2\tan^3(x)}{3a} + \frac{\tan^5(x)}{5a}$$

output
```
tan(x)/a+2/3*tan(x)^3/a+1/5*tan(x)^5/a
```

3.272.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\sec^4(x)}{a-a\sin^2(x)}\,dx = \frac{\tan(x) + \frac{2\tan^3(x)}{3} + \frac{\tan^5(x)}{5}}{a}$$

input
```
Integrate[Sec[x]^4/(a - a*Sin[x]^2),x]
```

output
```
(Tan[x] + (2*Tan[x]^3)/3 + Tan[x]^5/5)/a
```

3.272.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3042, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(x)}{a-a\sin^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(x)^4\left(a-a\sin(x)^2\right)}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sec^6(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^6dx}{a}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int \left(\tan^4(x)+2\tan^2(x)+1\right)d(-\tan(x))}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{5}\tan^5(x)-\frac{2\tan^3(x)}{3}-\tan(x)}{a}$$

input
```
Int[Sec[x]^4/(a - a*Sin[x]^2),x]
```

output
```
-((-Tan[x] - (2*Tan[x]^3)/3 - Tan[x]^5/5)/a)
```

3.272.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.272.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\frac{\left(\tan^{5}(x)\right)}{5}+\frac{2\left(\tan^{3}(x)\right)}{3}+\tan(x)}{a}$	20
parallelrisch	$\frac{\tan(x)\left(3\left(\sec^{4}(x)\right)+4\left(\sec^{2}(x)\right)+8\right)}{15a}$	22
risch	$\frac{16i\left(10\,\mathrm{e}^{4ix}+5\,\mathrm{e}^{2ix}+1\right)}{15\left(\mathrm{e}^{2ix}+1\right)^{5}a}$	32
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{8\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{3a}-\frac{116\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{15a}+\frac{8\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{3a}-\frac{2\left(\tan^{9}\left(\frac{x}{2}\right)\right)}{a}}{\left(\tan^{2}\left(\frac{x}{2}\right)-1\right)^{5}}$	66

input
```
int(sec(x)^4/(a-a*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/5*tan(x)^5+2/3*tan(x)^3+tan(x))
```

3.272.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{\sec^4(x)}{a - a\sin^2(x)}\, dx = \frac{(8 \cos(x)^4 + 4 \cos(x)^2 + 3) \sin(x)}{15\, a \cos(x)^5}$$

input
```
integrate(sec(x)^4/(a-a*sin(x)^2),x, algorithm="fricas")
```

output
```
1/15*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/(a*cos(x)^5)
```

3.272.6 Sympy [F]

$$\int \frac{\sec^4(x)}{a - a\sin^2(x)}\, dx = -\frac{\int \frac{\sec^4(x)}{\sin^2(x)-1}\, dx}{a}$$

input
```
integrate(sec(x)**4/(a-a*sin(x)**2),x)
```

output
```
-Integral(sec(x)**4/(sin(x)**2 - 1), x)/a
```

3.272.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{\sec^4(x)}{a - a\sin^2(x)}\, dx = \frac{3 \tan(x)^5 + 10 \tan(x)^3 + 15 \tan(x)}{15\, a}$$

input
```
integrate(sec(x)^4/(a-a*sin(x)^2),x, algorithm="maxima")
```

output
```
1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a
```

3.272.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{\sec^4(x)}{a - a\sin^2(x)}\, dx = \frac{3\,\tan\left(x\right)^5 + 10\,\tan\left(x\right)^3 + 15\,\tan\left(x\right)}{15\,a}$$

input
```
integrate(sec(x)^4/(a-a*sin(x)^2),x, algorithm="giac")
```

output
```
1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a
```

3.272.9 Mupad [B] (verification not implemented)

Time = 13.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{\sec^4(x)}{a - a\sin^2(x)}\, dx = \frac{\tan(x)\,\left(3\tan(x)^4 + 10\tan(x)^2 + 15\right)}{15\,a}$$

input
```
int(1/(cos(x)^4*(a - a*sin(x)^2)),x)
```

output
```
(tan(x)*(10*tan(x)^2 + 3*tan(x)^4 + 15))/(15*a)
```

3.273 $\int \frac{\cos^9(x)}{(a-a\sin^2(x))^2}\,dx$

3.273.1 Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{\cos^9(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\sin(x)}{a^2} - \frac{2\sin^3(x)}{3a^2} + \frac{\sin^5(x)}{5a^2}$$

output
```
sin(x)/a^2-2/3*sin(x)^3/a^2+1/5*sin(x)^5/a^2
```

3.273.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\cos^9(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\sin(x) - \frac{2\sin^3(x)}{3} + \frac{\sin^5(x)}{5}}{a^2}$$

input
```
Integrate[Cos[x]^9/(a - a*Sin[x]^2)^2,x]
```

output
```
(Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5)/a^2
```

3.273.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3042, 3654, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^9(x)}{\left(a-a\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^9}{\left(a-a\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \cos^5(x)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(x+\frac{\pi}{2}\right)^5dx}{a^2}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\int \left(\sin^4(x)-2\sin^2(x)+1\right)d(-\sin(x))}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{5}\sin^5(x)+\frac{2\sin^3(x)}{3}-\sin(x)}{a^2}$$

input
```
Int[Cos[x]^9/(a - a*Sin[x]^2)^2,x]
```

output
```
-((-Sin[x] + (2*Sin[x]^3)/3 - Sin[x]^5/5)/a^2)
```

3.273.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.273.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{\frac{\left(\sin^{5}(x)\right)}{5}-\frac{2\left(\sin^{3}(x)\right)}{3}+\sin(x)}{a^{2}}$	20
default	$\frac{\frac{\left(\sin^{5}(x)\right)}{5}-\frac{2\left(\sin^{3}(x)\right)}{3}+\sin(x)}{a^{2}}$	20
parallelrisch	$\frac{150\sin(x)+3\sin(5x)+25\sin(3x)}{240a^{2}}$	23
risch	$\frac{5\sin(x)}{8a^{2}}+\frac{\sin(5x)}{80a^{2}}+\frac{5\sin(3x)}{48a^{2}}$	27

input
```
int(cos(x)^9/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/5*sin(x)^5-2/3*sin(x)^3+sin(x))
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{\cos^9(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\left(3\,\cos\left(x\right)^4+4\,\cos\left(x\right)^2+8\right)\sin\left(x\right)}{15\,a^2}$$

input
```
integrate(cos(x)^9/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)/a^2
```

3.273.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(27) = 54.

Time = 32.28 (sec) , antiderivative size = 362, normalized size of antiderivative = 12.48

$$\int \frac{\cos^9(x)}{(a-a\sin^2(x))^2}\,dx$$

$$= \frac{30\tan^9\left(\frac{x}{2}\right)}{15a^2\tan^{10}\left(\frac{x}{2}\right)+75a^2\tan^8\left(\frac{x}{2}\right)+150a^2\tan^6\left(\frac{x}{2}\right)+150a^2\tan^4\left(\frac{x}{2}\right)+75a^2\tan^2\left(\frac{x}{2}\right)+15a^2}$$
$$+ \frac{40\tan^7\left(\frac{x}{2}\right)}{15a^2\tan^{10}\left(\frac{x}{2}\right)+75a^2\tan^8\left(\frac{x}{2}\right)+150a^2\tan^6\left(\frac{x}{2}\right)+150a^2\tan^4\left(\frac{x}{2}\right)+75a^2\tan^2\left(\frac{x}{2}\right)+15a^2}$$
$$+ \frac{116\tan^5\left(\frac{x}{2}\right)}{15a^2\tan^{10}\left(\frac{x}{2}\right)+75a^2\tan^8\left(\frac{x}{2}\right)+150a^2\tan^6\left(\frac{x}{2}\right)+150a^2\tan^4\left(\frac{x}{2}\right)+75a^2\tan^2\left(\frac{x}{2}\right)+15a^2}$$
$$+ \frac{40\tan^3\left(\frac{x}{2}\right)}{15a^2\tan^{10}\left(\frac{x}{2}\right)+75a^2\tan^8\left(\frac{x}{2}\right)+150a^2\tan^6\left(\frac{x}{2}\right)+150a^2\tan^4\left(\frac{x}{2}\right)+75a^2\tan^2\left(\frac{x}{2}\right)+15a^2}$$
$$+ \frac{30\tan\left(\frac{x}{2}\right)}{15a^2\tan^{10}\left(\frac{x}{2}\right)+75a^2\tan^8\left(\frac{x}{2}\right)+150a^2\tan^6\left(\frac{x}{2}\right)+150a^2\tan^4\left(\frac{x}{2}\right)+75a^2\tan^2\left(\frac{x}{2}\right)+15a^2}$$

input
```
integrate(cos(x)**9/(a-a*sin(x)**2)**2,x)
```

output
```
30*tan(x/2)**9/(15*a**2*tan(x/2)**10 + 75*a**2*tan(x/2)**8 + 150*a**2*tan(
x/2)**6 + 150*a**2*tan(x/2)**4 + 75*a**2*tan(x/2)**2 + 15*a**2) + 40*tan(x
/2)**7/(15*a**2*tan(x/2)**10 + 75*a**2*tan(x/2)**8 + 150*a**2*tan(x/2)**6
+ 150*a**2*tan(x/2)**4 + 75*a**2*tan(x/2)**2 + 15*a**2) + 116*tan(x/2)**5/
(15*a**2*tan(x/2)**10 + 75*a**2*tan(x/2)**8 + 150*a**2*tan(x/2)**6 + 150*a
**2*tan(x/2)**4 + 75*a**2*tan(x/2)**2 + 15*a**2) + 40*tan(x/2)**3/(15*a**2
*tan(x/2)**10 + 75*a**2*tan(x/2)**8 + 150*a**2*tan(x/2)**6 + 150*a**2*tan(
x/2)**4 + 75*a**2*tan(x/2)**2 + 15*a**2) + 30*tan(x/2)/(15*a**2*tan(x/2)**
10 + 75*a**2*tan(x/2)**8 + 150*a**2*tan(x/2)**6 + 150*a**2*tan(x/2)**4 + 7
5*a**2*tan(x/2)**2 + 15*a**2)
```

3.273.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{\cos^9(x)}{(a-a\sin^2(x))^2}\,dx = \frac{3\,\sin{(x)}^5-10\,\sin{(x)}^3+15\,\sin{(x)}}{15\,a^2}$$

input
```
integrate(cos(x)^9/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/15*(3*sin(x)^5 - 10*sin(x)^3 + 15*sin(x))/a^2
```

3.273.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{\cos^9(x)}{(a-a\sin^2(x))^2}\,dx = \frac{3\,\sin{(x)}^5-10\,\sin{(x)}^3+15\,\sin{(x)}}{15\,a^2}$$

input
```
integrate(cos(x)^9/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/15*(3*sin(x)^5 - 10*sin(x)^3 + 15*sin(x))/a^2
```

3.273.9 Mupad [B] (verification not implemented)

Time = 13.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{\cos^9(x)}{\left(a - a\sin^2(x)\right)^2}\, dx = \frac{\frac{\sin(x)^5}{5} - \frac{2\sin(x)^3}{3} + \sin(x)}{a^2}$$

input
```
int(cos(x)^9/(a - a*sin(x)^2)^2,x)
```

output
```
(sin(x) - (2*sin(x)^3)/3 + sin(x)^5/5)/a^2
```

3.274 $\int \frac{\cos^7(x)}{(a-a\sin^2(x))^2}\,dx$

3.274.1 Optimal result

Integrand size = 16, antiderivative size = 18

$$\int \frac{\cos^7(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\sin(x)}{a^2} - \frac{\sin^3(x)}{3a^2}$$

output
```
sin(x)/a^2-1/3*sin(x)^3/a^2
```

3.274.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\cos^7(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\sin(x) - \frac{\sin^3(x)}{3}}{a^2}$$

input
```
Integrate[Cos[x]^7/(a - a*Sin[x]^2)^2,x]
```

output
```
(Sin[x] - Sin[x]^3/3)/a^2
```

3.274.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3042, 3654, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^7(x)}{(a-a\sin^2(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^7}{(a-a\sin(x)^2)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \cos^3(x)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(x+\frac{\pi}{2}\right)^3dx}{a^2}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\int (1-\sin^2(x))\,d(-\sin(x))}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\sin^3(x)}{3}-\sin(x)}{a^2}$$

input
```
Int[Cos[x]^7/(a - a*Sin[x]^2)^2,x]
```

output
```
-((-Sin[x] + Sin[x]^3/3)/a^2)
```

3.274.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.274.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{-\frac{\left(\sin^{3}(x)\right)}{3}+\sin(x)}{a^{2}}$	14
default	$\frac{-\frac{\left(\sin^{3}(x)\right)}{3}+\sin(x)}{a^{2}}$	14
parallelrisch	$\frac{9\sin(x)+\sin(3x)}{12a^{2}}$	15
risch	$\frac{3\sin(x)}{4a^{2}}+\frac{\sin(3x)}{12a^{2}}$	18

input
```
int(cos(x)^7/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/3*sin(x)^3+sin(x))
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{\cos^7(x)}{(a-a\sin^2(x))^2}\,dx = \frac{(\cos(x)^2+2)\sin(x)}{3\,a^2}$$

input
```
integrate(cos(x)^7/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(cos(x)^2 + 2)*sin(x)/a^2
```

3.274.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(15) = 30.

Time = 13.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 8.00

$$\int \frac{\cos^7(x)}{(a-a\sin^2(x))^2}\,dx = \frac{6\tan^5\left(\frac{x}{2}\right)}{3a^2\tan^6\left(\frac{x}{2}\right)+9a^2\tan^4\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+3a^2} + \frac{4\tan^3\left(\frac{x}{2}\right)}{3a^2\tan^6\left(\frac{x}{2}\right)+9a^2\tan^4\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+3a^2} + \frac{6\tan\left(\frac{x}{2}\right)}{3a^2\tan^6\left(\frac{x}{2}\right)+9a^2\tan^4\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+3a^2}$$

input
```
integrate(cos(x)**7/(a-a*sin(x)**2)**2,x)
```

output
```
6*tan(x/2)**5/(3*a**2*tan(x/2)**6 + 9*a**2*tan(x/2)**4 + 9*a**2*tan(x/2)**
2 + 3*a**2) + 4*tan(x/2)**3/(3*a**2*tan(x/2)**6 + 9*a**2*tan(x/2)**4 + 9*a
**2*tan(x/2)**2 + 3*a**2) + 6*tan(x/2)/(3*a**2*tan(x/2)**6 + 9*a**2*tan(x/
2)**4 + 9*a**2*tan(x/2)**2 + 3*a**2)
```

3.274.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\cos^7(x)}{\left(a - a\sin^2(x)\right)^2}\,dx = -\frac{\sin(x)^3 - 3\sin(x)}{3a^2}$$

input
```
integrate(cos(x)^7/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
-1/3*(sin(x)^3 - 3*sin(x))/a^2
```

3.274.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\cos^7(x)}{\left(a - a\sin^2(x)\right)^2}\,dx = -\frac{\sin(x)^3 - 3\sin(x)}{3a^2}$$

input
```
integrate(cos(x)^7/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

output
```
-1/3*(sin(x)^3 - 3*sin(x))/a^2
```

3.274.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\cos^7(x)}{\left(a - a\sin^2(x)\right)^2}\,dx = \frac{3\sin(x) - \sin(x)^3}{3a^2}$$

input
```
int(cos(x)^7/(a - a*sin(x)^2)^2,x)
```

output
```
(3*sin(x) - sin(x)^3)/(3*a^2)
```

3.275 $\int \frac{\cos^5(x)}{(a-a\sin^2(x))^2}\,dx$

3.275.1 Optimal result

Integrand size = 16, antiderivative size = 6

$$\int \frac{\cos^5(x)}{(a-a\sin^2(x))^2}\,dx=\frac{\sin(x)}{a^2}$$

output
```
sin(x)/a^2
```

3.275.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos^5(x)}{(a-a\sin^2(x))^2}\,dx=\frac{\sin(x)}{a^2}$$

input
```
Integrate[Cos[x]^5/(a - a*Sin[x]^2)^2,x]
```

output
```
Sin[x]/a^2
```

3.275.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3654, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(x)}{\left(a-a\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)^5}{\left(a-a\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \text{ 3654}$$

$$\frac{\int \cos(x)dx}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sin\left(x+\frac{\pi}{2}\right)dx}{a^2}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{\sin(x)}{a^2}$$

input
```
Int[Cos[x]^5/(a - a*Sin[x]^2)^2,x]
```

output
```
Sin[x]/a^2
```

3.275.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.275.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	siz
derivativedivides	$\frac{\sin(x)}{a^2}$	7
default	$\frac{\sin(x)}{a^2}$	7
risch	$\frac{\sin(x)}{a^2}$	7
parallelrisch	$\frac{\sin(x)}{a^2}$	7
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}-\frac{2\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}+\frac{6\left(\tan^5\left(\frac{x}{2}\right)\right)}{a}+\frac{6\left(\tan^7\left(\frac{x}{2}\right)\right)}{a}-\frac{6\left(\tan^9\left(\frac{x}{2}\right)\right)}{a}-\frac{6\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{a}+\frac{2\left(\tan^{13}\left(\frac{x}{2}\right)\right)}{a}+\frac{2\left(\tan^{15}\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^5 a\left(\tan^2\left(\frac{x}{2}\right)-1\right)^3}$	11

input
```
int(cos(x)^5/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
sin(x)/a^2
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos^5(x)}{\left(a-a\sin^2(x)\right)^2}\,dx=\frac{\sin(x)}{a^2}$$

input
```
integrate(cos(x)^5/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
sin(x)/a^2
```

3.275.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(5) = 10.

Time = 6.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 3.17

$$\int \frac{\cos^5(x)}{(a-a\sin^2(x))^2}\,dx = \frac{2\tan\left(\frac{x}{2}\right)}{a^2\tan^2\left(\frac{x}{2}\right)+a^2}$$

input
```
integrate(cos(x)**5/(a-a*sin(x)**2)**2,x)
```

output
```
2*tan(x/2)/(a**2*tan(x/2)**2 + a**2)
```

3.275.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos^5(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\sin(x)}{a^2}$$

input
```
integrate(cos(x)^5/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
sin(x)/a^2
```

3.275.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos^5(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\sin(x)}{a^2}$$

input
```
integrate(cos(x)^5/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

output
```
sin(x)/a^2
```

3.275.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos^5(x)}{\left(a - a\sin^2(x)\right)^2}\,dx = \frac{\sin(x)}{a^2}$$

input
```
int(cos(x)^5/(a - a*sin(x)^2)^2,x)
```

output
```
sin(x)/a^2
```

3.276 $\int \frac{\cos^3(x)}{(a-a\sin^2(x))^2}\,dx$

3.276.1 Optimal result

Integrand size = 16, antiderivative size = 7

$$\int \frac{\cos^3(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\operatorname{arctanh}(\sin(x))}{a^2}$$

output
```
arctanh(sin(x))/a^2
```

3.276.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\operatorname{arctanh}(\sin(x))}{a^2}$$

input
```
Integrate[Cos[x]^3/(a - a*Sin[x]^2)^2,x]
```

output
```
ArcTanh[Sin[x]]/a^2
```

3.276.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3654, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(x)}{\left(a-a\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^3}{\left(a-a\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sec(x)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)dx}{a^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\text{arctanh}(\sin(x))}{a^2}$$

input
```
Int[Cos[x]^3/(a - a*Sin[x]^2)^2,x]
```

output
```
ArcTanh[Sin[x]]/a^2
```

3.276.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.276.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{\operatorname{arctanh}(\sin(x))}{a^2}$	8
parallelrisch	$\frac{-\ln(\tan(\frac{x}{2})-1)+\ln(\tan(\frac{x}{2})+1)}{a^2}$	22
norman	$\frac{\ln(\tan(\frac{x}{2})+1)}{a^2}-\frac{\ln(\tan(\frac{x}{2})-1)}{a^2}$	25
risch	$\frac{\ln(e^{ix}+i)}{a^2}-\frac{\ln(e^{ix}-i)}{a^2}$	29

input
```
int(cos(x)^3/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
arctanh(sin(x))/a^2
```

3.276.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(7) = 14.

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.86

$$\int \frac{\cos^3(x)}{\left(a-a\sin^2(x)\right)^2}\,dx=\frac{\log\left(\sin\left(x\right)+1\right)-\log\left(-\sin\left(x\right)+1\right)}{2\,a^2}$$

input
```
integrate(cos(x)^3/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
1/2*(log(sin(x) + 1) - log(-sin(x) + 1))/a^2
```

3.276.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(7) = 14.

Time = 2.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 3.14

$$\int \frac{\cos^3(x)}{\left(a-a\sin^2(x)\right)^2}\,dx=-\frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{a^2}+\frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{a^2}$$

input
```
integrate(cos(x)**3/(a-a*sin(x)**2)**2,x)
```

output
```
-log(tan(x/2) - 1)/a**2 + log(tan(x/2) + 1)/a**2
```

3.276.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(7) = 14.

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 3.00

$$\int \frac{\cos^3(x)}{\left(a-a\sin^2(x)\right)^2}\,dx=\frac{\log\left(\sin\left(x\right)+1\right)}{2\,a^2}-\frac{\log\left(\sin\left(x\right)-1\right)}{2\,a^2}$$

input
```
integrate(cos(x)^3/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/2*log(sin(x) + 1)/a^2 - 1/2*log(sin(x) - 1)/a^2
```

3.276.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(7) = 14.

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 3.29

$$\int \frac{\cos^3(x)}{\left(a-a\sin^2(x)\right)^2}\,dx=\frac{\log\left(\sin\left(x\right)+1\right)}{2\,a^2}-\frac{\log\left(-\sin\left(x\right)+1\right)}{2\,a^2}$$

input
```
integrate(cos(x)^3/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/2*log(sin(x) + 1)/a^2 - 1/2*log(-sin(x) + 1)/a^2
```

3.276.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(x)}{\left(a - a\sin^2(x)\right)^2}\,dx = \frac{\operatorname{atanh}(\sin(x))}{a^2}$$

input
```
int(cos(x)^3/(a - a*sin(x)^2)^2,x)
```

output
```
atanh(sin(x))/a^2
```

3.277 $\int \frac{\cos(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$

3.277.1 Optimal result

Integrand size = 14, antiderivative size = 22

$$\int \frac{\cos(x)}{\left(a-a\sin^2(x)\right)^2}\,dx=\frac{\operatorname{arctanh}(\sin(x))}{2a^2}+\frac{\sec(x)\tan(x)}{2a^2}$$

output
```
1/2*arctanh(sin(x))/a^2+1/2*sec(x)*tan(x)/a^2
```

3.277.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cos(x)}{\left(a-a\sin^2(x)\right)^2}\,dx=\frac{\frac{1}{2}\operatorname{arctanh}(\sin(x))+\frac{1}{2}\sec(x)\tan(x)}{a^2}$$

input
```
Integrate[Cos[x]/(a - a*Sin[x]^2)^2,x]
```

output
```
(ArcTanh[Sin[x]]/2 + (Sec[x]*Tan[x])/2)/a^2
```

3.277.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3654, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{(a - a\sin^2(x))^2} dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)}{(a - a\sin(x)^2)^2}dx$$

$$\Big\downarrow \text{ 3654}$$

$$\frac{\int \sec^3(x)dx}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc\left(x + \frac{\pi}{2}\right)^3 dx}{a^2}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{\int \sec(x)dx}{2} + \frac{1}{2}\tan(x)\sec(x)}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{2}\int \csc\left(x + \frac{\pi}{2}\right) dx + \frac{1}{2}\tan(x)\sec(x)}{a^2}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\frac{1}{2}\text{arctanh}(\sin(x)) + \frac{1}{2}\tan(x)\sec(x)}{a^2}$$

input
```
Int[Cos[x]/(a - a*Sin[x]^2)^2,x]
```

output
```
(ArcTanh[Sin[x]]/2 + (Sec[x]*Tan[x])/2)/a^2
```

3.277.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.277.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

method	result	size
parallelrisch	$\frac{\sec(x)\tan(x)-\ln(-\cot(x)+\csc(x)-1)+\ln(-\cot(x)+\csc(x)+1)}{2a^2}$	32
derivativedivides	$\frac{-\frac{1}{4(1+\sin(x))}+\frac{\ln(1+\sin(x))}{4}-\frac{1}{4(\sin(x)-1)}-\frac{\ln(\sin(x)-1)}{4}}{a^2}$	36
default	$\frac{-\frac{1}{4(1+\sin(x))}+\frac{\ln(1+\sin(x))}{4}-\frac{1}{4(\sin(x)-1)}-\frac{\ln(\sin(x)-1)}{4}}{a^2}$	36
risch	$-\frac{i(e^{3ix}-e^{ix})}{(e^{2ix}+1)^2a^2}-\frac{\ln(e^{ix}-i)}{2a^2}+\frac{\ln(e^{ix}+i)}{2a^2}$	58
norman	$\frac{\frac{\tan^5(\frac{x}{2})}{a}+\frac{\tan^7(\frac{x}{2})}{a}-\frac{\tan(\frac{x}{2})}{a}-\frac{\tan^3(\frac{x}{2})}{a}}{(1+\tan^2(\frac{x}{2}))a(\tan^2(\frac{x}{2})-1)^3}-\frac{\ln(\tan(\frac{x}{2})-1)}{2a^2}+\frac{\ln(\tan(\frac{x}{2})+1)}{2a^2}$	91

input
```
int(cos(x)/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(sec(x)*tan(x)-ln(-cot(x)+csc(x)-1)+ln(-cot(x)+csc(x)+1))/a^2
```

3.277.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{\cos(x)}{(a - a\sin^2(x))^2}\,dx = \frac{\cos(x)^2\log(\sin(x)+1) - \cos(x)^2\log(-\sin(x)+1) + 2\sin(x)}{4\,a^2\cos(x)^2}$$

input
```
integrate(cos(x)/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) + 2*sin(x))/(a^2
*cos(x)^2)
```

3.277.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(20) = 40$.

Time = 0.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 5.32

$$\int \frac{\cos(x)}{(a - a\sin^2(x))^2}\,dx = -\frac{\log(\sin(x)-1)\sin^2(x)}{4a^2\sin^2(x)-4a^2} + \frac{\log(\sin(x)-1)}{4a^2\sin^2(x)-4a^2} + \frac{\log(\sin(x)+1)\sin^2(x)}{4a^2\sin^2(x)-4a^2} - \frac{\log(\sin(x)+1)}{4a^2\sin^2(x)-4a^2} - \frac{2\sin(x)}{4a^2\sin^2(x)-4a^2}$$

input
```
integrate(cos(x)/(a-a*sin(x)**2)**2,x)
```

output
```
-log(sin(x) - 1)*sin(x)**2/(4*a**2*sin(x)**2 - 4*a**2) + log(sin(x) - 1)/(
4*a**2*sin(x)**2 - 4*a**2) + log(sin(x) + 1)*sin(x)**2/(4*a**2*sin(x)**2 -
 4*a**2) - log(sin(x) + 1)/(4*a**2*sin(x)**2 - 4*a**2) - 2*sin(x)/(4*a**2*
sin(x)**2 - 4*a**2)
```

3.277.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(18) = 36.

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \frac{\cos(x)}{(a-a\sin^2(x))^2}\,dx = -\frac{\sin(x)}{2\left(a^2\sin(x)^2-a^2\right)}+\frac{\log(\sin(x)+1)}{4\,a^2}-\frac{\log(\sin(x)-1)}{4\,a^2}$$

input
```
integrate(cos(x)/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*sin(x)/(a^2*sin(x)^2 - a^2) + 1/4*log(sin(x) + 1)/a^2 - 1/4*log(sin(x
) - 1)/a^2
```

3.277.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(18) = 36.

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{\cos(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\log(\sin(x)+1)}{4\,a^2}-\frac{\log(-\sin(x)+1)}{4\,a^2}-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)a^2}$$

input
```
integrate(cos(x)/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/4*log(sin(x) + 1)/a^2 - 1/4*log(-sin(x) + 1)/a^2 - 1/2*sin(x)/((sin(x)^2
 - 1)*a^2)
```

3.277.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{\cos(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\operatorname{atanh}(\sin(x))}{2\,a^2}-\frac{\sin(x)}{2\,\left(a^2\sin(x)^2-a^2\right)}$$

input
```
int(cos(x)/(a - a*sin(x)^2)^2,x)
```

output
```
atanh(sin(x))/(2*a^2) - sin(x)/(2*(a^2*sin(x)^2 - a^2))
```

3.278 $\int \frac{\sec(x)}{(a-a\sin^2(x))^2}\,dx$

3.278.1 Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{\sec(x)}{(a-a\sin^2(x))^2}\,dx=\frac{3\text{arctanh}(\sin(x))}{8a^2}+\frac{3\sec(x)\tan(x)}{8a^2}+\frac{\sec^3(x)\tan(x)}{4a^2}$$

output
```
3/8*arctanh(sin(x))/a^2+3/8*sec(x)*tan(x)/a^2+1/4*sec(x)^3*tan(x)/a^2
```

3.278.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{\sec(x)}{(a-a\sin^2(x))^2}\,dx=\frac{\frac{3}{8}\text{arctanh}(\sin(x))+\frac{3}{8}\sec(x)\tan(x)+\frac{1}{4}\sec^3(x)\tan(x)}{a^2}$$

input
```
Integrate[Sec[x]/(a - a*Sin[x]^2)^2,x]
```

output
```
((3*ArcTanh[Sin[x]])/8 + (3*Sec[x]*Tan[x])/8 + (Sec[x]^3*Tan[x])/4)/a^2
```

3.278.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3654, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(x)}{(a-a\sin^2(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(x)\,(a-a\sin(x)^2)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sec^5(x)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^5dx}{a^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{3}{4}\int \sec^3(x)dx+\frac{1}{4}\tan(x)\sec^3(x)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\int \csc\left(x+\frac{\pi}{2}\right)^3dx+\frac{1}{4}\tan(x)\sec^3(x)}{a^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{3}{4}\left(\frac{\int \sec(x)dx}{2}+\frac{1}{2}\tan(x)\sec(x)\right)+\frac{1}{4}\tan(x)\sec^3(x)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int \csc\left(x+\frac{\pi}{2}\right)dx+\frac{1}{2}\tan(x)\sec(x)\right)+\frac{1}{4}\tan(x)\sec^3(x)}{a^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\sin(x))+\frac{1}{2}\tan(x)\sec(x)\right)+\frac{1}{4}\tan(x)\sec^3(x)}{a^2}$$

input
```
Int[Sec[x]/(a - a*Sin[x]^2)^2,x]
```

output
```
((Sec[x]^3*Tan[x])/4 + (3*(ArcTanh[Sin[x]]/2 + (Sec[x]*Tan[x])/2))/4)/a^2
```

3.278.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.278.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

method	result	size
parallelrisch	$\frac{3\ln(-\cot(x)+\csc(x)+1)-3\ln(-\cot(x)+\csc(x)-1)+3\sec(x)\tan(x)+2\tan(x)\left(\sec^3(x)\right)}{8a^2}$	43
default	$\frac{-\frac{1}{16(1+\sin(x))^2}-\frac{3}{16(1+\sin(x))}+\frac{3\ln(1+\sin(x))}{16}+\frac{1}{16(\sin(x)-1)^2}-\frac{3}{16(\sin(x)-1)}-\frac{3\ln(\sin(x)-1)}{16}}{a^2}$	52
risch	$-\frac{i\left(3\,\mathrm{e}^{7ix}+11\,\mathrm{e}^{5ix}-11\,\mathrm{e}^{3ix}-3\,\mathrm{e}^{ix}\right)}{4\left(\mathrm{e}^{2ix}+1\right)^4a^2}-\frac{3\ln(\mathrm{e}^{ix}-i)}{8a^2}+\frac{3\ln(\mathrm{e}^{ix}+i)}{8a^2}$	74
norman	$\frac{\frac{5\tan\left(\frac{x}{2}\right)}{4a}+\frac{3\left(\tan^3\left(\frac{x}{2}\right)\right)}{4a}+\frac{3\left(\tan^5\left(\frac{x}{2}\right)\right)}{4a}+\frac{5\left(\tan^7\left(\frac{x}{2}\right)\right)}{4a}}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)^4a}-\frac{3\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{8a^2}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{8a^2}$	83

input
```
int(sec(x)/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(3*ln(-cot(x)+csc(x)+1)-3*ln(-cot(x)+csc(x)-1)+3*sec(x)*tan(x)+2*tan(x
)*sec(x)^3)/a^2
```

3.278.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \frac{\sec(x)}{(a-a\sin^2(x))^2}\,dx = \frac{3\cos(x)^4\log(\sin(x)+1)-3\cos(x)^4\log(-\sin(x)+1)+2\left(3\cos(x)^2+2\right)\sin(x)}{16\,a^2\cos(x)^4}$$

input
```
integrate(sec(x)/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
1/16*(3*cos(x)^4*log(sin(x) + 1) - 3*cos(x)^4*log(-sin(x) + 1) + 2*(3*cos(
x)^2 + 2)*sin(x))/(a^2*cos(x)^4)
```

3.278.6 Sympy [F]

$$\int \frac{\sec(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\int \frac{\sec(x)}{\sin^4(x)-2\sin^2(x)+1}\,dx}{a^2}$$

input
```
integrate(sec(x)/(a-a*sin(x)**2)**2,x)
```

output
```
Integral(sec(x)/(sin(x)**4 - 2*sin(x)**2 + 1), x)/a**2
```

3.278.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.63

$$\int \frac{\sec(x)}{(a-a\sin^2(x))^2}\,dx = -\frac{3\sin(x)^3-5\sin(x)}{8\left(a^2\sin(x)^4-2a^2\sin(x)^2+a^2\right)} + \frac{3\log(\sin(x)+1)}{16\,a^2} - \frac{3\log(\sin(x)-1)}{16\,a^2}$$

input
```
integrate(sec(x)/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
-1/8*(3*sin(x)^3 - 5*sin(x))/(a^2*sin(x)^4 - 2*a^2*sin(x)^2 + a^2) + 3/16*
log(sin(x) + 1)/a^2 - 3/16*log(sin(x) - 1)/a^2
```

3.278.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int \frac{\sec(x)}{(a-a\sin^2(x))^2}\,dx = \frac{3\,\log\left(\sin\left(x\right)+1\right)}{16\,a^2} - \frac{3\,\log\left(-\sin\left(x\right)+1\right)}{16\,a^2} - \frac{3\,\sin\left(x\right)^3-5\,\sin\left(x\right)}{8\,\left(\sin\left(x\right)^2-1\right)^2a^2}$$

input
```
integrate(sec(x)/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

output
```
3/16*log(sin(x) + 1)/a^2 - 3/16*log(-sin(x) + 1)/a^2 - 1/8*(3*sin(x)^3 - 5
*sin(x))/((sin(x)^2 - 1)^2*a^2)
```

3.278.9 Mupad [B] (verification not implemented)

Time = 13.47 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{\sec(x)}{(a-a\sin^2(x))^2}\,dx = \frac{3\,\mathrm{atanh}(\sin\left(x\right))}{8\,a^2} + \frac{3\,\sin\left(x\right)}{8\,a^2\cos\left(x\right)^2} + \frac{\sin\left(x\right)}{4\,a^2\cos\left(x\right)^4}$$

input
```
int(1/(cos(x)*(a - a*sin(x)^2)^2),x)
```

output
```
(3*atanh(sin(x)))/(8*a^2) + (3*sin(x))/(8*a^2*cos(x)^2) + sin(x)/(4*a^2*co
s(x)^4)
```

3.279 $\int \frac{\cos^8(x)}{(a-a\sin^2(x))^2}\,dx$

3.279.1 Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{\cos^8(x)}{(a-a\sin^2(x))^2}\,dx=\frac{3x}{8a^2}+\frac{3\cos(x)\sin(x)}{8a^2}+\frac{\cos^3(x)\sin(x)}{4a^2}$$

output
```
3/8*x/a^2+3/8*cos(x)*sin(x)/a^2+1/4*cos(x)^3*sin(x)/a^2
```

3.279.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{\cos^8(x)}{(a-a\sin^2(x))^2}\,dx=\frac{\frac{3x}{8}+\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)}{a^2}$$

input
```
Integrate[Cos[x]^8/(a - a*Sin[x]^2)^2,x]
```

output
```
((3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32)/a^2
```

3.279.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 3654, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^8(x)}{\left(a-a\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^8}{\left(a-a\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \cos^4(x)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(x+\frac{\pi}{2}\right)^4dx}{a^2}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)}{a^2}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)}{a^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2}$$

input
```
Int[Cos[x]^8/(a - a*Sin[x]^2)^2,x]
```

3.279. $\int \frac{\cos^8(x)}{\left(a-a\sin^2(x)\right)^2}dx$

output
```
((Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[x]*Sin[x])/2))/4)/a^2
```

3.279.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.279.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

method	result	size
parallelrisch	$\frac{12x+\sin(4x)+8\sin(2x)}{32a^2}$	20
risch	$\frac{3x}{8a^2}+\frac{\sin(4x)}{32a^2}+\frac{\sin(2x)}{4a^2}$	26
default	$\frac{\frac{\tan(x)}{4\left(1+\tan^2(x)\right)^2}+\frac{3\tan(x)}{8\left(1+\tan^2(x)\right)}+\frac{3\arctan(\tan(x))}{8}}{a^2}$	35

input
```
int(cos(x)^8/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*(12*x+sin(4*x)+8*sin(2*x))/a^2
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{\cos^8(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\left(2\cos(x)^3+3\cos(x)\right)\sin(x)+3x}{8a^2}$$

input
```
integrate(cos(x)^8/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
1/8*((2*cos(x)^3 + 3*cos(x))*sin(x) + 3*x)/a^2
```

3.279.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. 2(34) = 68.

Time = 20.78 (sec) , antiderivative size = 549, normalized size of antiderivative = 16.64

$$\begin{aligned}
&\int \frac{\cos^8(x)}{(a-a\sin^2(x))^2}\,dx\\
&= \frac{3x\tan^8\left(\frac{x}{2}\right)}{8a^2\tan^8\left(\frac{x}{2}\right)+32a^2\tan^6\left(\frac{x}{2}\right)+48a^2\tan^4\left(\frac{x}{2}\right)+32a^2\tan^2\left(\frac{x}{2}\right)+8a^2}\\
&\quad+\frac{12x\tan^6\left(\frac{x}{2}\right)}{8a^2\tan^8\left(\frac{x}{2}\right)+32a^2\tan^6\left(\frac{x}{2}\right)+48a^2\tan^4\left(\frac{x}{2}\right)+32a^2\tan^2\left(\frac{x}{2}\right)+8a^2}\\
&\quad+\frac{18x\tan^4\left(\frac{x}{2}\right)}{8a^2\tan^8\left(\frac{x}{2}\right)+32a^2\tan^6\left(\frac{x}{2}\right)+48a^2\tan^4\left(\frac{x}{2}\right)+32a^2\tan^2\left(\frac{x}{2}\right)+8a^2}\\
&\quad+\frac{12x\tan^2\left(\frac{x}{2}\right)}{8a^2\tan^8\left(\frac{x}{2}\right)+32a^2\tan^6\left(\frac{x}{2}\right)+48a^2\tan^4\left(\frac{x}{2}\right)+32a^2\tan^2\left(\frac{x}{2}\right)+8a^2}\\
&\quad+\frac{3x}{8a^2\tan^8\left(\frac{x}{2}\right)+32a^2\tan^6\left(\frac{x}{2}\right)+48a^2\tan^4\left(\frac{x}{2}\right)+32a^2\tan^2\left(\frac{x}{2}\right)+8a^2}\\
&\quad-\frac{10\tan^7\left(\frac{x}{2}\right)}{8a^2\tan^8\left(\frac{x}{2}\right)+32a^2\tan^6\left(\frac{x}{2}\right)+48a^2\tan^4\left(\frac{x}{2}\right)+32a^2\tan^2\left(\frac{x}{2}\right)+8a^2}\\
&\quad+\frac{6\tan^5\left(\frac{x}{2}\right)}{8a^2\tan^8\left(\frac{x}{2}\right)+32a^2\tan^6\left(\frac{x}{2}\right)+48a^2\tan^4\left(\frac{x}{2}\right)+32a^2\tan^2\left(\frac{x}{2}\right)+8a^2}\\
&\quad-\frac{6\tan^3\left(\frac{x}{2}\right)}{8a^2\tan^8\left(\frac{x}{2}\right)+32a^2\tan^6\left(\frac{x}{2}\right)+48a^2\tan^4\left(\frac{x}{2}\right)+32a^2\tan^2\left(\frac{x}{2}\right)+8a^2}\\
&\quad+\frac{10\tan\left(\frac{x}{2}\right)}{8a^2\tan^8\left(\frac{x}{2}\right)+32a^2\tan^6\left(\frac{x}{2}\right)+48a^2\tan^4\left(\frac{x}{2}\right)+32a^2\tan^2\left(\frac{x}{2}\right)+8a^2}
\end{aligned}$$

input
```
integrate(cos(x)**8/(a-a*sin(x)**2)**2,x)
```

output
```
3*x*tan(x/2)**8/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/
2)**4 + 32*a**2*tan(x/2)**2 + 8*a**2) + 12*x*tan(x/2)**6/(8*a**2*tan(x/2)*
*8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 + 32*a**2*tan(x/2)**2 + 8*a
**2) + 18*x*tan(x/2)**4/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**
2*tan(x/2)**4 + 32*a**2*tan(x/2)**2 + 8*a**2) + 12*x*tan(x/2)**2/(8*a**2*t
an(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 + 32*a**2*tan(x/2)*
*2 + 8*a**2) + 3*x/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan
(x/2)**4 + 32*a**2*tan(x/2)**2 + 8*a**2) - 10*tan(x/2)**7/(8*a**2*tan(x/2)
**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 + 32*a**2*tan(x/2)**2 + 8*
a**2) + 6*tan(x/2)**5/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*
tan(x/2)**4 + 32*a**2*tan(x/2)**2 + 8*a**2) - 6*tan(x/2)**3/(8*a**2*tan(x/
2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 + 32*a**2*tan(x/2)**2 +
8*a**2) + 10*tan(x/2)/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*
tan(x/2)**4 + 32*a**2*tan(x/2)**2 + 8*a**2)
```

3.279.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \frac{\cos^8(x)}{(a-a\sin^2(x))^2}\,dx = \frac{3\,\tan(x)^3+5\,\tan(x)}{8\,(a^2\tan(x)^4+2\,a^2\tan(x)^2+a^2)}+\frac{3\,x}{8\,a^2}$$

input
```
integrate(cos(x)^8/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/8*(3*tan(x)^3 + 5*tan(x))/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 + a^2) + 3/8*x/
a^2
```

3.279.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{\cos^8(x)}{(a-a\sin^2(x))^2}\,dx = \frac{3\,x}{8\,a^2}+\frac{3\,\tan(x)^3+5\,\tan(x)}{8\,(\tan(x)^2+1)^2a^2}$$

input
```
integrate(cos(x)^8/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

output
```
3/8*x/a^2 + 1/8*(3*tan(x)^3 + 5*tan(x))/((tan(x)^2 + 1)^2*a^2)
```

3.279.9 Mupad [B] (verification not implemented)

Time = 13.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{\cos^8(x)}{(a-a\sin^2(x))^2}\,dx = \frac{3x}{8a^2} + \frac{3\cos(x)\sin(x)^3}{8a^2} + \frac{5\cos(x)^3\sin(x)}{8a^2}$$

input
```
int(cos(x)^8/(a - a*sin(x)^2)^2,x)
```

output
```
(3*x)/(8*a^2) + (3*cos(x)*sin(x)^3)/(8*a^2) + (5*cos(x)^3*sin(x))/(8*a^2)
```

3.280 $\int \frac{\cos^6(x)}{(a-a\sin^2(x))^2}\,dx$

3.280.1 Optimal result

Integrand size = 16, antiderivative size = 20

$$\int \frac{\cos^6(x)}{(a-a\sin^2(x))^2}\,dx = \frac{x}{2a^2}+\frac{\cos(x)\sin(x)}{2a^2}$$

output
```
1/2*x/a^2+1/2*cos(x)*sin(x)/a^2
```

3.280.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\cos^6(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\frac{x}{2}+\frac{1}{4}\sin(2x)}{a^2}$$

input
```
Integrate[Cos[x]^6/(a - a*Sin[x]^2)^2,x]
```

output
```
(x/2 + Sin[2*x]/4)/a^2
```

3.280.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3042, 3654, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(x)}{\left(a-a\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)^6}{\left(a-a\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \text{ 3654}$$

$$\frac{\int \cos^2(x)dx}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)}{a^2}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)}{a^2}$$

input
```
Int[Cos[x]^6/(a - a*Sin[x]^2)^2,x]
```

output
```
(x/2 + (Cos[x]*Sin[x])/2)/a^2
```

3.280.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.280.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

method	result	size
parallelrisch	$\frac{2x+\sin(2x)}{4a^2}$	14
risch	$\frac{x}{2a^2}+\frac{\sin(2x)}{4a^2}$	17
default	$\frac{\frac{\tan(x)}{2+2\left(\tan^2(x)\right)}+\frac{\arctan(\tan(x))}{2}}{a^2}$	23

input
```
int(cos(x)^6/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*x+sin(2*x))/a^2
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{\cos^6(x)}{\left(a - a\sin^2(x)\right)^2}\,dx = \frac{\cos(x)\sin(x) + x}{2\,a^2}$$

input
```
integrate(cos(x)^6/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
1/2*(cos(x)*sin(x) + x)/a^2
```

3.280.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(17) = 34.

Time = 9.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 8.90

$$\begin{aligned}\int \frac{\cos^6(x)}{\left(a - a\sin^2(x)\right)^2}\,dx &= \frac{x\tan^4\left(\frac{x}{2}\right)}{2a^2\tan^4\left(\frac{x}{2}\right) + 4a^2\tan^2\left(\frac{x}{2}\right) + 2a^2} \\ &+ \frac{2x\tan^2\left(\frac{x}{2}\right)}{2a^2\tan^4\left(\frac{x}{2}\right) + 4a^2\tan^2\left(\frac{x}{2}\right) + 2a^2} \\ &+ \frac{x}{2a^2\tan^4\left(\frac{x}{2}\right) + 4a^2\tan^2\left(\frac{x}{2}\right) + 2a^2} \\ &- \frac{2\tan^3\left(\frac{x}{2}\right)}{2a^2\tan^4\left(\frac{x}{2}\right) + 4a^2\tan^2\left(\frac{x}{2}\right) + 2a^2} \\ &+ \frac{2\tan\left(\frac{x}{2}\right)}{2a^2\tan^4\left(\frac{x}{2}\right) + 4a^2\tan^2\left(\frac{x}{2}\right) + 2a^2}\end{aligned}$$

input
```
integrate(cos(x)**6/(a-a*sin(x)**2)**2,x)
```

output
```
x*tan(x/2)**4/(2*a**2*tan(x/2)**4 + 4*a**2*tan(x/2)**2 + 2*a**2) + 2*x*tan
(x/2)**2/(2*a**2*tan(x/2)**4 + 4*a**2*tan(x/2)**2 + 2*a**2) + x/(2*a**2*ta
n(x/2)**4 + 4*a**2*tan(x/2)**2 + 2*a**2) - 2*tan(x/2)**3/(2*a**2*tan(x/2)*
*4 + 4*a**2*tan(x/2)**2 + 2*a**2) + 2*tan(x/2)/(2*a**2*tan(x/2)**4 + 4*a**
2*tan(x/2)**2 + 2*a**2)
```

3.280.7 Maxima [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{\cos^6(x)}{\left(a-a\sin^2(x)\right)^2}\,dx=\frac{\tan(x)}{2\left(a^2\tan(x)^2+a^2\right)}+\frac{x}{2\,a^2}$$

input
```
integrate(cos(x)^6/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/2*tan(x)/(a^2*tan(x)^2 + a^2) + 1/2*x/a^2
```

3.280.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos^6(x)}{\left(a-a\sin^2(x)\right)^2}\,dx=\frac{x}{2\,a^2}+\frac{\tan(x)}{2\left(\tan(x)^2+1\right)a^2}$$

input
```
integrate(cos(x)^6/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/2*x/a^2 + 1/2*tan(x)/((tan(x)^2 + 1)*a^2)
```

3.280.9 Mupad [B] (verification not implemented)

Time = 13.59 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \frac{\cos^6(x)}{\left(a-a\sin^2(x)\right)^2}\,dx=\frac{2\,x+\sin(2\,x)}{4\,a^2}$$

input
```
int(cos(x)^6/(a - a*sin(x)^2)^2,x)
```

output
```
(2*x + sin(2*x))/(4*a^2)
```

3.281 $\int \frac{\cos^4(x)}{(a-a\sin^2(x))^2}\,dx$

3.281.1 Optimal result

Integrand size = 16, antiderivative size = 5

$$\int \frac{\cos^4(x)}{(a-a\sin^2(x))^2}\,dx = \frac{x}{a^2}$$

output
```
x/a^2
```

3.281.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\cos^4(x)}{(a-a\sin^2(x))^2}\,dx = \frac{x}{a^2}$$

input
```
Integrate[Cos[x]^4/(a - a*Sin[x]^2)^2,x]
```

output
```
x/a^2
```

3.281.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 3654, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(x)}{\left(a-a\sin^2(x)\right)^2}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(x)^4}{\left(a-a\sin(x)^2\right)^2}dx$$
$$\Big\downarrow \text{ 3654}$$
$$\frac{\int 1dx}{a^2}$$
$$\Big\downarrow \text{ 24}$$
$$\frac{x}{a^2}$$

input
```
Int[Cos[x]^4/(a - a*Sin[x]^2)^2,x]
```

output
```
x/a^2
```

3.281.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.281.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
risch	$\frac{x}{a^2}$	6
default	$\frac{\arctan(\tan(x))}{a^2}$	8
norman	$\frac{\frac{x\left(\tan^{12}\left(\frac{x}{2}\right)\right)}{a}+\frac{x\left(\tan^{14}\left(\frac{x}{2}\right)\right)}{a}-\frac{x}{a}-\frac{x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{a}+\frac{3x\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{a}+\frac{3x\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{a}-\frac{3x\left(\tan^{8}\left(\frac{x}{2}\right)\right)}{a}-\frac{3x\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4 a\left(\tan^2\left(\frac{x}{2}\right)-1\right)^3}$	114

input
```
int(cos(x)^4/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x/a^2
```

3.281.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\cos^4(x)}{\left(a-a\sin^2(x)\right)^2}\,dx = \frac{x}{a^2}$$

input
```
integrate(cos(x)^4/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
x/a^2
```

3.281.6 Sympy [A] (verification not implemented)

Time = 3.78 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int \frac{\cos^4(x)}{\left(a-a\sin^2(x)\right)^2}\,dx = \frac{x}{a^2}$$

input
```
integrate(cos(x)**4/(a-a*sin(x)**2)**2,x)
```

output
```
x/a**2
```

3.281.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\cos^4(x)}{(a - a\sin^2(x))^2}\,dx = \frac{x}{a^2}$$

input
```
integrate(cos(x)^4/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
x/a^2
```

3.281.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\cos^4(x)}{(a - a\sin^2(x))^2}\,dx = \frac{x}{a^2}$$

input
```
integrate(cos(x)^4/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

output
```
x/a^2
```

3.281.9 Mupad [B] (verification not implemented)

Time = 13.95 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\cos^4(x)}{(a - a\sin^2(x))^2}\,dx = \frac{x}{a^2}$$

input
```
int(cos(x)^4/(a - a*sin(x)^2)^2,x)
```

output
```
x/a^2
```

3.282 $\int \frac{\cos^2(x)}{(a-a\sin^2(x))^2} dx$

3.282.1 Optimal result

Integrand size = 16, antiderivative size = 6

$$\int \frac{\cos^2(x)}{(a-a\sin^2(x))^2} dx = \frac{\tan(x)}{a^2}$$

output
```
tan(x)/a^2
```

3.282.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(x)}{(a-a\sin^2(x))^2} dx = \frac{\tan(x)}{a^2}$$

input
```
Integrate[Cos[x]^2/(a - a*Sin[x]^2)^2,x]
```

output
```
Tan[x]/a^2
```

3.282.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3042, 3654, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(x)}{\left(a-a\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^2}{\left(a-a\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sec^2(x)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{a^2}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d(-\tan(x))}{a^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan(x)}{a^2}$$

input
```
Int[Cos[x]^2/(a - a*Sin[x]^2)^2,x]
```

output
```
Tan[x]/a^2
```

3.282.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.282.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{\tan(x)}{a^2}$	7
parallelrisch	$\frac{\tan(x)}{a^2}$	7
risch	$\frac{2i}{a^2(\mathrm{e}^{2ix}+1)}$	16
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{4\left(\tan^5\left(\frac{x}{2}\right)\right)}{a}-\frac{2\left(\tan^9\left(\frac{x}{2}\right)\right)}{a}}{a\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2\left(\tan^2\left(\frac{x}{2}\right)-1\right)^3}$	57

input
```
int(cos(x)^2/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
tan(x)/a^2
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.67

$$\int \frac{\cos^2(x)}{(a - a\sin^2(x))^2}\, dx = \frac{\sin(x)}{a^2\cos(x)}$$

input
```
integrate(cos(x)^2/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
sin(x)/(a^2*cos(x))
```

3.282.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(5) = 10.

Time = 1.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 3.33

$$\int \frac{\cos^2(x)}{(a - a\sin^2(x))^2}\, dx = -\frac{2\tan\left(\frac{x}{2}\right)}{a^2\tan^2\left(\frac{x}{2}\right) - a^2}$$

input
```
integrate(cos(x)**2/(a-a*sin(x)**2)**2,x)
```

output
```
-2*tan(x/2)/(a**2*tan(x/2)**2 - a**2)
```

3.282.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(x)}{(a - a\sin^2(x))^2}\, dx = \frac{\tan(x)}{a^2}$$

input
```
integrate(cos(x)^2/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
tan(x)/a^2
```

3.282.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(x)}{(a - a\sin^2(x))^2}\,dx = \frac{\tan(x)}{a^2}$$

input
```
integrate(cos(x)^2/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

output
```
tan(x)/a^2
```

3.282.9 Mupad [B] (verification not implemented)

Time = 13.89 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(x)}{(a - a\sin^2(x))^2}\,dx = \frac{\tan(x)}{a^2}$$

input
```
int(cos(x)^2/(a - a*sin(x)^2)^2,x)
```

output
```
tan(x)/a^2
```

3.283 $\int \frac{\sec^2(x)}{(a-a\sin^2(x))^2}\,dx$

3.283.1 Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{\sec^2(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\tan(x)}{a^2} + \frac{2\tan^3(x)}{3a^2} + \frac{\tan^5(x)}{5a^2}$$

output
```
tan(x)/a^2+2/3*tan(x)^3/a^2+1/5*tan(x)^5/a^2
```

3.283.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\sec^2(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\tan(x) + \frac{2\tan^3(x)}{3} + \frac{\tan^5(x)}{5}}{a^2}$$

input
```
Integrate[Sec[x]^2/(a - a*Sin[x]^2)^2,x]
```

output
```
(Tan[x] + (2*Tan[x]^3)/3 + Tan[x]^5/5)/a^2
```

3.283.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3042, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{\left(a-a\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(x)^2\left(a-a\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \sec^6(x)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^6dx}{a^2}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int \left(\tan^4(x)+2\tan^2(x)+1\right)d(-\tan(x))}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{5}\tan^5(x)-\frac{2\tan^3(x)}{3}-\tan(x)}{a^2}$$

input
```
Int[Sec[x]^2/(a - a*Sin[x]^2)^2,x]
```

output
```
-((-Tan[x] - (2*Tan[x]^3)/3 - Tan[x]^5/5)/a^2)
```

3.283.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.283.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\frac{\left(\tan^5(x)\right)}{5}+\frac{2\left(\tan^3(x)\right)}{3}+\tan(x)}{a^2}$	20
parallelrisch	$\frac{\tan(x)\left(3\left(\sec^4(x)\right)+4\left(\sec^2(x)\right)+8\right)}{15a^2}$	22
risch	$\frac{16i\left(10\,\mathrm{e}^{4ix}+5\,\mathrm{e}^{2ix}+1\right)}{15\left(\mathrm{e}^{2ix}+1\right)^5 a^2}$	32
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{8\left(\tan^3\left(\frac{x}{2}\right)\right)}{3a}-\frac{116\left(\tan^5\left(\frac{x}{2}\right)\right)}{15a}+\frac{8\left(\tan^7\left(\frac{x}{2}\right)\right)}{3a}-\frac{2\left(\tan^9\left(\frac{x}{2}\right)\right)}{a}}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)^5 a}$	69

input
```
int(sec(x)^2/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/5*tan(x)^5+2/3*tan(x)^3+tan(x))
```

3.283.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{\sec^2(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\left(8\,\cos\left(x\right)^4+4\,\cos\left(x\right)^2+3\right)\sin\left(x\right)}{15\,a^2\cos\left(x\right)^5}$$

input
```
integrate(sec(x)^2/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
1/15*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/(a^2*cos(x)^5)
```

3.283.6 Sympy [F]

$$\int \frac{\sec^2(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\int \frac{\sec^2(x)}{\sin^4(x)-2\sin^2(x)+1}\,dx}{a^2}$$

input
```
integrate(sec(x)**2/(a-a*sin(x)**2)**2,x)
```

output
```
Integral(sec(x)**2/(sin(x)**4 - 2*sin(x)**2 + 1), x)/a**2
```

3.283.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{\sec^2(x)}{(a-a\sin^2(x))^2}\,dx = \frac{3\,\tan\left(x\right)^5+10\,\tan\left(x\right)^3+15\,\tan\left(x\right)}{15\,a^2}$$

input
```
integrate(sec(x)^2/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^2
```

3.283.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{\sec^2(x)}{(a - a\sin^2(x))^2}\,dx = \frac{3\,\tan{(x)}^5 + 10\,\tan{(x)}^3 + 15\,\tan{(x)}}{15\,a^2}$$

input
```
integrate(sec(x)^2/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^2
```

3.283.9 Mupad [B] (verification not implemented)

Time = 13.66 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{\sec^2(x)}{(a - a\sin^2(x))^2}\,dx = \frac{\tan(x)\,\left(3\tan(x)^4 + 10\tan(x)^2 + 15\right)}{15\,a^2}$$

input
```
int(1/(cos(x)^2*(a - a*sin(x)^2)^2),x)
```

output
```
(tan(x)*(10*tan(x)^2 + 3*tan(x)^4 + 15))/(15*a^2)
```

3.284 $\int \frac{\sec^4(x)}{(a-a\sin^2(x))^2}\,dx$

3.284.1 Optimal result

Integrand size = 16, antiderivative size = 37

$$\int \frac{\sec^4(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\tan(x)}{a^2} + \frac{\tan^3(x)}{a^2} + \frac{3\tan^5(x)}{5a^2} + \frac{\tan^7(x)}{7a^2}$$

output
```
tan(x)/a^2+tan(x)^3/a^2+3/5*tan(x)^5/a^2+1/7*tan(x)^7/a^2
```

3.284.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{\sec^4(x)}{(a-a\sin^2(x))^2}\,dx = \frac{\tan(x)+\tan^3(x)+\frac{3\tan^5(x)}{5}+\frac{\tan^7(x)}{7}}{a^2}$$

input
```
Integrate[Sec[x]^4/(a - a*Sin[x]^2)^2,x]
```

output
```
(Tan[x] + Tan[x]^3 + (3*Tan[x]^5)/5 + Tan[x]^7/7)/a^2
```

3.284.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(x)}{\left(a-a\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos(x)^4\left(a-a\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \text{ 3654}$$

$$\frac{\int \sec^8(x)dx}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc\left(x+\frac{\pi}{2}\right)^8dx}{a^2}$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{\int \left(\tan^6(x)+3\tan^4(x)+3\tan^2(x)+1\right)d(-\tan(x))}{a^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-\frac{1}{7}\tan^7(x)-\frac{3\tan^5(x)}{5}-\tan^3(x)-\tan(x)}{a^2}$$

input
```
Int[Sec[x]^4/(a - a*Sin[x]^2)^2,x]
```

output
```
-((-Tan[x] - Tan[x]^3 - (3*Tan[x]^5)/5 - Tan[x]^7/7)/a^2)
```

3.284.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.284.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\frac{\left(\tan^{7}\left(x\right)\right)}{7}+\frac{3\left(\tan^{5}\left(x\right)\right)}{5}+\tan^{3}\left(x\right)+\tan\left(x\right)}{a^{2}}$	24
parallelrisch	$\frac{\tan\left(x\right)\left(\sec^{6}\left(x\right)\right)\left(32+\cos\left(6x\right)+8\cos\left(4x\right)+29\cos\left(2x\right)\right)}{70a^{2}}$	30
risch	$\frac{32i\left(35\,{\mathrm e}^{6ix}+21\,{\mathrm e}^{4ix}+7\,{\mathrm e}^{2ix}+1\right)}{35\left({\mathrm e}^{2ix}+1\right)^{7}a^{2}}$	39
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{4\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{a}-\frac{86\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{5a}+\frac{424\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{35a}-\frac{86\left(\tan^{9}\left(\frac{x}{2}\right)\right)}{5a}+\frac{4\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{a}-\frac{2\left(\tan^{13}\left(\frac{x}{2}\right)\right)}{a}}{\left(\tan^{2}\left(\frac{x}{2}\right)-1\right)^{7}a}$	91

input
```
int(sec(x)^4/(a-a*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/7*tan(x)^7+3/5*tan(x)^5+tan(x)^3+tan(x))
```

3.284.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{\sec^4(x)}{\left(a-a\sin^2(x)\right)^2}\,dx = \frac{\left(16\,\cos{(x)}^6+8\,\cos{(x)}^4+6\,\cos{(x)}^2+5\right)\sin{(x)}}{35\,a^2\cos{(x)}^7}$$

input
```
integrate(sec(x)^4/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
1/35*(16*cos(x)^6 + 8*cos(x)^4 + 6*cos(x)^2 + 5)*sin(x)/(a^2*cos(x)^7)
```

3.284.6 Sympy [F]

$$\int \frac{\sec^4(x)}{\left(a-a\sin^2(x)\right)^2}\,dx = \frac{\int \frac{\sec^4{(x)}}{\sin^4{(x)}-2\sin^2{(x)}+1}\,dx}{a^2}$$

input
```
integrate(sec(x)**4/(a-a*sin(x)**2)**2,x)
```

output
```
Integral(sec(x)**4/(sin(x)**4 - 2*sin(x)**2 + 1), x)/a**2
```

3.284.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{\sec^4(x)}{\left(a-a\sin^2(x)\right)^2}\,dx = \frac{5\,\tan{(x)}^7+21\,\tan{(x)}^5+35\,\tan{(x)}^3+35\,\tan{(x)}}{35\,a^2}$$

input
```
integrate(sec(x)^4/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/35*(5*tan(x)^7 + 21*tan(x)^5 + 35*tan(x)^3 + 35*tan(x))/a^2
```

3.284.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{\sec^4(x)}{\left(a - a\sin^2(x)\right)^2}\,dx = \frac{5\,\tan\left(x\right)^7 + 21\,\tan\left(x\right)^5 + 35\,\tan\left(x\right)^3 + 35\,\tan\left(x\right)}{35\,a^2}$$

input
```
integrate(sec(x)^4/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/35*(5*tan(x)^7 + 21*tan(x)^5 + 35*tan(x)^3 + 35*tan(x))/a^2
```

3.284.9 Mupad [B] (verification not implemented)

Time = 13.66 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{\sec^4(x)}{\left(a - a\sin^2(x)\right)^2}\,dx = \frac{\tan(x)}{a^2} + \frac{\tan(x)^3}{a^2} + \frac{3\tan(x)^5}{5\,a^2} + \frac{\tan(x)^7}{7\,a^2}$$

input
```
int(1/(cos(x)^4*(a - a*sin(x)^2)^2),x)
```

output
```
tan(x)/a^2 + tan(x)^3/a^2 + (3*tan(x)^5)/(5*a^2) + tan(x)^7/(7*a^2)
```

3.285 $\int \cos^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

3.285.1 Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \cos^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \frac{5}{128}(8a+b)x + \frac{5(8a+b)\cos(e+fx)\sin(e+fx)}{128f} + \frac{5(8a+b)\cos^3(e+fx)\sin(e+fx)}{192f} + \frac{(8a+b)\cos^5(e+fx)\sin(e+fx)}{48f} - \frac{b\cos^7(e+fx)\sin(e+fx)}{8f}$$

output
```
5/128*(8*a+b)*x+5/128*(8*a+b)*cos(f*x+e)*sin(f*x+e)/f+5/192*(8*a+b)*cos(f*
x+e)^3*sin(f*x+e)/f+1/48*(8*a+b)*cos(f*x+e)^5*sin(f*x+e)/f-1/8*b*cos(f*x+e
)^7*sin(f*x+e)/f
```

3.285.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.80

$$\int \cos^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \frac{960ae+960afx+120bfx+48(15a+b)\sin(2(e+fx))+24(6a-b)\sin(4(e+fx))+16a\sin(6(e+fx)}{3072f}$$

input
```
Integrate[Cos[e + f*x]^6*(a + b*Sin[e + f*x]^2),x]
```

output
```
(960*a*e + 960*a*f*x + 120*b*f*x + 48*(15*a + b)*Sin[2*(e + f*x)] + 24*(6*
a - b)*Sin[4*(e + f*x)] + 16*a*Sin[6*(e + f*x)] - 16*b*Sin[6*(e + f*x)] -
3*b*Sin[8*(e + f*x)])/(3072*f)
```

3.285.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3670, 298, 215, 215, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(e+fx)\left(a+b\sin^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^6\left(a+b\sin(e+fx)^2\right)dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{(a+b)\tan^2(e+fx)+a}{\left(\tan^2(e+fx)+1\right)^5}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{8}(8a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)^4}d\tan(e+fx)-\frac{b\tan(e+fx)}{8\left(\tan^2(e+fx)+1\right)^4}}{f}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{8}(8a+b)\left(\frac{5}{6}\int \frac{1}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)+\frac{\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}\right)-\frac{b\tan(e+fx)}{8\left(\tan^2(e+fx)+1\right)^4}}{f}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{8}(8a+b)\left(\frac{5}{6}\left(\frac{3}{4}\int \frac{1}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)+\frac{\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}\right)+\frac{\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}\right)-\frac{b\tan(e+fx)}{8\left(\tan^2(e+fx)+1\right)^4}}{f}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{8}(8a+b)\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)+\frac{\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}\right)+\frac{\tan(e+fx)}{6(\tan^2(e+fx)+1)^3}\right)-\frac{b\tan}{8(\tan^2(}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{8}(8a+b)\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\arctan(\tan(e+fx))+\frac{\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)+\frac{\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}\right)+\frac{\tan(e+fx)}{6(\tan^2(e+fx)+1)^3}\right)-\frac{b\tan(e+fx)}{8(\tan^2(e+fx)+1}}{f}$$

input
```
Int[Cos[e + f*x]^6*(a + b*Sin[e + f*x]^2),x]
```

output
```
(-1/8*(b*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^4 + ((8*a + b)*(Tan[e + f*x]/(
6*(1 + Tan[e + f*x]^2)^3) + (5*(Tan[e + f*x]/(4*(1 + Tan[e + f*x]^2)^2) +
(3*(ArcTan[Tan[e + f*x]]/2 + Tan[e + f*x]/(2*(1 + Tan[e + f*x]^2))))/4))/6
))/8)/f
```

3.285.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.285.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{48(15a+b)\sin(2fx+2e)+24(6a-b)\sin(4fx+4e)+16(a-b)\sin(6fx+6e)-3b\sin(8fx+8e)+960f\left(a+\frac{b}{8}\right)x}{3072f}$
derivativedivides	$\frac{b\left(-\frac{\left(\cos^7(fx+e)\right)\sin(fx+e)}{8}+\frac{\left(\cos^5(fx+e)+\frac{5\left(\cos^3(fx+e)\right)}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{48}+\frac{5fx}{128}+\frac{5e}{128}\right)+a\left(\frac{\left(\cos^5(fx+e)+\frac{5(c}{}\right.}{}\right.}{f}$
default	$\frac{b\left(-\frac{\left(\cos^7(fx+e)\right)\sin(fx+e)}{8}+\frac{\left(\cos^5(fx+e)+\frac{5\left(\cos^3(fx+e)\right)}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{48}+\frac{5fx}{128}+\frac{5e}{128}\right)+a\left(\frac{\left(\cos^5(fx+e)+\frac{5(c}{}\right.}{}\right.}{f}$
risch	$\frac{5ax}{16}+\frac{5bx}{128}-\frac{b\sin(8fx+8e)}{1024f}+\frac{\sin(6fx+6e)a}{192f}-\frac{\sin(6fx+6e)b}{192f}+\frac{3\sin(4fx+4e)a}{64f}-\frac{\sin(4fx+4e)b}{128f}+\frac{15\sin(2}{6}$
norman	$\left(\frac{5a}{16}+\frac{5b}{128}\right)x+\left(\frac{5a}{2}+\frac{5b}{16}\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{5a}{2}+\frac{5b}{16}\right)x\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{5a}{16}+\frac{5b}{128}\right)x\left(\tan^{16}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{35a}{2}+\frac{35b}{16}\right.$

input
```
int(cos(f*x+e)^6*(a+b*sin(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/3072*(48*(15*a+b)*sin(2*f*x+2*e)+24*(6*a-b)*sin(4*f*x+4*e)+16*(a-b)*sin(
6*f*x+6*e)-3*b*sin(8*f*x+8*e)+960*f*(a+1/8*b)*x)/f
```

3.285.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

$$\int \cos^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$$
$$=\frac{15\,(8\,a+b)fx-\left(48\,b\cos\left(fx+e\right)^7-8\,(8\,a+b)\cos\left(fx+e\right)^5-10\,(8\,a+b)\cos\left(fx+e\right)^3-15\,(8\,a+b)\,c\right.}{384\,f}$$

input
```
integrate(cos(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/384*(15*(8*a + b)*f*x - (48*b*cos(f*x + e)^7 - 8*(8*a + b)*cos(f*x + e)^
5 - 10*(8*a + b)*cos(f*x + e)^3 - 15*(8*a + b)*cos(f*x + e))*sin(f*x + e))
/f
```

3.285.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(107) = 214$.

Time = 0.83 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.25

$$\int \cos^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$$
$$= \begin{cases} \frac{5ax\sin^6(e+fx)}{16} + \frac{15ax\sin^4(e+fx)\cos^2(e+fx)}{16} + \frac{15ax\sin^2(e+fx)\cos^4(e+fx)}{16} + \frac{5ax\cos^6(e+fx)}{16} + \frac{5a\sin^5(e+fx)\cos(e+fx)}{16f} + \\ x(a+b\sin^2(e))\cos^6(e) \end{cases}$$

input
```
integrate(cos(f*x+e)**6*(a+b*sin(f*x+e)**2),x)
```

output
```
Piecewise((5*a*x*sin(e + f*x)**6/16 + 15*a*x*sin(e + f*x)**4*cos(e + f*x)*
*2/16 + 15*a*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 5*a*x*cos(e + f*x)**6/
16 + 5*a*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 5*a*sin(e + f*x)**3*cos(e +
 f*x)**3/(6*f) + 11*a*sin(e + f*x)*cos(e + f*x)**5/(16*f) + 5*b*x*sin(e +
f*x)**8/128 + 5*b*x*sin(e + f*x)**6*cos(e + f*x)**2/32 + 15*b*x*sin(e + f*
x)**4*cos(e + f*x)**4/64 + 5*b*x*sin(e + f*x)**2*cos(e + f*x)**6/32 + 5*b*
x*cos(e + f*x)**8/128 + 5*b*sin(e + f*x)**7*cos(e + f*x)/(128*f) + 55*b*si
n(e + f*x)**5*cos(e + f*x)**3/(384*f) + 73*b*sin(e + f*x)**3*cos(e + f*x)*
*5/(384*f) - 5*b*sin(e + f*x)*cos(e + f*x)**7/(128*f), Ne(f, 0)), (x*(a +
b*sin(e)**2)*cos(e)**6, True))
```

3.285.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.12

$$\int \cos^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \frac{15\,(fx+e)(8\,a+b)+\frac{15\,(8\,a+b)\tan(fx+e)^7+55\,(8\,a+b)\tan(fx+e)^5+73\,(8\,a+b)\tan(fx+e)^3+3\,(88\,a-5\,b)\tan(fx+e)}{\tan(fx+e)^8+4\,\tan(fx+e)^6+6\,\tan(fx+e)^4+4\,\tan(fx+e)^2+1}}{384\,f}$$

input
```
integrate(cos(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/384*(15*(f*x + e)*(8*a + b) + (15*(8*a + b)*tan(f*x + e)^7 + 55*(8*a + b
)*tan(f*x + e)^5 + 73*(8*a + b)*tan(f*x + e)^3 + 3*(88*a - 5*b)*tan(f*x +
e))/(tan(f*x + e)^8 + 4*tan(f*x + e)^6 + 6*tan(f*x + e)^4 + 4*tan(f*x + e)
^2 + 1))/f
```

3.285.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.76

$$\int \cos^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \frac{5}{128}\,(8\,a+b)x-\frac{b\sin(8\,fx+8\,e)}{1024\,f}+\frac{(a-b)\sin(6\,fx+6\,e)}{192\,f}+\frac{(6\,a-b)\sin(4\,fx+4\,e)}{128\,f}+\frac{(15\,a+b)\sin(2\,fx+2\,e)}{64\,f}$$

input
```
integrate(cos(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

output
```
5/128*(8*a + b)*x - 1/1024*b*sin(8*f*x + 8*e)/f + 1/192*(a - b)*sin(6*f*x
+ 6*e)/f + 1/128*(6*a - b)*sin(4*f*x + 4*e)/f + 1/64*(15*a + b)*sin(2*f*x
+ 2*e)/f
```

3.285.9 Mupad [B] (verification not implemented)

Time = 15.60 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.09

$$\int \cos^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = x\left(\frac{5a}{16}+\frac{5b}{128}\right) + \frac{\left(\frac{5a}{16}+\frac{5b}{128}\right)\tan(e+fx)^7+\left(\frac{55a}{48}+\frac{55b}{384}\right)\tan(e+fx)^5+\left(\frac{73a}{48}+\frac{73b}{384}\right)\tan(e+fx)^3+\left(\frac{11a}{16}-\frac{5b}{128}\right)\tan(e+fx)}{f\left(\tan(e+fx)^8+4\tan(e+fx)^6+6\tan(e+fx)^4+4\tan(e+fx)^2+1\right)}$$

input
```
int(cos(e + f*x)^6*(a + b*sin(e + f*x)^2),x)
```

output
```
x*((5*a)/16 + (5*b)/128) + (tan(e + f*x)^7*((5*a)/16 + (5*b)/128) + tan(e
+ f*x)^5*((55*a)/48 + (55*b)/384) + tan(e + f*x)^3*((73*a)/48 + (73*b)/384
) + tan(e + f*x)*((11*a)/16 - (5*b)/128))/(f*(4*tan(e + f*x)^2 + 6*tan(e +
 f*x)^4 + 4*tan(e + f*x)^6 + tan(e + f*x)^8 + 1))
```

3.286 $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

3.286.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \frac{1}{16}(6a+b)x+\frac{(6a+b)\cos(e+fx)\sin(e+fx)}{16f}+\frac{(6a+b)\cos^3(e+fx)\sin(e+fx)}{24f}-\frac{b\cos^5(e+fx)\sin(e+fx)}{6f}$$

output
```
1/16*(6*a+b)*x+1/16*(6*a+b)*cos(f*x+e)*sin(f*x+e)/f+1/24*(6*a+b)*cos(f*x+e
)^3*sin(f*x+e)/f-1/6*b*cos(f*x+e)^5*sin(f*x+e)/f
```

3.286.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \frac{72ae+72afx+12bfx+3(16a+b)\sin(2(e+fx))+(6a-3b)\sin(4(e+fx))-b\sin(6(e+fx))}{192f}$$

input
```
Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2),x]
```

output
```
(72*a*e + 72*a*f*x + 12*b*f*x + 3*(16*a + b)*Sin[2*(e + f*x)] + (6*a - 3*b
)*Sin[4*(e + f*x)] - b*Sin[6*(e + f*x)])/(192*f)
```

3.286.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3670, 298, 215, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^4\left(a+b\sin(e+fx)^2\right)dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{(a+b)\tan^2(e+fx)+a}{\left(\tan^2(e+fx)+1\right)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{6}(6a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)-\frac{b\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{6}(6a+b)\left(\frac{3}{4}\int \frac{1}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)+\frac{\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}\right)-\frac{b\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{6}(6a+b)\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}\right)-\frac{b\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{6}(6a+b)\left(\frac{3}{4}\left(\frac{1}{2}\arctan(\tan(e+fx))+\frac{\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}\right)-\frac{b\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

input
```
Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2),x]
```

output
```
(-1/6*(b*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^3 + ((6*a + b)*(Tan[e + f*x]/(
4*(1 + Tan[e + f*x]^2)^2) + (3*(ArcTan[Tan[e + f*x]]/2 + Tan[e + f*x]/(2*(
1 + Tan[e + f*x]^2))))/4))/6)/f
```

3.286.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.286.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

method	result
parallelrisch	$\frac{(48a+3b)\sin(2fx+2e)+(6a-3b)\sin(4fx+4e)-\sin(6fx+6e)b+72f\left(a+\frac{b}{6}\right)x}{192f}$
risch	$\frac{3ax}{8}+\frac{bx}{16}-\frac{\sin(6fx+6e)b}{192f}+\frac{\sin(4fx+4e)a}{32f}-\frac{\sin(4fx+4e)b}{64f}+\frac{\sin(2fx+2e)a}{4f}+\frac{\sin(2fx+2e)b}{64f}$
derivativedivides	$\frac{b\left(-\frac{\left(\cos^5(fx+e)\right)\sin(fx+e)}{6}+\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{24}+\frac{fx}{16}+\frac{e}{16}\right)+a\left(\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\right.}{f}$
default	$\frac{b\left(-\frac{\left(\cos^5(fx+e)\right)\sin(fx+e)}{6}+\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{24}+\frac{fx}{16}+\frac{e}{16}\right)+a\left(\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\right.}{f}$
norman	$\left(\frac{3a}{8}+\frac{b}{16}\right)x+\left(\frac{3a}{8}+\frac{b}{16}\right)x\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{9a}{4}+\frac{3b}{8}\right)x\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{9a}{4}+\frac{3b}{8}\right)x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{15a}{2}+\frac{5b}{4}\right)x\left(\right.$

input
```
int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/192*((48*a+3*b)*sin(2*f*x+2*e)+(6*a-3*b)*sin(4*f*x+4*e)-sin(6*f*x+6*e)*b
+72*f*(a+1/6*b)*x)/f
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$$
$$=\frac{3\,(6\,a+b)fx-\left(8\,b\cos\left(fx+e\right)^5-2\,(6\,a+b)\cos\left(fx+e\right)^3-3\,(6\,a+b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48\,f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/48*(3*(6*a + b)*f*x - (8*b*cos(f*x + e)^5 - 2*(6*a + b)*cos(f*x + e)^3 -
 3*(6*a + b)*cos(f*x + e))*sin(f*x + e))/f
```

3.286.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(76) = 152$.

Time = 0.40 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.01

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$$
$$= \begin{cases} \frac{3ax\sin^4(e+fx)}{8} + \frac{3ax\sin^2(e+fx)\cos^2(e+fx)}{4} + \frac{3ax\cos^4(e+fx)}{8} + \frac{3a\sin^3(e+fx)\cos(e+fx)}{8f} + \frac{5a\sin(e+fx)\cos^3(e+fx)}{8f} + bx\sin \\ x(a+b\sin^2(e))\cos^4(e) \end{cases}$$

input
```
integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2),x)
```

output
```
Piecewise((3*a*x*sin(e + f*x)**4/8 + 3*a*x*sin(e + f*x)**2*cos(e + f*x)**2
/4 + 3*a*x*cos(e + f*x)**4/8 + 3*a*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 5*
a*sin(e + f*x)*cos(e + f*x)**3/(8*f) + b*x*sin(e + f*x)**6/16 + 3*b*x*sin(
e + f*x)**4*cos(e + f*x)**2/16 + 3*b*x*sin(e + f*x)**2*cos(e + f*x)**4/16
+ b*x*cos(e + f*x)**6/16 + b*sin(e + f*x)**5*cos(e + f*x)/(16*f) + b*sin(e
 + f*x)**3*cos(e + f*x)**3/(6*f) - b*sin(e + f*x)*cos(e + f*x)**5/(16*f),
Ne(f, 0)), (x*(a + b*sin(e)**2)*cos(e)**4, True))
```

3.286.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$$
$$= \frac{3(fx+e)(6a+b) + \frac{3(6a+b)\tan(fx+e)^5 + 8(6a+b)\tan(fx+e)^3 + 3(10a-b)\tan(fx+e)}{\tan(fx+e)^6 + 3\tan(fx+e)^4 + 3\tan(fx+e)^2 + 1}}{48f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/48*(3*(f*x + e)*(6*a + b) + (3*(6*a + b)*tan(f*x + e)^5 + 8*(6*a + b)*ta
n(f*x + e)^3 + 3*(10*a - b)*tan(f*x + e))/(tan(f*x + e)^6 + 3*tan(f*x + e)
^4 + 3*tan(f*x + e)^2 + 1))/f
```

3.286.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \frac{1}{16}\,(6\,a+b)x - \frac{b\sin\left(6\,fx+6\,e\right)}{192\,f} + \frac{(2\,a-b)\sin\left(4\,fx+4\,e\right)}{64\,f} + \frac{(16\,a+b)\sin\left(2\,fx+2\,e\right)}{64\,f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

output
```
1/16*(6*a + b)*x - 1/192*b*sin(6*f*x + 6*e)/f + 1/64*(2*a - b)*sin(4*f*x +
 4*e)/f + 1/64*(16*a + b)*sin(2*f*x + 2*e)/f
```

3.286.9 Mupad [B] (verification not implemented)

Time = 14.56 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = x\left(\frac{3\,a}{8}+\frac{b}{16}\right) + \frac{\left(\frac{3\,a}{8}+\frac{b}{16}\right)\,\tan(e+f\,x)^5+\left(a+\frac{b}{6}\right)\,\tan(e+f\,x)^3+\left(\frac{5\,a}{8}-\frac{b}{16}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^6+3\tan\left(e+f\,x\right)^4+3\tan\left(e+f\,x\right)^2+1\right)}$$

input
```
int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2),x)
```

output
```
x*((3*a)/8 + b/16) + (tan(e + f*x)^5*((3*a)/8 + b/16) + tan(e + f*x)*((5*a
)/8 - b/16) + tan(e + f*x)^3*(a + b/6))/(f*(3*tan(e + f*x)^2 + 3*tan(e + f
*x)^4 + tan(e + f*x)^6 + 1))
```

3.287 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

3.287.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=\frac{1}{8}(4a+b)x+\frac{(4a+b)\cos(e+fx)\sin(e+fx)}{8f}-\frac{b\cos^3(e+fx)\sin(e+fx)}{4f}$$

output
```
1/8*(4*a+b)*x+1/8*(4*a+b)*cos(f*x+e)*sin(f*x+e)/f-1/4*b*cos(f*x+e)^3*sin(f
*x+e)/f
```

3.287.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=\frac{4(4ae+4afx+bfx)+8a\sin(2(e+fx))-b\sin(4(e+fx))}{32f}$$

input
```
Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2),x]
```

output
```
(4*(4*a*e + 4*a*f*x + b*f*x) + 8*a*Sin[2*(e + f*x)] - b*Sin[4*(e + f*x)])/
(32*f)
```

3.287.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3670, 298, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^2\left(a+b\sin(e+fx)^2\right)dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{(a+b)\tan^2(e+fx)+a}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{4}(4a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)-\frac{b\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{4}(4a+b)\left(\frac{1}{2}\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right)-\frac{b\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{4}(4a+b)\left(\frac{1}{2}\arctan(\tan(e+fx))+\frac{\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right)-\frac{b\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

input
```
Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2),x]
```

output
```
(-1/4*(b*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^2 + ((4*a + b)*(ArcTan[Tan[e +
 f*x]]/2 + Tan[e + f*x]/(2*(1 + Tan[e + f*x]^2))))/4)/f
```

3.287.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.287.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.70

method	result
risch	$\frac{ax}{2}+\frac{bx}{8}-\frac{\sin(4fx+4e)b}{32f}+\frac{\sin(2fx+2e)a}{4f}$
parallelrisch	$\frac{8\sin(2fx+2e)a-\sin(4fx+4e)b+16f\left(a+\frac{b}{4}\right)x}{32f}$
derivativedivides	$\frac{b\left(-\frac{\left(\cos^3(fx+e)\right)\sin(fx+e)}{4}+\frac{\cos(fx+e)\sin(fx+e)}{8}+\frac{fx}{8}+\frac{e}{8}\right)+a\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$
default	$\frac{b\left(-\frac{\left(\cos^3(fx+e)\right)\sin(fx+e)}{4}+\frac{\cos(fx+e)\sin(fx+e)}{8}+\frac{fx}{8}+\frac{e}{8}\right)+a\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$
norman	$\frac{\left(\frac{a}{2}+\frac{b}{8}\right)x+\left(2a+\frac{b}{2}\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(2a+\frac{b}{2}\right)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(3a+\frac{3b}{4}\right)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{a}{2}+\frac{b}{8}\right)x\left(\tan^8\left(\frac{fx}{2}+\right.\right.}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^4}$

input
```
int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x+1/8*b*x-1/32/f*sin(4*f*x+4*e)*b+1/4/f*sin(2*f*x+2*e)*a
```

3.287.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \frac{(4\,a+b)fx-\left(2\,b\cos\left(fx+e\right)^3-(4\,a+b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8\,f}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/8*((4*a + b)*f*x - (2*b*cos(f*x + e)^3 - (4*a + b)*cos(f*x + e))*sin(f*x
 + e))/f
```

3.287.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(49) = 98$.

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.63

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \begin{cases} \frac{ax\sin^2(e+fx)}{2} + \frac{ax\cos^2(e+fx)}{2} + \frac{a\sin(e+fx)\cos(e+fx)}{2f} + \frac{bx\sin^4(e+fx)}{8} + \frac{bx\sin^2(e+fx)\cos^2(e+fx)}{4} + \frac{bx\cos^4(e+fx)}{8} + \frac{b\sin \\ x(a+b\sin^2(e))\cos^2(e) \end{cases}$$

input
```
integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2),x)
```

output
```
Piecewise((a*x*sin(e + f*x)**2/2 + a*x*cos(e + f*x)**2/2 + a*sin(e + f*x)*
cos(e + f*x)/(2*f) + b*x*sin(e + f*x)**4/8 + b*x*sin(e + f*x)**2*cos(e + f
*x)**2/4 + b*x*cos(e + f*x)**4/8 + b*sin(e + f*x)**3*cos(e + f*x)/(8*f) -
b*sin(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e)**2)*cos(
e)**2, True))
```

3.287.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \frac{(fx+e)(4a+b) + \frac{(4a+b)\tan(fx+e)^3+(4a-b)\tan(fx+e)}{\tan(fx+e)^4+2\tan(fx+e)^2+1}}{8f}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/8*((f*x + e)*(4*a + b) + ((4*a + b)*tan(f*x + e)^3 + (4*a - b)*tan(f*x +
 e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

3.287.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.68

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=\frac{1}{8}\,(4\,a+b)x-\frac{b\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{a\sin\left(2\,fx+2\,e\right)}{4\,f}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

output
```
1/8*(4*a + b)*x - 1/32*b*sin(4*f*x + 4*e)/f + 1/4*a*sin(2*f*x + 2*e)/f
```

3.287.9 Mupad [B] (verification not implemented)

Time = 13.82 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=x\left(\frac{a}{2}+\frac{b}{8}\right)+\frac{\left(\frac{a}{2}+\frac{b}{8}\right)\,\tan(e+f\,x)^3+\left(\frac{a}{2}-\frac{b}{8}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^4+2\tan\left(e+f\,x\right)^2+1\right)}$$

input
```
int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2),x)
```

output
```
x*(a/2 + b/8) + (tan(e + f*x)^3*(a/2 + b/8) + tan(e + f*x)*(a/2 - b/8))/(f
*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1))
```

3.288 $\int (a+b\sin^2(e+fx))\,dx$

3.288.1 Optimal result

Integrand size = 12, antiderivative size = 30

$$\int (a+b\sin^2(e+fx))\,dx = ax+\frac{bx}{2}-\frac{b\cos(e+fx)\sin(e+fx)}{2f}$$

output
```
a*x+1/2*b*x-1/2*b*cos(f*x+e)*sin(f*x+e)/f
```

3.288.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

$$\int (a+b\sin^2(e+fx))\,dx = ax+\frac{b(e+fx)}{2f}-\frac{b\sin(2(e+fx))}{4f}$$

input
```
Integrate[a + b*Sin[e + f*x]^2,x]
```

output
```
a*x + (b*(e + f*x))/(2*f) - (b*Sin[2*(e + f*x)])/(4*f)
```

3.288.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + b\sin^2(e + fx)\right) dx$$
$$\Big\downarrow \; 2009$$
$$ax - \frac{b\sin(e + fx)\cos(e + fx)}{2f} + \frac{bx}{2}$$

input
```
Int[a + b*Sin[e + f*x]^2,x]
```

output
```
a*x + (b*x)/2 - (b*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

3.288.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.288.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
risch	$ax + \frac{bx}{2} - \frac{\sin(2fx+2e)b}{4f}$	24
parallelrisch	$\frac{b(2fx-\sin(2fx+2e))}{4f} + ax$	27
default	$ax + \frac{b\left(-\frac{\cos(fx+e)\sin(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2}\right)}{f}$	32
parts	$ax + \frac{b\left(-\frac{\cos(fx+e)\sin(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2}\right)}{f}$	32
derivativedivides	$\frac{a(fx+e)+b\left(-\frac{\cos(fx+e)\sin(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2}\right)}{f}$	37
norman	$\frac{\left(a+\frac{b}{2}\right)x+\frac{b\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+\left(a+\frac{b}{2}\right)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(2a+b)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$	92

input
```
int(a+b*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/2*b*x-1/4/f*sin(2*f*x+2*e)*b
```

3.288.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \left(a+b\sin^2(e+fx)\right)\,dx=\frac{(2\,a+b)fx-b\cos\left(fx+e\right)\sin\left(fx+e\right)}{2\,f}$$

input
```
integrate(a+b*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
1/2*((2*a + b)*f*x - b*cos(f*x + e)*sin(f*x + e))/f
```

3.288.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.70

$$\int \left(a+b\sin^2(e+fx)\right)\,dx$$
$$=ax+b\left(\begin{cases}\frac{x\sin^2\left(e+fx\right)}{2}+\frac{x\cos^2\left(e+fx\right)}{2}-\frac{\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} & \text{for } f\neq 0\\ x\sin^2\left(e\right) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*sin(f*x+e)**2,x)
```

output
```
a*x + b*Piecewise((x*sin(e + f*x)**2/2 + x*cos(e + f*x)**2/2 - sin(e + f*x
)*cos(e + f*x)/(2*f), Ne(f, 0)), (x*sin(e)**2, True))
```

3.288.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \left(a + b\sin^2(e + fx)\right)\,dx = ax + \frac{(2\,fx + 2\,e - \sin\left(2\,fx + 2\,e\right))b}{4\,f}$$

input
```
integrate(a+b*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
a*x + 1/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*b/f
```

3.288.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \left(a + b\sin^2(e + fx)\right)\,dx = \frac{1}{4}\,b\left(2\,x - \frac{\sin\left(2\,fx + 2\,e\right)}{f}\right) + ax$$

input
```
integrate(a+b*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
1/4*b*(2*x - sin(2*f*x + 2*e)/f) + a*x
```

3.288.9 Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \left(a + b\sin^2(e + fx)\right)\,dx = -\frac{\frac{b\,\sin(2\,e+2\,f\,x)}{4} - f\,x\,\left(a + \frac{b}{2}\right)}{f}$$

input
```
int(a + b*sin(e + f*x)^2,x)
```

output
```
-((b*sin(2*e + 2*f*x))/4 - f*x*(a + b/2))/f
```

3.289 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

3.289.1 Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=-bx+\frac{(a+b)\tan(e+fx)}{f}$$

output
```
-b*x+(a+b)*tan(f*x+e)/f
```

3.289.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=-\frac{b\arctan(\tan(e+fx))}{f}+\frac{a\tan(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

input
```
Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2),x]
```

output
```
-((b*ArcTan[Tan[e + f*x]])/f) + (a*Tan[e + f*x])/f + (b*Tan[e + f*x])/f
```

3.289.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3670, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(e+fx)^2}{\cos(e+fx)^2}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{(a+b)\tan^2(e+fx)+a}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{(a+b)\tan(e+fx)-b\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{(a+b)\tan(e+fx)-b\arctan(\tan(e+fx))}{f}$$

input
```
Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2),x]
```

output
```
(-(b*ArcTan[Tan[e + f*x]]) + (a + b)*Tan[e + f*x])/f
```

3.289.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.289.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

method	result
derivativedivides	$\frac{\tan(fx+e)a+b(\tan(fx+e)-fx-e)}{f}$
default	$\frac{\tan(fx+e)a+b(\tan(fx+e)-fx-e)}{f}$
risch	$-bx+\frac{2ia}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)}+\frac{2ib}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)}$
parallelrisch	$\frac{-\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)xfb+(-2a-2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+fxb}{f\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-f}$
norman	$\frac{bx+bx\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-bx\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-bx\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{4(a+b)\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{2(a+b)(\cdots}{\cdots}}{\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{2}}$

input
```
int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)*a+b*(tan(f*x+e)-f*x-e))
```

3.289.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.94

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = -\frac{bfx\cos(fx+e)-(a+b)\sin(fx+e)}{f\cos(fx+e)}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

output
```
-(b*f*x*cos(f*x + e) - (a + b)*sin(f*x + e))/(f*cos(f*x + e))
```

3.289.6 Sympy [F]

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \int \left(a+b\sin^2(e+fx)\right)\sec^2(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2),x)
```

output
```
Integral((a + b*sin(e + f*x)**2)*sec(e + f*x)**2, x)
```

3.289.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = -\frac{(fx+e-\tan(fx+e))b-a\tan(fx+e)}{f}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

output
```
-((f*x + e - tan(f*x + e))*b - a*tan(f*x + e))/f
```

3.289.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=-\frac{(fx+e)b-a\tan\left(fx+e\right)-b\tan\left(fx+e\right)}{f}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

output
```
-((f*x + e)*b - a*tan(f*x + e) - b*tan(f*x + e))/f
```

3.289.9 Mupad [B] (verification not implemented)

Time = 13.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=\frac{a\tan(e+f\,x)+b\tan(e+f\,x)-b\,f\,x}{f}$$

input
```
int((a + b*sin(e + f*x)^2)/cos(e + f*x)^2,x)
```

output
```
(a*tan(e + f*x) + b*tan(e + f*x) - b*f*x)/f
```

3.290 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

3.290.1 Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=\frac{a\tan(e+fx)}{f}+\frac{(a+b)\tan^3(e+fx)}{3f}$$

output
```
a*tan(f*x+e)/f+1/3*(a+b)*tan(f*x+e)^3/f
```

3.290.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=\frac{b\tan^3(e+fx)}{3f}+\frac{a\left(\tan(e+fx)+\frac{1}{3}\tan^3(e+fx)\right)}{f}$$

input
```
Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2),x]
```

output
```
(b*Tan[e + f*x]^3)/(3*f) + (a*(Tan[e + f*x] + Tan[e + f*x]^3/3))/f
```

3.290.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3670, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{a+b\sin(e+fx)^2}{\cos(e+fx)^4}dx$$

$$\Big\downarrow \; 3670$$

$$\frac{\int \left((a+b)\tan^2(e+fx)+a\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{3}(a+b)\tan^3(e+fx)+a\tan(e+fx)}{f}$$

input
```
Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2),x]
```

output
```
(a*Tan[e + f*x] + ((a + b)*Tan[e + f*x]^3)/3)/f
```

3.290.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.290.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

method	result	size
derivativedivides	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+\frac{b\left(\sin^3(fx+e)\right)}{3\cos(fx+e)^3}}{f}$	46
default	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+\frac{b\left(\sin^3(fx+e)\right)}{3\cos(fx+e)^3}}{f}$	46
risch	$-\frac{2i\left(3\,\mathrm{e}^{4i(fx+e)}b-6a\,\mathrm{e}^{2i(fx+e)}-2a+b\right)}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^3}$	49
parallelrisch	$-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+\frac{2(-a+2b)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+a\right)}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$	76
norman	$\frac{-\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2a\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{8(a+b)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{8(a+b)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{4(a+4b)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$	124

input
```
int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+1/3*b*sin(f*x+e)^3/cos(f*x+e)^3
)
```

3.290.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=\frac{\left((2\,a-b)\cos\left(fx+e\right)^2+a+b\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/3*((2*a - b)*cos(f*x + e)^2 + a + b)*sin(f*x + e)/(f*cos(f*x + e)^3)
```

3.290.6 Sympy [F]

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=\int \left(a+b\sin^2\left(e+fx\right)\right)\sec^4\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2),x)
```

output
```
Integral((a + b*sin(e + f*x)**2)*sec(e + f*x)**4, x)
```

3.290.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=\frac{(a+b)\tan\left(fx+e\right)^3+3\,a\tan\left(fx+e\right)}{3\,f}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/3*((a + b)*tan(f*x + e)^3 + 3*a*tan(f*x + e))/f
```

3.290.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=\frac{a\tan\left(fx+e\right)^3+b\tan\left(fx+e\right)^3+3\,a\tan\left(fx+e\right)}{3\,f}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

output
```
1/3*(a*tan(f*x + e)^3 + b*tan(f*x + e)^3 + 3*a*tan(f*x + e))/f
```

3.290.9 Mupad [B] (verification not implemented)

Time = 13.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=\frac{\tan(e+f\,x)^3\,\left(\frac{a}{3}+\frac{b}{3}\right)}{f}+\frac{a\tan(e+f\,x)}{f}$$

input
```
int((a + b*sin(e + f*x)^2)/cos(e + f*x)^4,x)
```

output
```
(tan(e + f*x)^3*(a/3 + b/3))/f + (a*tan(e + f*x))/f
```

3.291 $\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

3.291.1 Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=\frac{a\tan(e+fx)}{f}+\frac{(2a+b)\tan^3(e+fx)}{3f}+\frac{(a+b)\tan^5(e+fx)}{5f}$$

output
```
a*tan(f*x+e)/f+1/3*(2*a+b)*tan(f*x+e)^3/f+1/5*(a+b)*tan(f*x+e)^5/f
```

3.291.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.28

$$\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx =\frac{\tan(e+fx)\left(15a-2b-b\sec^2(e+fx)+3b\sec^4(e+fx)+10a\tan^2(e+fx)+3a\tan^4(e+fx)\right)}{15f}$$

input
```
Integrate[Sec[e + f*x]^6*(a + b*Sin[e + f*x]^2),x]
```

output
```
(Tan[e + f*x]*(15*a - 2*b - b*Sec[e + f*x]^2 + 3*b*Sec[e + f*x]^4 + 10*a*T
an[e + f*x]^2 + 3*a*Tan[e + f*x]^4))/(15*f)
```

3.291.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3670, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx) \left(a+b\sin^2(e+fx)\right) \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{a+b\sin(e+fx)^2}{\cos(e+fx)^6}dx$$
$$\Big\downarrow \text{3670}$$
$$\frac{\int \left(\tan^2(e+fx)+1\right) \left((a+b)\tan^2(e+fx)+a\right)d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{290}$$
$$\frac{\int \left((a+b)\tan^4(e+fx)+(2a+b)\tan^2(e+fx)+a\right)d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{1}{5}(a+b)\tan^5(e+fx)+\frac{1}{3}(2a+b)\tan^3(e+fx)+a\tan(e+fx)}{f}$$

input
```
Int[Sec[e + f*x]^6*(a + b*Sin[e + f*x]^2),x]
```

output
```
(a*Tan[e + f*x] + ((2*a + b)*Tan[e + f*x]^3)/3 + ((a + b)*Tan[e + f*x]^5)/
5)/f
```

3.291.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.291.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.52

method	result
derivativedivides	$\frac{-a\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)+b\left(\frac{\sin^3(fx+e)}{5\cos(fx+e)^5}+\frac{2\left(\sin^3(fx+e)\right)}{15\cos(fx+e)^3}\right)}{f}$
default	$\frac{-a\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)+b\left(\frac{\sin^3(fx+e)}{5\cos(fx+e)^5}+\frac{2\left(\sin^3(fx+e)\right)}{15\cos(fx+e)^3}\right)}{f}$
risch	$-\frac{4i\left(15b\,e^{6i(fx+e)}-40a\,e^{4i(fx+e)}-5\,e^{4i(fx+e)}b-20a\,e^{2i(fx+e)}+5\,e^{2i(fx+e)}b-4a+b\right)}{15f\left(e^{2i(fx+e)}+1\right)^5}$
parallelrisch	$-\frac{2\left(a\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{4(-a+b)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\frac{2(29a+4b)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15}+\frac{4(-a+b)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+a\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}$
norman	$\frac{-\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2a\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{4(a+2b)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{4(a+2b)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{2(11a+16b)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5f}-\frac{2(}{}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$

input
```
int(sec(f*x+e)^6*(a+b*sin(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)+b*(1/5*sin(f
*x+e)^3/cos(f*x+e)^5+2/15*sin(f*x+e)^3/cos(f*x+e)^3))
```

3.291.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \frac{(2(4a-b)\cos(fx+e)^4+(4a-b)\cos(fx+e)^2+3a+3b)\sin(fx+e)}{15f\cos(fx+e)^5}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/15*(2*(4*a - b)*cos(f*x + e)^4 + (4*a - b)*cos(f*x + e)^2 + 3*a + 3*b)*s
in(f*x + e)/(f*cos(f*x + e)^5)
```

3.291.6 Sympy [F(-1)]

Timed out.

$$\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**6*(a+b*sin(f*x+e)**2),x)
```

output
```
Timed out
```

3.291.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \frac{3(a+b)\tan(fx+e)^5+5(2a+b)\tan(fx+e)^3+15a\tan(fx+e)}{15f}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/15*(3*(a + b)*tan(f*x + e)^5 + 5*(2*a + b)*tan(f*x + e)^3 + 15*a*tan(f*x
 + e))/f
```

3.291.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$$
$$=\frac{3\,a\tan\left(fx+e\right)^5+3\,b\tan\left(fx+e\right)^5+10\,a\tan\left(fx+e\right)^3+5\,b\tan\left(fx+e\right)^3+15\,a\tan\left(fx+e\right)}{15\,f}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

output
```
1/15*(3*a*tan(f*x + e)^5 + 3*b*tan(f*x + e)^5 + 10*a*tan(f*x + e)^3 + 5*b*
tan(f*x + e)^3 + 15*a*tan(f*x + e))/f
```

3.291.9 Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$$
$$=\frac{\left(\frac{a}{5}+\frac{b}{5}\right)\,\tan(e+f\,x)^5+\left(\frac{2\,a}{3}+\frac{b}{3}\right)\,\tan(e+f\,x)^3+a\tan(e+f\,x)}{f}$$

input
```
int((a + b*sin(e + f*x)^2)/cos(e + f*x)^6,x)
```

output
```
(tan(e + f*x)^3*((2*a)/3 + b/3) + tan(e + f*x)^5*(a/5 + b/5) + a*tan(e + f
*x))/f
```

3.292 $\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

3.292.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx=\frac{a\tan(e+fx)}{f}+\frac{(3a+b)\tan^3(e+fx)}{3f}+\frac{(3a+2b)\tan^5(e+fx)}{5f}+\frac{(a+b)\tan^7(e+fx)}{7f}$$

output
```
a*tan(f*x+e)/f+1/3*(3*a+b)*tan(f*x+e)^3/f+1/5*(3*a+2*b)*tan(f*x+e)^5/f+1/7
*(a+b)*tan(f*x+e)^7/f
```

3.292.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19

$$\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$$
$$=\frac{\tan(e+fx)\left(105a-8b-4b\sec^2(e+fx)-3b\sec^4(e+fx)+15b\sec^6(e+fx)+105a\tan^2(e+fx)+63\right.}{105f}$$

input
```
Integrate[Sec[e + f*x]^8*(a + b*Sin[e + f*x]^2),x]
```

output
```
(Tan[e + f*x]*(105*a - 8*b - 4*b*Sec[e + f*x]^2 - 3*b*Sec[e + f*x]^4 + 15*
b*Sec[e + f*x]^6 + 105*a*Tan[e + f*x]^2 + 63*a*Tan[e + f*x]^4 + 15*a*Tan[e
 + f*x]^6))/(105*f)
```

3.292.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3670, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(e+fx)^2}{\cos(e+fx)^8}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \left(\tan^2(e+fx)+1\right)^2\left((a+b)\tan^2(e+fx)+a\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{290}$$

$$\frac{\int \left((a+b)\tan^6(e+fx)+(3a+2b)\tan^4(e+fx)+(3a+b)\tan^2(e+fx)+a\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{7}(a+b)\tan^7(e+fx)+\frac{1}{5}(3a+2b)\tan^5(e+fx)+\frac{1}{3}(3a+b)\tan^3(e+fx)+a\tan(e+fx)}{f}$$

input
```
Int[Sec[e + f*x]^8*(a + b*Sin[e + f*x]^2),x]
```

output
```
(a*Tan[e + f*x] + ((3*a + b)*Tan[e + f*x]^3)/3 + ((3*a + 2*b)*Tan[e + f*x]
^5)/5 + ((a + b)*Tan[e + f*x]^7)/7)/f
```

3.292.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.292.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{-a\left(-\frac{16}{35}-\frac{\left(\sec^{6}(fx+e)\right)}{7}-\frac{6\left(\sec^{4}(fx+e)\right)}{35}-\frac{8\left(\sec^{2}(fx+e)\right)}{35}\right)\tan(fx+e)+b\left(\frac{\sin^{3}(fx+e)}{7\cos(fx+e)^{7}}+\frac{4\left(\sin^{3}(fx+e)\right)}{35\cos(fx+e)^{5}}+\frac{8\left(\sin^{3}(fx+e)\right)}{105\cos(fx+e)^{3}}\right.}{f}$
default	$\frac{-a\left(-\frac{16}{35}-\frac{\left(\sec^{6}(fx+e)\right)}{7}-\frac{6\left(\sec^{4}(fx+e)\right)}{35}-\frac{8\left(\sec^{2}(fx+e)\right)}{35}\right)\tan(fx+e)+b\left(\frac{\sin^{3}(fx+e)}{7\cos(fx+e)^{7}}+\frac{4\left(\sin^{3}(fx+e)\right)}{35\cos(fx+e)^{5}}+\frac{8\left(\sin^{3}(fx+e)\right)}{105\cos(fx+e)^{3}}\right.}{f}$
risch	$-\frac{16i(70b\,e^{8i(fx+e)}-210a\,e^{6i(fx+e)}-35b\,e^{6i(fx+e)}-126a\,e^{4i(fx+e)}+21\,e^{4i(fx+e)}b-42a\,e^{2i(fx+e)}+7\,e^{2i(fx+e)}b-6a+b}{105f\left(e^{2i(fx+e)}+1\right)^{7}}$
parallelrisch	$-\frac{2\left(a\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-2a+\frac{4b}{3}\right)\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{43a}{5}+\frac{16b}{15}\right)\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-\frac{212a}{35}+\frac{152b}{35}\right)\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right.\right.}{f\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}$

input
```
int(sec(f*x+e)^8*(a+b*sin(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a*(-16/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(
f*x+e)+b*(1/7*sin(f*x+e)^3/cos(f*x+e)^7+4/35*sin(f*x+e)^3/cos(f*x+e)^5+8/1
05*sin(f*x+e)^3/cos(f*x+e)^3))
```

3.292.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \frac{\left(8\,(6\,a-b)\cos\left(fx+e\right)^6+4\,(6\,a-b)\cos\left(fx+e\right)^4+3\,(6\,a-b)\cos\left(fx+e\right)^2+15\,a+15\,b\right)\sin\left(fx+e\right)}{105\,f\cos\left(fx+e\right)^7}$$

input
```
integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/105*(8*(6*a - b)*cos(f*x + e)^6 + 4*(6*a - b)*cos(f*x + e)^4 + 3*(6*a -
b)*cos(f*x + e)^2 + 15*a + 15*b)*sin(f*x + e)/(f*cos(f*x + e)^7)
```

3.292.6 Sympy [F(-1)]

Timed out.

$$\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**8*(a+b*sin(f*x+e)**2),x)
```

output
```
Timed out
```

3.292.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx = \frac{15\,(a+b)\tan\left(fx+e\right)^7+21\,(3\,a+2\,b)\tan\left(fx+e\right)^5+35\,(3\,a+b)\tan\left(fx+e\right)^3+105\,a\tan\left(fx+e\right)}{105\,f}$$

input
```
integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/105*(15*(a + b)*tan(f*x + e)^7 + 21*(3*a + 2*b)*tan(f*x + e)^5 + 35*(3*a
 + b)*tan(f*x + e)^3 + 105*a*tan(f*x + e))/f
```

3.292.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12

$$\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$$
$$= \frac{15\,a\tan(fx+e)^7+15\,b\tan(fx+e)^7+63\,a\tan(fx+e)^5+42\,b\tan(fx+e)^5+105\,a\tan(fx+e)^3+3}{105\,f}$$

input
```
integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

output
```
1/105*(15*a*tan(f*x + e)^7 + 15*b*tan(f*x + e)^7 + 63*a*tan(f*x + e)^5 + 4
2*b*tan(f*x + e)^5 + 105*a*tan(f*x + e)^3 + 35*b*tan(f*x + e)^3 + 105*a*ta
n(f*x + e))/f
```

3.292.9 Mupad [B] (verification not implemented)

Time = 13.65 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$$
$$= \frac{\left(\frac{a}{7}+\frac{b}{7}\right)\tan(e+f\,x)^7+\left(\frac{3\,a}{5}+\frac{2\,b}{5}\right)\tan(e+f\,x)^5+\left(a+\frac{b}{3}\right)\tan(e+f\,x)^3+a\tan(e+f\,x)}{f}$$

input
```
int((a + b*sin(e + f*x)^2)/cos(e + f*x)^8,x)
```

output
```
(tan(e + f*x)^5*((3*a)/5 + (2*b)/5) + tan(e + f*x)^7*(a/7 + b/7) + a*tan(e
 + f*x) + tan(e + f*x)^3*(a + b/3))/f
```

3.293 $\int \cos^4(e+fx) \left(a+b\sin^2(e+fx)\right)^2 \, dx$

3.293.1 Optimal result

Integrand size = 23, antiderivative size = 156

$$\begin{aligned}
&\int \cos^4(e+fx) \left(a+b\sin^2(e+fx)\right)^2 \, dx \\
&= \frac{1}{128}\left(48a^2+16ab+3b^2\right)x + \frac{(48a^2+16ab+3b^2)\cos(e+fx)\sin(e+fx)}{128f} \\
&\quad + \frac{(48a^2+16ab+3b^2)\cos^3(e+fx)\sin(e+fx)}{192f} - \frac{b(10a+3b)\cos^5(e+fx)\sin(e+fx)}{48f} \\
&\quad - \frac{b\cos^7(e+fx)\sin(e+fx)\left(a+(a+b)\tan^2(e+fx)\right)}{8f}
\end{aligned}$$

output
```
1/128*(48*a^2+16*a*b+3*b^2)*x+1/128*(48*a^2+16*a*b+3*b^2)*cos(f*x+e)*sin(f
*x+e)/f+1/192*(48*a^2+16*a*b+3*b^2)*cos(f*x+e)^3*sin(f*x+e)/f-1/48*b*(10*a
+3*b)*cos(f*x+e)^5*sin(f*x+e)/f-1/8*b*cos(f*x+e)^7*sin(f*x+e)*(a+(a+b)*tan
(f*x+e)^2)/f
```

3.293.2 Mathematica [A] (verified)

Time = 2.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.62

$$\begin{aligned}
&\int \cos^4(e+fx) \left(a+b\sin^2(e+fx)\right)^2 \, dx \\
&= \frac{24(48a^2+16ab+3b^2)(e+fx)+96a(8a+b)\sin(2(e+fx))+24(4a^2-4ab-b^2)\sin(4(e+fx))-32ab}{3072f}
\end{aligned}$$

input
```
Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^2,x]
```

output
```
(24*(48*a^2 + 16*a*b + 3*b^2)*(e + f*x) + 96*a*(8*a + b)*Sin[2*(e + f*x)]
+ 24*(4*a^2 - 4*a*b - b^2)*Sin[4*(e + f*x)] - 32*a*b*Sin[6*(e + f*x)] + 3*
b^2*Sin[8*(e + f*x)])/(3072*f)
```

3.293.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3670, 315, 298, 215, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^4\left(a+b\sin(e+fx)^2\right)^2dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{\left((a+b)\tan^2(e+fx)+a\right)^2}{\left(\tan^2(e+fx)+1\right)^5}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{1}{8}\int \frac{(a+b)(8a+3b)\tan^2(e+fx)+a(8a+b)}{\left(\tan^2(e+fx)+1\right)^4}d\tan(e+fx)-\frac{b\tan(e+fx)\left((a+b)\tan^2(e+fx)+a\right)}{8\left(\tan^2(e+fx)+1\right)^4}}{f}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(48a^2+16ab+3b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)-\frac{b(10a+3b)\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}\right)-\frac{b\tan(e+fx)\left((a+b)\tan^2(e+fx)+a\right)}{8\left(\tan^2(e+fx)+1\right)^4}}{f}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(48a^2+16ab+3b^2\right)\left(\frac{3}{4}\int \frac{1}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)+\frac{\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}\right)-\frac{b(10a+3b)\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}\right)-\frac{b\tan(e+fx)(\cdots}{8(\tan\cdots}}{f}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(48a^2+16ab+3b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)+\frac{\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}\right)-\frac{b(10a+3b)\tan(e\text{-}}{6(\tan^2(e+fx)+}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(48a^2+16ab+3b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\arctan(\tan(e+fx))+\frac{\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)+\frac{\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}\right)-\frac{b(10a+3b)\tan(e+fx)}{6(\tan^2(e+fx)+1)^3}\right)-}{f}$$

input
```
Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^2,x]
```

output
```
(-1/8*(b*Tan[e + f*x]*(a + (a + b)*Tan[e + f*x]^2))/(1 + Tan[e + f*x]^2)^4
 + (-1/6*(b*(10*a + 3*b)*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^3 + ((48*a^2 +
 16*a*b + 3*b^2)*(Tan[e + f*x]/(4*(1 + Tan[e + f*x]^2)^2) + (3*(ArcTan[Tan
[e + f*x]]/2 + Tan[e + f*x]/(2*(1 + Tan[e + f*x]^2))))/4))/6)/8)/f
```

3.293.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.293.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.62

method	result
parallelrisch	$\frac{\left(96a^2-96ab-24b^2\right)\sin(4fx+4e)+\left(768a^2+96ab\right)\sin(2fx+2e)-32ab\sin(6fx+6e)+3b^2\sin(8fx+8e)+1152f\left(a^2+\frac{1}{3}ab+\right.}{3072f}$
risch	$\frac{3a^2x}{8}+\frac{abx}{8}+\frac{3b^2x}{128}+\frac{b^2\sin(8fx+8e)}{1024f}-\frac{ab\sin(6fx+6e)}{96f}+\frac{\sin(4fx+4e)a^2}{32f}-\frac{\sin(4fx+4e)ab}{32f}-\frac{\sin(4fx+4e)}{128f}$
derivativedivides	$a^2\left(\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(-\frac{\left(\cos^5(fx+e)\right)\sin(fx+e)}{6}+\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx}{24}\right.$
default	$a^2\left(\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(-\frac{\left(\cos^5(fx+e)\right)\sin(fx+e)}{6}+\frac{\left(\cos^3(fx+e)+\frac{3\cos(fx+e)}{2}\right)\sin(fx}{24}\right.$
norman	$\left(\frac{3}{8}a^2+\frac{1}{8}ab+\frac{3}{128}b^2\right)x+\left(3a^2+ab+\frac{3}{16}b^2\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(3a^2+ab+\frac{3}{16}b^2\right)x\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(21a^2+7ab+\frac{21}{16}b^2\right)x\left(t\right.$

input
```
int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3072*((96*a^2-96*a*b-24*b^2)*sin(4*f*x+4*e)+(768*a^2+96*a*b)*sin(2*f*x+2
*e)-32*a*b*sin(6*f*x+6*e)+3*b^2*sin(8*f*x+8*e)+1152*f*(a^2+1/3*a*b+1/16*b^
2)*x)/f
```

3.293.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.73

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$
$$=\frac{3\,(48\,a^2+16\,ab+3\,b^2)fx+\left(48\,b^2\cos\left(fx+e\right)^7-8\,(16\,ab+9\,b^2)\cos\left(fx+e\right)^5+2\,(48\,a^2+16\,ab+3\,b^2)\right.}{384\,f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/384*(3*(48*a^2 + 16*a*b + 3*b^2)*f*x + (48*b^2*cos(f*x + e)^7 - 8*(16*a*
b + 9*b^2)*cos(f*x + e)^5 + 2*(48*a^2 + 16*a*b + 3*b^2)*cos(f*x + e)^3 + 3
*(48*a^2 + 16*a*b + 3*b^2)*cos(f*x + e))*sin(f*x + e))/f
```

3.293.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(146) = 292.

Time = 0.73 (sec) , antiderivative size = 481, normalized size of antiderivative = 3.08

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$
$$=\begin{cases}\frac{3a^2x\sin^4\left(e+fx\right)}{8}+\frac{3a^2x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}+\frac{3a^2x\cos^4\left(e+fx\right)}{8}+\frac{3a^2\sin^3\left(e+fx\right)\cos\left(e+fx\right)}{8f}+\frac{5a^2\sin\left(e+fx\right)\cos^3\left(e+fx\right)}{8f}+ \\ x(a+b\sin^2\left(e\right))^2\cos^4\left(e\right)\end{cases}$$

input
```
integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2)**2,x)
```

output
```
Piecewise((3*a**2*x*sin(e + f*x)**4/8 + 3*a**2*x*sin(e + f*x)**2*cos(e + f
*x)**2/4 + 3*a**2*x*cos(e + f*x)**4/8 + 3*a**2*sin(e + f*x)**3*cos(e + f*x
)/(8*f) + 5*a**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) + a*b*x*sin(e + f*x)**
6/8 + 3*a*b*x*sin(e + f*x)**4*cos(e + f*x)**2/8 + 3*a*b*x*sin(e + f*x)**2*
cos(e + f*x)**4/8 + a*b*x*cos(e + f*x)**6/8 + a*b*sin(e + f*x)**5*cos(e +
f*x)/(8*f) + a*b*sin(e + f*x)**3*cos(e + f*x)**3/(3*f) - a*b*sin(e + f*x)*
cos(e + f*x)**5/(8*f) + 3*b**2*x*sin(e + f*x)**8/128 + 3*b**2*x*sin(e + f*
x)**6*cos(e + f*x)**2/32 + 9*b**2*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 3
*b**2*x*sin(e + f*x)**2*cos(e + f*x)**6/32 + 3*b**2*x*cos(e + f*x)**8/128
+ 3*b**2*sin(e + f*x)**7*cos(e + f*x)/(128*f) + 11*b**2*sin(e + f*x)**5*co
s(e + f*x)**3/(128*f) - 11*b**2*sin(e + f*x)**3*cos(e + f*x)**5/(128*f) -
3*b**2*sin(e + f*x)*cos(e + f*x)**7/(128*f), Ne(f, 0)), (x*(a + b*sin(e)**
2)**2*cos(e)**4, True))
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{3\left(48a^2+16ab+3b^2\right)(fx+e)+\frac{3\left(48a^2+16ab+3b^2\right)\tan(fx+e)^7+11\left(48a^2+16ab+3b^2\right)\tan(fx+e)^5+\left(624a^2+80ab-33b^2\right)\tan}{\tan(fx+e)^8+4\tan(fx+e)^6+6\tan(fx+e)^4+4\tan(fx+e)}}{384f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/384*(3*(48*a^2 + 16*a*b + 3*b^2)*(f*x + e) + (3*(48*a^2 + 16*a*b + 3*b^2
)*tan(f*x + e)^7 + 11*(48*a^2 + 16*a*b + 3*b^2)*tan(f*x + e)^5 + (624*a^2
+ 80*a*b - 33*b^2)*tan(f*x + e)^3 + 3*(80*a^2 - 16*a*b - 3*b^2)*tan(f*x +
e))/(tan(f*x + e)^8 + 4*tan(f*x + e)^6 + 6*tan(f*x + e)^4 + 4*tan(f*x + e)
^2 + 1))/f
```

3.293.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.67

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{1}{128}\left(48\,a^2+16\,ab+3\,b^2\right)x + \frac{b^2\sin\left(8\,fx+8\,e\right)}{1024\,f} - \frac{ab\sin\left(6\,fx+6\,e\right)}{96\,f} + \frac{\left(4\,a^2-4\,ab-b^2\right)\sin\left(4\,fx+4\,e\right)}{128\,f} + \frac{\left(8\,a^2+ab\right)\sin\left(2\,fx+2\,e\right)}{32\,f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/128*(48*a^2 + 16*a*b + 3*b^2)*x + 1/1024*b^2*sin(8*f*x + 8*e)/f - 1/96*a
*b*sin(6*f*x + 6*e)/f + 1/128*(4*a^2 - 4*a*b - b^2)*sin(4*f*x + 4*e)/f + 1
/32*(8*a^2 + a*b)*sin(2*f*x + 2*e)/f
```

3.293.9 Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = x\left(\frac{3\,a^2}{8}+\frac{a\,b}{8}+\frac{3\,b^2}{128}\right) + \frac{\left(\frac{3\,a^2}{8}+\frac{a\,b}{8}+\frac{3\,b^2}{128}\right)\,\tan(e+f\,x)^7+\left(\frac{11\,a^2}{8}+\frac{11\,a\,b}{24}+\frac{11\,b^2}{128}\right)\,\tan(e+f\,x)^5+\left(\frac{13\,a^2}{8}+\frac{5\,a\,b}{24}-\frac{11\,b^2}{128}\right)\,\tan(e+}{f\,\left(\tan\left(e+f\,x\right)^8+4\tan\left(e+f\,x\right)^6+6\tan\left(e+f\,x\right)^4+4\tan\left(e+f\,x\right.}$$

input
```
int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^2,x)
```

output
```
x*((a*b)/8 + (3*a^2)/8 + (3*b^2)/128) + (tan(e + f*x)^7*((a*b)/8 + (3*a^2)
/8 + (3*b^2)/128) - tan(e + f*x)*((a*b)/8 - (5*a^2)/8 + (3*b^2)/128) + tan
(e + f*x)^3*((5*a*b)/24 + (13*a^2)/8 - (11*b^2)/128) + tan(e + f*x)^5*((11
*a*b)/24 + (11*a^2)/8 + (11*b^2)/128))/(f*(4*tan(e + f*x)^2 + 6*tan(e + f*
x)^4 + 4*tan(e + f*x)^6 + tan(e + f*x)^8 + 1))
```

3.294 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

3.294.1 Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{1}{16}\left(8a^2+4ab+b^2\right)x+\frac{(8a^2+4ab+b^2)\cos(e+fx)\sin(e+fx)}{16f} - \frac{b(8a+3b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{b\cos^5(e+fx)\sin(e+fx)\left(a+(a+b)\tan^2(e+fx)\right)}{6f}$$

output
```
1/16*(8*a^2+4*a*b+b^2)*x+1/16*(8*a^2+4*a*b+b^2)*cos(f*x+e)*sin(f*x+e)/f-1/
24*b*(8*a+3*b)*cos(f*x+e)^3*sin(f*x+e)/f-1/6*b*cos(f*x+e)^5*sin(f*x+e)*(a+
(a+b)*tan(f*x+e)^2)/f
```

3.294.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{12((2-2i)a+b)((2+2i)a+b)(e+fx)+3(4a-b)(4a+b)\sin(2(e+fx))-3b(4a+b)\sin(4(e+fx))}{192f}$$

input
```
Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2,x]
```

output
```
(12*((2 - 2*I)*a + b)*((2 + 2*I)*a + b)*(e + f*x) + 3*(4*a - b)*(4*a + b)*
Sin[2*(e + f*x)] - 3*b*(4*a + b)*Sin[4*(e + f*x)] + b^2*Sin[6*(e + f*x)])/
(192*f)
```

3.294.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3670, 315, 298, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(e+fx)^2\left(a+b\sin(e+fx)^2\right)^2dx$$

$$\Big\downarrow \; 3670$$

$$\frac{\int \frac{\left((a+b)\tan^2(e+fx)+a\right)^2}{\left(\tan^2(e+fx)+1\right)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 315$$

$$\frac{\frac{1}{6}\int \frac{3(a+b)(2a+b)\tan^2(e+fx)+a(6a+b)}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)-\frac{b\tan(e+fx)\left((a+b)\tan^2(e+fx)+a\right)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\Big\downarrow \; 298$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2+4ab+b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)-\frac{b(8a+3b)\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}\right)-\frac{b\tan(e+fx)\left((a+b)\tan^2(e+fx)+a\right)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2+4ab+b^2\right)\left(\frac{1}{2}\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right)-\frac{b(8a+3b)\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}\right)-\frac{b\tan(e+fx)\left((a+b)\tan^2(e+fx)+a\right)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2+4ab+b^2\right)\left(\frac{1}{2}\arctan(\tan(e+fx))+\frac{\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)-\frac{b(8a+3b)\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}\right)-\frac{b\tan(e+fx)\left((a+b)\tan^2(e+fx)+\right.}{6(\tan^2(e+fx)+1)^3}}{f}$$

input
```
Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2,x]
```

output
```
(-1/6*(b*Tan[e + f*x]*(a + (a + b)*Tan[e + f*x]^2))/(1 + Tan[e + f*x]^2)^3
 + (-1/4*(b*(8*a + 3*b)*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^2 + (3*(8*a^2 +
 4*a*b + b^2)*(ArcTan[Tan[e + f*x]]/2 + Tan[e + f*x]/(2*(1 + Tan[e + f*x]^
2))))/4)/6)/f
```

3.294.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.294.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{(48a^2-3b^2)\sin(2fx+2e)+(-12ab-3b^2)\sin(4fx+4e)+b^2\sin(6fx+6e)+96f\left(a^2+\frac{1}{2}ab+\frac{1}{8}b^2\right)x}{192f}$
risch	$\frac{a^2x}{2}+\frac{abx}{4}+\frac{b^2x}{16}+\frac{b^2\sin(6fx+6e)}{192f}-\frac{\sin(4fx+4e)ab}{16f}-\frac{\sin(4fx+4e)b^2}{64f}+\frac{\sin(2fx+2e)a^2}{4f}-\frac{\sin(2fx+2e)b^2}{64f}$
derivativedivides	$\frac{a^2\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab\left(-\frac{\left(\cos^3(fx+e)\right)\sin(fx+e)}{4}+\frac{\cos(fx+e)\sin(fx+e)}{8}+\frac{fx}{8}+\frac{e}{8}\right)+b^2\left(-\frac{\left(\cos^3(fx+e)\right)\left(\mathrm{si}\right.}{6}\right.}{f}$
default	$\frac{a^2\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab\left(-\frac{\left(\cos^3(fx+e)\right)\sin(fx+e)}{4}+\frac{\cos(fx+e)\sin(fx+e)}{8}+\frac{fx}{8}+\frac{e}{8}\right)+b^2\left(-\frac{\left(\cos^3(fx+e)\right)\left(\mathrm{si}\right.}{6}\right.}{f}$
norman	$\left(\frac{1}{2}a^2+\frac{1}{4}ab+\frac{1}{16}b^2\right)x+\left(3a^2+\frac{3}{2}ab+\frac{3}{8}b^2\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(3a^2+\frac{3}{2}ab+\frac{3}{8}b^2\right)x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(10a^2+5ab+\frac{5}{4}b^2\right)x\left(\mathrm{ta}\right.$

input
```
int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/192*((48*a^2-3*b^2)*sin(2*f*x+2*e)+(-12*a*b-3*b^2)*sin(4*f*x+4*e)+b^2*si
n(6*f*x+6*e)+96*f*(a^2+1/2*a*b+1/8*b^2)*x)/f
```

3.294.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

$$\int \cos^2(e+fx) \left(a+b\sin^2(e+fx)\right)^2 dx$$
$$= \frac{3\,(8\,a^2+4\,ab+b^2)fx+\left(8\,b^2\cos\left(fx+e\right)^5-2\,(12\,ab+7\,b^2)\cos\left(fx+e\right)^3+3\,(8\,a^2+4\,ab+b^2)\cos\left(fx+\right.\right.}{48\,f}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/48*(3*(8*a^2 + 4*a*b + b^2)*f*x + (8*b^2*cos(f*x + e)^5 - 2*(12*a*b + 7*
b^2)*cos(f*x + e)^3 + 3*(8*a^2 + 4*a*b + b^2)*cos(f*x + e))*sin(f*x + e))/
f
```

3.294.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(107) = 214.

Time = 0.38 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.71

$$\int \cos^2(e+fx) \left(a+b\sin^2(e+fx)\right)^2 dx$$
$$= \begin{cases} \frac{a^2x\sin^2(e+fx)}{2} + \frac{a^2x\cos^2(e+fx)}{2} + \frac{a^2\sin(e+fx)\cos(e+fx)}{2f} + \frac{abx\sin^4(e+fx)}{4} + \frac{abx\sin^2(e+fx)\cos^2(e+fx)}{2} + \frac{abx\cos^4(e+fx)}{4} \\ x(a+b\sin^2(e))^2\cos^2(e) \end{cases}$$

input
```
integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2)**2,x)
```

output
```
Piecewise((a**2*x*sin(e + f*x)**2/2 + a**2*x*cos(e + f*x)**2/2 + a**2*sin(
e + f*x)*cos(e + f*x)/(2*f) + a*b*x*sin(e + f*x)**4/4 + a*b*x*sin(e + f*x)
**2*cos(e + f*x)**2/2 + a*b*x*cos(e + f*x)**4/4 + a*b*sin(e + f*x)**3*cos(
e + f*x)/(4*f) - a*b*sin(e + f*x)*cos(e + f*x)**3/(4*f) + b**2*x*sin(e + f
*x)**6/16 + 3*b**2*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*b**2*x*sin(e +
 f*x)**2*cos(e + f*x)**4/16 + b**2*x*cos(e + f*x)**6/16 + b**2*sin(e + f*x
)**5*cos(e + f*x)/(16*f) - b**2*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - b*
*2*sin(e + f*x)*cos(e + f*x)**5/(16*f), Ne(f, 0)), (x*(a + b*sin(e)**2)**2
*cos(e)**2, True))
```

3.294.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$
$$=\frac{3\left(8a^2+4ab+b^2\right)(fx+e)+\frac{3\left(8a^2+4ab+b^2\right)\tan(fx+e)^5+8\left(6a^2-b^2\right)\tan(fx+e)^3+3\left(8a^2-4ab-b^2\right)\tan(fx+e)}{\tan(fx+e)^6+3\tan(fx+e)^4+3\tan(fx+e)^2+1}}{48f}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/48*(3*(8*a^2 + 4*a*b + b^2)*(f*x + e) + (3*(8*a^2 + 4*a*b + b^2)*tan(f*x
 + e)^5 + 8*(6*a^2 - b^2)*tan(f*x + e)^3 + 3*(8*a^2 - 4*a*b - b^2)*tan(f*x
 + e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

3.294.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx=\frac{1}{16}\left(8a^2+4ab+b^2\right)x+\frac{b^2\sin(6fx+6e)}{192f}-\frac{\left(4ab+b^2\right)\sin(4fx+4e)}{64f}+\frac{\left(16a^2-b^2\right)\sin(2fx+2e)}{64f}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/16*(8*a^2 + 4*a*b + b^2)*x + 1/192*b^2*sin(6*f*x + 6*e)/f - 1/64*(4*a*b
+ b^2)*sin(4*f*x + 4*e)/f + 1/64*(16*a^2 - b^2)*sin(2*f*x + 2*e)/f
```

3.294.9 Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = x\left(\frac{a^2}{2}+\frac{a\,b}{4}+\frac{b^2}{16}\right) + \frac{\left(\frac{a^2}{2}+\frac{a\,b}{4}+\frac{b^2}{16}\right)\tan(e+f\,x)^5+\left(a^2-\frac{b^2}{6}\right)\tan(e+f\,x)^3+\left(\frac{a^2}{2}-\frac{a\,b}{4}-\frac{b^2}{16}\right)\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^6+3\tan\left(e+f\,x\right)^4+3\tan\left(e+f\,x\right)^2+1\right)}$$

input
```
int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^2,x)
```

output
```
x*((a*b)/4 + a^2/2 + b^2/16) + (tan(e + f*x)^3*(a^2 - b^2/6) - tan(e + f*x
)*((a*b)/4 - a^2/2 + b^2/16) + tan(e + f*x)^5*((a*b)/4 + a^2/2 + b^2/16))/
(f*(3*tan(e + f*x)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1))
```

3.295 $\int (a+b\sin^2(e+fx))^2\,dx$

3.295.1 Optimal result

Integrand size = 14, antiderivative size = 72

$$\int (a+b\sin^2(e+fx))^2\,dx=\frac{1}{8}\left(8a^2+8ab+3b^2\right)x-\frac{b(8a+3b)\cos(e+fx)\sin(e+fx)}{8f}-\frac{b^2\cos(e+fx)\sin^3(e+fx)}{4f}$$

output
```
1/8*(8*a^2+8*a*b+3*b^2)*x-1/8*b*(8*a+3*b)*cos(f*x+e)*sin(f*x+e)/f-1/4*b^2*
cos(f*x+e)*sin(f*x+e)^3/f
```

3.295.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int (a+b\sin^2(e+fx))^2\,dx=\frac{4(8a^2+8ab+3b^2)(e+fx)-8b(2a+b)\sin(2(e+fx))+b^2\sin(4(e+fx))}{32f}$$

input
```
Integrate[(a + b*Sin[e + f*x]^2)^2,x]
```

output
```
(4*(8*a^2 + 8*a*b + 3*b^2)*(e + f*x) - 8*b*(2*a + b)*Sin[2*(e + f*x)] + b^
2*Sin[4*(e + f*x)])/(32*f)
```

3.295.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3658}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin(e+fx)^2\right)^2 dx$$

$$\Big\downarrow \text{3658}$$

$$\frac{1}{8}x\left(8a^2+8ab+3b^2\right)-\frac{b(8a+3b)\sin(e+fx)\cos(e+fx)}{8f}-\frac{b^2\sin^3(e+fx)\cos(e+fx)}{4f}$$

input
```
Int[(a + b*Sin[e + f*x]^2)^2,x]
```

output
```
((8*a^2 + 8*a*b + 3*b^2)*x)/8 - (b*(8*a + 3*b)*Cos[e + f*x]*Sin[e + f*x])/
(8*f) - (b^2*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f)
```

3.295.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3658
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^2, x_Symbol] :> Simp[(8*a^2 +
8*a*b + 3*b^2)*(x/8), x] + (-Simp[b^2*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)),
x] - Simp[b*(8*a + 3*b)*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a,
 b, e, f}, x]
```

3.295.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

method	result
parallelrisch	$\frac{(-16ab-8b^2)\sin(2fx+2e)+\sin(4fx+4e)b^2+32f\left(a^2+ab+\frac{3}{8}b^2\right)x}{32f}$
risch	$a^2x+abx+\frac{3b^2x}{8}+\frac{\sin(4fx+4e)b^2}{32f}-\frac{\sin(2fx+2e)ab}{2f}-\frac{\sin(2fx+2e)b^2}{4f}$
parts	$a^2x+\frac{b^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}+\frac{2ab\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$
derivativedivides	$\frac{b^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+a^2(fx+e)}{f}$
default	$\frac{b^2\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+a^2(fx+e)}{f}$
norman	$\left(a^2+ab+\frac{3}{8}b^2\right)x+\left(a^2+ab+\frac{3}{8}b^2\right)x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(4a^2+4ab+\frac{3}{2}b^2\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(4a^2+4ab+\frac{3}{2}b^2\right)x\left(\tan^6\left(\frac{fx}{2}+\right.\right.$

input
```
int((a+b*sin(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*((-16*a*b-8*b^2)*sin(2*f*x+2*e)+sin(4*f*x+4*e)*b^2+32*f*(a^2+a*b+3/8*
b^2)*x)/f
```

3.295.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int \left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{(8\,a^2+8\,ab+3\,b^2)fx+\left(2\,b^2\cos\left(fx+e\right)^3-(8\,ab+5\,b^2)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8\,f}$$

input
```
integrate((a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/8*((8*a^2 + 8*a*b + 3*b^2)*f*x + (2*b^2*cos(f*x + e)^3 - (8*a*b + 5*b^2)
*cos(f*x + e))*sin(f*x + e))/f
```

3.295.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(61) = 122.

Time = 0.19 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.33

$$\int \left(a+b\sin^2(e+fx)\right)^2 \,dx = \begin{cases} a^2x+abx\sin^2\left(e+fx\right)+abx\cos^2\left(e+fx\right)-\frac{ab\sin\left(e+fx\right)\cos\left(e+fx\right)}{f}+\frac{3b^2x\sin^4\left(e+fx\right)}{8}+\frac{3b^2x\sin^2\left(e+fx\right)\cos^2\left(e+f\right.}{4} \\ x\left(a+b\sin^2\left(e\right)\right)^2 \end{cases}$$

input
```
integrate((a+b*sin(f*x+e)**2)**2,x)
```

output
```
Piecewise((a**2*x + a*b*x*sin(e + f*x)**2 + a*b*x*cos(e + f*x)**2 - a*b*si
n(e + f*x)*cos(e + f*x)/f + 3*b**2*x*sin(e + f*x)**4/8 + 3*b**2*x*sin(e +
f*x)**2*cos(e + f*x)**2/4 + 3*b**2*x*cos(e + f*x)**4/8 - 5*b**2*sin(e + f*
x)**3*cos(e + f*x)/(8*f) - 3*b**2*sin(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f
, 0)), (x*(a + b*sin(e)**2)**2, True))
```

3.295.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \left(a+b\sin^2(e+fx)\right)^2 \,dx = a^2x+\frac{\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)ab}{2\,f} + \frac{\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\,\sin\left(2\,fx+2\,e\right)\right)b^2}{32\,f}$$

input
```
integrate((a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
a^2*x + 1/2*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b/f + 1/32*(12*f*x + 12*e +
 sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b^2/f
```

3.295.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \left(a+b\sin^2(e+fx)\right)^2 \, dx = \frac{1}{8}\left(8a^2+8ab+3b^2\right)x + \frac{b^2\sin\left(4fx+4e\right)}{32f} - \frac{\left(2ab+b^2\right)\sin\left(2fx+2e\right)}{4f}$$

input
```
integrate((a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/8*(8*a^2 + 8*a*b + 3*b^2)*x + 1/32*b^2*sin(4*f*x + 4*e)/f - 1/4*(2*a*b +
 b^2)*sin(2*f*x + 2*e)/f
```

3.295.9 Mupad [B] (verification not implemented)

Time = 14.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int \left(a+b\sin^2(e+fx)\right)^2 \, dx = x\left(a^2+a\,b+\frac{3\,b^2}{8}\right) - \frac{\left(\frac{5\,b^2}{8}+a\,b\right)\,\tan(e+f\,x)^3+\left(\frac{3\,b^2}{8}+a\,b\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^4+2\tan\left(e+f\,x\right)^2+1\right)}$$

input
```
int((a + b*sin(e + f*x)^2)^2,x)
```

output
```
x*(a*b + a^2 + (3*b^2)/8) - (tan(e + f*x)*(a*b + (3*b^2)/8) + tan(e + f*x)
^3*(a*b + (5*b^2)/8))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1))
```

3.296 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

3.296.1 Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx=-\frac{1}{2}b(4a+3b)x+\frac{b^2\cos(e+fx)\sin(e+fx)}{2f}+\frac{(a+b)^2\tan(e+fx)}{f}$$

output
```
-1/2*b*(4*a+3*b)*x+1/2*b^2*cos(f*x+e)*sin(f*x+e)/f+(a+b)^2*tan(f*x+e)/f
```

3.296.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx=\frac{-2b(4a+3b)(e+fx)+b^2\sin(2(e+fx))+4(a+b)^2\tan(e+fx)}{4f}$$

input
```
Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2,x]
```

output
```
(-2*b*(4*a + 3*b)*(e + f*x) + b^2*Sin[2*(e + f*x)] + 4*(a + b)^2*Tan[e + f
*x])/(4*f)
```

3.296.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^2}{\cos(e+fx)^2}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \frac{\left((a+b)\tan^2(e+fx)+a\right)^2}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left((a+b)^2-\frac{2b(a+b)\tan^2(e+fx)+b(2a+b)}{\left(\tan^2(e+fx)+1\right)^2}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}b(4a+3b)\arctan(\tan(e+fx))+(a+b)^2\tan(e+fx)+\frac{b^2\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}}{f}$$

input
```
Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2,x]
```

output
```
(-1/2*(b*(4*a + 3*b)*ArcTan[Tan[e + f*x]]) + (a + b)^2*Tan[e + f*x] + (b^2
*Tan[e + f*x])/(2*(1 + Tan[e + f*x]^2)))/f
```

3.296.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.296.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

method	result
parallelrisch	$\frac{b^2\sin(3fx+3e)-16bf\left(a+\frac{3b}{4}\right)x\cos(fx+e)+8\sin(fx+e)\left(a^2+2ab+\frac{9}{8}b^2\right)}{8f\cos(fx+e)}$
derivativedivides	$\frac{a^2\tan(fx+e)+2ab(\tan(fx+e)-fx-e)+b^2\left(\frac{\sin^5(fx+e)}{\cos(fx+e)}+\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\frac{3e}{2}\right)}{f}$
default	$\frac{a^2\tan(fx+e)+2ab(\tan(fx+e)-fx-e)+b^2\left(\frac{\sin^5(fx+e)}{\cos(fx+e)}+\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\frac{3e}{2}\right)}{f}$
risch	$-2abx-\frac{3b^2x}{2}-\frac{ib^2\mathrm{e}^{2i(fx+e)}}{8f}+\frac{ib^2\mathrm{e}^{-2i(fx+e)}}{8f}+\frac{2ia^2}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)}+\frac{4iab}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)}+\frac{2ib^2}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)}$
norman	$(2ab+\frac{3}{2}b^2)x+(-6ab-\frac{9}{2}b^2)x\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-2ab-\frac{3}{2}b^2)x\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(6ab+\frac{9}{2}b^2)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(-4ab$

input
```
int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(b^2*sin(3*f*x+3*e)-16*b*f*(a+3/4*b)*x*cos(f*x+e)+8*sin(f*x+e)*(a^2+2*
a*b+9/8*b^2))/f/cos(f*x+e)
```

3.296.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.33

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$
$$= -\frac{(4\,ab+3\,b^2)fx\cos(fx+e)-\left(b^2\cos(fx+e)^2+2\,a^2+4\,ab+2\,b^2\right)\sin(fx+e)}{2\,f\cos(fx+e)}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*((4*a*b + 3*b^2)*f*x*cos(f*x + e) - (b^2*cos(f*x + e)^2 + 2*a^2 + 4*a
*b + 2*b^2)*sin(f*x + e))/(f*cos(f*x + e))
```

3.296.6 Sympy [F]

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx=\int \left(a+b\sin^2(e+fx)\right)^2\sec^2(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sin(e + f*x)**2)**2*sec(e + f*x)**2, x)
```

3.296.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.45

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx=$$
$$-\frac{4\,(fx+e-\tan(fx+e))ab+\left(3\,fx+3\,e-\frac{\tan(fx+e)}{\tan(fx+e)^2+1}-2\,\tan(fx+e)\right)b^2-2\,a^2\tan(fx+e)}{2\,f}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(4*(f*x + e - tan(f*x + e))*a*b + (3*f*x + 3*e - tan(f*x + e)/(tan(f*
x + e)^2 + 1) - 2*tan(f*x + e))*b^2 - 2*a^2*tan(f*x + e))/f
```

3.296.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.51

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$
$$=\frac{2\,a^2\tan\left(fx+e\right)+4\,ab\tan\left(fx+e\right)+2\,b^2\tan\left(fx+e\right)-(4\,ab+3\,b^2)(fx+e)+\frac{b^2\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(2*a^2*tan(f*x + e) + 4*a*b*tan(f*x + e) + 2*b^2*tan(f*x + e) - (4*a*b
 + 3*b^2)*(f*x + e) + b^2*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

3.296.9 Mupad [B] (verification not implemented)

Time = 13.84 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.45

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx=\frac{\tan(e+f\,x)\,\left(a+b\right)^2}{f}+\frac{b^2\,\sin\left(2\,e+2\,f\,x\right)}{4\,f}-\frac{b\,\mathrm{atan}\left(\frac{b\tan(e+f\,x)\,(4\,a+3\,b)}{2\,\left(\frac{3\,b^2}{2}+2\,a\,b\right)}\right)\,\left(4\,a+3\,b\right)}{2\,f}$$

input
```
int((a + b*sin(e + f*x)^2)^2/cos(e + f*x)^2,x)
```

output
```
(tan(e + f*x)*(a + b)^2)/f + (b^2*sin(2*e + 2*f*x))/(4*f) - (b*atan((b*tan
(e + f*x)*(4*a + 3*b))/(2*(2*a*b + (3*b^2)/2)))*(4*a + 3*b))/(2*f)
```

3.297 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

3.297.1 Optimal result

Integrand size = 23, antiderivative size = 45

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx=b^2x+\frac{(a^2-b^2)\tan(e+fx)}{f}+\frac{(a+b)^2\tan^3(e+fx)}{3f}$$

output
```
b^2*x+(a^2-b^2)*tan(f*x+e)/f+1/3*(a+b)^2*tan(f*x+e)^3/f
```

3.297.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$
$$=\frac{3b^2\arctan(\tan(e+fx))+(a+b)(2a-b+(a-2b)\cos(2(e+fx)))\sec^2(e+fx)\tan(e+fx)}{3f}$$

input
```
Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^2,x]
```

output
```
(3*b^2*ArcTan[Tan[e + f*x]] + (a + b)*(2*a - b + (a - 2*b)*Cos[2*(e + f*x)
])*Sec[e + f*x]^2*Tan[e + f*x])/(3*f)
```

3.297.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^2}{\cos(e+fx)^4}dx$$

$$\Big\downarrow \ 3670$$

$$\frac{\int \frac{\left((a+b)\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 300$$

$$\frac{\int \left(a^2-b^2+(a+b)^2\tan^2(e+fx)+\frac{b^2}{\tan^2(e+fx)+1}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 2009$$

$$\frac{\left(a^2-b^2\right)\tan(e+fx)+\frac{1}{3}(a+b)^2\tan^3(e+fx)+b^2\arctan(\tan(e+fx))}{f}$$

input
```
Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^2,x]
```

output
```
(b^2*ArcTan[Tan[e + f*x]] + (a^2 - b^2)*Tan[e + f*x] + ((a + b)^2*Tan[e +
f*x]^3)/3)/f
```

3.297.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.297.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.69

method	result
derivativedivides	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+\frac{2ab\left(\sin^3(fx+e)\right)}{3\cos(fx+e)^3}+b^2\left(\frac{\left(\tan^3(fx+e)\right)}{3}-\tan(fx+e)+fx+e\right)}{f}$
default	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+\frac{2ab\left(\sin^3(fx+e)\right)}{3\cos(fx+e)^3}+b^2\left(\frac{\left(\tan^3(fx+e)\right)}{3}-\tan(fx+e)+fx+e\right)}{f}$
risch	$b^2x+\frac{4i\left(-3ab\,e^{4i(fx+e)}-3b^2e^{4i(fx+e)}+3a^2e^{2i(fx+e)}-3b^2e^{2i(fx+e)}+a^2-ab-2b^2\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$
parallelrisch	$\frac{3x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)b^2f+\left(-6a^2+6b^2\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-9x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)b^2f+4(a+b)(a-5b)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+9x\left(\text{ta}\right.}{3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$
norman	$\frac{b^2x\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+b^2x\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-b^2x-b^2x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+3b^2x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+3b^2x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-}{}$

input
```
int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2/3*a*b*sin(f*x+e)^3/cos(f*x+
e)^3+b^2*(1/3*tan(f*x+e)^3-tan(f*x+e)+f*x+e))
```

3.297.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.56

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{3\,b^2fx\cos\left(fx+e\right)^3+\left(2\left(a^2-ab-2\,b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*b^2*f*x*cos(f*x + e)^3 + (2*(a^2 - a*b - 2*b^2)*cos(f*x + e)^2 + a^
2 + 2*a*b + b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

3.297.6 Sympy [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.297.7 Maxima [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{\left(a^2+2\,ab+b^2\right)\tan\left(fx+e\right)^3+3\left(fx+e\right)b^2+3\left(a^2-b^2\right)\tan\left(fx+e\right)}{3\,f}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/3*((a^2 + 2*a*b + b^2)*tan(f*x + e)^3 + 3*(f*x + e)*b^2 + 3*(a^2 - b^2)*
tan(f*x + e))/f
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.64

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$
$$=\frac{a^2\tan\left(fx+e\right)^3+2\,ab\tan\left(fx+e\right)^3+b^2\tan\left(fx+e\right)^3+3\left(fx+e\right)b^2+3\,a^2\tan\left(fx+e\right)-3\,b^2\tan\left(fx\right.}{3\,f}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/3*(a^2*tan(f*x + e)^3 + 2*a*b*tan(f*x + e)^3 + b^2*tan(f*x + e)^3 + 3*(f
*x + e)*b^2 + 3*a^2*tan(f*x + e) - 3*b^2*tan(f*x + e))/f
```

3.297.9 Mupad [B] (verification not implemented)

Time = 13.77 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$
$$=\frac{\frac{\tan(e+f\,x)^3\,(a+b)^2}{3}-\tan(e+f\,x)\,\left((a+b)^2-2\,a\,(a+b)\right)+b^2\,f\,x}{f}$$

input
```
int((a + b*sin(e + f*x)^2)^2/cos(e + f*x)^4,x)
```

output
```
((tan(e + f*x)^3*(a + b)^2)/3 - tan(e + f*x)*((a + b)^2 - 2*a*(a + b)) + b
^2*f*x)/f
```

3.298 $\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

3.298.1 Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx=\frac{a^2\tan(e+fx)}{f}+\frac{2a(a+b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan^5(e+fx)}{5f}$$

output
```
a^2*tan(f*x+e)/f+2/3*a*(a+b)*tan(f*x+e)^3/f+1/5*(a+b)^2*tan(f*x+e)^5/f
```

3.298.2 Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

$$\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx=\frac{(8a^2-4ab+3b^2+(4a^2-2ab-6b^2)\sec^2(e+fx)+3(a+b)^2\sec^4(e+fx))\tan(e+fx)}{15f}$$

input
```
Integrate[Sec[e + f*x]^6*(a + b*Sin[e + f*x]^2)^2,x]
```

output
```
((8*a^2 - 4*a*b + 3*b^2 + (4*a^2 - 2*a*b - 6*b^2)*Sec[e + f*x]^2 + 3*(a +
b)^2*Sec[e + f*x]^4)*Tan[e + f*x])/(15*f)
```

3.298.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3670, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^2}{\cos(e+fx)^6}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \left((a+b)\tan^2(e+fx)+a\right)^2d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{210}$$

$$\frac{\int \left((a+b)^2\tan^4(e+fx)+2a(a+b)\tan^2(e+fx)+a^2\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\tan(e+fx)+\frac{1}{5}(a+b)^2\tan^5(e+fx)+\frac{2}{3}a(a+b)\tan^3(e+fx)}{f}$$

input
```
Int[Sec[e + f*x]^6*(a + b*Sin[e + f*x]^2)^2,x]
```

output
```
(a^2*Tan[e + f*x] + (2*a*(a + b)*Tan[e + f*x]^3)/3 + ((a + b)^2*Tan[e + f*
x]^5)/5)/f
```

3.298.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.298.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(49) = 98$.

Time = 1.01 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.91

method	result
derivativedivides	$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)+2ab\left(\frac{\sin^3(fx+e)}{5\cos(fx+e)^5}+\frac{2\left(\sin^3(fx+e)\right)}{15\cos(fx+e)^3}\right)+\frac{b^2\left(\sin^5(fx+e)\right)}{5\cos(fx+e)^5}}{f}$
default	$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)+2ab\left(\frac{\sin^3(fx+e)}{5\cos(fx+e)^5}+\frac{2\left(\sin^3(fx+e)\right)}{15\cos(fx+e)^3}\right)+\frac{b^2\left(\sin^5(fx+e)\right)}{5\cos(fx+e)^5}}{f}$
parallelrisch	$-\frac{2\left(a^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{4a(a-2b)\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\left(\frac{58}{15}a^2+\frac{16}{15}ab+\frac{16}{5}b^2\right)\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{4a(a-2b)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+a\right.}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}$
risch	$\frac{2i(15b^2e^{8i(fx+e)}-60ab\,e^{6i(fx+e)}+80a^2e^{4i(fx+e)}+20ab\,e^{4i(fx+e)}+30b^2e^{4i(fx+e)}+40a^2e^{2i(fx+e)}-20ab\,e^{2i(fx+e)}+8a^2}{15f\left(e^{2i(fx+e)}+1\right)^5}$

input
```
int(sec(f*x+e)^6*(a+b*sin(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a^2*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)+2*a*b*(1/5
*sin(f*x+e)^3/cos(f*x+e)^5+2/15*sin(f*x+e)^3/cos(f*x+e)^3)+1/5*b^2*sin(f*x
+e)^5/cos(f*x+e)^5)
```

3.298.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.57

$$\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{\left((8a^2-4ab+3b^2)\cos(fx+e)^4+2(2a^2-ab-3b^2)\cos(fx+e)^2+3a^2+6ab+3b^2\right)\sin(fx+e)}{15f\cos(fx+e)^5}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/15*((8*a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^4 + 2*(2*a^2 - a*b - 3*b^2)*cos
(f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)*sin(f*x + e)/(f*cos(f*x + e)^5)
```

3.298.6 Sympy [F(-1)]

Timed out.

$$\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**6*(a+b*sin(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.298.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{3(a^2+2ab+b^2)\tan(fx+e)^5+10(a^2+ab)\tan(fx+e)^3+15a^2\tan(fx+e)}{15f}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/15*(3*(a^2 + 2*a*b + b^2)*tan(f*x + e)^5 + 10*(a^2 + a*b)*tan(f*x + e)^3
 + 15*a^2*tan(f*x + e))/f
```

3.298.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$
$$=\frac{3\,a^2\tan\left(fx+e\right)^5+6\,ab\tan\left(fx+e\right)^5+3\,b^2\tan\left(fx+e\right)^5+10\,a^2\tan\left(fx+e\right)^3+10\,ab\tan\left(fx+e\right)^3+1}{15\,f}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/15*(3*a^2*tan(f*x + e)^5 + 6*a*b*tan(f*x + e)^5 + 3*b^2*tan(f*x + e)^5 +
 10*a^2*tan(f*x + e)^3 + 10*a*b*tan(f*x + e)^3 + 15*a^2*tan(f*x + e))/f
```

3.298.9 Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx=\frac{a^2\tan(e+f\,x)+\frac{\tan(e+f\,x)^5\,(a+b)^2}{5}+\frac{2\,a\tan(e+f\,x)^3\,(a+b)}{3}}{f}$$

input
```
int((a + b*sin(e + f*x)^2)^2/cos(e + f*x)^6,x)
```

output
```
(a^2*tan(e + f*x) + (tan(e + f*x)^5*(a + b)^2)/5 + (2*a*tan(e + f*x)^3*(a
+ b))/3)/f
```

3.299 $\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

3.299.1 Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx=\frac{a^2\tan(e+fx)}{f}+\frac{a(3a+2b)\tan^3(e+fx)}{3f}+\frac{(a+b)(3a+b)\tan^5(e+fx)}{5f}+\frac{(a+b)^2\tan^7(e+fx)}{7f}$$

output
```
a^2*tan(f*x+e)/f+1/3*a*(3*a+2*b)*tan(f*x+e)^3/f+1/5*(a+b)*(3*a+b)*tan(f*x+
e)^5/f+1/7*(a+b)^2*tan(f*x+e)^7/f
```

3.299.2 Mathematica [A] (verified)

Time = 3.01 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.15

$$\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$
$$=\frac{(48a^2-16ab+6b^2+(24a^2-8ab+3b^2)\sec^2(e+fx)+6(3a^2-ab-4b^2)\sec^4(e+fx)+15(a+b)^2\sec^6}{105f}$$

input
```
Integrate[Sec[e + f*x]^8*(a + b*Sin[e + f*x]^2)^2,x]
```

output
```
((48*a^2 - 16*a*b + 6*b^2 + (24*a^2 - 8*a*b + 3*b^2)*Sec[e + f*x]^2 + 6*(3
*a^2 - a*b - 4*b^2)*Sec[e + f*x]^4 + 15*(a + b)^2*Sec[e + f*x]^6)*Tan[e +
f*x])/(105*f)
```

3.299.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3670, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^2}{\cos(e+fx)^8}dx$$

$$\Big\downarrow \text{3670}$$

$$\frac{\int \left(\tan^2(e+fx)+1\right)\left((a+b)\tan^2(e+fx)+a\right)^2 d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{290}$$

$$\frac{\int \left((a+b)^2\tan^6(e+fx)+(a+b)(3a+b)\tan^4(e+fx)+a(3a+2b)\tan^2(e+fx)+a^2\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\tan(e+fx)+\frac{1}{7}(a+b)^2\tan^7(e+fx)+\frac{1}{5}(a+b)(3a+b)\tan^5(e+fx)+\frac{1}{3}a(3a+2b)\tan^3(e+fx)}{f}$$

input
```
Int[Sec[e + f*x]^8*(a + b*Sin[e + f*x]^2)^2,x]
```

output
```
(a^2*Tan[e + f*x] + (a*(3*a + 2*b)*Tan[e + f*x]^3)/3 + ((a + b)*(3*a + b)*
Tan[e + f*x]^5)/5 + ((a + b)^2*Tan[e + f*x]^7)/7)/f
```

3.299.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.299.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.86

method	result
derivativedivides	$\frac{-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(fx+e)\right)}{7}-\frac{6\left(\sec^4(fx+e)\right)}{35}-\frac{8\left(\sec^2(fx+e)\right)}{35}\right)\tan(fx+e)+2ab\left(\frac{\sin^3(fx+e)}{7\cos(fx+e)^7}+\frac{4\left(\sin^3(fx+e)\right)}{35\cos(fx+e)^5}+\frac{8\left(\sin^3(fx+\cdots}{105\cos(fx+\cdots}\cdots\right.}{f}$
default	$\frac{-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(fx+e)\right)}{7}-\frac{6\left(\sec^4(fx+e)\right)}{35}-\frac{8\left(\sec^2(fx+e)\right)}{35}\right)\tan(fx+e)+2ab\left(\frac{\sin^3(fx+e)}{7\cos(fx+e)^7}+\frac{4\left(\sin^3(fx+e)\right)}{35\cos(fx+e)^5}+\frac{8\left(\sin^3(fx+\cdots}{105\cos(fx+\cdots}\cdots\right.}{f}$
parallelrisch	$-\frac{2\left(\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a^2-2\left(a-\frac{4b}{3}\right)a\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{\left(43a^2+\frac{32}{3}ab+16b^2\right)\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{4\left(-53a^2+76ab+24b^2\right)\left(\tan\cdots\right.}{35}\cdots\right.}{f\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cdots}$
risch	$\frac{4i\left(105b^2e^{10i(fx+e)}-560ab\,e^{8i(fx+e)}-105b^2e^{8i(fx+e)}+840a^2e^{6i(fx+e)}+280ab\,e^{6i(fx+e)}+210b^2e^{6i(fx+e)}+504a^2e^{4i(fx\cdots}\right.}{105f\left(e^{2i(fx+e)}+1\cdots\right.}$

input
```
int(sec(f*x+e)^8*(a+b*sin(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a^2*(-16/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*ta
n(f*x+e)+2*a*b*(1/7*sin(f*x+e)^3/cos(f*x+e)^7+4/35*sin(f*x+e)^3/cos(f*x+e)
^5+8/105*sin(f*x+e)^3/cos(f*x+e)^3)+b^2*(1/7*sin(f*x+e)^5/cos(f*x+e)^7+2/3
5*sin(f*x+e)^5/cos(f*x+e)^5))
```

3.299.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.35

$$\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$
$$= \frac{\left(2\left(24a^2-8ab+3b^2\right)\cos\left(fx+e\right)^6+\left(24a^2-8ab+3b^2\right)\cos\left(fx+e\right)^4+6\left(3a^2-ab-4b^2\right)\cos\left(fx+\right.\right.}{105f\cos\left(fx+e\right)^7}$$

input
```
integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/105*(2*(24*a^2 - 8*a*b + 3*b^2)*cos(f*x + e)^6 + (24*a^2 - 8*a*b + 3*b^2
)*cos(f*x + e)^4 + 6*(3*a^2 - a*b - 4*b^2)*cos(f*x + e)^2 + 15*a^2 + 30*a*
b + 15*b^2)*sin(f*x + e)/(f*cos(f*x + e)^7)
```

3.299.6 Sympy [F(-1)]

Timed out.

$$\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**8*(a+b*sin(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.299.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$
$$=\frac{15\left(a^2+2\,ab+b^2\right)\tan\left(fx+e\right)^7+21\left(3\,a^2+4\,ab+b^2\right)\tan\left(fx+e\right)^5+35\left(3\,a^2+2\,ab\right)\tan\left(fx+e\right)^3+10}{105\,f}$$

input
```
integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/105*(15*(a^2 + 2*a*b + b^2)*tan(f*x + e)^7 + 21*(3*a^2 + 4*a*b + b^2)*ta
n(f*x + e)^5 + 35*(3*a^2 + 2*a*b)*tan(f*x + e)^3 + 105*a^2*tan(f*x + e))/f
```

3.299.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.48

$$\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$
$$=\frac{15\,a^2\tan\left(fx+e\right)^7+30\,ab\tan\left(fx+e\right)^7+15\,b^2\tan\left(fx+e\right)^7+63\,a^2\tan\left(fx+e\right)^5+84\,ab\tan\left(fx+e\right)^5}{105\,f}$$

input
```
integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/105*(15*a^2*tan(f*x + e)^7 + 30*a*b*tan(f*x + e)^7 + 15*b^2*tan(f*x + e)
^7 + 63*a^2*tan(f*x + e)^5 + 84*a*b*tan(f*x + e)^5 + 21*b^2*tan(f*x + e)^5
 + 105*a^2*tan(f*x + e)^3 + 70*a*b*tan(f*x + e)^3 + 105*a^2*tan(f*x + e))/
f
```

3.299.9 Mupad [B] (verification not implemented)

Time = 14.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$
$$=\frac{a^2\tan(e+f\,x)+\frac{\tan(e+f\,x)^7\,(a+b)^2}{7}+\tan(e+f\,x)^5\left(\frac{3\,a^2}{5}+\frac{4\,a\,b}{5}+\frac{b^2}{5}\right)+\frac{a\tan(e+f\,x)^3\,(3\,a+2\,b)}{3}}{f}$$

input
```
int((a + b*sin(e + f*x)^2)^2/cos(e + f*x)^8,x)
```

output
```
(a^2*tan(e + f*x) + (tan(e + f*x)^7*(a + b)^2)/7 + tan(e + f*x)^5*((4*a*b)
/5 + (3*a^2)/5 + b^2/5) + (a*tan(e + f*x)^3*(3*a + 2*b))/3)/f
```

3.299. $\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

3.300 $\int \sec^{10}(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

3.300.1 Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \sec^{10}(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx=\frac{a^2\tan(e+fx)}{f}+\frac{2a(2a+b)\tan^3(e+fx)}{3f}+\frac{(6a^2+6ab+b^2)\tan^5(e+fx)}{5f}+\frac{2(a+b)(2a+b)\tan^7(e+fx)}{7f}+\frac{(a+b)^2\tan^9(e+fx)}{9f}$$

output
```
a^2*tan(f*x+e)/f+2/3*a*(2*a+b)*tan(f*x+e)^3/f+1/5*(6*a^2+6*a*b+b^2)*tan(f*
x+e)^5/f+2/7*(a+b)*(2*a+b)*tan(f*x+e)^7/f+1/9*(a+b)^2*tan(f*x+e)^9/f
```

3.300.2 Mathematica [A] (verified)

Time = 6.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.01

$$\int \sec^{10}(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$
$$=\frac{\sec^9(e+fx)\left(252(8a^2+8ab+3b^2)\sin(e+fx)+336(4a^2-ab-b^2)\sin(3(e+fx))+(16a^2-4ab+b^2)\right.}{10080f}$$

input
```
Integrate[Sec[e + f*x]^10*(a + b*Sin[e + f*x]^2)^2,x]
```

output
```
(Sec[e + f*x]^9*(252*(8*a^2 + 8*a*b + 3*b^2)*Sin[e + f*x] + 336*(4*a^2 - a
*b - b^2)*Sin[3*(e + f*x)] + (16*a^2 - 4*a*b + b^2)*(36*Sin[5*(e + f*x)] +
 9*Sin[7*(e + f*x)] + Sin[9*(e + f*x)])))/(10080*f)
```

3.300.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3670, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{10}(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^2}{\cos(e+fx)^{10}}dx$$

$$\Big\downarrow \text{ 3670}$$

$$\frac{\int \left(\tan^2(e+fx)+1\right)^2\left((a+b)\tan^2(e+fx)+a\right)^2d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 290}$$

$$\frac{\int \left((a+b)^2\tan^8(e+fx)+2(a+b)(2a+b)\tan^6(e+fx)+\left(6a^2+6ba+b^2\right)\tan^4(e+fx)+2a(2a+b)\tan^2(e+\right.}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{5}\left(6a^2+6ab+b^2\right)\tan^5(e+fx)+a^2\tan(e+fx)+\frac{1}{9}(a+b)^2\tan^9(e+fx)+\frac{2}{7}(a+b)(2a+b)\tan^7(e+fx)+\frac{2}{3}a}{f}$$

input
```
Int[Sec[e + f*x]^10*(a + b*Sin[e + f*x]^2)^2,x]
```

output
```
(a^2*Tan[e + f*x] + (2*a*(2*a + b)*Tan[e + f*x]^3)/3 + ((6*a^2 + 6*a*b + b
^2)*Tan[e + f*x]^5)/5 + (2*(a + b)*(2*a + b)*Tan[e + f*x]^7)/7 + ((a + b)^
2*Tan[e + f*x]^9)/9)/f
```

3.300.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.300.4 Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.84

method	result
derivativedivides	$\frac{-a^2\left(-\frac{128}{315}-\frac{\left(\sec^8(fx+e)\right)}{9}-\frac{8\left(\sec^6(fx+e)\right)}{63}-\frac{16\left(\sec^4(fx+e)\right)}{105}-\frac{64\left(\sec^2(fx+e)\right)}{315}\right)\tan(fx+e)+2ab\left(\frac{\sin^3(fx+e)}{9\cos(fx+e)^9}+\frac{2(\sin^3(\ldots}{21\cos(\ldots}\right.}{f}$
default	$\frac{-a^2\left(-\frac{128}{315}-\frac{\left(\sec^8(fx+e)\right)}{9}-\frac{8\left(\sec^6(fx+e)\right)}{63}-\frac{16\left(\sec^4(fx+e)\right)}{105}-\frac{64\left(\sec^2(fx+e)\right)}{315}\right)\tan(fx+e)+2ab\left(\frac{\sin^3(fx+e)}{9\cos(fx+e)^9}+\frac{2(\sin^3(\ldots}{21\cos(\ldots}\right.}{f}$
parallelrisch	$-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(a^2\left(\tan^{16}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{8a(a-b)\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\frac{4\left(19a^2+4ab+4b^2\right)\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{8\left(-71a^2+79ab+2\ldots\right.}{}\right.}{}$
risch	$\frac{16i\left(210b^2e^{12i(fx+e)}-1260ab\,e^{10i(fx+e)}-315b^2e^{10i(fx+e)}+2016a^2e^{8i(fx+e)}+756ab\,e^{8i(fx+e)}+441b^2e^{8i(fx+e)}+1344a^{\ldots}\right.}{}$

input
```
int(sec(f*x+e)^10*(a+b*sin(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a^2*(-128/315-1/9*sec(f*x+e)^8-8/63*sec(f*x+e)^6-16/105*sec(f*x+e)^4
-64/315*sec(f*x+e)^2)*tan(f*x+e)+2*a*b*(1/9*sin(f*x+e)^3/cos(f*x+e)^9+2/21
*sin(f*x+e)^3/cos(f*x+e)^7+8/105*sin(f*x+e)^3/cos(f*x+e)^5+16/315*sin(f*x+
e)^3/cos(f*x+e)^3)+b^2*(1/9*sin(f*x+e)^5/cos(f*x+e)^9+4/63*sin(f*x+e)^5/co
s(f*x+e)^7+8/315*sin(f*x+e)^5/cos(f*x+e)^5))
```

3.300.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.21

$$\int \sec^{10}(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{\left(8\left(16a^2-4ab+b^2\right)\cos\left(fx+e\right)^8+4\left(16a^2-4ab+b^2\right)\cos\left(fx+e\right)^6+3\left(16a^2-4ab+b^2\right)\cos\left(fx+e\right.}{315f\cos\left(fx+e\right)^9}$$

input
```
integrate(sec(f*x+e)^10*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/315*(8*(16*a^2 - 4*a*b + b^2)*cos(f*x + e)^8 + 4*(16*a^2 - 4*a*b + b^2)*
cos(f*x + e)^6 + 3*(16*a^2 - 4*a*b + b^2)*cos(f*x + e)^4 + 10*(4*a^2 - a*b
 - 5*b^2)*cos(f*x + e)^2 + 35*a^2 + 70*a*b + 35*b^2)*sin(f*x + e)/(f*cos(f
*x + e)^9)
```

3.300.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{10}(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**10*(a+b*sin(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.300.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.97

$$\int \sec^{10}(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{35\left(a^2+2ab+b^2\right)\tan\left(fx+e\right)^9+90\left(2a^2+3ab+b^2\right)\tan\left(fx+e\right)^7+63\left(6a^2+6ab+b^2\right)\tan\left(fx+e\right)^5}{315f}$$

input
```
integrate(sec(f*x+e)^10*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/315*(35*(a^2 + 2*a*b + b^2)*tan(f*x + e)^9 + 90*(2*a^2 + 3*a*b + b^2)*ta
n(f*x + e)^7 + 63*(6*a^2 + 6*a*b + b^2)*tan(f*x + e)^5 + 210*(2*a^2 + a*b)
*tan(f*x + e)^3 + 315*a^2*tan(f*x + e))/f
```

3.300.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.47

$$\int \sec^{10}(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{35a^2\tan\left(fx+e\right)^9+70ab\tan\left(fx+e\right)^9+35b^2\tan\left(fx+e\right)^9+180a^2\tan\left(fx+e\right)^7+270ab\tan\left(fx+e\right.}{}$$

input
```
integrate(sec(f*x+e)^10*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/315*(35*a^2*tan(f*x + e)^9 + 70*a*b*tan(f*x + e)^9 + 35*b^2*tan(f*x + e)
^9 + 180*a^2*tan(f*x + e)^7 + 270*a*b*tan(f*x + e)^7 + 90*b^2*tan(f*x + e)
^7 + 378*a^2*tan(f*x + e)^5 + 378*a*b*tan(f*x + e)^5 + 63*b^2*tan(f*x + e)
^5 + 420*a^2*tan(f*x + e)^3 + 210*a*b*tan(f*x + e)^3 + 315*a^2*tan(f*x + e
))/f
```

3.300.9 Mupad [B] (verification not implemented)

Time = 14.00 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \sec^{10}(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$$
$$=\frac{a^2\tan(e+fx)+\frac{\tan(e+fx)^9(a+b)^2}{9}+\tan(e+fx)^5\left(\frac{6a^2}{5}+\frac{6ab}{5}+\frac{b^2}{5}\right)+\tan(e+fx)^7\left(\frac{4a^2}{7}+\frac{6ab}{7}+\frac{2b^2}{7}\right)+}{f}$$

input
```
int((a + b*sin(e + f*x)^2)^2/cos(e + f*x)^10,x)
```

output
```
(a^2*tan(e + f*x) + (tan(e + f*x)^9*(a + b)^2)/9 + tan(e + f*x)^5*((6*a*b)
/5 + (6*a^2)/5 + b^2/5) + tan(e + f*x)^7*((6*a*b)/7 + (4*a^2)/7 + (2*b^2)/
7) + (2*a*tan(e + f*x)^3*(2*a + b))/3)/f
```

3.301 $\int \frac{\cos^7(x)}{a+b\sin^2(x)}\,dx$

3.301.1 Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{\cos^7(x)}{a+b\sin^2(x)}\,dx=\frac{(a+b)^3\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}-\frac{(a^2+3ab+3b^2)\sin(x)}{b^3}+\frac{(a+3b)\sin^3(x)}{3b^2}-\frac{\sin^5(x)}{5b}$$

output
```
-(a^2+3*a*b+3*b^2)*sin(x)/b^3+1/3*(a+3*b)*sin(x)^3/b^2-1/5*sin(x)^5/b+(a+b
)^3*arctan(sin(x)*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

3.301.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.40

$$\int \frac{\cos^7(x)}{a+b\sin^2(x)}\,dx=\frac{-120(a+b)^3\arctan\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)+120(a+b)^3\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)-2\sqrt{a}\sqrt{b}(120a^2+340ab+309b^2+4b(5a}{240\sqrt{a}b^{7/2}}$$

input
```
Integrate[Cos[x]^7/(a + b*Sin[x]^2),x]
```

output
```
(-120*(a + b)^3*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]] + 120*(a + b)^3*ArcTan[(S
qrt[b]*Sin[x])/Sqrt[a]] - 2*Sqrt[a]*Sqrt[b]*(120*a^2 + 340*a*b + 309*b^2 +
 4*b*(5*a + 12*b)*Cos[2*x] + 3*b^2*Cos[4*x])*Sin[x])/(240*Sqrt[a]*b^(7/2))
```

3.301.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3669, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^7(x)}{a+b\sin^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^7}{a+b\sin(x)^2}dx$$

$$\Big\downarrow \text{3669}$$

$$\int \frac{\left(1-\sin^2(x)\right)^3}{a+b\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{300}$$

$$\int \left(-\frac{a^2+3ab+3b^2}{b^3}+\frac{a^3+3a^2b+3ab^2+b^3}{b^3\left(a+b\sin^2(x)\right)}+\frac{(a+3b)\sin^2(x)}{b^2}-\frac{\sin^4(x)}{b}\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(a^2+3ab+3b^2\right)\sin(x)}{b^3}+\frac{(a+b)^3\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}+\frac{(a+3b)\sin^3(x)}{3b^2}-\frac{\sin^5(x)}{5b}$$

input
```
Int[Cos[x]^7/(a + b*Sin[x]^2),x]
```

output
```
((a + b)^3*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(7/2)) - ((a^2 + 3
*a*b + 3*b^2)*Sin[x])/b^3 + ((a + 3*b)*Sin[x]^3)/(3*b^2) - Sin[x]^5/(5*b)
```

3.301.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.301.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.23

method	result
derivativedivides	$-\frac{\frac{\left(\sin^5(x)\right)b^2}{5}-\frac{ab\left(\sin^3(x)\right)}{3}-b^2\left(\sin^3(x)\right)+\sin(x)a^2+3ab\sin(x)+3b^2\sin(x)}{b^3}-\frac{\left(-a^3-3a^2b-3a\,b^2-b^3\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$
default	$-\frac{\frac{\left(\sin^5(x)\right)b^2}{5}-\frac{ab\left(\sin^3(x)\right)}{3}-b^2\left(\sin^3(x)\right)+\sin(x)a^2+3ab\sin(x)+3b^2\sin(x)}{b^3}-\frac{\left(-a^3-3a^2b-3a\,b^2-b^3\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$
risch	$-\frac{ie^{-ix}a^2}{2b^3}-\frac{11ie^{-ix}a}{8b^2}-\frac{19ie^{-ix}}{16b}+\frac{11ie^{ix}a}{8b^2}+\frac{19ie^{ix}}{16b}+\frac{ie^{ix}a^2}{2b^3}-\frac{\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)a^3}{2\sqrt{-ab}\,b^3}-\frac{3\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{-ab}}\right.}{2\sqrt{-ab}\,b}$

input
```
int(cos(x)^7/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^3*(1/5*sin(x)^5*b^2-1/3*a*b*sin(x)^3-b^2*sin(x)^3+sin(x)*a^2+3*a*b*si
n(x)+3*b^2*sin(x))-(-a^3-3*a^2*b-3*a*b^2-b^3)/b^3/(a*b)^(1/2)*arctan(b*sin
(x)/(a*b)^(1/2))
```

3.301.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.99

$$\int \frac{\cos^7(x)}{a+b\sin^2(x)}\,dx$$
$$= \left[-\frac{15\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+2\left(3ab^3\cos(x)^4+15a^3b+40a^2b^2\right.}{30ab^4}\right.$$

input
```
integrate(cos(x)^7/(a+b*sin(x)^2),x, algorithm="fricas")
```

output
```
[-1/30*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2
*sqrt(-a*b)*sin(x) + a - b)/(b*cos(x)^2 - a - b)) + 2*(3*a*b^3*cos(x)^4 +
15*a^3*b + 40*a^2*b^2 + 33*a*b^3 + (5*a^2*b^2 + 9*a*b^3)*cos(x)^2)*sin(x))
/(a*b^4), 1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)*arctan(sqrt(a
*b)*sin(x)/a) - (3*a*b^3*cos(x)^4 + 15*a^3*b + 40*a^2*b^2 + 33*a*b^3 + (5*
a^2*b^2 + 9*a*b^3)*cos(x)^2)*sin(x))/(a*b^4)]
```

3.301.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^7(x)}{a+b\sin^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**7/(a+b*sin(x)**2),x)
```

output
```
Timed out
```

3.301.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int \frac{\cos^7(x)}{a+b\sin^2(x)}\,dx = \frac{\left(a^3+3a^2b+3ab^2+b^3\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} - \frac{3b^2\sin(x)^5-5\left(ab+3b^2\right)\sin(x)^3+15\left(a^2+3ab+3b^2\right)\sin(x)}{15b^3}$$

input
```
integrate(cos(x)^7/(a+b*sin(x)^2),x, algorithm="maxima")
```

output
```
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b^3)
 - 1/15*(3*b^2*sin(x)^5 - 5*(a*b + 3*b^2)*sin(x)^3 + 15*(a^2 + 3*a*b + 3*b
^2)*sin(x))/b^3
```

3.301.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.26

$$\int \frac{\cos^7(x)}{a+b\sin^2(x)}\,dx = \frac{(a^3+3\,a^2b+3\,ab^2+b^3)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} - \frac{3\,b^4\sin(x)^5-5\,ab^3\sin(x)^3-15\,b^4\sin(x)^3+15\,a^2b^2\sin(x)+45\,ab^3\sin(x)+45\,b^4\sin(x)}{15\,b^5}$$

input
```
integrate(cos(x)^7/(a+b*sin(x)^2),x, algorithm="giac")
```

output
```
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b^3)
 - 1/15*(3*b^4*sin(x)^5 - 5*a*b^3*sin(x)^3 - 15*b^4*sin(x)^3 + 15*a^2*b^2*
sin(x) + 45*a*b^3*sin(x) + 45*b^4*sin(x))/b^5
```

3.301.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

$$\int \frac{\cos^7(x)}{a+b\sin^2(x)}\,dx = \sin(x)^3\left(\frac{a}{3\,b^2}+\frac{1}{b}\right)-\sin(x)\left(\frac{3}{b}+\frac{a\left(\frac{a}{b^2}+\frac{3}{b}\right)}{b}\right) - \frac{\sin(x)^5}{5\,b}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sin(x)\,(a+b)^3}{\sqrt{a}\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}\right)\,(a+b)^3}{\sqrt{a}\,b^{7/2}}$$

input
```
int(cos(x)^7/(a + b*sin(x)^2),x)
```

output
```
sin(x)^3*(a/(3*b^2) + 1/b) - sin(x)*(3/b + (a*(a/b^2 + 3/b))/b) - sin(x)^5
/(5*b) + (atan((b^(1/2)*sin(x)*(a + b)^3)/(a^(1/2)*(3*a*b^2 + 3*a^2*b + a^
3 + b^3)))*(a + b)^3)/(a^(1/2)*b^(7/2))
```

3.302 $\int \frac{\cos^6(x)}{a+b\sin^2(x)}\,dx$

3.302.1 Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \frac{\cos^6(x)}{a+b\sin^2(x)}\,dx = -\frac{(8a^2+20ab+15b^2)\,x}{8b^3}+\frac{(a+b)^{5/2}\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b^3} - \frac{(4a+7b)\cos(x)\sin(x)}{8b^2}-\frac{\cos^3(x)\sin(x)}{4b}$$

output
```
-1/8*(8*a^2+20*a*b+15*b^2)*x/b^3-1/8*(4*a+7*b)*cos(x)*sin(x)/b^2-1/4*cos(x
)^3*sin(x)/b+(a+b)^(5/2)*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/b^3/a^(1/2)
```

3.302.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \frac{\cos^6(x)}{a+b\sin^2(x)}\,dx = \frac{(a+b)^{5/2}\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b^3} - \frac{4(8a^2+20ab+15b^2)\,x+8b(a+2b)\sin(2x)+b^2\sin(4x)}{32b^3}$$

input
```
Integrate[Cos[x]^6/(a + b*Sin[x]^2),x]
```

output
```
((a + b)^(5/2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*b^3) - (4*(8
*a^2 + 20*a*b + 15*b^2)*x + 8*b*(a + 2*b)*Sin[2*x] + b^2*Sin[4*x])/(32*b^3
)
```

3.302.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 3670, 316, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(x)}{a+b\sin^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^6}{a+b\sin(x)^2}dx$$

$$\Big\downarrow \text{3670}$$

$$\int \frac{1}{\left(\tan^2(x)+1\right)^3\left((a+b)\tan^2(x)+a\right)}d\tan(x)$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{-3(a+b)\tan^2(x)+a+4b}{\left(\tan^2(x)+1\right)^2\left((a+b)\tan^2(x)+a\right)}d\tan(x)}{4b}-\frac{\tan(x)}{4b\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\int \frac{4a^2+9ba+8b^2-(a+b)(4a+7b)\tan^2(x)}{\left(\tan^2(x)+1\right)\left((a+b)\tan^2(x)+a\right)}d\tan(x)}{2b}-\frac{(4a+7b)\tan(x)}{2b(\tan^2(x)+1)}}{4b}-\frac{\tan(x)}{4b\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{8(a+b)^3\int \frac{1}{(a+b)\tan^2(x)+a}d\tan(x)}{b}-\frac{\left(8a^2+20ab+15b^2\right)\int \frac{1}{\tan^2(x)+1}d\tan(x)}{b}}{2b}-\frac{(4a+7b)\tan(x)}{2b(\tan^2(x)+1)}}{4b}-\frac{\tan(x)}{4b\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{8(a+b)^3\int \frac{1}{(a+b)\tan^2(x)+a}d\tan(x)}{b}-\frac{\left(8a^2+20ab+15b^2\right)\arctan(\tan(x))}{b}}{2b}-\frac{(4a+7b)\tan(x)}{2b(\tan^2(x)+1)}}{4b}-\frac{\tan(x)}{4b\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{8(a+b)^{5/2}\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{\left(8a^2+20ab+15b^2\right)\arctan(\tan(x))}{b}}{2b}-\frac{(4a+7b)\tan(x)}{2b(\tan^2(x)+1)}}{4b}-\frac{\tan(x)}{4b\left(\tan^2(x)+1\right)^2}$$

input
```
Int[Cos[x]^6/(a + b*Sin[x]^2),x]
```

output
```
-1/4*Tan[x]/(b*(1 + Tan[x]^2)^2) + ((-(((8*a^2 + 20*a*b + 15*b^2)*ArcTan[T
an[x]])/b) + (8*(a + b)^(5/2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[
a]*b))/(2*b) - ((4*a + 7*b)*Tan[x])/(2*b*(1 + Tan[x]^2)))/(4*b)
```

3.302.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.302.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10

method	result
default	$\frac{(a+b)^3 \arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{b^3\sqrt{a(a+b)}} - \frac{\frac{\left(\frac{1}{2}ab+\frac{7}{8}b^2\right)\left(\tan^3(x)\right)+\left(\frac{1}{2}ab+\frac{9}{8}b^2\right)\tan(x)}{\left(1+\tan^2(x)\right)^2}+\frac{\left(8a^2+20ab+15b^2\right)\arctan(\tan(x))}{8}}{b^3}$
risch	$-\frac{x\,a^2}{b^3}-\frac{5ax}{2b^2}-\frac{15x}{8b}+\frac{i e^{2ix}a}{8b^2}+\frac{i e^{2ix}}{4b}-\frac{i e^{-2ix}a}{8b^2}-\frac{i e^{-2ix}}{4b}+\frac{a\sqrt{-a(a+b)}\,\ln\left(e^{2ix}+\frac{2i\sqrt{-a(a+b)}-2a-b}{b}\right)}{2b^3}+\frac{\sqrt{-a(a+b)}}{}$

input
```
int(cos(x)^6/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
(a+b)^3/b^3/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))-1/b^3*(((
1/2*a*b+7/8*b^2)*tan(x)^3+(1/2*a*b+9/8*b^2)*tan(x))/(1+tan(x)^2)^2+1/8*(8*
a^2+20*a*b+15*b^2)*arctan(tan(x)))
```

3.302.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 312, normalized size of antiderivative = 3.59

$$\int \frac{\cos^6(x)}{a+b\sin^2(x)}\,dx$$
$$= \left[\frac{2\,(a^2+2\,ab+b^2)\sqrt{-\frac{a+b}{a}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+5\,ab+b^2)\cos(x)^2-4\left((2\,a^2+ab)\cos(x)^3-(a^2+ab)\cos(x)\right)\sqrt{-\frac{a+b}{a}}\cdots}{b^2\cos(x)^4-2\,(ab+b^2)\cos(x)^2+a^2+2\,ab+b^2}\right.\cdots}{8\,b^3}\right.$$
$$-\frac{4\,(a^2+2\,ab+b^2)\sqrt{\frac{a+b}{a}}\arctan\left(\frac{\left((2\,a+b)\cos(x)^2-a-b\right)\sqrt{\frac{a+b}{a}}}{2\,(a+b)\cos(x)\sin(x)}\right)+(8\,a^2+20\,ab+15\,b^2)x+\left(2\,b^2\cos(x)^3+(\cdots\right.}{8\,b^3}$$

input
```
integrate(cos(x)^6/(a+b*sin(x)^2),x, algorithm="fricas")
```

output
```
[1/8*(2*(a^2 + 2*a*b + b^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*co
s(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - (a
^2 + a*b)*cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)
^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - (8*a^2 + 20*a*b + 15*b
^2)*x - (2*b^2*cos(x)^3 + (4*a*b + 7*b^2)*cos(x))*sin(x))/b^3, -1/8*(4*(a^
2 + 2*a*b + b^2)*sqrt((a + b)/a)*arctan(1/2*((2*a + b)*cos(x)^2 - a - b)*s
qrt((a + b)/a)/((a + b)*cos(x)*sin(x))) + (8*a^2 + 20*a*b + 15*b^2)*x + (2
*b^2*cos(x)^3 + (4*a*b + 7*b^2)*cos(x))*sin(x))/b^3]
```

3.302.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(x)}{a+b\sin^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**6/(a+b*sin(x)**2),x)
```

output
```
Timed out
```

3.302.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.31

$$\int \frac{\cos^6(x)}{a+b\sin^2(x)}\,dx = -\frac{(4\,a+7\,b)\tan{(x)}^3+(4\,a+9\,b)\tan{(x)}}{8\,\left(b^2\tan{(x)}^4+2\,b^2\tan{(x)}^2+b^2\right)} - \frac{(8\,a^2+20\,ab+15\,b^2)x}{8\,b^3} + \frac{(a^3+3\,a^2b+3\,ab^2+b^3)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}b^3}$$

input
```
integrate(cos(x)^6/(a+b*sin(x)^2),x, algorithm="maxima")
```

output
```
-1/8*((4*a + 7*b)*tan(x)^3 + (4*a + 9*b)*tan(x))/(b^2*tan(x)^4 + 2*b^2*tan
(x)^2 + b^2) - 1/8*(8*a^2 + 20*a*b + 15*b^2)*x/b^3 + (a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^3)
```

3.302.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.51

$$\begin{aligned}\int \frac{\cos^6(x)}{a+b\sin^2(x)}\,dx &= -\frac{(8\,a^2+20\,ab+15\,b^2)x}{8\,b^3} \\ &\quad+ \frac{(a^3+3\,a^2b+3\,ab^2+b^3)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)}{\sqrt{a^2+ab}b^3} \\ &\quad- \frac{4\,a\tan{(x)}^3+7\,b\tan{(x)}^3+4\,a\tan{(x)}+9\,b\tan{(x)}}{8\,\left(\tan{(x)}^2+1\right)^2b^2}\end{aligned}$$

input
```
integrate(cos(x)^6/(a+b*sin(x)^2),x, algorithm="giac")
```

output
```
-1/8*(8*a^2 + 20*a*b + 15*b^2)*x/b^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(pi
*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a^2
+ a*b)))/(sqrt(a^2 + a*b)*b^3) - 1/8*(4*a*tan(x)^3 + 7*b*tan(x)^3 + 4*a*ta
n(x) + 9*b*tan(x))/((tan(x)^2 + 1)^2*b^2)
```

3.302.9 Mupad [B] (verification not implemented)

Time = 14.52 (sec) , antiderivative size = 1804, normalized size of antiderivative = 20.74

$$\int \frac{\cos^6(x)}{a + b\sin^2(x)}\,dx = \text{Too large to display}$$

input
```
int(cos(x)^6/(a + b*sin(x)^2),x)
```

output
```
(atan(((((tan(x)*(1723*a*b^6 + 960*a^6*b + 128*a^7 + 289*b^7 + 4459*a^2*b^
5 + 6505*a^3*b^4 + 5784*a^4*b^3 + 3136*a^5*b^2))/(32*b^4) - ((((25*a*b^9)/
2 + 4*b^10 + 15*a^2*b^8 + (17*a^3*b^7)/2 + 2*a^4*b^6)/b^6 - (tan(x)*(a*b*2
0i + a^2*8i + b^2*15i)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6)
)/(512*b^7))*(a*b*20i + a^2*8i + b^2*15i))/(16*b^3))*(a*b*20i + a^2*8i + b
^2*15i)*1i)/(16*b^3) + (((tan(x)*(1723*a*b^6 + 960*a^6*b + 128*a^7 + 289*b
^7 + 4459*a^2*b^5 + 6505*a^3*b^4 + 5784*a^4*b^3 + 3136*a^5*b^2))/(32*b^4)
+ ((((25*a*b^9)/2 + 4*b^10 + 15*a^2*b^8 + (17*a^3*b^7)/2 + 2*a^4*b^6)/b^6
+ (tan(x)*(a*b*20i + a^2*8i + b^2*15i)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^
7 + 512*a^3*b^6))/(512*b^7))*(a*b*20i + a^2*8i + b^2*15i))/(16*b^3))*(a*b*
20i + a^2*8i + b^2*15i)*1i)/(16*b^3))/(((725*a*b^7)/32 + (37*a^7*b)/4 + a^
8 + (105*b^8)/32 + (1093*a^2*b^6)/16 + (1881*a^3*b^5)/16 + (4045*a^4*b^4)/
32 + (2785*a^5*b^3)/32 + (75*a^6*b^2)/2)/b^6 - (((tan(x)*(1723*a*b^6 + 960
*a^6*b + 128*a^7 + 289*b^7 + 4459*a^2*b^5 + 6505*a^3*b^4 + 5784*a^4*b^3 +
3136*a^5*b^2))/(32*b^4) - ((((25*a*b^9)/2 + 4*b^10 + 15*a^2*b^8 + (17*a^3*
b^7)/2 + 2*a^4*b^6)/b^6 - (tan(x)*(a*b*20i + a^2*8i + b^2*15i)*(1024*a*b^8
 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(512*b^7))*(a*b*20i + a^2*8i + b
^2*15i))/(16*b^3))*(a*b*20i + a^2*8i + b^2*15i))/(16*b^3) + (((tan(x)*(172
3*a*b^6 + 960*a^6*b + 128*a^7 + 289*b^7 + 4459*a^2*b^5 + 6505*a^3*b^4 + 57
84*a^4*b^3 + 3136*a^5*b^2))/(32*b^4) + ((((25*a*b^9)/2 + 4*b^10 + 15*a^...
```

3.303 $\int \frac{\cos^5(x)}{a+b\sin^2(x)}\,dx$

3.303.1 Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{\cos^5(x)}{a+b\sin^2(x)}\,dx=\frac{(a+b)^2\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}-\frac{(a+2b)\sin(x)}{b^2}+\frac{\sin^3(x)}{3b}$$

output
```
-(a+2*b)*sin(x)/b^2+1/3*sin(x)^3/b+(a+b)^2*arctan(sin(x)*b^(1/2)/a^(1/2))/
b^(5/2)/a^(1/2)
```

3.303.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.56

$$\int \frac{\cos^5(x)}{a+b\sin^2(x)}\,dx$$
$$=\frac{-6(a+b)^2\arctan\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)+6(a+b)^2\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)-2\sqrt{a}\sqrt{b}(6a+11b+b\cos(2x))\sin(x)}{12\sqrt{a}b^{5/2}}$$

input
```
Integrate[Cos[x]^5/(a + b*Sin[x]^2),x]
```

output
```
(-6*(a + b)^2*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]] + 6*(a + b)^2*ArcTan[(Sqrt[
b]*Sin[x])/Sqrt[a]] - 2*Sqrt[a]*Sqrt[b]*(6*a + 11*b + b*Cos[2*x])*Sin[x])/
(12*Sqrt[a]*b^(5/2))
```

3.303.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3669, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(x)}{a+b\sin^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^5}{a+b\sin(x)^2}dx$$

$$\Big\downarrow \text{3669}$$

$$\int \frac{\left(1-\sin^2(x)\right)^2}{a+b\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{a^2+2ab+b^2}{b^2\left(a+b\sin^2(x)\right)}-\frac{a+2b}{b^2}+\frac{\sin^2(x)}{b}\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+b)^2\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}-\frac{(a+2b)\sin(x)}{b^2}+\frac{\sin^3(x)}{3b}$$

input
```
Int[Cos[x]^5/(a + b*Sin[x]^2),x]
```

output
```
((a + b)^2*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)) - ((a + 2*b
)*Sin[x])/b^2 + Sin[x]^3/(3*b)
```

3.303.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.303.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

method	result
derivativedivides	$-\frac{-\frac{b\left(\sin^3(x)\right)}{3}+\sin(x)a+2\sin(x)b}{b^2}+\frac{(a^2+2ab+b^2)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$
default	$-\frac{-\frac{b\left(\sin^3(x)\right)}{3}+\sin(x)a+2\sin(x)b}{b^2}+\frac{(a^2+2ab+b^2)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$
risch	$\frac{ie^{ix}a}{2b^2}+\frac{7ie^{ix}}{8b}-\frac{ie^{-ix}a}{2b^2}-\frac{7ie^{-ix}}{8b}-\frac{\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)a^2}{2\sqrt{-ab}\,b^2}-\frac{\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)a}{\sqrt{-ab}\,b}-\frac{\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)}{2\sqrt{-ab}}$

input
```
int(cos(x)^5/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(-1/3*b*sin(x)^3+sin(x)*a+2*sin(x)*b)+(a^2+2*a*b+b^2)/b^2/(a*b)^(1/
2)*arctan(b*sin(x)/(a*b)^(1/2))
```

3.303.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.94

$$\int \frac{\cos^5(x)}{a+b\sin^2(x)}\,dx = \left[-\frac{3\,(a^2+2\,ab+b^2)\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+2\,(ab^2\cos(x)^2+3\,a^2b+5\,ab^2)\sin(x)}{6\,ab^3}, \frac{3\,(a^2$$

input
```
integrate(cos(x)^5/(a+b*sin(x)^2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(a^2 + 2*a*b + b^2)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*si
n(x) + a - b)/(b*cos(x)^2 - a - b)) + 2*(a*b^2*cos(x)^2 + 3*a^2*b + 5*a*b^
2)*sin(x))/(a*b^3), 1/3*(3*(a^2 + 2*a*b + b^2)*sqrt(a*b)*arctan(sqrt(a*b)*
sin(x)/a) - (a*b^2*cos(x)^2 + 3*a^2*b + 5*a*b^2)*sin(x))/(a*b^3)]
```

3.303.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(x)}{a+b\sin^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**5/(a+b*sin(x)**2),x)
```

output
```
Timed out
```

3.303.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{\cos^5(x)}{a+b\sin^2(x)}\,dx = \frac{(a^2+2\,ab+b^2)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{b\sin(x)^3-3\,(a+2\,b)\sin(x)}{3\,b^2}$$

input
```
integrate(cos(x)^5/(a+b*sin(x)^2),x, algorithm="maxima")
```

output
```
(a^2 + 2*a*b + b^2)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*si
n(x)^3 - 3*(a + 2*b)*sin(x))/b^2
```

3.303.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{\cos^5(x)}{a+b\sin^2(x)}\,dx = \frac{(a^2+2\,ab+b^2)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{b^2\sin{(x)}^3-3\,ab\sin{(x)}-6\,b^2\sin{(x)}}{3\,b^3}$$

input
```
integrate(cos(x)^5/(a+b*sin(x)^2),x, algorithm="giac")
```

output
```
(a^2 + 2*a*b + b^2)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b^2*
sin(x)^3 - 3*a*b*sin(x) - 6*b^2*sin(x))/b^3
```

3.303.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int \frac{\cos^5(x)}{a+b\sin^2(x)}\,dx = \frac{\sin{(x)}^3}{3\,b} - \sin{(x)}\,\left(\frac{a}{b^2}+\frac{2}{b}\right) + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sin(x)\,(a+b)^2}{\sqrt{a}\,(a^2+2\,a\,b+b^2)}\right)\,{(a+b)}^2}{\sqrt{a}\,b^{5/2}}$$

input
```
int(cos(x)^5/(a + b*sin(x)^2),x)
```

output
```
sin(x)^3/(3*b) - sin(x)*(a/b^2 + 2/b) + (atan((b^(1/2)*sin(x)*(a + b)^2)/(
a^(1/2)*(2*a*b + a^2 + b^2)))*(a + b)^2)/(a^(1/2)*b^(5/2))
```

3.304 $\int \frac{\cos^4(x)}{a+b\sin^2(x)}\,dx$

3.304.1 Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{\cos^4(x)}{a+b\sin^2(x)}\,dx=-\frac{(2a+3b)x}{2b^2}+\frac{(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b^2}-\frac{\cos(x)\sin(x)}{2b}$$

output
```
-1/2*(2*a+3*b)*x/b^2-1/2*cos(x)*sin(x)/b+(a+b)^(3/2)*arctan((a+b)^(1/2)*ta
n(x)/a^(1/2))/b^2/a^(1/2)
```

3.304.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{\cos^4(x)}{a+b\sin^2(x)}\,dx=\frac{\frac{4(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}}-2(2ax+3bx+b\cos(x)\sin(x))}{4b^2}$$

input
```
Integrate[Cos[x]^4/(a + b*Sin[x]^2),x]
```

output
```
((4*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/Sqrt[a] - 2*(2*a*x
 + 3*b*x + b*Cos[x]*Sin[x]))/(4*b^2)
```

3.304.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3670, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(x)}{a+b\sin^2(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)^4}{a+b\sin(x)^2}dx$$

$$\Big\downarrow \text{ 3670}$$

$$\int \frac{1}{\left(\tan^2(x)+1\right)^2\left((a+b)\tan^2(x)+a\right)}d\tan(x)$$

$$\Big\downarrow \text{ 316}$$

$$\frac{\int \frac{-((a+b)\tan^2(x))+a+2b}{(\tan^2(x)+1)((a+b)\tan^2(x)+a)}d\tan(x)}{2b}-\frac{\tan(x)}{2b\left(\tan^2(x)+1\right)}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{\frac{2(a+b)^2\int \frac{1}{(a+b)\tan^2(x)+a}d\tan(x)}{b}-\frac{(2a+3b)\int \frac{1}{\tan^2(x)+1}d\tan(x)}{b}}{2b}-\frac{\tan(x)}{2b\left(\tan^2(x)+1\right)}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{2(a+b)^2\int \frac{1}{(a+b)\tan^2(x)+a}d\tan(x)}{b}-\frac{(2a+3b)\arctan(\tan(x))}{b}}{2b}-\frac{\tan(x)}{2b\left(\tan^2(x)+1\right)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{2(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{(2a+3b)\arctan(\tan(x))}{b}}{2b}-\frac{\tan(x)}{2b\left(\tan^2(x)+1\right)}$$

input
```
Int[Cos[x]^4/(a + b*Sin[x]^2),x]
```

output
```
(-(((2*a + 3*b)*ArcTan[Tan[x]])/b) + (2*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*
Tan[x])/Sqrt[a]])/(Sqrt[a]*b))/(2*b) - Tan[x]/(2*b*(1 + Tan[x]^2))
```

3.304.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.304.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

method	result
default	$-\frac{\frac{b\tan(x)}{2+2(\tan^2(x))}+\frac{(2a+3b)\arctan(\tan(x))}{2}}{b^2}+\frac{(a+b)^2\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{b^2\sqrt{a(a+b)}}$
risch	$-\frac{ax}{b^2}-\frac{3x}{2b}+\frac{ie^{2ix}}{8b}-\frac{ie^{-2ix}}{8b}+\frac{\sqrt{-a(a+b)}\ \ln\left(e^{2ix}+\frac{2i\sqrt{-a(a+b)}-2a-b}{b}\right)}{2b^2}+\frac{\sqrt{-a(a+b)}\ \ln\left(e^{2ix}+\frac{2i\sqrt{-a(a+b)}-2a-b}{b}\right)}{2ab}-$

input
```
int(cos(x)^4/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(1/2*b*tan(x)/(1+tan(x)^2)+1/2*(2*a+3*b)*arctan(tan(x)))+(a+b)^2/b^
2/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

3.304.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 239, normalized size of antiderivative = 4.05

$$\int \frac{\cos^4(x)}{a+b\sin^2(x)}\,dx$$

$$= \left[-\frac{2\,b\cos\left(x\right)\sin\left(x\right)-(a+b)\sqrt{-\frac{a+b}{a}}\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(x)^2-4\left(\left(2\,a^2+ab\right)\cos(x)^3-\left(a^2+ab\right)\right.}{b^2\cos(x)^4-2\left(ab+b^2\right)\cos(x)^2+a^2+2\,ab+b^2}\right.}{4\,b^2}\right.$$

$$\left.-\frac{b\cos\left(x\right)\sin\left(x\right)+(a+b)\sqrt{\frac{a+b}{a}}\arctan\left(\frac{\left((2\,a+b)\cos(x)^2-a-b\right)\sqrt{\frac{a+b}{a}}}{2\,(a+b)\cos(x)\sin(x)}\right)+(2\,a+3\,b)x}{2\,b^2}\right]$$

input
```
integrate(cos(x)^4/(a+b*sin(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*(2*b*cos(x)*sin(x) - (a + b)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b +
b^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)
^3 - (a^2 + a*b)*cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2
*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) + 2*(2*a + 3*b)*x
)/b^2, -1/2*(b*cos(x)*sin(x) + (a + b)*sqrt((a + b)/a)*arctan(1/2*((2*a +
b)*cos(x)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(x)*sin(x))) + (2*a + 3*b
)*x)/b^2]
```

3.304.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(x)}{a+b\sin^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**4/(a+b*sin(x)**2),x)
```

output
```
Timed out
```

3.304.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int \frac{\cos^4(x)}{a+b\sin^2(x)}\,dx = -\frac{(2a+3b)x}{2b^2} - \frac{\tan(x)}{2\left(b\tan(x)^2+b\right)} + \frac{(a^2+2ab+b^2)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}b^2}$$

input
```
integrate(cos(x)^4/(a+b*sin(x)^2),x, algorithm="maxima")
```

output
```
-1/2*(2*a + 3*b)*x/b^2 - 1/2*tan(x)/(b*tan(x)^2 + b) + (a^2 + 2*a*b + b^2)
*arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^2)
```

3.304.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.56

$$\int \frac{\cos^4(x)}{a+b\sin^2(x)}\,dx = -\frac{(2\,a+3\,b)x}{2\,b^2} + \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)(a^2+2\,ab+b^2)}{\sqrt{a^2+ab}b^2} - \frac{\tan(x)}{2\left(\tan(x)^2+1\right)b}$$

input
```
integrate(cos(x)^4/(a+b*sin(x)^2),x, algorithm="giac")
```

output
```
-1/2*(2*a + 3*b)*x/b^2 + (pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*
tan(x) + b*tan(x))/sqrt(a^2 + a*b)))*(a^2 + 2*a*b + b^2)/(sqrt(a^2 + a*b)*
b^2) - 1/2*tan(x)/((tan(x)^2 + 1)*b)
```

3.304.9 Mupad [B] (verification not implemented)

Time = 13.65 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.02

$$\int \frac{\cos^4(x)}{a+b\sin^2(x)}\,dx = -\frac{3\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)}{2\,b} - \frac{a\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)}{b^2} - \frac{\cos(x)\,\sin(x)}{2\,b} - \frac{\mathrm{atanh}\left(\frac{\sin(x)\,\sqrt{-a^4-3\,a^3\,b-3\,a^2\,b^2-a\,b^3}}{\cos(x)\,a^2+b\,\cos(x)\,a}\right)\sqrt{-a^4-3\,a^3\,b-3\,a^2\,b^2-a\,b^3}}{a\,b^2}$$

input
```
int(cos(x)^4/(a + b*sin(x)^2),x)
```

output
```
- (3*atan(sin(x)/cos(x)))/(2*b) - (a*atan(sin(x)/cos(x)))/b^2 - (cos(x)*si
n(x))/(2*b) - (atanh((sin(x)*(- a*b^3 - 3*a^3*b - a^4 - 3*a^2*b^2)^(1/2))/
(a^2*cos(x) + a*b*cos(x)))*(- a*b^3 - 3*a^3*b - a^4 - 3*a^2*b^2)^(1/2))/(a
*b^2)
```

3.305 $\int \frac{\cos^3(x)}{a+b\sin^2(x)}\,dx$

3.305.1 Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{\cos^3(x)}{a+b\sin^2(x)}\,dx=\frac{(a+b)\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{\sin(x)}{b}$$

output
```
-sin(x)/b+(a+b)*arctan(sin(x)*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

3.305.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(x)}{a+b\sin^2(x)}\,dx=\frac{(a+b)\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{\sin(x)}{b}$$

input
```
Integrate[Cos[x]^3/(a + b*Sin[x]^2),x]
```

output
```
((a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)) - Sin[x]/b
```

3.305.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3669, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(x)}{a+b\sin^2(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos(x)^3}{a+b\sin(x)^2}dx$$

$$\Big\downarrow \ 3669$$

$$\int \frac{1-\sin^2(x)}{a+b\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \ 299$$

$$\frac{(a+b)\int \frac{1}{b\sin^2(x)+a}d\sin(x)}{b}-\frac{\sin(x)}{b}$$

$$\Big\downarrow \ 218$$

$$\frac{(a+b)\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{\sin(x)}{b}$$

input
```
Int[Cos[x]^3/(a + b*Sin[x]^2),x]
```

output
```
((a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)) - Sin[x]/b
```

3.305.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.305.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

method	result
derivativedivides	$-\frac{\sin(x)}{b}+\frac{(a+b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{b\sqrt{ab}}$
default	$-\frac{\sin(x)}{b}+\frac{(a+b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{b\sqrt{ab}}$
risch	$\frac{i\mathrm{e}^{ix}}{2b}-\frac{i\mathrm{e}^{-ix}}{2b}-\frac{\ln\left(\mathrm{e}^{2ix}-\frac{2ia\,\mathrm{e}^{ix}}{\sqrt{-ab}}-1\right)a}{2\sqrt{-ab}\,b}-\frac{\ln\left(\mathrm{e}^{2ix}-\frac{2ia\,\mathrm{e}^{ix}}{\sqrt{-ab}}-1\right)}{2\sqrt{-ab}}+\frac{\ln\left(\mathrm{e}^{2ix}+\frac{2ia\,\mathrm{e}^{ix}}{\sqrt{-ab}}-1\right)a}{2\sqrt{-ab}\,b}+\frac{\ln\left(\mathrm{e}^{2ix}+\frac{2ia\,\mathrm{e}^{ix}}{\sqrt{-ab}}-1\right)}{2\sqrt{-ab}}$

input
```
int(cos(x)^3/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-sin(x)/b+(a+b)/b/(a*b)^(1/2)*arctan(b*sin(x)/(a*b)^(1/2))
```

3.305.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.81

$$\int \frac{\cos^3(x)}{a+b\sin^2(x)}\,dx = \left[-\frac{2\,ab\sin(x)+\sqrt{-ab}(a+b)\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)}{2\,ab^2},\ -\frac{ab\sin(x)-\sqrt{ab}(a+b)\arctan\left(\frac{\sqrt{ab}\sin(x)}{a}\right)}{ab^2}\right]$$

input
```
integrate(cos(x)^3/(a+b*sin(x)^2),x, algorithm="fricas")
```

output
```
[-1/2*(2*a*b*sin(x) + sqrt(-a*b)*(a + b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*s
in(x) + a - b)/(b*cos(x)^2 - a - b)))/(a*b^2), -(a*b*sin(x) - sqrt(a*b)*(a
 + b)*arctan(sqrt(a*b)*sin(x)/a))/(a*b^2)]
```

3.305.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)}{a+b\sin^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**3/(a+b*sin(x)**2),x)
```

output
```
Timed out
```

3.305.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{\cos^3(x)}{a+b\sin^2(x)}\,dx = \frac{(a+b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b} - \frac{\sin(x)}{b}$$

input
```
integrate(cos(x)^3/(a+b*sin(x)^2),x, algorithm="maxima")
```

output
```
(a + b)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b) - sin(x)/b
```

3.305.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{\cos^3(x)}{a+b\sin^2(x)}\,dx = \frac{(a+b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b} - \frac{\sin(x)}{b}$$

input
```
integrate(cos(x)^3/(a+b*sin(x)^2),x, algorithm="giac")
```

output
```
(a + b)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b) - sin(x)/b
```

3.305.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{\cos^3(x)}{a+b\sin^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)\,(a+b)}{\sqrt{a}\,b^{3/2}} - \frac{\sin(x)}{b}$$

input
```
int(cos(x)^3/(a + b*sin(x)^2),x)
```

output
```
(atan((b^(1/2)*sin(x))/a^(1/2))*(a + b))/(a^(1/2)*b^(3/2)) - sin(x)/b
```

3.306 $\int \frac{\cos^2(x)}{a+b\sin^2(x)}\,dx$

3.306.1 Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{\cos^2(x)}{a+b\sin^2(x)}\,dx=-\frac{x}{b}+\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b}$$

output
```
-x/b+arctan((a+b)^(1/2)*tan(x)/a^(1/2))*(a+b)^(1/2)/b/a^(1/2)
```

3.306.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(x)}{a+b\sin^2(x)}\,dx=-\frac{x}{b}+\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b}$$

input
```
Integrate[Cos[x]^2/(a + b*Sin[x]^2),x]
```

output
```
-(x/b) + (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*b)
```

3.306.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3670, 303, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(x)}{a+b\sin^2(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)^2}{a+b\sin(x)^2}dx$$

$$\Big\downarrow \text{ 3670}$$

$$\int \frac{1}{(\tan^2(x)+1)\left((a+b)\tan^2(x)+a\right)}d\tan(x)$$

$$\Big\downarrow \text{ 303}$$

$$\frac{(a+b)\int \frac{1}{(a+b)\tan^2(x)+a}d\tan(x)}{b}-\frac{\int \frac{1}{\tan^2(x)+1}d\tan(x)}{b}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{(a+b)\int \frac{1}{(a+b)\tan^2(x)+a}d\tan(x)}{b}-\frac{\arctan(\tan(x))}{b}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{\arctan(\tan(x))}{b}$$

input
```
Int[Cos[x]^2/(a + b*Sin[x]^2),x]
```

output
```
-(ArcTan[Tan[x]]/b) + (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(
Sqrt[a]*b)
```

3.306.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.306.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\arctan(\tan(x))}{b}+\frac{(a+b)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{b\sqrt{a(a+b)}}$	38
risch	$-\frac{x}{b}+\frac{\sqrt{-a(a+b)}\,\ln\left(\mathrm{e}^{2ix}+\frac{2i\sqrt{-a(a+b)}-2a-b}{b}\right)}{2ab}-\frac{\sqrt{-a(a+b)}\,\ln\left(\mathrm{e}^{2ix}-\frac{2i\sqrt{-a(a+b)}+2a+b}{b}\right)}{2ab}$	97

input
```
int(cos(x)^2/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b*arctan(tan(x))+(a+b)/b/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^
(1/2))
```

3.306.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 206, normalized size of antiderivative = 5.28

$$\int \frac{\cos^2(x)}{a+b\sin^2(x)}\,dx = \left[\frac{\sqrt{-\frac{a+b}{a}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+5\,ab+b^2)\cos(x)^2-4\left((2\,a^2+ab)\cos(x)^3-(a^2+ab)\cos(x)\right)\sqrt{-\frac{a+b}{a}}\sin(x)+a^2+2\,ab+b^2}{b^2\cos(x)^4-2\,(ab+b^2)\cos(x)^2+a^2+2\,ab+b^2}\right)}{4\,b}, -\frac{\sqrt{\frac{a+b}{a}}\arctan\left(\frac{\left((2\,a+b)\cos(x)^2-a-b\right)\sqrt{\frac{a+b}{a}}}{2\,(a+b)\cos(x)\sin(x)}\right)+2\,x}{2\,b}\right]$$

input
```
integrate(cos(x)^2/(a+b*sin(x)^2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*
a*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - (a^2 + a*b)*cos(x))*sqrt
(-(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos
(x)^2 + a^2 + 2*a*b + b^2)) - 4*x)/b, -1/2*(sqrt((a + b)/a)*arctan(1/2*((2
*a + b)*cos(x)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(x)*sin(x))) + 2*x)/
b]
```

3.306.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(x)}{a+b\sin^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**2/(a+b*sin(x)**2),x)
```

output
```
Timed out
```

3.306.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{\cos^2(x)}{a+b\sin^2(x)}\,dx = \frac{(a+b)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)ab}} - \frac{x}{b}$$

input
```
integrate(cos(x)^2/(a+b*sin(x)^2),x, algorithm="maxima")
```

output
```
(a + b)*arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b) - x/b
```

3.306.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.59

$$\int \frac{\cos^2(x)}{a+b\sin^2(x)}\,dx = \frac{\left(\pi\left\lfloor \frac{x}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)(a+b)}{\sqrt{a^2+ab}b} - \frac{x}{b}$$

input
```
integrate(cos(x)^2/(a+b*sin(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a
^2 + a*b)))*(a + b)/(sqrt(a^2 + a*b)*b) - x/b
```

3.306.9 Mupad [B] (verification not implemented)

Time = 14.02 (sec) , antiderivative size = 272, normalized size of antiderivative = 6.97

$$\int \frac{\cos^2(x)}{a+b\sin^2(x)}\,dx = -\frac{\operatorname{atan}\left(\frac{2\,a^2\tan(x)}{2\,a^2+4\,a\,b+2\,b^2}+\frac{2\,b^2\tan(x)}{2\,a^2+4\,a\,b+2\,b^2}+\frac{4\,a\,b\tan(x)}{2\,a^2+4\,a\,b+2\,b^2}\right)}{b} - \frac{\operatorname{atanh}\left(\frac{6\,b^2\tan(x)\sqrt{-a^2-b\,a}}{2\,a^3+6\,a^2\,b+6\,a\,b^2+2\,b^3}+\frac{2\,a\tan(x)\sqrt{-a^2-b\,a}}{6\,a\,b+2\,a^2+6\,b^2+\frac{2\,b^3}{a}}+\frac{6\,b\tan(x)\sqrt{-a^2-b\,a}}{6\,a\,b+2\,a^2+6\,b^2+\frac{2\,b^3}{a}}+\frac{2\,b^3\tan(x)\sqrt{-a^2-b\,a}}{2\,a^4+6\,a^3\,b+6\,a^2\,b^2+2\,a\,b^3}\right)\sqrt{-a\,(a+b)}}{a\,b}$$

input
```
int(cos(x)^2/(a + b*sin(x)^2),x)
```

output
```
- atan((2*a^2*tan(x))/(4*a*b + 2*a^2 + 2*b^2) + (2*b^2*tan(x))/(4*a*b + 2*
a^2 + 2*b^2) + (4*a*b*tan(x))/(4*a*b + 2*a^2 + 2*b^2))/b - (atanh((6*b^2*t
an(x)*(- a*b - a^2)^(1/2))/(6*a*b^2 + 6*a^2*b + 2*a^3 + 2*b^3) + (2*a*tan(
x)*(- a*b - a^2)^(1/2))/(6*a*b + 2*a^2 + 6*b^2 + (2*b^3)/a) + (6*b*tan(x)*
(- a*b - a^2)^(1/2))/(6*a*b + 2*a^2 + 6*b^2 + (2*b^3)/a) + (2*b^3*tan(x)*(
- a*b - a^2)^(1/2))/(2*a*b^3 + 6*a^3*b + 2*a^4 + 6*a^2*b^2))*(-a*(a + b))^
(1/2))/(a*b)
```

3.306. $\int \frac{\cos^2(x)}{a+b\sin^2(x)}\,dx$

3.307 $\int \frac{\cos(x)}{a+b\sin^2(x)} dx$

3.307.1 Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{\cos(x)}{a+b\sin^2(x)} dx = \frac{\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
arctan(sin(x)*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

3.307.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a+b\sin^2(x)} dx = \frac{\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[Cos[x]/(a + b*Sin[x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

3.307.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3669, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{a+b\sin^2(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(x)}{a+b\sin(x)^2}dx$$
$$\Big\downarrow \text{ 3669}$$
$$\int \frac{1}{a+b\sin^2(x)}d\sin(x)$$
$$\Big\downarrow \text{ 218}$$
$$\frac{\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[Cos[x]/(a + b*Sin[x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

3.307.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.307.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$	17
default	$\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$	17
risch	$-\frac{\ln\left(\mathrm{e}^{2ix}-\frac{2ia\,\mathrm{e}^{ix}}{\sqrt{-ab}}-1\right)}{2\sqrt{-ab}}+\frac{\ln\left(\mathrm{e}^{2ix}+\frac{2ia\,\mathrm{e}^{ix}}{\sqrt{-ab}}-1\right)}{2\sqrt{-ab}}$	64

input
```
int(cos(x)/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*b)^(1/2)*arctan(b*sin(x)/(a*b)^(1/2))
```

3.307.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.12

$$\int \frac{\cos(x)}{a+b\sin^2(x)}\,dx = \left[-\frac{\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\sin(x)}{a}\right)}{ab}\right]$$

input
```
integrate(cos(x)/(a+b*sin(x)^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*sin(x) + a - b)/(b*cos(x)
^2 - a - b))/(a*b), sqrt(a*b)*arctan(sqrt(a*b)*sin(x)/a)/(a*b)]
```

3.307.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(24) = 48$.

Time = 0.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.64

$$\int \frac{\cos(x)}{a+b\sin^2(x)}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sin(x)} & \text{for } a=0 \wedge b=0 \\ \frac{\sin(x)}{a} & \text{for } b=0 \\ -\frac{1}{b\sin(x)} & \text{for } a=0 \\ \frac{\log\left(-\sqrt{-\frac{a}{b}}+\sin(x)\right)}{2b\sqrt{-\frac{a}{b}}} - \frac{\log\left(\sqrt{-\frac{a}{b}}+\sin(x)\right)}{2b\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(x)/(a+b*sin(x)**2),x)
```

output
```
Piecewise((zoo/sin(x), Eq(a, 0) & Eq(b, 0)), (sin(x)/a, Eq(b, 0)), (-1/(b*
sin(x)), Eq(a, 0)), (log(-sqrt(-a/b) + sin(x))/(2*b*sqrt(-a/b)) - log(sqrt
(-a/b) + sin(x))/(2*b*sqrt(-a/b)), True))
```

3.307.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \frac{\cos(x)}{a+b\sin^2(x)}\,dx = \frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(cos(x)/(a+b*sin(x)^2),x, algorithm="maxima")
```

output
```
arctan(b*sin(x)/sqrt(a*b))/sqrt(a*b)
```

3.307.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \frac{\cos(x)}{a + b\sin^2(x)}\,dx = \frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(cos(x)/(a+b*sin(x)^2),x, algorithm="giac")
```

output
```
arctan(b*sin(x)/sqrt(a*b))/sqrt(a*b)
```

3.307.9 Mupad [B] (verification not implemented)

Time = 14.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{\cos(x)}{a + b\sin^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int(cos(x)/(a + b*sin(x)^2),x)
```

output
```
atan((b^(1/2)*sin(x))/a^(1/2))/(a^(1/2)*b^(1/2))
```

3.308 $\int \frac{\sec(x)}{a+b\sin^2(x)}\,dx$

3.308.1 Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{\sec(x)}{a+b\sin^2(x)}\,dx=\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{\operatorname{arctanh}(\sin(x))}{a+b}$$

output
```
arctanh(sin(x))/(a+b)+arctan(sin(x)*b^(1/2)/a^(1/2))*b^(1/2)/(a+b)/a^(1/2)
```

3.308.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 96 vs. 2(40) = 80.

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.40

$$\int \frac{\sec(x)}{a+b\sin^2(x)}\,dx =\frac{-\sqrt{b}\arctan\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)+\sqrt{b}\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)+2\sqrt{a}\left(-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)}{2\sqrt{a}(a+b)}$$

input
```
Integrate[Sec[x]/(a + b*Sin[x]^2),x]
```

output
```
(-(Sqrt[b]*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]]) + Sqrt[b]*ArcTan[(Sqrt[b]*Sin
[x])/Sqrt[a]] + 2*Sqrt[a]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[
x/2]]))/(2*Sqrt[a]*(a + b))
```

3.308.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3669, 303, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(x)}{a+b\sin^2(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\cos(x)\left(a+b\sin(x)^2\right)}dx$$

$$\Big\downarrow \; 3669$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)\left(a+b\sin^2(x)\right)}d\sin(x)$$

$$\Big\downarrow \; 303$$

$$\frac{\int \frac{1}{1-\sin^2(x)}d\sin(x)}{a+b}+\frac{b\int \frac{1}{b\sin^2(x)+a}d\sin(x)}{a+b}$$

$$\Big\downarrow \; 218$$

$$\frac{\int \frac{1}{1-\sin^2(x)}d\sin(x)}{a+b}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{\text{arctanh}(\sin(x))}{a+b}$$

input
```
Int[Sec[x]/(a + b*Sin[x]^2),x]
```

output
```
(Sqrt[b]*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + ArcTanh[Sin
[x]]/(a + b)
```

3.308.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.308.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

method	result	size
default	$-\frac{\ln(\sin(x)-1)}{2a+2b}+\frac{b\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}+\frac{\ln(1+\sin(x))}{2a+2b}$	55
risch	$\frac{\ln(e^{ix}+i)}{a+b}-\frac{\ln(e^{ix}-i)}{a+b}+\frac{\sqrt{-ab}\,\ln\left(e^{2ix}+\frac{2i\sqrt{-ab}\,e^{ix}}{b}-1\right)}{2a(a+b)}-\frac{\sqrt{-ab}\,\ln\left(e^{2ix}-\frac{2i\sqrt{-ab}\,e^{ix}}{b}-1\right)}{2a(a+b)}$	115

input
```
int(sec(x)/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/(2*a+2*b)*ln(sin(x)-1)+b/(a+b)/(a*b)^(1/2)*arctan(b*sin(x)/(a*b)^(1/2))
+1/(2*a+2*b)*ln(1+sin(x))
```

3.308.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.90

$$\int \frac{\sec(x)}{a+b\sin^2(x)}\,dx = \left[\frac{\sqrt{-\frac{b}{a}}\log\left(-\frac{b\cos(x)^2-2\,a\sqrt{-\frac{b}{a}}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+\log\left(\sin\left(x\right)+1\right)-\log\left(-\sin\left(x\right)+1\right)}{2\left(a+b\right)}, \frac{2\sqrt{\frac{b}{a}}\arctan\left(\sqrt{\frac{b}{a}}\sin\left(x\right.\right.}{}\right.$$

input
```
integrate(sec(x)/(a+b*sin(x)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-b/a)*log(-(b*cos(x)^2 - 2*a*sqrt(-b/a)*sin(x) + a - b)/(b*cos(
x)^2 - a - b)) + log(sin(x) + 1) - log(-sin(x) + 1))/(a + b), 1/2*(2*sqrt(
b/a)*arctan(sqrt(b/a)*sin(x)) + log(sin(x) + 1) - log(-sin(x) + 1))/(a + b
)]
```

3.308.6 Sympy [F]

$$\int \frac{\sec(x)}{a+b\sin^2(x)}\,dx = \int \frac{\sec\left(x\right)}{a+b\sin^2\left(x\right)}\,dx$$

input
```
integrate(sec(x)/(a+b*sin(x)**2),x)
```

output
```
Integral(sec(x)/(a + b*sin(x)**2), x)
```

3.308.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int \frac{\sec(x)}{a+b\sin^2(x)}\,dx = \frac{b\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}(a+b)}+\frac{\log\left(\sin\left(x\right)+1\right)}{2\left(a+b\right)}-\frac{\log\left(\sin\left(x\right)-1\right)}{2\left(a+b\right)}$$

input
```
integrate(sec(x)/(a+b*sin(x)^2),x, algorithm="maxima")
```

output
```
b*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*(a + b)) + 1/2*log(sin(x) + 1)/(a
+ b) - 1/2*log(sin(x) - 1)/(a + b)
```

3.308.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \frac{\sec(x)}{a+b\sin^2(x)}\,dx=\frac{b\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}(a+b)}+\frac{\log\left(\sin\left(x\right)+1\right)}{2\left(a+b\right)}-\frac{\log\left(-\sin\left(x\right)+1\right)}{2\left(a+b\right)}$$

input
```
integrate(sec(x)/(a+b*sin(x)^2),x, algorithm="giac")
```

output
```
b*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*(a + b)) + 1/2*log(sin(x) + 1)/(a
+ b) - 1/2*log(-sin(x) + 1)/(a + b)
```

3.308.9 Mupad [B] (verification not implemented)

Time = 15.20 (sec) , antiderivative size = 856, normalized size of antiderivative = 21.40

$$\int \frac{\sec(x)}{a+b\sin^2(x)}\,dx=$$

$$-\frac{\mathrm{atan}\left(\frac{\frac{\left(4\,b^3\,\sin(x)+\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2-\frac{\sin(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}\right)\,1{}\mathrm{i}}{2\,(a+b)}+\frac{\left(4\,b^3\,\sin(x)-\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2+\frac{\sin(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\right.}{2\,(a+b)}}{2\,(a+b)}\right.}{2\,(a+b)}}{\frac{4\,b^3\,\sin(x)+\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2-\frac{\sin(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}}{2\,(a+b)}-\frac{4\,b^3\,\sin(x)-\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2+\frac{\sin(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\right.}{2\,(a+b)}}{2\,(a+b)}}{2\,(a+b)}}\right.}{a+b}$$

$$-\frac{\mathrm{atan}\left(\frac{\frac{\left(2\,b^3\,\sin(x)+\frac{\sqrt{-a\,b}\,\left(4\,a\,b^3+2\,b^4+2\,a^2\,b^2-\frac{\sin(x)\,\sqrt{-a\,b}\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{4\,\left(a^2+b\,a\right)}\right)}{2\,\left(a^2+b\,a\right)}\right)\,\sqrt{-a\,b}\,1{}\mathrm{i}}{a^2+b\,a}+\frac{\left(2\,b^3\,\sin(x)-\frac{\sqrt{-a\,b}\,\left(4\,a\,b^3+2\,b^4+2\,a^2\right.}{}\right.}{}}{\frac{\left(2\,b^3\,\sin(x)+\frac{\sqrt{-a\,b}\,\left(4\,a\,b^3+2\,b^4+2\,a^2\,b^2-\frac{\sin(x)\,\sqrt{-a\,b}\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{4\,\left(a^2+b\,a\right)}\right)}{2\,\left(a^2+b\,a\right)}\right)\,\sqrt{-a\,b}}{a^2+b\,a}-\frac{\left(2\,b^3\,\sin(x)-\frac{\sqrt{-a\,b}\,\left(4\,a\,b^3+2\,b^4+2\,a^2\right.}{}\right.}{}}\right.}{a\,(a+b)}$$

input
```
int(1/(cos(x)*(a + b*sin(x)^2)),x)
```

output
```
- (atan((((4*b^3*sin(x) + (8*a*b^3 + 4*b^4 + 4*a^2*b^2 - (sin(x)*(8*a*b^4
+ 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(2*(a + b)))/(2*(a + b)))*1i)/(2*(a + b)
) + ((4*b^3*sin(x) - (8*a*b^3 + 4*b^4 + 4*a^2*b^2 + (sin(x)*(8*a*b^4 + 8*b
^5 - 8*a^2*b^3 - 8*a^3*b^2))/(2*(a + b)))/(2*(a + b)))*1i)/(2*(a + b)))/((
4*b^3*sin(x) + (8*a*b^3 + 4*b^4 + 4*a^2*b^2 - (sin(x)*(8*a*b^4 + 8*b^5 - 8
*a^2*b^3 - 8*a^3*b^2))/(2*(a + b)))/(2*(a + b)))/(2*(a + b)) - (4*b^3*sin(
x) - (8*a*b^3 + 4*b^4 + 4*a^2*b^2 + (sin(x)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 -
 8*a^3*b^2))/(2*(a + b)))/(2*(a + b)))/(2*(a + b))))*1i)/(a + b) - (atan((
((2*b^3*sin(x) + ((-a*b)^(1/2)*(4*a*b^3 + 2*b^4 + 2*a^2*b^2 - (sin(x)*(-a*
b)^(1/2)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b + a^2))))/(2*(
a*b + a^2)))*(-a*b)^(1/2)*1i)/(a*b + a^2) + ((2*b^3*sin(x) - ((-a*b)^(1/2)
*(4*a*b^3 + 2*b^4 + 2*a^2*b^2 + (sin(x)*(-a*b)^(1/2)*(8*a*b^4 + 8*b^5 - 8*
a^2*b^3 - 8*a^3*b^2))/(4*(a*b + a^2))))/(2*(a*b + a^2)))*(-a*b)^(1/2)*1i)/
(a*b + a^2))/(((2*b^3*sin(x) + ((-a*b)^(1/2)*(4*a*b^3 + 2*b^4 + 2*a^2*b^2
- (sin(x)*(-a*b)^(1/2)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b
+ a^2))))/(2*(a*b + a^2)))*(-a*b)^(1/2))/(a*b + a^2) - ((2*b^3*sin(x) - ((
-a*b)^(1/2)*(4*a*b^3 + 2*b^4 + 2*a^2*b^2 + (sin(x)*(-a*b)^(1/2)*(8*a*b^4 +
 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b + a^2))))/(2*(a*b + a^2)))*(-a*b)
^(1/2))/(a*b + a^2)))*(-a*b)^(1/2)*1i)/(a*(a + b))
```

3.308. $\int \frac{\sec(x)}{a+b\sin^2(x)}\,dx$

3.309 $\int \frac{\sec^2(x)}{a+b\sin^2(x)}\,dx$

3.309.1 Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{\sec^2(x)}{a+b\sin^2(x)}\,dx = \frac{b\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}} + \frac{\tan(x)}{a+b}$$

output
```
b*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/(a+b)^(3/2)/a^(1/2)+tan(x)/(a+b)
```

3.309.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(x)}{a+b\sin^2(x)}\,dx = \frac{b\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}} + \frac{\tan(x)}{a+b}$$

input
```
Integrate[Sec[x]^2/(a + b*Sin[x]^2),x]
```

output
```
(b*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)) + Tan[x]/
(a + b)
```

3.309.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3670, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{a+b\sin^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(x)^2\left(a+b\sin(x)^2\right)}dx$$

$$\Big\downarrow \text{3670}$$

$$\int \frac{\tan^2(x)+1}{(a+b)\tan^2(x)+a}d\tan(x)$$

$$\Big\downarrow \text{299}$$

$$\frac{b\int \frac{1}{(a+b)\tan^2(x)+a}d\tan(x)}{a+b}+\frac{\tan(x)}{a+b}$$

$$\Big\downarrow \text{218}$$

$$\frac{b\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}}+\frac{\tan(x)}{a+b}$$

input
```
Int[Sec[x]^2/(a + b*Sin[x]^2),x]
```

output
```
(b*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)) + Tan[x]/
(a + b)
```

3.309.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.309.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\tan(x)}{a+b}+\frac{b\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{(a+b)\sqrt{a(a+b)}}$	38
risch	$\frac{2i}{(\mathrm{e}^{2ix}+1)(a+b)}-\frac{b\ln\left(\mathrm{e}^{2ix}-\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,(a+b)}+\frac{b\ln\left(\mathrm{e}^{2ix}+\frac{2ia^2+2iab-2a\sqrt{-a^2-ab}-b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,(a+b)}$	189

input
```
int(sec(x)^2/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
tan(x)/(a+b)+b/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

3.309.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(31) = 62.

Time = 0.32 (sec) , antiderivative size = 255, normalized size of antiderivative = 6.54

$$\int \frac{\sec^2(x)}{a+b\sin^2(x)}\,dx$$
$$= \left[-\frac{\sqrt{-a^2-ab}b\cos\left(x\right)\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos\left(x\right)^4-2\,\left(4\,a^2+5\,ab+b^2\right)\cos\left(x\right)^2+4\,\left(\left(2\,a+b\right)\cos\left(x\right)^3-\left(a+b\right)\cos\left(x\right)\right)\sqrt{-a^2-ab}\sin\left(x\right)+}{b^2\cos\left(x\right)^4-2\,\left(ab+b^2\right)\cos\left(x\right)^2+a^2+2\,ab+b^2}\right.}{4\,\left(a^3+2\,a^2b+ab^2\right)\cos\left(x\right)}\right.$$
$$\left.-\frac{\sqrt{a^2+ab}b\arctan\left(\frac{\left(2\,a+b\right)\cos\left(x\right)^2-a-b}{2\,\sqrt{a^2+ab}\cos\left(x\right)\sin\left(x\right)}\right)\cos\left(x\right)-2\,\left(a^2+ab\right)\sin\left(x\right)}{2\,\left(a^3+2\,a^2b+ab^2\right)\cos\left(x\right)}\right]$$

input
```
integrate(sec(x)^2/(a+b*sin(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(-a^2 - a*b)*b*cos(x)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(
4*a^2 + 5*a*b + b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sq
rt(-a^2 - a*b)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*c
os(x)^2 + a^2 + 2*a*b + b^2)) - 4*(a^2 + a*b)*sin(x))/((a^3 + 2*a^2*b + a*
b^2)*cos(x)), -1/2*(sqrt(a^2 + a*b)*b*arctan(1/2*((2*a + b)*cos(x)^2 - a -
 b)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))*cos(x) - 2*(a^2 + a*b)*sin(x))/((a^3
+ 2*a^2*b + a*b^2)*cos(x))]
```

3.309.6 Sympy [F]

$$\int \frac{\sec^2(x)}{a+b\sin^2(x)}\,dx = \int \frac{\sec^2\left(x\right)}{a+b\sin^2\left(x\right)}\,dx$$

input
```
integrate(sec(x)**2/(a+b*sin(x)**2),x)
```

output
```
Integral(sec(x)**2/(a + b*sin(x)**2), x)
```

3.309.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{\sec^2(x)}{a+b\sin^2(x)}\,dx = \frac{b\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}(a+b)} + \frac{\tan(x)}{a+b}$$

input
```
integrate(sec(x)^2/(a+b*sin(x)^2),x, algorithm="maxima")
```

output
```
b*arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a + b)) + tan(x
)/(a + b)
```

3.309.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{\sec^2(x)}{a+b\sin^2(x)}\,dx = \frac{b\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}(a+b)} + \frac{\tan(x)}{a+b}$$

input
```
integrate(sec(x)^2/(a+b*sin(x)^2),x, algorithm="giac")
```

output
```
b*arctan((a*tan(x) + b*tan(x))/sqrt(a^2 + a*b))/(sqrt(a^2 + a*b)*(a + b))
+ tan(x)/(a + b)
```

3.309.9 Mupad [B] (verification not implemented)

Time = 14.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(x)}{a+b\sin^2(x)}\,dx = \frac{\tan(x)}{a+b} + \frac{b\,\mathrm{atan}\left(\frac{\tan(x)\,(2\,a+2\,b)}{2\,\sqrt{a}\,\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{3/2}}$$

input
```
int(1/(cos(x)^2*(a + b*sin(x)^2)),x)
```

output
```
tan(x)/(a + b) + (b*atan((tan(x)*(2*a + 2*b))/(2*a^(1/2)*(a + b)^(1/2))))/
(a^(1/2)*(a + b)^(3/2))
```

3.310 $\int \frac{\sec^3(x)}{a+b\sin^2(x)}\,dx$

3.310.1 Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \frac{\sec^3(x)}{a+b\sin^2(x)}\,dx=\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^2}+\frac{(a+3b)\text{arctanh}(\sin(x))}{2(a+b)^2}+\frac{\sec(x)\tan(x)}{2(a+b)}$$

output
```
1/2*(a+3*b)*arctanh(sin(x))/(a+b)^2+b^(3/2)*arctan(sin(x)*b^(1/2)/a^(1/2))
/(a+b)^2/a^(1/2)+1/2*sec(x)*tan(x)/(a+b)
```

3.310.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 147 vs. 2(61) = 122.

Time = 0.35 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.41

$$\int \frac{\sec^3(x)}{a+b\sin^2(x)}\,dx$$
$$=\frac{-\frac{2b^{3/2}\arctan\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)}{\sqrt{a}}+\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}}-2(a+3b)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+2(a+3b)\log\left(\cos\left(\frac{x}{2}\right)+\text{si}\right.}{4(a+b)^2}$$

input
```
Integrate[Sec[x]^3/(a + b*Sin[x]^2),x]
```

output
```
((-2*b^(3/2)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/Sqrt[a] + (2*b^(3/2)*ArcTan
[(Sqrt[b]*Sin[x])/Sqrt[a]])/Sqrt[a] - 2*(a + 3*b)*Log[Cos[x/2] - Sin[x/2]]
 + 2*(a + 3*b)*Log[Cos[x/2] + Sin[x/2]] + (a + b)/(Cos[x/2] - Sin[x/2])^2
- (a + b)/(Cos[x/2] + Sin[x/2])^2)/(4*(a + b)^2)
```

3.310.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3669, 316, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(x)}{a+b\sin^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(x)^3\left(a+b\sin(x)^2\right)}dx$$

$$\Big\downarrow \text{3669}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2\left(a+b\sin^2(x)\right)}d\sin(x)$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{b\sin^2(x)+a+2b}{\left(1-\sin^2(x)\right)\left(b\sin^2(x)+a\right)}d\sin(x)}{2(a+b)}+\frac{\sin(x)}{2(a+b)\left(1-\sin^2(x)\right)}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{2b^2\int \frac{1}{b\sin^2(x)+a}d\sin(x)}{a+b}+\frac{(a+3b)\int \frac{1}{1-\sin^2(x)}d\sin(x)}{a+b}}{2(a+b)}+\frac{\sin(x)}{2(a+b)\left(1-\sin^2(x)\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(a+3b)\int \frac{1}{1-\sin^2(x)}d\sin(x)}{a+b}+\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2(a+b)}+\frac{\sin(x)}{2(a+b)\left(1-\sin^2(x)\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{(a+3b)\text{arctanh}(\sin(x))}{a+b}}{2(a+b)}+\frac{\sin(x)}{2(a+b)\left(1-\sin^2(x)\right)}$$

input
```
Int[Sec[x]^3/(a + b*Sin[x]^2),x]
```

output
```
((2*b^(3/2)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + ((a + 3*
b)*ArcTanh[Sin[x]])/(a + b))/(2*(a + b)) + Sin[x]/(2*(a + b)*(1 - Sin[x]^2
))
```

3.310.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.310.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.57

method	result
default	$-\frac{1}{(4a+4b)(1+\sin(x))}+\frac{(a+3b)\ln(1+\sin(x))}{4(a+b)^2}-\frac{1}{(4a+4b)(\sin(x)-1)}+\frac{(-a-3b)\ln(\sin(x)-1)}{4(a+b)^2}+\frac{b^2\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{(a+b)^2\sqrt{ab}}$
risch	$-\frac{i(e^{3ix}-e^{ix})}{(e^{2ix}+1)^2(a+b)}-\frac{\ln(e^{ix}-i)a}{2(a^2+2ab+b^2)}-\frac{3\ln(e^{ix}-i)b}{2(a^2+2ab+b^2)}+\frac{\ln(e^{ix}+i)a}{2a^2+4ab+2b^2}+\frac{3\ln(e^{ix}+i)b}{2(a^2+2ab+b^2)}+\frac{\sqrt{-ab}\,b\ln\left(e^{2ix}+\frac{2i\sqrt{-ab}\,e^{ix}}{b}-1\right.}{2a(a+b)^2}$

input
```
int(sec(x)^3/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/(4*a+4*b)/(1+sin(x))+1/4*(a+3*b)/(a+b)^2*ln(1+sin(x))-1/(4*a+4*b)/(sin(
x)-1)+1/4/(a+b)^2*(-a-3*b)*ln(sin(x)-1)+b^2/(a+b)^2/(a*b)^(1/2)*arctan(b*s
in(x)/(a*b)^(1/2))
```

3.310.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.33

$$\int \frac{\sec^3(x)}{a+b\sin^2(x)}\,dx$$

$$= \left[\frac{2\,b\sqrt{-\frac{b}{a}}\cos{(x)}^2\log\left(-\frac{b\cos(x)^2-2\,a\sqrt{-\frac{b}{a}}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+(a+3\,b)\cos{(x)}^2\log{(\sin{(x)}+1)}-(a+3\,b)\cos{(x)}}{4\,(a^2+2\,ab+b^2)\cos{(x)}^2}\right.$$

input
```
integrate(sec(x)^3/(a+b*sin(x)^2),x, algorithm="fricas")
```

output
```
[1/4*(2*b*sqrt(-b/a)*cos(x)^2*log(-(b*cos(x)^2 - 2*a*sqrt(-b/a)*sin(x) + a
 - b)/(b*cos(x)^2 - a - b)) + (a + 3*b)*cos(x)^2*log(sin(x) + 1) - (a + 3*
b)*cos(x)^2*log(-sin(x) + 1) + 2*(a + b)*sin(x))/((a^2 + 2*a*b + b^2)*cos(
x)^2), 1/4*(4*b*sqrt(b/a)*arctan(sqrt(b/a)*sin(x))*cos(x)^2 + (a + 3*b)*co
s(x)^2*log(sin(x) + 1) - (a + 3*b)*cos(x)^2*log(-sin(x) + 1) + 2*(a + b)*s
in(x))/((a^2 + 2*a*b + b^2)*cos(x)^2)]
```

3.310.6 Sympy [F]

$$\int \frac{\sec^3(x)}{a+b\sin^2(x)}\,dx = \int \frac{\sec^3{(x)}}{a+b\sin^2{(x)}}\,dx$$

input
```
integrate(sec(x)**3/(a+b*sin(x)**2),x)
```

output
```
Integral(sec(x)**3/(a + b*sin(x)**2), x)
```

3.310.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(49) = 98.

Time = 0.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.70

$$\int \frac{\sec^3(x)}{a+b\sin^2(x)}\,dx = \frac{b^2\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}} + \frac{(a+3\,b)\log\left(\sin\left(x\right)+1\right)}{4\,(a^2+2\,ab+b^2)} - \frac{(a+3\,b)\log\left(\sin\left(x\right)-1\right)}{4\,(a^2+2\,ab+b^2)} - \frac{\sin\left(x\right)}{2\,\left((a+b)\sin\left(x\right)^2-a-b\right)}$$

input
```
integrate(sec(x)^3/(a+b*sin(x)^2),x, algorithm="maxima")
```

output
```
b^2*arctan(b*sin(x)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) + 1/4*(a +
3*b)*log(sin(x) + 1)/(a^2 + 2*a*b + b^2) - 1/4*(a + 3*b)*log(sin(x) - 1)/(
a^2 + 2*a*b + b^2) - 1/2*sin(x)/((a + b)*sin(x)^2 - a - b)
```

3.310.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(49) = 98$.

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.67

$$\int \frac{\sec^3(x)}{a+b\sin^2(x)}\,dx = \frac{b^2\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}} + \frac{(a+3\,b)\log\left(\sin\left(x\right)+1\right)}{4\,(a^2+2\,ab+b^2)} - \frac{(a+3\,b)\log\left(-\sin\left(x\right)+1\right)}{4\,(a^2+2\,ab+b^2)} - \frac{\sin\left(x\right)}{2\left(\sin\left(x\right)^2-1\right)(a+b)}$$

input
```
integrate(sec(x)^3/(a+b*sin(x)^2),x, algorithm="giac")
```

output
```
b^2*arctan(b*sin(x)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) + 1/4*(a +
3*b)*log(sin(x) + 1)/(a^2 + 2*a*b + b^2) - 1/4*(a + 3*b)*log(-sin(x) + 1)/
(a^2 + 2*a*b + b^2) - 1/2*sin(x)/((sin(x)^2 - 1)*(a + b))
```

3.310.9 Mupad [B] (verification not implemented)

Time = 13.90 (sec) , antiderivative size = 1139, normalized size of antiderivative = 18.67

$$\int \frac{\sec^3(x)}{a+b\sin^2(x)}\,dx$$
$$= \frac{\sin\left(x\right)}{2\cos\left(x\right)^2\,\left(a+b\right)} - \ln\left(\sin\left(x\right)-1\right)\,\left(\frac{b}{2\left(a+b\right)^2}+\frac{1}{4\,\left(a+b\right)}\right) + \frac{\ln\left(\sin\left(x\right)+1\right)\,\left(a+3\,b\right)}{4\left(a+b\right)^2}$$
$$+ \frac{\mathrm{atan}\left(\frac{\frac{\sqrt{-a\,b^3}\left(\frac{\sin(x)\left(a^2\,b^3+6\,a\,b^4+13\,b^5\right)}{4\left(a^2+2\,a\,b+b^2\right)}+\frac{\left(\frac{2\,a^5\,b^2+12\,a^4\,b^3+28\,a^3\,b^4+32\,a^2\,b^5+18\,a\,b^6+4\,b^7}{2\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}-\frac{\sin(x)\,\sqrt{-a\,b^3}\left(-16\,a^5\,b^2-48\,a^4\,b^3-32\,a^3 \cdots\right.}{8\left(a^2+2\,a\,b+b^2\right)\left(a^3+2 \cdots\right.}\cdots}{2\left(a^3+2\,a^2\,b+a\,b^2\right)}\cdots}{a^3+2\,a^2\,b+a\,b^2}\cdots}{\frac{\frac{3\,b^5}{2}+\frac{a\,b^4}{2}}{a^3+3\,a^2\,b+3\,a\,b^2+b^3}-\frac{\sqrt{-a\,b^3}\left(\frac{\sin(x)\left(a^2\,b^3+6\,a\,b^4+13\,b^5\right)}{4\left(a^2+2\,a\,b+b^2\right)}+\frac{\left(\frac{2\,a^5\,b^2+12\,a^4\,b^3+28\,a^3\,b^4+32\,a^2\,b^5+18\,a\,b^6+4\,b^7}{2\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}-\frac{\sin(x)\,\sqrt{-a\,b^3}\left(-16\,a^5\,b^2-4 \cdots\right.}{8\left(a^2+2\,a \cdots\right.}\cdots}{2\left(a^3+2\,a^2\,b+a\,b^2\right)}\cdots}{a^3+2\,a^2\,b+a\,b^2}\cdots}\right)\cdots}{\cdots}$$

input
```
int(1/(cos(x)^3*(a + b*sin(x)^2)),x)
```

output
```
sin(x)/(2*cos(x)^2*(a + b)) - log(sin(x) - 1)*(b/(2*(a + b)^2) + 1/(4*(a +
 b))) + (log(sin(x) + 1)*(a + 3*b))/(4*(a + b)^2) + (atan((((-a*b^3)^(1/2)
*((sin(x)*(6*a*b^4 + 13*b^5 + a^2*b^3))/(4*(2*a*b + a^2 + b^2)) + (((18*a*
b^6 + 4*b^7 + 32*a^2*b^5 + 28*a^3*b^4 + 12*a^4*b^3 + 2*a^5*b^2)/(2*(3*a*b^
2 + 3*a^2*b + a^3 + b^3)) - (sin(x)*(-a*b^3)^(1/2)*(48*a*b^6 + 16*b^7 + 32
*a^2*b^5 - 32*a^3*b^4 - 48*a^4*b^3 - 16*a^5*b^2))/(8*(2*a*b + a^2 + b^2)*(
a*b^2 + 2*a^2*b + a^3)))*(-a*b^3)^(1/2))/(2*(a*b^2 + 2*a^2*b + a^3)))*1i)/
(a*b^2 + 2*a^2*b + a^3) + ((-a*b^3)^(1/2)*((sin(x)*(6*a*b^4 + 13*b^5 + a^2
*b^3))/(4*(2*a*b + a^2 + b^2)) - (((18*a*b^6 + 4*b^7 + 32*a^2*b^5 + 28*a^3
*b^4 + 12*a^4*b^3 + 2*a^5*b^2)/(2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (sin(
x)*(-a*b^3)^(1/2)*(48*a*b^6 + 16*b^7 + 32*a^2*b^5 - 32*a^3*b^4 - 48*a^4*b^
3 - 16*a^5*b^2))/(8*(2*a*b + a^2 + b^2)*(a*b^2 + 2*a^2*b + a^3)))*(-a*b^3)
^(1/2))/(2*(a*b^2 + 2*a^2*b + a^3)))*1i)/(a*b^2 + 2*a^2*b + a^3))/(((a*b^4
)/2 + (3*b^5)/2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) - ((-a*b^3)^(1/2)*((sin(x
)*(6*a*b^4 + 13*b^5 + a^2*b^3))/(4*(2*a*b + a^2 + b^2)) + (((18*a*b^6 + 4*
b^7 + 32*a^2*b^5 + 28*a^3*b^4 + 12*a^4*b^3 + 2*a^5*b^2)/(2*(3*a*b^2 + 3*a^
2*b + a^3 + b^3)) - (sin(x)*(-a*b^3)^(1/2)*(48*a*b^6 + 16*b^7 + 32*a^2*b^5
 - 32*a^3*b^4 - 48*a^4*b^3 - 16*a^5*b^2))/(8*(2*a*b + a^2 + b^2)*(a*b^2 +
2*a^2*b + a^3)))*(-a*b^3)^(1/2))/(2*(a*b^2 + 2*a^2*b + a^3))))/(a*b^2 + 2*
a^2*b + a^3) + ((-a*b^3)^(1/2)*((sin(x)*(6*a*b^4 + 13*b^5 + a^2*b^3))/(...
```

3.310. $\int \frac{\sec^3(x)}{a+b\sin^2(x)}\,dx$

3.311 $\int \frac{\sec^4(x)}{a+b\sin^2(x)}\,dx$

3.311.1 Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{\sec^4(x)}{a+b\sin^2(x)}\,dx=\frac{b^2\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}}+\frac{(a+2b)\tan(x)}{(a+b)^2}+\frac{\tan^3(x)}{3(a+b)}$$

output
```
b^2*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/(a+b)^(5/2)/a^(1/2)+(a+2*b)*tan(x)/
(a+b)^2+1/3*tan(x)^3/(a+b)
```

3.311.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{\sec^4(x)}{a+b\sin^2(x)}\,dx=\frac{b^2\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}}+\frac{(2a+5b+(a+b)\sec^2(x))\tan(x)}{3(a+b)^2}$$

input
```
Integrate[Sec[x]^4/(a + b*Sin[x]^2),x]
```

output
```
(b^2*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2)) + ((2*a
 + 5*b + (a + b)*Sec[x]^2)*Tan[x])/(3*(a + b)^2)
```

3.311.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(x)}{a+b\sin^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(x)^4\left(a+b\sin(x)^2\right)}dx$$

$$\Big\downarrow \text{3670}$$

$$\int \frac{\left(\tan^2(x)+1\right)^2}{(a+b)\tan^2(x)+a}d\tan(x)$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{b^2}{(a+b)^2\left((a+b)\tan^2(x)+a\right)}+\frac{\tan^2(x)}{a+b}+\frac{a+2b}{(a+b)^2}\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}}+\frac{\tan^3(x)}{3(a+b)}+\frac{(a+2b)\tan(x)}{(a+b)^2}$$

input
```
Int[Sec[x]^4/(a + b*Sin[x]^2),x]
```

output
```
(b^2*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2)) + ((a +
 2*b)*Tan[x])/(a + b)^2 + Tan[x]^3/(3*(a + b))
```

3.311.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.311.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

method	result
default	$\frac{\frac{a\left(\tan^{3}(x)\right)}{3}+\frac{b\left(\tan^{3}(x)\right)}{3}+\tan(x)a+2\tan(x)b}{(a+b)^{2}}+\frac{b^{2}\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{(a+b)^{2}\sqrt{a(a+b)}}$
risch	$\frac{2i\left(3b\,e^{4ix}+6a\,e^{2ix}+12b\,e^{2ix}+2a+5b\right)}{3\left(e^{2ix}+1\right)^{3}(a+b)^{2}}-\frac{b^{2}\ln\left(e^{2ix}-\frac{2ia^{2}+2iab+2a\sqrt{-a^{2}-ab}+b\sqrt{-a^{2}-ab}}{b\sqrt{-a^{2}-ab}}\right)}{2\sqrt{-a^{2}-ab}\,(a+b)^{2}}+\frac{b^{2}\ln\left(e^{2ix}+\frac{2ia^{2}+2iab-2a\sqrt{-a^{2}-ab}-}{b\sqrt{-a^{2}-ab}}\right.}{2\sqrt{-a^{2}-ab}\,(a+b)^{2}}$

input
```
int(sec(x)^4/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/(a+b)^2*(1/3*a*tan(x)^3+1/3*b*tan(x)^3+tan(x)*a+2*tan(x)*b)+b^2/(a+b)^2/
(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

3.311.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(49) = 98.

Time = 0.32 (sec) , antiderivative size = 343, normalized size of antiderivative = 5.81

$$\int \frac{\sec^4(x)}{a+b\sin^2(x)}\,dx = \left[-\frac{3\sqrt{-a^2-ab}b^2\cos(x)^3\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+5ab+b^2)\cos(x)^2+4\left((2a+b)\cos(x)^3-(a+b)\cos(x)\right)\sqrt{-a^2-ab}\sin}{b^2\cos(x)^4-2(ab+b^2)\cos(x)^2+a^2+2ab+b^2}\right.}{12(a^4+3a^3b+3a^2b^2+ab^3)\cos}\right.$$

$$-\frac{3\sqrt{a^2+ab}b^2\arctan\left(\frac{(2a+b)\cos(x)^2-a-b}{2\sqrt{a^2+ab}\cos(x)\sin(x)}\right)\cos(x)^3-2\left(a^3+2a^2b+ab^2+(2a^3+7a^2b+5ab^2)\cos(x)^2\right.}{6(a^4+3a^3b+3a^2b^2+ab^3)\cos(x)^3}$$

input
```
integrate(sec(x)^4/(a+b*sin(x)^2),x, algorithm="fricas")
```

output
```
[-1/12*(3*sqrt(-a^2 - a*b)*b^2*cos(x)^3*log(((8*a^2 + 8*a*b + b^2)*cos(x)^
4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos
(x))*sqrt(-a^2 - a*b)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b +
 b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - 4*(a^3 + 2*a^2*b + a*b^2 + (2*a^3 +
 7*a^2*b + 5*a*b^2)*cos(x)^2)*sin(x))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)
*cos(x)^3), -1/6*(3*sqrt(a^2 + a*b)*b^2*arctan(1/2*((2*a + b)*cos(x)^2 - a
 - b)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))*cos(x)^3 - 2*(a^3 + 2*a^2*b + a*b^2
 + (2*a^3 + 7*a^2*b + 5*a*b^2)*cos(x)^2)*sin(x))/((a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*cos(x)^3)]
```

3.311.6 Sympy [F]

$$\int \frac{\sec^4(x)}{a+b\sin^2(x)}\,dx = \int \frac{\sec^4(x)}{a+b\sin^2(x)}\,dx$$

input
```
integrate(sec(x)**4/(a+b*sin(x)**2),x)
```

output
```
Integral(sec(x)**4/(a + b*sin(x)**2), x)
```

3.311.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.22

$$\int \frac{\sec^4(x)}{a+b\sin^2(x)}\,dx = \frac{b^2\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}(a^2+2\,ab+b^2)} + \frac{(a+b)\tan{(x)}^3+3\,(a+2\,b)\tan{(x)}}{3\,(a^2+2\,ab+b^2)}$$

input
```
integrate(sec(x)^4/(a+b*sin(x)^2),x, algorithm="maxima")
```

output
```
b^2*arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a^2 + 2*a*b +
 b^2)) + 1/3*((a + b)*tan(x)^3 + 3*(a + 2*b)*tan(x))/(a^2 + 2*a*b + b^2)
```

3.311.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(49) = 98.

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.27

$$\int \frac{\sec^4(x)}{a+b\sin^2(x)}\,dx = \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)b^2}{(a^2+2\,ab+b^2)\sqrt{a^2+ab}} + \frac{a^2\tan{(x)}^3+2\,ab\tan{(x)}^3+b^2\tan{(x)}^3+3\,a^2\tan{(x)}+9\,ab\tan{(x)}+6\,b^2\tan{(x)}}{3\,(a^3+3\,a^2b+3\,ab^2+b^3)}$$

input
```
integrate(sec(x)^4/(a+b*sin(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a
^2 + a*b)))*b^2/((a^2 + 2*a*b + b^2)*sqrt(a^2 + a*b)) + 1/3*(a^2*tan(x)^3
+ 2*a*b*tan(x)^3 + b^2*tan(x)^3 + 3*a^2*tan(x) + 9*a*b*tan(x) + 6*b^2*tan(
x))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

3.311.9 Mupad [B] (verification not implemented)

Time = 14.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.31

$$\int \frac{\sec^4(x)}{a+b\sin^2(x)}\,dx = \frac{\tan(x)^3}{3\ (a+b)} - \tan(x)\ \left(\frac{a}{(a+b)^2} - \frac{2}{a+b}\right) + \frac{b^2\,\mathrm{atan}\left(\frac{\tan(x)\ (2\,a+2\,b)\ (a^2+2\,a\,b+b^2)}{2\,\sqrt{a}\,(a+b)^{5/2}}\right)}{\sqrt{a}\,(a+b)^{5/2}}$$

input
```
int(1/(cos(x)^4*(a + b*sin(x)^2)),x)
```

output
```
tan(x)^3/(3*(a + b)) - tan(x)*(a/(a + b)^2 - 2/(a + b)) + (b^2*atan((tan(x
)*(2*a + 2*b)*(2*a*b + a^2 + b^2))/(2*a^(1/2)*(a + b)^(5/2))))/(a^(1/2)*(a
 + b)^(5/2))
```

3.312 $\int \frac{\sec^5(x)}{a+b\sin^2(x)}\,dx$

3.312.1 Optimal result

Integrand size = 15, antiderivative size = 93

$$\int \frac{\sec^5(x)}{a+b\sin^2(x)}\,dx = \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^3} + \frac{(3a^2+10ab+15b^2)\,\text{arctanh}(\sin(x))}{8(a+b)^3} + \frac{(3a+7b)\sec(x)\tan(x)}{8(a+b)^2} + \frac{\sec^3(x)\tan(x)}{4(a+b)}$$

output
```
1/8*(3*a^2+10*a*b+15*b^2)*arctanh(sin(x))/(a+b)^3+b^(5/2)*arctan(sin(x)*b^
(1/2)/a^(1/2))/(a+b)^3/a^(1/2)+1/8*(3*a+7*b)*sec(x)*tan(x)/(a+b)^2+1/4*sec
(x)^3*tan(x)/(a+b)
```

3.312.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 214 vs. 2(93) = 186.

Time = 1.33 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.30

$$\int \frac{\sec^5(x)}{a+b\sin^2(x)}\,dx = -\frac{\frac{8b^{5/2}\arctan\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)}{\sqrt{a}} - \frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}} + 2(3a^2+10ab+15b^2)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) - 2(3a^2+10ab}{16(a+b}$$

input
```
Integrate[Sec[x]^5/(a + b*Sin[x]^2),x]
```

output
```
-1/16*((8*b^(5/2)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/Sqrt[a] - (8*b^(5/2)*A
rcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/Sqrt[a] + 2*(3*a^2 + 10*a*b + 15*b^2)*Log
[Cos[x/2] - Sin[x/2]] - 2*(3*a^2 + 10*a*b + 15*b^2)*Log[Cos[x/2] + Sin[x/2
]] - (a + b)^2/(Cos[x/2] - Sin[x/2])^4 + (a + b)^2/(Cos[x/2] + Sin[x/2])^4
 + ((a + b)*(3*a + 7*b))/(Cos[x/2] + Sin[x/2])^2 + ((a + b)*(3*a + 7*b))/(
-1 + Sin[x]))/(a + b)^3
```

3.312.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.34, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 3669, 316, 402, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(x)}{a+b\sin^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(x)^5\left(a+b\sin(x)^2\right)}dx$$

$$\Big\downarrow \text{3669}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^3\left(a+b\sin^2(x)\right)}d\sin(x)$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{3b\sin^2(x)+3a+4b}{\left(1-\sin^2(x)\right)^2\left(b\sin^2(x)+a\right)}d\sin(x)}{4(a+b)}+\frac{\sin(x)}{4(a+b)\left(1-\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\int \frac{3a^2+7ba+8b^2+b(3a+7b)\sin^2(x)}{\left(1-\sin^2(x)\right)\left(b\sin^2(x)+a\right)}d\sin(x)}{2(a+b)}+\frac{(3a+7b)\sin(x)}{2(a+b)\left(1-\sin^2(x)\right)}}{4(a+b)}+\frac{\sin(x)}{4(a+b)\left(1-\sin^2(x)\right)^2}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{(3a^2+10ab+15b^2)\int \frac{1}{1-\sin^2(x)}d\sin(x)}{a+b}+\frac{8b^3\int \frac{1}{b\sin^2(x)+a}d\sin(x)}{a+b}}{2(a+b)}+\frac{(3a+7b)\sin(x)}{2(a+b)(1-\sin^2(x))}}{4(a+b)}+\frac{\sin(x)}{4(a+b)\left(1-\sin^2(x)\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{(3a^2+10ab+15b^2)\int \frac{1}{1-\sin^2(x)}d\sin(x)}{a+b}+\frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2(a+b)}+\frac{(3a+7b)\sin(x)}{2(a+b)(1-\sin^2(x))}}{4(a+b)}+\frac{\sin(x)}{4(a+b)\left(1-\sin^2(x)\right)^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{(3a^2+10ab+15b^2)\text{arctanh}(\sin(x))}{a+b}+\frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2(a+b)}+\frac{(3a+7b)\sin(x)}{2(a+b)(1-\sin^2(x))}}{4(a+b)}+\frac{\sin(x)}{4(a+b)\left(1-\sin^2(x)\right)^2}$$

input
```
Int[Sec[x]^5/(a + b*Sin[x]^2),x]
```

output
```
Sin[x]/(4*(a + b)*(1 - Sin[x]^2)^2) + (((8*b^(5/2)*ArcTan[(Sqrt[b]*Sin[x])
/Sqrt[a]])/(Sqrt[a]*(a + b)) + ((3*a^2 + 10*a*b + 15*b^2)*ArcTanh[Sin[x]])
/(a + b))/(2*(a + b)) + ((3*a + 7*b)*Sin[x])/(2*(a + b)*(1 - Sin[x]^2)))/(
4*(a + b))
```

3.312.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.312.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.66

method	result
default	$-\frac{1}{2(8a+8b)(1+\sin(x))^2}-\frac{3a+7b}{16(a+b)^2(1+\sin(x))}+\frac{(3a^2+10ab+15b^2)\ln(1+\sin(x))}{16(a+b)^3}+\frac{1}{2(8a+8b)(\sin(x)-1)^2}-\frac{3a+7b}{16(a+b)^2(\sin(x}$
risch	$-\frac{i(3a\,e^{7ix}+7b\,e^{7ix}+11a\,e^{5ix}+15b\,e^{5ix}-11a\,e^{3ix}-15b\,e^{3ix}-3\,e^{ix}a-7\,e^{ix}b)}{4(e^{2ix}+1)^4(a+b)^2}+\frac{3\ln(e^{ix}+i)a^2}{8(a^3+3a^2b+3a\,b^2+b^3)}+\frac{5\ln(e^{ix}+i)ab}{4(a^3+3a^2b+3a\,b^2+b^3)}$

3.312. $\int\frac{\sec^5(x)}{a+b\sin^2(x)}\,dx$

input
```
int(sec(x)^5/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(8*a+8*b)/(1+sin(x))^2-1/16*(3*a+7*b)/(a+b)^2/(1+sin(x))+1/16*(3*a^2+
10*a*b+15*b^2)/(a+b)^3*ln(1+sin(x))+1/2/(8*a+8*b)/(sin(x)-1)^2-1/16*(3*a+7
*b)/(a+b)^2/(sin(x)-1)+1/16/(a+b)^3*(-3*a^2-10*a*b-15*b^2)*ln(sin(x)-1)+b^
3/(a+b)^3/(a*b)^(1/2)*arctan(b*sin(x)/(a*b)^(1/2))
```

3.312.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.52

$$\int \frac{\sec^5(x)}{a+b\sin^2(x)}\,dx = \left[\frac{8\,b^2\sqrt{-\frac{b}{a}}\cos\left(x\right)^4\log\left(-\frac{b\cos(x)^2-2\,a\sqrt{-\frac{b}{a}}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+\left(3\,a^2+10\,ab+15\,b^2\right)\cos\left(x\right)^4\log\left(\sin\left(x\right)+1\right)-(3}{16\,(a^3+3\,a^2b+}\right.$$

input
```
integrate(sec(x)^5/(a+b*sin(x)^2),x, algorithm="fricas")
```

output
```
[1/16*(8*b^2*sqrt(-b/a)*cos(x)^4*log(-(b*cos(x)^2 - 2*a*sqrt(-b/a)*sin(x)
+ a - b)/(b*cos(x)^2 - a - b)) + (3*a^2 + 10*a*b + 15*b^2)*cos(x)^4*log(si
n(x) + 1) - (3*a^2 + 10*a*b + 15*b^2)*cos(x)^4*log(-sin(x) + 1) + 2*((3*a^
2 + 10*a*b + 7*b^2)*cos(x)^2 + 2*a^2 + 4*a*b + 2*b^2)*sin(x))/((a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*cos(x)^4), 1/16*(16*b^2*sqrt(b/a)*arctan(sqrt(b/a)*si
n(x))*cos(x)^4 + (3*a^2 + 10*a*b + 15*b^2)*cos(x)^4*log(sin(x) + 1) - (3*a
^2 + 10*a*b + 15*b^2)*cos(x)^4*log(-sin(x) + 1) + 2*((3*a^2 + 10*a*b + 7*b
^2)*cos(x)^2 + 2*a^2 + 4*a*b + 2*b^2)*sin(x))/((a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cos(x)^4)]
```

3.312.6 Sympy [F]

$$\int \frac{\sec^5(x)}{a+b\sin^2(x)}\,dx=\int \frac{\sec^5(x)}{a+b\sin^2(x)}\,dx$$

input
```
integrate(sec(x)**5/(a+b*sin(x)**2),x)
```

output
```
Integral(sec(x)**5/(a + b*sin(x)**2), x)
```

3.312.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(79) = 158.

Time = 0.33 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.14

$$\begin{aligned}\int \frac{\sec^5(x)}{a+b\sin^2(x)}\,dx &= \frac{b^3\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}}+\frac{(3\,a^2+10\,ab+15\,b^2)\log(\sin(x)+1)}{16\,(a^3+3\,a^2b+3\,ab^2+b^3)}\\&\quad-\frac{(3\,a^2+10\,ab+15\,b^2)\log(\sin(x)-1)}{16\,(a^3+3\,a^2b+3\,ab^2+b^3)}\\&\quad-\frac{(3\,a+7\,b)\sin(x)^3-(5\,a+9\,b)\sin(x)}{8\,\left((a^2+2\,ab+b^2)\sin(x)^4-2\,(a^2+2\,ab+b^2)\sin(x)^2+a^2+2\,ab+b^2\right)}\end{aligned}$$

input
```
integrate(sec(x)^5/(a+b*sin(x)^2),x, algorithm="maxima")
```

output
```
b^3*arctan(b*sin(x)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b))
 + 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(sin(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2
 + b^3) - 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(sin(x) - 1)/(a^3 + 3*a^2*b +
3*a*b^2 + b^3) - 1/8*((3*a + 7*b)*sin(x)^3 - (5*a + 9*b)*sin(x))/((a^2 + 2
*a*b + b^2)*sin(x)^4 - 2*(a^2 + 2*a*b + b^2)*sin(x)^2 + a^2 + 2*a*b + b^2)
```

3.312.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(79) = 158.

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.90

$$\int \frac{\sec^5(x)}{a+b\sin^2(x)}\,dx = \frac{b^3\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab}} + \frac{(3a^2+10ab+15b^2)\log(\sin(x)+1)}{16(a^3+3a^2b+3ab^2+b^3)} - \frac{(3a^2+10ab+15b^2)\log(-\sin(x)+1)}{16(a^3+3a^2b+3ab^2+b^3)} - \frac{3a\sin(x)^3+7b\sin(x)^3-5a\sin(x)-9b\sin(x)}{8(a^2+2ab+b^2)\left(\sin(x)^2-1\right)^2}$$

input
```
integrate(sec(x)^5/(a+b*sin(x)^2),x, algorithm="giac")
```

output
```
b^3*arctan(b*sin(x)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b))
 + 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(sin(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2
 + b^3) - 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(-sin(x) + 1)/(a^3 + 3*a^2*b +
 3*a*b^2 + b^3) - 1/8*(3*a*sin(x)^3 + 7*b*sin(x)^3 - 5*a*sin(x) - 9*b*sin(
x))/((a^2 + 2*a*b + b^2)*(sin(x)^2 - 1)^2)
```

3.312.9 Mupad [B] (verification not implemented)

Time = 17.83 (sec) , antiderivative size = 832, normalized size of antiderivative = 8.95

$$\int \frac{\sec^5(x)}{a+b\sin^2(x)}\,dx$$
$$= \frac{5a^3\sin(x)-3a^3\sin(x)^3+3a^3\operatorname{atanh}(\sin(x))+9ab^2\sin(x)+14a^2b\sin(x)-6a^3\operatorname{atanh}(\sin(x))\sin(\ldots}{\ldots}$$

input
```
int(1/(cos(x)^5*(a + b*sin(x)^2)),x)
```

output
```
(5*a^3*sin(x) - 3*a^3*sin(x)^3 + 3*a^3*atanh(sin(x)) + atan((a*sin(x)*(-a*
b^5)^(3/2)*64i - b*sin(x)*(-a*b^5)^(3/2)*64i + a^6*b*sin(x)*(-a*b^5)^(1/2)
*9i + a^2*b^5*sin(x)*(-a*b^5)^(1/2)*289i + a^3*b^4*sin(x)*(-a*b^5)^(1/2)*3
00i + a^4*b^3*sin(x)*(-a*b^5)^(1/2)*190i + a^5*b^2*sin(x)*(-a*b^5)^(1/2)*6
0i)/(64*a^2*b^8 + 225*a^3*b^7 + 300*a^4*b^6 + 190*a^5*b^5 + 60*a^6*b^4 + 9
*a^7*b^3))*(-a*b^5)^(1/2)*8i + 9*a*b^2*sin(x) + 14*a^2*b*sin(x) - 6*a^3*at
anh(sin(x))*sin(x)^2 + 3*a^3*atanh(sin(x))*sin(x)^4 - 7*a*b^2*sin(x)^3 - 1
0*a^2*b*sin(x)^3 + 15*a*b^2*atanh(sin(x)) + 10*a^2*b*atanh(sin(x)) - atan(
(a*sin(x)*(-a*b^5)^(3/2)*64i - b*sin(x)*(-a*b^5)^(3/2)*64i + a^6*b*sin(x)*
(-a*b^5)^(1/2)*9i + a^2*b^5*sin(x)*(-a*b^5)^(1/2)*289i + a^3*b^4*sin(x)*(-
a*b^5)^(1/2)*300i + a^4*b^3*sin(x)*(-a*b^5)^(1/2)*190i + a^5*b^2*sin(x)*(-
a*b^5)^(1/2)*60i)/(64*a^2*b^8 + 225*a^3*b^7 + 300*a^4*b^6 + 190*a^5*b^5 +
60*a^6*b^4 + 9*a^7*b^3))*sin(x)^2*(-a*b^5)^(1/2)*16i + atan((a*sin(x)*(-a*
b^5)^(3/2)*64i - b*sin(x)*(-a*b^5)^(3/2)*64i + a^6*b*sin(x)*(-a*b^5)^(1/2)
*9i + a^2*b^5*sin(x)*(-a*b^5)^(1/2)*289i + a^3*b^4*sin(x)*(-a*b^5)^(1/2)*3
00i + a^4*b^3*sin(x)*(-a*b^5)^(1/2)*190i + a^5*b^2*sin(x)*(-a*b^5)^(1/2)*6
0i)/(64*a^2*b^8 + 225*a^3*b^7 + 300*a^4*b^6 + 190*a^5*b^5 + 60*a^6*b^4 + 9
*a^7*b^3))*sin(x)^4*(-a*b^5)^(1/2)*8i - 30*a*b^2*atanh(sin(x))*sin(x)^2 -
20*a^2*b*atanh(sin(x))*sin(x)^2 + 15*a*b^2*atanh(sin(x))*sin(x)^4 + 10*a^2
*b*atanh(sin(x))*sin(x)^4)/(8*a^4*sin(x)^4 - 16*a^4*sin(x)^2 + 8*a*b^3 ...
```

3.312. $\int \frac{\sec^5(x)}{a+b\sin^2(x)}\,dx$

3.313 $\int \frac{\sec^6(x)}{a+b\sin^2(x)}\,dx$

3.313.1 Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \frac{\sec^6(x)}{a+b\sin^2(x)}\,dx = \frac{b^3\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{7/2}} + \frac{(a^2+3ab+3b^2)\tan(x)}{(a+b)^3} + \frac{(2a+3b)\tan^3(x)}{3(a+b)^2} + \frac{\tan^5(x)}{5(a+b)}$$

output
```
b^3*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/(a+b)^(7/2)/a^(1/2)+(a^2+3*a*b+3*b^
2)*tan(x)/(a+b)^3+1/3*(2*a+3*b)*tan(x)^3/(a+b)^2+1/5*tan(x)^5/(a+b)
```

3.313.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{\sec^6(x)}{a+b\sin^2(x)}\,dx = \frac{b^3\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{7/2}} + \frac{(8a^2+26ab+33b^2+(4a^2+13ab+9b^2)\sec^2(x)+3(a+b)^2\sec^4(x))\tan(x)}{15(a+b)^3}$$

input
```
Integrate[Sec[x]^6/(a + b*Sin[x]^2),x]
```

output
```
(b^3*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(7/2)) + ((8*a
^2 + 26*a*b + 33*b^2 + (4*a^2 + 13*a*b + 9*b^2)*Sec[x]^2 + 3*(a + b)^2*Sec
[x]^4)*Tan[x])/(15*(a + b)^3)
```

3.313.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(x)}{a+b\sin^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(x)^6\left(a+b\sin(x)^2\right)}dx$$

$$\Big\downarrow \text{3670}$$

$$\int \frac{\left(\tan^2(x)+1\right)^3}{(a+b)\tan^2(x)+a}d\tan(x)$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{a^2+3ab+3b^2}{(a+b)^3}+\frac{b^3}{(a+b)^3\left((a+b)\tan^2(x)+a\right)}+\frac{\tan^4(x)}{a+b}+\frac{(2a+3b)\tan^2(x)}{(a+b)^2}\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a^2+3ab+3b^2\right)\tan(x)}{(a+b)^3}+\frac{b^3\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{7/2}}+\frac{\tan^5(x)}{5(a+b)}+\frac{(2a+3b)\tan^3(x)}{3(a+b)^2}$$

input
```
Int[Sec[x]^6/(a + b*Sin[x]^2),x]
```

output
```
(b^3*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(7/2)) + ((a^2
 + 3*a*b + 3*b^2)*Tan[x])/(a + b)^3 + ((2*a + 3*b)*Tan[x]^3)/(3*(a + b)^2)
 + Tan[x]^5/(5*(a + b))
```

3.313.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.313.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.25

method	result
default	$\frac{\frac{a^2\left(\tan^5(x)\right)}{5}+\frac{2ab\left(\tan^5(x)\right)}{5}+\frac{b^2\left(\tan^5(x)\right)}{5}+\frac{2a^2\left(\tan^3(x)\right)}{3}+\frac{5ab\left(\tan^3(x)\right)}{3}+b^2\left(\tan^3(x)\right)+a^2\tan(x)+3ab\tan(x)+3b^2\tan(x)}{(a+b)^3}+\frac{b^3\operatorname{arcta}}{(a+}$
risch	$\frac{2i\left(15b^2\mathrm{e}^{8ix}+30ab\,\mathrm{e}^{6ix}+90b^2\mathrm{e}^{6ix}+80a^2\mathrm{e}^{4ix}+230ab\,\mathrm{e}^{4ix}+240b^2\mathrm{e}^{4ix}+40\,\mathrm{e}^{2ix}a^2+130ab\,\mathrm{e}^{2ix}+150\,\mathrm{e}^{2ix}b^2+8a^2+26ab+33b^2\right)}{15(a+b)^3\left(\mathrm{e}^{2ix}+1\right)^5}-\frac{b^3\ln(}{}$

input
```
int(sec(x)^6/(a+b*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/(a+b)^3*(1/5*a^2*tan(x)^5+2/5*a*b*tan(x)^5+1/5*b^2*tan(x)^5+2/3*a^2*tan(
x)^3+5/3*a*b*tan(x)^3+b^2*tan(x)^3+a^2*tan(x)+3*a*b*tan(x)+3*b^2*tan(x))+b
^3/(a+b)^3/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

3.313.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(75) = 150$.

Time = 0.33 (sec) , antiderivative size = 459, normalized size of antiderivative = 5.28

$$\int \frac{\sec^6(x)}{a+b\sin^2(x)}\,dx$$
$$= \left[-\frac{15\sqrt{-a^2-ab}b^3\cos(x)^5\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+5ab+b^2)\cos(x)^2+4\left((2a+b)\cos(x)^3-(a+b)\cos(x)\right)\sqrt{-a^2-ab}\,\mathrm{si}}{b^2\cos(x)^4-2(ab+b^2)\cos(x)^2+a^2+2ab+b^2}\right.}{60}\right.$$

$$-\frac{15\sqrt{a^2+ab}b^3\arctan\left(\frac{(2a+b)\cos(x)^2-a-b}{2\sqrt{a^2+ab}\cos(x)\sin(x)}\right)\cos(x)^5-2\left((8a^4+34a^3b+59a^2b^2+33ab^3)\cos(x)^4+3a\right.}{30(a^5+4a^4b+6a^3b^2+4a^2b^3+ab}$$

input
```
integrate(sec(x)^6/(a+b*sin(x)^2),x, algorithm="fricas")
```

output
```
[-1/60*(15*sqrt(-a^2 - a*b)*b^3*cos(x)^5*log(((8*a^2 + 8*a*b + b^2)*cos(x)
^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*co
s(x))*sqrt(-a^2 - a*b)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b
+ b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - 4*((8*a^4 + 34*a^3*b + 59*a^2*b^2
+ 33*a*b^3)*cos(x)^4 + 3*a^4 + 9*a^3*b + 9*a^2*b^2 + 3*a*b^3 + (4*a^4 + 17
*a^3*b + 22*a^2*b^2 + 9*a*b^3)*cos(x)^2)*sin(x))/((a^5 + 4*a^4*b + 6*a^3*b
^2 + 4*a^2*b^3 + a*b^4)*cos(x)^5), -1/30*(15*sqrt(a^2 + a*b)*b^3*arctan(1/
2*((2*a + b)*cos(x)^2 - a - b)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))*cos(x)^5 -
 2*((8*a^4 + 34*a^3*b + 59*a^2*b^2 + 33*a*b^3)*cos(x)^4 + 3*a^4 + 9*a^3*b
+ 9*a^2*b^2 + 3*a*b^3 + (4*a^4 + 17*a^3*b + 22*a^2*b^2 + 9*a*b^3)*cos(x)^2
)*sin(x))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^5)]
```

3.313.6 Sympy [F]

$$\int \frac{\sec^6(x)}{a+b\sin^2(x)}\,dx = \int \frac{\sec^6(x)}{a+b\sin^2(x)}\,dx$$

input
```
integrate(sec(x)**6/(a+b*sin(x)**2),x)
```

output
```
Integral(sec(x)**6/(a + b*sin(x)**2), x)
```

3.313.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \frac{\sec^6(x)}{a+b\sin^2(x)}\,dx = \frac{b^3\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{(a^3+3a^2b+3ab^2+b^3)\sqrt{(a+b)a}} + \frac{3(a^2+2ab+b^2)\tan(x)^5+5(2a^2+5ab+3b^2)\tan(x)^3+15(a^2+3ab+3b^2)\tan(x)}{15(a^3+3a^2b+3ab^2+b^3)}$$

input
```
integrate(sec(x)^6/(a+b*sin(x)^2),x, algorithm="maxima")
```

output
```
b^3*arctan((a + b)*tan(x)/sqrt((a + b)*a))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*sqrt((a + b)*a)) + 1/15*(3*(a^2 + 2*a*b + b^2)*tan(x)^5 + 5*(2*a^2 + 5*a
*b + 3*b^2)*tan(x)^3 + 15*(a^2 + 3*a*b + 3*b^2)*tan(x))/(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)
```

3.313.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(75) = 150.

Time = 0.30 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.92

$$\int \frac{\sec^6(x)}{a+b\sin^2(x)}\,dx = \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)b^3}{(a^3+3a^2b+3ab^2+b^3)\sqrt{a^2+ab}} + \frac{3a^4\tan(x)^5+12a^3b\tan(x)^5+18a^2b^2\tan(x)^5+12ab^3\tan(x)^5+3b^4\tan(x)^5+10a^4\tan(x)^3+45a}{15}$$

input
```
integrate(sec(x)^6/(a+b*sin(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a
^2 + a*b)))*b^3/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a^2 + a*b)) + 1/15*(
3*a^4*tan(x)^5 + 12*a^3*b*tan(x)^5 + 18*a^2*b^2*tan(x)^5 + 12*a*b^3*tan(x)
^5 + 3*b^4*tan(x)^5 + 10*a^4*tan(x)^3 + 45*a^3*b*tan(x)^3 + 75*a^2*b^2*tan
(x)^3 + 55*a*b^3*tan(x)^3 + 15*b^4*tan(x)^3 + 15*a^4*tan(x) + 75*a^3*b*tan
(x) + 150*a^2*b^2*tan(x) + 135*a*b^3*tan(x) + 45*b^4*tan(x))/(a^5 + 5*a^4*
b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)
```

3.313.9 Mupad [B] (verification not implemented)

Time = 14.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.39

$$\int \frac{\sec^6(x)}{a+b\sin^2(x)}\,dx = \frac{\tan(x)^5}{5\,(a+b)} - \tan(x)^3\left(\frac{a}{3\,(a+b)^2} - \frac{1}{a+b}\right) + \tan(x)\left(\frac{3}{a+b} + \frac{a\left(\frac{a}{(a+b)^2} - \frac{3}{a+b}\right)}{a+b}\right) + \frac{b^3\,\mathrm{atan}\left(\frac{\tan(x)\,(2\,a+2\,b)\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}{2\,\sqrt{a}\,(a+b)^{7/2}}\right)}{\sqrt{a}\,(a+b)^{7/2}}$$

input
```
int(1/(cos(x)^6*(a + b*sin(x)^2)),x)
```

output
```
tan(x)^5/(5*(a + b)) - tan(x)^3*(a/(3*(a + b)^2) - 1/(a + b)) + tan(x)*(3/
(a + b) + (a*(a/(a + b)^2 - 3/(a + b)))/(a + b)) + (b^3*atan((tan(x)*(2*a
+ 2*b)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(2*a^(1/2)*(a + b)^(7/2))))/(a^(1/
2)*(a + b)^(7/2))
```

3.314 $\int \frac{\cos^6(x)}{(a+b\sin^2(x))^2}\,dx$

3.314.1 Optimal result

Integrand size = 15, antiderivative size = 113

$$\int \frac{\cos^6(x)}{(a+b\sin^2(x))^2}\,dx = \frac{(4a+5b)x}{2b^3} - \frac{(4a-b)(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^3} - \frac{\cos(x)\sin(x)}{2b\left(a+(a+b)\tan^2(x)\right)} + \frac{(a+b)(2a+b)\tan(x)}{2ab^2\left(a+(a+b)\tan^2(x)\right)}$$

output
```
1/2*(4*a+5*b)*x/b^3-1/2*(4*a-b)*(a+b)^(3/2)*arctan((a+b)^(1/2)*tan(x)/a^(1
/2))/a^(3/2)/b^3-1/2*cos(x)*sin(x)/b/(a+(a+b)*tan(x)^2)+1/2*(a+b)*(2*a+b)*
tan(x)/a/b^2/(a+(a+b)*tan(x)^2)
```

3.314.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80

$$\int \frac{\cos^6(x)}{(a+b\sin^2(x))^2}\,dx = \frac{2(4a+5b)x - \frac{2(4a-b)(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{a^{3/2}} + b\sin(2x) + \frac{2b(a+b)^2\sin(2x)}{a(2a+b-b\cos(2x))}}{4b^3}$$

input
```
Integrate[Cos[x]^6/(a + b*Sin[x]^2)^2,x]
```

output
```
(2*(4*a + 5*b)*x - (2*(4*a - b)*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[x])/
Sqrt[a]])/a^(3/2) + b*Sin[2*x] + (2*b*(a + b)^2*Sin[2*x])/(a*(2*a + b - b*
Cos[2*x])))/(4*b^3)
```

3.314.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3670, 316, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(x)}{\left(a+b\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^6}{\left(a+b\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \text{3670}$$

$$\int \frac{1}{\left(\tan^2(x)+1\right)^2\left((a+b)\tan^2(x)+a\right)^2}d\tan(x)$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{-3(a+b)\tan^2(x)+a+2b}{\left(\tan^2(x)+1\right)\left((a+b)\tan^2(x)+a\right)^2}d\tan(x)}{2b}-\frac{\tan(x)}{2b\left(\tan^2(x)+1\right)\left((a+b)\tan^2(x)+a\right)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{(a+b)(2a+b)\tan(x)}{ab((a+b)\tan^2(x)+a)}-\frac{\int \frac{2\left(2a^2+2ba-b^2-(a+b)(2a+b)\tan^2(x)\right)}{\left(\tan^2(x)+1\right)\left((a+b)\tan^2(x)+a\right)}d\tan(x)}{2ab}}{2b}-\frac{\tan(x)}{2b\left(\tan^2(x)+1\right)\left((a+b)\tan^2(x)+a\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(a+b)(2a+b)\tan(x)}{ab((a+b)\tan^2(x)+a)}-\frac{\int \frac{2a^2+2ba-b^2-(a+b)(2a+b)\tan^2(x)}{\left(\tan^2(x)+1\right)\left((a+b)\tan^2(x)+a\right)}d\tan(x)}{ab}}{2b}-\frac{\tan(x)}{2b\left(\tan^2(x)+1\right)\left((a+b)\tan^2(x)+a\right)}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{(a+b)(2a+b)\tan(x)}{ab((a+b)\tan^2(x)+a)}-\frac{\frac{(4a-b)(a+b)^2\int\frac{1}{(a+b)\tan^2(x)+a}d\tan(x)}{b}-\frac{a(4a+5b)\int\frac{1}{\tan^2(x)+1}d\tan(x)}{b}}{ab}}{2b}-\frac{\tan(x)}{2b\left(\tan^2(x)+1\right)\left((a+b)\tan^2(x)+a\right)}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{\frac{(a+b)(2a+b)\tan(x)}{ab((a+b)\tan^2(x)+a)}-\frac{\frac{(4a-b)(a+b)^2\int\frac{1}{(a+b)\tan^2(x)+a}d\tan(x)}{b}-\frac{a(4a+5b)\arctan(\tan(x))}{b}}{ab}}{2b}-\frac{\tan(x)}{2b\left(\tan^2(x)+1\right)\left((a+b)\tan^2(x)+a\right)}$$

$$\Big\downarrow \textbf{218}$$

$$\frac{\frac{(a+b)(2a+b)\tan(x)}{ab((a+b)\tan^2(x)+a)}-\frac{\frac{(4a-b)(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{a(4a+5b)\arctan(\tan(x))}{b}}{ab}}{2b}-\frac{\tan(x)}{2b\left(\tan^2(x)+1\right)\left((a+b)\tan^2(x)+a\right)}$$

input
```
Int[Cos[x]^6/(a + b*Sin[x]^2)^2,x]
```

output
```
-1/2*Tan[x]/(b*(1 + Tan[x]^2)*(a + (a + b)*Tan[x]^2)) + (-((-(-((a*(4*a + 5*
b)*ArcTan[Tan[x]])/b) + ((4*a - b)*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[x
])/Sqrt[a]])/(Sqrt[a]*b))/(a*b)) + ((a + b)*(2*a + b)*Tan[x])/(a*b*(a + (a
 + b)*Tan[x]^2)))/(2*b)
```

3.314.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.314. $\int \frac{\cos^6(x)}{(a+b\sin^2(x))^2}\,dx$

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.314. $\int \frac{\cos^6(x)}{(a+b\sin^2(x))^2}\,dx$

3.314.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

method	result
default	$\frac{\frac{b\tan(x)}{2+2\left(\tan^2(x)\right)}+\frac{(4a+5b)\arctan(\tan(x))}{2}}{b^3}-\frac{(a+b)^2\left(-\frac{b\tan(x)}{2a\left(a\left(\tan^2(x)\right)+\left(\tan^2(x)\right)b+a\right)}+\frac{(4a-b)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{2a\sqrt{a(a+b)}}\right)}{b^3}$
risch	$\frac{2ax}{b^3}+\frac{5x}{2b^2}-\frac{ie^{2ix}}{8b^2}+\frac{ie^{-2ix}}{8b^2}-\frac{i(2a^3e^{2ix}+5a^2b\,e^{2ix}+4a\,b^2e^{2ix}+b^3e^{2ix}-a^2b-2a\,b^2-b^3)}{a\,b^3(-b\,e^{4ix}+4a\,e^{2ix}+2b\,e^{2ix}-b)}+\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2ix}-\frac{2i\sqrt{-a(a+b)}}{b}\right.}{b^3}$

input
```
int(cos(x)^6/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/2*b*tan(x)/(1+tan(x)^2)+1/2*(4*a+5*b)*arctan(tan(x)))-(a+b)^2/b^3
*(-1/2/a*b*tan(x)/(a*tan(x)^2+tan(x)^2*b+a)+1/2*(4*a-b)/a/(a*(a+b))^(1/2)*
arctan((a+b)*tan(x)/(a*(a+b))^(1/2)))
```

3.314.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(97) = 194.

Time = 0.34 (sec) , antiderivative size = 491, normalized size of antiderivative = 4.35

$$\int \frac{\cos^6(x)}{(a+b\sin^2(x))^2}\,dx$$

$$= \left[\frac{4\,(4\,a^2b+5\,ab^2)x\cos(x)^2+\left(4\,a^3+7\,a^2b+2\,ab^2-b^3-(4\,a^2b+3\,ab^2-b^3)\cos(x)^2\right)\sqrt{-\frac{a+b}{a}}\log\left(\frac{(8\,a^2+\right.}{}\right.$$

input
```
integrate(cos(x)^6/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*(4*a^2*b + 5*a*b^2)*x*cos(x)^2 + (4*a^3 + 7*a^2*b + 2*a*b^2 - b^3
- (4*a^2*b + 3*a*b^2 - b^3)*cos(x)^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b
 + b^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos
(x)^3 - (a^2 + a*b)*cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(
b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - 4*(4*a^3 + 9
*a^2*b + 5*a*b^2)*x + 4*(a*b^2*cos(x)^3 - (2*a^2*b + 3*a*b^2 + b^3)*cos(x)
)*sin(x))/(a*b^4*cos(x)^2 - a^2*b^3 - a*b^4), 1/4*(2*(4*a^2*b + 5*a*b^2)*x
*cos(x)^2 - (4*a^3 + 7*a^2*b + 2*a*b^2 - b^3 - (4*a^2*b + 3*a*b^2 - b^3)*c
os(x)^2)*sqrt((a + b)/a)*arctan(1/2*((2*a + b)*cos(x)^2 - a - b)*sqrt((a +
 b)/a)/((a + b)*cos(x)*sin(x))) - 2*(4*a^3 + 9*a^2*b + 5*a*b^2)*x + 2*(a*b
^2*cos(x)^3 - (2*a^2*b + 3*a*b^2 + b^3)*cos(x))*sin(x))/(a*b^4*cos(x)^2 -
a^2*b^3 - a*b^4)]
```

3.314.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**6/(a+b*sin(x)**2)**2,x)
```

output
```
Timed out
```

3.314.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.33

$$\int \frac{\cos^6(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{\left(2\,a^2+3\,ab+b^2\right)\tan\left(x\right)^3+\left(2\,a^2+2\,ab+b^2\right)\tan\left(x\right)}{2\left(\left(a^2b^2+ab^3\right)\tan\left(x\right)^4+a^2b^2+\left(2\,a^2b^2+ab^3\right)\tan\left(x\right)^2\right)} + \frac{(4\,a+5\,b)x}{2\,b^3} - \frac{\left(4\,a^3+7\,a^2b+2\,ab^2-b^3\right)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{2\,\sqrt{(a+b)a}ab^3}$$

input
```
integrate(cos(x)^6/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/2*((2*a^2 + 3*a*b + b^2)*tan(x)^3 + (2*a^2 + 2*a*b + b^2)*tan(x))/((a^2*
b^2 + a*b^3)*tan(x)^4 + a^2*b^2 + (2*a^2*b^2 + a*b^3)*tan(x)^2) + 1/2*(4*a
 + 5*b)*x/b^3 - 1/2*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*arctan((a + b)*tan(x
)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a*b^3)
```

3.314.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.55

$$\int \frac{\cos^6(x)}{(a+b\sin^2(x))^2}\,dx = \frac{(4a+5b)x}{2b^3} - \frac{(4a^3+7a^2b+2ab^2-b^3)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)}{2\sqrt{a^2+ab}ab^3} + \frac{2a^2\tan(x)^3+3ab\tan(x)^3+b^2\tan(x)^3+2a^2\tan(x)+2ab\tan(x)+b^2\tan(x)}{2\left(a\tan(x)^4+b\tan(x)^4+2a\tan(x)^2+b\tan(x)^2+a\right)ab^2}$$

input
```
integrate(cos(x)^6/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/2*(4*a + 5*b)*x/b^3 - 1/2*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*(pi*floor(x/
pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a^2 + a*b)))/
(sqrt(a^2 + a*b)*a*b^3) + 1/2*(2*a^2*tan(x)^3 + 3*a*b*tan(x)^3 + b^2*tan(x
)^3 + 2*a^2*tan(x) + 2*a*b*tan(x) + b^2*tan(x))/((a*tan(x)^4 + b*tan(x)^4
+ 2*a*tan(x)^2 + b*tan(x)^2 + a)*a*b^2)
```

3.314.9 Mupad [B] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 463, normalized size of antiderivative = 4.10

$$\int \frac{\cos^6(x)}{(a+b\sin^2(x))^2}\,dx = \frac{\frac{\tan(x)\,(2\,a^2+2\,a\,b+b^2)}{2\,a\,b^2}+\frac{\tan(x)^3\,(a+b)\,(2\,a+b)}{2\,a\,b^2}}{(a+b)\,\tan(x)^4+(2\,a+b)\,\tan(x)^2+a}$$
$$-\frac{\ln\left(a^2\,b-\tan(x)\,\sqrt{-a^3\,(a+b)^3}+a^3\right)\sqrt{-a^3\,(a+b)^3}\,(4\,a-b)}{4\,a^3\,b^3}$$
$$+\frac{\ln\left(\tan(x)\,\sqrt{-a^3\,(a+b)^3}+a^2\,b+a^3\right)\,\left(a-\frac{b}{4}\right)\,\sqrt{-a^3\,(a+b)^3}}{a^3\,b^3}$$
$$-\frac{\operatorname{atan}\left(\frac{41\tan(x)}{2\left(\frac{131\,a}{4\,b}+\frac{11\,b}{4\,a}-\frac{5\,b^2}{4\,a^2}+\frac{85\,a^2}{4\,b^2}+\frac{5\,a^3}{b^3}+\frac{41}{2}\right)}+\frac{11\tan(x)}{4\left(\frac{41\,a}{2\,b}-\frac{5\,b}{4\,a}+\frac{131\,a^2}{4\,b^2}+\frac{85\,a^3}{4\,b^3}+\frac{5\,a^4}{b^4}+\frac{11}{4}\right)}+\frac{131\,a\tan(x)}{4\left(\frac{131\,a}{4}+\frac{41\,b}{2}+\frac{11\,b^2}{4\,a}+\frac{85\,a^2}{4\,b}-\frac{5\,b^3}{4\,a^2}+\frac{5\,a^3}{b^2}\right)}-\cdots$$

input
```
int(cos(x)^6/(a + b*sin(x)^2)^2,x)
```

output
```
((tan(x)*(2*a*b + 2*a^2 + b^2))/(2*a*b^2) + (tan(x)^3*(a + b)*(2*a + b))/(
2*a*b^2))/(a + tan(x)^2*(2*a + b) + tan(x)^4*(a + b)) - (atan((41*tan(x))/
(2*((131*a)/(4*b) + (11*b)/(4*a) - (5*b^2)/(4*a^2) + (85*a^2)/(4*b^2) + (5
*a^3)/b^3 + 41/2)) + (11*tan(x))/(4*((41*a)/(2*b) - (5*b)/(4*a) + (131*a^2
)/(4*b^2) + (85*a^3)/(4*b^3) + (5*a^4)/b^4 + 11/4)) + (131*a*tan(x))/(4*((
131*a)/4 + (41*b)/2 + (11*b^2)/(4*a) + (85*a^2)/(4*b) - (5*b^3)/(4*a^2) +
(5*a^3)/b^2)) - (5*b*tan(x))/(4*((11*a)/4 - (5*b)/4 + (41*a^2)/(2*b) + (13
1*a^3)/(4*b^2) + (85*a^4)/(4*b^3) + (5*a^5)/b^4)) + (85*a^2*tan(x))/(4*((1
31*a*b)/4 + (85*a^2)/4 + (41*b^2)/2 + (11*b^3)/(4*a) + (5*a^3)/b - (5*b^4)
/(4*a^2))) + (5*a^3*tan(x))/((131*a*b^2)/4 + (85*a^2*b)/4 + 5*a^3 + (41*b^
3)/2 + (11*b^4)/(4*a) - (5*b^5)/(4*a^2)))*(a*1i + (b*5i)/4)*2i)/b^3 - (log
(a^2*b - tan(x)*(-a^3*(a + b)^3)^(1/2) + a^3)*(-a^3*(a + b)^3)^(1/2)*(4*a
- b))/(4*a^3*b^3) + (log(tan(x)*(-a^3*(a + b)^3)^(1/2) + a^2*b + a^3)*(a -
 b/4)*(-a^3*(a + b)^3)^(1/2))/(a^3*b^3)
```

3.315 $\int \frac{\cos^5(x)}{(a+b\sin^2(x))^2}\,dx$

3.315.1 Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \frac{\cos^5(x)}{(a+b\sin^2(x))^2}\,dx = -\frac{(3a-b)(a+b)\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}} + \frac{\sin(x)}{b^2} + \frac{(a+b)^2\sin(x)}{2ab^2\left(a+b\sin^2(x)\right)}$$

output
```
-1/2*(3*a-b)*(a+b)*arctan(sin(x)*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)+sin(x)/b
^2+1/2*(a+b)^2*sin(x)/a/b^2/(a+b*sin(x)^2)
```

3.315.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.64

$$\int \frac{\cos^5(x)}{(a+b\sin^2(x))^2}\,dx = \frac{\frac{(3a^2+2ab-b^2)\arctan\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)}{a^{3/2}} + \frac{(-3a^2-2ab+b^2)\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{a^{3/2}} + 4\sqrt{b}\sin(x) + \frac{4\sqrt{b}(a+b)^2\sin(x)}{a(2a+b-b\cos(2x))}}{4b^{5/2}}$$

input
```
Integrate[Cos[x]^5/(a + b*Sin[x]^2)^2,x]
```

output
```
(((3*a^2 + 2*a*b - b^2)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/a^(3/2) + ((-3*a
^2 - 2*a*b + b^2)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/a^(3/2) + 4*Sqrt[b]*Si
n[x] + (4*Sqrt[b]*(a + b)^2*Sin[x])/(a*(2*a + b - b*Cos[2*x])))/(4*b^(5/2)
)
```

3.315.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3669, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(x)^5}{\left(a+b\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \; 3669$$

$$\int \frac{\left(1-\sin^2(x)\right)^2}{\left(a+b\sin^2(x)\right)^2}d\sin(x)$$

$$\Big\downarrow \; 300$$

$$\int \left(\frac{1}{b^2}-\frac{a^2+2b(a+b)\sin^2(x)-b^2}{b^2\left(a+b\sin^2(x)\right)^2}\right)d\sin(x)$$

$$\Big\downarrow \; 2009$$

$$-\frac{(3a-b)(a+b)\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{(a+b)^2\sin(x)}{2ab^2\left(a+b\sin^2(x)\right)}+\frac{\sin(x)}{b^2}$$

input
```
Int[Cos[x]^5/(a + b*Sin[x]^2)^2,x]
```

output
```
-1/2*((3*a - b)*(a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(a^(3/2)*b^(5/2)
) + Sin[x]/b^2 + ((a + b)^2*Sin[x])/(2*a*b^2*(a + b*Sin[x]^2))
```

3.315.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.315.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{\sin(x)}{b^2}-\frac{-\frac{(a^2+2ab+b^2)\sin(x)}{2a\left(a+b\left(\sin^2(x)\right)\right)}+\frac{(3a^2+2ab-b^2)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^2}$
default	$\frac{\sin(x)}{b^2}-\frac{-\frac{(a^2+2ab+b^2)\sin(x)}{2a\left(a+b\left(\sin^2(x)\right)\right)}+\frac{(3a^2+2ab-b^2)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^2}$
risch	$-\frac{i e^{ix}}{2b^2}+\frac{i e^{-ix}}{2b^2}-\frac{i(a^2+2ab+b^2)(e^{3ix}-e^{ix})}{a\,b^2(-b\,e^{4ix}+4a\,e^{2ix}+2b\,e^{2ix}-b)}-\frac{3a\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,b^2}-\frac{\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)}{2\sqrt{-ab}\,b}+\frac{\ln\left(e^{2i\ldots\right.}{4\ldots}$

input
```
int(cos(x)^5/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
sin(x)/b^2-1/b^2*(-1/2*(a^2+2*a*b+b^2)/a*sin(x)/(a+b*sin(x)^2)+1/2*(3*a^2+
2*a*b-b^2)/a/(a*b)^(1/2)*arctan(b*sin(x)/(a*b)^(1/2)))
```

3.315.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(60) = 120.

Time = 0.32 (sec) , antiderivative size = 296, normalized size of antiderivative = 4.11

$$\int \frac{\cos^5(x)}{(a+b\sin^2(x))^2}\,dx$$
$$= \left[-\frac{\left(3\,a^3+5\,a^2b+ab^2-b^3-\left(3\,a^2b+2\,ab^2-b^3\right)\cos\left(x\right)^2\right)\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)-2\left(2\,a^2\right.}{4\left(a^2b^4\cos\left(x\right)^2-a^3b^3-a^2b^4\right)}\right.$$

input
```
integrate(cos(x)^5/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*((3*a^3 + 5*a^2*b + a*b^2 - b^3 - (3*a^2*b + 2*a*b^2 - b^3)*cos(x)^2
)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*sin(x) + a - b)/(b*cos(x)^2 -
 a - b)) - 2*(2*a^2*b^2*cos(x)^2 - 3*a^3*b - 4*a^2*b^2 - a*b^3)*sin(x))/(a
^2*b^4*cos(x)^2 - a^3*b^3 - a^2*b^4), 1/2*((3*a^3 + 5*a^2*b + a*b^2 - b^3
- (3*a^2*b + 2*a*b^2 - b^3)*cos(x)^2)*sqrt(a*b)*arctan(sqrt(a*b)*sin(x)/a)
 + (2*a^2*b^2*cos(x)^2 - 3*a^3*b - 4*a^2*b^2 - a*b^3)*sin(x))/(a^2*b^4*cos
(x)^2 - a^3*b^3 - a^2*b^4)]
```

3.315.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(x)}{(a+b\sin^2(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**5/(a+b*sin(x)**2)**2,x)
```

output
```
Timed out
```

3.315.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \frac{\cos^5(x)}{(a+b\sin^2(x))^2}\,dx = \frac{(a^2+2\,ab+b^2)\sin(x)}{2\left(ab^3\sin(x)^2+a^2b^2\right)}+\frac{\sin(x)}{b^2}-\frac{(3\,a^2+2\,ab-b^2)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\sqrt{ab}ab^2}$$

input
```
integrate(cos(x)^5/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(a^2 + 2*a*b + b^2)*sin(x)/(a*b^3*sin(x)^2 + a^2*b^2) + sin(x)/b^2 - 1
/2*(3*a^2 + 2*a*b - b^2)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

3.315.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.14

$$\int \frac{\cos^5(x)}{(a+b\sin^2(x))^2}\,dx = \frac{\sin(x)}{b^2}-\frac{(3\,a^2+2\,ab-b^2)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\sqrt{ab}ab^2} + \frac{a^2\sin(x)+2\,ab\sin(x)+b^2\sin(x)}{2\left(b\sin(x)^2+a\right)ab^2}$$

input
```
integrate(cos(x)^5/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

output
```
sin(x)/b^2 - 1/2*(3*a^2 + 2*a*b - b^2)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*
b)*a*b^2) + 1/2*(a^2*sin(x) + 2*a*b*sin(x) + b^2*sin(x))/((b*sin(x)^2 + a)
*a*b^2)
```

3.315.9 Mupad [B] (verification not implemented)

Time = 14.74 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.33

$$\int \frac{\cos^5(x)}{(a+b\sin^2(x))^2}\,dx = \frac{\sin(x)}{b^2}+\frac{\sin(x)\,\left(a^2+2\,a\,b+b^2\right)}{2\,a\,\left(b^3\sin(x)^2+a\,b^2\right)} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sin(x)\,(a+b)\,(3\,a-b)}{\sqrt{a}\,(3\,a^2+2\,a\,b-b^2)}\right)\,(a+b)\,(3\,a-b)}{2\,a^{3/2}\,b^{5/2}}$$

input
```
int(cos(x)^5/(a + b*sin(x)^2)^2,x)
```

output
```
sin(x)/b^2 + (sin(x)*(2*a*b + a^2 + b^2))/(2*a*(b^3*sin(x)^2 + a*b^2)) - (
atan((b^(1/2)*sin(x)*(a + b)*(3*a - b))/(a^(1/2)*(2*a*b + 3*a^2 - b^2)))*(
a + b)*(3*a - b))/(2*a^(3/2)*b^(5/2))
```

3.315. $\int \frac{\cos^5(x)}{(a+b\sin^2(x))^2}\,dx$

3.316 $\int \frac{\cos^4(x)}{(a+b\sin^2(x))^2} dx$

3.316.1 Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \frac{\cos^4(x)}{(a+b\sin^2(x))^2} dx = \frac{x}{b^2} - \frac{(2a-b)\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^2} + \frac{(a+b)\tan(x)}{2ab\,(a+(a+b)\tan^2(x))}$$

output
```
x/b^2-1/2*(2*a-b)*arctan((a+b)^(1/2)*tan(x)/a^(1/2))*(a+b)^(1/2)/a^(3/2)/b
^2+1/2*(a+b)*tan(x)/a/b/(a+(a+b)*tan(x)^2)
```

3.316.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \frac{\cos^4(x)}{(a+b\sin^2(x))^2} dx = \frac{2x + \frac{(-2a^2-ab+b^2)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}} + \frac{b(a+b)\sin(2x)}{a(2a+b-b\cos(2x))}}{2b^2}$$

input
```
Integrate[Cos[x]^4/(a + b*Sin[x]^2)^2,x]
```

output
```
(2*x + ((-2*a^2 - a*b + b^2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(a^(3/2
)*Sqrt[a + b]) + (b*(a + b)*Sin[2*x])/(a*(2*a + b - b*Cos[2*x])))/(2*b^2)
```

3.316.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3670, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(x)}{\left(a+b\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos(x)^4}{\left(a+b\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \ 3670$$

$$\int \frac{1}{\left(\tan^2(x)+1\right)\left((a+b)\tan^2(x)+a\right)^2}d\tan(x)$$

$$\Big\downarrow \ 316$$

$$\frac{(a+b)\tan(x)}{2ab\left((a+b)\tan^2(x)+a\right)}-\frac{\int \frac{-((a+b)\tan^2(x))+a-b}{(\tan^2(x)+1)((a+b)\tan^2(x)+a)}d\tan(x)}{2ab}$$

$$\Big\downarrow \ 397$$

$$\frac{(a+b)\tan(x)}{2ab\left((a+b)\tan^2(x)+a\right)}-\frac{\frac{(2a-b)(a+b)\int \frac{1}{(a+b)\tan^2(x)+a}d\tan(x)}{b}-\frac{2a\int \frac{1}{\tan^2(x)+1}d\tan(x)}{b}}{2ab}$$

$$\Big\downarrow \ 216$$

$$\frac{(a+b)\tan(x)}{2ab\left((a+b)\tan^2(x)+a\right)}-\frac{\frac{(2a-b)(a+b)\int \frac{1}{(a+b)\tan^2(x)+a}d\tan(x)}{b}-\frac{2a\arctan(\tan(x))}{b}}{2ab}$$

$$\Big\downarrow \ 218$$

$$\frac{(a+b)\tan(x)}{2ab\left((a+b)\tan^2(x)+a\right)}-\frac{\frac{(2a-b)\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{2a\arctan(\tan(x))}{b}}{2ab}$$

input
```
Int[Cos[x]^4/(a + b*Sin[x]^2)^2,x]
```

output
```
-1/2*((-2*a*ArcTan[Tan[x]])/b + ((2*a - b)*Sqrt[a + b]*ArcTan[(Sqrt[a + b]
*Tan[x])/Sqrt[a]])/(Sqrt[a]*b))/(a*b) + ((a + b)*Tan[x])/(2*a*b*(a + (a +
b)*Tan[x]^2))
```

3.316.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.316. $\int \frac{\cos^4(x)}{(a+b\sin^2(x))^2}\,dx$

3.316.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

method	result
default	$-\frac{(a+b)\left(-\frac{b\tan(x)}{2a\left(a\left(\tan^2(x)\right)+\left(\tan^2(x)\right)b+a\right)}+\frac{(2a-b)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{2a\sqrt{a(a+b)}}\right)}{b^2}+\frac{\arctan(\tan(x))}{b^2}$
risch	$\frac{x}{b^2}-\frac{i(2\,e^{2ix}a^2+3ab\,e^{2ix}+e^{2ix}b^2-ab-b^2)}{a\,b^2(-b\,e^{4ix}+4a\,e^{2ix}+2b\,e^{2ix}-b)}+\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2ix}-\frac{2i\sqrt{-a(a+b)}+2a+b}{b}\right)}{2a\,b^2}-\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2ix}-\frac{2i\sqrt{-a(a+b)}+2a}{b}\right.}{4a^2b}$

input
```
int(cos(x)^4/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-(a+b)/b^2*(-1/2/a*b*tan(x)/(a*tan(x)^2+tan(x)^2*b+a)+1/2*(2*a-b)/a/(a*(a+
b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2)))+1/b^2*arctan(tan(x))
```

3.316.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(63) = 126$.

Time = 0.33 (sec) , antiderivative size = 367, normalized size of antiderivative = 4.89

$$\int \frac{\cos^4(x)}{(a+b\sin^2(x))^2}\,dx$$
$$= \left[\frac{8\,abx\cos(x)^2-4\,(ab+b^2)\cos(x)\sin(x)-\left((2\,ab-b^2)\cos(x)^2-2\,a^2-ab+b^2\right)\sqrt{-\frac{a+b}{a}}\log\left(\frac{(8\,a^2+8\,ab-}{}\right.}{8\,(ab^3\cos(x)^2-a}\right.$$

input
```
integrate(cos(x)^4/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(8*a*b*x*cos(x)^2 - 4*(a*b + b^2)*cos(x)*sin(x) - ((2*a*b - b^2)*cos(
x)^2 - 2*a^2 - a*b + b^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(
x)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - (a^2
 + a*b)*cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4
 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - 8*(a^2 + a*b)*x)/(a*b^3*
cos(x)^2 - a^2*b^2 - a*b^3), 1/4*(4*a*b*x*cos(x)^2 - 2*(a*b + b^2)*cos(x)*
sin(x) + ((2*a*b - b^2)*cos(x)^2 - 2*a^2 - a*b + b^2)*sqrt((a + b)/a)*arct
an(1/2*((2*a + b)*cos(x)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(x)*sin(x)
)) - 4*(a^2 + a*b)*x)/(a*b^3*cos(x)^2 - a^2*b^2 - a*b^3)]
```

3.316.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**4/(a+b*sin(x)**2)**2,x)
```

output
```
Timed out
```

3.316.7 Maxima [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

$$\int \frac{\cos^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{(a+b)\tan(x)}{2\left(a^2b+(a^2b+ab^2)\tan(x)^2\right)}+\frac{x}{b^2} - \frac{(2a^2+ab-b^2)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}\,ab^2}$$

input
```
integrate(cos(x)^4/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(a + b)*tan(x)/(a^2*b + (a^2*b + a*b^2)*tan(x)^2) + x/b^2 - 1/2*(2*a^2
 + a*b - b^2)*arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a*b^
2)
```

3.316.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.45

$$\int \frac{\cos^4(x)}{(a+b\sin^2(x))^2}\,dx = \frac{x}{b^2} - \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(2\,a+2\,b) + \arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)(2\,a^2+ab-b^2)}{2\,\sqrt{a^2+ab}ab^2} + \frac{a\tan(x)+b\tan(x)}{2\,\left(a\tan(x)^2+b\tan(x)^2+a\right)ab}$$

input
```
integrate(cos(x)^4/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

output
```
x/b^2 - 1/2*(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*ta
n(x))/sqrt(a^2 + a*b)))*(2*a^2 + a*b - b^2)/(sqrt(a^2 + a*b)*a*b^2) + 1/2*
(a*tan(x) + b*tan(x))/((a*tan(x)^2 + b*tan(x)^2 + a)*a*b)
```

3.316.9 Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 533, normalized size of antiderivative = 7.11

$$\int \frac{\cos^4(x)}{(a+b\sin^2(x))^2}\,dx = \frac{\operatorname{atan}\left(\frac{5\tan(x)}{2\left(\frac{3a}{2b}+\frac{b}{2a}-\frac{b^2}{2a^2}+\frac{5}{2}\right)}+\frac{\tan(x)}{2\left(\frac{5a}{2b}-\frac{b}{2a}+\frac{3a^2}{2b^2}+\frac{1}{2}\right)}+\frac{3a\tan(x)}{2\left(\frac{3a}{2}+\frac{5b}{2}+\frac{b^2}{2a}-\frac{b^3}{2a^2}\right)}-\frac{b\tan(x)}{2\left(\frac{a}{2}-\frac{b}{2}+\frac{5a^2}{2b}+\frac{3a^3}{2b^2}\right)}\right)}{b^2} + \frac{\operatorname{atanh}\left(\frac{\tan(x)\sqrt{-a^4-b\,a^3}}{a^2-\frac{3ab}{2}-\frac{b^2}{2}+\frac{b^3}{4a}+\frac{13a^3}{4b}+\frac{3a^4}{2b^2}}+\frac{3\tan(x)\sqrt{-a^4-b\,a^3}}{2\left(\frac{13ab}{4}+\frac{3a^2}{2}+b^2-\frac{3b^3}{2a}-\frac{b^4}{2a^2}+\frac{b^5}{4a^3}\right)}+\frac{13\tan(x)\sqrt{-a^4-b\,a^3}}{4\left(ab+\frac{13a^2}{4}-\frac{3b^2}{2}-\frac{b^3}{2a}+\frac{3a^3}{2b}+\frac{b^4}{4a^2}\right)}-\frac{3}{2\left(a^3-\frac{3a}{}\right.}\right.}{} + \frac{\tan(x)\,(a+b)}{2\,a\,b\,\left((a+b)\,\tan(x)^2+a\right)}$$

input
```
int(cos(x)^4/(a + b*sin(x)^2)^2,x)
```

output
```
atan((5*tan(x))/(2*((3*a)/(2*b) + b/(2*a) - b^2/(2*a^2) + 5/2)) + tan(x)/(
2*((5*a)/(2*b) - b/(2*a) + (3*a^2)/(2*b^2) + 1/2)) + (3*a*tan(x))/(2*((3*a
)/2 + (5*b)/2 + b^2/(2*a) - b^3/(2*a^2))) - (b*tan(x))/(2*(a/2 - b/2 + (5*
a^2)/(2*b) + (3*a^3)/(2*b^2))))/b^2 + (atanh((tan(x)*(- a^3*b - a^4)^(1/2)
)/(a^2 - (3*a*b)/2 - b^2/2 + b^3/(4*a) + (13*a^3)/(4*b) + (3*a^4)/(2*b^2))
 + (3*tan(x)*(- a^3*b - a^4)^(1/2))/(2*((13*a*b)/4 + (3*a^2)/2 + b^2 - (3*
b^3)/(2*a) - b^4/(2*a^2) + b^5/(4*a^3))) + (13*tan(x)*(- a^3*b - a^4)^(1/2
))/(4*(a*b + (13*a^2)/4 - (3*b^2)/2 - b^3/(2*a) + (3*a^3)/(2*b) + b^4/(4*a
^2))) - (3*b*tan(x)*(- a^3*b - a^4)^(1/2))/(2*(a^3 - (3*a^2*b)/2 - (a*b^2)
/2 + b^3/4 + (13*a^4)/(4*b) + (3*a^5)/(2*b^2))) - (b^2*tan(x)*(- a^3*b - a
^4)^(1/2))/(2*((a*b^3)/4 - (3*a^3*b)/2 + a^4 - (a^2*b^2)/2 + (13*a^5)/(4*b
) + (3*a^6)/(2*b^2))) + (b^3*tan(x)*(- a^3*b - a^4)^(1/2))/(4*(a^5 - (3*a^
4*b)/2 + (a^2*b^3)/4 - (a^3*b^2)/2 + (13*a^6)/(4*b) + (3*a^7)/(2*b^2))))*(
-a^3*(a + b))^(1/2)*(2*a - b))/(2*a^3*b^2) + (tan(x)*(a + b))/(2*a*b*(a +
tan(x)^2*(a + b)))
```

3.316. $\int \frac{\cos^4(x)}{(a+b\sin^2(x))^2}\,dx$

3.317 $\int \frac{\cos^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

3.317.1 Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{\cos^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx=-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)}$$

output
```
-1/2*(a-b)*arctan(sin(x)*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)+1/2*(a+b)*sin(x)
/a/b/(a+b*sin(x)^2)
```

3.317.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx=-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)}$$

input
```
Integrate[Cos[x]^3/(a + b*Sin[x]^2)^2,x]
```

output
```
-1/2*((a - b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(a^(3/2)*b^(3/2)) + ((a +
b)*Sin[x])/(2*a*b*(a + b*Sin[x]^2))
```

3.317.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3669, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(x)}{\left(a+b\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^3}{\left(a+b\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \text{3669}$$

$$\int \frac{1-\sin^2(x)}{\left(a+b\sin^2(x)\right)^2}d\sin(x)$$

$$\Big\downarrow \text{298}$$

$$\frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)}-\frac{(a-b)\int \frac{1}{b\sin^2(x)+a}d\sin(x)}{2ab}$$

$$\Big\downarrow \text{218}$$

$$\frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)}-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

input
```
Int[Cos[x]^3/(a + b*Sin[x]^2)^2,x]
```

output
```
-1/2*((a - b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(a^(3/2)*b^(3/2)) + ((a +
b)*Sin[x])/(2*a*b*(a + b*Sin[x]^2))
```

3.317.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.317.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{(a+b)\sin(x)}{2ab\left(a+b\left(\sin^2(x)\right)\right)}-\frac{(a-b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}$
default	$\frac{(a+b)\sin(x)}{2ab\left(a+b\left(\sin^2(x)\right)\right)}-\frac{(a-b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}$
risch	$\frac{i(a+b)\left(e^{3ix}-e^{ix}\right)}{ab\left(b\,e^{4ix}-4a\,e^{2ix}-2b\,e^{2ix}+b\right)}-\frac{\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,b}+\frac{\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,a}+\frac{\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,b}-\frac{\ln\left(e^{2ix}-\right.}{4\sqrt{}}$

input
```
int(cos(x)^3/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a+b)*sin(x)/a/b/(a+b*sin(x)^2)-1/2*(a-b)/a/b/(a*b)^(1/2)*arctan(b*sin
(x)/(a*b)^(1/2))
```

3.317.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.49

$$\int \frac{\cos^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$$
$$= \left[\frac{\left((ab-b^2)\cos(x)^2-a^2+b^2\right)\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)-2\left(a^2b+ab^2\right)\sin(x)}{4\left(a^2b^3\cos(x)^2-a^3b^2-a^2b^3\right)},\right.$$
$$\left. -\frac{\left((ab-b^2)\cos(x)^2-a^2+b^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\sin(x)}{a}\right)+\left(a^2b+ab^2\right)\sin(x)}{2\left(a^2b^3\cos(x)^2-a^3b^2-a^2b^3\right)}\right]$$

input
```
integrate(cos(x)^3/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(((a*b - b^2)*cos(x)^2 - a^2 + b^2)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*s
qrt(-a*b)*sin(x) + a - b)/(b*cos(x)^2 - a - b)) - 2*(a^2*b + a*b^2)*sin(x)
)/(a^2*b^3*cos(x)^2 - a^3*b^2 - a^2*b^3), -1/2*(((a*b - b^2)*cos(x)^2 - a^
2 + b^2)*sqrt(a*b)*arctan(sqrt(a*b)*sin(x)/a) + (a^2*b + a*b^2)*sin(x))/(a
^2*b^3*cos(x)^2 - a^3*b^2 - a^2*b^3)]
```

3.317.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**3/(a+b*sin(x)**2)**2,x)
```

output
```
Timed out
```

3.317.7 Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\cos^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{(a+b)\sin(x)}{2\left(ab^2\sin(x)^2+a^2b\right)} - \frac{(a-b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\sqrt{ab}ab}$$

input
```
integrate(cos(x)^3/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(a + b)*sin(x)/(a*b^2*sin(x)^2 + a^2*b) - 1/2*(a - b)*arctan(b*sin(x)/
sqrt(a*b))/(sqrt(a*b)*a*b)
```

3.317.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{\cos^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = -\frac{(a-b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\sqrt{ab}ab} + \frac{a\sin(x)+b\sin(x)}{2\left(b\sin(x)^2+a\right)ab}$$

input
```
integrate(cos(x)^3/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(a - b)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/2*(a*sin(x) +
b*sin(x))/((b*sin(x)^2 + a)*a*b)
```

3.317.9 Mupad [B] (verification not implemented)

Time = 14.58 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{\cos^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{\sin(x)\ (a+b)}{2\,a\,b\ \left(b\sin(x)^2+a\right)} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)\ (a-b)}{2\,a^{3/2}\,b^{3/2}}$$

input
```
int(cos(x)^3/(a + b*sin(x)^2)^2,x)
```

output
```
(sin(x)*(a + b))/(2*a*b*(a + b*sin(x)^2)) - (atan((b^(1/2)*sin(x))/a^(1/2)
)*(a - b))/(2*a^(3/2)*b^(3/2))
```

3.318 $\int \frac{\cos^2(x)}{(a+b\sin^2(x))^2} dx$

3.318.1 Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{\cos^2(x)}{(a+b\sin^2(x))^2}\,dx=\frac{\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}}+\frac{\tan(x)}{2a\left(a+(a+b)\tan^2(x)\right)}$$

output
```
1/2*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/a^(3/2)/(a+b)^(1/2)+1/2*tan(x)/a/(a
+(a+b)*tan(x)^2)
```

3.318.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(x)}{(a+b\sin^2(x))^2}\,dx=\frac{\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}}-\frac{\sin(2x)}{2a(-2a-b+b\cos(2x))}$$

input
```
Integrate[Cos[x]^2/(a + b*Sin[x]^2)^2,x]
```

output
```
ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a + b]) - Sin[2*x]/(2
*a*(-2*a - b + b*Cos[2*x]))
```

3.318.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3670, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(x)}{\left(a+b\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos(x)^2}{\left(a+b\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \ 3670$$

$$\int \frac{1}{\left((a+b)\tan^2(x)+a\right)^2}d\tan(x)$$

$$\Big\downarrow \ 215$$

$$\frac{\int \frac{1}{(a+b)\tan^2(x)+a}d\tan(x)}{2a}+\frac{\tan(x)}{2a\left((a+b)\tan^2(x)+a\right)}$$

$$\Big\downarrow \ 218$$

$$\frac{\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}}+\frac{\tan(x)}{2a\left((a+b)\tan^2(x)+a\right)}$$

input
```
Int[Cos[x]^2/(a + b*Sin[x]^2)^2,x]
```

output
```
ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a + b]) + Tan[x]/(2*a
*(a + (a + b)*Tan[x]^2))
```

3.318.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.318.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

method	result
default	$\frac{\tan(x)}{2a(a(\tan^2(x))+(\tan^2(x))b+a)}+\frac{\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{2a\sqrt{a(a+b)}}$
risch	$-\frac{i(2a\,e^{2ix}+b\,e^{2ix}-b)}{ab(-b\,e^{4ix}+4a\,e^{2ix}+2b\,e^{2ix}-b)}-\frac{\ln\left(e^{2ix}-\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{4\sqrt{-a^2-ab}\,a}+\frac{\ln\left(e^{2ix}-\frac{-2ia^2-2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2}}{b\sqrt{-a^2-ab}}\right.}{4\sqrt{-a^2-ab}\,a}$

input
```
int(cos(x)^2/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*tan(x)/a/(a*tan(x)^2+tan(x)^2*b+a)+1/2/a/(a*(a+b))^(1/2)*arctan((a+b)*
tan(x)/(a*(a+b))^(1/2))
```

3.318.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(42) = 84.

Time = 0.34 (sec) , antiderivative size = 313, normalized size of antiderivative = 5.80

$$\int \frac{\cos^2(x)}{(a+b\sin^2(x))^2}\,dx = \left[\frac{4\,(a^2+ab)\cos(x)\sin(x) + \left(b\cos(x)^2 - a - b\right)\sqrt{-a^2-ab}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4 - 2\,(4\,a^2+5\,ab+b^2)\cos(x)^2 + 4}{b^2\cos(x)^4 - 2\,(ab}\right.}{8\,\left(a^4+2\,a^3b+a^2b^2-(a^3b+a^2b^2)\cos(x)^2\right)}\right.$$

input
```
integrate(cos(x)^2/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*(a^2 + a*b)*cos(x)*sin(x) + (b*cos(x)^2 - a - b)*sqrt(-a^2 - a*b)*
log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 + 4
*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt(-a^2 - a*b)*sin(x) + a^2 + 2*a
*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)))/(a
^4 + 2*a^3*b + a^2*b^2 - (a^3*b + a^2*b^2)*cos(x)^2), 1/4*(2*(a^2 + a*b)*c
os(x)*sin(x) + (b*cos(x)^2 - a - b)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*
cos(x)^2 - a - b)/(sqrt(a^2 + a*b)*cos(x)*sin(x))))/(a^4 + 2*a^3*b + a^2*b
^2 - (a^3*b + a^2*b^2)*cos(x)^2)]
```

3.318.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(x)}{(a+b\sin^2(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**2/(a+b*sin(x)**2)**2,x)
```

output
```
Timed out
```

3.318.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{\cos^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{\tan(x)}{2\left((a^2+ab)\tan(x)^2+a^2\right)}+\frac{\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}a}$$

input
```
integrate(cos(x)^2/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/2*tan(x)/((a^2 + a*b)*tan(x)^2 + a^2) + 1/2*arctan((a + b)*tan(x)/sqrt((
a + b)*a))/(sqrt((a + b)*a)*a)
```

3.318.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.43

$$\int \frac{\cos^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{\pi\left\lfloor \frac{x}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)}{2\sqrt{a^2+ab}a} + \frac{\tan(x)}{2\left(a\tan(x)^2+b\tan(x)^2+a\right)a}$$

input
```
integrate(cos(x)^2/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/2*(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sq
rt(a^2 + a*b)))/(sqrt(a^2 + a*b)*a) + 1/2*tan(x)/((a*tan(x)^2 + b*tan(x)^2
 + a)*a)
```

3.318.9 Mupad [B] (verification not implemented)

Time = 13.87 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{\cos^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\tan(x)\,(2a+2b)}{2\sqrt{a}\sqrt{a+b}}\right)}{2a^{3/2}\sqrt{a+b}}+\frac{\tan(x)}{2a\,\left((a+b)\,\tan(x)^2+a\right)}$$

input
```
int(cos(x)^2/(a + b*sin(x)^2)^2,x)
```

output
```
atan((tan(x)*(2*a + 2*b))/(2*a^(1/2)*(a + b)^(1/2)))/(2*a^(3/2)*(a + b)^(1
/2)) + tan(x)/(2*a*(a + tan(x)^2*(a + b)))
```

3.318. $\int \frac{\cos^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

3.319 $\int \frac{\cos(x)}{(a+b\sin^2(x))^2}\,dx$

3.319.1 Optimal result

Integrand size = 13, antiderivative size = 48

$$\int \frac{\cos(x)}{(a+b\sin^2(x))^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{\sin(x)}{2a\,(a+b\sin^2(x))}$$

output
```
1/2*sin(x)/a/(a+b*sin(x)^2)+1/2*arctan(sin(x)*b^(1/2)/a^(1/2))/a^(3/2)/b^(
1/2)
```

3.319.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{(a+b\sin^2(x))^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{\sin(x)}{2a\,(a+b\sin^2(x))}$$

input
```
Integrate[Cos[x]/(a + b*Sin[x]^2)^2,x]
```

output
```
ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Sin[x]/(2*a*(a + b*
Sin[x]^2))
```

3.319.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3669, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\left(a+b\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos(x)}{\left(a+b\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \ 3669$$

$$\int \frac{1}{\left(a+b\sin^2(x)\right)^2}d\sin(x)$$

$$\Big\downarrow \ 215$$

$$\frac{\int \frac{1}{b\sin^2(x)+a}d\sin(x)}{2a}+\frac{\sin(x)}{2a\left(a+b\sin^2(x)\right)}$$

$$\Big\downarrow \ 218$$

$$\frac{\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\sin(x)}{2a\left(a+b\sin^2(x)\right)}$$

input
```
Int[Cos[x]/(a + b*Sin[x]^2)^2,x]
```

output
```
ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Sin[x]/(2*a*(a + b*
Sin[x]^2))
```

3.319.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.319.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\sin(x)}{2a(a+b(\sin^2(x)))}+\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$	39
default	$\frac{\sin(x)}{2a(a+b(\sin^2(x)))}+\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$	39
risch	$\frac{i(\mathrm{e}^{3ix}-\mathrm{e}^{ix})}{a(b\,\mathrm{e}^{4ix}-4a\,\mathrm{e}^{2ix}-2b\,\mathrm{e}^{2ix}+b)}-\frac{\ln\left(\mathrm{e}^{2ix}-\frac{2ia\,\mathrm{e}^{ix}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,a}+\frac{\ln\left(\mathrm{e}^{2ix}+\frac{2ia\,\mathrm{e}^{ix}}{\sqrt{-ab}}-1\right)}{4\sqrt{-ab}\,a}$	116

input
```
int(cos(x)/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(x)/a/(a+b*sin(x)^2)+1/2/a/(a*b)^(1/2)*arctan(b*sin(x)/(a*b)^(1/2))
```

3.319.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.44

$$\int \frac{\cos(x)}{(a+b\sin^2(x))^2}\,dx = \left[-\frac{2\,ab\sin(x)+\left(b\cos(x)^2-a-b\right)\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)}{4\left(a^2b^2\cos(x)^2-a^3b-a^2b^2\right)}, -\frac{ab\sin(x)-\left(b\cos(x)^2-a-b\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\sin(x)}{a}\right)}{2\left(a^2b^2\cos(x)^2-a^3b-a^2b^2\right)}\right]$$

input
```
integrate(cos(x)/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*b*sin(x) + (b*cos(x)^2 - a - b)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2
*sqrt(-a*b)*sin(x) + a - b)/(b*cos(x)^2 - a - b)))/(a^2*b^2*cos(x)^2 - a^3
*b - a^2*b^2), -1/2*(a*b*sin(x) - (b*cos(x)^2 - a - b)*sqrt(a*b)*arctan(sq
rt(a*b)*sin(x)/a))/(a^2*b^2*cos(x)^2 - a^3*b - a^2*b^2)]
```

3.319.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(41) = 82.

Time = 3.26 (sec) , antiderivative size = 289, normalized size of antiderivative = 6.02

$$\int \frac{\cos(x)}{(a+b\sin^2(x))^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sin^3(x)} \\ \frac{\sin(x)}{a^2} \\ -\frac{1}{3b^2\sin^3(x)} \\ \frac{a\log\left(-\sqrt{-\frac{a}{b}}+\sin(x)\right)}{4a^2b\sqrt{-\frac{a}{b}}+4ab^2\sqrt{-\frac{a}{b}}\sin^2(x)} - \frac{a\log\left(\sqrt{-\frac{a}{b}}+\sin(x)\right)}{4a^2b\sqrt{-\frac{a}{b}}+4ab^2\sqrt{-\frac{a}{b}}\sin^2(x)} + \frac{2b\sqrt{-\frac{a}{b}}\sin(x)}{4a^2b\sqrt{-\frac{a}{b}}+4ab^2\sqrt{-\frac{a}{b}}\sin^2(x)} + \frac{b\log\left(-\sqrt{-\frac{a}{b}}+\sin(x)\right)\sin^2(x)}{4a^2b\sqrt{-\frac{a}{b}}+4ab^2\sqrt{-\frac{a}{b}}\sin^2(x)} - \end{cases}$$

input
```
integrate(cos(x)/(a+b*sin(x)**2)**2,x)
```

output
```
Piecewise((zoo/sin(x)**3, Eq(a, 0) & Eq(b, 0)), (sin(x)/a**2, Eq(b, 0)), (
-1/(3*b**2*sin(x)**3), Eq(a, 0)), (a*log(-sqrt(-a/b) + sin(x))/(4*a**2*b*s
qrt(-a/b) + 4*a*b**2*sqrt(-a/b)*sin(x)**2) - a*log(sqrt(-a/b) + sin(x))/(4
*a**2*b*sqrt(-a/b) + 4*a*b**2*sqrt(-a/b)*sin(x)**2) + 2*b*sqrt(-a/b)*sin(x
)/(4*a**2*b*sqrt(-a/b) + 4*a*b**2*sqrt(-a/b)*sin(x)**2) + b*log(-sqrt(-a/b
) + sin(x))*sin(x)**2/(4*a**2*b*sqrt(-a/b) + 4*a*b**2*sqrt(-a/b)*sin(x)**2
) - b*log(sqrt(-a/b) + sin(x))*sin(x)**2/(4*a**2*b*sqrt(-a/b) + 4*a*b**2*s
qrt(-a/b)*sin(x)**2), True))
```

3.319.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \frac{\cos(x)}{(a+b\sin^2(x))^2}\,dx = \frac{\sin(x)}{2\left(ab\sin(x)^2+a^2\right)} + \frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\sqrt{ab}a}$$

input
```
integrate(cos(x)/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/2*sin(x)/(a*b*sin(x)^2 + a^2) + 1/2*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b
)*a)
```

3.319.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \frac{\cos(x)}{(a+b\sin^2(x))^2}\,dx = \frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\sqrt{ab}a} + \frac{\sin(x)}{2\left(b\sin(x)^2+a\right)a}$$

input
```
integrate(cos(x)/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/2*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*sin(x)/((b*sin(x)^2 + a
)*a)
```

3.319.9 Mupad [B] (verification not implemented)

Time = 13.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{\cos(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}} + \frac{\sin\left(x\right)}{2\,a\,\left(b\sin\left(x\right)^2+a\right)}$$

input
```
int(cos(x)/(a + b*sin(x)^2)^2,x)
```

output
```
atan((b^(1/2)*sin(x))/a^(1/2))/(2*a^(3/2)*b^(1/2)) + sin(x)/(2*a*(a + b*si
n(x)^2))
```

3.320 $\int \frac{\sec(x)}{(a+b\sin^2(x))^2}\,dx$

3.320.1 Optimal result

Integrand size = 13, antiderivative size = 73

$$\int \frac{\sec(x)}{(a+b\sin^2(x))^2}\,dx = \frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^2} + \frac{\operatorname{arctanh}(\sin(x))}{(a+b)^2} + \frac{b\sin(x)}{2a(a+b)(a+b\sin^2(x))}$$

output
```
arctanh(sin(x))/(a+b)^2+1/2*b*sin(x)/a/(a+b)/(a+b*sin(x)^2)+1/2*(3*a+b)*ar
ctan(sin(x)*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)/(a+b)^2
```

3.320.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.78

$$\int \frac{\sec(x)}{(a+b\sin^2(x))^2}\,dx = \frac{-\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)}{a^{3/2}} + \frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{a^{3/2}} + 4\left(-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right) \cdots}{4(a+b)^2}$$

input
```
Integrate[Sec[x]/(a + b*Sin[x]^2)^2,x]
```

output
```
(-((Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/a^(3/2)) + (Sqrt[b
]*(3*a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/a^(3/2) + 4*(-Log[Cos[x/2] -
 Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] + (b*(a + b)*Sin[x])/(a*(2*a + b - b
*Cos[2*x]))))/(4*(a + b)^2)
```

3.320.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 3669, 316, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(x)}{\left(a+b\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(x)\left(a+b\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \text{3669}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)\left(a+b\sin^2(x)\right)^2}d\sin(x)$$

$$\Big\downarrow \text{316}$$

$$\frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}-\frac{\int \frac{b\sin^2(x)+b-2(a+b)}{\left(1-\sin^2(x)\right)\left(b\sin^2(x)+a\right)}d\sin(x)}{2a(a+b)}$$

$$\Big\downarrow \text{397}$$

$$\frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}-\frac{-\frac{2a\int \frac{1}{1-\sin^2(x)}d\sin(x)}{a+b}-\frac{b(3a+b)\int \frac{1}{b\sin^2(x)+a}d\sin(x)}{a+b}}{2a(a+b)}$$

$$\Big\downarrow \text{218}$$

$$\frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}-\frac{-\frac{2a\int \frac{1}{1-\sin^2(x)}d\sin(x)}{a+b}-\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2a(a+b)}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}-\frac{-\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}-\frac{2a\text{arctanh}(\sin(x))}{a+b}}{2a(a+b)}$$

input
```
Int[Sec[x]/(a + b*Sin[x]^2)^2,x]
```

output
```
-1/2*(-((Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*(a +
 b))) - (2*a*ArcTanh[Sin[x]])/(a + b))/(a*(a + b)) + (b*Sin[x])/(2*a*(a +
b)*(a + b*Sin[x]^2))
```

3.320.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.320. $\int \frac{\sec(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.320.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

method	result
default	$\frac{\ln(1+\sin(x))}{2(a+b)^2}+\frac{b\left(\frac{(a+b)\sin(x)}{2a\left(a+b\left(\sin^2(x)\right)\right)}+\frac{(3a+b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^2}-\frac{\ln(\sin(x)-1)}{2(a+b)^2}$
risch	$\frac{ib(\mathrm{e}^{3ix}-\mathrm{e}^{ix})}{a(a+b)(b\,\mathrm{e}^{4ix}-4a\,\mathrm{e}^{2ix}-2b\,\mathrm{e}^{2ix}+b)}+\frac{\ln(\mathrm{e}^{ix}+i)}{a^2+2ab+b^2}-\frac{\ln(\mathrm{e}^{ix}-i)}{a^2+2ab+b^2}+\frac{3\sqrt{-ab}\,\ln\left(\mathrm{e}^{2ix}+\frac{2i\sqrt{-ab}\,\mathrm{e}^{ix}}{b}-1\right)}{4a(a+b)^2}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2ix}+\frac{2i\sqrt{-}}{}\right.}{4a^2(a+b)^2}$

input
```
int(sec(x)/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(a+b)^2*ln(1+sin(x))+b/(a+b)^2*(1/2*(a+b)/a*sin(x)/(a+b*sin(x)^2)+1/2*
(3*a+b)/a/(a*b)^(1/2)*arctan(b*sin(x)/(a*b)^(1/2)))-1/2/(a+b)^2*ln(sin(x)-
1)
```

3.320.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(61) = 122.

Time = 0.35 (sec) , antiderivative size = 354, normalized size of antiderivative = 4.85

$$\int \frac{\sec(x)}{(a+b\sin^2(x))^2}\,dx$$
$$= \left[-\frac{\left((3\,ab+b^2)\cos(x)^2-3\,a^2-4\,ab-b^2\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{b\cos(x)^2-2\,a\sqrt{-\frac{b}{a}}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+2\left(ab\cos(x)^2-a^2-\right.}{4\left(a^4+3\,a^3b+3\,a^2b^2+ab^3-(a^3b+\right.}\right.$$
$$-\frac{\left((3\,ab+b^2)\cos(x)^2-3\,a^2-4\,ab-b^2\right)\sqrt{\frac{b}{a}}\arctan\left(\sqrt{\frac{b}{a}}\sin(x)\right)+\left(ab\cos(x)^2-a^2-ab\right)\log(\sin(x)-}{2\left(a^4+3\,a^3b+3\,a^2b^2+ab^3-(a^3b+2\,a^2b^2+a\right.}$$

input
```
integrate(sec(x)/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(((3*a*b + b^2)*cos(x)^2 - 3*a^2 - 4*a*b - b^2)*sqrt(-b/a)*log(-(b*c
os(x)^2 - 2*a*sqrt(-b/a)*sin(x) + a - b)/(b*cos(x)^2 - a - b)) + 2*(a*b*co
s(x)^2 - a^2 - a*b)*log(sin(x) + 1) - 2*(a*b*cos(x)^2 - a^2 - a*b)*log(-si
n(x) + 1) - 2*(a*b + b^2)*sin(x))/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - (a^
3*b + 2*a^2*b^2 + a*b^3)*cos(x)^2), -1/2*(((3*a*b + b^2)*cos(x)^2 - 3*a^2
- 4*a*b - b^2)*sqrt(b/a)*arctan(sqrt(b/a)*sin(x)) + (a*b*cos(x)^2 - a^2 -
a*b)*log(sin(x) + 1) - (a*b*cos(x)^2 - a^2 - a*b)*log(-sin(x) + 1) - (a*b
+ b^2)*sin(x))/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - (a^3*b + 2*a^2*b^2 + a
*b^3)*cos(x)^2)]
```

3.320.6 Sympy [F]

$$\int \frac{\sec(x)}{(a+b\sin^2(x))^2}\,dx = \int \frac{\sec{(x)}}{\left(a+b\sin^2{(x)}\right)^2}\,dx$$

input
```
integrate(sec(x)/(a+b*sin(x)**2)**2,x)
```

output
```
Integral(sec(x)/(a + b*sin(x)**2)**2, x)
```

3.320.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.58

$$\int \frac{\sec(x)}{(a+b\sin^2(x))^2}\,dx = \frac{b\sin{(x)}}{2\left(a^3+a^2b+(a^2b+ab^2)\sin{(x)}^2\right)} + \frac{(3\,ab+b^2)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\left(a^3+2\,a^2b+ab^2\right)\sqrt{ab}} + \frac{\log{(\sin{(x)}+1)}}{2\left(a^2+2\,ab+b^2\right)} - \frac{\log{(\sin{(x)}-1)}}{2\left(a^2+2\,ab+b^2\right)}$$

input
```
integrate(sec(x)/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/2*b*sin(x)/(a^3 + a^2*b + (a^2*b + a*b^2)*sin(x)^2) + 1/2*(3*a*b + b^2)*
arctan(b*sin(x)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)) + 1/2*log(s
in(x) + 1)/(a^2 + 2*a*b + b^2) - 1/2*log(sin(x) - 1)/(a^2 + 2*a*b + b^2)
```

3.320.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.49

$$\int \frac{\sec(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{(3\,ab+b^2)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\,(a^3+2\,a^2b+ab^2)\sqrt{ab}} + \frac{\log\left(\sin\left(x\right)+1\right)}{2\,(a^2+2\,ab+b^2)} - \frac{\log\left(-\sin\left(x\right)+1\right)}{2\,(a^2+2\,ab+b^2)} + \frac{b\sin\left(x\right)}{2\left(b\sin\left(x\right)^2+a\right)(a^2+ab)}$$

input
```
integrate(sec(x)/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/2*(3*a*b + b^2)*arctan(b*sin(x)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt
(a*b)) + 1/2*log(sin(x) + 1)/(a^2 + 2*a*b + b^2) - 1/2*log(-sin(x) + 1)/(a
^2 + 2*a*b + b^2) + 1/2*b*sin(x)/((b*sin(x)^2 + a)*(a^2 + a*b))
```

3.320.9 Mupad [B] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 2213, normalized size of antiderivative = 30.32

$$\int \frac{\sec(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(x)*(a + b*sin(x)^2)^2),x)
```

output
```
(b*sin(x))/(2*a*(a + b)*(a + b*sin(x)^2)) - (atan((((3*a + b)*(-a^3*b)^(1/
2)*((sin(x)*(6*a*b^4 + b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) +
((3*a + b)*(-a^3*b)^(1/2)*((2*a*b^7 + 12*a^2*b^6 + 28*a^3*b^5 + 32*a^4*b^4
 + 18*a^5*b^3 + 4*a^6*b^2)/(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2) - (sin(x)
*(3*a + b)*(-a^3*b)^(1/2)*(16*a^2*b^7 + 48*a^3*b^6 + 32*a^4*b^5 - 32*a^5*b
^4 - 48*a^6*b^3 - 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(2*a^4*b + a^5
 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2)))*1i)/(4*(2*a^4*b + a^5 + a^3*
b^2)) + ((3*a + b)*(-a^3*b)^(1/2)*((sin(x)*(6*a*b^4 + b^5 + 13*a^2*b^3))/(
2*(2*a^3*b + a^4 + a^2*b^2)) - ((3*a + b)*(-a^3*b)^(1/2)*((2*a*b^7 + 12*a^
2*b^6 + 28*a^3*b^5 + 32*a^4*b^4 + 18*a^5*b^3 + 4*a^6*b^2)/(3*a^4*b + a^5 +
 a^2*b^3 + 3*a^3*b^2) + (sin(x)*(3*a + b)*(-a^3*b)^(1/2)*(16*a^2*b^7 + 48*
a^3*b^6 + 32*a^4*b^5 - 32*a^5*b^4 - 48*a^6*b^3 - 16*a^7*b^2))/(8*(2*a^3*b
+ a^4 + a^2*b^2)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2)
))*1i)/(4*(2*a^4*b + a^5 + a^3*b^2)))/(((3*a*b^3)/2 + b^4/2)/(3*a^4*b + a^
5 + a^2*b^3 + 3*a^3*b^2) + ((3*a + b)*(-a^3*b)^(1/2)*((sin(x)*(6*a*b^4 + b
^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) + ((3*a + b)*(-a^3*b)^(1/2
)*((2*a*b^7 + 12*a^2*b^6 + 28*a^3*b^5 + 32*a^4*b^4 + 18*a^5*b^3 + 4*a^6*b^
2)/(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2) - (sin(x)*(3*a + b)*(-a^3*b)^(1/2
)*(16*a^2*b^7 + 48*a^3*b^6 + 32*a^4*b^5 - 32*a^5*b^4 - 48*a^6*b^3 - 16*a^7
*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*...
```

3.320. $\int \frac{\sec(x)}{(a+b\sin^2(x))^2}\,dx$

3.321 $\int \frac{\sec^2(x)}{(a+b\sin^2(x))^2}\,dx$

3.321.1 Optimal result

Integrand size = 15, antiderivative size = 76

$$\int \frac{\sec^2(x)}{(a+b\sin^2(x))^2}\,dx = \frac{b(4a+b)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}} + \frac{\tan(x)}{(a+b)^2} + \frac{b^2\tan(x)}{2a(a+b)^2\,(a+(a+b)\tan^2(x))}$$

output
```
1/2*b*(4*a+b)*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/a^(3/2)/(a+b)^(5/2)+tan(x
)/(a+b)^2+1/2*b^2*tan(x)/a/(a+b)^2/(a+(a+b)*tan(x)^2)
```

3.321.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(x)}{(a+b\sin^2(x))^2}\,dx = \frac{1}{2}\left(\frac{b(4a+b)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{a^{3/2}(a+b)^{5/2}} + \frac{\frac{b^2\sin(2x)}{a(2a+b-b\cos(2x))}+2\tan(x)}{(a+b)^2}\right)$$

input
```
Integrate[Sec[x]^2/(a + b*Sin[x]^2)^2,x]
```

output
```
((b*(4*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(a^(3/2)*(a + b)^(5/2)
) + ((b^2*Sin[2*x])/(a*(2*a + b - b*Cos[2*x])) + 2*Tan[x])/(a + b)^2)/2
```

3.321.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{\left(a+b\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(x)^2\left(a+b\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \text{3670}$$

$$\int \frac{\left(\tan^2(x)+1\right)^2}{\left((a+b)\tan^2(x)+a\right)^2}d\tan(x)$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{2b(a+b)\tan^2(x)+b(2a+b)}{(a+b)^2\left((a+b)\tan^2(x)+a\right)^2}+\frac{1}{(a+b)^2}\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{b(4a+b)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}}+\frac{b^2\tan(x)}{2a(a+b)^2\left((a+b)\tan^2(x)+a\right)}+\frac{\tan(x)}{(a+b)^2}$$

input
```
Int[Sec[x]^2/(a + b*Sin[x]^2)^2,x]
```

output
```
(b*(4*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(5/2
)) + Tan[x]/(a + b)^2 + (b^2*Tan[x])/(2*a*(a + b)^2*(a + (a + b)*Tan[x]^2)
)
```

3.321.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.321.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07

method	result
default	$\frac{\tan(x)}{a^2+2ab+b^2}+\frac{b\left(\frac{b\tan(x)}{2a\left(a\left(\tan^2(x)\right)+\left(\tan^2(x)\right)b+a\right)}+\frac{(4a+b)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{2a\sqrt{a(a+b)}}\right)}{(a+b)^2}$
risch	$\frac{i\left(-4ab\,e^{4ix}-b^2e^{4ix}+8\,e^{2ix}a^2+2ab\,e^{2ix}-2ab+b^2\right)}{a(a+b)^2\left(-b\,e^{4ix}+4a\,e^{2ix}+2b\,e^{2ix}-b\right)\left(e^{2ix}+1\right)}-\frac{b\ln\left(e^{2ix}-\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,(a+b)^2}-\frac{b^2\ln\left(e^{2ix}-\frac{2ia^2+2iab+2a}{b}\right)}{4\sqrt{-a^2-a}}$

input
```
int(sec(x)^2/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
tan(x)/(a^2+2*a*b+b^2)+b/(a+b)^2*(1/2/a*b*tan(x)/(a*tan(x)^2+tan(x)^2*b+a)
+1/2*(4*a+b)/a/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2)))
```

3.321.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(64) = 128.

Time = 0.32 (sec) , antiderivative size = 505, normalized size of antiderivative = 6.64

$$\int \frac{\sec^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$$
$$= \left[-\frac{\left(\left(4\,ab^2+b^3\right)\cos\left(x\right)^3-\left(4\,a^2b+5\,ab^2+b^3\right)\cos\left(x\right)\right)\sqrt{-a^2-ab}\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos}{b^2\cos(x)}\right.}{8\left(\left(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4\right)\cos\right.}\right.$$
$$-\frac{\left(\left(4\,ab^2+b^3\right)\cos\left(x\right)^3-\left(4\,a^2b+5\,ab^2+b^3\right)\cos\left(x\right)\right)\sqrt{a^2+ab}\arctan\left(\frac{(2\,a+b)\cos(x)^2-a-b}{2\,\sqrt{a^2+ab}\cos(x)\sin(x)}\right)+2\left(2\,a^4+4\,a\right.}{4\left(\left(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4\right)\cos\left(x\right)^3-\left(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b\right.\right.}$$

input
```
integrate(sec(x)^2/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(((4*a*b^2 + b^3)*cos(x)^3 - (4*a^2*b + 5*a*b^2 + b^3)*cos(x))*sqrt(
-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*
cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt(-a^2 - a*b)*sin(x)
 + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b
 + b^2)) + 4*(2*a^4 + 4*a^3*b + 2*a^2*b^2 - (2*a^3*b + a^2*b^2 - a*b^3)*co
s(x)^2)*sin(x))/((a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*cos(x)^3 - (a^6
 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(x)), -1/4*(((4*a*b^2 + b
^3)*cos(x)^3 - (4*a^2*b + 5*a*b^2 + b^3)*cos(x))*sqrt(a^2 + a*b)*arctan(1/
2*((2*a + b)*cos(x)^2 - a - b)/(sqrt(a^2 + a*b)*cos(x)*sin(x))) + 2*(2*a^4
 + 4*a^3*b + 2*a^2*b^2 - (2*a^3*b + a^2*b^2 - a*b^3)*cos(x)^2)*sin(x))/((a
^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*cos(x)^3 - (a^6 + 4*a^5*b + 6*a^4*
b^2 + 4*a^3*b^3 + a^2*b^4)*cos(x))]
```

3.321.6 Sympy [F]

$$\int \frac{\sec^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \int \frac{\sec^2\left(x\right)}{\left(a+b\sin^2\left(x\right)\right)^2}\,dx$$

input
```
integrate(sec(x)**2/(a+b*sin(x)**2)**2,x)
```

output
```
Integral(sec(x)**2/(a + b*sin(x)**2)**2, x)
```

3.321.7 Maxima [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.57

$$\int \frac{\sec^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{b^2\tan(x)}{2\left(a^4+2\,a^3b+a^2b^2+\left(a^4+3\,a^3b+3\,a^2b^2+ab^3\right)\tan(x)^2\right)} + \frac{\left(4\,ab+b^2\right)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{2\left(a^3+2\,a^2b+ab^2\right)\sqrt{(a+b)a}} + \frac{\tan(x)}{a^2+2\,ab+b^2}$$

input
```
integrate(sec(x)^2/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/2*b^2*tan(x)/(a^4 + 2*a^3*b + a^2*b^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b
^3)*tan(x)^2) + 1/2*(4*a*b + b^2)*arctan((a + b)*tan(x)/sqrt((a + b)*a))/(
(a^3 + 2*a^2*b + a*b^2)*sqrt((a + b)*a)) + tan(x)/(a^2 + 2*a*b + b^2)
```

3.321.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.49

$$\int \frac{\sec^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{b^2\tan(x)}{2\left(a^3+2\,a^2b+ab^2\right)\left(a\tan(x)^2+b\tan(x)^2+a\right)} + \frac{\left(4\,ab+b^2\right)\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)}{2\left(a^3+2\,a^2b+ab^2\right)\sqrt{a^2+ab}} + \frac{\tan(x)}{a^2+2\,ab+b^2}$$

input
```
integrate(sec(x)^2/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/2*b^2*tan(x)/((a^3 + 2*a^2*b + a*b^2)*(a*tan(x)^2 + b*tan(x)^2 + a)) + 1
/2*(4*a*b + b^2)*arctan((a*tan(x) + b*tan(x))/sqrt(a^2 + a*b))/((a^3 + 2*a
^2*b + a*b^2)*sqrt(a^2 + a*b)) + tan(x)/(a^2 + 2*a*b + b^2)
```

3.321.9 Mupad [B] (verification not implemented)

Time = 13.91 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.62

$$\int \frac{\sec^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{\tan(x)}{(a+b)^2} + \frac{b^2\tan(x)}{2\,a\,\left(a\,b^2+2\,a^2\,b+\tan\left(x\right)^2\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)+a^3\right)} + \frac{b\,\mathrm{atan}\left(\frac{b\tan(x)\,(4\,a+b)\,(2\,a+2\,b)\,\left(a^2+2\,a\,b+b^2\right)}{2\,\sqrt{a}\,(a+b)^{5/2}\,\left(b^2+4\,a\,b\right)}\right)\,(4\,a+b)}{2\,a^{3/2}\,(a+b)^{5/2}}$$

input
```
int(1/(cos(x)^2*(a + b*sin(x)^2)^2),x)
```

output
```
tan(x)/(a + b)^2 + (b^2*tan(x))/(2*a*(a*b^2 + 2*a^2*b + tan(x)^2*(3*a*b^2
+ 3*a^2*b + a^3 + b^3) + a^3)) + (b*atan((b*tan(x)*(4*a + b)*(2*a + 2*b)*(
2*a*b + a^2 + b^2))/(2*a^(1/2)*(a + b)^(5/2)*(4*a*b + b^2)))*(4*a + b))/(2
*a^(3/2)*(a + b)^(5/2))
```

3.322 $\int \frac{\sec^3(x)}{(a+b\sin^2(x))^2}\,dx$

3.322.1 Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \frac{\sec^3(x)}{(a+b\sin^2(x))^2}\,dx = \frac{b^{3/2}(5a+b)\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^3}+\frac{(a+5b)\text{arctanh}(\sin(x))}{2(a+b)^3} - \frac{(a-b)b\sin(x)}{2a(a+b)^2\,(a+b\sin^2(x))}+\frac{\sec(x)\tan(x)}{2(a+b)\,(a+b\sin^2(x))}$$

output
```
1/2*b^(3/2)*(5*a+b)*arctan(sin(x)*b^(1/2)/a^(1/2))/a^(3/2)/(a+b)^3+1/2*(a+
5*b)*arctanh(sin(x))/(a+b)^3-1/2*(a-b)*b*sin(x)/a/(a+b)^2/(a+b*sin(x)^2)+1
/2*sec(x)*tan(x)/(a+b)/(a+b*sin(x)^2)
```

3.322.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.68

$$\int \frac{\sec^3(x)}{(a+b\sin^2(x))^2}\,dx = \frac{-\frac{b^{3/2}(5a+b)\arctan\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)}{a^{3/2}}+\frac{b^{3/2}(5a+b)\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{a^{3/2}}-2(a+5b)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+2(a+5b)\log\left(\cos\right.}{4(a+b)^3}$$

input
```
Integrate[Sec[x]^3/(a + b*Sin[x]^2)^2,x]
```

output
```
(-((b^(3/2)*(5*a + b)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/a^(3/2)) + (b^(3/2
)*(5*a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/a^(3/2) - 2*(a + 5*b)*Log[Co
s[x/2] - Sin[x/2]] + 2*(a + 5*b)*Log[Cos[x/2] + Sin[x/2]] + (a + b)/(Cos[x
/2] - Sin[x/2])^2 - (a + b)/(Cos[x/2] + Sin[x/2])^2 + (4*b^2*(a + b)*Sin[x
])/(a*(2*a + b - b*Cos[2*x])))/(4*(a + b)^3)
```

3.322.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3669, 316, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(x)}{\left(a+b\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(x)^3\left(a+b\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \text{3669}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2\left(a+b\sin^2(x)\right)^2}d\sin(x)$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{3b\sin^2(x)+a+2b}{\left(1-\sin^2(x)\right)\left(b\sin^2(x)+a\right)^2}d\sin(x)}{2(a+b)}+\frac{\sin(x)}{2(a+b)\left(1-\sin^2(x)\right)\left(a+b\sin^2(x)\right)}$$

$$\Big\downarrow \text{402}$$

$$\frac{-\frac{\int -\frac{2\left(a^2+4ba+b^2+(a-b)b\sin^2(x)\right)}{\left(1-\sin^2(x)\right)\left(b\sin^2(x)+a\right)}d\sin(x)}{2a(a+b)}-\frac{b(a-b)\sin(x)}{a(a+b)\left(a+b\sin^2(x)\right)}}{2(a+b)}+\frac{\sin(x)}{2(a+b)\left(1-\sin^2(x)\right)\left(a+b\sin^2(x)\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{a^2+4ba+b^2+(a-b)b\sin^2(x)}{\left(1-\sin^2(x)\right)\left(b\sin^2(x)+a\right)}d\sin(x)}{a(a+b)}-\frac{b(a-b)\sin(x)}{a(a+b)\left(a+b\sin^2(x)\right)}}{2(a+b)}+\frac{\sin(x)}{2(a+b)\left(1-\sin^2(x)\right)\left(a+b\sin^2(x)\right)}$$

$$\Big\downarrow \ 397$$

$$\frac{\frac{\frac{b^2(5a+b)\int \frac{1}{b\sin^2(x)+a}d\sin(x)}{a+b}+\frac{a(a+5b)\int \frac{1}{1-\sin^2(x)}d\sin(x)}{a+b}}{a(a+b)}-\frac{b(a-b)\sin(x)}{a(a+b)(a+b\sin^2(x))}}{2(a+b)}+\frac{\sin(x)}{2(a+b)\left(1-\sin^2(x)\right)\left(a+b\sin^2(x)\right)}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{\frac{a(a+5b)\int \frac{1}{1-\sin^2(x)}d\sin(x)}{a+b}+\frac{b^{3/2}(5a+b)\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a(a+b)}-\frac{b(a-b)\sin(x)}{a(a+b)(a+b\sin^2(x))}}{2(a+b)}+\frac{\sin(x)}{2(a+b)\left(1-\sin^2(x)\right)\left(a+b\sin^2(x)\right)}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\frac{b^{3/2}(5a+b)\arctan\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{a(a+5b)\text{arctanh}(\sin(x))}{a+b}}{a(a+b)}-\frac{b(a-b)\sin(x)}{a(a+b)(a+b\sin^2(x))}}{2(a+b)}+\frac{\sin(x)}{2(a+b)\left(1-\sin^2(x)\right)\left(a+b\sin^2(x)\right)}$$

input
```
Int[Sec[x]^3/(a + b*Sin[x]^2)^2,x]
```

output
```
Sin[x]/(2*(a + b)*(1 - Sin[x]^2)*(a + b*Sin[x]^2)) + (((b^(3/2)*(5*a + b)*
ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + (a*(a + 5*b)*ArcTanh
[Sin[x]])/(a + b))/(a*(a + b)) - ((a - b)*b*Sin[x])/(a*(a + b)*(a + b*Sin[
x]^2)))/(2*(a + b))
```

3.322.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.322. $\int \frac{\sec^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.322. $\int \frac{\sec^3(x)}{(a+b\sin^2(x))^2} dx$

3.322.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.09

method	result
default	$\frac{b^2\left(\frac{(a+b)\sin(x)}{2a\left(a+b\left(\sin^2(x)\right)\right)}+\frac{(5a+b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^3}-\frac{1}{4(a+b)^2(\sin(x)-1)}+\frac{(-a-5b)\ln(\sin(x)-1)}{4(a+b)^3}-\frac{1}{4(a+b)^2(1+\sin(x))}+\frac{(a+$
risch	$-\frac{i(ab\,e^{7ix}-b^2e^{7ix}-4a^2e^{5ix}-3ab\,e^{5ix}-b^2e^{5ix}+4\,e^{3ix}a^2+3ab\,e^{3ix}+b^2e^{3ix}-e^{ix}ab+e^{ix}b^2)}{(a+b)^2(e^{2ix}+1)^2a(b\,e^{4ix}-4a\,e^{2ix}-2b\,e^{2ix}+b)}+\frac{\ln(e^{ix}+i)a}{2a^3+6a^2b+6a\,b^2+2b^3}+\frac{5\ln(e}{2(a^3+3a^2b}$

input
```
int(sec(x)^3/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a+b)^3*b^2*(1/2*(a+b)/a*sin(x)/(a+b*sin(x)^2)+1/2*(5*a+b)/a/(a*b)^(1/2)
*arctan(b*sin(x)/(a*b)^(1/2)))-1/4/(a+b)^2/(sin(x)-1)+1/4/(a+b)^3*(-a-5*b)
*ln(sin(x)-1)-1/4/(a+b)^2/(1+sin(x))+1/4*(a+5*b)/(a+b)^3*ln(1+sin(x))
```

3.322.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(93) = 186.

Time = 0.41 (sec) , antiderivative size = 560, normalized size of antiderivative = 5.14

$$\int \frac{\sec^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$$
$$=\left[\frac{\left((5\,ab^2+b^3)\cos(x)^4-(5\,a^2b+6\,ab^2+b^3)\cos(x)^2\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{b\cos(x)^2-2\,a\sqrt{-\frac{b}{a}}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+\left((a^2b+5\right.}{4\,(}\right.$$

input
```
integrate(sec(x)^3/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(((5*a*b^2 + b^3)*cos(x)^4 - (5*a^2*b + 6*a*b^2 + b^3)*cos(x)^2)*sqrt
(-b/a)*log(-(b*cos(x)^2 - 2*a*sqrt(-b/a)*sin(x) + a - b)/(b*cos(x)^2 - a -
 b)) + ((a^2*b + 5*a*b^2)*cos(x)^4 - (a^3 + 6*a^2*b + 5*a*b^2)*cos(x)^2)*l
og(sin(x) + 1) - ((a^2*b + 5*a*b^2)*cos(x)^4 - (a^3 + 6*a^2*b + 5*a*b^2)*c
os(x)^2)*log(-sin(x) + 1) - 2*(a^3 + 2*a^2*b + a*b^2 - (a^2*b - b^3)*cos(x
)^2)*sin(x))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(x)^4 - (a^5 + 4*
a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^2), 1/4*(2*((5*a*b^2 + b^3)*
cos(x)^4 - (5*a^2*b + 6*a*b^2 + b^3)*cos(x)^2)*sqrt(b/a)*arctan(sqrt(b/a)*
sin(x)) + ((a^2*b + 5*a*b^2)*cos(x)^4 - (a^3 + 6*a^2*b + 5*a*b^2)*cos(x)^2
)*log(sin(x) + 1) - ((a^2*b + 5*a*b^2)*cos(x)^4 - (a^3 + 6*a^2*b + 5*a*b^2
)*cos(x)^2)*log(-sin(x) + 1) - 2*(a^3 + 2*a^2*b + a*b^2 - (a^2*b - b^3)*co
s(x)^2)*sin(x))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(x)^4 - (a^5 +
 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^2)]
```

3.322.6 Sympy [F]

$$\int \frac{\sec^3(x)}{(a+b\sin^2(x))^2}\,dx = \int \frac{\sec^3(x)}{(a+b\sin^2(x))^2}\,dx$$

input
```
integrate(sec(x)**3/(a+b*sin(x)**2)**2,x)
```

output
```
Integral(sec(x)**3/(a + b*sin(x)**2)**2, x)
```

3.322.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(93) = 186$.

Time = 0.48 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.02

$$\begin{aligned}\int \frac{\sec^3(x)}{(a+b\sin^2(x))^2}\,dx &= \frac{(a+5b)\log(\sin(x)+1)}{4(a^3+3a^2b+3ab^2+b^3)} - \frac{(a+5b)\log(\sin(x)-1)}{4(a^3+3a^2b+3ab^2+b^3)} \\ &\quad + \frac{(5ab^2+b^3)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2(a^4+3a^3b+3a^2b^2+ab^3)\sqrt{ab}} \\ &\quad - \frac{(ab-b^2)\sin(x)^3+(a^2+b^2)\sin(x)}{2\left((a^3b+2a^2b^2+ab^3)\sin(x)^4-a^4-2a^3b-a^2b^2+(a^4+a^3b-a^2b^2-ab^3)\sin(x)^2\right)}\end{aligned}$$

input
```
integrate(sec(x)^3/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/4*(a + 5*b)*log(sin(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/4*(a + 5
*b)*log(sin(x) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/2*(5*a*b^2 + b^3)*
arctan(b*sin(x)/sqrt(a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b))
 - 1/2*((a*b - b^2)*sin(x)^3 + (a^2 + b^2)*sin(x))/((a^3*b + 2*a^2*b^2 + a
*b^3)*sin(x)^4 - a^4 - 2*a^3*b - a^2*b^2 + (a^4 + a^3*b - a^2*b^2 - a*b^3)
*sin(x)^2)
```

3.322.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(93) = 186$.

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.78

$$\int \frac{\sec^3(x)}{(a+b\sin^2(x))^2}\,dx = \frac{(a+5b)\log(\sin(x)+1)}{4(a^3+3a^2b+3ab^2+b^3)} - \frac{(a+5b)\log(-\sin(x)+1)}{4(a^3+3a^2b+3ab^2+b^3)} + \frac{(5ab^2+b^3)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2(a^4+3a^3b+3a^2b^2+ab^3)\sqrt{ab}} - \frac{ab\sin(x)^3-b^2\sin(x)^3+a^2\sin(x)+b^2\sin(x)}{2\left(b\sin(x)^4+a\sin(x)^2-b\sin(x)^2-a\right)(a^3+2a^2b+ab^2)}$$

input
```
integrate(sec(x)^3/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/4*(a + 5*b)*log(sin(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/4*(a + 5
*b)*log(-sin(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/2*(5*a*b^2 + b^3)
*arctan(b*sin(x)/sqrt(a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)
) - 1/2*(a*b*sin(x)^3 - b^2*sin(x)^3 + a^2*sin(x) + b^2*sin(x))/((b*sin(x)
^4 + a*sin(x)^2 - b*sin(x)^2 - a)*(a^3 + 2*a^2*b + a*b^2))
```

3.322.9 Mupad [B] (verification not implemented)

Time = 14.64 (sec) , antiderivative size = 2009, normalized size of antiderivative = 18.43

$$\int \frac{\sec^3(x)}{(a+b\sin^2(x))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(x)^3*(a + b*sin(x)^2)^2),x)
```

output
```
(log(sin(x) + 1)*(a + 5*b))/(4*(a + b)^3) - log(sin(x) - 1)*(b/(a + b)^3 +
 1/(4*(a + b)^2)) - ((sin(x)*(a^2 + b^2))/(2*a*(2*a*b + a^2 + b^2)) + (b*s
in(x)^3*(a - b))/(2*a*(2*a*b + a^2 + b^2)))/(b*sin(x)^4 - a + sin(x)^2*(a
- b)) - (atan(((((sin(x)*(10*a*b^6 + b^7 + 50*a^2*b^5 + 10*a^3*b^4 + a^4*b
^3))/(2*(4*a^5*b + a^6 + a^2*b^4 + 4*a^3*b^3 + 6*a^4*b^2)) + ((5*a + b)*(-
a^3*b^3)^(1/2)*((2*a*b^10 + 20*a^2*b^9 + 80*a^3*b^8 + 172*a^4*b^7 + 220*a^
5*b^6 + 172*a^6*b^5 + 80*a^7*b^4 + 20*a^8*b^3 + 2*a^9*b^2)/(6*a^7*b + a^8
+ a^2*b^6 + 6*a^3*b^5 + 15*a^4*b^4 + 20*a^5*b^3 + 15*a^6*b^2) - (sin(x)*(5
*a + b)*(-a^3*b^3)^(1/2)*(16*a^2*b^9 + 80*a^3*b^8 + 144*a^4*b^7 + 80*a^5*b
^6 - 80*a^6*b^5 - 144*a^7*b^4 - 80*a^8*b^3 - 16*a^9*b^2))/(8*(3*a^5*b + a^
6 + a^3*b^3 + 3*a^4*b^2)*(4*a^5*b + a^6 + a^2*b^4 + 4*a^3*b^3 + 6*a^4*b^2)
)))/(4*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*(5*a + b)*(-a^3*b^3)^(1/2)*
1i)/(4*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)) + (((sin(x)*(10*a*b^6 + b^7
+ 50*a^2*b^5 + 10*a^3*b^4 + a^4*b^3))/(2*(4*a^5*b + a^6 + a^2*b^4 + 4*a^3*
b^3 + 6*a^4*b^2)) - ((5*a + b)*(-a^3*b^3)^(1/2)*((2*a*b^10 + 20*a^2*b^9 +
80*a^3*b^8 + 172*a^4*b^7 + 220*a^5*b^6 + 172*a^6*b^5 + 80*a^7*b^4 + 20*a^8
*b^3 + 2*a^9*b^2)/(6*a^7*b + a^8 + a^2*b^6 + 6*a^3*b^5 + 15*a^4*b^4 + 20*a
^5*b^3 + 15*a^6*b^2) + (sin(x)*(5*a + b)*(-a^3*b^3)^(1/2)*(16*a^2*b^9 + 80
*a^3*b^8 + 144*a^4*b^7 + 80*a^5*b^6 - 80*a^6*b^5 - 144*a^7*b^4 - 80*a^8*b^
3 - 16*a^9*b^2))/(8*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)*(4*a^5*b + a^...
```

3.323 $\int \frac{\sec^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

3.323.1 Optimal result

Integrand size = 15, antiderivative size = 96

$$\int \frac{\sec^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{b^2(6a+b)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}} + \frac{(a+3b)\tan(x)}{(a+b)^3} + \frac{\tan^3(x)}{3(a+b)^2} + \frac{b^3\tan(x)}{2a(a+b)^3\left(a+(a+b)\tan^2(x)\right)}$$

output
```
1/2*b^2*(6*a+b)*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/a^(3/2)/(a+b)^(7/2)+(a+
3*b)*tan(x)/(a+b)^3+1/3*tan(x)^3/(a+b)^2+1/2*b^3*tan(x)/a/(a+b)^3/(a+(a+b)
*tan(x)^2)
```

3.323.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01

$$\int \frac{\sec^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{1}{6}\left(\frac{3b^2(6a+b)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{a^{3/2}(a+b)^{7/2}} + \frac{\frac{3b^3\sin(2x)}{a(2a+b-b\cos(2x))} + 4a\tan(x) + 16b\tan(x) + 2(a+b)\sec^2(x)\tan(x)}{(a+b)^3}\right)$$

input
```
Integrate[Sec[x]^4/(a + b*Sin[x]^2)^2,x]
```

output
```
((3*b^2*(6*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(a^(3/2)*(a + b)^(
7/2)) + ((3*b^3*Sin[2*x])/(a*(2*a + b - b*Cos[2*x])) + 4*a*Tan[x] + 16*b*T
an[x] + 2*(a + b)*Sec[x]^2*Tan[x])/(a + b)^3)/6
```

3.323.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(x)}{\left(a+b\sin^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(x)^4\left(a+b\sin(x)^2\right)^2}dx$$

$$\Big\downarrow \text{3670}$$

$$\int \frac{\left(\tan^2(x)+1\right)^3}{\left((a+b)\tan^2(x)+a\right)^2}d\tan(x)$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{3b^2(a+b)\tan^2(x)+b^2(3a+b)}{(a+b)^3\left((a+b)\tan^2(x)+a\right)^2}+\frac{\tan^2(x)}{(a+b)^2}+\frac{a+3b}{(a+b)^3}\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2(6a+b)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}}+\frac{b^3\tan(x)}{2a(a+b)^3\left((a+b)\tan^2(x)+a\right)}+\frac{\tan^3(x)}{3(a+b)^2}+\frac{(a+3b)\tan(x)}{(a+b)^3}$$

input
```
Int[Sec[x]^4/(a + b*Sin[x]^2)^2,x]
```

output
```
(b^2*(6*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(7
/2)) + ((a + 3*b)*Tan[x])/(a + b)^3 + Tan[x]^3/(3*(a + b)^2) + (b^3*Tan[x]
)/(2*a*(a + b)^3*(a + (a + b)*Tan[x]^2))
```

3.323.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.323.4 Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.15

method	result
default	$\frac{\frac{a\left(\tan^{3}\left(x\right)\right)}{3}+\frac{b\left(\tan^{3}\left(x\right)\right)}{3}+\tan\left(x\right)a+3\tan\left(x\right)b}{\left(a^{2}+2ab+b^{2}\right)\left(a+b\right)}+\frac{b^{2}\left(\frac{b\tan\left(x\right)}{2a\left(a\left(\tan^{2}\left(x\right)\right)+\left(\tan^{2}\left(x\right)\right)b+a\right)}+\frac{\left(6a+b\right)\arctan\left(\frac{\left(a+b\right)\tan\left(x\right)}{\sqrt{a\left(a+b\right)}}\right)}{2a\sqrt{a\left(a+b\right)}}\right)}{\left(a+b\right)^{3}}$
risch	$\frac{i\left(-18a\,b^{2}e^{8ix}-3b^{3}e^{8ix}+36a^{2}b\,e^{6ix}-30a\,b^{2}e^{6ix}-6b^{3}e^{6ix}+48a^{3}e^{4ix}+164a^{2}b\,e^{4ix}+26a\,b^{2}e^{4ix}+16a^{3}e^{2ix}+60a^{2}b\,e^{2ix}-10a\,b^{2}e^{2ix}+6b\right.}{3\left(e^{2ix}+1\right)^{3}\left(a+b\right)^{3}a\left(-b\,e^{4ix}+4a\,e^{2ix}+2b\,e^{2ix}-b\right)}$

input
```
int(sec(x)^4/(a+b*sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

3.323. $\int \frac{\sec^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

output
```
1/(a^2+2*a*b+b^2)/(a+b)*(1/3*a*tan(x)^3+1/3*b*tan(x)^3+tan(x)*a+3*tan(x)*b
)+b^2/(a+b)^3*(1/2/a*b*tan(x)/(a*tan(x)^2+tan(x)^2*b+a)+1/2*(6*a+b)/a/(a*(
a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2)))
```

3.323.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(82) = 164$.

Time = 0.34 (sec) , antiderivative size = 653, normalized size of antiderivative = 6.80

$$\int \frac{\sec^4(x)}{(a+b\sin^2(x))^2}\,dx$$
$$= \left[-\frac{3\left((6\,ab^3+b^4)\cos{(x)}^5-(6\,a^2b^2+7\,ab^3+b^4)\cos{(x)}^3\right)\sqrt{-a^2-ab}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+5\,ab+b^2}{b^2\cos}\right.}{24\,((a^6b+}\right.$$
$$-\frac{3\left((6\,ab^3+b^4)\cos{(x)}^5-(6\,a^2b^2+7\,ab^3+b^4)\cos{(x)}^3\right)\sqrt{a^2+ab}\arctan\left(\frac{(2\,a+b)\cos(x)^2-a-b}{2\,\sqrt{a^2+ab}\cos(x)\sin(x)}\right)+2\,(2\,a^5+}{12\,((a^6b+4\,a^5b^2+6\,a^4b^3+4\,a^3b^4+a^2b^5)\cos}$$

input
```
integrate(sec(x)^4/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

output
```
[-1/24*(3*((6*a*b^3 + b^4)*cos(x)^5 - (6*a^2*b^2 + 7*a*b^3 + b^4)*cos(x)^3
)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b
+ b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt(-a^2 - a*b)
*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2
+ 2*a*b + b^2)) + 4*(2*a^5 + 6*a^4*b + 6*a^3*b^2 + 2*a^2*b^3 - (4*a^4*b +
20*a^3*b^2 + 13*a^2*b^3 - 3*a*b^4)*cos(x)^4 + 2*(2*a^5 + 11*a^4*b + 16*a^3
*b^2 + 7*a^2*b^3)*cos(x)^2)*sin(x))/((a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^
3*b^4 + a^2*b^5)*cos(x)^5 - (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a
^3*b^4 + a^2*b^5)*cos(x)^3), -1/12*(3*((6*a*b^3 + b^4)*cos(x)^5 - (6*a^2*b
^2 + 7*a*b^3 + b^4)*cos(x)^3)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)
^2 - a - b)/(sqrt(a^2 + a*b)*cos(x)*sin(x))) + 2*(2*a^5 + 6*a^4*b + 6*a^3*
b^2 + 2*a^2*b^3 - (4*a^4*b + 20*a^3*b^2 + 13*a^2*b^3 - 3*a*b^4)*cos(x)^4 +
 2*(2*a^5 + 11*a^4*b + 16*a^3*b^2 + 7*a^2*b^3)*cos(x)^2)*sin(x))/((a^6*b +
 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*cos(x)^5 - (a^7 + 5*a^6*b +
10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cos(x)^3)]
```

3.323.6 Sympy [F]

$$\int \frac{\sec^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx=\int \frac{\sec^4{(x)}}{\left(a+b\sin^2{(x)}\right)^2}\,dx$$

input
```
integrate(sec(x)**4/(a+b*sin(x)**2)**2,x)
```

output
```
Integral(sec(x)**4/(a + b*sin(x)**2)**2, x)
```

3.323.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(82) = 164.

Time = 0.41 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.77

$$\int \frac{\sec^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{b^3\tan(x)}{2\left(a^5+3a^4b+3a^3b^2+a^2b^3+\left(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4\right)\tan(x)^2\right)} + \frac{\left(6ab^2+b^3\right)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{2\left(a^4+3a^3b+3a^2b^2+ab^3\right)\sqrt{(a+b)a}} + \frac{(a+b)\tan(x)^3+3(a+3b)\tan(x)}{3\left(a^3+3a^2b+3ab^2+b^3\right)}$$

input
```
integrate(sec(x)^4/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

output
```
1/2*b^3*tan(x)/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^5 + 4*a^4*b + 6*a
^3*b^2 + 4*a^2*b^3 + a*b^4)*tan(x)^2) + 1/2*(6*a*b^2 + b^3)*arctan((a + b)
*tan(x)/sqrt((a + b)*a))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt((a + b)
*a)) + 1/3*((a + b)*tan(x)^3 + 3*(a + 3*b)*tan(x))/(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)
```

3.323.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(82) = 164$.

Time = 0.31 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.81

$$\int \frac{\sec^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{b^3\tan(x)}{2\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\left(a\tan(x)^2+b\tan(x)^2+a\right)} + \frac{(6\,ab^2+b^3)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)}{2\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{a^2+ab}} + \frac{a^4\tan(x)^3+4\,a^3b\tan(x)^3+6\,a^2b^2\tan(x)^3+4\,ab^3\tan(x)^3+b^4\tan(x)^3+3\,a^4\tan(x)+18\,a^3b\tan(x)}{3\,(a^6+6\,a^5b+15\,a^4b^2+20\,a^3b^3+15\,a^2b^4+6\,ab^5+b^6)}$$

input
```
integrate(sec(x)^4/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/2*b^3*tan(x)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(a*tan(x)^2 + b*tan(x)
^2 + a)) + 1/2*(6*a*b^2 + b^3)*(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arct
an((a*tan(x) + b*tan(x))/sqrt(a^2 + a*b)))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a
*b^3)*sqrt(a^2 + a*b)) + 1/3*(a^4*tan(x)^3 + 4*a^3*b*tan(x)^3 + 6*a^2*b^2*
tan(x)^3 + 4*a*b^3*tan(x)^3 + b^4*tan(x)^3 + 3*a^4*tan(x) + 18*a^3*b*tan(x
) + 36*a^2*b^2*tan(x) + 30*a*b^3*tan(x) + 9*b^4*tan(x))/(a^6 + 6*a^5*b + 1
5*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)
```

3.323.9 Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.83

$$\int \frac{\sec^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \frac{\tan(x)^3}{3\,(a+b)^2} - \tan(x)\left(\frac{2\,a}{(a+b)^3}-\frac{3}{(a+b)^2}\right) + \frac{b^3\tan(x)}{2\,a\,\left(\tan(x)^2\,(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4)+a\,b^3+3\,a^3\,b+a^4+3\,a^2\,b^2\right)} + \frac{b^2\,\mathrm{atan}\left(\frac{b^2\tan(x)\,(6\,a+b)\,(2\,a+2\,b)\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}{2\,\sqrt{a}\,(a+b)^{7/2}\,(b^3+6\,a\,b^2)}\right)\,(6\,a+b)}{2\,a^{3/2}\,(a+b)^{7/2}}$$

input
```
int(1/(cos(x)^4*(a + b*sin(x)^2)^2),x)
```

output
```
tan(x)^3/(3*(a + b)^2) - tan(x)*((2*a)/(a + b)^3 - 3/(a + b)^2) + (b^3*tan
(x))/(2*a*(tan(x)^2*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2) + a*b^3 +
3*a^3*b + a^4 + 3*a^2*b^2)) + (b^2*atan((b^2*tan(x)*(6*a + b)*(2*a + 2*b)*
(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(2*a^(1/2)*(a + b)^(7/2)*(6*a*b^2 + b^3))
)*(6*a + b))/(2*a^(3/2)*(a + b)^(7/2))
```

3.323. $\int \frac{\sec^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

3.324 $\int \cos^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.324.1 Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \cos^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{a(a+4b)\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8b^{3/2}f} + \frac{(a+4b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8bf} - \frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4bf}$$

output
```
1/8*a*(a+4*b)*arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/b^(3/2)
/f-1/4*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/b/f+1/8*(a+4*b)*sin(f*x+e)*(a+b
*sin(f*x+e)^2)^(1/2)/b/f
```

3.324.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int \cos^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\sqrt{a+b\sin^2(e+fx)}\left(\sqrt{a}(a+4b)\text{arcsinh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a}}\right)-\sqrt{b}\sin(e+fx)\left(a-4b+2b\sin^2(e+fx)\right)\sqrt{1+\cdots}\right.}{8b^{3/2}f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

input
```
Integrate[Cos[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[a + b*Sin[e + f*x]^2]*(Sqrt[a]*(a + 4*b)*ArcSinh[(Sqrt[b]*Sin[e + f*
x])/Sqrt[a]] - Sqrt[b]*Sin[e + f*x]*(a - 4*b + 2*b*Sin[e + f*x]^2)*Sqrt[1
+ (b*Sin[e + f*x]^2)/a]))/(8*b^(3/2)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
```

3.324.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3669, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^3\sqrt{a+b\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{(a+4b)\int \sqrt{b\sin^2(e+fx)+a}d\sin(e+fx)}{4b}-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4b}}{f}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{(a+4b)\left(\frac{1}{2}a\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{1}{2}\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)}{4b}-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4b}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{(a+4b)\left(\frac{1}{2}a\int \frac{1}{1-\frac{b\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}+\frac{1}{2}\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)}{4b}-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4b}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(a+4b)\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{b}}+\frac{1}{2}\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)}{4b}-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4b}}{f}$$

input
```
Int[Cos[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-1/4*(Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/b + ((a + 4*b)*((a*ArcTa
nh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2*Sqrt[b]) + (Sin[
e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/2))/(4*b))/f
```

3.324.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.324.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\frac{\sin(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}}{2}+\frac{a\ln\left(\sqrt{b}\,\sin(fx+e)+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{2\sqrt{b}}-\frac{\sin(fx+e)\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}{4b}+\frac{a\left(\frac{\sin(fx+e)\sqrt{a+b}}{2}\right.}{}}{f}$
default	$\frac{\frac{\sin(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}}{2}+\frac{a\ln\left(\sqrt{b}\,\sin(fx+e)+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{2\sqrt{b}}-\frac{\sin(fx+e)\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}{4b}+\frac{a\left(\frac{\sin(fx+e)\sqrt{a+b}}{2}\right.}{}}{f}$

input
```
int(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*sin(
f*x+e)+(a+b*sin(f*x+e)^2)^(1/2))-1/4*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/b
+1/4*a/b*(1/2*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)
*sin(f*x+e)+(a+b*sin(f*x+e)^2)^(1/2))))
```

3.324.5 Fricas [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 511, normalized size of antiderivative = 4.37

$$\int \cos^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$= \left[\frac{(a^2+4\,ab)\sqrt{b}\log\left(128\,b^4\cos\left(fx+e\right)^8-256\,(ab^3+2\,b^4)\cos\left(fx+e\right)^6+32\,(5\,a^2b^2+24\,ab^3+24\,b^4)\cos\right.}{}\right.$$

$$-\frac{(a^2+4\,ab)\sqrt{-b}\arctan\left(\frac{\left(8\,b^2\cos(fx+e)^4-8\,(ab+2\,b^2)\cos(fx+e)^2+a^2+8\,ab+8\,b^2\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-b}}{4\left(2\,b^3\cos(fx+e)^4+a^2b+3\,ab^2+2\,b^3-(3\,ab^2+4\,b^3)\cos(fx+e)^2\right)\sin(fx+e)}\right)-4\,(2\,b^2\cos}{32\,b^2f}$$

input
```
integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/64*((a^2 + 4*a*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b
^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a
^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2
 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^
2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*
b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqr
t(b)*sin(f*x + e)) + 8*(2*b^2*cos(f*x + e)^2 - a*b + 2*b^2)*sqrt(-b*cos(f*
x + e)^2 + a + b)*sin(f*x + e))/(b^2*f), -1/32*((a^2 + 4*a*b)*sqrt(-b)*arc
tan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a
*b + 8*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^
4 + a^2*b + 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x +
e))) - 4*(2*b^2*cos(f*x + e)^2 - a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a +
 b)*sin(f*x + e))/(b^2*f)]
```

3.324.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.324.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int \cos^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$= \frac{\frac{a^2 \operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} + \frac{4\,a \operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{\sqrt{b}} + 4\sqrt{b\sin(fx+e)^2+a}\sin(fx+e) - \frac{2\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}\sin(fx+e)}{b} +}{8\,f}$$

input
```
integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/8*(a^2*arcsinh(b*sin(f*x + e)/sqrt(a*b))/b^(3/2) + 4*a*arcsinh(b*sin(f*x
 + e)/sqrt(a*b))/sqrt(b) + 4*sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e) - 2*(
b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)/b + sqrt(b*sin(f*x + e)^2 + a)*a*
sin(f*x + e)/b)/f
```

3.324.8 Giac [F]

$$\int \cos^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin(fx+e)^2+a}\cos(fx+e)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

3.324.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx=\int \cos\left(e+f\,x\right)^3\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.325 $\int \cos(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.325.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \cos(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx=\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{b}f}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}$$

output
```
1/2*a*arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f/b^(1/2)+1/2*s
in(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.325.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.33

$$\int \cos(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx=\frac{\sqrt{b}\sin(e+fx)\left(a+b\sin^2(e+fx)\right)+a^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a}}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{2\sqrt{b}f\sqrt{a+b\sin^2(e+fx)}}$$

input
```
Integrate[Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[b]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2) + a^(3/2)*ArcSinh[(Sqrt[b]*Si
n[e + f*x])/Sqrt[a]]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(2*Sqrt[b]*f*Sqrt[a +
 b*Sin[e + f*x]^2])
```

3.325.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3669, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)\sqrt{a+b\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \sqrt{b\sin^2(e+fx)+a}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{1}{2}a\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{1}{2}\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}a\int \frac{1}{1-\frac{b\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}+\frac{1}{2}\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{b}}+\frac{1}{2}\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}$$

input
```
Int[Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
((a*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2*Sqrt[b]
) + (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/2)/f
```

3.325.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.325.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\frac{\sin(fx+e)\sqrt{a+b\left(\sin^{2}(fx+e)\right)}}{2}+\frac{a\ln\left(\sqrt{b}\,\sin(fx+e)+\sqrt{a+b\left(\sin^{2}(fx+e)\right)}\right)}{2\sqrt{b}}}{f}$	60
default	$\frac{\frac{\sin(fx+e)\sqrt{a+b\left(\sin^{2}(fx+e)\right)}}{2}+\frac{a\ln\left(\sqrt{b}\,\sin(fx+e)+\sqrt{a+b\left(\sin^{2}(fx+e)\right)}\right)}{2\sqrt{b}}}{f}$	60

input
```
int(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*sin(
f*x+e)+(a+b*sin(f*x+e)^2)^(1/2)))
```

3.325.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(60) = 120$.

Time = 0.37 (sec) , antiderivative size = 453, normalized size of antiderivative = 6.29

$$\int \cos(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$= \left[\frac{a\sqrt{b}\log\left(128\,b^4\cos\left(fx+e\right)^8 - 256\left(ab^3+2\,b^4\right)\cos\left(fx+e\right)^6 + 32\left(5\,a^2b^2+24\,ab^3+24\,b^4\right)\cos\left(fx+e\right)\right.}{}\right.$$

$$-\frac{a\sqrt{-b}\arctan\left(\frac{\left(8\,b^2\cos(fx+e)^4-8\left(ab+2\,b^2\right)\cos(fx+e)^2+a^2+8\,ab+8\,b^2\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-b}}{4\left(2\,b^3\cos(fx+e)^4+a^2b+3\,ab^2+2\,b^3-(3\,ab^2+4\,b^3)\cos(fx+e)^2\right)\sin(fx+e)}\right)-4\sqrt{-b\cos\left(fx+e\right.}}{8\,bf}$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(a*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x
+ e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*
b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3
+ 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*c
os(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2
 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x
 + e)) + 8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/(b*f), -1/8*(a*
sqrt(-b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2
 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*c
os(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2
)*sin(f*x + e))) - 4*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/(b*f)
]
```

3.325.6 Sympy [F]

$$\int \cos(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2{(e+fx)}}\cos{(e+fx)}\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*cos(e + f*x), x)
```

3.325.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

$$\int \cos(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\frac{a\,\text{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{\sqrt{b}} + \sqrt{b\sin{(fx+e)}^2+a}\,\sin{(fx+e)}}{2\,f}$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(a*arcsinh(b*sin(f*x + e)/sqrt(a*b))/sqrt(b) + sqrt(b*sin(f*x + e)^2 +
 a)*sin(f*x + e))/f
```

3.325.8 Giac [F]

$$\int \cos(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\cos{(fx+e)}\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*cos(f*x + e), x)
```

3.325.9 Mupad [B] (verification not implemented)

Time = 13.43 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int \cos(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\sin(e+fx)\sqrt{b\sin(e+fx)^2+a}}{2f} + \frac{a\ln\left(\sqrt{b}\sin(e+fx)+\sqrt{b\sin(e+fx)^2+a}\right)}{2\sqrt{b}f}$$

input
```
int(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2))/(2*f) + (a*log(b^(1/2)*sin(e +
 f*x) + (a + b*sin(e + f*x)^2)^(1/2)))/(2*b^(1/2)*f)
```

3.326 $\int \sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.326.1 Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f} + \frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

output
```
-arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))*b^(1/2)/f+arctanh(si
n(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))*(a+b)^(1/2)/f
```

3.326.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.57

$$\int \sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{2a+2b}\sin(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right) + \frac{\sqrt{a}\sqrt{-b}\arcsin\left(\frac{\sqrt{-b}\sin(e+fx)}{\sqrt{a}}\right)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}}{\sqrt{2a+b-b\cos(2(e+fx))}}}{f}$$

input
```
Integrate[Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[a + b]*ArcTanh[(Sqrt[2*a + 2*b]*Sin[e + f*x])/Sqrt[2*a + b - b*Cos[2
*(e + f*x)]]] + (Sqrt[a]*Sqrt[-b]*ArcSin[(Sqrt[-b]*Sin[e + f*x])/Sqrt[a]]*
Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
)/f
```

3.326.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3669, 301, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin(e+fx)^2}}{\cos(e+fx)}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}}{1-\sin^2(e+fx)}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{301}$$

$$\frac{(a+b)\int \frac{1}{(1-\sin^2(e+fx))\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-b\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{(a+b)\int \frac{1}{(1-\sin^2(e+fx))\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-b\int \frac{1}{1-\frac{b\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{(a+b)\int \frac{1}{(1-\sin^2(e+fx))\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{(a+b)\int \frac{1}{1-\frac{(a+b)\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}-\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)-\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

input
```
Int[Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-(Sqrt[b]*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]) + S
qrt[a + b]*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])
/f
```

3.326.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.326.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(70) = 140$.

Time = 1.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.83

method	result
default	$\frac{-\sqrt{b}\,\ln\left(\frac{\sqrt{a+b-b\left(\cos^2(fx+e)\right)}\,\sqrt{b}+b\sin(fx+e)}{\sqrt{b}}\right)-\frac{\sqrt{a+b}\,\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}-2b\sin(fx+e)+2a}{1+\sin(fx+e)}\right)}{2}+\frac{\sqrt{a+b}\,\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b}}{}\right.}{}}{f}$

input
```
int(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-b^(1/2)*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))-1/
2*(a+b)^(1/2)*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-
b*sin(f*x+e)+a))+1/2*(a+b)^(1/2)*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*c
os(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a)))/f
```

3.326.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(70) = 140$.

Time = 0.52 (sec) , antiderivative size = 1246, normalized size of antiderivative = 15.20

$$\int \sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e
)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b +
 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 1
6*b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(
f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 +
24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x +
e)) + 2*sqrt(a + b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3
*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sq
rt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*a*b +
8*b^2)/cos(f*x + e)^4))/f, -1/8*(4*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(
f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*
b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) - sqrt(b)*log(
128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^
2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*b^2 + 256
*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e
)^2 + 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^4 - a^3 -
 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*x +
e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e)))/f, 1/4*(sqrt(
-b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^
2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos...
```

3.326.6 Sympy [F]

$$\int \sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2{(e+fx)}}\sec{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*sec(e + f*x), x)
```

3.326.7 Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.54

$$\int \sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{2\sqrt{b}\,\text{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)-\sqrt{a+b}\,\text{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}(\sin(fx+e)+1)}-\frac{a}{\sqrt{ab}(\sin(fx+e)+1)}\right)-\sqrt{a+b}\,\text{arsinh}\left(-\frac{b\sin}{\sqrt{ab}(\sin}\right.}{2f}$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*sqrt(b)*arcsinh(b*sin(f*x + e)/sqrt(a*b)) - sqrt(a + b)*arcsinh(b*
sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a/(sqrt(a*b)*(sin(f*x + e) +
 1))) - sqrt(a + b)*arcsinh(-b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) - 1))
 - a/(sqrt(a*b)*(sin(f*x + e) - 1))))/f
```

3.326.8 Giac [F]

$$\int \sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin(fx+e)^2+a}\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e), x)
```

3.326.9 Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \frac{\sqrt{b\sin(e+f\,x)^2+a}}{\cos(e+f\,x)}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x),x)
```

output
```
int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x), x)
```

3.327 $\int \sec^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.327.1 Optimal result

Integrand size = 25, antiderivative size = 82

$$\int \sec^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{a\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{a+b}f} + \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{2f}$$

output
```
1/2*a*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f/(a+b)^(1/
2)+1/2*sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

3.327.2 Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.00

$$\int \sec^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\sin(e+fx)\left(\sqrt{2}a\operatorname{arctanh}\left(\frac{\sqrt{\frac{(a+b)\sin^2(e+fx)}{a}}}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\right)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}} + (2a+b-b\cos(2(e+fx)))\sec^2(e+fx\right.}{4f\sqrt{\frac{(a+b)\sin^2(e+fx)}{a}}\sqrt{a+b\sin^2(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sin[e + f*x]*(Sqrt[2]*a*ArcTanh[Sqrt[((a + b)*Sin[e + f*x]^2)/a]/Sqrt[1 +
 (b*Sin[e + f*x]^2)/a]]*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a] + (2*a + b
- b*Cos[2*(e + f*x)])*Sec[e + f*x]^2*Sqrt[((a + b)*Sin[e + f*x]^2)/a]))/(4
*f*Sqrt[((a + b)*Sin[e + f*x]^2)/a]*Sqrt[a + b*Sin[e + f*x]^2])
```

3.327.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3669, 292, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin(e+fx)^2}}{\cos(e+fx)^3}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{292}$$

$$\frac{\frac{1}{2}a\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2\left(1-\sin^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{1}{2}a\int \frac{1}{1-\frac{(a+b)\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2\left(1-\sin^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{a\,\text{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{a+b}}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2\left(1-\sin^2(e+fx)\right)}}{f}$$

input
```
Int[Sec[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
((a*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2*Sqr
t[a + b]) + (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*(1 - Sin[e + f*x]
^2)))/f
```

3.327.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.327.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(70) = 140$.

Time = 1.34 (sec) , antiderivative size = 290, normalized size of antiderivative = 3.54

method	result
default	$\frac{2b\sqrt{a+b-b(\cos^2(fx+e))}\,\sqrt{a+b}\,(\cos^2(fx+e))\sin(fx+e)+a\left(\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}+2b\sin(fx+e)+2a}{\sin(fx+e)-1}\right)a+\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b}}{}\right.\right.}{}$

input
```
int(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*b*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)^2*sin(f*x+e)+a*
(ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+
a))*a+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*
x+e)+a))*b-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*s
in(f*x+e)+a))*a-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2
)-b*sin(f*x+e)+a))*b)*cos(f*x+e)^2+2*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(1/2
)*sin(f*x+e))/(a+b)^(3/2)/cos(f*x+e)^2/f
```

3.327.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(70) = 140$.

Time = 0.33 (sec) , antiderivative size = 337, normalized size of antiderivative = 4.11

$$\int \sec^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{a+b}a\cos(fx+e)^2\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-8\,(a^2+3\,ab+2\,b^2)\cos(fx+e)^2-4\,\left((a+2\,b)\cos(fx+e)^2-2\,a-2\,b\right)\sqrt{-b\cos(}}{\cos(fx+e)^4}\right.}{8\,(a+b)f\cos(fx+}\right.$$
$$-\frac{a\sqrt{-a-b}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-2\,a-2\,b\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a-b}}{2\,\left((ab+b^2)\cos(fx+e)^2-a^2-2\,ab-b^2\right)\sin(fx+e)}\right)\cos(fx+e)^2-2\sqrt{-b\cos(fx+e}}{4\,(a+b)f\cos(fx+e)^2}$$

input
```
integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a + b)*a*cos(f*x + e)^2*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)
^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2
- 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*
a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*sqrt(-b*cos(f*x + e)^2 + a + b)*
(a + b)*sin(f*x + e))/((a + b)*f*cos(f*x + e)^2), -1/4*(a*sqrt(-a - b)*arc
tan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a
+ b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*
x + e)))*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*sin(f*
x + e))/((a + b)*f*cos(f*x + e)^2)]
```

3.327.6 Sympy [F]

$$\int \sec^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx=\int \sqrt{a+b\sin^2{(e+fx)}}\sec^3{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*sec(e + f*x)**3, x)
```

3.327.7 Maxima [F]

$$\int \sec^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx=\int \sqrt{b\sin{(fx+e)}^2+a}\sec{(fx+e)}^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

3.327.8 Giac [F]

$$\int \sec^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx=\int \sqrt{b\sin\left(fx+e\right)^2+a}\sec\left(fx+e\right)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx=\int \frac{\sqrt{b\sin\left(e+f\,x\right)^2+a}}{\cos\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^3,x)
```

output
```
int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^3, x)
```

3.328 $\int \sec^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.328.1 Optimal result

Integrand size = 25, antiderivative size = 143

$$\int \sec^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{a(3a+4b)\text{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8(a+b)^{3/2}f} + \frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{8(a+b)f} + \frac{\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)}{4(a+b)f}$$

output
```
1/8*a*(3*a+4*b)*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/(
a+b)^(3/2)/f+1/4*sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)/(a+b)/f+
1/8*(3*a+4*b)*sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f
```

3.328.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 14.16 (sec) , antiderivative size = 669, normalized size of antiderivative = 4.68

$$\int \sec^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx =$$

$$-\frac{\sec^3(e+fx)\left(1+\frac{b\sin^2(e+fx)}{a}\right)\tan(e+fx)\left(-15a\arcsin\left(\sqrt{-\frac{(a+b)\tan^2(e+fx)}{a}}\right)-10b\arcsin\left(\sqrt{-\frac{(a+b)}{}}\right.\right.}{}$$

input
```
Integrate[Sec[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-1/40*(Sec[e + f*x]^3*(1 + (b*Sin[e + f*x]^2)/a)*Tan[e + f*x]*(-15*a*ArcSi
n[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]] - 10*b*ArcSin[Sqrt[-(((a + b)*Tan[e
 + f*x]^2)/a)]]*Sin[e + f*x]^2 - 30*a*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e +
f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2) - 20*b*Sin[e + f*x]^2*Sq
rt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/
a))^(3/2) - 32*a*Hypergeometric2F1[2, 4, 7/2, -(((a + b)*Tan[e + f*x]^2)/a
)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x
]^2)/a))^(5/2) - 32*b*Hypergeometric2F1[2, 4, 7/2, -(((a + b)*Tan[e + f*x]
^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(
((a + b)*Tan[e + f*x]^2)/a))^(5/2) + 32*a*Hypergeometric2F1[2, 4, 7/2, -((
(a + b)*Tan[e + f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a
]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2) + 32*b*Hypergeometric2F1[2, 4, 7/2
, -(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a +
b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2) + 15*a*Sqrt[-(
((a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)] + 10*
b*Sin[e + f*x]^2*Sqrt[-(((a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan
[e + f*x]^2)/a^2)]))/(f*Sqrt[a + b*Sin[e + f*x]^2]*Sqrt[(Sec[e + f*x]^2*(a
 + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2))
```

3.328.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3669, 296, 292, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin(e+fx)^2}}{\cos(e+fx)^5}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\left(1-\sin^2(e+fx)\right)^3}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{296}$$

$$\frac{\frac{(3a+4b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)}{4(a+b)}+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4(a+b)\left(1-\sin^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \text{292}$$

$$\frac{\frac{(3a+4b)\left(\frac{1}{2}a\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2\left(1-\sin^2(e+fx)\right)}\right)}{4(a+b)}+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4(a+b)\left(1-\sin^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{(3a+4b)\left(\frac{1}{2}a\int \frac{1}{1-\frac{(a+b)\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2\left(1-\sin^2(e+fx)\right)}\right)}{4(a+b)}+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4(a+b)\left(1-\sin^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(3a+4b)\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{a+b}}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2\left(1-\sin^2(e+fx)\right)}\right)}{4(a+b)}+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4(a+b)\left(1-\sin^2(e+fx)\right)^2}}{f}$$

input
```
Int[Sec[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
((Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(4*(a + b)*(1 - Sin[e + f*x]^
2)^2) + ((3*a + 4*b)*((a*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin
[e + f*x]^2]])/(2*Sqrt[a + b]) + (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])
/(2*(1 - Sin[e + f*x]^2))))/(4*(a + b)))/f
```

3.328. $\int \sec^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.328.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.328. $\int \sec^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.328.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(127) = 254$.

Time = 1.59 (sec) , antiderivative size = 570, normalized size of antiderivative = 3.99

method	result
default	$\frac{2(a+b)^{\frac{3}{2}}\sqrt{a+b-b(\cos^2(fx+e))}\,b(3a+4b)(\cos^4(fx+e))\sin(fx+e)+a\left(3\ln\left(\frac{2\sqrt{a+b}\sqrt{a+b-b\left(\cos^2(fx+e)\right)}+2b\sin(fx+e)+2a}{\sin(fx+e)-1}\right)a^3+10\ln\right.}{}$

input
```
int(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(2*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*b*(3*a+4*b)*cos(f*x+e)^4*si
n(f*x+e)+a*(3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+
b*sin(f*x+e)+a))*a^3+10*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)
^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b+11*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^2+4*ln(2/(sin(f*x+e)-1)*((a+b)^
(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^3-3*ln(2/(1+sin(f*x+e)
)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3-10*ln(2/(1+
sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b
-11*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+
e)+a))*a*b^2-4*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)
-b*sin(f*x+e)+a))*b^3)*cos(f*x+e)^4+2*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(3/
2)*(3*a+4*b)*cos(f*x+e)^2*sin(f*x+e)+4*(a+b)^(5/2)*(a+b-b*cos(f*x+e)^2)^(3
/2)*sin(f*x+e))/(a+b)^(3/2)/cos(f*x+e)^4/(a^2+2*a*b+b^2)/f
```

3.328.5 Fricas [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.10

$$\int \sec^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$= \left[\frac{(3\,a^2+4\,ab)\sqrt{a+b}\cos\left(fx+e\right)^4\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-8\,(a^2+3\,ab+2\,b^2)\cos(fx+e)^2-4\left((a+2\,b)\cos(fx+e)^2-2\,a\right.}{\cos(fx+e)^4}\right.}{} \right.$$
$$-\frac{(3\,a^2+4\,ab)\sqrt{-a-b}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-2\,a-2\,b\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a-b}}{2\left((ab+b^2)\cos(fx+e)^2-a^2-2\,ab-b^2\right)\sin(fx+e)}\right)\cos\left(fx+e\right)^4-2\left((3\,a^2\right.}{16\,(a^2+2\,ab+b^2)f\cos}$$

input
```
integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/32*((3*a^2 + 4*a*b)*sqrt(a + b)*cos(f*x + e)^4*log(((a^2 + 8*a*b + 8*b^
2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*
cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*si
n(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*((3*a^2 + 5*a*b +
 2*b^2)*cos(f*x + e)^2 + 2*a^2 + 4*a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a
 + b)*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4), -1/16*((3*a^2
+ 4*a*b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sq
rt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 -
a^2 - 2*a*b - b^2)*sin(f*x + e)))*cos(f*x + e)^4 - 2*((3*a^2 + 5*a*b + 2*b
^2)*cos(f*x + e)^2 + 2*a^2 + 4*a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b
)*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4)]
```

3.328.6 Sympy [F]

$$\int \sec^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2\left(e+fx\right)}\sec^5\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**5*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*sec(e + f*x)**5, x)
```

3.328.7 Maxima [F]

$$\int \sec^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\sec{(fx+e)}^5\,dx$$

input
```
integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

3.328.8 Giac [F]

$$\int \sec^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\sec{(fx+e)}^5\,dx$$

input
```
integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int \sec^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \frac{\sqrt{b\sin{(e+f\,x)}^2+a}}{\cos{(e+f\,x)}^5}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^5,x)
```

output
```
int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^5, x)
```

3.329 $\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.329.1 Optimal result

Integrand size = 25, antiderivative size = 220

$$\begin{aligned}\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx &= \frac{2(a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} \\ &- \frac{\cos(e+fx)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5bf} \\ &- \frac{(2a^2+7ab-3b^2)\,E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{15b^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} \\ &+ \frac{2a(a+b)(a+3b)\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{15b^2f\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

output
```
-1/5*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/b/f+2/15*(a+3*b)*cos(f
*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f-1/15*(2*a^2+7*a*b-3*b^2)*(co
s(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f
*x+e)^2)^(1/2)/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+2/15*a*(a+b)*(a+3*b)*(cos(
f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x
+e)^2/a)^(1/2)/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.329.2 Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.90

$$\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$= \frac{-16a(2a^2+7ab-3b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+32a(a^2+4ab+3b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,\text{Ellip}}{240b^2f\sqrt{2a+}}$$

input
```
Integrate[Cos[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-16*a*(2*a^2 + 7*a*b - 3*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Elli
pticE[e + f*x, -(b/a)] + 32*a*(a^2 + 4*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[
2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] - Sqrt[2]*b*(8*a^2 - 32*a*b -
15*b^2 - 4*(4*a - 3*b)*b*Cos[2*(e + f*x)] + 3*b^2*Cos[4*(e + f*x)])*Sin[2*
(e + f*x)])/(240*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.329.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3671, 318, 403, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \cos(e+fx)^4\sqrt{a+b\sin(e+fx)^2}dx$$
$$\Big\downarrow \text{3671}$$
$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{b\sin^2(e+fx)+a}\,d\sin(e+fx)}{f}$$
$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}\left(-2(a+3b)\sin^2(e+fx)+a+5b\right)}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{5b}-\frac{\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)}{5b}\right.}{f}$$

$\downarrow$ 403

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{2}{3}(a+3b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}-\frac{1}{3}\int -\frac{a(a+9b)-\left(2a^2+7ba-3b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{5b}\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{1}{3}\int \frac{a(a+9b)-\left(2a^2+7ba-3b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{2}{3}(a+3b)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{5b}\right.}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{1}{3}\left(\frac{2a(a+b)(a+3b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{\left(2a^2+7ab-3b^2\right)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right.}{5b}\right.}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{1}{3}\left(\frac{2a(a+b)(a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(2a^2+7ab-3b^2\right)\int \frac{\sqrt{b\sin^2(e+}}{\sqrt{1-\sin^2(}}}{b}\right.}{5b}\right.}{f}$$

$\downarrow$ 321

3.329. $\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{1}{3}\left(\frac{2a(a+b)(a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(2a^2+7ab-3b^2\right)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right.}{5b}\right.}{f}$$

$$\downarrow 330$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{1}{3}\left(\frac{2a(a+b)(a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(2a^2+7ab-3b^2\right)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(}{}}}{\sqrt{1-\sin}}}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.}{5b}\right.}{f}$$

$$\downarrow 327$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{1}{3}\left(\frac{2a(a+b)(a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(2a^2+7ab-3b^2\right)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\text{si}\right.}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.}{5b}\right.}{f}$$

input
```
Int[Cos[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-1/5*(Sin[e + f*x]*Sqrt[1 - Sin[e + f*
x]^2]*(a + b*Sin[e + f*x]^2)^(3/2))/b + ((2*(a + 3*b)*Sin[e + f*x]*Sqrt[1
- Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/3 + (-(((2*a^2 + 7*a*b - 3*b
^2)*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b
*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) + (2*a*(a + b)*(a + 3*b)*EllipticF[ArcSi
n[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin
[e + f*x]^2]))/3)/(5*b)))/f
```

3.329. $\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.329.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

3.329. $\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.329.4 Maple [A] (verified)

Time = 3.78 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.96

method	result
default	$\frac{-3b^3\left(\cos^6(fx+e)\right)\sin(fx+e)+4\left(\cos^4(fx+e)\right)\sin(fx+e)a\,b^2+\left(-a^2b+2a\,b^2+3b^3\right)\left(\cos^2(fx+e)\right)\sin(fx+e)+2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-}}{}$

input
```
int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(-3*b^3*cos(f*x+e)^6*sin(f*x+e)+4*cos(f*x+e)^4*sin(f*x+e)*a*b^2+(-a^2
*b+2*a*b^2+3*b^3)*cos(f*x+e)^2*sin(f*x+e)+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos
(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+8*(cos(f
*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/
a*b)^(1/2))*a^2*b+6*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)
*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2-2*(cos(f*x+e)^2)^(1/2)*(-b/a*c
os(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-7*(cos
(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-
1/a*b)^(1/2))*a^2*b+3*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/
2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2)/b^2/cos(f*x+e)/(a+b*sin(f*x
+e)^2)^(1/2)/f
```

3.329.5 Fricas [F]

$$\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin(fx+e)^2+a}\cos(fx+e)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^4, x)
```

3.329.6 Sympy [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.329.7 Maxima [F]

$$\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin(fx+e)^2+a}\cos(fx+e)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*cos(f*x + e)^4, x)
```

3.329.8 Giac [F]

$$\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx=\int \sqrt{b\sin\left(fx+e\right)^2+a}\cos\left(fx+e\right)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*cos(f*x + e)^4, x)
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx=\int \cos\left(e+f\,x\right)^4\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.330 $\int \cos^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.330.1 Optimal result

Integrand size = 25, antiderivative size = 159

$$\int \cos^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{(a-b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3bf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{a(a+b)\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3bf\sqrt{a+b\sin^2(e+fx)}}$$

output
```
1/3*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-1/3*(a-b)*(cos(f*x+e)
^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)
^(1/2)/b/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*a*(a+b)*(cos(f*x+e)^2)^(1/2)/cos
(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/b/f/
(a+b*sin(f*x+e)^2)^(1/2)
```

3.330.2 Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.99

$$\int \cos^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$= \frac{-2\sqrt{2}a(a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+2\sqrt{2}a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\operatorname{EllipticF}\left(e+fx,-\frac{b}{a}\right.}{6\sqrt{2}bf\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Cos[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-2*Sqrt[2]*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
 f*x, -(b/a)] + 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]
*EllipticF[e + f*x, -(b/a)] + b*(2*a + b - b*Cos[2*(e + f*x)])*Sin[2*(e +
f*x)])/(6*Sqrt[2]*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.330.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 3671, 319, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \cos(e+fx)^2\sqrt{a+b\sin(e+fx)^2}dx$$
$$\Big\downarrow \text{3671}$$
$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}d\sin(e+fx)}{f}$$
$$\Big\downarrow \text{319}$$
$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{2}{3}\int\frac{2a-(a-b)\sin^2(e+fx)}{2\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{1}{3}\sqrt{1-\sin^2(e+fx)}\sin(e+fx)\sqrt{a\right.}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\int\frac{2a-(a-b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{1}{3}\sqrt{1-\sin^2(e+fx)}\sin(e+fx)\sqrt{a-}\right.}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{a(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{(a-b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)+\frac{1}{3}\right.}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin}{b}\right.\right.}{f}$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx}{a}}}{\sqrt{1-\sin^2(e+}}}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.\right.}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e-\right.}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.\right.}{f}$$

input
```
Int[Cos[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

3.330. $\int \cos^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]
*Sqrt[a + b*Sin[e + f*x]^2])/3 + (-(((a - b)*EllipticE[ArcSin[Sin[e + f*x]
], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e + f*x]^2)/a]))
 + (a*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e +
f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2]))/3))/f
```

3.330.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 319
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[x*(a + b*x^2)^p*((c + d*x^2)^q/(2*(p + q) + 1)), x] + Simp[2/(2*(p + q) +
 1)   Int[(a + b*x^2)^(p - 1)*(c + d*x^2)^(q - 1)*Simp[a*c*(p + q) + (q*(b*
c - a*d) + a*d*(p + q))*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 2, p, q,
x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.330.4 Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.67

method	result
default	$\frac{-b^2(\sin^5(fx+e))+\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2+a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx\right.}{}$

input
```
int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-b^2*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f
*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-(cos(f*x+e)^2)^(1
/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+
(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1
/a*b)^(1/2))*a*b-a*b*sin(f*x+e)^3+b^2*sin(f*x+e)^3+a*b*sin(f*x+e))/b/cos(f
*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.330.5 Fricas [F]

$$\int \cos^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin\left(fx+e\right)^2+a}\cos\left(fx+e\right)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^2, x)
```

3.330.6 Sympy [F]

$$\int \cos^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2\left(e+fx\right)}\cos^2\left(e+fx\right)dx$$

input
```
integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*cos(e + f*x)**2, x)
```

3.330.7 Maxima [F]

$$\int \cos^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin\left(fx+e\right)^2+a}\cos\left(fx+e\right)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*cos(f*x + e)^2, x)
```

3.330.8 Giac [F]

$$\int \cos^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin\left(fx+e\right)^2+a}\cos\left(fx+e\right)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*cos(f*x + e)^2, x)
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \cos\left(e+f\,x\right)^2\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.331 $\int \sqrt{a+b\sin^2(e+fx)}\,dx$

3.331.1 Optimal result

Integrand size = 16, antiderivative size = 51

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

output
```
(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*si
n(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)
```

3.331.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \frac{a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)])/(f*S
qrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.331.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin^2(e+fx)}{a}+1}dx}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin(e+fx)^2}{a}+1}dx}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

input
```
Int[Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin
[e + f*x]^2)/a])
```

3.331.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.331.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

method	result	size
default	$\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,E\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)}{\cos(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}\,f}$	71

input
```
int((a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.331.5 Fricas [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin\left(fx+e\right)^2+a}\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-b*cos(f*x + e)^2 + a + b), x)
```

3.331.6 Sympy [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2{(e+fx)}}\,dx$$

input
```
integrate((a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2), x)
```

3.331.7 Maxima [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

3.331.8 Giac [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \begin{cases} \frac{\sqrt{a}\,\mathrm{E}\left(e+f\,x\middle|-\frac{b}{a}\right)}{f} & \text{if } 0<a \\ \int \sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx & \text{if } \neg 0<a \end{cases}$$

input
```
int((a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
piecewise(0 < a, (a^(1/2)*ellipticE(e + f*x, -b/a))/f, ~0 < a, int((a + b*
sin(e + f*x)^2)^(1/2), x))
```

3.332 $\int \sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.332.1 Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{a\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f}$$

output
```
-(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*s
in(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+a*(cos(f*x+e)^2)^(1/2)/cos
(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a
+b*sin(f*x+e)^2)^(1/2)+(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

3.332.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02

$$\int \sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{-2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right) + 2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right) + \sqrt{2}(2a+b-b\,\text{co}}{2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] +
2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sq
rt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(2*f*Sqrt[2*a + b - b*C
os[2*(e + f*x)]])
```

3.332.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.33, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3671, 314, 27, 389, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin(e+fx)^2}}{\cos(e+fx)^2}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\left(1-\sin^2(e+fx)\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{314}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}-\int \frac{b\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)\right)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}-b\int \frac{\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)\right)}{f}$$

$$\Big\downarrow \text{389}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}-b\left(\frac{\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d}{b}\right.\right.}{f}$$

$\Big\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}-b\left(\frac{\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{}{\sqrt{1-\sin^2(e+fx)}}}{b\sqrt{a+b\sin^2(}}\right.\right.}{f}$$

$\Big\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}-b\left(\frac{\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\Big(\arcsin(}{b\sqrt{a+b\sin^2(e+fx)}}\right.\right.}{f}$$

$\Big\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}-b\left(\frac{\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+}}{}\right.\right.}{f}$$

$\Big\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}-b\left(\frac{\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{El}}{b\sqrt{a}}\right.\right.}{f}$$

input
```
Int[Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^
2])/Sqrt[1 - Sin[e + f*x]^2] - b*((EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]
*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (a*Ellip
ticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt
[a + b*Sin[e + f*x]^2]))))/f
```

3.332. $\int \sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.332.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.332.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.69

method	result
default	$\frac{\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\left(a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)-\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{\cdot}\right.}{\sqrt{-(a+b(\sin^2(fx+e)))(\sin(fx+e)-1)(1+\sin(fx+e))}\,\cos}$

input
```
int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a*(cos(f*x+e)^2)^(1/2)*(-b/a*c
os(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))-(cos(f*x+e
)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*EllipticE(sin(f*x+e),(-1/a*
b)^(1/2))-cos(f*x+e)^2*sin(f*x+e)*b+a*sin(f*x+e)+b*sin(f*x+e))/(-(a+b*sin(
f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^
2)^(1/2)/f
```

3.332.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 640, normalized size of antiderivative = 4.89

$$\int \sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx =$$

$$-\frac{-4i\sqrt{-b}b\sqrt{\frac{2b\sqrt{\frac{a^2+ab}{b^2}}+2a+b}{b}}\sqrt{\frac{a^2+ab}{b^2}}\cos(fx+e)\,F\left(\arcsin\left(\sqrt{\frac{2b\sqrt{\frac{a^2+ab}{b^2}}+2a+b}{b}}(\cos(fx+e)+i\sin(fx+\right.\right.}{}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(-4*I*sqrt(-b)*b*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*sqrt((
a^2 + a*b)/b^2)*cos(f*x + e)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/
b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2
- 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 4*I*sqrt(-b)*b*sqrt((2*b*s
qrt((a^2 + a*b)/b^2) + 2*a + b)/b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)*elli
ptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e)
- I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b
)/b^2))/b^2) + (2*I*sqrt(-b)*b*sqrt((a^2 + a*b)/b^2)*cos(f*x + e) + (2*I*a
 + I*b)*sqrt(-b)*cos(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/
b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*
x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^
2 + a*b)/b^2))/b^2) + (-2*I*sqrt(-b)*b*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)
+ (-2*I*a - I*b)*sqrt(-b)*cos(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) +
2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b
)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)
*sqrt((a^2 + a*b)/b^2))/b^2) - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x
 + e))/(b*f*cos(f*x + e))
```

3.332.6 Sympy [F]

$$\int \sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2{(e+fx)}}\sec^2{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*sec(e + f*x)**2, x)
```

3.332.7 Maxima [F]

$$\int \sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin(fx+e)^2+a}\,\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^2, x)
```

3.332.8 Giac [F]

$$\int \sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin\left(fx+e\right)^2+a}\sec\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^2, x)
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \frac{\sqrt{b\sin\left(e+f\,x\right)^2+a}}{\cos\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^2,x)
```

output
```
int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^2, x)
```

3.333 $\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.333.1 Optimal result

Integrand size = 25, antiderivative size = 196

$$\begin{aligned}\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx &= -\frac{(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\\ &\quad+\frac{2a\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3f\sqrt{a+b\sin^2(e+fx)}}\\ &\quad+\frac{(2a+b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)f}\\ &\quad+\frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3f}\end{aligned}$$

output
```
-1/3*(2*a+b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(
1/2))*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+2/3*a*(c
os(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(
f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*(2*a+b)*(a+b*sin(f*x+e)^2
)^(1/2)*tan(f*x+e)/(a+b)/f+1/3*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)*tan(f
*x+e)/f
```

3.333.2 Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.95

$$\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$= \frac{-2a(2a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+4a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\operatorname{EllipticF}\left(e+fx,-\frac{b}{a}\right)+\frac{(}{}}{6(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Sec[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-2*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x,
-(b/a)] + 4*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e +
 f*x, -(b/a)] + (((8*a^2 - 4*b^2)*Cos[2*(e + f*x)] + (2*a + b)*(8*a + 5*b
- b*Cos[4*(e + f*x)]))*Sec[e + f*x]^2*Tan[e + f*x])/(2*Sqrt[2]))/(6*(a + b
)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.333.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.24, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3671, 314, 25, 402, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a+b\sin(e+fx)^2}}{\cos(e+fx)^4}dx$$
$$\Big\downarrow \text{3671}$$
$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\left(1-\sin^2(e+fx)\right)^{5/2}}d\sin(e+fx)}{f}$$
$$\Big\downarrow \text{314}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{1}{3}\int -\frac{b\sin^2(e+fx)+2a}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)\right)}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\int \frac{b\sin^2(e+fx)+2a}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3\left(1-\sin^2(e+fx)\right)^{3/2}}\right)}{f}$$

$\downarrow$ 402

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{\int \frac{b\left(a-(2a+b)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a+b}+\frac{(2a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)\sqrt{1-\sin^2(e+fx)}}\right)+\frac{\sin(e+fx)}{3(1-\text{si}}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{b\int \frac{a-(2a+b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a+b}+\frac{(2a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)\sqrt{1-\sin^2(e+fx)}}\right)+\frac{\sin(e+fx}{3(1-}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{b\left(\frac{2a(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{(2a+b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)}{a+b}+\frac{(2a+}{}\right.\right.}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{b\left(\frac{2a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(}{b}\right.}{a+b}\right.\right.}{f}$$

3.333. $\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{b\left(\frac{2a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)}{a+b}+\frac{(\cdots}{}\right.\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{b\left(\frac{2a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\,s\cdots}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.}{a+b}\right.\right.}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{b\left(\frac{2a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\cdots\right.}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.}{a+b}\right.\right.}{f}$$

input
```
Int[Sec[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^
2])/(3*(1 - Sin[e + f*x]^2)^(3/2)) + (((2*a + b)*Sin[e + f*x]*Sqrt[a + b*S
in[e + f*x]^2])/((a + b)*Sqrt[1 - Sin[e + f*x]^2]) + (b*(-(((2*a + b)*Elli
pticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1
+ (b*Sin[e + f*x]^2)/a])) + (2*a*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], -
(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2])))/(a
 + b))/3))/f
```

3.333. $\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.333.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

3.333. $\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.333.4 Maple [A] (verified)

Time = 3.69 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.88

method	result
default	$\frac{\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\,b(2a+b)\left(\cos^4(fx+e)\right)\sin(fx+e)-2\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\,a(a+b)\left(\cos^2(f\right.}{}$

input
```
int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(2*a+b)*cos(f*x+e)^4*sin
(f*x+e)-2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(a+b)*cos(f*x+e)^2*
sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*
x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2
))*a+2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a-EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2-(-b*cos(
f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*x+e))/(-(a+b*sin(
f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(a+b)/(sin(f*x+e)-1)/(1+sin
(f*x+e))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.333.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 789, normalized size of antiderivative = 4.03

$$\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$= \frac{\left(2\left(-2i\,ab-i\,b^2\right)\sqrt{-b}\sqrt{\frac{a^2+ab}{b^2}}\cos\left(fx+e\right)^3-\left(4i\,a^2+4i\,ab+i\,b^2\right)\sqrt{-b}\cos\left(fx+e\right)^3\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}E}{}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*((2*(-2*I*a*b - I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)^3 -
 (4*I*a^2 + 4*I*a*b + I*b^2)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a^2
+ a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2
) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4
*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(2*I*a*b + I*b^2)*sqrt(-b)
*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)^3 - (-4*I*a^2 - 4*I*a*b - I*b^2)*sqrt(
-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic
_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*
sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^
2))/b^2) - 2*(2*(-I*a*b - I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x +
e)^3 + (-2*I*a^2 - I*a*b)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a^2 + a
*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) +
 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2
*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*(2*(I*a*b + I*b^2)*sqrt(-b)*sq
rt((a^2 + a*b)/b^2)*cos(f*x + e)^3 + (2*I*a^2 + I*a*b)*sqrt(-b)*cos(f*x +
e)^3)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt
((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e)))
, (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 2*(
(2*a*b + b^2)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*
sin(f*x + e))/((a*b + b^2)*f*cos(f*x + e)^3)
```

3.333.6 Sympy [F]

$$\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2\left(e+fx\right)}\sec^4\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*sec(e + f*x)**4, x)
```

3.333.7 Maxima [F]

$$\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin\left(fx+e\right)^2+a}\sec\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^4, x)
```

3.333.8 Giac [F]

$$\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin\left(fx+e\right)^2+a}\sec\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^4, x)
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \frac{\sqrt{b\sin\left(e+f\,x\right)^2+a}}{\cos\left(e+f\,x\right)^4}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^4,x)
```

output
```
int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^4, x)
```

3.334 $\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.334.1 Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{a^2(a+6b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16b^{3/2}f} + \frac{a(a+6b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{16bf} + \frac{(a+6b)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24bf} - \frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6bf}$$

output
```
1/16*a^2*(a+6*b)*arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/b^(3
/2)/f+1/24*(a+6*b)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/b/f-1/6*sin(f*x+e)*
(a+b*sin(f*x+e)^2)^(5/2)/b/f+1/16*a*(a+6*b)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^
(1/2)/b/f
```

3.334.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.95

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{a+b\sin^2(e+fx)}\left(3a^{3/2}(a+6b)\operatorname{arcsinh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a}}\right)+\sqrt{b}\sin(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}(-\right.}{48b^{3/2}f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

input
```
Integrate[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[a + b*Sin[e + f*x]^2]*(3*a^(3/2)*(a + 6*b)*ArcSinh[(Sqrt[b]*Sin[e +
f*x])/Sqrt[a]] + Sqrt[b]*Sin[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a]*(-3*a
*(a - 10*b) - 2*(7*a - 6*b)*b*Sin[e + f*x]^2 - 8*b^2*Sin[e + f*x]^4)))/(48
*b^(3/2)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
```

3.334.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 3669, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(e+fx)^3\left(a+b\sin(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \; 3669$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)\left(b\sin^2(e+fx)+a\right)^{3/2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{(a+6b)\int \left(b\sin^2(e+fx)+a\right)^{3/2}d\sin(e+fx)}{6b}-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6b}}{f}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{(a+6b)\left(\frac{3}{4}a\int \sqrt{b\sin^2(e+fx)+a}d\sin(e+fx)+\frac{1}{4}\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\right)}{6b}-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6b}}{f}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{(a+6b)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{1}{2}\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)+\frac{1}{4}\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\right)}{6b}-\frac{\sin(e+fx)\left(a+b\sin^2(e\right.}{6b}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{(a+6b)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{b\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}+\frac{1}{2}\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)+\frac{1}{4}\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\right)}{6b}-\frac{\sin(e+fx)(a+}{}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(a+6b)\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{b}}+\frac{1}{2}\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)+\frac{1}{4}\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\right)}{6b}-\frac{\sin(e+fx)(a+b\sin^2(e+}{6b}}{f}$$

input
```
Int[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-1/6*(Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(5/2))/b + ((a + 6*b)*((Sin[e +
 f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/4 + (3*a*((a*ArcTanh[(Sqrt[b]*Sin[e +
f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2*Sqrt[b]) + (Sin[e + f*x]*Sqrt[a + b*
Sin[e + f*x]^2])/2))/4))/(6*b))/f
```

3.334.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.334. $\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.334.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{\sin(fx+e)\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}{4f}+\frac{3a\sin(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}}{8f}+\frac{3a^2\ln\left(\sqrt{b}\,\sin(fx+e)+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{8f\sqrt{b}}$
default	$\frac{\sin(fx+e)\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}{4f}+\frac{3a\sin(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}}{8f}+\frac{3a^2\ln\left(\sqrt{b}\,\sin(fx+e)+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{8f\sqrt{b}}$

input
```
int(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/f+3/8*a*sin(f*x+e)*(a+b*sin(f*x+e)
^2)^(1/2)/f+3/8/f*a^2/b^(1/2)*ln(b^(1/2)*sin(f*x+e)+(a+b*sin(f*x+e)^2)^(1/
2))-1/6*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(5/2)/b/f+1/24/f/b*a*sin(f*x+e)*(a+b
*sin(f*x+e)^2)^(3/2)+1/16/f/b*a^2*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)+1/16
/f/b^(3/2)*a^3*ln(b^(1/2)*sin(f*x+e)+(a+b*sin(f*x+e)^2)^(1/2))
```

3.334.5 Fricas [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.68

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\,(a^3+6\,a^2b)\sqrt{b}\log\left(128\,b^4\cos(fx+e)^8-256\,(ab^3+2\,b^4)\cos(fx+e)^6+32\,(5\,a^2b^2+24\ldots\right.}{\ldots}\right.$$

$$-\frac{3\,(a^3+6\,a^2b)\sqrt{-b}\arctan\left(\frac{\left(8\,b^2\cos(fx+e)^4-8\,(ab+2\,b^2)\cos(fx+e)^2+a^2+8\,ab+8\,b^2\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-b}}{4\left(2\,b^3\cos(fx+e)^4+a^2b+3\,ab^2+2\,b^3-(3\,ab^2+4\,b^3)\cos(fx+e)^2\right)\sin(fx+e)}\right)+4\,(8\,b^3\,\mathrm{co}\ldots}{192\ldots}$$

input
```
integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/384*(3*(a^3 + 6*a^2*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3
+ 2*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^
4 + a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^
2*b^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*
(a*b^2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5
*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b
)*sqrt(b)*sin(f*x + e)) - 8*(8*b^3*cos(f*x + e)^4 + 3*a^2*b - 16*a*b^2 - 4
*b^3 - 2*(7*a*b^2 + 2*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)
*sin(f*x + e))/(b^2*f), -1/192*(3*(a^3 + 6*a^2*b)*sqrt(-b)*arctan(1/4*(8*b
^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b +
3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(8*
b^3*cos(f*x + e)^4 + 3*a^2*b - 16*a*b^2 - 4*b^3 - 2*(7*a*b^2 + 2*b^3)*cos(
f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/(b^2*f)]
```

3.334.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.334.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.11

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{\frac{3a^3\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}+\frac{18a^2\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{\sqrt{b}}+12\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}\sin(fx+e)+18\sqrt{b\sin}$$

input
```
integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/48*(3*a^3*arcsinh(b*sin(f*x + e)/sqrt(a*b))/b^(3/2) + 18*a^2*arcsinh(b*s
in(f*x + e)/sqrt(a*b))/sqrt(b) + 12*(b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x +
 e) + 18*sqrt(b*sin(f*x + e)^2 + a)*a*sin(f*x + e) - 8*(b*sin(f*x + e)^2 +
 a)^(5/2)*sin(f*x + e)/b + 2*(b*sin(f*x + e)^2 + a)^(3/2)*a*sin(f*x + e)/b
 + 3*sqrt(b*sin(f*x + e)^2 + a)*a^2*sin(f*x + e)/b)/f
```

3.334.8 Giac [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \cos\left(e+f\,x\right)^3\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

input
```
int(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.335 $\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.335.1 Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8\sqrt{b}f}+\frac{3a\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8f}+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f}$$

output
```
1/4*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/f+3/8*a^2*arctanh(sin(f*x+e)*b^(1/
2)/(a+b*sin(f*x+e)^2)^(1/2))/f/b^(1/2)+3/8*a*sin(f*x+e)*(a+b*sin(f*x+e)^2)
^(1/2)/f
```

3.335.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{\sqrt{a+b\sin^2(e+fx)}\left(5a\sin(e+fx)+2b\sin^3(e+fx)+\frac{3a^{3/2}\text{arcsinh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\right)}{8f}$$

input
```
Integrate[Cos[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[a + b*Sin[e + f*x]^2]*(5*a*Sin[e + f*x] + 2*b*Sin[e + f*x]^3 + (3*a^
(3/2)*ArcSinh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*Sin[e
+ f*x]^2)/a])))/(8*f)
```

3.335.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3669, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)\left(a+b\sin(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \left(b\sin^2(e+fx)+a\right)^{3/2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{3}{4}a\int \sqrt{b\sin^2(e+fx)+a}d\sin(e+fx)+\frac{1}{4}\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{f}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{1}{2}\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)+\frac{1}{4}\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{b\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}+\frac{1}{2}\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)+\frac{1}{4}\sin(e+fx)\left(a+b\sin^2(e+fx\right.}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3}{4}a\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{b}}+\frac{1}{2}\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)+\frac{1}{4}\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{f}$$

input
```
Int[Cos[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
((Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/4 + (3*a*((a*ArcTanh[(Sqrt[b]
*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2*Sqrt[b]) + (Sin[e + f*x]*Sq
rt[a + b*Sin[e + f*x]^2])/2))/4)/f
```

3.335.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.335.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\sin(fx+e)\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}{4f}+\frac{3a\sin(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}}{8f}+\frac{3a^2\ln\left(\sqrt{b}\,\sin(fx+e)+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{8f\sqrt{b}}$
default	$\frac{\sin(fx+e)\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}{4f}+\frac{3a\sin(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}}{8f}+\frac{3a^2\ln\left(\sqrt{b}\,\sin(fx+e)+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{8f\sqrt{b}}$

input
```
int(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/f+3/8*a*sin(f*x+e)*(a+b*sin(f*x+e)
^2)^(1/2)/f+3/8/f*a^2/b^(1/2)*ln(b^(1/2)*sin(f*x+e)+(a+b*sin(f*x+e)^2)^(1/
2))
```

3.335.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(88) = 176.

Time = 0.51 (sec) , antiderivative size = 503, normalized size of antiderivative = 4.84

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\,a^2\sqrt{b}\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+2\,b^4\right)\cos\left(fx+e\right)^6+32\left(5\,a^2b^2+24\,ab^3+24\right.\right.}{}\right.$$

$$-\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\left(8\,b^2\cos(fx+e)^4-8\left(ab+2\,b^2\right)\cos(fx+e)^2+a^2+8\,ab+8\,b^2\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-b}}{4\left(2\,b^3\cos(fx+e)^4+a^2b+3\,ab^2+2\,b^3-\left(3\,ab^2+4\,b^3\right)\cos(fx+e)^2\right)\sin(fx+e)}\right)+4\left(2\,b^2\cos\left(fx+e\right.\right.}{32\,bf}$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/64*(3*a^2*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(
f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*
a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*
b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^
3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a
*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin
(f*x + e)) - 8*(2*b^2*cos(f*x + e)^2 - 5*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)
^2 + a + b)*sin(f*x + e))/(b*f), -1/32*(3*a^2*sqrt(-b)*arctan(1/4*(8*b^2*c
os(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*
b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(2*b^2*
cos(f*x + e)^2 - 5*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x +
e))/(b*f)]
```

3.335.6 Sympy [F(-1)]

Timed out.

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.335.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\frac{3\,a^2 \operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{\sqrt{b}} + 2\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right) + 3\sqrt{b\sin\left(fx+e\right)^2+a}\,a\sin\left(fx+\right.}{8\,f}$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(3*a^2*arcsinh(b*sin(f*x + e)/sqrt(a*b))/sqrt(b) + 2*(b*sin(f*x + e)^2
 + a)^(3/2)*sin(f*x + e) + 3*sqrt(b*sin(f*x + e)^2 + a)*a*sin(f*x + e))/f
```

3.335.8 Giac [F(-1)]

Timed out.

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.335.9 Mupad [B] (verification not implemented)

Time = 14.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{\sin\left(e+f\,x\right)\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\sin(e+f\,x)^2}{a}\right)}{f\,\left(\frac{b\sin(e+f\,x)^2}{a}+1\right)^{3/2}}$$

input
```
int(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b
*sin(e + f*x)^2)/a))/(f*((b*sin(e + f*x)^2)/a + 1)^(3/2))
```

3.336 $\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.336.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=-\frac{\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f}+\frac{(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}-\frac{b\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}$$

output
```
(a+b)^(3/2)*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f-1/2
*(3*a+2*b)*arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))*b^(1/2)/f-
1/2*b*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.336.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.93

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{\sqrt{2}b(3a+2b)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a+b}\sin(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)+\sqrt{2}\left(4a^2+5ab+2b^2\right)\text{arctanh}\left(\frac{\sqrt{2a+2b}\sin(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx}\right.$$

input
```
Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*b*(3*a + 2*b)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Sin[e + f*x])/Sqrt[2*a
 + b - b*Cos[2*(e + f*x)]]] + Sqrt[2]*(4*a^2 + 5*a*b + 2*b^2)*ArcTanh[(Sqr
t[2*a + 2*b]*Sin[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]]] - 2*Sqrt[b]
*Sqrt[a + b]*(Sqrt[2]*(3*a + 2*b)*Log[Sqrt[2*a + b - b*Cos[2*(e + f*x)]] +
 Sqrt[2]*Sqrt[b]*Sin[e + f*x]] + Sqrt[b]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]
]*Sin[e + f*x]))/(4*Sqrt[2]*Sqrt[a + b]*f)
```

3.336.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3669, 318, 25, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^{3/2}}{\cos(e+fx)}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\left(b\sin^2(e+fx)+a\right)^{3/2}}{1-\sin^2(e+fx)}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{-\frac{1}{2}\int -\frac{b(3a+2b)\sin^2(e+fx)+a(2a+b)}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\frac{1}{2}b\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\int \frac{b(3a+2b)\sin^2(e+fx)+a(2a+b)}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\frac{1}{2}b\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{1}{2}\left(2(a+b)^2\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-b(3a+2b)\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)\right)-\frac{1}{2}b\sin(e+}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}\left(2(a+b)^2\int \frac{1}{(1-\sin^2(e+fx))\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-b(3a+2b)\int \frac{1}{1-\frac{b\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}\right)-\frac{1}{2}b\sin(e}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left(2(a+b)^2\int \frac{1}{(1-\sin^2(e+fx))\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)\right)-\frac{1}{2}b\sin(e+fx}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{1}{2}\left(2(a+b)^2\int \frac{1}{1-\frac{(a+b)\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}-\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)\right)-\frac{1}{2}b\sin(e+fx)\sqrt{a+b}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left(2(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)-\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)\right)-\frac{1}{2}b\sin(e+fx)\sqrt{a+b\sin^2(e}{f}$$

input
```
Int[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
((-(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e +
f*x]^2]]) + 2*(a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*
Sin[e + f*x]^2]])/2 - (b*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/2)/f
```

3.336.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.336.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(103) = 206$.

Time = 1.31 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.22

method	result
default	$\frac{-b^{\frac{3}{2}} \ln\left(\sqrt{b}\, \sin(fx+e)+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)-b^2\left(\frac{\sin(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}}{2b}-\frac{a \ln\left(\sqrt{b}\, \sin(fx+e)+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{2b^{\frac{3}{2}}}\right)-2a\sqrt{b}\, \ln}{}$

input
```
int(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-b^(3/2)*ln(b^(1/2)*sin(f*x+e)+(a+b*sin(f*x+e)^2)^(1/2))-b^2*(1/2*sin(f*x
+e)/b*(a+b*sin(f*x+e)^2)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*sin(f*x+e)+(a+b*si
n(f*x+e)^2)^(1/2)))-2*a*b^(1/2)*ln(b^(1/2)*sin(f*x+e)+(a+b*sin(f*x+e)^2)^(
1/2))+(-1/2*a^2-a*b-1/2*b^2)/(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+
e)^2)^(1/2)-2*b*sin(f*x+e)+2*a)/(1+sin(f*x+e)))+(1/2*a^2+a*b+1/2*b^2)/(a+b
)^(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(
sin(f*x+e)-1)))/f
```

3.336.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(103) = 206.

Time = 0.69 (sec) , antiderivative size = 1381, normalized size of antiderivative = 11.41

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*((3*a + 2*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4
)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4
 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 +
 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2
+ 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b
+ 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(
b)*sin(f*x + e)) + 4*(a + b)^(3/2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)
^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2
- 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*
a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) - 8*sqrt(-b*cos(f*x + e)^2 + a + b)*
b*sin(f*x + e))/f, -1/16*(8*(a + b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos
(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a
*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) - (3*a + 2*b)
*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 +
 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 160*
a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4
)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x +
 e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^
3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e)) +
 8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/f, 1/8*((3*a + 2*b)*...
```

3.336.6 Sympy [F]

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}\sec\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sin(e + f*x)**2)**(3/2)*sec(e + f*x), x)
```

3.336.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.39

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx =$$

$$-\frac{3\,a\sqrt{b}\,\mathrm{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)+2\,b^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)-(a+b)^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}(\sin(fx+e)+1)}-\frac{a}{\sqrt{ab}(\sin(fx+e)+1)}\right)-}{2\,f}$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(3*a*sqrt(b)*arcsinh(b*sin(f*x + e)/sqrt(a*b)) + 2*b^(3/2)*arcsinh(b*
sin(f*x + e)/sqrt(a*b)) - (a + b)^(3/2)*arcsinh(b*sin(f*x + e)/(sqrt(a*b)*
(sin(f*x + e) + 1)) - a/(sqrt(a*b)*(sin(f*x + e) + 1))) - (a + b)^(3/2)*ar
csinh(-b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f
*x + e) - 1))) + sqrt(b*sin(f*x + e)^2 + a)*b*sin(f*x + e))/f
```

3.336.8 Giac [F(-1)]

Timed out.

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\cos\left(e+f\,x\right)}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x),x)
```

output
```
int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x), x)
```

3.337 $\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.337.1 Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}+\frac{(a-2b)\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f}+\frac{(a+b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{2f}$$

output
```
b^(3/2)*arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f+1/2*(a-2*b)
*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))*(a+b)^(1/2)/f+1/
2*(a+b)*sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

3.337.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.65

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{-2b^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a+b}\sin(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)+2(a^2-ab-b^2)\,\text{arctanh}\left(\frac{\sqrt{2a+2b}\sin(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)+\sqrt{a+b}(\ldots}{\ldots}$$

input
```
Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-2*b^2*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Sin[e + f*x])/Sqrt[2*a + b - b*Cos[2*
(e + f*x)]]] + 2*(a^2 - a*b - b^2)*ArcTanh[(Sqrt[2*a + 2*b]*Sin[e + f*x])/
Sqrt[2*a + b - b*Cos[2*(e + f*x)]]] + Sqrt[a + b]*(4*b^(3/2)*Log[Sqrt[2*a
+ b - b*Cos[2*(e + f*x)]] + Sqrt[2]*Sqrt[b]*Sin[e + f*x]] + Sqrt[2]*(a + b
)*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Sec[e + f*x]*Tan[e + f*x]))/(4*Sqrt[a
 + b]*f)
```

3.337.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 3669, 315, 25, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^{3/2}}{\cos(e+fx)^3}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\left(b\sin^2(e+fx)+a\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{(a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2\left(1-\sin^2(e+fx)\right)}-\frac{1}{2}\int -\frac{a(a-b)-2b^2\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\int \frac{a(a-b)-2b^2\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{(a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2\left(1-\sin^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{1}{2}\left(2b^2\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+(a-2b)(a+b)\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)\right)+\frac{(a+b)\sin(e+f}{2(1-}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}\left(2b^2\int \frac{1}{1-\frac{b\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}+(a-2b)(a+b)\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)\right)+\frac{(a+b)\sin(e}{2(}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left((a-2b)(a+b)\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)\right)+\frac{(a+b)\sin(e+fx)\sqrt{}}{2(1-\sin^2}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{1}{2}\left((a-2b)(a+b)\int \frac{1}{1-\frac{(a+b)\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}+2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)\right)+\frac{(a+b)\sin(e+fx)\sqrt{a+b\sin^2(e}}{2\left(1-\sin^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left(2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)+(a-2b)\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)\right)+\frac{(a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2\left(1-\sin^2(e+fx)\right)}}{f}$$

input
```
Int[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
((2*b^(3/2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]] + (
a - 2*b)*Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e +
 f*x]^2]])/2 + ((a + b)*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*(1 - S
in[e + f*x]^2)))/f
```

3.337. $\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.337.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.337. $\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.337.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(109) = 218.

Time = 1.48 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.17

method	result
default	$-\left(-4b^{\frac{3}{2}}\ln\left(\sqrt{b}\,\sin(fx+e)+\sqrt{a+b-b(\cos^2(fx+e))}\right)(a+b)^{\frac{3}{2}}-\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}+2b\sin(fx+e)+2a}{\sin(fx+e)-1}\right)a^3+3\ln\left(\frac{2\sqrt{a+b}\,\sqrt{\ }}{}\right.\right.$

input
```
int(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-(-4*b^(3/2)*ln(b^(1/2)*sin(f*x+e)+(a+b-b*cos(f*x+e)^2)^(1/2))*(a+b)^
(3/2)-ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*
x+e)+a))*a^3+3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)
+b*sin(f*x+e)+a))*a*b^2+2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+
e)^2)^(1/2)+b*sin(f*x+e)+a))*b^3+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*c
os(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3-3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)
*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^2-2*ln(2/(1+sin(f*x+e))*(
(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^3)*cos(f*x+e)^2+
2*sin(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2))/(a+b)^(3/2)/cos(f*x+e
)^2/f
```

3.337.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(109) = 218.

Time = 0.71 (sec) , antiderivative size = 1471, normalized size of antiderivative = 11.58

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(b^(3/2)*cos(f*x + e)^2*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b
^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a
^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2
 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^
2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*
b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqr
t(b)*sin(f*x + e)) - sqrt(a + b)*(a - 2*b)*cos(f*x + e)^2*log(((a^2 + 8*a*
b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 + 4*((a
 + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a
 + b)*sin(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*sqrt(-b*c
os(f*x + e)^2 + a + b)*(a + b)*sin(f*x + e))/(f*cos(f*x + e)^2), -1/8*(2*(
a - 2*b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sq
rt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 -
a^2 - 2*a*b - b^2)*sin(f*x + e)))*cos(f*x + e)^2 - b^(3/2)*cos(f*x + e)^2*
log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^
2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*b^2 +
 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*cos(f*x
 + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^4 - a
^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*
x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e)) - 4*sqr...
```

3.337.6 Sympy [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.337.7 Maxima [F]

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)
```

3.337.8 Giac [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.337.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\cos\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^3,x)
```

output
```
int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^3, x)
```

3.338 $\int \sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.338.1 Optimal result

Integrand size = 25, antiderivative size = 122

$$\int \sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{3a^2\text{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8\sqrt{a+b}f}+\frac{3a\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{8f}+\frac{\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)}{4f}$$

output
```
3/8*a^2*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f/(a+b)^(
1/2)+1/4*sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)/f+3/8*a*sec(f*x+
e)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

3.338.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

$$\int \sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{a^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},\frac{(a+b)\sin^2(e+fx)}{a+b\sin^2(e+fx)}\right)\sin(e+fx)}{f\sqrt{a+b\sin^2(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(a^2*Hypergeometric2F1[1/2, 3, 3/2, ((a + b)*Sin[e + f*x]^2)/(a + b*Sin[e
+ f*x]^2)]*Sin[e + f*x])/(f*Sqrt[a + b*Sin[e + f*x]^2])
```

3.338.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3669, 292, 292, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^{3/2}}{\cos(e+fx)^5}dx$$

$$\Big\downarrow \text{ 3669}$$

$$\frac{\int \frac{\left(b\sin^2(e+fx)+a\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^3}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 292}$$

$$\frac{\frac{3}{4}a\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4\left(1-\sin^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \text{ 292}$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2\left(1-\sin^2(e+fx)\right)}\right)+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4\left(1-\sin^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{(a+b)\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2\left(1-\sin^2(e+fx)\right)}\right)+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4\left(1-\sin^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow 219$$

$$\frac{\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{a+b}}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2(1-\sin^2(e+fx))}\right)+\frac{\sin(e+fx)(a+b\sin^2(e+fx))^{3/2}}{4(1-\sin^2(e+fx))^2}}{f}$$

input
```
Int[Sec[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
((Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(4*(1 - Sin[e + f*x]^2)^2) +
(3*a*((a*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(
2*Sqrt[a + b]) + (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*(1 - Sin[e +
 f*x]^2))))/4)/f
```

3.338.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.338.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(106) = 212$.

Time = 1.39 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.33

method	result
default	$\frac{3a^2\left(\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}+2b\sin(fx+e)+2a}{\sin(fx+e)-1}\right)a^2+2\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}+2b\sin(fx+e)+2a}{\sin(fx+e)-1}\right)ab+\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+}\right.\right.}{}$

input
```
int(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(3*a^2*(ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b
*sin(f*x+e)+a))*a^2+2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2
)^(1/2)+b*sin(f*x+e)+a))*a*b+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f
*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^2-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2-2*ln(2/(1+sin(f*x+e))*((a+b)^(1
/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b-ln(2/(1+sin(f*x+e))*((
a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^2)*cos(f*x+e)^4+2
*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2)*(3*a-2*b)*cos(f*x+e)^2*sin(f*x+e)+
4*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(7/2)*sin(f*x+e))/(a+b)^(5/2)/cos(f*x+e
)^4/f
```

3.338.5 Fricas [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.39

$$\int \sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\sqrt{a+b}a^2\cos\left(fx+e\right)^4\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos(fx+e)^4-8\left(a^2+3\,ab+2\,b^2\right)\cos(fx+e)^2-4\left((a+2\,b)\cos(fx+e)^2\right.}{\cos(fx+e)^4}\right.}{}\right.$$

$$-\frac{3\,a^2\sqrt{-a-b}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-2\,a-2\,b\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a-b}}{2\left(\left(ab+b^2\right)\cos(fx+e)^2-a^2-2\,ab-b^2\right)\sin(fx+e)}\right)\cos\left(fx+e\right)^4-2\left(\left(3\,a^2+ab-2\,b^2\right.\right.}{16\,(a+b)f\cos\left(fx+e\right)^4}$$

input
```
integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(3*sqrt(a + b)*a^2*cos(f*x + e)^4*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x
 + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x +
e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e)
 + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*((3*a^2 + a*b - 2*b^2)*cos(
f*x + e)^2 + 2*a^2 + 4*a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*
x + e))/((a + b)*f*cos(f*x + e)^4), -1/16*(3*a^2*sqrt(-a - b)*arctan(1/2*(
(a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt
(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e)))*
cos(f*x + e)^4 - 2*((3*a^2 + a*b - 2*b^2)*cos(f*x + e)^2 + 2*a^2 + 4*a*b +
 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a + b)*f*cos(f*x +
 e)^4)]
```

3.338.6 Sympy [F(-1)]

Timed out.

$$\int \sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**5*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.338.7 Maxima [F]

$$\int \sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^5\,dx$$

input
```
integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

3.338.8 Giac [F(-1)]

Timed out.

$$\int \sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int \sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\cos\left(e+f\,x\right)^5}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^5,x)
```

output
```
int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^5, x)
```

3.339 $\int \sec^7(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx$

3.339.1 Optimal result

Integrand size = 25, antiderivative size = 195

$$\int \sec^7(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \frac{a^2(5a+6b)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16(a+b)^{3/2}f} + \frac{a(5a+6b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{16(a+b)f} + \frac{(5a+6b)\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)}{24(a+b)f} + \frac{\sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}\tan(e+fx)}{6(a+b)f}$$

output
```
1/16*a^2*(5*a+6*b)*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2)
)/(a+b)^(3/2)/f+1/24*(5*a+6*b)*sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2)*tan(f
*x+e)/(a+b)/f+1/6*sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(5/2)*tan(f*x+e)/(a+b)/f
+1/16*a*(5*a+6*b)*sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f
```

3.339.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 14.94 (sec) , antiderivative size = 938, normalized size of antiderivative = 4.81

$$\int \sec^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{a^2\sec^3(e+fx)\left(1+\frac{b\sin^2(e+fx)}{a}\right)^2\tan(e+fx)\left(45a\arcsin\left(\sqrt{-\frac{(a+b)\tan^2(e+fx)}{a}}\right)+30b\arcsin\right.}{}$$

input
```
Integrate[Sec[e + f*x]^7*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(a^2*Sec[e + f*x]^3*(1 + (b*Sin[e + f*x]^2)/a)^2*Tan[e + f*x]*(45*a*ArcSin
[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]] + 30*b*ArcSin[Sqrt[-(((a + b)*Tan[e
+ f*x]^2)/a)]]*Sin[e + f*x]^2 + 210*a*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e +
f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2) + 140*b*Sin[e + f*x]^2*S
qrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)
/a))^(3/2) - 120*a*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a
+ b)*Tan[e + f*x]^2)/a))^(5/2) + 256*a*Hypergeometric2F1[2, 5, 7/2, -(((a
+ b)*Tan[e + f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(
-(((a + b)*Tan[e + f*x]^2)/a))^(5/2) - 80*b*Sin[e + f*x]^2*Sqrt[(Sec[e + f
*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(5/2) + 2
56*b*Hypergeometric2F1[2, 5, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e + f
*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e +
f*x]^2)/a))^(5/2) - 512*a*Hypergeometric2F1[2, 5, 7/2, -(((a + b)*Tan[e +
f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Ta
n[e + f*x]^2)/a))^(7/2) - 512*b*Hypergeometric2F1[2, 5, 7/2, -(((a + b)*Ta
n[e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^
2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2) + 256*a*Hypergeometric2F1[2,
5, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e +
 f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(9/2) + 256*b*Hypergeometric2
F1[2, 5, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e...
```

3.339.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 3669, 296, 292, 292, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^{3/2}}{\cos(e+fx)^7}dx$$

$$\Big\downarrow \text{ 3669}$$

$$\frac{\int \frac{\left(b\sin^2(e+fx)+a\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^4}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 296}$$

$$\frac{\frac{(5a+6b)\int \frac{\left(b\sin^2(e+fx)+a\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^3}d\sin(e+fx)}{6(a+b)}+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6(a+b)\left(1-\sin^2(e+fx)\right)^3}}{f}$$

$$\Big\downarrow \text{ 292}$$

$$\frac{\frac{(5a+6b)\left(\frac{3}{4}a\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4\left(1-\sin^2(e+fx)\right)^2}\right)}{6(a+b)}+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6(a+b)\left(1-\sin^2(e+fx)\right)^3}}{f}$$

$$\Big\downarrow \text{ 292}$$

$$\frac{\frac{(5a+6b)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2\left(1-\sin^2(e+fx)\right)}\right)+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4\left(1-\sin^2(e+fx)\right)^2}\right)}{6(a+b)}+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6(a+b)\left(1-\sin^2(e+fx)\right)^3}}{f}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\frac{(5a+6b)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{(a+b)\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2\left(1-\sin^2(e+fx)\right)}\right)+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4\left(1-\sin^2(e+fx)\right)^2}\right)}{6(a+b)}+\frac{\sin(e+fx)(a+b\sin^2(}{6(a+b)(1-\sin^2(e}}{f}$$

$\downarrow$ 219

$$\frac{\frac{(5a+6b)\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{a+b}}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2\left(1-\sin^2(e+fx)\right)}\right)+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4\left(1-\sin^2(e+fx)\right)^2}\right)}{6(a+b)}+\frac{\sin(e+fx)(a+b\sin^2(e+fx))^{5/2}}{6(a+b)(1-\sin^2(e+fx))^3}}{f}$$

input
```
Int[Sec[e + f*x]^7*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
((Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(5/2))/(6*(a + b)*(1 - Sin[e + f*x]^
2)^3) + ((5*a + 6*b)*((Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(4*(1 -
Sin[e + f*x]^2)^2) + (3*a*((a*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a +
b*Sin[e + f*x]^2]])/(2*Sqrt[a + b]) + (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x
]^2])/(2*(1 - Sin[e + f*x]^2))))/4))/(6*(a + b)))/f
```

3.339.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.339.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(175) = 350$.

Time = 2.08 (sec) , antiderivative size = 693, normalized size of antiderivative = 3.55

method	result
default	$\frac{-3a^2\left(5\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}-2b\sin(fx+e)+2a}{1+\sin(fx+e)}\right)a^4+21\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}-2b\sin(fx+e)+2a}{1+\sin(fx+e)}\right)a^3b+33\ln\left(\frac{2\sqrt{}}{}\right.\right.}{}$

input
```
int(sec(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/96*(-3*a^2*(5*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2
)-b*sin(f*x+e)+a))*a^4+21*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+
e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b+33*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+
b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^2+23*ln(2/(1+sin(f*x+e))*((
a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^3+6*ln(2/(1+sin
(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^4-5*ln
(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))
*a^4-21*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(
f*x+e)+a))*a^3*b-33*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^
(1/2)+b*sin(f*x+e)+a))*a^2*b^2-23*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*
cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^3-6*ln(2/(sin(f*x+e)-1)*((a+b)^(1
/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^4)*cos(f*x+e)^6+2*(a+b)^
(7/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*(15*a^2+8*a*b-4*b^2)*cos(f*x+e)^4*sin(f*x
+e)+4*(a+b)^(7/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*(5*a^2+3*a*b-2*b^2)*cos(f*x+e
)^2*sin(f*x+e)+16*(a+b)^(7/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*s
in(f*x+e))/(a+b)^(9/2)/cos(f*x+e)^6/f
```

3.339.5 Fricas [A] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 545, normalized size of antiderivative = 2.79

$$\int \sec^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\left(5a^3+6a^2b\right)\sqrt{a+b}\cos\left(fx+e\right)^6\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos(fx+e)^4-8\left(a^2+3ab+2b^2\right)\cos(fx+e)^2-4\left((a+\right.}{}\right.}{}\right.$$

$$-\frac{3\left(5a^3+6a^2b\right)\sqrt{-a-b}\arctan\left(\frac{\left((a+2b)\cos(fx+e)^2-2a-2b\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a-b}}{2\left((ab+b^2)\cos(fx+e)^2-a^2-2ab-b^2\right)\sin(fx+e)}\right)\cos\left(fx+e\right)^6-2\left(\left(15a\right.\right.}{}$$

input
```
integrate(sec(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/192*(3*(5*a^3 + 6*a^2*b)*sqrt(a + b)*cos(f*x + e)^6*log(((a^2 + 8*a*b +
 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a +
2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a +
b)*sin(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*((15*a^3 + 2
3*a^2*b + 4*a*b^2 - 4*b^3)*cos(f*x + e)^4 + 8*a^3 + 24*a^2*b + 24*a*b^2 +
8*b^3 + 2*(5*a^3 + 8*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*
x + e)^2 + a + b)*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^6), -1
/96*(3*(5*a^3 + 6*a^2*b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2
 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*c
os(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e)))*cos(f*x + e)^6 - 2*((15*
a^3 + 23*a^2*b + 4*a*b^2 - 4*b^3)*cos(f*x + e)^4 + 8*a^3 + 24*a^2*b + 24*a
*b^2 + 8*b^3 + 2*(5*a^3 + 8*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e)^2)*sqrt(-b
*cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)
^6)]
```

3.339.6 Sympy [F(-1)]

Timed out.

$$\int \sec^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**7*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.339.7 Maxima [F]

$$\int \sec^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^7\,dx$$

input
```
integrate(sec(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^7, x)
```

3.339.8 Giac [F(-1)]

Timed out.

$$\int \sec^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int \sec^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\cos\left(e+f\,x\right)^7}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^7,x)
```

output
```
int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^7, x)
```

3.340 $\int \cos^4(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx$

3.340.1 Optimal result

Integrand size = 25, antiderivative size = 321

$$\int \cos^4(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx = -\frac{(a^2-9ab-2b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf} + \frac{2(4a+b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35f} - \frac{b\cos^5(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{7f} - \frac{2(a-b)(a^2+6ab+b^2)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35b^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{a(a+b)(2a^2+9ab-b^2)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{35b^2f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-1/35*(a^2-9*a*b-2*b^2)*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f
+2/35*(4*a+b)*cos(f*x+e)^3*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-1/7*b*cos
(f*x+e)^5*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-2/35*(a-b)*(a^2+6*a*b+b^2)
*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*s
in(f*x+e)^2)^(1/2)/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/35*a*(a+b)*(2*a^2+9*
a*b-b^2)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2
)*(1+b*sin(f*x+e)^2/a)^(1/2)/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.340.2 Mathematica [A] (verified)

Time = 3.14 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.77

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{-128a(a^3+5a^2b-5ab^2-b^3)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+64a(2a^3+11a^2b+8ab^2-\cdots}{\cdots}$$

input
```
Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-128*a*(a^3 + 5*a^2*b - 5*a*b^2 - b^3)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)]
)/a]*EllipticE[e + f*x, -(b/a)] + 64*a*(2*a^3 + 11*a^2*b + 8*a*b^2 - b^3)*
Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sqrt[2
]*b*(-32*a^3 + 400*a^2*b + 212*a*b^2 + 30*b^3 + b*(144*a^2 - 192*a*b - 37*
b^2)*Cos[2*(e + f*x)] + 2*b^2*(-26*a + b)*Cos[4*(e + f*x)] + 5*b^3*Cos[6*(
e + f*x)])*Sin[2*(e + f*x)])/(2240*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]
])
```

3.340.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3671, 318, 25, 403, 27, 403, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^4\left(a+b\sin(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \left(1-\sin^2(e+fx)\right)^{3/2}\left(b\sin^2(e+fx)+a\right)^{3/2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{1}{7}\int -\frac{(1-\sin^2(e+fx))^{3/2}(2b(4a+b)\sin^2(e+fx)+a(7a+b))}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\frac{1}{7}b\sin(e+fx)\,(1-\right.}{f}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{7}\int \frac{(1-\sin^2(e+fx))^{3/2}(2b(4a+b)\sin^2(e+fx)+a(7a+b))}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\frac{1}{7}b\sin(e+fx)\,(1-\sin\right.}{f}$$

$$\Big\downarrow \ 403$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{7}\left(\frac{\int \frac{3b\sqrt{1-\sin^2(e+fx)}\left(a(9a+b)-\left(a^2-9ba-2b^2\right)\sin^2(e+fx)\right)}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{5b}+\frac{2}{5}(4a+b)\sin(e+fx)\,(1-\right.\right.}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\int \frac{\sqrt{1-\sin^2(e+fx)}(a(9a+b)-(a^2-9ba-2b^2)\sin^2(e+fx))}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{2}{5}(4a+b)\sin(e+\right.\right.}{f}$$

$$\Big\downarrow \ 403$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\int \frac{a\left(a^2+18ba+b^2\right)-2(a-b)\left(a^2+6ba+b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3b}-\frac{(a^2-9ab-2b^2)\sin(e+fx)\sqrt{1-\sin}}{3b}\right.\right.\right.}{}$$

$$\Big\downarrow \ 399$$

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\frac{a(a+b)\left(2a^2+9ab-b^2\right)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{2(a-b)\left(a^2+6ab+b^2\right)\int \frac{\sqrt{b\sin^2(e+}}{\sqrt{1-\sin^2(e}}}{b}}{3b}\right.\right.\right.$$

$$\Big\downarrow \ 323$$

3.340. $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\frac{a(a+b)\left(2a^2+9ab-b^2\right)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\left(a^2+6ab+\right.}{}}{3b}\right.\right.\right.$$

$\downarrow$ 321

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\frac{a(a+b)\left(2a^2+9ab-b^2\right)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\left(a^2+6ab+b^2\right)\int\frac{\sqrt{b\sin^2(}}{\sqrt{1-\sin}}}{b}}{3b}\right.\right.\right.$$

$\downarrow$ 330

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\frac{a(a+b)\left(2a^2+9ab-b^2\right)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\left(a^2+6ab+b^2\right)\sqrt{a+b\sin^2(}}{b\sqrt{b\,\text{si}}}}{3b}\right.\right.\right.$$

$\downarrow$ 327

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\frac{a(a+b)\left(2a^2+9ab-b^2\right)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\left(a^2+6ab+b^2\right)\sqrt{a+b\sin^2(}}{b\sqrt{\frac{b\sin^2}{}}}}{3b}\right.\right.\right.$$

input

```
Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

3.340. $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-1/7*(b*Sin[e + f*x]*(1 - Sin[e + f*x]
^2)^(5/2)*Sqrt[a + b*Sin[e + f*x]^2]) + ((2*(4*a + b)*Sin[e + f*x]*(1 - Si
n[e + f*x]^2)^(3/2)*Sqrt[a + b*Sin[e + f*x]^2])/5 + (3*(-1/3*((a^2 - 9*a*b
 - 2*b^2)*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2]
)/b + ((-2*(a - b)*(a^2 + 6*a*b + b^2)*EllipticE[ArcSin[Sin[e + f*x]], -(b
/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(
a + b)*(2*a^2 + 9*a*b - b^2)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[
1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2]))/(3*b)))/5)/7))/
f
```

3.340.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.340. $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.340.4 Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.84

method	result
default	$\frac{5b^4(\cos^8(fx+e))\sin(fx+e)+(-13a\,b^3-7b^4)(\cos^6(fx+e))\sin(fx+e)+(9a^2b^2+a\,b^3)(\cos^4(fx+e))\sin(fx+e)+(-a^3b+8a^2b^2+11a\,b}{}$

input
```
int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(5*b^4*cos(f*x+e)^8*sin(f*x+e)+(-13*a*b^3-7*b^4)*cos(f*x+e)^6*sin(f*x
+e)+(9*a^2*b^2+a*b^3)*cos(f*x+e)^4*sin(f*x+e)+(-a^3*b+8*a^2*b^2+11*a*b^3+2
*b^4)*cos(f*x+e)^2*sin(f*x+e)+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a
+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^4+11*(cos(f*x+e)^2)^(1
/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))
*a^3*b+8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(
sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2
+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3-2*(cos(f*x+e)^2
)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1
/2))*a^4-10*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Ellipti
cE(sin(f*x+e),(-1/a*b)^(1/2))*a^3*b+10*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+
e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2+2*(cos(f*
x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a*b^3)/b^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.340.5 Fricas [F]

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(b*cos(f*x + e)^6 - (a + b)*cos(f*x + e)^4)*sqrt(-b*cos(f*x + e)
^2 + a + b), x)
```

3.340.6 Sympy [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.340.7 Maxima [F]

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)
```

3.340.8 Giac [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \cos\left(e+f\,x\right)^4\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}\,dx$$

input
```
int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.341 $\int \cos^2(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx$

3.341.1 Optimal result

Integrand size = 25, antiderivative size = 259

$$\int \cos^2(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \frac{2(3a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f} - \frac{b\cos^3(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{5f} - \frac{(3a^2-7ab-2b^2)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{a(3a-b)(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{15bf\sqrt{a+b\sin^2(e+fx)}}$$

output
```
2/15*(3*a+b)*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-1/5*b*cos(f*
x+e)^3*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-1/15*(3*a^2-7*a*b-2*b^2)*Elli
pticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*
x+e)^2)^(1/2)/b/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/15*a*(3*a-b)*(a+b)*Elliptic
F(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)
^2/a)^(1/2)/b/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.341.2 Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.77

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{-16a(3a^2-7ab-2b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+16a(3a^2+2ab-b^2)\sqrt{\frac{2a+b-b\cos}{a}}\cdots}{2\cdots}$$

input
```
Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-16*a*(3*a^2 - 7*a*b - 2*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Elli
pticE[e + f*x, -(b/a)] + 16*a*(3*a^2 + 2*a*b - b^2)*Sqrt[(2*a + b - b*Cos[
2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sqrt[2]*b*(48*a^2 + 28*a*b +
 5*b^2 - 4*b*(9*a + 2*b)*Cos[2*(e + f*x)] + 3*b^2*Cos[4*(e + f*x)])*Sin[2*
(e + f*x)])/(240*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.341.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3671, 318, 25, 403, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(e+fx)^2\left(a+b\sin(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \; 3671$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 318$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{1}{5}\int -\frac{\sqrt{1-\sin^2(e+fx)}(2b(3a+b)\sin^2(e+fx)+a(5a+b))}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\frac{1}{5}b\sin(e+fx)\left(1-\sin\right.\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{5}\int \frac{\sqrt{1-\sin^2(e+fx)}(2b(3a+b)\sin^2(e+fx)+a(5a+b))}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\frac{1}{5}b\sin(e+fx)\left(1-\sin^2(\right.\right.}{f}$$

$\downarrow$ 403

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{5}\left(\frac{\int \frac{b\left(a(9a+b)-\left(3a^2-7ba-2b^2\right)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3b}+\frac{2}{3}(3a+b)\sqrt{1-\sin^2(e+fx)}\sin(e+\right.\right.}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{a(9a+b)-(3a^2-7ba-2b^2)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{2}{3}(3a+b)\sqrt{1-\sin^2(e+fx)}\sin\right.\right.}{f}$$

$\downarrow$ 399

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{a(3a-b)(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{(3a^2-7ab-2b^2)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}}{b}\right.\right.\right.$$

$\downarrow$ 323

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{a(3a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(3a^2-7ab-2b^2)}{}\right.\right.\right.$$

$\downarrow$ 321

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{a(3a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(3a^2-7ab-2b^2)\int \frac{\sqrt{b\sin^2(e-}}{\sqrt{1-\sin^2(}}}{b}\right.\right.\right.$$

3.341. $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$

$\downarrow$ 330

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{a(3a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(3a^2-7ab-2b^2)\sqrt{a+b\sin^2(}}{b\sqrt{\frac{b}{}}}\right.\right.\right.$$

$\downarrow$ 327

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{a(3a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(3a^2-7ab-2b^2)\sqrt{a+b\sin^2(e}}{b\sqrt{\frac{b\sin}{}}}\right.\right.\right.$$

input
```
Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-1/5*(b*Sin[e + f*x]*(1 - Sin[e + f*x]
^2)^(3/2)*Sqrt[a + b*Sin[e + f*x]^2]) + ((2*(3*a + b)*Sin[e + f*x]*Sqrt[1
- Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/3 + (-(((3*a^2 - 7*a*b - 2*b
^2)*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b
*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) + (a*(3*a - b)*(a + b)*EllipticF[ArcSin[
Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e
 + f*x]^2]))/3)/5))/f
```

3.341.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.341. $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

3.341. $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.341.4 Maple [A] (verified)

Time = 5.09 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.66

method	result
default	$\frac{-3b^3\left(\sin^7(fx+e)\right)-9a\,b^2\left(\sin^5(fx+e)\right)+4b^3\left(\sin^5(fx+e)\right)+3\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^3+2a^2\sqrt{\cdots}}{}$

input
```
int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(-3*b^3*sin(f*x+e)^7-9*a*b^2*sin(f*x+e)^5+4*b^3*sin(f*x+e)^5+3*(cos(f
*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^
(1/2))*a^3+2*a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Ellipti
cF(sin(f*x+e),(-1/a*b)^(1/2))*b-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)
/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-3*(cos(f*x+e)^2)^(1/2)*
((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3+7*(c
os(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a^2*b+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Ellip
ticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2-6*a^2*b*sin(f*x+e)^3+10*a*b^2*sin(f*
x+e)^3-b^3*sin(f*x+e)^3+6*a^2*b*sin(f*x+e)-a*b^2*sin(f*x+e))/b/cos(f*x+e)/
(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.341.5 Fricas [F]

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(b*cos(f*x + e)^4 - (a + b)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)
^2 + a + b), x)
```

3.341.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.341.7 Maxima [F]

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

3.341.8 Giac [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.341.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \cos\left(e+f\,x\right)^2\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

input
```
int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.342 $\int (a+b\sin^2(e+fx))^{3/2}\,dx$

3.342.1 Optimal result

Integrand size = 16, antiderivative size = 154

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=-\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}-\frac{a(a+b)\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-1/3*b*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+2/3*(2*a+b)*(cos(f
*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+
e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*a*(a+b)*(cos(f*x+e)^2)^(1/2)/
cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/f
/(a+b*sin(f*x+e)^2)^(1/2)
```

3.342.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01

$$\int (a + b\sin^2(e + fx))^{3/2} \, dx = \frac{4\sqrt{2}a(2a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right) - 2\sqrt{2}a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,\text{EllipticF}}{6\sqrt{2}f\sqrt{2a+b-b\cos(2(e+fx)}}$$

input
```
Integrate[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(4*Sqrt[2]*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e
+ f*x, -(b/a)] - 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a
]*EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e
+ f*x)])/(6*Sqrt[2]*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.342.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {3042, 3659, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\sin^2(e + fx))^{3/2} \, dx$$

$$\Big\downarrow \ 3042$$

$$\int (a + b\sin(e + fx)^2)^{3/2} \, dx$$

$$\Big\downarrow \ 3659$$

$$\frac{1}{3}\int \frac{2b(2a+b)\sin^2(e+fx) + a(3a+b)}{\sqrt{b\sin^2(e+fx)+a}}dx - \frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\int \frac{2b(2a+b)\sin(e+fx)^2 + a(3a+b)}{\sqrt{b\sin(e+fx)^2+a}}dx - \frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \ 3651$$

$$\frac{1}{3}\left(2(2a+b)\int \sqrt{b\sin^2(e+fx)+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}dx\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\left(2(2a+b)\int \sqrt{b\sin(e+fx)^2+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \ 3657$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin^2(e+fx)}{a}+1}dx}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin(e+fx)^2}{a}+1}dx}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \ 3656$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \ 3662$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}dx}{\sqrt{a+b\sin^2(e+fx)}}\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \ 3042$$

3.342. $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin(e+fx)^2}{a}+1}}dx}{\sqrt{a+b\sin^2(e+fx)}}\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3661}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

input
```
Int[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-1/3*(b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f + ((2*(2*a
 + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (
b*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*
Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2]))/3
```

3.342.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.342.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.73

method	result
default	$\frac{-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2}{3}-\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)b}{3}+\frac{4\sqrt{\frac{\cos(2f}{2}}}{}}{}$

input
```
int((a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/3*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+
e),(-1/a*b)^(1/2))*a^2-1/3*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(
1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b+4/3*(cos(f*x+e)^2)^(1/2)*((a+b
*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2/3*(cos(
f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)
^(1/2))*a*b+1/3*b^2*sin(f*x+e)^5+1/3*a*b*sin(f*x+e)^3-1/3*b^2*sin(f*x+e)^3
-1/3*a*b*sin(f*x+e))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.342.5 Fricas [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^(3/2), x)
```

3.342.6 Sympy [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sin(e + f*x)**2)**(3/2), x)
```

3.342.7 Maxima [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.342.8 Giac [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.342.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int((a + b*sin(e + f*x)^2)^(3/2), x)
```

3.343 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.343.1 Optimal result

Integrand size = 25, antiderivative size = 182

$$\begin{aligned}&\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \\ &-\frac{(a+2b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} \\ &+\frac{a(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}} \\ &+\frac{(a+b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f}\end{aligned}$$

output
```
-(a+2*b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2
)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+a*(a+b)*EllipticF(
sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2
/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)+(a+b)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*
x+e)/f
```

3.343.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.79

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{-2a(a+2b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+(a+b)\left(2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\operatorname{EllipticF}\left(e\right.\right.}{2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-2*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x,
-(b/a)] + (a + b)*(2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e
+ f*x, -(b/a)] + Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x]))/(2*
f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.343.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 3671, 315, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^{3/2}}{\cos(e+fx)^2}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\left(b\sin^2(e+fx)+a\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}-\int\frac{b((a+2b)\sin^2(e+fx)+a)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)\right)}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}-b\int\frac{(a+2b)\sin^2(e+fx)+a}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)\right)}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}-b\left(\frac{(a+2b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a(a+b)\int\frac{}{\sqrt{1-\sin^2(e+f}}}{}\right.\right.}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}-b\left(\frac{(a+2b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}}}{}\right.\right.}{f}$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}-b\left(\frac{(a+2b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}}}{b\sqrt{}}\right.\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}-b\left(\frac{(a+2b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a}{}\right.\right.}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}-b\left(\frac{(a+2b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)}{}\right.\right.}{f}$$

3.343. $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

input
```
Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(((a + b)*Sin[e + f*x]*Sqrt[a + b*Sin[e
 + f*x]^2])/Sqrt[1 - Sin[e + f*x]^2] - b*(((a + 2*b)*EllipticE[ArcSin[Sin[
e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e + f*x]
^2)/a]) - (a*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*S
in[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2]))))/f
```

3.343.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.343.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(170) = 340$.

Time = 3.87 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.01

method	result
default	$\frac{\sqrt{-b\left(\cos^4(fx+e)\right)+(a+b)\left(\cos^2(fx+e)\right)}\left(\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2+ab\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\right.}{}$

input
```
int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*((cos(f*x+e)^2)^(1/2)*(-b/a*cos
(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+a*b*(cos
(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-
1/a*b)^(1/2))-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Ellip
ticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e
)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b-b*cos(f*x+e)^2
*sin(f*x+e)*a-b^2*cos(f*x+e)^2*sin(f*x+e)+sin(f*x+e)*a^2+2*a*b*sin(f*x+e)+
b^2*sin(f*x+e))/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/
cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.343.5 Fricas [F]

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sec(f
*x + e)^2, x)
```

3.343.6 Sympy [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.343.7 Maxima [F]

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^2, x)
```

3.343.8 Giac [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\cos\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^2,x)
```

output
```
int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^2, x)
```

3.344 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.344.1 Optimal result

Integrand size = 25, antiderivative size = 236

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{2(a-b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{a(2a-b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3f\sqrt{a+b\sin^2(e+fx)}} + \frac{2(a-b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3f} + \frac{(a+b)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3f}$$

output
```
-2/3*(a-b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1
/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*a*(2*a-b)*El
lipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(
f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)+2/3*(a-b)*(a+b*sin(f*x+e)^2)^
(1/2)*tan(f*x+e)/f+1/3*(a+b)*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x
+e)/f
```

3.344.2 Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.81

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{-4a(a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+2a(2a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,\text{EllipticF}\left(e+\right.}{6f\sqrt{2a+b-b\cos(2}}$$

input
```
Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-4*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(
b/a)] + 2*a*(2*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e +
 f*x, -(b/a)] + ((8*a^2 + 3*a*b + b^2 + (4*a^2 - 6*a*b - 2*b^2)*Cos[2*(e +
 f*x)] + b*(-a + b)*Cos[4*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/Sqrt[2]
)/(6*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.344.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3671, 315, 25, 402, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^{3/2}}{\cos(e+fx)^4}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\left(b\sin^2(e+fx)+a\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{1}{3}\int -\frac{(a-2b)b\sin^2(e+fx)+a(2a-b)}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)\right)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\int \frac{(a-2b)b\sin^2(e+fx)+a(2a-b)}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{(a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3\left(1-\sin^2(e+fx)\right)^{3/2}}\right)}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{\int \frac{b\left(a(a+b)-2\left(a^2-b^2\right)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a+b}+\frac{2(a-b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}\right)+\frac{(a+b)\sin(\cdots}{3(1\cdots}\right.}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{b\int \frac{a(a+b)-2\left(a^2-b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a+b}+\frac{2(a-b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}\right)+\frac{(a+b)\sin\cdots}{3(\cdots}\right.}{f}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{b\left(\frac{a(2a-b)(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{2\left(a^2-b^2\right)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)}{a+b}\cdots\right.\right.}{f}$$

$$\Big\downarrow \text{323}$$

3.344. $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{b\left(\frac{a(2a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2\left(a^2-b^2\right)\int\frac{\sqrt{b\sin^2(e+fx}}{\sqrt{1-\sin^2(e+}}}{b}\right.}{a+b}\right.\right.}{f}$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{b\left(\frac{a(2a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2\left(a^2-b^2\right)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right.}{a+b}\right.\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{b\left(\frac{a(2a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2\left(a^2-b^2\right)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+}{a}}}{\sqrt{1-\sin^2(e}}}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.}{a+b}\right.\right.}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{b\left(\frac{a(2a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2\left(a^2-b^2\right)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e\right.}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.}{a+b}\right.\right.}{f}$$

input
```
Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

3.344. $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(((a + b)*Sin[e + f*x]*Sqrt[a + b*Sin[e
 + f*x]^2])/(3*(1 - Sin[e + f*x]^2)^(3/2)) + ((2*(a - b)*Sin[e + f*x]*Sqrt
[a + b*Sin[e + f*x]^2])/Sqrt[1 - Sin[e + f*x]^2] + (b*((-2*(a^2 - b^2)*Ell
ipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1
 + (b*Sin[e + f*x]^2)/a]) + (a*(2*a - b)*(a + b)*EllipticF[ArcSin[Sin[e +
f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^
2])))/(a + b))/3))/f
```

3.344.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.344.4 Maple [A] (verified)

Time = 3.73 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.59

method	result
default	$\frac{2\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\,b(a-b)(\cos^4(fx+e))\sin(fx+e)-\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\,(2a^2-ab-3b^2)\,(\cdots}{\cdots}$

input
```
int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(a-b)*cos(f*x+e)^4*sin
(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*a^2-a*b-3*b^2)*cos(f
*x+e)^2*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*
(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(2*EllipticF(sin(f*x+e),(-1/a
*b)^(1/2))*a-EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*EllipticE(sin(f*x+e)
,(-1/a*b)^(1/2))*a+2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2-
(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*x+e))/(-(
a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/(1+s
in(f*x+e))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.344.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 784, normalized size of antiderivative = 3.32

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\left(2\left(-i\,ab+i\,b^2\right)\sqrt{-b}\sqrt{\frac{a^2+ab}{b^2}}\cos\left(fx+e\right)^3-\left(2i\,a^2-i\,ab-i\,b^2\right)\sqrt{-b}\cos\left(fx+e\right)^3\right)\sqrt{\frac{2\,b\sqrt{a^2 \cdots}}{\cdots}}}{\cdots}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*((2*(-I*a*b + I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)^3 - (
2*I*a^2 - I*a*b - I*b^2)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a^2 + a*
b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) +
2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*
a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(I*a*b - I*b^2)*sqrt(-b)*sqrt(
(a^2 + a*b)/b^2)*cos(f*x + e)^3 - (-2*I*a^2 + I*a*b + I*b^2)*sqrt(-b)*cos(
f*x + e)^3)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsi
n(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x
+ e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2)
 + (2*(I*a*b - 2*I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)^3 - (-
2*I*a^2 - I*a*b)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a^2 + a*b)/b^2)
+ 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)
/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^
2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-I*a*b + 2*I*b^2)*sqrt(-b)*sqrt((a^2
+ a*b)/b^2)*cos(f*x + e)^3 - (2*I*a^2 + I*a*b)*sqrt(-b)*cos(f*x + e)^3)*sq
rt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sq
rt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2
 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(a*b - b
^2)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x +
e))/(b*f*cos(f*x + e)^3)
```

3.344.6 Sympy [F(-1)]

Timed out.

$$\int \sec^4(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.344.7 Maxima [F]

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^4, x)
```

3.344.8 Giac [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\cos\left(e+f\,x\right)^4}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^4,x)
```

output
```
int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^4, x)
```

3.345 $\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.345.1 Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2b^{3/2}f} - \frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2bf}$$

output
```
1/2*(a+2*b)*arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/b^(3/2)/f
-1/2*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f
```

3.345.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{-\frac{(-a-2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2b^{3/2}} - \frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2b}}{f}$$

input
```
Integrate[Cos[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-1/2*((-a - 2*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2
]])/b^(3/2) - (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*b))/f
```

3.345.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3669, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(e+fx)^3}{\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \; 3669$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{(a+2b)\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{2b}-\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2b}}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{(a+2b)\int \frac{1}{1-\frac{b\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}}{2b}-\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2b}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2b^{3/2}}-\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2b}}{f}$$

input
```
Int[Cos[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(((a + 2*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2
*b^(3/2)) - (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*b))/f
```

3.345.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.345.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.18

method	result	size
derivativedivides	$\frac{\frac{\ln\left(\sqrt{b}\,\sin(fx+e)+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{\sqrt{b}}-\frac{\sin(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}}{2b}+\frac{a\ln\left(\sqrt{b}\,\sin(fx+e)+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{2b^{\frac{3}{2}}}}{f}$	93
default	$\frac{\frac{\ln\left(\sqrt{b}\,\sin(fx+e)+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{\sqrt{b}}-\frac{\sin(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}}{2b}+\frac{a\ln\left(\sqrt{b}\,\sin(fx+e)+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{2b^{\frac{3}{2}}}}{f}$	93

input
```
int(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(ln(b^(1/2)*sin(f*x+e)+(a+b*sin(f*x+e)^2)^(1/2))/b^(1/2)-1/2*sin(f*x+e
)/b*(a+b*sin(f*x+e)^2)^(1/2)+1/2*a/b^(3/2)*ln(b^(1/2)*sin(f*x+e)+(a+b*sin(
f*x+e)^2)^(1/2)))
```

3.345.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(67) = 134$.

Time = 0.39 (sec) , antiderivative size = 461, normalized size of antiderivative = 5.84

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \left[\frac{(a+2\,b)\sqrt{b}\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+2\,b^4\right)\cos\left(fx+e\right)^6+32\left(5\,a^2b^2+24\,ab^3+24\,b^4\right)\cos\left(\right.\right.}{}\right.$$

$$-\frac{(a+2\,b)\sqrt{-b}\arctan\left(\frac{\left(8\,b^2\cos(fx+e)^4-8\left(ab+2\,b^2\right)\cos(fx+e)^2+a^2+8\,ab+8\,b^2\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-b}}{4\left(2\,b^3\cos(fx+e)^4+a^2b+3\,ab^2+2\,b^3-\left(3\,ab^2+4\,b^3\right)\cos(fx+e)^2\right)\sin(fx+e)}\right)+4\sqrt{-b\cos}}{8\,b^2f}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((a + 2*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*
cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 +
 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 2
4*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 +
2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b +
24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)
*sin(f*x + e)) - 8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/(b^2*f)
, -1/8*((a + 2*b)*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b
^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*
sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b
^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*sqrt(-b*cos(f*x + e)^2 + a + b)*b*s
in(f*x + e))/(b^2*f)]
```

3.345.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.345.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.87

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\frac{a\,\mathrm{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} + \frac{2\,\mathrm{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{\sqrt{b\sin(fx+e)^2+a}\,\sin(fx+e)}{b}}{2\,f}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(a*arcsinh(b*sin(f*x + e)/sqrt(a*b))/b^(3/2) + 2*arcsinh(b*sin(f*x + e
)/sqrt(a*b))/sqrt(b) - sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)/b)/f
```

3.345.8 Giac [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cos(fx+e)^3}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.345.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cos\left(e+f\,x\right)^3}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(cos(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.346 $\int \frac{\cos(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.346.1 Optimal result

Integrand size = 23, antiderivative size = 38

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{b}f}$$

output
```
arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f/b^(1/2)
```

3.346.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{b}f}$$

input
```
Integrate[Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(Sqrt[b]*f)
```

3.346.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3669, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \ 3669$$

$$\frac{\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \ 224$$

$$\frac{\int \frac{1}{1-\frac{b\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}}{f}$$

$$\Big\downarrow \ 219$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{b}f}$$

input
```
Int[Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(Sqrt[b]*f)
```

3.346.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.346.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\ln\left(\sqrt{b}\,\sin(fx+e)+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{f\sqrt{b}}$	34
default	$\frac{\ln\left(\sqrt{b}\,\sin(fx+e)+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{f\sqrt{b}}$	34

input
```
int(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*ln(b^(1/2)*sin(f*x+e)+(a+b*sin(f*x+e)^2)^(1/2))/b^(1/2)
```

3.346.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(32) = 64.

Time = 0.36 (sec) , antiderivative size = 394, normalized size of antiderivative = 10.37

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \left[\frac{\log\left(128\,b^4\cos\left(fx+e\right)^8 - 256\left(ab^3+2\,b^4\right)\cos\left(fx+e\right)^6 + 32\left(5\,a^2b^2+24\,ab^3+24\,b^4\right)\cos\left(fx+e\right)^4 + a\right.}{}\right.$$

$$\left. - \frac{\sqrt{-b}\arctan\left(\frac{\left(8\,b^2\cos(fx+e)^4 - 8\,(ab+2\,b^2)\cos(fx+e)^2 + a^2 + 8\,ab + 8\,b^2\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-b}}{4\left(2\,b^3\cos(fx+e)^4 + a^2b + 3\,ab^2 + 2\,b^3 - (3\,ab^2+4\,b^3)\cos(fx+e)^2\right)\sin(fx+e)}\right)}{4\,bf}\right]$$

input
```
integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*
(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*
b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*co
s(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^
4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*c
os(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e))/(sqrt
(b)*f), -1/4*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*c
os(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(
-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*c
os(f*x + e)^2)*sin(f*x + e)))/(b*f)]
```

3.346.6 Sympy [F]

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cos\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cos(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

3.346.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.55

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx=\frac{\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{\sqrt{b}f}$$

input
```
integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(b*sin(f*x + e)/sqrt(a*b))/(sqrt(b)*f)
```

3.346.8 Giac [F]

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx=\int \frac{\cos(fx+e)}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.346.9 Mupad [B] (verification not implemented)

Time = 13.62 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx=\frac{\ln\left(\sqrt{b}\,\sin(e+f\,x)+\sqrt{b\sin(e+f\,x)^2+a}\right)}{\sqrt{b}\,f}$$

input
```
int(cos(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
log(b^(1/2)*sin(e + f*x) + (a + b*sin(e + f*x)^2)^(1/2))/(b^(1/2)*f)
```

3.347 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.347.1 Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{a+b}f}$$

output
```
arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f/(a+b)^(1/2)
```

3.347.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{a+b}f}$$

input
```
Integrate[Sec[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(Sqrt[a + b
]*f)
```

3.347.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3669, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\cos(e+fx)\sqrt{a+b\sin(e+fx)^2}}dx$$
$$\Big\downarrow \text{3669}$$
$$\frac{\int \frac{1}{(1-\sin^2(e+fx))\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{f}$$
$$\Big\downarrow \text{291}$$
$$\frac{\int \frac{1}{1-\frac{(a+b)\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}}{f}$$
$$\Big\downarrow \text{219}$$
$$\frac{\text{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f\sqrt{a+b}}$$

input
```
Int[Sec[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(Sqrt[a + b
]*f)
```

3.347.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.347.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(36) = 72$.

Time = 1.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.50

method	result	size
default	$\frac{\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}+2b\sin(fx+e)+2a}{\sin(fx+e)-1}\right)-\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}-2b\sin(fx+e)+2a}{1+\sin(fx+e)}\right)}{2\sqrt{a+b}\,f}$	105

input
```
int(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x
+e)+a))-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(
f*x+e)+a)))/(a+b)^(1/2)/f
```

3.347.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(36) = 72$.

Time = 0.36 (sec) , antiderivative size = 240, normalized size of antiderivative = 5.71

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$= \left[\frac{\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-8\,(a^2+3\,ab+2\,b^2)\cos(fx+e)^2-4\left((a+2\,b)\cos(fx+e)^2-2\,a-2\,b\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a+b}\sin(fx+e)}{\cos(fx+e)^4}\right.}{4\sqrt{a+b}f}\right.$$
$$\left. -\frac{\sqrt{-a-b}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-2\,a-2\,b\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a-b}}{2\left((ab+b^2)\cos(fx+e)^2-a^2-2\,ab-b^2\right)\sin(fx+e)}\right)}{2\,(a+b)f}\right]$$

input
```
integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*c
os(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x +
 e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x
+ e)^4)/(sqrt(a + b)*f), -1/2*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x +
 e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b
^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e)))/((a + b)*f)]
```

3.347.6 Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sec\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sec(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

3.347.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(36) = 72.

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.50

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\frac{\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}(\sin(fx+e)+1)}-\frac{a}{\sqrt{ab}(\sin(fx+e)+1)}\right)}{\sqrt{a+b}}+\frac{\operatorname{arsinh}\left(-\frac{b\sin(fx+e)}{\sqrt{ab}(\sin(fx+e)-1)}-\frac{a}{\sqrt{ab}(\sin(fx+e)-1)}\right)}{\sqrt{a+b}}}{2f}$$

input
```
integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a/(sqrt(a*b)*
(sin(f*x + e) + 1)))/sqrt(a + b) + arcsinh(-b*sin(f*x + e)/(sqrt(a*b)*(sin
(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1)))/sqrt(a + b))/f
```

3.347.8 Giac [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.347.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)\,\sqrt{b\sin(e+f\,x)^2+a}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

3.348 $\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.348.1 Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2(a+b)^{3/2}f} + \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{2(a+b)f}$$

output
```
1/2*(a+2*b)*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/(a+b)
^(3/2)/f+1/2*sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f
```

3.348.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.14 (sec) , antiderivative size = 408, normalized size of antiderivative = 4.48

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$= \frac{\sec^3(e+fx)\left(1+\frac{b\sin^2(e+fx)}{a}\right)\tan(e+fx)\left(45a\arcsin\left(\sqrt{-\frac{(a+b)\tan^2(e+fx)}{a}}\right)+30b\arcsin\left(\sqrt{-\frac{(a+b)\tan^2(e}{a}}\right.\right.}{}$$

input
```
Integrate[Sec[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sec[e + f*x]^3*(1 + (b*Sin[e + f*x]^2)/a)*Tan[e + f*x]*(45*a*ArcSin[Sqrt[
-(((a + b)*Tan[e + f*x]^2)/a)]] + 30*b*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]
^2)/a)]]*Sin[e + f*x]^2 + 16*a*Hypergeometric2F1[2, 3, 7/2, -(((a + b)*Tan
[e + f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a +
b)*Tan[e + f*x]^2)/a))^(5/2) + 16*b*Hypergeometric2F1[2, 3, 7/2, -(((a + b
)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f
*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(5/2) - 45*a*Sqrt[-(((a + b)*Se
c[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)] - 30*b*Sin[e + f
*x]^2*Sqrt[-(((a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2
)/a^2)]))/(30*a*f*Sqrt[a + b*Sin[e + f*x]^2]*Sqrt[(Sec[e + f*x]^2*(a + b*S
in[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2))
```

3.348.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3669, 296, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(e+fx)^3\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)^2\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{296}$$

$$\frac{\frac{(a+2b)\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{2(a+b)}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)\left(1-\sin^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{(a+2b)\int \frac{1}{1-\frac{(a+b)\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}}{2(a+b)}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)\left(1-\sin^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2(a+b)^{3/2}}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)\left(1-\sin^2(e+fx)\right)}}{f}$$

input
```
Int[Sec[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(((a + 2*b)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]
)/(2*(a + b)^(3/2)) + (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*(a + b)
*(1 - Sin[e + f*x]^2)))/f
```

3.348.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.348. $\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.348.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(79) = 158$.

Time = 1.05 (sec) , antiderivative size = 360, normalized size of antiderivative = 3.96

method	result
default	$-\left(\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^{2}(fx+e)\right)}-2b\sin(fx+e)+2a}{1+\sin(fx+e)}\right)a^{2}+3\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^{2}(fx+e)\right)}-2b\sin(fx+e)+2a}{1+\sin(fx+e)}\right)ab+2\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+}}{}\right.\right.$

input
```
int(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-(ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f
*x+e)+a))*a^2+3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2
)-b*sin(f*x+e)+a))*a*b+2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e
)^2)^(1/2)-b*sin(f*x+e)+a))*b^2-ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*co
s(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2-3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*
(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b-2*ln(2/(sin(f*x+e)-1)*((a+
b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^2)*cos(f*x+e)^2+2*s
in(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2))/(a+b)^(5/2)/cos(f*x+e)^2
/f
```

3.348.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(79) = 158.

Time = 0.43 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.97

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$= \left[\frac{(a+2b)\sqrt{a+b}\cos(fx+e)^2\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-8(a^2+3ab+2b^2)\cos(fx+e)^2-4\left((a+2b)\cos(fx+e)^2-2a-2b\right)}{\cos(fx+e)^4}\right.}{8(a^2+2ab+b^2)f\,\mathrm{c}}\right.$$
$$-\frac{(a+2b)\sqrt{-a-b}\arctan\left(\frac{\left((a+2b)\cos(fx+e)^2-2a-2b\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a-b}}{2\left((ab+b^2)\cos(fx+e)^2-a^2-2ab-b^2\right)\sin(fx+e)}\right)\cos(fx+e)^2-2\sqrt{-b\cos}}{4(a^2+2ab+b^2)f\cos(fx+e)^2}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((a + 2*b)*sqrt(a + b)*cos(f*x + e)^2*log(((a^2 + 8*a*b + 8*b^2)*cos(
f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x
 + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x +
 e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*sqrt(-b*cos(f*x + e)^2 +
 a + b)*(a + b)*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2), -1/4
*((a + 2*b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)
*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2
 - a^2 - 2*a*b - b^2)*sin(f*x + e)))*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x +
e)^2 + a + b)*(a + b)*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2)
]
```

3.348.6 Sympy [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sec(e + f*x)**3/sqrt(a + b*sin(e + f*x)**2), x)
```

3.348.7 Maxima [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)^3}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^3/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.348.8 Giac [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)^3}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.348.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)^3\,\sqrt{b\sin(e+f\,x)^2+a}}\,dx$$

input
```
int(1/(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

3.349 $\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.349.1 Optimal result

Integrand size = 25, antiderivative size = 168

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{2(a+2b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3b^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{(a+b)(2a+3b)\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3b^2f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-1/3*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f-2/3*(a+2*b)*(cos(f
*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+
e)^2)^(1/2)/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*(a+b)*(2*a+3*b)*(cos(f*x+
e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^
2/a)^(1/2)/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.349.2 Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.01

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{-4\sqrt{2}a(a+2b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+2\sqrt{2}(2a^2+5ab+3b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,\text{EllipticF}\,(}{6\sqrt{2}b^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Cos[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-4*Sqrt[2]*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e
 + f*x, -(b/a)] + 2*Sqrt[2]*(2*a^2 + 5*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[
2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f
*x)])*Sin[2*(e + f*x)])/(6*Sqrt[2]*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]
])
```

3.349.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3671, 318, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(e+fx)^4}{\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \; 3671$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\left(1-\sin^2(e+fx)\right)^{3/2}}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 318$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int \frac{-2(a+2b)\sin^2(e+fx)+a+3b}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3b}-\frac{\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}{3b}\right)}{f}$$

$$\big\downarrow \; 399$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{(a+b)(2a+3b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{2(a+2b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}}{3b}-\sin(e+f\right.}{f}$$

$$\big\downarrow \; 323$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{(a+b)(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e\right.}{b}}{3b}}{f}$$

$$\big\downarrow \; 321$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{(a+b)(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}}{3b}-\sin(e\right.}{f}$$

$$\big\downarrow \; 330$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{(a+b)(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}\int \frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin\right.}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{3b}}{f}$$

$$\big\downarrow \; 327$$

3.349. $\displaystyle\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{(a+b)(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}\,E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right.}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{3b}\right.}{f}$$

input
```
Int[Cos[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-1/3*(Sin[e + f*x]*Sqrt[1 - Sin[e + f*
x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/b + ((-2*(a + 2*b)*EllipticE[ArcSin[Sin[
e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e + f*x]
^2)/a]) + ((a + b)*(2*a + 3*b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqr
t[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2]))/(3*b)))/f
```

3.349.3.1 Defintions of rubi rules used

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.349. $\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.349.4 Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.88

method	result
default	$\frac{b^2\left(\sin^5(fx+e)\right)+2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2+5a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(f\right.}{}$

input
```
int(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b^2*sin(f*x+e)^5+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*
EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+5*a*(cos(f*x+e)^2)^(1/2)*((a+b*si
n(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b+3*(cos(f*x+e)^
2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))
*b^2-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))*a^2-4*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2
)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b+a*b*sin(f*x+e)^3-b^2*sin(f*x+e)
^3-a*b*sin(f*x+e))/b^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.349.5 Fricas [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cos{(fx+e)}^4}{\sqrt{b\sin{(fx+e)}^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^4/(b*cos(f*x + e)^2
 - a - b), x)
```

3.349.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.349.7 Maxima [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx=\int \frac{\cos{(fx+e)}^4}{\sqrt{b\sin{(fx+e)}^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.349.8 Giac [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx=\int \frac{\cos{(fx+e)}^4}{\sqrt{b\sin{(fx+e)}^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.349.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx=\int \frac{\cos{(e+f\,x)}^4}{\sqrt{b\sin{(e+f\,x)}^2+a}}\,dx$$

input
```
int(cos(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.350 $\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.350.1 Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{aE\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{bf\sqrt{a+b\sin^2(e+fx)}} + \frac{(a+b)\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{bf\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-a*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(1+b
*sin(f*x+e)^2/a)^(1/2)/b/f/(a+b*sin(f*x+e)^2)^(1/2)+(a+b)*(cos(f*x+e)^2)^(
1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1
/2)/b/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.350.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\left(-aE\left(e+fx\middle|-\frac{b}{a}\right)+(a+b)\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\right)}{bf\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Cos[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*(-(a*EllipticE[e + f*x, -(b/a)]) +
 (a + b)*EllipticF[e + f*x, -(b/a)]))/(b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x
)]])
```

3.350.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3671, 326, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(e+fx)^2}{\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \; 3671$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sqrt{1-\sin^2(e+fx)}}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 326$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)}{f}$$

$$\Big\downarrow \; 323$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)}{f}$$

$$\Big\downarrow \; 321$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)}{f}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.}{f}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.}{f}$$

input
```
Int[Cos[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-((EllipticE[ArcSin[Sin[e + f*x]], -(b
/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) + ((a
 + b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/
a])/(b*Sqrt[a + b*Sin[e + f*x]^2])))/f
```

3.350.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

3.350. $\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.350.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\left(F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a+F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)b-E\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a\right)}{b\cos(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}\,f}$	111

input
```
int(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*(EllipticF(sin(f*x+e),(-
1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-EllipticE(sin(f*x+e
),(-1/a*b)^(1/2))*a)/b/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.350.5 Fricas [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^2/(b*cos(f*x + e)^2
 - a - b), x)
```

3.350.6 Sympy [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cos^2\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

input
```
integrate(cos(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cos(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

3.350.7 Maxima [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.350.8 Giac [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cos(fx+e)^2}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.350.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cos(e+f\,x)^2}{\sqrt{b\sin(e+f\,x)^2+a}}\,dx$$

input
```
int(cos(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.351 $\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.351.1 Optimal result

Integrand size = 16, antiderivative size = 51

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx=\frac{\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*si
n(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.351.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx=\frac{\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)}{f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(f*Sqr
t[2*a + b - b*Cos[2*(e + f*x)]])
```

3.351.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{ 3662}$$

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}dx}{\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin(e+fx)^2}{a}+1}}dx}{\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{ 3661}$$

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

input
```
Int[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*
Sin[e + f*x]^2])
```

3.351.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.351.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.47 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,\mathrm{am}^{-1}\left(fx+e\middle|\frac{i\sqrt{b}}{\sqrt{a}}\right)}{f\sqrt{a+b\left(\sin^2(fx+e)\right)}}$	52

input
```
int(1/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(a+b*sin(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*InverseJacobiAM(
f*x+e,I/a^(1/2)*b^(1/2))
```

3.351.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 305, normalized size of antiderivative = 5.98

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx =$$

$$-\frac{\left(2i\sqrt{-b}b\sqrt{\frac{a^2+ab}{b^2}}+(-2i\,a-i\,b)\sqrt{-b}\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}F(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}(\cos{(fx+e)}+i\,\mathrm{s}\right.}{}$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-((2*I*sqrt(-b)*b*sqrt((a^2 + a*b)/b^2) + (-2*I*a - I*b)*sqrt(-b))*sqrt((2
*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a
^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*
a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (-2*I*sqrt(-b)*b
*sqrt((a^2 + a*b)/b^2) + (2*I*a + I*b)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b
)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2
*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a
*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/(b^2*f)
```

3.351.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\sin^2{(e+fx)}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sin(e + f*x)**2), x)
```

3.351.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.351.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.351.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(1/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(1/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.352 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.352.1 Optimal result

Integrand size = 25, antiderivative size = 140

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{(a+b)f}$$

output
```
-(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*s
in(f*x+e)^2)^(1/2)/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+(cos(f*x+e)^2)^(1/2)
/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/
f/(a+b*sin(f*x+e)^2)^(1/2)+(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f
```

3.352.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$= \frac{-2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+2(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)+\sqrt{2}(2a+b}{2(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Sec[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] +
2*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)
] + Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(2*(a + b)*f*Sqrt
[2*a + b - b*Cos[2*(e + f*x)]])
```

3.352.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.32, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 3671, 316, 27, 326, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\cos(e+fx)^2\sqrt{a+b\sin(e+fx)^2}}dx$$
$$\Big\downarrow \; 3671$$
$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{1}{(1-\sin^2(e+fx))^{3/2}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{f}$$
$$\Big\downarrow \; 316$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int \frac{b\sqrt{1-\sin^2(e+fx)}}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a+b}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)\sqrt{1-\sin^2(e+fx)}}\right)}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\int \frac{\sqrt{1-\sin^2(e+fx)}}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a+b}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)\sqrt{1-\sin^2(e+fx)}}\right)}{f}$$

$\downarrow$ 326

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)}{a+b}+\frac{\sin(e+fx)\sqrt{a+b\cdots}}{(a+b)\sqrt{1-\cdots}}\cdots}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)}{a+b}+\cdots}{f}$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)}{a+b}+\frac{\sin(e+fx)\sqrt{a\cdots}}{(a+b)\sqrt{1-\cdots}}\cdots}{f}$$

$\downarrow$ 330

3.352. $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right)}{a+b} -\right.}{f}$$

$\Big\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right)}{a+b}+\frac{\sin}{}\right.}{f}$$

input
```
Int[Sec[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^
2])/((a + b)*Sqrt[1 - Sin[e + f*x]^2]) + (b*(-((EllipticE[ArcSin[Sin[e + f
*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e + f*x]^2)/a
])) + ((a + b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e +
 f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2])))/(a + b)))/f
```

3.352.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

3.352. $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.352. $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.352.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.99

method	result
default	$\frac{\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\left(a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)+b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{\right.}{(a+b)\sqrt{-(a+b(\sin^2(fx+e)}}$

input
```
int(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a*(cos(f*x+e)^2)^(1/2)*(-b/a*c
os(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))+b*(cos(f*x
+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*
b)^(1/2))-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*Ellipti
cE(sin(f*x+e),(-1/a*b)^(1/2))-cos(f*x+e)^2*sin(f*x+e)*b+a*sin(f*x+e)+b*sin
(f*x+e))/(a+b)/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/c
os(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.352.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 632, normalized size of antiderivative = 4.51

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx =$$

$$-\frac{2\,(-2i\,a-i\,b)\sqrt{-b}\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}\cos\left(fx+e\right)F(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}(\cos\left(fx+e\right)+i\,\sin\left(fx+\right.\right.}{}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(2*(-2*I*a - I*b)*sqrt(-b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)
/b)*cos(f*x + e)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a +
 b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b +
 b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 2*(2*I*a + I*b)*sqrt(-b)*sqrt((2*b*sqr
t((a^2 + a*b)/b^2) + 2*a + b)/b)*cos(f*x + e)*elliptic_f(arcsin(sqrt((2*b*
sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a
^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*I*sqrt
(-b)*b*sqrt((a^2 + a*b)/b^2)*cos(f*x + e) + (2*I*a + I*b)*sqrt(-b)*cos(f*x
 + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqr
t((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))
), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (-
2*I*sqrt(-b)*b*sqrt((a^2 + a*b)/b^2)*cos(f*x + e) + (-2*I*a - I*b)*sqrt(-b
)*cos(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(a
rcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(
f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/
b^2) - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/((a*b + b^2)*f*co
s(f*x + e))
```

3.352.6 Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sec(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

3.352.7 Maxima [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)^2}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.352.8 Giac [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)^2}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.352.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)^2\,\sqrt{b\sin(e+f\,x)^2+a}}\,dx$$

input
```
int(1/(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

3.353 $\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.353.1 Optimal result

Integrand size = 25, antiderivative size = 212

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{2(a+2b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{(2a+3b)\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)f}$$

output
```
-2/3*(a+2*b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(
1/2))*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*(2
*a+3*b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))
*(1+b*sin(f*x+e)^2/a)^(1/2)/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+2/3*(a+2*b)*(
a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)^2/f+1/3*sec(f*x+e)^2*(a+b*sin(f*x
+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f
```

3.353.2 Mathematica [A] (verified)

Time = 2.57 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.97

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$= \frac{-4a(a+2b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+2(2a^2+5ab+3b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,\text{EllipticF}\left(e+fx\right.}{6(a+b)^2f\sqrt{2a+b-b\cos(2(e-}}$$

input
```
Integrate[Sec[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-4*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x,
-(b/a)] + 2*(2*a^2 + 5*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]
*EllipticF[e + f*x, -(b/a)] + ((8*a^2 + 15*a*b + 4*b^2 + (4*a^2 + 6*a*b -
2*b^2)*Cos[2*(e + f*x)] - b*(a + 2*b)*Cos[4*(e + f*x)])*Sec[e + f*x]^2*Tan
[e + f*x])/Sqrt[2])/(6*(a + b)^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.353.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3671, 316, 402, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\cos(e+fx)^4\sqrt{a+b\sin(e+fx)^2}}dx$$
$$\Big\downarrow \text{3671}$$
$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{1}{(1-\sin^2(e+fx))^{5/2}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{f}$$
$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int \frac{b\sin^2(e+fx)+2a+3b}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3(a+b)}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}}\right)}{f}$$

$\downarrow$ 402

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{\int \frac{b\left(-2(a+2b)\sin^2(e+fx)+a+3b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a+b}+\frac{2(a+2b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)\sqrt{1-\sin^2(e+fx)}}}{3(a+b)}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}}\right)}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{b\int \frac{-2(a+2b)\sin^2(e+fx)+a+3b}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a+b}+\frac{2(a+2b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)\sqrt{1-\sin^2(e+fx)}}}{3(a+b)}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}}\right)}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{b\left(\frac{(a+b)(2a+3b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{2(a+2b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)}{a+b}+\frac{2(a+2b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)\sqrt{1-\sin^2(e+fx)}}}{3(a+b)}+\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}}\right)}{f}$$

$\downarrow$ 323

3.353. $\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{b\left(\frac{(a+b)(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin\cdots}{b}\cdots}{a+b}\cdots}{3(a+b)}\cdots\right.}{f}$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{b\left(\frac{(a+b)(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}\right)}{a+b}+\frac{2(\cdots}{\cdots}}{3(a+b)}\cdots\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{b\left(\frac{(a+b)(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\cdots}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\cdots}{a+b}\cdots}{3(a+b)}\cdots\right.}{f}$$

$\downarrow$ 327

3.353. $\displaystyle\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{b\left(\frac{(a+b)(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\right|}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{a+b}}{3(a+b)}}{f}$$

input
```
Int[Sec[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^
2])/(3*(a + b)*(1 - Sin[e + f*x]^2)^(3/2)) + ((2*(a + 2*b)*Sin[e + f*x]*Sq
rt[a + b*Sin[e + f*x]^2])/((a + b)*Sqrt[1 - Sin[e + f*x]^2]) + (b*((-2*(a
+ 2*b)*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])
/(b*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((a + b)*(2*a + 3*b)*EllipticF[ArcSi
n[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin
[e + f*x]^2])))/(a + b))/(3*(a + b))))/f
```

3.353.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.353. $\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.353. $\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.353.4 Maple [A] (verified)

Time = 3.89 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.91

method	result
default	$\frac{2\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\, b(a+2b)(\cos^4(fx+e))\sin(fx+e)-\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\,(2a^2+5ab+3b^2}{}$

input
```
int(sec(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(a+2*b)*cos(f*x+e)^4*s
in(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*a^2+5*a*b+3*b^2)*c
os(f*x+e)^2*sin(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(-b/a*co
s(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*(2*EllipticF(sin(f*x+e),(-1
/a*b)^(1/2))*a^2+5*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b+3*EllipticF(si
n(f*x+e),(-1/a*b)^(1/2))*b^2-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2-4*
EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b)*cos(f*x+e)^2-(-b*cos(f*x+e)^4+(a
+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*x+e))/(1+sin(f*x+e))/(sin(f*
x+e)-1)/(a+b)^2/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/
cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.353.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 808, normalized size of antiderivative = 3.81

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$= \frac{\left(2\left(-i\,ab-2i\,b^2\right)\sqrt{-b}\sqrt{\frac{a^2+ab}{b^2}}\cos\left(fx+e\right)^3-\left(2i\,a^2+5i\,ab+2i\,b^2\right)\sqrt{-b}\cos\left(fx+e\right)^3\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}}{}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*((2*(-I*a*b - 2*I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)^3 -
 (2*I*a^2 + 5*I*a*b + 2*I*b^2)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a^
2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b
^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 -
 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(I*a*b + 2*I*b^2)*sqrt(-
b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)^3 - (-2*I*a^2 - 5*I*a*b - 2*I*b^2)*s
qrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elli
ptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e)
- I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b
)/b^2))/b^2) + (2*(I*a*b + I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x +
 e)^3 - (-2*I*a^2 - 7*I*a*b - 3*I*b^2)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*
sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2
+ a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b
 + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-I*a*b - I*b^2)
*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)^3 - (2*I*a^2 + 7*I*a*b + 3*I*
b^2)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b
)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x
 + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2
 + a*b)/b^2))/b^2) + (2*(a*b + 2*b^2)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-b*
cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a^2*b + 2*a*b^2 + b^3)*f*cos(f*...
```

3.353.6 Sympy [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sec(e + f*x)**4/sqrt(a + b*sin(e + f*x)**2), x)
```

3.353.7 Maxima [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx=\int \frac{\sec\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.353.8 Giac [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx=\int \frac{\sec\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.353.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx=\int \frac{1}{\cos\left(e+f\,x\right)^4\,\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(1/(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

3.354 $\int \frac{\cos^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.354.1 Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{\cos^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{3/2}f} + \frac{(a+b)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/b^(3/2)/f+(a+b)*sin(
f*x+e)/a/b/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.354.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.17

$$\int \frac{\cos^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}(a+b)\sin(e+fx) - a^{3/2}\text{arcsinh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a}}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{ab^{3/2}f\sqrt{a+b\sin^2(e+fx)}}$$

input
```
Integrate[Cos[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[b]*(a + b)*Sin[e + f*x] - a^(3/2)*ArcSinh[(Sqrt[b]*Sin[e + f*x])/Sqr
t[a]]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(a*b^(3/2)*f*Sqrt[a + b*Sin[e + f*x]
^2])
```

3.354.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3669, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(e+fx)^3}{\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{(a+b)\sin(e+fx)}{ab\sqrt{a+b\sin^2(e+fx)}}-\frac{\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{(a+b)\sin(e+fx)}{ab\sqrt{a+b\sin^2(e+fx)}}-\frac{\int \frac{1}{1-\frac{b\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}}{b}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(a+b)\sin(e+fx)}{ab\sqrt{a+b\sin^2(e+fx)}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{3/2}}}{f}$$

input
```
Int[Cos[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-(ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/b^(3/2)) + (
(a + b)*Sin[e + f*x])/(a*b*Sqrt[a + b*Sin[e + f*x]^2]))/f
```

3.354. $\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$

3.354.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.354.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.13

method	result	size
derivativedivides	$\frac{\frac{\sin(fx+e)}{a\sqrt{a+b\left(\sin^2(fx+e)\right)}}+\frac{\sin(fx+e)}{b\sqrt{a+b\left(\sin^2(fx+e)\right)}}-\frac{\ln\left(\sqrt{b}\,\sin(fx+e)+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{b^{\frac{3}{2}}}}{f}$	85
default	$\frac{\frac{\sin(fx+e)}{a\sqrt{a+b\left(\sin^2(fx+e)\right)}}+\frac{\sin(fx+e)}{b\sqrt{a+b\left(\sin^2(fx+e)\right)}}-\frac{\ln\left(\sqrt{b}\,\sin(fx+e)+\sqrt{a+b\left(\sin^2(fx+e)\right)}\right)}{b^{\frac{3}{2}}}}{f}$	85

input
```
int(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(sin(f*x+e)/a/(a+b*sin(f*x+e)^2)^(1/2)+sin(f*x+e)/b/(a+b*sin(f*x+e)^2)
^(1/2)-1/b^(3/2)*ln(b^(1/2)*sin(f*x+e)+(a+b*sin(f*x+e)^2)^(1/2)))
```

3.354.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(67) = 134$.

Time = 0.42 (sec) , antiderivative size = 559, normalized size of antiderivative = 7.45

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \left[\frac{\left(ab\cos\left(fx+e\right)^2-a^2-ab\right)\sqrt{b}\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+2\,b^4\right)\right.}{}\right.$$

input
```
integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*((a*b*cos(f*x + e)^2 - a^2 - a*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8
- 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*
cos(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(
a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x
 + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 -
16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x +
 e)^2 + a + b)*sqrt(b)*sin(f*x + e)) - 8*sqrt(-b*cos(f*x + e)^2 + a + b)*(
a*b + b^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^2 - (a^2*b^2 + a*b^3)*f), 1
/4*((a*b*cos(f*x + e)^2 - a^2 - a*b)*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*x +
e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f
*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b
^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) - 4*sqrt(-b*cos(f*x
+ e)^2 + a + b)*(a*b + b^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^2 - (a^2*b
^2 + a*b^3)*f)]
```

3.354.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.354.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{\cos^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = -\frac{\frac{\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}a} - \frac{\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}b}}{f}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-(arcsinh(b*sin(f*x + e)/sqrt(a*b))/b^(3/2) - sin(f*x + e)/(sqrt(b*sin(f*x
 + e)^2 + a)*a) - sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*b))/f
```

3.354.8 Giac [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^3}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^3/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.354.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{{\cos\left(e+f\,x\right)}^3}{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

input
```
int(cos(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.355 $\int \frac{\cos(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.355.1 Optimal result

Integrand size = 23, antiderivative size = 29

$$\int \frac{\cos(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}$$

output
```
sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.355.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cos(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}$$

input
```
Integrate[Cos[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
Sin[e + f*x]/(a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

3.355.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3669, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(e+fx)}{\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{1}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}$$

input
```
Int[Cos[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
Sin[e + f*x]/(a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

3.355.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.355.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{\sin(fx+e)}{af\sqrt{a+b(\sin^2(fx+e))}}$	28
default	$\frac{\sin(fx+e)}{af\sqrt{a+b(\sin^2(fx+e))}}$	28

input
```
int(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.355.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{\cos(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = -\frac{\sqrt{-b\cos{(fx+e)}^2+a+b}\sin{(fx+e)}}{abf\cos{(fx+e)}^2-(a^2+ab)f}$$

input
```
integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e)/(a*b*f*cos(f*x + e)^2 - (a^2
 + a*b)*f)
```

3.355.6 Sympy [F]

$$\int \frac{\cos(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cos{(e+fx)}}{\left(a+b\sin^2{(e+fx)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cos(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.355.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\cos(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{\sin{(fx+e)}}{\sqrt{b\sin{(fx+e)}^2+a}\,af}$$

input
```
integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a*f)
```

3.355.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3392 vs. 2(27) = 54.

Time = 12.38 (sec) , antiderivative size = 3392, normalized size of antiderivative = 116.97

$$\int \frac{\cos(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
-2*(((a^3*b^2*tan(1/2*e)^25 + 2*a^2*b^3*tan(1/2*e)^25 + a*b^4*tan(1/2*e)^2
5 + 12*a^3*b^2*tan(1/2*e)^23 + 24*a^2*b^3*tan(1/2*e)^23 + 12*a*b^4*tan(1/2
*e)^23 + 66*a^3*b^2*tan(1/2*e)^21 + 132*a^2*b^3*tan(1/2*e)^21 + 66*a*b^4*t
an(1/2*e)^21 + 220*a^3*b^2*tan(1/2*e)^19 + 440*a^2*b^3*tan(1/2*e)^19 + 220
*a*b^4*tan(1/2*e)^19 + 495*a^3*b^2*tan(1/2*e)^17 + 990*a^2*b^3*tan(1/2*e)^
17 + 495*a*b^4*tan(1/2*e)^17 + 792*a^3*b^2*tan(1/2*e)^15 + 1584*a^2*b^3*ta
n(1/2*e)^15 + 792*a*b^4*tan(1/2*e)^15 + 924*a^3*b^2*tan(1/2*e)^13 + 1848*a
^2*b^3*tan(1/2*e)^13 + 924*a*b^4*tan(1/2*e)^13 + 792*a^3*b^2*tan(1/2*e)^11
 + 1584*a^2*b^3*tan(1/2*e)^11 + 792*a*b^4*tan(1/2*e)^11 + 495*a^3*b^2*tan(
1/2*e)^9 + 990*a^2*b^3*tan(1/2*e)^9 + 495*a*b^4*tan(1/2*e)^9 + 220*a^3*b^2
*tan(1/2*e)^7 + 440*a^2*b^3*tan(1/2*e)^7 + 220*a*b^4*tan(1/2*e)^7 + 66*a^3
*b^2*tan(1/2*e)^5 + 132*a^2*b^3*tan(1/2*e)^5 + 66*a*b^4*tan(1/2*e)^5 + 12*
a^3*b^2*tan(1/2*e)^3 + 24*a^2*b^3*tan(1/2*e)^3 + 12*a*b^4*tan(1/2*e)^3 + a
^3*b^2*tan(1/2*e) + 2*a^2*b^3*tan(1/2*e) + a*b^4*tan(1/2*e))*tan(1/2*f*x)/
(a^4*b^2*tan(1/2*e)^24 + 2*a^3*b^3*tan(1/2*e)^24 + a^2*b^4*tan(1/2*e)^24 +
 12*a^4*b^2*tan(1/2*e)^22 + 24*a^3*b^3*tan(1/2*e)^22 + 12*a^2*b^4*tan(1/2*
e)^22 + 66*a^4*b^2*tan(1/2*e)^20 + 132*a^3*b^3*tan(1/2*e)^20 + 66*a^2*b^4*
tan(1/2*e)^20 + 220*a^4*b^2*tan(1/2*e)^18 + 440*a^3*b^3*tan(1/2*e)^18 + 22
0*a^2*b^4*tan(1/2*e)^18 + 495*a^4*b^2*tan(1/2*e)^16 + 990*a^3*b^3*tan(1/2*
e)^16 + 495*a^2*b^4*tan(1/2*e)^16 + 792*a^4*b^2*tan(1/2*e)^14 + 1584*a^...
```

3.355.9 Mupad [B] (verification not implemented)

Time = 14.35 (sec) , antiderivative size = 117, normalized size of antiderivative = 4.03

$$\int \frac{\cos(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}\,\sqrt{2\,a+b-b\,\cos\left(2\,e+2\,f\,x\right)}\,\left(4\,a\,\sin\left(e+f\,x\right)+3\,b\,\sin\left(e+f\,x\right)-b\,\mathrm{si}\right.}{a\,f\,\left(8\,a\,b+8\,a^2+3\,b^2-4\,b^2\,\cos\left(2\,e+2\,f\,x\right)+b^2\,\cos\left(4\,e+4\,f\,x\right)-8\,a\,b\,\mathrm{c}\right.}$$

input
```
int(cos(e + f*x)/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
(2^(1/2)*(2*a + b - b*cos(2*e + 2*f*x))^(1/2)*(4*a*sin(e + f*x) + 3*b*sin(
e + f*x) - b*sin(3*e + 3*f*x)))/(a*f*(8*a*b + 8*a^2 + 3*b^2 - 4*b^2*cos(2*
e + 2*f*x) + b^2*cos(4*e + 4*f*x) - 8*a*b*cos(2*e + 2*f*x)))
```

3.356 $\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

3.356.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{(a+b)^{3/2}f}+\frac{b\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/(a+b)^(3/2)/f+b*s
in(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.356.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.52 (sec) , antiderivative size = 480, normalized size of antiderivative = 6.15

$$\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\frac{\sec(e+fx)\tan(e+fx)\left(-45\arcsin\left(\sqrt{-\frac{(a+b)\tan^2(e+fx)}{a}}\right)-\frac{30b\arcsin\left(\sqrt{-\frac{(a\cdots}{}}\right)}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sec[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sec[e + f*x]*Tan[e + f*x]*(-45*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]
] - (30*b*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^2)/a -
(45*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Tan[e + f*x]^2)/a
- (30*b*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^2
*Tan[e + f*x]^2)/a^2 + 4*Hypergeometric2F1[2, 2, 7/2, -(((a + b)*Tan[e + f
*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan
[e + f*x]^2)/a))^(5/2) + (4*b*Hypergeometric2F1[2, 2, 7/2, -(((a + b)*Tan[
e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)
)/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(5/2))/a + 45*Sqrt[-(((a + b)*Sec[e +
 f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)] + (30*b*Sin[e + f*x]^
2*Sqrt[-(((a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^
2)])/a))/(15*a*f*Sqrt[a + b*Sin[e + f*x]^2]*Sqrt[(Sec[e + f*x]^2*(a + b*Si
n[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2))
```

3.356.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3669, 296, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(e+fx)\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{296}$$

$$\frac{\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a+b}+\frac{b\sin(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\int \frac{1}{1-\frac{(a+b)\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}}{a+b}+\frac{b\sin(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{(a+b)^{3/2}}+\frac{b\sin(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}}{f}$$

input
```
Int[Sec[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(a + b)^(3
/2) + (b*Sin[e + f*x])/(a*(a + b)*Sqrt[a + b*Sin[e + f*x]^2]))/f
```

3.356.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.356. $\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.356.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(70) = 140$.

Time = 1.14 (sec) , antiderivative size = 397, normalized size of antiderivative = 5.09

method	result
default	$\frac{ab\left(-\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}+2b\sin(fx+e)+2a}{\sin(fx+e)-1}\right)+\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}-2b\sin(fx+e)+2a}{1+\sin(fx+e)}\right)\right)(\cos^2(fx+e))+2\sqrt{a}}{}$

input
```
int(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a+b)^(1/2)/a/(-cos(f*x+e)^2*a*b-cos(f*x+e)^2*b^2+a^2+2*a*b+b^2)*(a*b*
(-ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)
+a))+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x
+e)+a)))*cos(f*x+e)^2+2*(a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2
)*b*sin(f*x+e)-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)
-b*sin(f*x+e)+a))*a^2-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2
)^(1/2)-b*sin(f*x+e)+a))*a*b+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f
*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b)/f
```

3.356.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(70) = 140$.

Time = 0.39 (sec) , antiderivative size = 453, normalized size of antiderivative = 5.81

$$\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \left[\frac{\left(ab\cos\left(fx+e\right)^2-a^2-ab\right)\sqrt{a+b}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-8\,(a^2+3\,ab+2\,b}{}\right.}{4\,((a^3}\right.$$

$$-\frac{\left(ab\cos\left(fx+e\right)^2-a^2-ab\right)\sqrt{-a-b}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-2\,a-2\,b\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a-b}}{2\left((ab+b^2)\cos(fx+e)^2-a^2-2\,ab-b^2\right)\sin(fx+e)}\right)+2\sqrt{-b\,c}}{2\left((a^3b+2\,a^2b^2+ab^3)f\cos\left(fx+e\right)^2-(a^4+3\,a^3b+3\,a^2b^2+ab^3)\right.}$$

input
```
integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a*b*cos(f*x + e)^2 - a^2 - a*b)*sqrt(a + b)*log(((a^2 + 8*a*b + 8*b
^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)
*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*s
in(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) - 4*sqrt(-b*cos(f*x
+ e)^2 + a + b)*(a*b + b^2)*sin(f*x + e))/((a^3*b + 2*a^2*b^2 + a*b^3)*f*c
os(f*x + e)^2 - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f), -1/2*((a*b*cos(f*x
 + e)^2 - a^2 - a*b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2
*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f
*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) + 2*sqrt(-b*cos(f*x + e)^2 +
 a + b)*(a*b + b^2)*sin(f*x + e))/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x +
 e)^2 - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)]
```

3.356.6 Sympy [F]

$$\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sec\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sec(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.356.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(70) = 140.

Time = 0.33 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.95

$$\int \frac{\sec(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{\frac{2\,b\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}\,a^2+\sqrt{b\sin(fx+e)^2+a}\,ab} + \frac{\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}(\sin(fx+e)+1)}-\frac{a}{\sqrt{ab}(\sin(fx+e)+1)}\right)}{(a+b)^{\frac{3}{2}}} + \operatorname{arsinh}\cdots}{2\,f}$$

input
```
integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*b*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^2 + sqrt(b*sin(f*x + e
)^2 + a)*a*b) + arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a/
(sqrt(a*b)*(sin(f*x + e) + 1)))/(a + b)^(3/2) + arcsinh(-b*sin(f*x + e)/(s
qrt(a*b)*(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1)))/(a + b)^(
3/2))/f
```

3.356.8 Giac [F]

$$\int \frac{\sec(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.356.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos(e+f\,x)\,\left(b\sin(e+f\,x)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2)), x)
```

3.357 $\int \frac{\sec^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.357.1 Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{\sec^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{(a+4b)\text{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2(a+b)^{5/2}f} - \frac{(a-2b)b\sin(e+fx)}{2a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec(e+fx)\tan(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
1/2*(a+4*b)*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/(a+b)
^(5/2)/f-1/2*(a-2*b)*b*sin(f*x+e)/a/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+1/2
*sec(f*x+e)*tan(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.357.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.37 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.67

$$\int \frac{\sec^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = -\frac{\sec^5(e+fx)\left(16(a+b)\,{}_3F_2\left(2,2,3;1,\frac{9}{2};-\frac{(a+b)\tan^2(e+fx)}{a}\right)\sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2+16(a+b)\,\text{H}\right.}{}$$

input
```
Integrate[Sec[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-1/105*(Sec[e + f*x]^5*(16*(a + b)*HypergeometricPFQ[{2, 2, 3}, {1, 9/2},
-(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2 + 1
6*(a + b)*Hypergeometric2F1[2, 3, 9/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sin[
e + f*x]^2*(4*a^2 + 7*a*b*Sin[e + f*x]^2 + 3*b^2*Sin[e + f*x]^4) - 7*a*Cos
[e + f*x]^2*Hypergeometric2F1[1, 2, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*(1
5*a^2 + 20*a*b*Sin[e + f*x]^2 + 8*b^2*Sin[e + f*x]^4))*Tan[e + f*x])/(a^4*
f*Sqrt[a + b*Sin[e + f*x]^2])
```

3.357.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3669, 316, 402, 25, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\cos(e+fx)^3\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 3669$$

$$\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)^2\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 316$$

$$\frac{\frac{\int \frac{2b\sin^2(e+fx)+a+2b}{\left(1-\sin^2(e+fx)\right)\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{2(a+b)}+\frac{\sin(e+fx)}{2(a+b)\left(1-\sin^2(e+fx)\right)\sqrt{a+b\sin^2(e+fx)}}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{-\frac{\int -\frac{a(a+4b)}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a(a+b)}-\frac{b(a-2b)\sin(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}}{2(a+b)}+\frac{\sin(e+fx)}{2(a+b)\left(1-\sin^2(e+fx)\right)\sqrt{a+b\sin^2(e+fx)}}}{f}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\frac{\int \frac{a(a+4b)}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a(a+b)}-\frac{b(a-2b)\sin(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}}{2(a+b)}+\frac{\sin(e+fx)}{2(a+b)\left(1-\sin^2(e+fx)\right)\sqrt{a+b\sin^2(e+fx)}}}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{(a+4b)\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a+b}-\frac{b(a-2b)\sin(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}}{2(a+b)}+\frac{\sin(e+fx)}{2(a+b)\left(1-\sin^2(e+fx)\right)\sqrt{a+b\sin^2(e+fx)}}}{f}$$

$$\Big\downarrow \ 291$$

$$\frac{\frac{\frac{(a+4b)\int \frac{1}{1-\frac{(a+b)\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}}{a+b}-\frac{b(a-2b)\sin(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}}{2(a+b)}+\frac{\sin(e+fx)}{2(a+b)\left(1-\sin^2(e+fx)\right)\sqrt{a+b\sin^2(e+fx)}}}{f}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\frac{(a+4b)\text{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{(a+b)^{3/2}}-\frac{b(a-2b)\sin(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}}{2(a+b)}+\frac{\sin(e+fx)}{2(a+b)\left(1-\sin^2(e+fx)\right)\sqrt{a+b\sin^2(e+fx)}}}{f}$$

input
```
Int[Sec[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sin[e + f*x]/(2*(a + b)*(1 - Sin[e + f*x]^2)*Sqrt[a + b*Sin[e + f*x]^2])
+ (((a + 4*b)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2
]])/(a + b)^(3/2) - ((a - 2*b)*b*Sin[e + f*x])/(a*(a + b)*Sqrt[a + b*Sin[e
 + f*x]^2]))/(2*(a + b)))/f
```

3.357.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.357. $\int \frac{\sec^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.357.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3218 vs. 2(118) = 236.

Time = 3.54 (sec) , antiderivative size = 3219, normalized size of antiderivative = 24.02

method	result	size
default	Expression too large to display	3219

input
```
int(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(a+b)^(1/2)/a/b^5/cos(f*x+e)^2/(a^4*b^2*cos(f*x+e)^4+4*a^3*b^3*cos(f*x
+e)^4+6*a^2*b^4*cos(f*x+e)^4+4*a*b^5*cos(f*x+e)^4+b^6*cos(f*x+e)^4-2*a^5*b
*cos(f*x+e)^2-10*a^4*b^2*cos(f*x+e)^2-20*a^3*b^3*cos(f*x+e)^2-20*a^2*b^4*c
os(f*x+e)^2-10*a*b^5*cos(f*x+e)^2-2*b^6*cos(f*x+e)^2+a^6+6*a^5*b+15*a^4*b^
2+20*a^3*b^3+15*a^2*b^4+6*a*b^5+b^6)*(-a*(8*ln(((a+b-b*cos(f*x+e)^2)^(1/2)
*b^(1/2)+b*sin(f*x+e))/b^(1/2))*b^(19/2)*(a+b)^(1/2)-8*b^(19/2)*(a+b)^(1/2
)*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2)+b^2*sin(f*x+e))/b^(3
/2))+24*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))*b^(1
7/2)*(a+b)^(1/2)*a-24*b^(17/2)*(a+b)^(1/2)*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3
)/b^2)^(1/2)*b^(3/2)+b^2*sin(f*x+e))/b^(3/2))*a+24*ln(((a+b-b*cos(f*x+e)^2
)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))*b^(15/2)*(a+b)^(1/2)*a^2-24*b^(15/2
)*(a+b)^(1/2)*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2)+b^2*sin(
f*x+e))/b^(3/2))*a^2+8*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e)
)/b^(1/2))*b^(13/2)*(a+b)^(1/2)*a^3-8*b^(13/2)*(a+b)^(1/2)*ln(((-b*cos(f*x
+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2)+b^2*sin(f*x+e))/b^(3/2))*a^3+ln(2/(1+
sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^5*b
^5+8*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x
+e)+a))*a^4*b^6+22*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(
1/2)-b*sin(f*x+e)+a))*a^3*b^7+28*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*c
os(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^8+17*ln(2/(1+sin(f*x+e))*((a+...
```

3.357.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(118) = 236$.

Time = 0.60 (sec) , antiderivative size = 625, normalized size of antiderivative = 4.66

$$\int \frac{\sec^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \left[\frac{\left((a^2b+4\,ab^2)\cos\left(fx+e\right)^4-(a^3+5\,a^2b+4\,ab^2)\cos\left(fx+e\right)^2\right)\sqrt{a+b}\log}{}\right.$$

$$-\frac{\left((a^2b+4\,ab^2)\cos\left(fx+e\right)^4-(a^3+5\,a^2b+4\,ab^2)\cos\left(fx+e\right)^2\right)\sqrt{-a-b}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-2\,a-2\,b\right)}{2\left((ab+b^2)\cos(fx+e)^2-\right.}\right.}{4\left((a^4b+3\,a^3b^2+3\,a^2b^3+ab^4)f\cos\left(fx+\right.\right.}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(((a^2*b + 4*a*b^2)*cos(f*x + e)^4 - (a^3 + 5*a^2*b + 4*a*b^2)*cos(f*
x + e)^2)*sqrt(a + b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 +
 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*a*b
+ 8*b^2)/cos(f*x + e)^4) - 4*(a^3 + 2*a^2*b + a*b^2 - (a^2*b - a*b^2 - 2*b
^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a^4*b
+ 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*f*cos(f*x + e)^4 - (a^5 + 4*a^4*b + 6*a^3
*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x + e)^2), -1/4*(((a^2*b + 4*a*b^2)*cos(
f*x + e)^4 - (a^3 + 5*a^2*b + 4*a*b^2)*cos(f*x + e)^2)*sqrt(-a - b)*arctan
(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b
)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x +
 e))) + 2*(a^3 + 2*a^2*b + a*b^2 - (a^2*b - a*b^2 - 2*b^3)*cos(f*x + e)^2)
*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a^4*b + 3*a^3*b^2 + 3*a^2
*b^3 + a*b^4)*f*cos(f*x + e)^4 - (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 +
a*b^4)*f*cos(f*x + e)^2)]
```

3.357.6 Sympy [F]

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sec^3{(e+fx)}}{\left(a+b\sin^2{(e+fx)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sec(e + f*x)**3/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.357.7 Maxima [F]

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sec{(fx+e)}^3}{\left(b\sin{(fx+e)}^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^3/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.357.8 Giac [F]

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sec{(fx+e)}^3}{\left(b\sin{(fx+e)}^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^3/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.357.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\cos\left(e+f\,x\right)^3\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2)), x)
```

3.358 $\int \frac{\cos^6(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.358.1 Optimal result

Integrand size = 25, antiderivative size = 274

$$\int \frac{\cos^6(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(4a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3ab^2f} + \frac{(8a^2+13ab+3b^2)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3ab^3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{(a+b)(8a+9b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3b^3f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
(a+b)*cos(f*x+e)^3*sin(f*x+e)/a/b/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*(4*a+3*b)
*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a/b^2/f+1/3*(8*a^2+13*a*b+
3*b^2)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*
(a+b*sin(f*x+e)^2)^(1/2)/a/b^3/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*(a+b)*(8*a
+9*b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(
1+b*sin(f*x+e)^2/a)^(1/2)/b^3/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.358.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.67

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{4a(8a^2+13ab+3b^2)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-4a(8a^2+17ab+\cdots}{12\cdots}$$

input
```
Integrate[Cos[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(4*a*(8*a^2 + 13*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ellip
ticE[e + f*x, -(b/a)] - 4*a*(8*a^2 + 17*a*b + 9*b^2)*Sqrt[(2*a + b - b*Cos
[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sqrt[2]*b*(8*a^2 + 13*a*b +
 6*b^2 - a*b*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(12*a*b^3*f*Sqrt[2*a + b
- b*Cos[2*(e + f*x)]])
```

3.358.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3671, 315, 25, 403, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(e+fx)^6}{\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\left(1-\sin^2(e+fx)\right)^{5/2}}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int -\frac{\sqrt{1-\sin^2(e+fx)}\left(a-(4a+3b)\sin^2(e+fx)\right)}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{ab}+\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{ab\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{ab\sqrt{a+b\sin^2(e+fx)}}-\frac{\int \frac{\sqrt{1-\sin^2(e+fx)}\left(a-(4a+3b)\sin^2(e+fx)\right)}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{ab}\right)}{f}$$

$$\Big\downarrow \; 403$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{ab\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{\int \frac{2a(2a+3b)-\left(8a^2+13ba+3b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3b}-\frac{(4a+3b)\sin(e+\cdots}{\cdots}}{ab}\cdots\right.}{f}$$

$$\Big\downarrow \; 399$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{ab\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{a(a+b)(8a+9b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{\left(8a^2+1\cdots\right.}{\cdots}}{3b}\cdots\right.}{f}$$

$$\Big\downarrow \; 323$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{ab\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{a(a+b)(8a+9b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\,s\cdots}{b\sqrt{a+b\sin^2(e+fx)}}\cdots}{3b}\cdots\right.}{f}$$

$$\Big\downarrow \; 321$$

3.358. $\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{ab\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{a(a+b)(8a+9b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(8a^{\dots}}{\dots}}{3b}\right.}{f}$$

$$\Big\downarrow \ 330$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{ab\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{a(a+b)(8a+9b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(8a^{\dots}}{\dots}}{3b}\right.}{f}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{ab\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{a(a+b)(8a+9b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-(8a^{\dots}}{3b}\right.}{f}$$

input
```
Int[Cos[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(((a + b)*Sin[e + f*x]*(1 - Sin[e + f*x
]^2)^(3/2))/(a*b*Sqrt[a + b*Sin[e + f*x]^2]) - (-1/3*((4*a + 3*b)*Sin[e +
f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/b + (-(((8*a^2 +
 13*a*b + 3*b^2)*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e
+ f*x]^2])/(b*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) + (a*(a + b)*(8*a + 9*b)*El
lipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*S
qrt[a + b*Sin[e + f*x]^2]))/(3*b))/(a*b)))/f
```

3.358. $\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

3.358.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

3.358. $\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.358.4 Maple [A] (verified)

Time = 3.98 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.51

method	result
default	$-\frac{(\cos^4(fx+e))\sin(fx+e)a\,b^2+(-4a^2b-7a\,b^2-3b^3)(\cos^2(fx+e))\sin(fx+e)+8\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}\,F\Big(\sin(}{}$

input
```
int(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(cos(f*x+e)^4*sin(f*x+e)*a*b^2+(-4*a^2*b-7*a*b^2-3*b^3)*cos(f*x+e)^2*
sin(f*x+e)+8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Ellipt
icF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+17*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e
)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+9*(cos(f*x+e
)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)
^(1/2))*a*b^2-8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Ell
ipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-13*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*
x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-3*(cos(f*
x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a*b^2)/a/b^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.358.5 Fricas [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx=\int \frac{\cos(fx+e)^6}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^6/(b^2*cos(f*x + e)^
4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

3.358.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**6/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.358.7 Maxima [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx=\int \frac{\cos(fx+e)^6}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.358.8 Giac [F]

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cos\left(fx+e\right)^6}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.358.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cos\left(e+f\,x\right)^6}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(cos(e + f*x)^6/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^6/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.359 $\int \frac{\cos^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.359.1 Optimal result

Integrand size = 25, antiderivative size = 202

$$\begin{aligned}\int \frac{\cos^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx &= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}}\\ &+ \frac{(2a+b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{ab^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\\ &- \frac{2(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{b^2f\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

output
```
(a+b)*cos(f*x+e)*sin(f*x+e)/a/b/f/(a+b*sin(f*x+e)^2)^(1/2)+(2*a+b)*Ellipti
cE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e
)^2)^(1/2)/a/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)-2*(a+b)*EllipticF(sin(f*x+e)
,(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/
b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.359.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.69

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{2a(2a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-(a+b)\left(4a\sqrt{\frac{2a+b-b\cos(2(e+f}{a}}\right.}{2ab^2f\sqrt{2a+b-b\cos(2(e+f}}$$

input
```
Integrate[Cos[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(2*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] - (a + b)*(4*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e +
 f*x, -(b/a)] - Sqrt[2]*b*Sin[2*(e + f*x)]))/(2*a*b^2*f*Sqrt[2*a + b - b*C
os[2*(e + f*x)]])
```

3.359.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 3671, 315, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(e+fx)^4}{\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\left(1-\sin^2(e+fx)\right)^{3/2}}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int -\frac{a-(2a+b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{ab}+\frac{(a+b)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{ab\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{ab\sqrt{a+b\sin^2(e+fx)}}-\frac{\int\frac{a-(2a+b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{ab}\right)}{f}$$

$$\Big\downarrow 399$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{ab\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{2a(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{(2a+b)\int\frac{\sqrt{b\sin^2(}}{\sqrt{1-\sin}}}{}}{ab}\right.}{f}$$

$$\Big\downarrow 323$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{ab\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{2a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-}{ab}\right.}{f}$$

$$\Big\downarrow 321$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{ab\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{2a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+b)\int\frac{\sqrt{b\sin}}{\sqrt{1-}}}{}}{ab}\right.}{f}$$

$$\Big\downarrow 330$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{ab\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{2a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+b)\sqrt{a+b\,\mathrm{s}}}{}}{ab}\right.}{f}$$

$$\Big\downarrow 327$$

3.359. $\displaystyle\int\frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{ab\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{2a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+b)\sqrt{a+b\sin \cdots}}{}}{ab}\right.}{f}$$

input
```
Int[Cos[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(((a + b)*Sin[e + f*x]*Sqrt[1 - Sin[e +
 f*x]^2])/(a*b*Sqrt[a + b*Sin[e + f*x]^2]) - (-(((2*a + b)*EllipticE[ArcSi
n[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e
+ f*x]^2)/a])) + (2*a*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt
[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2]))/(a*b)))/f
```

3.359.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

3.359. $\int \frac{\cos^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.359.4 Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.42

method	result
default	$-\frac{2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2+2ab\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}\,F\left(\sin(fx+\right.}{}$

input
```
int(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f
*x+e),(-1/a*b)^(1/2))*a^2+2*a*b*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a
+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))-2*(cos(f*x+e)^2)^(1/2)*(
-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2-
(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e
),(-1/a*b)^(1/2))*a*b-b*cos(f*x+e)^2*sin(f*x+e)*a-b^2*cos(f*x+e)^2*sin(f*x
+e))/a/b^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.359.5 Fricas [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx=\int \frac{\cos(fx+e)^4}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^4/(b^2*cos(f*x + e)^
4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

3.359.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**4/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.359.7 Maxima [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^4}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.359.8 Giac [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^4}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.359.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(e+f\,x)^4}{(b\sin(e+f\,x)^2+a)^{3/2}}\,dx$$

input
```
int(cos(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.360 $\int \frac{\cos^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.360.1 Optimal result

Integrand size = 25, antiderivative size = 188

$$\int \frac{\cos^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{\cos(e+fx)\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} + \frac{\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{abf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{bf\sqrt{a+b\sin^2(e+fx)}}$$

output
```
cos(f*x+e)*sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(1/2)+EllipticE(sin(f*x+e),(-
b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a/b/f
/(1+b*sin(f*x+e)^2/a)^(1/2)-EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*
(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/b/f/(a+b*sin(f*x+e)^2)^(1/
2)
```

3.360.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.71

$$\int \frac{\cos^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,\text{EllipticF}\,(}{\sqrt{2}abf\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Cos[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a
)] - Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -
(b/a)] + b*Sin[2*(e + f*x)])/(Sqrt[2]*a*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*
x)]])
```

3.360.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3671, 314, 25, 389, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(e+fx)^2}{(a+b\sin(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \; 3671$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sqrt{1-\sin^2(e+fx)}}{(b\sin^2(e+fx)+a)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 314$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{a\sqrt{a+b\sin^2(e+fx)}}-\frac{\int -\frac{\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}\right)}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int \frac{\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}+\frac{\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{a\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

$\downarrow$ 389

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}}{a}+\frac{\sqrt{1-\sin^2(e+fx)}\sin(e+fx}{a\sqrt{a+b\sin^2(e+fx)}}\right.}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}}{a}+\frac{\sqrt{1-\sin^2(}{a\sqrt{a-}}\right.}{f}$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}}{a}+\frac{\sqrt{1-\sin^2(e+fx)}\sin(e+}{a\sqrt{a+b\sin^2(e+fx)}}\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{\sqrt{a+b\sin^2(e+fx)}\int \frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}}{a}+\frac{\sqrt{1-\text{si}}}{a\sqrt{}}\right.}{f}$$

$\downarrow$ 327

3.360. $\displaystyle\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}}{a}+\frac{\sqrt{1-\sin^2(e-}}{a\sqrt{a+b}}\right.}{f}$$

input
```
Int[Cos[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]
)/(a*Sqrt[a + b*Sin[e + f*x]^2]) + ((EllipticE[ArcSin[Sin[e + f*x]], -(b/a
)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (a*Ell
ipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sq
rt[a + b*Sin[e + f*x]^2]))/a))/f
```

3.360.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.360.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.77

method	result
default	$-\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)-a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,E\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)+(\sin}{ab\cos(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}\,f}$

input
```
int(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e)
,(-1/a*b)^(1/2))-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Ellip
ticE(sin(f*x+e),(-1/a*b)^(1/2))+sin(f*x+e)^3*b-b*sin(f*x+e))/a/b/cos(f*x+e
)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.360.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 777, normalized size of antiderivative = 4.13

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{-b\cos\left(fx+e\right)^2+a+b}b^2\cos\left(fx+e\right)\sin\left(fx+e\right)+4\left(i\,b^2\cos\left(fx+e\right)^2-i\,ab-i\,b^2\right)\sqrt{-b}\sqrt{\frac{2\,b\sqrt{\frac{a^2+}{b^2}}}{}}}{}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(-b*cos(f*x + e)^2 + a + b)*b^2*cos(f*x + e)*sin(f*x + e) + 4*
(I*b^2*cos(f*x + e)^2 - I*a*b - I*b^2)*sqrt(-b)*sqrt((2*b*sqrt((a^2 + a*b)
/b^2) + 2*a + b)/b)*sqrt((a^2 + a*b)/b^2)*elliptic_f(arcsin(sqrt((2*b*sqrt
((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 +
 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 4*(-I*b^2*cos
(f*x + e)^2 + I*a*b + I*b^2)*sqrt(-b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*
a + b)/b)*sqrt((a^2 + a*b)/b^2)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*
b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b
^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (2*(I*b^2*cos(f*x + e)^
2 - I*a*b - I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((-2*I*a*b - I*b^2)*co
s(f*x + e)^2 + 2*I*a^2 + 3*I*a*b + I*b^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 +
a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2)
+ 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(
2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (2*(-I*b^2*cos(f*x + e)^2 + I*a
*b + I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((2*I*a*b + I*b^2)*cos(f*x +
e)^2 - 2*I*a^2 - 3*I*a*b - I*b^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2
) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a +
b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b +
b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/(a*b^3*f*cos(f*x + e)^2 - (a^2*b^2 + a*b
^3)*f)
```

3.360.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.360.7 Maxima [F]

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cos\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.360.8 Giac [F]

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cos\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.360.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\cos\left(e+f\,x\right)^2}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(cos(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.361 $\int \frac{1}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.361.1 Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{1}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{a(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

output
```
b*cos(f*x+e)*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+(cos(f*x+e)^2)^
(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)^(1/
2)/a/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)
```

3.361.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}b\sin(2(e+fx))}{2a(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

output
```
(2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + S
qrt[2]*b*Sin[2*(e + f*x)])/(2*a*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)
]])
```

3.361.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 3663, 25, 3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sin^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\sin(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\int -\sqrt{b\sin^2(e+fx)+a}dx}{a(a+b)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sqrt{b\sin^2(e+fx)+a}dx}{a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sin(e+fx)^2+a}dx}{a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin^2(e+fx)}{a}+1}dx}{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin(e+fx)^2}{a}+1}dx}{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{af(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

input
```
Int[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

output
```
(b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + (
EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)*f*Sqrt[1
 + (b*Sin[e + f*x]^2)/a])
```

3.361.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.361.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}\,aE\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)+\left(\cos^2(fx+e)\right)\sin(fx+e)b}{a(a+b)\cos(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}\,f}$	103

3.361. $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

input
```
int(1/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*EllipticE(sin(f*
x+e),(-1/a*b)^(1/2))+cos(f*x+e)^2*sin(f*x+e)*b)/a/(a+b)/cos(f*x+e)/(a+b*si
n(f*x+e)^2)^(1/2)/f
```

3.361.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 938, normalized size of antiderivative = 9.29

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,b^3\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(2\left(i\,b^3\cos\left(fx+e\right)^2-i\,ab^2-i\,b^3\right)\sqrt{-b}\sqrt{\frac{a^2+a}{b^2}}\right.}{}$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(-b*cos(f*x + e)^2 + a + b)*b^3*cos(f*x + e)*sin(f*x + e) - (2
*(I*b^3*cos(f*x + e)^2 - I*a*b^2 - I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) -
 (2*I*a^2*b + 3*I*a*b^2 + I*b^3 + (-2*I*a*b^2 - I*b^3)*cos(f*x + e)^2)*sqr
t(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqr
t((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))
), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (2
*(-I*b^3*cos(f*x + e)^2 + I*a*b^2 + I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)
- (-2*I*a^2*b - 3*I*a*b^2 - I*b^3 + (2*I*a*b^2 + I*b^3)*cos(f*x + e)^2)*sq
rt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sq
rt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e)
)), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 2
*(2*(-I*a^2*b - 2*I*a*b^2 - I*b^3 + (I*a*b^2 + I*b^3)*cos(f*x + e)^2)*sqrt
(-b)*sqrt((a^2 + a*b)/b^2) + (2*I*a^3 + 3*I*a^2*b + I*a*b^2 + (-2*I*a^2*b
- I*a*b^2)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
 + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(
cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sq
rt((a^2 + a*b)/b^2))/b^2) + 2*(2*(I*a^2*b + 2*I*a*b^2 + I*b^3 + (-I*a*b^2
- I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + (-2*I*a^3 - 3*I*
a^2*b - I*a*b^2 + (2*I*a^2*b + I*a*b^2)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*
b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt(...
```

3.361.6 Sympy [F]

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(a+b\sin^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sin(e + f*x)**2)**(-3/2), x)
```

3.361.7 Maxima [F]

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

3.361.8 Giac [F]

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.361.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(1/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.362 $\int \frac{\sec^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.362.1 Optimal result

Integrand size = 25, antiderivative size = 240

$$\begin{aligned}\int \frac{\sec^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx &= -\frac{(a-b)b\cos(e+fx)\sin(e+fx)}{a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}\\ &\quad-\frac{(a-b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\\ &\quad+\frac{\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{(a+b)f\sqrt{a+b\sin^2(e+fx)}}\\ &\quad+\frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

output
```
-(a-b)*b*cos(f*x+e)*sin(f*x+e)/a/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)-(a-b)*
EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*si
n(f*x+e)^2)^(1/2)/a/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+EllipticF(sin(f*x
+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/
2)/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+tan(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)^
(1/2)
```

3.362.2 Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.70

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{-\sqrt{2}a(a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}a(a+b)\sqrt{\frac{2a+b-b\cos(2(e}{a}}}{\sqrt{2}a(a+b)^2f\sqrt{}}$$

input
```
Integrate[Sec[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-(Sqrt[2]*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
f*x, -(b/a)]) + Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*E
llipticF[e + f*x, -(b/a)] + (2*a^2 + a*b + b^2 + b*(-a + b)*Cos[2*(e + f*x
)])*Tan[e + f*x])/(Sqrt[2]*a*(a + b)^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]
])
```

3.362.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3671, 316, 27, 402, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(e+fx)^2\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{1}{\left(1-\sin^2(e+fx)\right)^{3/2}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int \frac{b\left(\sin^2(e+fx)+1\right)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{a+b}+\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\int \frac{\sin^2(e+fx)+1}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{a+b}+\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

$\downarrow$ 402

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(-\frac{\int -\frac{2a-(a-b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a(a+b)}-\frac{(a-b)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}\right)}{a+b}+\frac{\sin(}{(a+b)\sqrt{1-\sin^2(e+}}\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{\int \frac{2a-(a-b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a(a+b)}-\frac{(a-b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}\right)}{a+b}+\frac{\sin(e+}{(a+b)\sqrt{1-\sin^2(e+fx)}}\right.}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{\frac{a(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{(a-b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}}{a(a+b)}-\frac{(a-b)\sin(e+fx)}{a(a+b)\sqrt{a+}}\right.}{a+b}\right.}{f}$$

3.362. $\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}}{a(a+b)}\right)}{a+b}\right)}{f}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}}{a(a+b)}-\frac{(a-b)\sin(e+\cdots}{a(a+b)\sqrt{\cdots}}\right)}{a+b}\right)}{f}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+f\cdots}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{a(a+b)}\right)}{a+b}\right)}{f}$$

$$\Big\downarrow \text{327}$$

3.362. $\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{a(a+b)}-\cdots\right)}{a+b}\cdots\right.}{f}$$

input
```
Int[Sec[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(Sin[e + f*x]/((a + b)*Sqrt[1 - Sin[e +
 f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2]) + (b*(-(((a - b)*Sin[e + f*x]*Sqrt[1
- Sin[e + f*x]^2])/(a*(a + b)*Sqrt[a + b*Sin[e + f*x]^2])) + (-(((a - b)*E
llipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt
[1 + (b*Sin[e + f*x]^2)/a])) + (a*(a + b)*EllipticF[ArcSin[Sin[e + f*x]],
-(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2]))/(a
*(a + b))))/(a + b)))/f
```

3.362.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.362. $\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.362.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.50

method	result
default	$\frac{\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\left(\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2+ab\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\right.}{}$

input
```
int(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*((cos(f*x+e)^2)^(1/2)*(-b/a*cos
(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+a*b*(cos
(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-
1/a*b)^(1/2))-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Ellip
ticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^
2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b-b*cos(f*x+e)^2*s
in(f*x+e)*a+b^2*cos(f*x+e)^2*sin(f*x+e)+sin(f*x+e)*a^2+a*b*sin(f*x+e))/(a+
b)^2/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/a/cos(f*x+e
)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.362.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 1079, normalized size of antiderivative = 4.50

$$\int \frac{\sec^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((2*((-I*a*b^2 + I*b^3)*cos(f*x + e)^3 + (I*a^2*b - I*b^3)*cos(f*x + e
))*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((2*I*a^2*b - I*a*b^2 - I*b^3)*cos(f*x
 + e)^3 + (-2*I*a^3 - I*a^2*b + 2*I*a*b^2 + I*b^3)*cos(f*x + e))*sqrt(-b))
*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b
*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*
a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((I*a
*b^2 - I*b^3)*cos(f*x + e)^3 + (-I*a^2*b + I*b^3)*cos(f*x + e))*sqrt(-b)*s
qrt((a^2 + a*b)/b^2) - ((-2*I*a^2*b + I*a*b^2 + I*b^3)*cos(f*x + e)^3 + (2
*I*a^3 + I*a^2*b - 2*I*a*b^2 - I*b^3)*cos(f*x + e))*sqrt(-b))*sqrt((2*b*sq
rt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 +
a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b +
 b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 4*(((I*a*b^2 + I*b^3)
*cos(f*x + e)^3 + (-I*a^2*b - 2*I*a*b^2 - I*b^3)*cos(f*x + e))*sqrt(-b)*sq
rt((a^2 + a*b)/b^2) + ((-2*I*a^2*b - I*a*b^2)*cos(f*x + e)^3 + (2*I*a^3 +
3*I*a^2*b + I*a*b^2)*cos(f*x + e))*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^
2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a +
 b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b +
 b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 4*(((-I*a*b^2 - I*b^3)*cos(f*x + e)^3
+ (I*a^2*b + 2*I*a*b^2 + I*b^3)*cos(f*x + e))*sqrt(-b)*sqrt((a^2 + a*b)/b^
2) + ((2*I*a^2*b + I*a*b^2)*cos(f*x + e)^3 + (-2*I*a^3 - 3*I*a^2*b - I*...
```

3.362.6 Sympy [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\sec^2(e+fx)}{(a+b\sin^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sec(e + f*x)**2/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.362.7 Maxima [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^2}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.362.8 Giac [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^2}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.362.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos(e+f\,x)^2\,\left(b\sin(e+f\,x)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2)), x)
```

3.363 $\int \frac{\cos^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.363.1 Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{5/2}f} + \frac{(a+b)\cos^2(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)(a+b)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/b^(5/2)/f+1/3*(a+b)*c
os(f*x+e)^2*sin(f*x+e)/a/b/f/(a+b*sin(f*x+e)^2)^(3/2)-1/3*(3*a-2*b)*(a+b)*
sin(f*x+e)/a^2/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.363.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\frac{3\arctan\left(\frac{\sqrt{2}\sqrt{-b}\sin(e+fx)}{\sqrt{2a+b-b\cos(2(e+fx))}}\right)}{\sqrt{-b}} + \frac{2\sqrt{2}(a+b)\left(-3a^2+ab+b^2+(2a-b)b\cos(2(e+fx))\right)\sin(e+fx)}{a^2(2a+b-b\cos(2(e+fx)))^{3/2}}}{3b^2f}$$

input
```
Integrate[Cos[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
((3*ArcTan[(Sqrt[2]*Sqrt[-b]*Sin[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x
)]]])/Sqrt[-b] + (2*Sqrt[2]*(a + b)*(-3*a^2 + a*b + b^2 + (2*a - b)*b*Cos[
2*(e + f*x)])*Sin[e + f*x])/(a^2*(2*a + b - b*Cos[2*(e + f*x)])^(3/2)))/(3
*b^2*f)
```

3.363.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3669, 315, 25, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(e+fx)^5}{\left(a+b\sin(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^2}{\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{\int -\frac{-3a\sin^2(e+fx)+a-2b}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3ab}+\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\int \frac{-3a\sin^2(e+fx)+a-2b}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3ab}}{f}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\left(\frac{3a}{b}-\frac{2b}{a}+1\right)\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}-\frac{3a\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}}{3ab}}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{(a+b)\sin(e+fx)(1-\sin^2(e+fx))}{3ab(a+b\sin^2(e+fx))^{3/2}}-\frac{\frac{\left(\frac{3a}{b}-\frac{2b}{a}+1\right)\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}-\frac{3a\int\frac{1}{1-\frac{b\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}}{b}}{3ab}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{(a+b)\sin(e+fx)(1-\sin^2(e+fx))}{3ab(a+b\sin^2(e+fx))^{3/2}}-\frac{\frac{\left(\frac{3a}{b}-\frac{2b}{a}+1\right)\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}-\frac{3a\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{3/2}}}{3ab}}{f}$$

input
```
Int[Cos[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(((a + b)*Sin[e + f*x]*(1 - Sin[e + f*x]^2))/(3*a*b*(a + b*Sin[e + f*x]^2)
^(3/2)) - ((-3*a*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]
])/b^(3/2) + ((1 + (3*a)/b - (2*b)/a)*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x
]^2])/(3*a*b))/f
```

3.363.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.363. $\int \frac{\cos^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.363.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{\sin(fx+e)}{3af\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\sin(fx+e)}{3a^2f\sqrt{a+b\left(\sin^2(fx+e)\right)}}-\frac{\sin^3(fx+e)}{3fb\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}-\frac{\sin(fx+e)}{f\,b^2\sqrt{a+b\left(\sin^2(fx+e)\right)}}+$
default	$\frac{\sin(fx+e)}{3af\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\sin(fx+e)}{3a^2f\sqrt{a+b\left(\sin^2(fx+e)\right)}}-\frac{\sin^3(fx+e)}{3fb\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}-\frac{\sin(fx+e)}{f\,b^2\sqrt{a+b\left(\sin^2(fx+e)\right)}}+$

input
```
int(cos(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*sin(f*x+e)/a^2/f/(a+b*sin(
f*x+e)^2)^(1/2)-1/3/f*sin(f*x+e)^3/b/(a+b*sin(f*x+e)^2)^(3/2)-1/f/b^2*sin(
f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)+1/f/b^(5/2)*ln(b^(1/2)*sin(f*x+e)+(a+b*sin
(f*x+e)^2)^(1/2))+2/3/f/b*sin(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2)-2/3/f/b/a*si
n(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)
```

3.363.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(116) = 232.

Time = 1.11 (sec) , antiderivative size = 799, normalized size of antiderivative = 6.15

$$\int \frac{\cos^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \left[\frac{3\left(a^2b^2\cos(fx+e)^4+a^4+2a^3b+a^2b^2-2(a^3b+a^2b^2)\cos(fx+e)^2\right)\sqrt{b}}{}\right.$$

$$-\frac{3\left(a^2b^2\cos(fx+e)^4+a^4+2a^3b+a^2b^2-2(a^3b+a^2b^2)\cos(fx+e)^2\right)\sqrt{-b}\arctan\left(\frac{\left(8b^2\cos(fx+e)^4-8(ab+2\right.}{4\left(2b^3\cos(fx+e)^4+\right.}\right.}{12\left(a^2b^5f\cos(fx\right.}$$

input
```
integrate(cos(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(3*(a^2*b^2*cos(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 - 2*(a^3*b + a^
2*b^2)*cos(f*x + e)^2)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2
*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 +
 a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b
^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*
b^2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^
2*b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*s
qrt(b)*sin(f*x + e)) - 8*(3*a^3*b + 4*a^2*b^2 - a*b^3 - 2*b^4 - 2*(2*a^2*b
^2 + a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x
+ e))/(a^2*b^5*f*cos(f*x + e)^4 - 2*(a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^2 +
 (a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*f), -1/12*(3*(a^2*b^2*cos(f*x + e)^4 + a^
4 + 2*a^3*b + a^2*b^2 - 2*(a^3*b + a^2*b^2)*cos(f*x + e)^2)*sqrt(-b)*arcta
n(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b
 + 8*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4
+ a^2*b + 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e)
)) + 4*(3*a^3*b + 4*a^2*b^2 - a*b^3 - 2*b^4 - 2*(2*a^2*b^2 + a*b^3 - b^4)*
cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/(a^2*b^5*f*c
os(f*x + e)^4 - 2*(a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^2 + (a^4*b^3 + 2*a^3*
b^4 + a^2*b^5)*f)]
```

3.363.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

3.363.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.59

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx =$$

$$-\frac{\left(\frac{3\sin(fx+e)^2}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}b}+\frac{2a}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}b^2}\right)\sin(fx+e)-\frac{3\,\mathrm{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}-\frac{2\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}\,a^2}-\frac{\sin(fx+e)}{\left(b\sin(fx+e)^2+a\right.}}{3f}$$

input
```
integrate(cos(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*((3*sin(f*x + e)^2/((b*sin(f*x + e)^2 + a)^(3/2)*b) + 2*a/((b*sin(f*x
 + e)^2 + a)^(3/2)*b^2))*sin(f*x + e) - 3*arcsinh(b*sin(f*x + e)/sqrt(a*b)
)/b^(5/2) - 2*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^2) - sin(f*x + e)
/((b*sin(f*x + e)^2 + a)^(3/2)*a) + sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 +
a)*b^2) - 2*sin(f*x + e)/((b*sin(f*x + e)^2 + a)^(3/2)*b) + 2*sin(f*x + e)
/(sqrt(b*sin(f*x + e)^2 + a)*a*b))/f
```

3.363.8 Giac [F]

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cos(fx+e)^5}{\left(b\sin(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^5/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.363.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+f\,x)^5}{\left(b\sin(e+f\,x)^2+a\right)^{5/2}}\,dx$$

input
```
int(cos(e + f*x)^5/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(cos(e + f*x)^5/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.364 $\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.364.1 Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\frac{\cos^2(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
1/3*cos(f*x+e)^2*sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*sin(f*x+e)/a^
2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.364.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\frac{3a\sin(e+fx)-(a-2b)\sin^3(e+fx)}{3a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Cos[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(3*a*Sin[e + f*x] - (a - 2*b)*Sin[e + f*x]^3)/(3*a^2*f*(a + b*Sin[e + f*x]
^2)^(3/2))
```

3.364.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3669, 292, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(e+fx)^3}{\left(a+b\sin(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3669}$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 292}$$

$$\frac{\frac{2\int \frac{1}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3a}+\frac{\sin(e+fx)\left(1-\sin^2(e+fx)\right)}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{\frac{2\sin(e+fx)}{3a^2\sqrt{a+b\sin^2(e+fx)}}+\frac{\left(1-\sin^2(e+fx)\right)\sin(e+fx)}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{f}$$

input
```
Int[Cos[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
((Sin[e + f*x]*(1 - Sin[e + f*x]^2))/(3*a*(a + b*Sin[e + f*x]^2)^(3/2)) +
(2*Sin[e + f*x])/(3*a^2*Sqrt[a + b*Sin[e + f*x]^2]))/f
```

3.364.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.364.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(65) = 130$.

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.89

method	result
derivativedivides	$\frac{\frac{\sin(fx+e)}{3a\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\sin(fx+e)}{3a^2\sqrt{a+b\left(\sin^2(fx+e)\right)}}+\frac{\sin(fx+e)}{2b\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}-\frac{a\left(\frac{\sin(fx+e)}{3a\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\sin(fx+e)}{3a^2\sqrt{a+b\left(\sin^2(fx+\right.}}\right.}{2b}}{f}$
default	$\frac{\frac{\sin(fx+e)}{3a\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\sin(fx+e)}{3a^2\sqrt{a+b\left(\sin^2(fx+e)\right)}}+\frac{\sin(fx+e)}{2b\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}-\frac{a\left(\frac{\sin(fx+e)}{3a\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\sin(fx+e)}{3a^2\sqrt{a+b\left(\sin^2(fx+\right.}}\right.}{2b}}{f}$

input
```
int(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/3*sin(f*x+e)/a/(a+b*sin(f*x+e)^2)^(3/2)+2/3/a^2*sin(f*x+e)/(a+b*sin
(f*x+e)^2)^(1/2)+1/2*sin(f*x+e)/b/(a+b*sin(f*x+e)^2)^(3/2)-1/2*a/b*(1/3*si
n(f*x+e)/a/(a+b*sin(f*x+e)^2)^(3/2)+2/3/a^2*sin(f*x+e)/(a+b*sin(f*x+e)^2)^
(1/2)))
```

3.364.5 Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.47

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left((a-2\,b)\cos\left(fx+e\right)^2+2\,a+2\,b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sin\left(fx+e\right)}{3\left(a^2b^2f\cos\left(fx+e\right)^4-2\left(a^3b+a^2b^2\right)f\cos\left(fx+e\right)^2+\left(a^4+2\,a^3b+a^2b^2\right)f\right)}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((a - 2*b)*cos(f*x + e)^2 + 2*a + 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)
*sin(f*x + e)/(a^2*b^2*f*cos(f*x + e)^4 - 2*(a^3*b + a^2*b^2)*f*cos(f*x +
e)^2 + (a^4 + 2*a^3*b + a^2*b^2)*f)
```

3.364.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

3.364.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.47

$$\int \frac{\cos^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{\frac{2\,\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}a^2} + \frac{\sin(fx+e)}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}a} + \frac{\sin(fx+e)}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}b} - \frac{\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}ab}}{3\,f}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^2) + sin(f*x + e)/((b*si
n(f*x + e)^2 + a)^(3/2)*a) + sin(f*x + e)/((b*sin(f*x + e)^2 + a)^(3/2)*b)
 - sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a*b))/f
```

3.364.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35378 vs. 2(65) = 130.

Time = 104.20 (sec) , antiderivative size = 35378, normalized size of antiderivative = 484.63

$$\int \frac{\cos^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(((((((3*a^8*b^4*tan(1/2*e)^57 + 12*a^7*b^5*tan(1/2*e)^57 + 18*a^6*b^
6*tan(1/2*e)^57 + 12*a^5*b^7*tan(1/2*e)^57 + 3*a^4*b^8*tan(1/2*e)^57 + 80*
a^8*b^4*tan(1/2*e)^55 + 340*a^7*b^5*tan(1/2*e)^55 + 560*a^6*b^6*tan(1/2*e)
^55 + 440*a^5*b^7*tan(1/2*e)^55 + 160*a^4*b^8*tan(1/2*e)^55 + 20*a^3*b^9*t
an(1/2*e)^55 + 1030*a^8*b^4*tan(1/2*e)^53 + 4624*a^7*b^5*tan(1/2*e)^53 + 8
228*a^6*b^6*tan(1/2*e)^53 + 7272*a^5*b^7*tan(1/2*e)^53 + 3238*a^4*b^8*tan(
1/2*e)^53 + 632*a^3*b^9*tan(1/2*e)^53 + 32*a^2*b^10*tan(1/2*e)^53 + 8528*a
^8*b^4*tan(1/2*e)^51 + 40228*a^7*b^5*tan(1/2*e)^51 + 76400*a^6*b^6*tan(1/2
*e)^51 + 73880*a^5*b^7*tan(1/2*e)^51 + 37600*a^4*b^8*tan(1/2*e)^51 + 9188*
a^3*b^9*tan(1/2*e)^51 + 768*a^2*b^10*tan(1/2*e)^51 + 51025*a^8*b^4*tan(1/2
*e)^49 + 251684*a^7*b^5*tan(1/2*e)^49 + 505318*a^6*b^6*tan(1/2*e)^49 + 524
932*a^5*b^7*tan(1/2*e)^49 + 294353*a^4*b^8*tan(1/2*e)^49 + 82912*a^3*b^9*t
an(1/2*e)^49 + 8832*a^2*b^10*tan(1/2*e)^49 + 235040*a^8*b^4*tan(1/2*e)^47
+ 1206776*a^7*b^5*tan(1/2*e)^47 + 2541472*a^6*b^6*tan(1/2*e)^47 + 2798928*
a^5*b^7*tan(1/2*e)^47 + 1690112*a^4*b^8*tan(1/2*e)^47 + 525688*a^3*b^9*tan
(1/2*e)^47 + 64768*a^2*b^10*tan(1/2*e)^47 + 867100*a^8*b^4*tan(1/2*e)^45 +
 4613984*a^7*b^5*tan(1/2*e)^45 + 10124968*a^6*b^6*tan(1/2*e)^45 + 11702032
*a^5*b^7*tan(1/2*e)^45 + 7489628*a^4*b^8*tan(1/2*e)^45 + 2505712*a^3*b^9*t
an(1/2*e)^45 + 340032*a^2*b^10*tan(1/2*e)^45 + 2631200*a^8*b^4*tan(1/2*e)^
43 + 14449336*a^7*b^5*tan(1/2*e)^43 + 32845472*a^6*b^6*tan(1/2*e)^43 + ...
```

3.364.9 Mupad [B] (verification not implemented)

Time = 21.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.51

$$\int \frac{\cos^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{2\,e^{e\,1i+f\,x\,1i}\,\left(e^{e\,2i+f\,x\,2i}-1\right)\,\sqrt{a+b\,\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,\left(a\,1i-b\,2i+a\,e^{e\,2i+f\,x\,2i}\,10i+a\,e^{e\,4i+f\,x\,4i}\,1i\right.}{3\,a^2\,f\,\left(b-4\,a\,e^{e\,2i+f\,x\,2i}-2\,b\,e^{e\,2i+f\,x\,2i}+b\,e^{e\,4i+f\,x\,4i}\right)^2}$$

input
```
int(cos(e + f*x)^3/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
-(2*exp(e*1i + f*x*1i)*(exp(e*2i + f*x*2i) - 1)*(a + b*((exp(- e*1i - f*x*
1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*(a*1i - b*2i + a*exp(e*2i
+ f*x*2i)*10i + a*exp(e*4i + f*x*4i)*1i + b*exp(e*2i + f*x*2i)*4i - b*exp(
e*4i + f*x*4i)*2i))/(3*a^2*f*(b - 4*a*exp(e*2i + f*x*2i) - 2*b*exp(e*2i +
f*x*2i) + b*exp(e*4i + f*x*4i))^2)
```

3.365 $\int \frac{\cos(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.365.1 Optimal result

Integrand size = 23, antiderivative size = 65

$$\int \frac{\cos(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{\sin(e+fx)}{3af\,(a+b\sin^2(e+fx))^{3/2}} + \frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
1/3*sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*sin(f*x+e)/a^2/f/(a+b*sin(
f*x+e)^2)^(1/2)
```

3.365.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{\cos(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{\sin(e+fx)\,(3a+2b\sin^2(e+fx))}{3a^2f\,(a+b\sin^2(e+fx))^{3/2}}$$

input
```
Integrate[Cos[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(Sin[e + f*x]*(3*a + 2*b*Sin[e + f*x]^2))/(3*a^2*f*(a + b*Sin[e + f*x]^2)^
(3/2))
```

3.365.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3669, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(e+fx)}{\left(a+b\sin(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{1}{\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{2\int \frac{1}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3a}+\frac{\sin(e+fx)}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{2\sin(e+fx)}{3a^2\sqrt{a+b\sin^2(e+fx)}}+\frac{\sin(e+fx)}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{f}$$

input
```
Int[Cos[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(Sin[e + f*x]/(3*a*(a + b*Sin[e + f*x]^2)^(3/2)) + (2*Sin[e + f*x])/(3*a^2
*Sqrt[a + b*Sin[e + f*x]^2]))/f
```

3.365.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.365.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{\sin(fx+e)}{3a\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\sin(fx+e)}{3a^2\sqrt{a+b\left(\sin^2(fx+e)\right)}}}{f}$	56
default	$\frac{\frac{\sin(fx+e)}{3a\left(a+b\left(\sin^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\sin(fx+e)}{3a^2\sqrt{a+b\left(\sin^2(fx+e)\right)}}}{f}$	56

input
```
int(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/3*sin(f*x+e)/a/(a+b*sin(f*x+e)^2)^(3/2)+2/3/a^2*sin(f*x+e)/(a+b*sin
(f*x+e)^2)^(1/2))
```

3.365.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.60

$$\int \frac{\cos(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = -\frac{(2\,b\cos{(fx+e)}^2-3\,a-2\,b)\sqrt{-b\cos{(fx+e)}^2+a+b}\sin{(fx+e)}}{3\,(a^2b^2f\cos{(fx+e)}^4-2\,(a^3b+a^2b^2)f\cos{(fx+e)}^2+(a^4+2\,a^3b+a^2b^2)f)}$$

input
```
integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(2*b*cos(f*x + e)^2 - 3*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(
f*x + e)/(a^2*b^2*f*cos(f*x + e)^4 - 2*(a^3*b + a^2*b^2)*f*cos(f*x + e)^2
+ (a^4 + 2*a^3*b + a^2*b^2)*f)
```

3.365.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

3.365.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{\cos(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{\frac{2\,\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}a^2}+\frac{\sin(fx+e)}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}a}}{3\,f}$$

input
```
integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^2) + sin(f*x + e)/((b*si
n(f*x + e)^2 + a)^(3/2)*a))/f
```

3.365.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35375 vs. $2(57) = 114$.

Time = 64.00 (sec) , antiderivative size = 35375, normalized size of antiderivative = 544.23

$$\int \frac{\cos(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(((((((3*a^8*b^4*tan(1/2*e)^57 + 12*a^7*b^5*tan(1/2*e)^57 + 18*a^6*b^
6*tan(1/2*e)^57 + 12*a^5*b^7*tan(1/2*e)^57 + 3*a^4*b^8*tan(1/2*e)^57 + 84*
a^8*b^4*tan(1/2*e)^55 + 356*a^7*b^5*tan(1/2*e)^55 + 584*a^6*b^6*tan(1/2*e)
^55 + 456*a^5*b^7*tan(1/2*e)^55 + 164*a^4*b^8*tan(1/2*e)^55 + 20*a^3*b^9*t
an(1/2*e)^55 + 1134*a^8*b^4*tan(1/2*e)^53 + 5056*a^7*b^5*tan(1/2*e)^53 + 8
916*a^6*b^6*tan(1/2*e)^53 + 7784*a^5*b^7*tan(1/2*e)^53 + 3406*a^4*b^8*tan(
1/2*e)^53 + 648*a^3*b^9*tan(1/2*e)^53 + 32*a^2*b^10*tan(1/2*e)^53 + 9828*a
^8*b^4*tan(1/2*e)^51 + 45812*a^7*b^5*tan(1/2*e)^51 + 85736*a^6*b^6*tan(1/2
*e)^51 + 81384*a^5*b^7*tan(1/2*e)^51 + 40436*a^4*b^8*tan(1/2*e)^51 + 9572*
a^3*b^9*tan(1/2*e)^51 + 768*a^2*b^10*tan(1/2*e)^51 + 61425*a^8*b^4*tan(1/2
*e)^49 + 297700*a^7*b^5*tan(1/2*e)^49 + 585382*a^6*b^6*tan(1/2*e)^49 + 593
028*a^5*b^7*tan(1/2*e)^49 + 322417*a^4*b^8*tan(1/2*e)^49 + 87328*a^3*b^9*t
an(1/2*e)^49 + 8832*a^2*b^10*tan(1/2*e)^49 + 294840*a^8*b^4*tan(1/2*e)^47
+ 1478360*a^7*b^5*tan(1/2*e)^47 + 3029808*a^6*b^6*tan(1/2*e)^47 + 3232432*
a^5*b^7*tan(1/2*e)^47 + 1879448*a^4*b^8*tan(1/2*e)^47 + 558072*a^3*b^9*tan
(1/2*e)^47 + 64768*a^2*b^10*tan(1/2*e)^47 + 1130220*a^8*b^4*tan(1/2*e)^45
+ 5836480*a^7*b^5*tan(1/2*e)^45 + 12383752*a^6*b^6*tan(1/2*e)^45 + 1377460
8*a^5*b^7*tan(1/2*e)^45 + 8432812*a^4*b^8*tan(1/2*e)^45 + 2675728*a^3*b^9*
tan(1/2*e)^45 + 340032*a^2*b^10*tan(1/2*e)^45 + 3552120*a^8*b^4*tan(1/2*e)
^43 + 18813080*a^7*b^5*tan(1/2*e)^43 + 41091248*a^6*b^6*tan(1/2*e)^43 +...
```

3.365.9 Mupad [B] (verification not implemented)

Time = 19.36 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.52

$$\int \frac{\cos(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{4\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)\,\sqrt{a+b\,{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,\left(b\,1\mathrm{i}-a\,\mathrm{e}^{e\,2\mathrm{i}+\dots}\right.}{3\,a^2\,f\,\left(b-4\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-2\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+b\,\mathrm{e}^{e\,4\mathrm{i}\dots}\right.}$$

input
```
int(cos(e + f*x)/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
(4*exp(e*1i + f*x*1i)*(exp(e*2i + f*x*2i) - 1)*(a + b*((exp(- e*1i - f*x*1
i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*(b*1i - a*exp(e*2i + f*x*2i
)*6i - b*exp(e*2i + f*x*2i)*2i + b*exp(e*4i + f*x*4i)*1i))/(3*a^2*f*(b - 4
*a*exp(e*2i + f*x*2i) - 2*b*exp(e*2i + f*x*2i) + b*exp(e*4i + f*x*4i))^2)
```

3.365. $\int \frac{\cos(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.366 $\int \frac{\sec(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.366.1 Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{\sec(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{(a+b)^{5/2}f} + \frac{b\sin(e+fx)}{3a(a+b)f\,(a+b\sin^2(e+fx))^{3/2}} + \frac{b(5a+2b)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/(a+b)^(5/2)/f+1/3
*b*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+1/3*b*(5*a+2*b)*sin(f*x+e
)/a^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.366.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 9.57 (sec) , antiderivative size = 1291, normalized size of antiderivative = 10.25

$$\int \frac{\sec(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(Sec[e + f*x]*Tan[e + f*x]*(1575*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)
]] + (2100*b*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^2)/a
 + (840*b^2*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^4)/a^
2 + (3150*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Tan[e + f*x]
^2)/a + (4200*b*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e
+ f*x]^2*Tan[e + f*x]^2)/a^2 + (1680*b^2*(a + b)*ArcSin[Sqrt[-(((a + b)*Ta
n[e + f*x]^2)/a)]]*Sin[e + f*x]^4*Tan[e + f*x]^2)/a^3 + (1575*(a + b)^2*Ar
cSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Tan[e + f*x]^4)/a^2 + (2100*b*(a
 + b)^2*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^2*Tan[e +
 f*x]^4)/a^3 + (840*b^2*(a + b)^2*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a
)]]*Sin[e + f*x]^4*Tan[e + f*x]^4)/a^4 + 2100*Sqrt[(Sec[e + f*x]^2*(a + b*
Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2) + (2800*b*Sin[e
+ f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e
 + f*x]^2)/a))^(3/2))/a + (1120*b^2*Sin[e + f*x]^4*Sqrt[(Sec[e + f*x]^2*(a
 + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2))/a^2 + 96*H
ypergeometric2F1[2, 2, 9/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sqrt[(Sec[e + f
*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2) + 2
4*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Tan[e + f*x]^2)/a)]*Sq
rt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/
a))^(7/2) + (168*b*Hypergeometric2F1[2, 2, 9/2, -(((a + b)*Tan[e + f*x]...
```

3.366.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3669, 316, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(e+fx)\left(a+b\sin(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{b\sin(e+fx)}{3a(a+b)(a+b\sin^2(e+fx))^{3/2}}-\frac{\int\frac{2b\sin^2(e+fx)+b-3(a+b)}{(1-\sin^2(e+fx))(b\sin^2(e+fx)+a)^{3/2}}d\sin(e+fx)}{3a(a+b)}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{b\sin(e+fx)}{3a(a+b)(a+b\sin^2(e+fx))^{3/2}}-\frac{-\frac{\int\frac{3a^2}{(1-\sin^2(e+fx))\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a(a+b)}-\frac{b(5a+2b)\sin(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{b\sin(e+fx)}{3a(a+b)(a+b\sin^2(e+fx))^{3/2}}-\frac{-\frac{3a\int\frac{1}{(1-\sin^2(e+fx))\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a+b}-\frac{b(5a+2b)\sin(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{b\sin(e+fx)}{3a(a+b)(a+b\sin^2(e+fx))^{3/2}}-\frac{-\frac{3a\int\frac{1}{1-\frac{(a+b)\sin^2(e+fx)}{b\sin^2(e+fx)+a}}d\frac{\sin(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}}{a+b}-\frac{b(5a+2b)\sin(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{b\sin(e+fx)}{3a(a+b)(a+b\sin^2(e+fx))^{3/2}}-\frac{-\frac{3a\,\text{arctanh}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{(a+b)^{3/2}}-\frac{b(5a+2b)\sin(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}}{f}$$

input
```
Int[Sec[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
((b*Sin[e + f*x])/(3*a*(a + b)*(a + b*Sin[e + f*x]^2)^(3/2)) - ((-3*a*ArcT
anh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(a + b)^(3/2)
- (b*(5*a + 2*b)*Sin[e + f*x])/(a*(a + b)*Sqrt[a + b*Sin[e + f*x]^2]))/(3*
a*(a + b)))/f
```

3.366. $\int \frac{\sec(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.366.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.366. $\int \frac{\sec(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.366.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(112) = 224$.

Time = 1.63 (sec) , antiderivative size = 899, normalized size of antiderivative = 7.13

method	result
default	$\frac{-3a^4b^2 \ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}-2b\sin(fx+e)+2a}{1+\sin(fx+e)}\right)+3a^4b^2\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}+2b\sin(fx+e)+2a}{\sin(fx+e)-1}\right)-3\ln\left(\frac{2\sqrt{a+b}}{}\right.}{}$

input
```
int(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/b^2/(a+b)^(1/2)/a^2/(cos(f*x+e)^4*a^2*b^2+2*a*b^3*cos(f*x+e)^4+b^4*cos
(f*x+e)^4-2*cos(f*x+e)^2*a^3*b-6*cos(f*x+e)^2*a^2*b^2-6*a*b^3*cos(f*x+e)^2
-2*b^4*cos(f*x+e)^2+a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*(-3*a^4*b^2*ln(2/(1
+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))+3*a^
4*b^2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*
x+e)+a))-3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*s
in(f*x+e)+a))*a^2*b^4+3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)
^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^4-6*a^3*b^3*ln(2/(1+sin(f*x+e))*((a+b)^(1
/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))+6*a^3*b^3*ln(2/(sin(f*x+e)
-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))+3*a^2*b^4*(ln
(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))
-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+
a)))*cos(f*x+e)^4-2*cos(f*x+e)^2*sin(f*x+e)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b
^2)^(1/2)*(a+b)^(1/2)*b^4*(5*a+2*b)+4*sin(f*x+e)*(-b*cos(f*x+e)^2+(a*b^2+b
^3)/b^2)^(1/2)*(a+b)^(1/2)*b^3*(3*a^2+4*a*b+b^2)-6*a^2*b^3*(ln(2/(sin(f*x+
e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a+ln(2/(sin
(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b-ln(2
/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a
-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+
a))*b)*cos(f*x+e)^2)/f
```

3.366.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(112) = 224$.

Time = 0.59 (sec) , antiderivative size = 775, normalized size of antiderivative = 6.15

$$\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \left[\frac{3\left(a^2b^2\cos\left(fx+e\right)^4+a^4+2\,a^3b+a^2b^2-2\left(a^3b+a^2b^2\right)\cos\left(fx+e\right)^2\right)\sqrt{a}}{}\right.$$

$$-\frac{3\left(a^2b^2\cos\left(fx+e\right)^4+a^4+2\,a^3b+a^2b^2-2\left(a^3b+a^2b^2\right)\cos\left(fx+e\right)^2\right)\sqrt{-a-b}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2}{2\left((ab+b^2)\cos}\right.}{6\left(\left(a^5b^2+3\,a^4b^3+3\,a^3b^4+a^2b^5\right)f\cos\left(fx+e\right)^4-2\left(a^6b+\right.\right.}$$

input
```
integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a^2*b^2*cos(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 - 2*(a^3*b + a^
2*b^2)*cos(f*x + e)^2)*sqrt(a + b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)
^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2
- 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*
a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*(6*a^3*b + 14*a^2*b^2 + 10*a*b^3
 + 2*b^4 - (5*a^2*b^2 + 7*a*b^3 + 2*b^4)*cos(f*x + e)^2)*sqrt(-b*cos(f*x +
 e)^2 + a + b)*sin(f*x + e))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*
f*cos(f*x + e)^4 - 2*(a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)
*f*cos(f*x + e)^2 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 +
 a^2*b^5)*f), -1/6*(3*(a^2*b^2*cos(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 -
2*(a^3*b + a^2*b^2)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos
(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a
*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) - 2*(6*a^3*b
+ 14*a^2*b^2 + 10*a*b^3 + 2*b^4 - (5*a^2*b^2 + 7*a*b^3 + 2*b^4)*cos(f*x +
e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a^5*b^2 + 3*a^4*b^3
+ 3*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4 - 2*(a^6*b + 4*a^5*b^2 + 6*a^4*b^3
 + 4*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^2 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 1
0*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f)]
```

3.366.6 Sympy [F]

$$\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sec\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sec(e + f*x)/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.366.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(112) = 224$.

Time = 0.47 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.16

$$\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\frac{2\,b\sin(fx+e)}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}a^2+\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}ab} + \frac{6\,b\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}\,a^3+2\,\sqrt{b\sin(fx+e)^2+a}\,a^2b+\sqrt{bs}}}{}$$

input
```
integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(2*b*sin(f*x + e)/((b*sin(f*x + e)^2 + a)^(3/2)*a^2 + (b*sin(f*x + e)^
2 + a)^(3/2)*a*b) + 6*b*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^3 + 2*s
qrt(b*sin(f*x + e)^2 + a)*a^2*b + sqrt(b*sin(f*x + e)^2 + a)*a*b^2) + 4*b*
sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^3 + sqrt(b*sin(f*x + e)^2 + a)*
a^2*b) + 3*arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a/(sqrt
(a*b)*(sin(f*x + e) + 1)))/(a + b)^(5/2) + 3*arcsinh(-b*sin(f*x + e)/(sqrt
(a*b)*(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1)))/(a + b)^(5/2
))/f
```

3.366.8 Giac [F]

$$\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sec(fx+e)}{\left(b\sin(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.366.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\cos(e+f\,x)\,\left(b\sin(e+f\,x)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(5/2)), x)
```

3.367 $\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.367.1 Optimal result

Integrand size = 25, antiderivative size = 243

$$\begin{aligned}\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx &= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} \\ &- \frac{2(2a-b)(a+b)\cos(e+fx)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}} \\ &- \frac{(8a^2+3ab-2b^2)\,E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3a^2b^3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} \\ &+ \frac{(8a-b)(a+b)\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3ab^3f\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

output
```
1/3*(a+b)*cos(f*x+e)^3*sin(f*x+e)/a/b/f/(a+b*sin(f*x+e)^2)^(3/2)-2/3*(2*a-
b)*(a+b)*cos(f*x+e)*sin(f*x+e)/a^2/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(8*a
^2+3*a*b-2*b^2)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a
)^(1/2))*(a+b*sin(f*x+e)^2)^(1/2)/a^2/b^3/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3
*(8*a-b)*(a+b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)
^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/a/b^3/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.367.2 Mathematica [A] (verified)

Time = 2.85 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.80

$$\int \frac{\cos^6(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{-2a^2(8a^2+3ab-2b^2)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)+\frac{1}{2}(a+b)\left(4a\right.}{}$$

input
```
Integrate[Cos[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(-2*a^2*(8*a^2 + 3*a*b - 2*b^2)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*E
llipticE[e + f*x, -(b/a)] + ((a + b)*(4*a^2*(8*a - b)*((2*a + b - b*Cos[2*
(e + f*x)])/a)^(3/2)*EllipticF[e + f*x, -(b/a)] - 2*Sqrt[2]*b*(8*a^2 - a*b
 - 2*b^2 + b*(-5*a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/2)/(6*a^2*b
^3*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

3.367.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3671, 315, 25, 401, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(e+fx)^6}{(a+b\sin(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{(1-\sin^2(e+fx))^{5/2}}{(b\sin^2(e+fx)+a)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int -\frac{\sqrt{1-\sin^2(e+fx)}\left(-\left((4a+b)\sin^2(e+fx)\right)+a-2b\right)}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3ab}+\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\int \frac{\sqrt{1-\sin^2(e+fx)}\left(-\left((4a+b)\sin^2(e+fx)\right)+a-2b\right)}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3ab}\right)}{f}$$

$$\Big\downarrow \; 401$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2\left(\frac{2a}{b}-\frac{b}{a}+1\right)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{\sqrt{a+b\sin^2(e+fx)}}-\frac{\int\frac{a(4a+b)-\left(8a^2+3ba-2b^2\right)\sin}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+}}}{ab}}{3ab}\right.}{f}$$

$$\Big\downarrow \; 399$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2\left(\frac{2a}{b}-\frac{b}{a}+1\right)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{\sqrt{a+b\sin^2(e+fx)}}-\frac{a(8a-b)(a+b)\int\frac{}{\sqrt{1-\sin^2(e+f}}}{}}{}\right.}{f}$$

$$\Big\downarrow \; 323$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2\left(\frac{2a}{b}-\frac{b}{a}+1\right)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{\sqrt{a+b\sin^2(e+fx)}}-\frac{a(8a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+}}{}}{}\right.}{f}$$

$$\Big\downarrow \; 321$$

3.367. $\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2\left(\frac{2a}{b}-\frac{b}{a}+1\right)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{\sqrt{a+b\sin^2(e+fx)}}-\frac{a(8a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+}}{b\sqrt{a+b}}}{}\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2\left(\frac{2a}{b}-\frac{b}{a}+1\right)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{\sqrt{a+b\sin^2(e+fx)}}-\frac{a(8a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+}}{b\sqrt{a+b}}}{}\right.}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2\left(\frac{2a}{b}-\frac{b}{a}+1\right)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{\sqrt{a+b\sin^2(e+fx)}}-\frac{a(8a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+}}{b\sqrt{a+b}}}{}\right.}{f}$$

input
```
Int[Cos[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(((a + b)*Sin[e + f*x]*(1 - Sin[e + f*x
]^2)^(3/2))/(3*a*b*(a + b*Sin[e + f*x]^2)^(3/2)) - ((2*(1 + (2*a)/b - b/a)
*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2])/Sqrt[a + b*Sin[e + f*x]^2] - (-(((
8*a^2 + 3*a*b - 2*b^2)*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*
Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) + (a*(8*a - b)*(a + b
)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/
(b*Sqrt[a + b*Sin[e + f*x]^2]))/(a*b))/(3*a*b)))/f
```

3.367. $\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.367.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

3.367. $\int \frac{\cos^6(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.367.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(265) = 530$.

Time = 4.94 (sec) , antiderivative size = 712, normalized size of antiderivative = 2.93

method	result
default	$\frac{\left(5a^{2}b^{2}+3a\,b^{3}-2b^{4}\right)\left(\cos^{4}\left(fx+e\right)\right)\sin\left(fx+e\right)+\left(-4a^{3}b-6a^{2}b^{2}+2b^{4}\right)\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)-\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^{2}\left(fx+e\right)\right)}{a}+}}{}$

input
```
int(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((5*a^2*b^2+3*a*b^3-2*b^4)*cos(f*x+e)^4*sin(f*x+e)+(-4*a^3*b-6*a^2*b^2
+2*b^4)*cos(f*x+e)^2*sin(f*x+e)-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a
+b)/a)^(1/2)*a*b*(8*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+7*EllipticF(s
in(f*x+e),(-1/a*b)^(1/2))*a*b-EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-8*E
llipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2-3*EllipticE(sin(f*x+e),(-1/a*b)^(1
/2))*a*b+2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b^2)*cos(f*x+e)^2+8*(cos(f
*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/
a*b)^(1/2))*a^4+15*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*
EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3*b+6*(cos(f*x+e)^2)^(1/2)*(-b/a*co
s(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2-(co
s(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(
-1/a*b)^(1/2))*a*b^3-8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1
/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^4-11*(cos(f*x+e)^2)^(1/2)*(-b/a
*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3*b-(c
os(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),
(-1/a*b)^(1/2))*a^2*b^2+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)
^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3)/a^2/(a+b*sin(f*x+e)^2)^
(3/2)/b^3/cos(f*x+e)/f
```

3.367.5 Fricas [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^6}{(b\sin(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^6/(b^3*cos(f*x + e)
^6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a
^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

3.367.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**6/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

3.367.7 Maxima [F]

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cos\left(fx+e\right)^6}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.367.8 Giac [F]

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cos\left(fx+e\right)^6}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.367.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+f\,x)^6}{\left(b\sin(e+f\,x)^2+a\right)^{5/2}}\,dx$$

input
```
int(cos(e + f*x)^6/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(cos(e + f*x)^6/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.368 $\int \frac{\cos^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.368.1 Optimal result

Integrand size = 25, antiderivative size = 223

$$\int \frac{\cos^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{(a+b)\cos(e+fx)\sin(e+fx)}{3abf\,(a+b\sin^2(e+fx))^{3/2}} - \frac{2(a-b)\cos(e+fx)\sin(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} - \frac{2(a-b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3ab^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{(2a-b)\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3ab^2f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
1/3*(a+b)*cos(f*x+e)*sin(f*x+e)/a/b/f/(a+b*sin(f*x+e)^2)^(3/2)-2/3*(a-b)*c
os(f*x+e)*sin(f*x+e)/a^2/b/f/(a+b*sin(f*x+e)^2)^(1/2)-2/3*(a-b)*(cos(f*x+e
)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2
/a)^(1/2)/a/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*(2*a-b)*(cos(f*x+e)^2)^(1/2
)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)
/a/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.368.2 Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.77

$$\int \frac{\cos^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{-2a^2(a-b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)+a^2(2a-b)\left(\frac{2a+b-b\cos(2}{a}\right.}{3a^2b^2f}$$

input
```
Integrate[Cos[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(-2*a^2*(a - b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x
, -(b/a)] + a^2*(2*a - b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*Ellipti
cF[e + f*x, -(b/a)] - Sqrt[2]*b*(a^2 - 2*a*b - b^2 + b*(-a + b)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/(3*a^2*b^2*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2
))
```

3.368.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.22, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3671, 315, 25, 402, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(e+fx)^4}{(a+b\sin(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \; 3671$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{(1-\sin^2(e+fx))^{3/2}}{(b\sin^2(e+fx)+a)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 315$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int -\frac{-\left((2a-b)\sin^2(e+fx)\right)+a-2b}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3ab}+\frac{(a+b)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{f}$$

$\big\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\int \frac{-\left((2a-b)\sin^2(e+fx)\right)+a-2b}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3ab}\right)}{f}$$

$\big\downarrow$ 402

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{a\sqrt{a+b\sin^2(e+fx)}}-\frac{\int \frac{a(a+b)-2\left(a^2-b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin}{a(a+b)}}{3ab}\right.}{f}$$

$\big\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{a\sqrt{a+b\sin^2(e+fx)}}-\frac{a(2a-b)(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2}}}{b}}{3ab}\right.}{f}$$

$\big\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{a\sqrt{a+b\sin^2(e+fx)}}-\frac{a(2a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{}{\sqrt{1-s}}}{b\sqrt{a+b\sin}}}{}\right.}{f}$$

$\big\downarrow$ 321

3.368. $\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{a\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{a(2a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}}{b\sqrt{a+b\sin^2(e+f}}}{}}{3ab}\right.}{f}$$

$\big\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{a\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{a(2a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}}{b\sqrt{a+b\sin^2(e+f}}}{}}{}\right.}{f}$$

$\big\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{a\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{a(2a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}}{b\sqrt{a+b\sin^2(e+f}}}{}}{}\right.}{f}$$

input
```
Int[Cos[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(((a + b)*Sin[e + f*x]*Sqrt[1 - Sin[e +
 f*x]^2])/(3*a*b*(a + b*Sin[e + f*x]^2)^(3/2)) - ((2*(a - b)*Sin[e + f*x]*
Sqrt[1 - Sin[e + f*x]^2])/(a*Sqrt[a + b*Sin[e + f*x]^2]) - ((-2*(a^2 - b^2
)*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*S
qrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(2*a - b)*(a + b)*EllipticF[ArcSin[Sin
[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e +
f*x]^2]))/(a*(a + b)))/(3*a*b)))/f
```

3.368. $\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.368.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

3.368. $\int \frac{\cos^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.368.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.17

method	result
default	$\frac{(2a\,b^2-2b^3)\left(\cos^4(fx+e)\right)\sin(fx+e)+\left(-a^2b+a\,b^2+2b^3\right)\left(\cos^2(fx+e)\right)\sin(fx+e)-\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}\,ab\Big(2}{}$

input
```
int(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((2*a*b^2-2*b^3)*cos(f*x+e)^4*sin(f*x+e)+(-a^2*b+a*b^2+2*b^3)*cos(f*x+
e)^2*sin(f*x+e)-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*b
*(2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a-EllipticF(sin(f*x+e),(-1/a*b)^(
1/2))*b-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a+2*EllipticE(sin(f*x+e),(-
1/a*b)^(1/2))*b)*cos(f*x+e)^2+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a
+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+(cos(f*x+e)^2)^(1/2)
*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^
2*b-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f
*x+e),(-1/a*b)^(1/2))*a*b^2-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b
)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3+2*(cos(f*x+e)^2)^(1/2)
*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*
b^2)/a^2/(a+b*sin(f*x+e)^2)^(3/2)/b^2/cos(f*x+e)/f
```

3.368.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 1343, normalized size of antiderivative = 6.02

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((2*((-I*a*b^3 + I*b^4)*cos(f*x + e)^4 - I*a^3*b - I*a^2*b^2 + I*a*b^3
 + I*b^4 - 2*(-I*a^2*b^2 + I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b
)/b^2) - ((2*I*a^2*b^2 - I*a*b^3 - I*b^4)*cos(f*x + e)^4 + 2*I*a^4 + 3*I*a
^3*b - I*a^2*b^2 - 3*I*a*b^3 - I*b^4 + 2*(-2*I*a^3*b - I*a^2*b^2 + 2*I*a*b
^3 + I*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*
a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*
(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*s
qrt((a^2 + a*b)/b^2))/b^2) + (2*((I*a*b^3 - I*b^4)*cos(f*x + e)^4 + I*a^3*
b + I*a^2*b^2 - I*a*b^3 - I*b^4 - 2*(I*a^2*b^2 - I*b^4)*cos(f*x + e)^2)*sq
rt(-b)*sqrt((a^2 + a*b)/b^2) - ((-2*I*a^2*b^2 + I*a*b^3 + I*b^4)*cos(f*x +
 e)^4 - 2*I*a^4 - 3*I*a^3*b + I*a^2*b^2 + 3*I*a*b^3 + I*b^4 + 2*(2*I*a^3*b
 + I*a^2*b^2 - 2*I*a*b^3 - I*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt
((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*
b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b
^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((I*a*b^3 - 2*I*b^4)
*cos(f*x + e)^4 + I*a^3*b - 3*I*a*b^3 - 2*I*b^4 - 2*(I*a^2*b^2 - I*a*b^3 -
 2*I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((-2*I*a^2*b^2
- I*a*b^3)*cos(f*x + e)^4 - 2*I*a^4 - 5*I*a^3*b - 4*I*a^2*b^2 - I*a*b^3 +
2*(2*I*a^3*b + 3*I*a^2*b^2 + I*a*b^3)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*
sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a...
```

3.368.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**4/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

3.368.7 Maxima [F]

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.368.8 Giac [F]

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.368.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cos\left(e+f\,x\right)^4}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(cos(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(cos(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.369 $\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.369.1 Optimal result

Integrand size = 25, antiderivative size = 217

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\cos(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(a+2b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{(a+2b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3ab(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3abf\sqrt{a+b\sin^2(e+fx)}}$$

output
```
1/3*cos(f*x+e)*sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(3/2)+1/3*(a+2*b)*cos(f*x
+e)*sin(f*x+e)/a^2/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*(a+2*b)*(cos(f*x+e
)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2
/a)^(1/2)/a/b/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(cos(f*x+e)^2)^(1/2)/co
s(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/a/b
/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.369.2 Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.81

$$\int \frac{\cos^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{2a^2(a+2b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)-2a^2(a+b)\left(\frac{2a+b-b\cos(2(\right.}{a}}{6a^2b(a+}$$

input
```
Integrate[Cos[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(2*a^2*(a + 2*b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*
x, -(b/a)] - 2*a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*Ellipt
icF[e + f*x, -(b/a)] - Sqrt[2]*b*(-4*a^2 - 7*a*b - 2*b^2 + b*(a + 2*b)*Cos
[2*(e + f*x)])*Sin[2*(e + f*x)])/(6*a^2*b*(a + b)*f*(2*a + b - b*Cos[2*(e
+ f*x)])^(3/2))
```

3.369.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3671, 314, 25, 402, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(e+fx)^2}{(a+b\sin(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 3671}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sqrt{1-\sin^2(e+fx)}}{(b\sin^2(e+fx)+a)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 314}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\int -\frac{2-\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3a}\right)}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int \frac{2-\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3a}+\frac{\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{f}$$

$\downarrow$ 402

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{(a+2b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\int -\frac{(a+2b)\sin^2(e+fx)+a}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a(a+b)}}{3a}+\frac{\sqrt{1-\sin^2(e+fx)}\sin(e+f\cdots}{3a\left(a+b\sin^2(e+fx)\right)^{3/2\cdots}}\cdots\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{\int \frac{(a+2b)\sin^2(e+fx)+a}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a(a+b)}+\frac{(a+2b)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a}+\frac{\sqrt{1-\sin^2(e+fx)}\sin(e+fx\cdots}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}\cdots\right.}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{\frac{(a+2b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}}{a(a+b)}+\frac{(a+2b)\sqrt{1-\sin^2(e-\cdots}}{a(a+b)\sqrt{a+b\,\mathrm{s}\cdots}}}{3a}\cdots\right.}{f}$$

$\downarrow$ 323

3.369. $\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{\frac{(a+2b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}}{a(a+b)}+}{3a}\right.}{f}$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{\frac{(a+2b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}}{a(a+b)}+\frac{(a+2b)\sqrt{1-\sin^2}}{a(a+b)\sqrt{a}}}{3a}\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{\frac{(a+2b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}}{a(a+b)}}{3a}\right.}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{\frac{(a+2b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}}{a(a+b)}+\frac{(a-}{}}{3a}\right.}{f}$$

input
```
Int[Cos[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

3.369. $\int \frac{\cos^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]
)/(3*a*(a + b*Sin[e + f*x]^2)^(3/2)) + (((a + 2*b)*Sin[e + f*x]*Sqrt[1 - S
in[e + f*x]^2])/(a*(a + b)*Sqrt[a + b*Sin[e + f*x]^2]) + (((a + 2*b)*Ellip
ticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a
)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2]))/(a*(a +
 b)))/(3*a)))/f
```

3.369.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

3.369. $\int \frac{\cos^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.369.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(239) = 478$.

Time = 3.10 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.53

method	result
default	$-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2b\left(\sin^2(fx+e)\right)+\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e\right.}{}$

input
```
int(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+
e),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e
)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2-(cos
(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b
)^(1/2))*a^2*b*sin(f*x+e)^2-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^
(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2+a*b^2*sin(f*
x+e)^5+2*b^3*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2
)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+a^2*(cos(f*x+e)^2)^(1/2)*((a+b*
sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-(cos(f*x+e)^
2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))
*a^3-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))*a^2*b+2*a^2*b*sin(f*x+e)^3+2*a*b^2*sin(f*x+e)^3-2*b^3*
sin(f*x+e)^3-2*a^2*b*sin(f*x+e)-3*a*b^2*sin(f*x+e))/a^2/(a+b)/(a+b*sin(f*x
+e)^2)^(3/2)/b/cos(f*x+e)/f
```

3.369.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 1394, normalized size of antiderivative = 6.42

$$\int \frac{\cos^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/6*((2*((I*a*b^3 + 2*I*b^4)*cos(f*x + e)^4 + I*a^3*b + 4*I*a^2*b^2 + 5*I*
a*b^3 + 2*I*b^4 - 2*(I*a^2*b^2 + 3*I*a*b^3 + 2*I*b^4)*cos(f*x + e)^2)*sqrt
(-b)*sqrt((a^2 + a*b)/b^2) - ((-2*I*a^2*b^2 - 5*I*a*b^3 - 2*I*b^4)*cos(f*x
 + e)^4 - 2*I*a^4 - 9*I*a^3*b - 14*I*a^2*b^2 - 9*I*a*b^3 - 2*I*b^4 + 2*(2*
I*a^3*b + 7*I*a^2*b^2 + 7*I*a*b^3 + 2*I*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqr
t((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqr
t((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2
+ 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((-I*a*b^
3 - 2*I*b^4)*cos(f*x + e)^4 - I*a^3*b - 4*I*a^2*b^2 - 5*I*a*b^3 - 2*I*b^4
- 2*(-I*a^2*b^2 - 3*I*a*b^3 - 2*I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2
+ a*b)/b^2) - ((2*I*a^2*b^2 + 5*I*a*b^3 + 2*I*b^4)*cos(f*x + e)^4 + 2*I*a^
4 + 9*I*a^3*b + 14*I*a^2*b^2 + 9*I*a*b^3 + 2*I*b^4 + 2*(-2*I*a^3*b - 7*I*a
^2*b^2 - 7*I*a*b^3 - 2*I*b^4)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^
2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b
^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 -
 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*(4*((I*a*b^3 + I*b^4)*cos
(f*x + e)^4 + I*a^3*b + 3*I*a^2*b^2 + 3*I*a*b^3 + I*b^4 + 2*(-I*a^2*b^2 -
2*I*a*b^3 - I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + ((-2*I
*a^2*b^2 - I*a*b^3)*cos(f*x + e)^4 - 2*I*a^4 - 5*I*a^3*b - 4*I*a^2*b^2 - I
*a*b^3 + 2*(2*I*a^3*b + 3*I*a^2*b^2 + I*a*b^3)*cos(f*x + e)^2)*sqrt(-b)...
```

3.369.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

3.369.7 Maxima [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^2}{(b\sin(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.369.8 Giac [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^2}{(b\sin(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.369.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+f\,x)^2}{(b\sin(e+f\,x)^2+a)^{5/2}}\,dx$$

input
```
int(cos(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(cos(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.370 $\int \frac{1}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.370.1 Optimal result

Integrand size = 16, antiderivative size = 223

$$\int \frac{1}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\,(a+b\sin^2(e+fx))^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3a^2(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3a(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
1/3*b*cos(f*x+e)*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*b*(2*a+
b)*cos(f*x+e)*sin(f*x+e)/a^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+2/3*(2*a+b
)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*
sin(f*x+e)^2)^(1/2)/a^2/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*(cos(f*x+
e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^
2/a)^(1/2)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.370.2 Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{2a^2(2a+b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)-a^2(a+b)\left(\frac{2a+b-b\cos(2(e-}{a}\right.}{3a^2(a+b}$$

input
```
Integrate[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

output
```
(2*a^2*(2*a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*
x, -(b/a)] - a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*Elliptic
F[e + f*x, -(b/a)] - Sqrt[2]*b*(-5*a^2 - 5*a*b - b^2 + b*(2*a + b)*Cos[2*(
e + f*x)])*Sin[2*(e + f*x)])/(3*a^2*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*
x)])^(3/2))
```

3.370.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 3663, 25, 3042, 3652, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\int -\frac{-b\sin^2(e+fx)+3a+2b}{\left(b\sin^2(e+fx)+a\right)^{3/2}}dx}{3a(a+b)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-b\sin^2(e+fx)+3a+2b}{\left(b\sin^2(e+fx)+a\right)^{3/2}}dx}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-b\sin(e+fx)^2+3a+2b}{(b\sin(e+fx)^2+a)^{3/2}}dx}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3652}$$

$$\frac{\frac{\int \frac{2b(2a+b)\sin^2(e+fx)+a(3a+b)}{\sqrt{b\sin^2(e+fx)+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{2b(2a+b)\sin(e+fx)^2+a(3a+b)}{\sqrt{b\sin(e+fx)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3651}$$

$$\frac{\frac{2(2a+b)\int \sqrt{b\sin^2(e+fx)+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(2a+b)\int \sqrt{b\sin(e+fx)^2+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3657}$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin^2(e+fx)}{a}+1}dx}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

3.370. $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\int\sqrt{\frac{b\sin(e+fx)^2}{a}+1}dx}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-a(a+b)\int\frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-a(a+b)\int\frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3662}$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}dx}{\sqrt{a+b\sin^2(e+fx)}}}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin(e+fx)^2}{a}+1}}dx}{\sqrt{a+b\sin^2(e+fx)}}}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3661}$$

$$\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}}{a(a+b)}}{3a(a+b)}$$

input
```
Int[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

3.370. $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

output
```
(b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
 + ((2*b*(2*a + b)*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[
e + f*x]^2]) + ((2*(2*a + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e +
 f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*
x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2]))
/(a*(a + b)))/(3*a*(a + b))
```

3.370.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.370.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(245) = 490$.

Time = 1.62 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.45

method	result
default	$-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2b\left(\sin^2(fx+e)\right)+\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e\right.}{}$

input
```
int(1/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+
e),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e
)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2-4*(c
os(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a^2*b*sin(f*x+e)^2-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a
)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2+4*a*b^2*si
n(f*x+e)^5+2*b^3*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^
(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+a^2*(cos(f*x+e)^2)^(1/2)*((
a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-4*(cos(f
*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a^3-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(s
in(f*x+e),(-1/a*b)^(1/2))*a^2*b+5*a^2*b*sin(f*x+e)^3-a*b^2*sin(f*x+e)^3-2*
b^3*sin(f*x+e)^3-5*a^2*b*sin(f*x+e)-3*a*b^2*sin(f*x+e))/(a+b*sin(f*x+e)^2)
^(3/2)/a^2/(a+b)^2/cos(f*x+e)/f
```

3.370.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 1531, normalized size of antiderivative = 6.87

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((2*(2*I*a^3*b^2 + 5*I*a^2*b^3 + 4*I*a*b^4 + I*b^5 + (2*I*a*b^4 + I*b^
5)*cos(f*x + e)^4 - 2*(2*I*a^2*b^3 + 3*I*a*b^4 + I*b^5)*cos(f*x + e)^2)*sq
rt(-b)*sqrt((a^2 + a*b)/b^2) - (-4*I*a^4*b - 12*I*a^3*b^2 - 13*I*a^2*b^3 -
 6*I*a*b^4 - I*b^5 + (-4*I*a^2*b^3 - 4*I*a*b^4 - I*b^5)*cos(f*x + e)^4 + 2
*(4*I*a^3*b^2 + 8*I*a^2*b^3 + 5*I*a*b^4 + I*b^5)*cos(f*x + e)^2)*sqrt(-b))
*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b
*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*
a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-2*I
*a^3*b^2 - 5*I*a^2*b^3 - 4*I*a*b^4 - I*b^5 + (-2*I*a*b^4 - I*b^5)*cos(f*x
+ e)^4 - 2*(-2*I*a^2*b^3 - 3*I*a*b^4 - I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqr
t((a^2 + a*b)/b^2) - (4*I*a^4*b + 12*I*a^3*b^2 + 13*I*a^2*b^3 + 6*I*a*b^4
+ I*b^5 + (4*I*a^2*b^3 + 4*I*a*b^4 + I*b^5)*cos(f*x + e)^4 + 2*(-4*I*a^3*b
^2 - 8*I*a^2*b^3 - 5*I*a*b^4 - I*b^5)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*
sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2
+ a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b
 + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-3*I*a^4*b - 11
*I*a^3*b^2 - 15*I*a^2*b^3 - 9*I*a*b^4 - 2*I*b^5 + (-3*I*a^2*b^3 - 5*I*a*b^
4 - 2*I*b^5)*cos(f*x + e)^4 - 2*(-3*I*a^3*b^2 - 8*I*a^2*b^3 - 7*I*a*b^4 -
2*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - (-6*I*a^5 - 17*I
*a^4*b - 17*I*a^3*b^2 - 7*I*a^2*b^3 - I*a*b^4 + (-6*I*a^3*b^2 - 5*I*a^2...
```

3.370.6 Sympy [F]

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Integral((a + b*sin(e + f*x)**2)**(-5/2), x)
```

3.370.7 Maxima [F]

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

3.370.8 Giac [F]

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.370.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(1/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.371 $\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.371.1 Optimal result

Integrand size = 25, antiderivative size = 288

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{(3a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{b(3a^2-7ab-2b^2)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{(3a^2-7ab-2b^2)\,E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3a(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} + \frac{(3a-b)\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

output
```
-1/3*(3*a-b)*b*cos(f*x+e)*sin(f*x+e)/a/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(3/2)-
1/3*b*(3*a^2-7*a*b-2*b^2)*cos(f*x+e)*sin(f*x+e)/a^2/(a+b)^3/f/(a+b*sin(f*x
+e)^2)^(1/2)-1/3*(3*a^2-7*a*b-2*b^2)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*Ellip
ticE(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/a/(a+b)^3/f/(a+b*
sin(f*x+e)^2)^(1/2)+1/3*(3*a-b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(
sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/a/(a+b)^2/f/(a+b*sin(f
*x+e)^2)^(1/2)+tan(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)
```

3.371.2 Mathematica [A] (verified)

Time = 4.01 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.85

$$\int \frac{\sec^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{-2a^2(3a^2-7ab-2b^2)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2} E\left(e+fx\middle|-\frac{b}{a}\right)+2a^2(3a^2+2a$$

input
```
Integrate[Sec[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(-2*a^2*(3*a^2 - 7*a*b - 2*b^2)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*E
llipticE[e + f*x, -(b/a)] + 2*a^2*(3*a^2 + 2*a*b - b^2)*((2*a + b - b*Cos[
2*(e + f*x)])/a)^(3/2)*EllipticF[e + f*x, -(b/a)] + ((24*a^4 + 24*a^3*b +
41*a^2*b^2 + 19*a*b^3 + 2*b^4 - 4*a*b*(6*a^2 - 5*a*b - 3*b^2)*Cos[2*(e + f
*x)] + b^2*(3*a^2 - 7*a*b - 2*b^2)*Cos[4*(e + f*x)])*Tan[e + f*x])/Sqrt[2]
)/(6*a^2*(a + b)^3*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

3.371.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.22, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 3671, 316, 27, 402, 25, 402, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(e+fx)^2\,(a+b\sin(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{1}{(1-\sin^2(e+fx))^{3/2}(b\sin^2(e+fx)+a)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int \frac{b\left(3\sin^2(e+fx)+1\right)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin(e+fx)}{a+b}+\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\int \frac{3\sin^2(e+fx)+1}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin(e+fx)}{a+b}+\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{f}$$

$\downarrow$ 402

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(-\frac{\int -\frac{(3a-b)\sin^2(e+fx)+2(3a+b)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}-\frac{(3a-b)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{3a(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{a+b}+\frac{\cdots}{(a+b)\sqrt{1-\sin^2\cdots}}\cdots\right)}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{\int \frac{(3a-b)\sin^2(e+fx)+2(3a+b)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}-\frac{(3a-b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3a(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{a+b}+\frac{\mathrm{s}\cdots}{(a+b)\sqrt{1-\sin^2(e\cdots}}\cdots\right)}{f}$$

$\downarrow$ 402

3.371. $\displaystyle\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{-\frac{\int -\frac{a(9a+b)-\left(3a^2-7ba-2b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a(a+b)}-\frac{\left(3a^2-7ab-2b^2\right)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}-\frac{(3a-b)\sin(}{3a(a+b)(}}{a+b}}{f}$$

$$\downarrow 25$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{\frac{\int \frac{a(9a+b)-\left(3a^2-7ba-2b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a(a+b)}-\frac{\left(3a^2-7ab-2b^2\right)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}-\frac{(3a-b)\sin(e+}{3a(a+b)(a+}}{a+b}}{f}$$

$$\downarrow 399$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{\frac{\frac{a(3a-b)(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{b}-\frac{\left(3a^2-7ab-2b^2\right)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b}}{a(a+b)}}{3a(a+b)}}{a+b}}{f}$$

$$\downarrow 323$$

3.371. $\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{\frac{a(3a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(3a^2-7ab-2b^2\right)\int\frac{\sqrt{b\sin^2(e+f}}{\sqrt{1-\sin^2(e}}}{b}}{a(a+b)}}{3a(a+b)}}{a+b}}{f}$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{\frac{a(3a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(3a^2-7ab-2b^2\right)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+f}{b}}{a(a+b)}}{3a(a+b)}}{a+b}}{f}$$

$\downarrow$ 330

3.371. $\int\frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{\frac{a(3a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(3a^2-7ab-2b^2\right)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e}{a}}}{\sqrt{1-\sin^2}}}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{a(a+b)}\right.}{3a(a+b)}}{a+b}\right.}{}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{b\left(\frac{\frac{a(3a-b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{b\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(3a^2-7ab-2b^2\right)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin\right.}{b\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}}{a(a+b)}\right.}{3a(a+b)}}{a+b}\right.}{f}$$

input
```
Int[Sec[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

3.371. $\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(Sin[e + f*x]/((a + b)*Sqrt[1 - Sin[e +
 f*x]^2]*(a + b*Sin[e + f*x]^2)^(3/2)) + (b*(-1/3*((3*a - b)*Sin[e + f*x]*
Sqrt[1 - Sin[e + f*x]^2])/(a*(a + b)*(a + b*Sin[e + f*x]^2)^(3/2)) + (-(((
3*a^2 - 7*a*b - 2*b^2)*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2])/(a*(a + b)*S
qrt[a + b*Sin[e + f*x]^2])) + (-(((3*a^2 - 7*a*b - 2*b^2)*EllipticE[ArcSin
[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*Sqrt[1 + (b*Sin[e +
 f*x]^2)/a])) + (a*(3*a - b)*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a
)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*Sqrt[a + b*Sin[e + f*x]^2]))/(a*(a +
 b)))/(3*a*(a + b))))/(a + b)))/f
```

3.371.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

3.371. $\int \frac{\sec^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.371. $\int \frac{\sec^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.371.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1081 vs. $2(308) = 616$.

Time = 4.79 (sec) , antiderivative size = 1082, normalized size of antiderivative = 3.76

method	result	size
default	Expression too large to display	1082

input
```
int(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*(3*a^2-7*a*b-2*b^2)*co
s(f*x+e)^4*sin(f*x+e)-2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(3*a^
3-a^2*b-5*a*b^2-b^3)*cos(f*x+e)^2*sin(f*x+e)-(cos(f*x+e)^2)^(1/2)*(-b*cos(
f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*b*(
3*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2*EllipticF(sin(f*x+e),(-1/a*b)
^(1/2))*a*b-EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-3*EllipticE(sin(f*x+e
),(-1/a*b)^(1/2))*a^2+7*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b+2*Ellipti
cE(sin(f*x+e),(-1/a*b)^(1/2))*b^2)*cos(f*x+e)^2+3*(-b*cos(f*x+e)^4+(a+b)*c
os(f*x+e)^2)^(1/2)*a^2*(a^2+2*a*b+b^2)*sin(f*x+e)+3*(-b*cos(f*x+e)^4+(a+b)
*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2
)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^4+5*(-b*cos(f*x+e)^4+(a+b)*cos(f*
x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Ellip
ticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3*b+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)
^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(si
n(f*x+e),(-1/a*b)^(1/2))*a^2*b^2-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2
)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*a*b^3-3*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(co
s(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^4+4*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e
)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a...
```

3.371.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 1746, normalized size of antiderivative = 6.06

$$\int \frac{\sec^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/6*((2*((-3*I*a^2*b^3 + 7*I*a*b^4 + 2*I*b^5)*cos(f*x + e)^5 - 2*(-3*I*a^3
*b^2 + 4*I*a^2*b^3 + 9*I*a*b^4 + 2*I*b^5)*cos(f*x + e)^3 + (-3*I*a^4*b + I
*a^3*b^2 + 13*I*a^2*b^3 + 11*I*a*b^4 + 2*I*b^5)*cos(f*x + e))*sqrt(-b)*sqr
t((a^2 + a*b)/b^2) - ((6*I*a^3*b^2 - 11*I*a^2*b^3 - 11*I*a*b^4 - 2*I*b^5)*
cos(f*x + e)^5 + 2*(-6*I*a^4*b + 5*I*a^3*b^2 + 22*I*a^2*b^3 + 13*I*a*b^4 +
 2*I*b^5)*cos(f*x + e)^3 + (6*I*a^5 + I*a^4*b - 27*I*a^3*b^2 - 35*I*a^2*b^
3 - 15*I*a*b^4 - 2*I*b^5)*cos(f*x + e))*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*
b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) +
2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*
a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((3*I*a^2*b^3 - 7*I*a*b^4 - 2*
I*b^5)*cos(f*x + e)^5 - 2*(3*I*a^3*b^2 - 4*I*a^2*b^3 - 9*I*a*b^4 - 2*I*b^5
)*cos(f*x + e)^3 + (3*I*a^4*b - I*a^3*b^2 - 13*I*a^2*b^3 - 11*I*a*b^4 - 2*
I*b^5)*cos(f*x + e))*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((-6*I*a^3*b^2 + 11*
I*a^2*b^3 + 11*I*a*b^4 + 2*I*b^5)*cos(f*x + e)^5 + 2*(6*I*a^4*b - 5*I*a^3*
b^2 - 22*I*a^2*b^3 - 13*I*a*b^4 - 2*I*b^5)*cos(f*x + e)^3 + (-6*I*a^5 - I*
a^4*b + 27*I*a^3*b^2 + 35*I*a^2*b^3 + 15*I*a*b^4 + 2*I*b^5)*cos(f*x + e))*
sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(
sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x +
e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) -
 2*(4*((3*I*a^2*b^3 + 4*I*a*b^4 + I*b^5)*cos(f*x + e)^5 + 2*(-3*I*a^3*b...
```

3.371.6 Sympy [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\sec^2(e+fx)}{(a+b\sin^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sec(e + f*x)**2/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.371.7 Maxima [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)^2}{(b\sin(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.371.8 Giac [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)^2}{(b\sin(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.371.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos(e+f\,x)^2\,(b\sin(e+f\,x)^2+a)^{5/2}}\,dx$$

input
```
int(1/(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(5/2)), x)
```

3.372 $\int (d\cos(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p dx$

3.372.1 Optimal result

Integrand size = 25, antiderivative size = 115

$$\int (d\cos(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p dx$$
$$= \frac{d\,\text{AppellF1}\left(\frac{1}{2},\frac{1-m}{2},-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)(d\cos(e+fx))^{-1+m}\cos^2(e+fx)^{\frac{1-m}{2}}\sin(e+fx)\,(\cdots}{f}$$

output
```
d*AppellF1(1/2,-1/2*m+1/2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(d*cos(f*
x+e))^(-1+m)*(cos(f*x+e)^2)^(-1/2*m+1/2)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^p/f
/((1+b*sin(f*x+e)^2/a)^p)
```

3.372.2 Mathematica [A] (warning: unable to verify)

Time = 1.43 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.98

$$\int (d\cos(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p dx$$
$$= \frac{3a\,\text{AppellF1}\left(\frac{1}{2},\frac{1-m}{2},-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)(\cdots}{f\left(3a\,\text{AppellF1}\left(\frac{1}{2},\frac{1-m}{2},-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)+\left(2bp\,\text{AppellF1}\left(\frac{3}{2},\frac{1-m}{2},1-p,\frac{5}{2},\sin^2(e+fx\cdots\right.\right.\right.}$$

input
```
Integrate[(d*Cos[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(3*a*AppellF1[1/2, (1 - m)/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2
)/a)]*(d*Cos[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*a*Ap
pellF1[1/2, (1 - m)/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] +
 (2*b*p*AppellF1[3/2, (1 - m)/2, 1 - p, 5/2, Sin[e + f*x]^2, -((b*Sin[e +
f*x]^2)/a)] - a*(-1 + m)*AppellF1[3/2, (3 - m)/2, -p, 5/2, Sin[e + f*x]^2,
 -((b*Sin[e + f*x]^2)/a)])*Sin[e + f*x]^2))
```

3.372.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3672, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\cos(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\cos(e+fx))^m \left(a+b\sin(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{3672}$$

$$\frac{d\cos^2(e+fx)^{\frac{1-m}{2}}(d\cos(e+fx))^{m-1}\int \left(1-\sin^2(e+fx)\right)^{\frac{m-1}{2}}\left(b\sin^2(e+fx)+a\right)^p d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{d\cos^2(e+fx)^{\frac{1-m}{2}}(d\cos(e+fx))^{m-1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\int \left(1-\sin^2(e+fx)\right)^{\frac{m-1}{2}}\left(\frac{b\sin^2(e+}{a}\right.}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{d\sin(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(d\cos(e+fx))^{m-1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},\frac{1-m}{2},-p\right.}{f}$$

input
```
Int[(d*Cos[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(d*AppellF1[1/2, (1 - m)/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/
a)]*(d*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*Sin[e + f*x]*(a
 + b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

3.372.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3672
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[f
f*d^(2*IntPart[(m - 1)/2] + 1)*((d*Cos[e + f*x])^(2*FracPart[(m - 1)/2])/(f
*(Cos[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e,
f, m, p}, x] &&  !IntegerQ[m]
```

3.372.4 Maple [F]

$$\int (\cos (fx+e)\, d)^{m} \left(a+b \left(\sin^{2}(fx+e)\right)\right)^{p} dx$$

input
```
int((cos(f*x+e)*d)^m*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int((cos(f*x+e)*d)^m*(a+b*sin(f*x+e)^2)^p,x)
```

3.372.5 Fricas [F]

$$\int (d\cos(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p \,dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p (d\cos\left(fx+e\right))^m \,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*(d*cos(f*x + e))^m, x)
```

3.372.6 Sympy [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p \,dx = \text{Timed out}$$

input
```
integrate((d*cos(f*x+e))**m*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.372.7 Maxima [F]

$$\int (d\cos(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p \,dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p (d\cos\left(fx+e\right))^m \,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

3.372.8 Giac [F]

$$\int (d\cos(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p \, dx = \int (b\sin(fx+e)^2+a)^p(d\cos(fx+e))^m \, dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

3.372.9 Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p \, dx = \int (d\cos(e+f\,x))^m \left(b\sin(e+f\,x)^2+a\right)^p \, dx$$

input
```
int((d*cos(e + f*x))^m*(a + b*sin(e + f*x)^2)^p,x)
```

output
```
int((d*cos(e + f*x))^m*(a + b*sin(e + f*x)^2)^p, x)
```

3.373 $\int \cos^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.373.1 Optimal result

Integrand size = 23, antiderivative size = 214

$$\int \cos^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{(3a+b(7+2p))\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} - \frac{\cos^2(e+fx)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{bf(5+2p)} + \frac{(3a^2+2ab(5+2p)+b^2(15+16p+4p^2))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+\right.}{b^2f(3+2p)(5+2p)}$$

output
```
-(3*a+b*(7+2*p))*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(p+1)/b^2/f/(4*p^2+16*p+15)
-cos(f*x+e)^2*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(p+1)/b/f/(5+2*p)+(3*a^2+2*a*b
*(5+2*p)+b^2*(4*p^2+16*p+15))*hypergeom([1/2, -p],[3/2],-b*sin(f*x+e)^2/a)
*sin(f*x+e)*(a+b*sin(f*x+e)^2)^p/b^2/f/(4*p^2+16*p+15)/((1+b*sin(f*x+e)^2/
a)^p)
```

3.373.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.44 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.89

$$\int \cos^5(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx$$
$$= \frac{3a \operatorname{AppellF1}\left(\frac{1}{2},-2,-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\cos^4(}{f\left(3a \operatorname{AppellF1}\left(\frac{1}{2},-2,-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)+2\left(bp \operatorname{AppellF1}\left(\frac{3}{2},-2,1-p,\frac{5}{2},\sin^2(e+fx)\right.\right.\right.}$$

input
```
Integrate[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(3*a*AppellF1[1/2, -2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*C
os[e + f*x]^4*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(3*a*AppellF1[1/2,
 -2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] + 2*(b*p*AppellF1[3
/2, -2, 1 - p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] - 2*a*AppellF
1[3/2, -1, -p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)])*Sin[e + f*x]
^2))
```

3.373.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3669, 318, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \cos(e+fx)^5 \left(a+b\sin(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{ 3669}$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)^2 \left(b\sin^2(e+fx)+a\right)^p d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 318}$$

$$\frac{\frac{\int (b\sin^2(e+fx)+a)^p(-((3a+b(2p+7))\sin^2(e+fx))+a+b(2p+5))d\sin(e+fx)}{b(2p+5)}-\frac{\sin(e+fx)(1-\sin^2(e+fx))(a+b\sin^2(e+fx))^{p+1}}{b(2p+5)}}{f}$$

$\Big\downarrow$ 299

$$\frac{\frac{\frac{\left(3a^2+2ab(2p+5)+b^2\left(4p^2+16p+15\right)\right)\int \left(b\sin^2(e+fx)+a\right)^p d\sin(e+fx)}{b(2p+3)}-\frac{(3a+b(2p+7))\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{p+1}}{b(2p+3)}}{b(2p+5)}-\frac{\sin(e+fx)(1-\sin^2(e+fx))(a+b\cdots}{b(2p+5)}}{f}$$

$\Big\downarrow$ 238

$$\frac{\frac{\frac{\left(3a^2+2ab(2p+5)+b^2\left(4p^2+16p+15\right)\right)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\int \left(\frac{b\sin^2(e+fx)}{a}+1\right)^p d\sin(e+fx)}{b(2p+3)}-\frac{(3a+b(2p+7))\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{p+\cdots}}{b(2p+3)}}{b(2p+5)}\cdots}{f}$$

$\Big\downarrow$ 237

$$\frac{\frac{\frac{\left(3a^2+2ab(2p+5)+b^2\left(4p^2+16p+15\right)\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\sin^2(e+fx)}{a}\right)}{b(2p+3)}-\frac{(3a+b(2p+7))\sin(e+f\cdots}{b(\cdots}}{b(2p+5)}\cdots}{f}$$

input
```
Int[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(-((Sin[e + f*x]*(1 - Sin[e + f*x]^2)*(a + b*Sin[e + f*x]^2)^(1 + p))/(b*(
5 + 2*p))) + (-(((3*a + b*(7 + 2*p))*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(
1 + p))/(b*(3 + 2*p))) + ((3*a^2 + 2*a*b*(5 + 2*p) + b^2*(15 + 16*p + 4*p^
2))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Sin[e + f*x]*
(a + b*Sin[e + f*x]^2)^p)/(b*(3 + 2*p)*(1 + (b*Sin[e + f*x]^2)/a)^p))/(b*(
5 + 2*p)))/f
```

3.373.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.373.4 Maple [F]

$$\int \left(\cos^{5}\left(fx+e\right)\right)\left(a+b\left(\sin^{2}\left(fx+e\right)\right)\right)^{p}dx$$

input
```
int(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

3.373.5 Fricas [F]

$$\int \cos^5(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p \cos\left(fx+e\right)^5 \, dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^5, x)
```

3.373.6 Sympy [F(-1)]

Timed out.

$$\int \cos^5(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.373.7 Maxima [F]

$$\int \cos^5(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p \cos\left(fx+e\right)^5 \, dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

3.373.8 Giac [F]

$$\int \cos^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

3.373.9 Mupad [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^5\left(b\sin\left(e+f\,x\right)^2+a\right)^p\,dx$$

input
```
int(cos(e + f*x)^5*(a + b*sin(e + f*x)^2)^p,x)
```

output
```
int(cos(e + f*x)^5*(a + b*sin(e + f*x)^2)^p, x)
```

3.374 $\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.374.1 Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{bf(3+2p)}+\frac{(a+b(3+2p))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2}{}\right.}{bf(3+2p)}$$

output
```
-sin(f*x+e)*(a+b*sin(f*x+e)^2)^(p+1)/b/f/(3+2*p)+(a+b*(3+2*p))*hypergeom([
1/2, -p],[3/2],-b*sin(f*x+e)^2/a)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^p/b/f/(3+2
*p)/((1+b*sin(f*x+e)^2/a)^p)
```

3.374.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.97

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\left(-\left((a+b(3+2p))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2}\right.\right.\right.}{bf(3+2p)}$$

input
```
Integrate[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
-((Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p*(-((a + b*(3 + 2*p))*Hypergeometr
ic2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]) + (a + b*Sin[e + f*x]^2)*(1
+ (b*Sin[e + f*x]^2)/a)^p))/(b*f*(3 + 2*p)*(1 + (b*Sin[e + f*x]^2)/a)^p))
```

3.374.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3669, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^3\left(a+b\sin(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)\left(b\sin^2(e+fx)+a\right)^p d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{\left(\frac{a}{2bp+3b}+1\right)\int \left(b\sin^2(e+fx)+a\right)^p d\sin(e+fx)-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{p+1}}{b(2p+3)}}{f}$$

$$\Big\downarrow \text{238}$$

$$\frac{\left(\frac{a}{2bp+3b}+1\right)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\int \left(\frac{b\sin^2(e+fx)}{a}+1\right)^p d\sin(e+fx)-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)}{b(2p+3)}}{f}$$

$$\Big\downarrow \text{237}$$

$$\frac{\left(\frac{a}{2bp+3b}+1\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\sin^2(e+fx)}{a}\right)-}{f}$$

input
```
Int[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(-((Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(1 + p))/(b*(3 + 2*p))) + ((1 + a/
(3*b + 2*b*p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Si
n[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(1 + (b*Sin[e + f*x]^2)/a)^p)/f
```

3.374.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.374.4 Maple [F]

$$\int \left(\cos^3\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p dx$$

input
```
int(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

3.374.5 Fricas [F]

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^3, x)
```

3.374.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.374.7 Maxima [F]

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

3.374.8 Giac [F]

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

3.374.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^3\left(b\sin\left(e+f\,x\right)^2+a\right)^p\,dx$$

input
```
int(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^p,x)
```

output
```
int(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^p, x)
```

3.375 $\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.375.1 Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}}{f}$$

output
```
hypergeom([1/2, -p],[3/2],-b*sin(f*x+e)^2/a)*sin(f*x+e)*(a+b*sin(f*x+e)^2)
^p/f/((1+b*sin(f*x+e)^2/a)^p)
```

3.375.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}}{f}$$

input
```
Integrate[Cos[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Sin[e + f*x]*(a
+ b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

3.375.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3669, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \cos(e+fx)\left(a+b\sin(e+fx)^2\right)^p dx$$
$$\Big\downarrow \; 3669$$
$$\frac{\int \left(b\sin^2(e+fx)+a\right)^p d\sin(e+fx)}{f}$$
$$\Big\downarrow \; 238$$
$$\frac{\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\int\left(\frac{b\sin^2(e+fx)}{a}+1\right)^p d\sin(e+fx)}{f}$$
$$\Big\downarrow \; 237$$
$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

input
```
Int[Cos[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Sin[e + f*x]*(a
+ b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

3.375.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.375.4 Maple [F]

$$\int \cos(fx+e)\left(a+b\left(\sin^2(fx+e)\right)\right)^p dx$$

input
```
int(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

3.375.5 Fricas [F]

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \int \left(b\sin(fx+e)^2+a\right)^p\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e), x)
```

3.375.6 Sympy [F(-1)]

Timed out.

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.375.7 Maxima [F]

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

3.375.8 Giac [F]

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

3.375.9 Mupad [B] (verification not implemented)

Time = 15.62 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\sin\left(e+f\,x\right)\,\left(b\sin\left(e+f\,x\right)^2+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\sin(e+f\,x)^2}{a}\right)}{f\left(\frac{b\sin(e+f\,x)^2}{a}+1\right)^p}$$

input
```
int(cos(e + f*x)*(a + b*sin(e + f*x)^2)^p,x)
```

output
```
(sin(e + f*x)*(a + b*sin(e + f*x)^2)^p*hypergeom([1/2, -p], 3/2, -(b*sin(e
 + f*x)^2)/a))/(f*((b*sin(e + f*x)^2)/a + 1)^p)
```

3.376 $\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.376.1 Optimal result

Integrand size = 21, antiderivative size = 76

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\operatorname{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}}{f}$$

output
```
AppellF1(1/2,1,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*sin(f*x+e)*(a+b*sin(
f*x+e)^2)^p/f/((1+b*sin(f*x+e)^2/a)^p)
```

3.376.2 Mathematica [F]

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

input
```
Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p, x]
```

3.376.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3669, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^p}{\cos(e+fx)}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\left(b\sin^2(e+fx)+a\right)^p}{1-\sin^2(e+fx)}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\int \frac{\left(\frac{b\sin^2(e+fx)}{a}+1\right)^p}{1-\sin^2(e+fx)}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

input
```
Int[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 1, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sin[e
+ f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

3.376.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.376.4 Maple [F]

$$\int \sec \left(f x +e \right) \left(a +b \left(\sin^{2}\left(f x +e \right)\right)\right)^{p}d x$$

input
```
int(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

3.376.5 Fricas [F]

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e), x)
```

3.376.6 Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.376.7 Maxima [F]

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

3.376.8 Giac [F]

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

3.376.9 Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\cos\left(e+f\,x\right)}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^p/cos(e + f*x),x)
```

output
```
int((a + b*sin(e + f*x)^2)^p/cos(e + f*x), x)
```

3.377 $\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.377.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\operatorname{AppellF1}\left(\frac{1}{2},2,-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}}{f}$$

output
```
AppellF1(1/2,2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*sin(f*x+e)*(a+b*sin(
f*x+e)^2)^p/f/((1+b*sin(f*x+e)^2/a)^p)
```

3.377.2 Mathematica [F]

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

input
```
Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p, x]
```

3.377.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3669, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^p}{\cos(e+fx)^3}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{\int \frac{\left(b\sin^2(e+fx)+a\right)^p}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\int \frac{\left(\frac{b\sin^2(e+fx)}{a}+1\right)^p}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},2,-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

input
```
Int[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sin[e
+ f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

3.377.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.377.4 Maple [F]

$$\int \left(\sec^3(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)^p dx$$

input
```
int(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

3.377.5 Fricas [F]

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^3, x)
```

3.377.6 Sympy [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.377.7 Maxima [F]

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

3.377.8 Giac [F]

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

3.377.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\cos\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^p/cos(e + f*x)^3,x)
```

output
```
int((a + b*sin(e + f*x)^2)^p/cos(e + f*x)^3, x)
```

3.378 $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.378.1 Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

output
```
AppellF1(1/2,-3/2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(a+b*sin(f*x+e)^2
)^p*(cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f/((1+b*sin(f*x+e)^2/a)^p)
```

3.378.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 199 vs. 2(90) = 180.

Time = 0.85 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.21

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{3a\,\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\cos^3(}{f\left(3a\,\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)+\left(2bp\,\text{AppellF1}\left(\frac{3}{2},-\frac{3}{2},1-p,\frac{5}{2},\sin^2(e+fx)\right.\right.\right.}$$

input
```
Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(3*a*AppellF1[1/2, -3/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]
*Cos[e + f*x]^3*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(3*a*AppellF1[1/
2, -3/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] + (2*b*p*Appell
F1[3/2, -3/2, 1 - p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] - 3*a*A
ppellF1[3/2, -1/2, -p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)])*Sin[
e + f*x]^2))
```

3.378.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3671, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^4\left(a+b\sin(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int\left(1-\sin^2(e+fx)\right)^{3/2}\left(b\sin^2(e+fx)+a\right)^p d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\int\left(1-\sin^2(e+fx)\right)^{3/2}\left(\frac{b\sin^2(e+fx)}{a}+1\right)^p d}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin}{}\right.}{f}$$

input
```
Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, -3/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqr
t[Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e
+ f*x]^2)/a)^p)
```

3.378.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.378.4 Maple [F]

$$\int \left(\cos^4\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p dx$$

input
```
int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

3.378.5 Fricas [F]

$$\int \cos^4(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p \cos\left(fx+e\right)^4 \, dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^4, x)
```

3.378.6 Sympy [F(-1)]

Timed out.

$$\int \cos^4(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.378.7 Maxima [F]

$$\int \cos^4(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p \cos\left(fx+e\right)^4 \, dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

3.378.8 Giac [F]

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

3.378.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^4\left(b\sin\left(e+f\,x\right)^2+a\right)^p\,dx$$

input
```
int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^p,x)
```

output
```
int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^p, x)
```

3.379 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.379.1 Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$
$$=\frac{\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)$$
$$f$$

output
```
AppellF1(1/2,-1/2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(a+b*sin(f*x+e)^2
)^p*(cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f/((1+b*sin(f*x+e)^2/a)^p)
```

3.379.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 195 vs. $2(90) = 180$.

Time = 0.75 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.17

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$
$$=\frac{3a\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)(}{2f\left(3a\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)+\left(2bp\,\text{AppellF1}\left(\frac{3}{2},-\frac{1}{2},1-p,\frac{5}{2},\sin^2(e+fx\right.\right.\right.}$$

input
```
Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(3*a*AppellF1[1/2, -1/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]
*(a + b*Sin[e + f*x]^2)^p*Sin[2*(e + f*x)])/(2*f*(3*a*AppellF1[1/2, -1/2,
-p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, -
1/2, 1 - p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] - a*AppellF1[3/2
, 1/2, -p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)])*Sin[e + f*x]^2))
```

3.379.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3671, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^2\left(a+b\sin(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^p d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\int\sqrt{1-\sin^2(e+fx)}\left(\frac{b\sin^2(e+fx)}{a}+1\right)^p d\sin}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin}{}\right.}{f}$$

input
```
Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, -1/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqr
t[Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e
+ f*x]^2)/a)^p)
```

3.379.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.379.4 Maple [F]

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p dx$$

input
```
int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

3.379.5 Fricas [F]

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^2, x)
```

3.379.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.379.7 Maxima [F]

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

3.379.8 Giac [F]

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

3.379.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^2\,\left(b\sin\left(e+f\,x\right)^2+a\right)^p\,dx$$

input
```
int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^p,x)
```

output
```
int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^p, x)
```

3.380 $\int (a+b\sin^2(e+fx))^p\,dx$

3.380.1 Optimal result

Integrand size = 14, antiderivative size = 90

$$\int (a+b\sin^2(e+fx))^p\,dx$$
$$=\frac{\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}(a+b\sin^2(e+fx))^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}}{f}$$

output
```
AppellF1(1/2,1/2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(a+b*sin(f*x+e)^2)
^p*(cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f/((1+b*sin(f*x+e)^2/a)^p)
```

3.380.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.61

$$\int (a+b\sin^2(e+fx))^p\,dx$$
$$=\frac{2^{-1-p}\operatorname{AppellF1}\left(1+p,\frac{1}{2},\frac{1}{2},2+p,\frac{2a+b-b\cos(2(e+fx))}{2(a+b)},\frac{2a+b-b\cos(2(e+fx))}{2a}\right)\sqrt{\frac{b\cos^2(e+fx)}{a+b}}(2a+b-b\cos(2(e+}{bf(1+p)}$$

input
```
Integrate[(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(2^(-1 - p)*AppellF1[1 + p, 1/2, 1/2, 2 + p, (2*a + b - b*Cos[2*(e + f*x)]
)/(2*(a + b)), (2*a + b - b*Cos[2*(e + f*x)])/(2*a)]*Sqrt[(b*Cos[e + f*x]^
2)/(a + b)]*(2*a + b - b*Cos[2*(e + f*x)])^(1 + p)*Csc[2*(e + f*x)]*Sqrt[-
((b*Sin[e + f*x]^2)/a)])/(b*f*(1 + p))
```

3.380.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3664, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \text{3664}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\left(b\sin^2(e+fx)+a\right)^p}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\int \frac{\left(\frac{b\sin^2(e+fx)}{a}+1\right)^p}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e}{a}\right.}{f}$$

input
```
Int[(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 1/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt
[Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e +
 f*x]^2)/a)^p)
```

3.380.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3664
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*
x]))   Subst[Int[(a + b*ff^2*x^2)^p/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &&  !IntegerQ[p]
```

3.380.4 Maple [F]

$$\int \left(a + b\left(\sin^2\left(fx + e\right)\right)\right)^p dx$$

input
```
int((a+b*sin(f*x+e)^2)^p,x)
```

output
```
int((a+b*sin(f*x+e)^2)^p,x)
```

3.380.5 Fricas [F]

$$\int \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p \, dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p, x)
```

3.380.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate((a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.380.7 Maxima [F]

$$\int \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p \, dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p, x)
```

3.380.8 Giac [F]

$$\int \left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p, x)
```

3.380.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(e+f\,x\right)^2+a\right)^p\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^p,x)
```

output
```
int((a + b*sin(e + f*x)^2)^p, x)
```

3.381 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.381.1 Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$
$$=\frac{\text{AppellF1}\left(\frac{1}{2},\frac{3}{2},-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}}{f}$$

output
```
AppellF1(1/2,3/2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(a+b*sin(f*x+e)^2)
^p*(cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f/((1+b*sin(f*x+e)^2/a)^p)
```

3.381.2 Mathematica [F]

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

input
```
Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p, x]
```

3.381.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3671, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^p}{\cos(e+fx)^2}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\left(b\sin^2(e+fx)+a\right)^p}{\left(1-\sin^2(e+fx)\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\int \frac{\left(\frac{b\sin^2(e+fx)}{a}+1\right)^p}{\left(1-\sin^2(e+fx)\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},\frac{3}{2},-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e}{a}\right.}{f}$$

input
```
Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 3/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt
[Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e +
 f*x]^2)/a)^p)
```

3.381.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.381.4 Maple [F]

$$\int \left(\sec^2(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)^p dx$$

input
```
int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

3.381.5 Fricas [F]

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^2, x)
```

3.381.6 Sympy [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.381.7 Maxima [F]

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

3.381.8 Giac [F]

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

3.381.9 Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\cos\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^p/cos(e + f*x)^2,x)
```

output
```
int((a + b*sin(e + f*x)^2)^p/cos(e + f*x)^2, x)
```

3.382 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

3.382.1 Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$
$$=\frac{\text{AppellF1}\left(\frac{1}{2},\frac{5}{2},-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}}{f}$$

output
```
AppellF1(1/2,5/2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(a+b*sin(f*x+e)^2)
^p*(cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f/((1+b*sin(f*x+e)^2/a)^p)
```

3.382.2 Mathematica [F]

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

input
```
Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p, x]
```

3.382.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3671, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^p}{\cos(e+fx)^4}dx$$

$$\Big\downarrow \text{3671}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\left(b\sin^2(e+fx)+a\right)^p}{\left(1-\sin^2(e+fx)\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\int \frac{\left(\frac{b\sin^2(e+fx)}{a}+1\right)^p}{\left(1-\sin^2(e+fx)\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},\frac{5}{2},-p,\frac{3}{2},\sin^2(e+fx),-\frac{b\sin^2(e}{a}\right.}{f}$$

input
```
Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 5/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt
[Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e +
 f*x]^2)/a)^p)
```

3.382.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3671
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff*(Sqrt[
Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
 + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
 && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.382.4 Maple [F]

$$\int \left(\sec^4\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p dx$$

input
```
int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

3.382.5 Fricas [F]

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^4, x)
```

3.382.6 Sympy [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.382.7 Maxima [F]

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

3.382.8 Giac [F]

$$\int \sec^4(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p \sec\left(fx+e\right)^4 \, dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

3.382.9 Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx) \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\cos\left(e+f\,x\right)^4} \, dx$$

input
```
int((a + b*sin(e + f*x)^2)^p/cos(e + f*x)^4,x)
```

output
```
int((a + b*sin(e + f*x)^2)^p/cos(e + f*x)^4, x)
```

3.383 $\int \frac{\cos^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.383.1 Optimal result

Integrand size = 23, antiderivative size = 219

$$\int \frac{\cos^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\left(a^{4/3}-b^{4/3}\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{5/3}d} + \frac{\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}b^{5/3}d} - \frac{\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}b^{5/3}d} - \frac{2\log\left(a+b\sin^3(c+dx)\right)}{3bd} + \frac{\sin^2(c+dx)}{2bd}$$

output
```
1/3*(a^(4/3)+b^(4/3))*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(2/3)/b^(5/3)/d-1/6
*(a^(4/3)+b^(4/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c
)^2)/a^(2/3)/b^(5/3)/d-2/3*ln(a+b*sin(d*x+c)^3)/b/d+1/2*sin(d*x+c)^2/b/d+1
/3*(a^(4/3)-b^(4/3))*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(
1/2))/a^(2/3)/b^(5/3)/d*3^(1/2)
```

3.383.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.93

$$\int \frac{\cos^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$= \frac{-2\sqrt{3}b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)+2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)-b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c-\right.}{}$$

input
```
Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

output
```
(-2*Sqrt[3]*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(
1/3))] + 2*b^(2/3)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] - b^(2/3)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2] - 4*a^(2/3)*Lo
g[a + b*Sin[c + d*x]^3] + 3*a^(2/3)*Sin[c + d*x]^2 - 3*a^(2/3)*Hypergeomet
ric2F1[2/3, 1, 5/3, -((b*Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(6*a^(2/3)*b*
d)
```

3.383.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3702, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(c+dx)^5}{a+b\sin(c+dx)^3}dx$$
$$\Big\downarrow \text{ 3702}$$
$$\frac{\int \frac{\left(1-\sin^2(c+dx)\right)^2}{b\sin^3(c+dx)+a}d\sin(c+dx)}{d}$$
$$\Big\downarrow \text{ 2426}$$

$$\frac{\int \left(\frac{\sin(c+dx)}{b}+\frac{-2b\sin^2(c+dx)-a\sin(c+dx)+b}{b(b\sin^3(c+dx)+a)}\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a^{4/3}-b^{4/3})\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{5/3}}-\frac{(a^{4/3}+b^{4/3})\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}b^{5/3}}+\frac{(a^{4/3}+b^{4/3})\log\left(\sqrt[3]{a}+\sqrt[3]{b}s\right.}{3a^{2/3}b^{5/3}}}{d}$$

input
```
Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

output
```
(((a^(4/3) - b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a
^(1/3))])/(Sqrt[3]*a^(2/3)*b^(5/3)) + ((a^(4/3) + b^(4/3))*Log[a^(1/3) + b
^(1/3)*Sin[c + d*x]])/(3*a^(2/3)*b^(5/3)) - ((a^(4/3) + b^(4/3))*Log[a^(2/
3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(6*a^(2/3)*b^
(5/3)) - (2*Log[a + b*Sin[c + d*x]^3])/(3*b) + Sin[c + d*x]^2/(2*b))/d
```

3.383.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.383. $\int \frac{\cos^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.383.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{\sin^2(dx+c)}{2b}+\frac{b\left(\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-a}{}$
default	$\frac{\sin^2(dx+c)}{2b}+\frac{b\left(\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-a}{}$
risch	$\frac{2ix}{b}-\frac{e^{2i(dx+c)}}{8bd}-\frac{e^{-2i(dx+c)}}{8bd}+\frac{4ic}{bd}+\left(\sum_{\_R=\mathrm{RootOf}\left(27a^2b^5d^3\_Z^3+54a^2b^4d^2\_Z^2+27a^2b^3d\_Z-a^4+2a^2b^2-b\right.}\right.$

input
```
int(cos(d*x+c)^5/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*sin(d*x+c)^2/b+(b*(1/3/b/(1/b*a)^(2/3)*ln(sin(d*x+c)+(1/b*a)^(1/3
))-1/6/b/(1/b*a)^(2/3)*ln(sin(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2
/3))+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d
*x+c)-1)))-a*(-1/3/b/(1/b*a)^(1/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))+1/6/b/(1/b
*a)^(1/3)*ln(sin(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3*3^(1
/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))-2/
3*ln(a+b*sin(d*x+c)^3))/b)
```

3.383.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 3216, normalized size of antiderivative = 14.68

$$\int \frac{\cos^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/12*(2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^
4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2
/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*
b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*b*d*log(1/4*((1/2)^(1/3)*(I*
sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 -
 b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2
*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a
^2*b^5*d^3))^(1/3)))^2*a^3*b^3*d^2 + 2*a^3*b - 2*a*b^3 - 1/2*(4*a^3*b^2 -
a*b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)
/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3
)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^
5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*d + (a^4 - b^4)*sin(d*x + c))
+ 6*cos(d*x + c)^2 - (((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2
*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d)
 + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2
 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*b*d + 3*sqrt(1/
3)*b*d*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2
 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/
2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/
(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))^2*b*d - 8*(1/2)^(1/3...
```

3.383.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5/(a+b*sin(d*x+c)**3),x)
```

output
```
Timed out
```

3.383.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.96

$$\int \frac{\cos^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$= \frac{\frac{9\sin(dx+c)^2}{b} - \frac{2\sqrt{3}\left(a\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\right)-b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{4a}{b}\right)\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab} - \frac{3\left(b\left(4\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)+a\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(\sin(dx+c\cdots\right.}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}\cdots}{18\,d}$$

input
```
integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/18*(9*sin(d*x + c)^2/b - 2*sqrt(3)*(a*(3*(a/b)^(2/3) - 4) - b*(3*(a/b)^(
1/3) - 4*a/b))*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1
/3))/(a*b) - 3*(b*(4*(a/b)^(2/3) + 1) + a*(a/b)^(1/3))*log(sin(d*x + c)^2
- (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) - 6*(b*(2*(a/b
)^(2/3) - 1) - a*(a/b)^(1/3))*log((a/b)^(1/3) + sin(d*x + c))/(b^2*(a/b)^(
2/3)))/d
```

3.383.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.01

$$\int \frac{\cos^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$= \frac{\frac{3\sin(dx+c)^2}{b} - \frac{4\log\left(\left|b\sin(dx+c)^3+a\right|\right)}{b} + \frac{2\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2+\left(-ab^2\right)^{\frac{2}{3}}a\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^3} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2-\left(-ab^2\right)^{\cdots}\right.}{\cdots}\cdots}{6\,d}$$

input
```
integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
1/6*(3*sin(d*x + c)^2/b - 4*log(abs(b*sin(d*x + c)^3 + a))/b + 2*sqrt(3)*(
(-a*b^2)^(1/3)*b^2 + (-a*b^2)^(2/3)*a)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) +
2*sin(d*x + c))/(-a/b)^(1/3))/(a*b^3) + ((-a*b^2)^(1/3)*b^2 - (-a*b^2)^(2/
3)*a)*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a*b^
3) + 2*(a*b^4*(-a/b)^(1/3) - b^5)*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin
(d*x + c)))/(a*b^5))/d
```

3.383.9 Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.05

$$\int \frac{\cos^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$= \frac{\left(\sum_{k=1}^{3}\ln\left(3\,a+\mathrm{root}(27\,a^2\,b^5\,d^3+54\,a^2\,b^4\,d^2+27\,a^2\,b^3\,d+2\,a^2\,b^2-b^4-a^4,d,k)\,\left(12\,a\,b+3\,b^2\,\sin\left(c+d\right.\right.\right.\right.}{}$$

input
```
int(cos(c + d*x)^5/(a + b*sin(c + d*x)^3),x)
```

output
```
(symsum(log(3*a + root(27*a^2*b^5*d^3 + 54*a^2*b^4*d^2 + 27*a^2*b^3*d + 2*
a^2*b^2 - b^4 - a^4, d, k)*(12*a*b + 3*b^2*sin(c + d*x) + 9*root(27*a^2*b^
5*d^3 + 54*a^2*b^4*d^2 + 27*a^2*b^3*d + 2*a^2*b^2 - b^4 - a^4, d, k)*a*b^2
) + (sin(c + d*x)*(a^2 + 2*b^2))/b)*root(27*a^2*b^5*d^3 + 54*a^2*b^4*d^2 +
 27*a^2*b^3*d + 2*a^2*b^2 - b^4 - a^4, d, k), k, 1, 3) + sin(c + d*x)^2/(2
*b))/d
```

3.384 $\int \frac{\cos^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.384.1 Optimal result

Integrand size = 23, antiderivative size = 167

$$\int \frac{\cos^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\sqrt[3]{b}d} - \frac{\log\left(a+b\sin^3(c+dx)\right)}{3bd}$$

output
```
1/3*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(2/3)/b^(1/3)/d-1/6*ln(a^(2/3)-a^(1/3
)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(2/3)/b^(1/3)/d-1/3*ln(a+b*si
n(d*x+c)^3)/b/d-1/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1
/2))/a^(2/3)/b^(1/3)/d*3^(1/2)
```

3.384.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.83

$$\int \frac{\cos^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\left(-a^{2/3}+(-1)^{2/3}b^{2/3}\right)\log\left(-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)+\left(-a^{2/3}+b^{2/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}bd}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

output
```
((-a^(2/3) + (-1)^(2/3)*b^(2/3))*Log[-((-1)^(2/3)*a^(1/3)) - b^(1/3)*Sin[c
 + d*x]] + (-a^(2/3) + b^(2/3))*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] - (a^(
2/3) + (-1)^(1/3)*b^(2/3))*Log[a^(1/3) + (-1)^(2/3)*b^(1/3)*Sin[c + d*x]])
/(3*a^(2/3)*b*d)
```

3.384.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3042, 3702, 2410, 750, 16, 792, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{a+b\sin^3(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^3}{a+b\sin(c+dx)^3}dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int \frac{1-\sin^2(c+dx)}{b\sin^3(c+dx)+a}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2410}$$

$$\frac{\int \frac{1}{b\sin^3(c+dx)+a}d\sin(c+dx)-\int \frac{\sin^2(c+dx)}{b\sin^3(c+dx)+a}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{750}$$

$$\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}\sin(c+dx)+\sqrt[3]{a}}d\sin(c+dx)}{3a^{2/3}}-\int \frac{\sin^2(c+dx)}{b\sin^3(c+dx)+a}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{3a^{2/3}}-\int \frac{\sin^2(c+dx)}{b\sin^3(c+dx)+a}d\sin(c+dx)+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\Big\downarrow \; 792$$

$$\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\log\left(a+b\sin^3(c+dx)\right)}{3b}}{d}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)\right)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+d\right.}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)\right)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right.}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+\right.}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}-}{d}$$

$$\Big\downarrow \; 217$$

3.384. $\int \frac{\cos^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\log(a+b\sin^3(c+dx))}{3b}}{d}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\log(a+b\sin^3(c+dx))}{3b}}{d}$$

input
```
Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

output
```
(Log[a^(1/3) + b^(1/3)*Sin[c + d*x]]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*Arc
Tan[(1 - (2*b^(1/3)*Sin[c + d*x])/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3
) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]/(2*b^(1/3)))/(3
*a^(2/3)) - Log[a + b*Sin[c + d*x]^3]/(3*b))/d
```

3.384.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2410
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Si
mp[C   Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[
a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.384. $\int \frac{\cos^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.384.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.54 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.63

method	result
risch	$\frac{ix}{b}+\frac{2ic}{bd}+\left(\sum_{\_R=\operatorname{RootOf}\left(27a^2\_Z^3d^3b^3+27a^2b^2d^2\_Z^2+9a^2d\_Zb+a^2-b^2\right)}\_R\ln\left(\mathrm{e}^{2i(dx+c)}+\left(6iad\_R\right.\right.\right.$
derivativedivides	$\frac{\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(a+b\left(\sin^3(dx+c)\right.\right.}{3b}}{d}$
default	$\frac{\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(a+b\left(\sin^3(dx+c)\right.\right.}{3b}}{d}$

input
```
int(cos(d*x+c)^3/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
I*x/b+2*I/b/d*c+sum(_R*ln(exp(2*I*(d*x+c))+(6*I*a*d*_R+2*I/b*a)*exp(I*(d*x
+c))-1),_R=RootOf(27*_Z^3*a^2*b^3*d^3+27*_Z^2*a^2*b^2*d^2+9*_Z*a^2*b*d+a^2
-b^2))
```

3.384.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 1049, normalized size of antiderivative = 6.28

$$\int \frac{\cos^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/12*(2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2
- b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*b*d*log(-1/2*((1/2)^(1/3)*(I*sqrt(3
) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2
/(b*d))*a*b*d + b*sin(d*x + c) + a) - (((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^
3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*b*d +
 3*sqrt(1/3)*b*d*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2
*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*b^2*d^2 - 4*((1/2)
^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3
*d^3))^(1/3) + 2/(b*d))*b*d + 4)/(b^2*d^2)) - 6)*log(1/2*((1/2)^(1/3)*(I*s
qrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3
) + 2/(b*d))*a*b*d + 3/2*sqrt(1/3)*a*b*d*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) +
1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*
d))^2*b^2*d^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3
) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*b*d + 4)/(b^2*d^2)) + 2*b*
sin(d*x + c) - a) - (((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*
d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*b*d - 3*sqrt(1/3)*b*d*s
qrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b
^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*b^2*d^2 - 4*((1/2)^(1/3)*(I*sqrt(3)
+ 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(
b*d))*b*d + 4)/(b^2*d^2)) - 6)*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1...
```

3.384.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+b*sin(d*x+c)**3),x)
```

output
```
Timed out
```

3.384.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.95

$$\int \frac{\cos^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$= \frac{\frac{2\sqrt{3}\left(b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{2a}{b}\right)+2a\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab} - \frac{3\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{6\left(\left(\frac{a}{b}\right)^{\frac{2}{3}} \cdots\right.}{\cdots}}{18\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/18*(2*sqrt(3)*(b*(3*(a/b)^(1/3) - 2*a/b) + 2*a)*arctan(-1/3*sqrt(3)*((a/
b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1/3))/(a*b) - 3*(2*(a/b)^(2/3) + 1)*log(
sin(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(b*(a/b)^(2/3)) -
 6*((a/b)^(2/3) - 1)*log((a/b)^(1/3) + sin(d*x + c))/(b*(a/b)^(2/3)))/d
```

3.384.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.93

$$\int \frac{\cos^3(c+dx)}{a+b\sin^3(c+dx)}\,dx =$$
$$-\frac{\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a} + \frac{2\log\left(\left|b\sin(dx+c)^3+a\right|\right)}{b} - \frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab} - \frac{\left(-ab^2\right)^{\frac{1}{3}}\log \cdots}{\cdots}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
-1/6*(2*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/a + 2*log(abs(
b*sin(d*x + c)^3 + a))/b - 2*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*((-
a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(a*b) - (-a*b^2)^(1/3)*log(sin(
d*x + c)^2 + (-a/b)^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a*b))/d
```

3.384.9 Mupad [B] (verification not implemented)

Time = 13.99 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.92

$$\int \frac{\cos^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$= \frac{\sum_{k=1}^{3} \ln\left(\left(\mathrm{root}(27\,a^2\,b^3\,d^3+27\,a^2\,b^2\,d^2+9\,a^2\,b\,d-b^2+a^2,d,k)\;b\,3+1\right)\,\left(a+b\,\sin\left(c+d\,x\right)+\mathrm{root}(27\,a^2\right.\right.}{d}$$

input
```
int(cos(c + d*x)^3/(a + b*sin(c + d*x)^3),x)
```

output
```
symsum(log((3*root(27*a^2*b^3*d^3 + 27*a^2*b^2*d^2 + 9*a^2*b*d - b^2 + a^2
, d, k)*b + 1)*(a + b*sin(c + d*x) + 3*root(27*a^2*b^3*d^3 + 27*a^2*b^2*d^
2 + 9*a^2*b*d - b^2 + a^2, d, k)*a*b))*root(27*a^2*b^3*d^3 + 27*a^2*b^2*d^
2 + 9*a^2*b*d - b^2 + a^2, d, k), k, 1, 3)/d
```

3.385 $\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.385.1 Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\sqrt[3]{b}d}$$

output
```
1/3*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(2/3)/b^(1/3)/d-1/6*ln(a^(2/3)-a^(1/3
)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(2/3)/b^(1/3)/d-1/3*arctan(1/
3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/b^(1/3)/d*3^(1/2
)
```

3.385.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.81

$$\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/}\right.}{6a^{2/3}\sqrt[3]{b}d}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3)
)] - 2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] + Log[a^(2/3) - a^(1/3)*b^(1/3)
*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(a^(2/3)*b^(1/3)*d)
```

3.385.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3702, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)}{a+b\sin(c+dx)^3}dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int \frac{1}{b\sin^3(c+dx)+a}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{750}$$

$$\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}\sin(c+dx)+\sqrt[3]{a}}d\sin(c+dx)}{3a^{2/3}}}{d}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)\right)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$\big\downarrow$ 25

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)\right)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$\big\downarrow$ 1082

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$\big\downarrow$ 217

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$\big\downarrow$ 1103

3.385. $\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}\,dx$

$$\frac{\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

input
```
Int[Cos[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

output
```
(Log[a^(1/3) + b^(1/3)*Sin[c + d*x]]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*Arc
Tan[(1 - (2*b^(1/3)*Sin[c + d*x])/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3
) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]/(2*b^(1/3)))/(3
*a^(2/3)))/d
```

3.385.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

3.385. $\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.385.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.33

3.385. $\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}\,dx$

method	result	size
risch	$\sum_{\_R=\mathrm{RootOf}(27a^2b\,d^3\_Z^3-1)} \_R\ln\left(\mathrm{e}^{2i(dx+c)}+6iad\_R\,\mathrm{e}^{i(dx+c)}-1\right)$	48
derivativedivides	$\frac{\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{d}$	115
default	$\frac{\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{d}$	115

input
```
int(cos(d*x+c)/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(exp(2*I*(d*x+c))+6*I*a*d*_R*exp(I*(d*x+c))-1),_R=RootOf(27*_Z^3*
a^2*b*d^3-1))
```

3.385.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.78

$$\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$= \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(-\frac{3\left(a^2b\right)^{\frac{1}{3}}a\sin(dx+c)+a^2+3\sqrt{\frac{1}{3}}\left(2ab\cos(dx+c)^2-2ab-\left(a^2b\right)^{\frac{2}{3}}\sin(dx+c)+\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}+2\left(ab\right.}{\left(b\cos(dx+c)^2-b\right)\sin(dx+c)-a}\right.}{}\right.$$

input
```
integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log(-(3*(a^2*b)^(1/3)*a*sin(d
*x + c) + a^2 + 3*sqrt(1/3)*(2*a*b*cos(d*x + c)^2 - 2*a*b - (a^2*b)^(2/3)*
sin(d*x + c) + (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) + 2*(a*b*cos(d*x +
c)^2 - a*b)*sin(d*x + c))/((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)) - (a^
2*b)^(2/3)*log(-a*b*cos(d*x + c)^2 + a*b - (a^2*b)^(2/3)*sin(d*x + c) + (a
^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*sin(d*x + c) + (a^2*b)^(2/3)))/(a
^2*b*d), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a
^2*b)^(2/3)*sin(d*x + c) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (
a^2*b)^(2/3)*log(-a*b*cos(d*x + c)^2 + a*b - (a^2*b)^(2/3)*sin(d*x + c) +
(a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*sin(d*x + c) + (a^2*b)^(2/3)))/
(a^2*b*d)]
```

3.385.6 Sympy [A] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.28

$$\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty}x\cos(c)}{\sin^3(c)} \\ -\frac{1}{2bd\sin^2(c+dx)} \\ \frac{\sin(c+dx)}{ad} \\ \frac{x\cos(c)}{a+b\sin^3(c)} \\ -\frac{\sqrt[3]{-\frac{a}{b}}\log\left(-\sqrt[3]{-\frac{a}{b}}+\sin(c+dx)\right)}{3ad} + \frac{\sqrt[3]{-\frac{a}{b}}\log\left(4\left(-\frac{a}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-\frac{a}{b}}\sin(c+dx)+4\sin^2(c+dx)\right)}{6ad} + \frac{\sqrt{3}\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\sin}{3\sqrt[3]{}}\right.}{3ad} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+b*sin(d*x+c)**3),x)
```

output
```
Piecewise((zoo*x*cos(c)/sin(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-1/(2
*b*d*sin(c + d*x)**2), Eq(a, 0)), (sin(c + d*x)/(a*d), Eq(b, 0)), (x*cos(c
)/(a + b*sin(c)**3), Eq(d, 0)), (-(-a/b)**(1/3)*log(-(-a/b)**(1/3) + sin(c
 + d*x))/(3*a*d) + (-a/b)**(1/3)*log(4*(-a/b)**(2/3) + 4*(-a/b)**(1/3)*sin
(c + d*x) + 4*sin(c + d*x)**2)/(6*a*d) + sqrt(3)*(-a/b)**(1/3)*atan(sqrt(3
)/3 + 2*sqrt(3)*sin(c + d*x)/(3*(-a/b)**(1/3)))/(3*a*d), True))
```

3.385.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.84

$$\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{6d}$$

input
```
integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/6*(2*sqrt(3)*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1
/3))/(b*(a/b)^(2/3)) - log(sin(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/
b)^(2/3))/(b*(a/b)^(2/3)) + 2*log((a/b)^(1/3) + sin(d*x + c))/(b*(a/b)^(2/
3)))/d
```

3.385.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95

$$\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}\,dx =$$

$$-\frac{\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a} - \frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab} - \frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(\sin(dx+c)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx\right.}{ab}}{6d}$$

input
```
integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
-1/6*(2*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/a - 2*sqrt(3)*
(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(
1/3))/(a*b) - (-a*b^2)^(1/3)*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x + c
) + (-a/b)^(2/3))/(a*b))/d
```

3.385.9 Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\ln\left(b^{1/3}\,\sin\left(c+d\,x\right)+a^{1/3}\right)}{3\,a^{2/3}\,b^{1/3}\,d} + \frac{\ln\left(3\,b^2\,\sin\left(c+d\,x\right)+\frac{3\,a^{1/3}\,b^{5/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d} - \frac{\ln\left(3\,b^2\,\sin\left(c+d\,x\right)-\frac{3\,a^{1/3}\,b^{5/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d}$$

input
```
int(cos(c + d*x)/(a + b*sin(c + d*x)^3),x)
```

output
```
log(b^(1/3)*sin(c + d*x) + a^(1/3))/(3*a^(2/3)*b^(1/3)*d) + (log(3*b^2*sin
(c + d*x) + (3*a^(1/3)*b^(5/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a
^(2/3)*b^(1/3)*d) - (log(3*b^2*sin(c + d*x) - (3*a^(1/3)*b^(5/3)*(3^(1/2)*
1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)*b^(1/3)*d)
```

3.386 $\int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.386.1 Optimal result

Integrand size = 21, antiderivative size = 290

$$\begin{aligned}\int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx &= -\frac{\sqrt[3]{b}\left(a^{4/3}-b^{4/3}\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a^2-b^2\right)d}-\frac{\log(1-\sin(c+dx))}{2(a+b)d}\\&\quad+\frac{\log(1+\sin(c+dx))}{2(a-b)d}-\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\left(a^2-b^2\right)d}\\&\quad+\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\left(a^2-b^2\right)d}\\&\quad-\frac{b\log\left(a+b\sin^3(c+dx)\right)}{3\left(a^2-b^2\right)d}\end{aligned}$$

output
```
-1/2*ln(1-sin(d*x+c))/(a+b)/d+1/2*ln(1+sin(d*x+c))/(a-b)/d-1/3*b^(1/3)*(a^
(4/3)+b^(4/3))*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(2/3)/(a^2-b^2)/d+1/6*b^(1
/3)*(a^(4/3)+b^(4/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*
x+c)^2)/a^(2/3)/(a^2-b^2)/d-1/3*b*ln(a+b*sin(d*x+c)^3)/(a^2-b^2)/d-1/3*b^(
1/3)*(a^(4/3)-b^(4/3))*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3
^(1/2))/a^(2/3)/(a^2-b^2)/d*3^(1/2)
```

3.386.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.92

$$\int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$= \frac{2\sqrt{3}b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)-3a^{5/3}\log(1-\sin(c+dx))+3a^{2/3}b\log(1-\sin(c+dx))+3a^{5/3}\log}{}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

output
```
(2*Sqrt[3]*b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1
/3))] - 3*a^(5/3)*Log[1 - Sin[c + d*x]] + 3*a^(2/3)*b*Log[1 - Sin[c + d*x]
] + 3*a^(5/3)*Log[1 + Sin[c + d*x]] + 3*a^(2/3)*b*Log[1 + Sin[c + d*x]] -
2*b^(5/3)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] + b^(5/3)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2] - 2*a^(2/3)*b*Log[a + b
*Sin[c + d*x]^3] + 3*a^(2/3)*b*Hypergeometric2F1[2/3, 1, 5/3, -((b*Sin[c +
 d*x]^3)/a)]*Sin[c + d*x]^2)/(6*a^(2/3)*(a - b)*(a + b)*d)
```

3.386.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3702, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\cos(c+dx)\left(a+b\sin(c+dx)^3\right)}dx$$
$$\Big\downarrow \text{3702}$$
$$\frac{\int \frac{1}{(1-\sin^2(c+dx))\left(b\sin^3(c+dx)+a\right)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{7276}$$

$$\frac{\int \left(\frac{b(b\sin^2(c+dx)-a\sin(c+dx)+b)}{(b^2-a^2)(b\sin^3(c+dx)+a)}-\frac{1}{2(a+b)(\sin(c+dx)-1)}+\frac{1}{2(a-b)(\sin(c+dx)+1)}\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b\log\left(a+b\sin^3(c+dx)\right)}{3(a^2-b^2)}-\frac{\sqrt[3]{b}(a^{4/3}-b^{4/3})\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(a^2-b^2)}+\frac{\sqrt[3]{b}(a^{4/3}+b^{4/3})\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+d\right)}{6a^{2/3}(a^2-b^2)}}{d}$$

input
```
Int[Sec[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

output
```
(-((b^(1/3)*(a^(4/3) - b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/
(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a^2 - b^2))) - Log[1 - Sin[c + d*x]]
/(2*(a + b)) + Log[1 + Sin[c + d*x]]/(2*(a - b)) - (b^(1/3)*(a^(4/3) + b^(
4/3))*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(3*a^(2/3)*(a^2 - b^2)) + (b^(1
/3)*(a^(4/3) + b^(4/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/
3)*Sin[c + d*x]^2])/(6*a^(2/3)*(a^2 - b^2)) - (b*Log[a + b*Sin[c + d*x]^3]
)/(3*(a^2 - b^2)))/d
```

3.386.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.386.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{-\frac{\ln(\sin(dx+c)-1)}{2a+2b}+\frac{\ln(1+\sin(dx+c))}{2a-2b}+\frac{\left(-b\left(\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\sqrt{3}\arctan\left(\right.\right.\right.}{}}{}$
default	$\frac{-\frac{\ln(\sin(dx+c)-1)}{2a+2b}+\frac{\ln(1+\sin(dx+c))}{2a-2b}+\frac{\left(-b\left(\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\sqrt{3}\arctan\left(\right.\right.\right.}{}}{}$
risch	$-\frac{ix}{a-b}-\frac{ic}{d(a-b)}+\frac{ix}{a+b}+\frac{ic}{d(a+b)}+\frac{2ia^2b\,d^3x}{a^4d^3-a^2b^2d^3}+\frac{2ia^2b\,d^2c}{a^4d^3-a^2b^2d^3}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{d(a-b)}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{d(a+b)}+$

input
```
int(sec(d*x+c)/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(2*a+2*b)*ln(sin(d*x+c)-1)+1/(2*a-2*b)*ln(1+sin(d*x+c))+(-b*(1/3/b
/(1/b*a)^(2/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*ln(sin(d*x
+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))+a*(-1/3/b/(1/b*a)^(1/3
)*ln(sin(d*x+c)+(1/b*a)^(1/3))+1/6/b/(1/b*a)^(1/3)*ln(sin(d*x+c)^2-(1/b*a)
^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^
(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))-1/3*ln(a+b*sin(d*x+c)^3))*b/(a+b)/(
a-b))
```

3.386.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 4396, normalized size of antiderivative = 15.16

$$\int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/36*(2*(a^2 - b^2)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) -
 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^
(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^
2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^
2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d*log(-1/36*(a^5 - a^3*b^2)*(9*(I*sq
rt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 +
 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/
((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^
2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d -
 b^2*d))^2*d^2 + a*b^2 + 1/6*(2*a^3*b + a*b^3)*(9*(I*sqrt(3) + 1)*(-1/54*b
/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b
/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*
(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2
+ b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d - (a^2*b
 + b^3)*sin(d*x + c)) - ((a^2 - b^2)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3
- a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b
^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(
a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/(
(a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d - 3*sqrt(1/3)*(a^2
 - b^2)*d*sqrt(-((a^4 - 2*a^2*b^2 + b^4)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a...
```

3.386.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sin(d*x+c)**3),x)
```

output
```
Integral(sec(c + d*x)/(a + b*sin(c + d*x)**3), x)
```

3.386.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.99

$$\int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\frac{2\sqrt{3}\left(a\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\right)-b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{2a}{b}\right)\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{3\left(b\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}-1\right)-a\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+\right.}{a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{18\,d}$$

input
```
integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/18*(2*sqrt(3)*(a*(3*(a/b)^(2/3) + 2) - b*(3*(a/b)^(1/3) + 2*a/b))*arctan
(-1/3*sqrt(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1/3))/((a^2*(a/b)^(2/3
) - b^2*(a/b)^(2/3))*(a/b)^(1/3)) - 3*(b*(2*(a/b)^(2/3) - 1) - a*(a/b)^(1/
3))*log(sin(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(a^2*(a/b
)^(2/3) - b^2*(a/b)^(2/3)) - 6*(b*((a/b)^(2/3) + 1) + a*(a/b)^(1/3))*log((
a/b)^(1/3) + sin(d*x + c))/(a^2*(a/b)^(2/3) - b^2*(a/b)^(2/3)) + 9*log(sin
(d*x + c) + 1)/(a - b) - 9*log(sin(d*x + c) - 1)/(a + b))/d
```

3.386.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.07

$$\int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx = -\frac{\frac{2\left(a^3b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^3+b^5\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^5b-2a^3b^3+ab^5} + \frac{6\left(\left(-ab^2\right)^{\frac{1}{3}}b^2+\left(-ab^2\right)^{\frac{2}{3}}a\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx\right.}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{\sqrt{3}a^3b-\sqrt{3}ab^3}}{}$$

input
```
integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
-1/6*(2*(a^3*b^2*(-a/b)^(1/3) - a*b^4*(-a/b)^(1/3) - a^2*b^3 + b^5)*(-a/b)
^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/(a^5*b - 2*a^3*b^3 + a*b^5)
+ 6*((-a*b^2)^(1/3)*b^2 + (-a*b^2)^(2/3)*a)*arctan(1/3*sqrt(3)*((-a/b)^(1/
3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(sqrt(3)*a^3*b - sqrt(3)*a*b^3) + ((-a*
b^2)^(1/3)*b^2 - (-a*b^2)^(2/3)*a)*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d
*x + c) + (-a/b)^(2/3))/(a^3*b - a*b^3) + 2*b*log(abs(b*sin(d*x + c)^3 + a
))/(a^2 - b^2) - 3*log(abs(sin(d*x + c) + 1))/(a - b) + 3*log(abs(sin(d*x
+ c) - 1))/(a + b))/d
```

3.386.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.07

$$\int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$= \frac{\left(\sum_{k=1}^{3} \ln\left(-\mathrm{root}(27\,a^2\,b^2\,z^3-27\,a^4\,z^3-27\,a^2\,b\,z^2-b,z,k)^2\,a\,b^4\,13-\mathrm{root}(27\,a^2\,b^2\,z^3-27\,a^4\,z^3-27\,a^2\,\right.\right.}{}$$

input
```
int(1/(cos(c + d*x)*(a + b*sin(c + d*x)^3)),x)
```

output
```
(symsum(log(- 13*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k
)^2*a*b^4 - 36*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k)^
3*a*b^5 - 36*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k)^4*
a*b^6 - 16*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k)^2*b^
5*sin(c + d*x) - 12*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z
, k)^3*b^6*sin(c + d*x) - 27*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z
^2 - b, z, k)^3*a^3*b^3 - 180*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*
z^2 - b, z, k)^4*a^3*b^4 - 5*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z
^2 - b, z, k)*b^4*sin(c + d*x) - 69*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*
a^2*b*z^2 - b, z, k)^3*a^2*b^4*sin(c + d*x) - 162*root(27*a^2*b^2*z^3 - 27
*a^4*z^3 - 27*a^2*b*z^2 - b, z, k)^4*a^2*b^5*sin(c + d*x) - 54*root(27*a^2
*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k)^4*a^4*b^3*sin(c + d*x))*ro
ot(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k), k, 1, 3) - log(s
in(c + d*x) - 1)/(2*a + 2*b) + log(sin(c + d*x) + 1)/(2*a - 2*b))/d
```

3.387 $\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.387.1 Optimal result

Integrand size = 23, antiderivative size = 385

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx &= -\frac{b^{5/3}\left(2a^2-3a^{4/3}b^{2/3}+b^2\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a^2-b^2\right)^2 d}-\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d}\\&\quad+\frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^2d}+\frac{b^{5/3}\left(2a^2+3a^{4/3}b^{2/3}+b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\left(a^2-b^2\right)^2 d}\\&\quad-\frac{b^{5/3}\left(2a^2+3a^{4/3}b^{2/3}+b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\left(a^2-b^2\right)^2 d}\\&\quad+\frac{b(a^2+2b^2)\log\left(a+b\sin^3(c+dx)\right)}{3\left(a^2-b^2\right)^2 d}\\&\quad+\frac{1}{4(a+b)d(1-\sin(c+dx))}-\frac{1}{4(a-b)d(1+\sin(c+dx))}\end{aligned}$$

output
```
-1/4*(a+4*b)*ln(1-sin(d*x+c))/(a+b)^2/d+1/4*(a-4*b)*ln(1+sin(d*x+c))/(a-b)
^2/d+1/3*b^(5/3)*(2*a^2+3*a^(4/3)*b^(2/3)+b^2)*ln(a^(1/3)+b^(1/3)*sin(d*x+
c))/a^(2/3)/(a^2-b^2)^2/d-1/6*b^(5/3)*(2*a^2+3*a^(4/3)*b^(2/3)+b^2)*ln(a^(
2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(2/3)/(a^2-b^2)^2/
d+1/3*b*(a^2+2*b^2)*ln(a+b*sin(d*x+c)^3)/(a^2-b^2)^2/d+1/4/(a+b)/d/(1-sin(
d*x+c))-1/4/(a-b)/d/(1+sin(d*x+c))-1/3*b^(5/3)*(2*a^2-3*a^(4/3)*b^(2/3)+b^
2)*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(a^2
-b^2)^2/d*3^(1/2)
```

3.387.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.26 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.94

$$\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx =$$
$$-\frac{\frac{4\sqrt{3}b^{5/3}(2a^2+b^2)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}(a^2-b^2)^2}+\frac{3(a+4b)\log(1-\sin(c+dx))}{(a+b)^2}-\frac{3(a-4b)\log(1+\sin(c+dx))}{(a-b)^2}-\frac{4b^{5/3}(2a^2+b^2)\log\left(\sqrt[3]{\ldots}\right.}{a^{2/3}(a^2\ldots}}{}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

output
```
-1/12*((4*Sqrt[3]*b^(5/3)*(2*a^2 + b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c
+ d*x])/(Sqrt[3]*a^(1/3))])/(a^(2/3)*(a^2 - b^2)^2) + (3*(a + 4*b)*Log[1 -
 Sin[c + d*x]])/(a + b)^2 - (3*(a - 4*b)*Log[1 + Sin[c + d*x]])/(a - b)^2
- (4*b^(5/3)*(2*a^2 + b^2)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(a^(2/3)*(
a^2 - b^2)^2) + (2*b^(5/3)*(2*a^2 + b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin
[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(a^(2/3)*(a^2 - b^2)^2) - (4*b*(a^2 +
 2*b^2)*Log[a + b*Sin[c + d*x]^3])/(a^2 - b^2)^2 + 3/((a + b)*(-1 + Sin[c
+ d*x])) + (18*b^3*Hypergeometric2F1[2/3, 1, 5/3, -((b*Sin[c + d*x]^3)/a)]
*Sin[c + d*x]^2)/(a^2 - b^2)^2 + 3/((a - b)*(1 + Sin[c + d*x])))/d
```

3.387.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3702, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\cos(c+dx)^3\,(a+b\sin(c+dx)^3)}dx$$
$$\Big\downarrow \; 3702$$

$$\frac{\int \frac{1}{(1-\sin^2(c+dx))^2(b\sin^3(c+dx)+a)}d\sin(c+dx)}{d}$$

$\big\downarrow$ 7276

$$\frac{\int \left(\frac{(2a^2-3b\sin(c+dx)a+b^2+(a^2+2b^2)\sin^2(c+dx))b^2}{(a^2-b^2)^2(b\sin^3(c+dx)+a)}+\frac{-a-4b}{4(a+b)^2(\sin(c+dx)-1)}+\frac{a-4b}{4(a-b)^2(\sin(c+dx)+1)}+\frac{1}{4(a+b)(\sin(c+dx)-1)^2}+\frac{}{4(a}\right.}{d}$$

$\big\downarrow$ 2009

$$\frac{\frac{b(a^2+2b^2)\log(a+b\sin^3(c+dx))}{3(a^2-b^2)^2}-\frac{b^{5/3}(-3a^{4/3}b^{2/3}+2a^2+b^2)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(a^2-b^2)^2}-\frac{b^{5/3}(3a^{4/3}b^{2/3}+2a^2+b^2)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\right.}{6a^{2/3}(a^2-b^2)^2}}{}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

output
```
(-((b^(5/3)*(2*a^2 - 3*a^(4/3)*b^(2/3) + b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*
Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a^2 - b^2)^2)) - ((a +
 4*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2) + ((a - 4*b)*Log[1 + Sin[c + d*
x]])/(4*(a - b)^2) + (b^(5/3)*(2*a^2 + 3*a^(4/3)*b^(2/3) + b^2)*Log[a^(1/3
) + b^(1/3)*Sin[c + d*x]])/(3*a^(2/3)*(a^2 - b^2)^2) - (b^(5/3)*(2*a^2 + 3
*a^(4/3)*b^(2/3) + b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/
3)*Sin[c + d*x]^2])/(6*a^(2/3)*(a^2 - b^2)^2) + (b*(a^2 + 2*b^2)*Log[a + b
*Sin[c + d*x]^3])/(3*(a^2 - b^2)^2) + 1/(4*(a + b)*(1 - Sin[c + d*x])) - 1
/(4*(a - b)*(1 + Sin[c + d*x])))/d
```

3.387.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.387. $\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.387.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.97

method	result
derivativedivides	$-\frac{1}{(4a+4b)(\sin(dx+c)-1)}+\frac{(-a-4b)\ln(\sin(dx+c)-1)}{4(a+b)^2}-\frac{1}{(4a-4b)(1+\sin(dx+c))}+\frac{(a-4b)\ln(1+\sin(dx+c))}{4(a-b)^2}+\frac{\left((2a^2+b^2)\left(\frac{\ln\left(\sin(dx\right.}{3b(}\right.\right.}{}$
default	$-\frac{1}{(4a+4b)(\sin(dx+c)-1)}+\frac{(-a-4b)\ln(\sin(dx+c)-1)}{4(a+b)^2}-\frac{1}{(4a-4b)(1+\sin(dx+c))}+\frac{(a-4b)\ln(1+\sin(dx+c))}{4(a-b)^2}+\frac{\left((2a^2+b^2)\left(\frac{\ln\left(\sin(dx\right.}{3b(}\right.\right.}{}$
risch	$\frac{2ibx}{a^2-2ab+b^2}-\frac{iac}{2d(a^2-2ab+b^2)}+\frac{2ibc}{d(a^2-2ab+b^2)}-\frac{iax}{2(a^2-2ab+b^2)}-\frac{4id^2b^3a^2c}{a^6d^3-2a^4b^2d^3+d^3b^4a^2}+\frac{iax}{2a^2+4ab+2b^2}$

input
```
int(sec(d*x+c)^3/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(4*a+4*b)/(sin(d*x+c)-1)+1/4/(a+b)^2*(-a-4*b)*ln(sin(d*x+c)-1)-1/(
4*a-4*b)/(1+sin(d*x+c))+1/4*(a-4*b)/(a-b)^2*ln(1+sin(d*x+c))+((2*a^2+b^2)*
(1/3/b/(1/b*a)^(2/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*ln(s
in(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))-3*a*b*(-1/3/b/(1
/b*a)^(1/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))+1/6/b/(1/b*a)^(1/3)*ln(sin(d*x+c)
^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3*3^(1/2)/b/(1/b*a)^(1/3)*arc
tan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))+1/3*(a^2+2*b^2)/b*ln(a+b*
sin(d*x+c)^3))*b^2/(a-b)^2/(a+b)^2)
```

3.387.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.13 (sec) , antiderivative size = 10135, normalized size of antiderivative = 26.32

$$\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.387.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sin(d*x+c)**3),x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*sin(c + d*x)**3), x)
```

3.387.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.22

$$\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx =$$

$$-\frac{\frac{4\sqrt{3}\left(ab^2\left(9\left(\frac{a}{b}\right)^{\frac{2}{3}}+4\right)-b^3\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{4a}{b}\right)-2a^2b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{a}{b}\right)+2a^3\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}-2a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{6\left(b^3\left(4\left(\frac{a}{b}\right)^{\frac{2}{3}}-1\right)+2a^2b\left(\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
-1/36*(4*sqrt(3)*(a*b^2*(9*(a/b)^(2/3) + 4) - b^3*(3*(a/b)^(1/3) + 4*a/b)
- 2*a^2*b*(3*(a/b)^(1/3) + a/b) + 2*a^3)*arctan(-1/3*sqrt(3)*((a/b)^(1/3)
- 2*sin(d*x + c))/(a/b)^(1/3))/((a^4*(a/b)^(2/3) - 2*a^2*b^2*(a/b)^(2/3) +
 b^4*(a/b)^(2/3))*(a/b)^(1/3)) - 6*(b^3*(4*(a/b)^(2/3) - 1) + 2*a^2*b*((a/
b)^(2/3) - 1) - 3*a*b^2*(a/b)^(1/3))*log(sin(d*x + c)^2 - (a/b)^(1/3)*sin(
d*x + c) + (a/b)^(2/3))/(a^4*(a/b)^(2/3) - 2*a^2*b^2*(a/b)^(2/3) + b^4*(a/
b)^(2/3)) - 12*(b^3*(2*(a/b)^(2/3) + 1) + a^2*b*((a/b)^(2/3) + 2) + 3*a*b^
2*(a/b)^(1/3))*log((a/b)^(1/3) + sin(d*x + c))/(a^4*(a/b)^(2/3) - 2*a^2*b^
2*(a/b)^(2/3) + b^4*(a/b)^(2/3)) - 9*(a - 4*b)*log(sin(d*x + c) + 1)/(a^2
- 2*a*b + b^2) + 9*(a + 4*b)*log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^2) + 1
8*(a*sin(d*x + c) - b)/((a^2 - b^2)*sin(d*x + c)^2 - a^2 + b^2))/d
```

3.387.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.32

$$\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$= \frac{\frac{4\left(3a^5b^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}-6a^3b^6\left(-\frac{a}{b}\right)^{\frac{1}{3}}+3ab^8\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2a^6b^3+3a^4b^5-b^9\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^9b-4a^7b^3+6a^5b^5-4a^3b^7+ab^9} + \frac{12\left(3\left(-ab^2\right)^{\frac{2}{3}}ab+\left(2a^2b+b^3\right)\left(-\right.\right.}{\sqrt{3}a^5-}}{}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
1/12*(4*(3*a^5*b^4*(-a/b)^(1/3) - 6*a^3*b^6*(-a/b)^(1/3) + 3*a*b^8*(-a/b)^
(1/3) - 2*a^6*b^3 + 3*a^4*b^5 - b^9)*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) +
sin(d*x + c)))/(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9) + 12*(3
*(-a*b^2)^(2/3)*a*b + (2*a^2*b + b^3)*(-a*b^2)^(1/3))*arctan(1/3*sqrt(3)*(
(-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(sqrt(3)*a^5 - 2*sqrt(3)*a^3*
b^2 + sqrt(3)*a*b^4) - 2*(3*(-a*b^2)^(2/3)*a*b - (2*a^2*b + b^3)*(-a*b^2)^
(1/3))*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a^5
 - 2*a^3*b^2 + a*b^4) + 4*(a^2*b + 2*b^3)*log(abs(b*sin(d*x + c)^3 + a))/(
a^4 - 2*a^2*b^2 + b^4) + 3*(a - 4*b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a
*b + b^2) - 3*(a + 4*b)*log(abs(sin(d*x + c) - 1))/(a^2 + 2*a*b + b^2) + 6
*(a^2*b*sin(d*x + c)^2 + 2*b^3*sin(d*x + c)^2 - a^3*sin(d*x + c) + a*b^2*s
in(d*x + c) - 3*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1)))/d
```

3.387.9 Mupad [B] (verification not implemented)

Time = 14.96 (sec) , antiderivative size = 898, normalized size of antiderivative = 2.33

$$\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$= \frac{\left(\sum_{k=1}^{3} \ln\left(-\mathrm{root}(54\,a^4\,b^2\,z^3 - 27\,a^2\,b^4\,z^3 - 27\,a^6\,z^3 + 54\,a^2\,b^3\,z^2 + 27\,a^4\,b\,z^2 - 9\,a^2\,b^2\,z + b^3, z, k)\,\left(-\frac{28\,\cdots}{a^4\cdots}\right.\right.\right.}{}$$

input
```
int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x)^3)),x)
```

output
```
(symsum(log((a*b^6)/(2*(a^4 + b^4 - 2*a^2*b^2)) - root(54*a^4*b^2*z^3 - 27
*a^2*b^4*z^3 - 27*a^6*z^3 + 54*a^2*b^3*z^2 + 27*a^4*b*z^2 - 9*a^2*b^2*z +
b^3, z, k)*(root(54*a^4*b^2*z^3 - 27*a^2*b^4*z^3 - 27*a^6*z^3 + 54*a^2*b^3
*z^2 + 27*a^4*b*z^2 - 9*a^2*b^2*z + b^3, z, k)*(root(54*a^4*b^2*z^3 - 27*a
^2*b^4*z^3 - 27*a^6*z^3 + 54*a^2*b^3*z^2 + 27*a^4*b*z^2 - 9*a^2*b^2*z + b^
3, z, k)*((48*a*b^9 + (51*a^3*b^7)/2 - 87*a^5*b^5 + (27*a^7*b^3)/2)/(a^4 +
 b^4 - 2*a^2*b^2) + root(54*a^4*b^2*z^3 - 27*a^2*b^4*z^3 - 27*a^6*z^3 + 54
*a^2*b^3*z^2 + 27*a^4*b*z^2 - 9*a^2*b^2*z + b^3, z, k)*((36*a*b^10 + 108*a
^3*b^8 - 324*a^5*b^6 + 180*a^7*b^4)/(a^4 + b^4 - 2*a^2*b^2) + (sin(c + d*x
)*(648*a^2*b^9 - 1080*a^4*b^7 + 216*a^6*b^5 + 216*a^8*b^3))/(4*(a^4 + b^4
- 2*a^2*b^2))) + (sin(c + d*x)*(48*b^10 + 552*a^2*b^8 - 600*a^4*b^6))/(4*(
a^4 + b^4 - 2*a^2*b^2))) - (12*a*b^8 - (219*a^3*b^6)/4 + 18*a^5*b^4)/(a^4
+ b^4 - 2*a^2*b^2) + (sin(c + d*x)*(96*b^9 + 120*a^2*b^7 - 171*a^4*b^5))/(
4*(a^4 + b^4 - 2*a^2*b^2))) - (28*a*b^7 - 6*a^3*b^5)/(a^4 + b^4 - 2*a^2*b^
2) + (sin(c + d*x)*(40*b^8 + 61*a^2*b^6))/(4*(a^4 + b^4 - 2*a^2*b^2))) + (
2*b^7*sin(c + d*x))/(a^4 + b^4 - 2*a^2*b^2))*root(54*a^4*b^2*z^3 - 27*a^2*
b^4*z^3 - 27*a^6*z^3 + 54*a^2*b^3*z^2 + 27*a^4*b*z^2 - 9*a^2*b^2*z + b^3,
z, k), k, 1, 3) + (b/(2*(a^2 - b^2)) - (a*sin(c + d*x))/(2*(a^2 - b^2)))/(
sin(c + d*x)^2 - 1) - (log(sin(c + d*x) - 1)*(a + 4*b))/(8*a*b + 4*a^2 + 4
*b^2) + (log(sin(c + d*x) + 1)*(a - 4*b))/(4*a^2 - 8*a*b + 4*b^2))/d
```

3.387. $\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.388 $\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.388.1 Optimal result

Integrand size = 23, antiderivative size = 764

$$\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = -\frac{2(-1)^{2/3}a^{2/3}\arctan\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}b^{4/3}d} + \frac{2\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}d} + \frac{2a^{2/3}\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}b^{4/3}d} - \frac{4\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}b^{2/3}d} + \frac{2\arctan\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}d} - \frac{2\sqrt[3]{-1}a^{2/3}\arctan\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}b^{4/3}d} - \frac{2\arctan\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}d} + \frac{4\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}b^{2/3}d} + \frac{4\text{arctanh}\left(\frac{\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}b^{2/3}d} - \frac{\cos(c+dx)}{bd}$$

output
```
-cos(d*x+c)/b/d+2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b
^(2/3))^(1/2))/a^(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*a^(2/3)*arctan((b^(1/
3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/b^(4/3)/d/(a^(2/3)
-b^(2/3))^(1/2)-4/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b
^(2/3))^(1/2))/b^(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)+4/3*arctanh((b^(1/3)+(-1)
^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2))/b^(
2/3)/d/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2)+4/3*arctanh((b^(1/3)-(-1)^(1/3)*
a^(1/3)*tan(1/2*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2))/b^(2/3)/d
/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2)+2/3*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3
)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(2/3)/d/(a^(2/
3)+(-1)^(1/3)*b^(2/3))^(1/2)-2/3*(-1)^(1/3)*a^(2/3)*arctan(((-1)^(2/3)*b^(
1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/b^(4/
3)/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)-2/3*(-1)^(2/3)*a^(2/3)*arctan(((-1
)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(
1/2))/b^(4/3)/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)-2/3*arctan((-1)^(1/3)*(
b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3)
)^(1/2))/a^(2/3)/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

3.388.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.54 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.39

$$\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx =$$

$$-\frac{3\cos(c+dx)+i\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&,\frac{2b\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-ib\log\left(1-2\right.}{}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

output
```
-1/3*(3*Cos[c + d*x] + I*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*
b*#1^4 + I*b*#1^6 & , (2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*b*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - (2*I)*a*ArcTan[Sin[c + d*x]/(Cos[c + d
*x] - #1)]*#1 - a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 + (2*I)*a*ArcTan[Si
n[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 + a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]
*#1^3 + 2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*b*Log[1 - 2*
Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) &
])/(b*d)
```

3.388.3 Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 767, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3705, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^4}{a+b\sin(c+dx)^3}dx$$

$$\Big\downarrow \text{3705}$$

$$\int \left(\frac{1}{a+b\sin^3(c+dx)}+\frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}-\frac{2\sin^2(c+dx)}{a+b\sin^3(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{4\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2a^{2/3}\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}-\frac{2\sqrt[3]{-1}a^{2/3}\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}-\frac{2\arctan\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2(-1)^{2/3}a^{2/3}\arctan\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{4\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3b^{2/3}d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}+\frac{4\text{arctanh}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3b^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}-\frac{\cos(c+dx)}{bd}$$

input
```
Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

3.388. $\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$

output
```
(2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(
3*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*a^(2/3)*ArcTan[(b^(1/3) + a^(1/3
)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b
^(4/3)*d) - (4*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) -
b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(2/3)*d) + (2*ArcTan[((-1)^(2/3)*b
^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3
*a^(2/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*(-1)^(1/3)*a^(2/3)*Arc
Tan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1
/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^(4/3)*d) - (2*ArcTa
n[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3
) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*d)
 - (2*(-1)^(2/3)*a^(2/3)*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*
Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) -
(-1)^(2/3)*b^(2/3)]*b^(4/3)*d) + (4*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*
Tan[(c + d*x)/2])/Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*Sqrt[-((-1)^(
2/3)*a^(2/3)) + b^(2/3)]*b^(2/3)*d) + (4*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(
1/3)*Tan[(c + d*x)/2])/Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*Sqrt[(-1)^(
1/3)*a^(2/3) + b^(2/3)]*b^(2/3)*d) - Cos[c + d*x]/(b*d)
```

3.388.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3705
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_)
), x_Symbol] :> Int[Expand[(1 - Sin[e + f*x]^2)^(m/2)/(a + b*Sin[e + f*x]^n
), x], x] /; FreeQ[{a, b, e, f}, x] && IGtQ[m/2, 0] && IntegerQ[(n - 1)/2]
```

3.388. $\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.388.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.74 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.16

method	result
derivativedivides	$\frac{-\frac{2}{b\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4b-2\_R^3a-6\_R^2b-2\_Ra+b\right)\ln\left(\tan\left(\ldots\right.\right.}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra\ldots}}{3b}}{d}$
default	$\frac{-\frac{2}{b\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4b-2\_R^3a-6\_R^2b-2\_Ra+b\right)\ln\left(\tan\left(\ldots\right.\right.}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra\ldots}}{3b}}{d}$
risch	$-\frac{e^{i(dx+c)}}{2bd}-\frac{e^{-i(dx+c)}}{2bd}+\left(\sum_{\_R=\mathrm{RootOf}\left(729a^4b^8d^6\_Z^6-729a^4b^6d^4\_Z^4+\left(162a^4b^4d^2+81a^2b^6d^2\right)\_Z^2+a^6-3a^4b^2\ldots\right.}\right.$

input
```
int(cos(d*x+c)^4/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/b/(1+tan(1/2*d*x+1/2*c)^2)+1/3/b*sum((_R^4*b-2*_R^3*a-6*_R^2*b-2*_
R*a+b)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf
(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a)))
```

3.388.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.71 (sec) , antiderivative size = 23437, normalized size of antiderivative = 30.68

$$\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.388.6 Sympy [F]

$$\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**4/(a+b*sin(d*x+c)**3),x)
```

output
```
Integral(cos(c + d*x)**4/(a + b*sin(c + d*x)**3), x)
```

3.388.7 Maxima [F]

$$\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\cos(dx+c)^4}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
-(b*d*integrate(4*(3*a*b*cos(4*d*x + 4*c)^2 + 3*a*b*cos(2*d*x + 2*c)^2 + 3
*a*b*sin(4*d*x + 4*c)^2 + 3*b^2*cos(d*x + c)*sin(2*d*x + 2*c) + 3*a*b*sin(
2*d*x + 2*c)^2 + b^2*sin(d*x + c) - (a*b*cos(4*d*x + 4*c) - a*b*cos(2*d*x
+ 2*c) + b^2*sin(5*d*x + 5*c) + b^2*sin(d*x + c))*cos(6*d*x + 6*c) - (8*a*
b*cos(3*d*x + 3*c) + 3*b^2*sin(4*d*x + 4*c) - 3*b^2*sin(2*d*x + 2*c))*cos(
5*d*x + 5*c) - (6*a*b*cos(2*d*x + 2*c) + 8*a^2*sin(3*d*x + 3*c) - 3*b^2*si
n(d*x + c) - a*b)*cos(4*d*x + 4*c) - 8*(a*b*cos(d*x + c) + a^2*sin(2*d*x +
 2*c))*cos(3*d*x + 3*c) - (3*b^2*sin(d*x + c) + a*b)*cos(2*d*x + 2*c) + (b
^2*cos(5*d*x + 5*c) + b^2*cos(d*x + c) - a*b*sin(4*d*x + 4*c) + a*b*sin(2*
d*x + 2*c))*sin(6*d*x + 6*c) + (3*b^2*cos(4*d*x + 4*c) - 3*b^2*cos(2*d*x +
 2*c) - 8*a*b*sin(3*d*x + 3*c) + b^2)*sin(5*d*x + 5*c) + (8*a^2*cos(3*d*x
+ 3*c) - 3*b^2*cos(d*x + c) - 6*a*b*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) + 8
*(a^2*cos(2*d*x + 2*c) - a*b*sin(d*x + c))*sin(3*d*x + 3*c))/(b^3*cos(6*d*
x + 6*c)^2 + 9*b^3*cos(4*d*x + 4*c)^2 + 64*a^2*b*cos(3*d*x + 3*c)^2 + 9*b^
3*cos(2*d*x + 2*c)^2 + b^3*sin(6*d*x + 6*c)^2 + 9*b^3*sin(4*d*x + 4*c)^2 +
 64*a^2*b*sin(3*d*x + 3*c)^2 - 48*a*b^2*cos(3*d*x + 3*c)*sin(2*d*x + 2*c)
+ 9*b^3*sin(2*d*x + 2*c)^2 - 6*b^3*cos(2*d*x + 2*c) + b^3 - 2*(3*b^3*cos(4
*d*x + 4*c) - 3*b^3*cos(2*d*x + 2*c) - 8*a*b^2*sin(3*d*x + 3*c) + b^3)*cos
(6*d*x + 6*c) - 6*(3*b^3*cos(2*d*x + 2*c) + 8*a*b^2*sin(3*d*x + 3*c) - b^3
)*cos(4*d*x + 4*c) - 2*(8*a*b^2*cos(3*d*x + 3*c) + 3*b^3*sin(4*d*x + 4*...
```

3.388.8 Giac [F]

$$\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\cos(dx+c)^4}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.388.9 Mupad [B] (verification not implemented)

Time = 17.29 (sec) , antiderivative size = 2338, normalized size of antiderivative = 3.06

$$\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^4/(a + b*sin(c + d*x)^3),x)
```

output
```
symsum(log(172032*a^4*b^9 - 81920*a^2*b^11 - 98304*a^6*b^7 + 8192*a^8*b^5
- 319488*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2
*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)*a^2*b^12 - 688128*root
(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*
a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)*a^4*b^10 + 344064*root(729*a^4*b^8*
d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a
^2*b^4 + a^6 - b^6, d, k)*a^6*b^8 - 98304*a*b^12*tan(c/2 + (d*x)/2) - 2949
12*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d
^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^2*a^2*b^13 - 1400832*root(72
9*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4
*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^2*a^4*b^11 + 1695744*root(729*a^4*b^8*
d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a
^2*b^4 + a^6 - b^6, d, k)^2*a^6*b^9 + 5750784*root(729*a^4*b^8*d^6 - 729*a
^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^
6 - b^6, d, k)^3*a^4*b^12 - 3760128*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4
 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d
, k)^3*a^6*b^10 + 3317760*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4
*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^4*a^4
*b^13 - 3317760*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 +
 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^4*a^6*b^11 -...
```

3.389 $\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.389.1 Optimal result

Integrand size = 23, antiderivative size = 484

$$\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{2\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}d} - \frac{2\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}b^{2/3}d} + \frac{2\arctan\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}d} - \frac{2\arctan\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}d} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}b^{2/3}d} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}b^{2/3}d}$$

output
```
2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a
^(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1
/2*c))/(a^(2/3)-b^(2/3))^(1/2))/b^(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*arct
anh((b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)+b^
(2/3))^(1/2))/b^(2/3)/d/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2)+2/3*arctanh((b^
(1/3)-(-1)^(1/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)+b^(2/3))
^(1/2))/b^(2/3)/d/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2)+2/3*arctan(((-1)^(2/
3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))
/a^(2/3)/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)-2/3*arctan((-1)^(1/3)*(b^(1/
3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/
2))/a^(2/3)/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

3.389.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.48

$$\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx =$$

$$-\frac{i\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&,\frac{2\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)}{\cdots}\right]}{\cdots}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

output
```
((-1/6*I)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1
^6 & , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d
*x]*#1 + #1^2] + 4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (2*I)*L
og[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x
] - #1)]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*
#1^2 - 2*b*#1^3 + b*#1^5) & ])/d
```

3.389.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3705, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(c+dx)^2}{a+b\sin(c+dx)^3}dx$$
$$\Big\downarrow \text{3705}$$
$$\int \left(\frac{1}{a+b\sin^3(c+dx)}-\frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}-\frac{2\arctan\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\text{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3b^{2/3}d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}+\frac{2\text{arctanh}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3b^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

output
```
(2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(
3*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) - (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c
 + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(2/3)*d
) + (2*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3)
 + (-1)^(1/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d)
- (2*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/S
qrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b
^(2/3)]*d) + (2*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sq
rt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2
/3)]*b^(2/3)*d) + (2*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2
])/Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/
3)]*b^(2/3)*d)
```

3.389.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3705
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_)
), x_Symbol] :> Int[Expand[(1 - Sin[e + f*x]^2)^(m/2)/(a + b*Sin[e + f*x]^n
), x], x] /; FreeQ[{a, b, e, f}, x] && IGtQ[m/2, 0] && IntegerQ[(n - 1)/2]
```

3.389.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.89 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.17

method	result
derivativedivides	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}+3a\_Z^{4}+8b\_Z^{3}+3a\_Z^{2}+a\right)}\frac{\left(\_R^{4}-2\_R^{2}+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a+2\_R^{3}a+4\_R^{2}b+\_Ra}}{3d}$
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}+3a\_Z^{4}+8b\_Z^{3}+3a\_Z^{2}+a\right)}\frac{\left(\_R^{4}-2\_R^{2}+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a+2\_R^{3}a+4\_R^{2}b+\_Ra}}{3d}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(729a^{4}b^{4}d^{6}\_Z^{6}+27a^{2}b^{2}d^{2}\_Z^{2}+a^{2}-b^{2}\right)}\_R\ln\left(\mathrm{e}^{i(dx+c)}-\frac{243a^{4}b^{3}d^{5}\_R^{5}}{a^{2}+b^{2}}+\frac{81id^{4}b^{3}a^{3}\_R^{4}}{a^{2}+b^{2}}-\right.$

input
```
int(cos(d*x+c)^2/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*sum((_R^4-2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1
/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

3.389.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 8236, normalized size of antiderivative = 17.02

$$\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.389.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*sin(d*x+c)**3),x)
```

output
```
Integral(cos(c + d*x)**2/(a + b*sin(c + d*x)**3), x)
```

3.389.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\cos(dx+c)^2}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

3.389.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\cos(dx+c)^2}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

3.389.9 Mupad [B] (verification not implemented)

Time = 15.55 (sec) , antiderivative size = 951, normalized size of antiderivative = 1.96

$$\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$= \frac{\sum_{k=1}^{6} \ln\left(24576\,a^4 - 24576\,a^2\,b^2 - \mathrm{root}(729\,a^4\,b^4\,d^6 + 27\,a^2\,b^2\,d^2 + a^2 - b^2, d, k)\ a^2\,b^3\,122880 - \mathrm{root}(729\right.}{}$$

input
```
int(cos(c + d*x)^2/(a + b*sin(c + d*x)^3),x)
```

output
```
symsum(log(24576*a^4 - 24576*a^2*b^2 - 122880*root(729*a^4*b^4*d^6 + 27*a^
2*b^2*d^2 + a^2 - b^2, d, k)*a^2*b^3 - 24576*root(729*a^4*b^4*d^6 + 27*a^2
*b^2*d^2 + a^2 - b^2, d, k)*a^5*tan(c/2 + (d*x)/2) - 32768*a*b^3*tan(c/2 +
 (d*x)/2) + 32768*a^3*b*tan(c/2 + (d*x)/2) - 294912*root(729*a^4*b^4*d^6 +
 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^2*a^2*b^4 + 294912*root(729*a^4*b^4*d^6
 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^2*a^4*b^2 + 663552*root(729*a^4*b^4*d
^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^4*a^4*b^4 - 663552*root(729*a^4*b^4
*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^4*a^6*b^2 - 7962624*root(729*a^4*
b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^5*a^4*b^5 + 5971968*root(729*a
^4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^5*a^6*b^3 + 49152*root(729*
a^4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)*a^4*b + 147456*root(729*a^
4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)*a^3*b^2*tan(c/2 + (d*x)/2) +
 294912*root(729*a^4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^2*a^5*b*t
an(c/2 + (d*x)/2) - 294912*root(729*a^4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b
^2, d, k)^2*a^3*b^3*tan(c/2 + (d*x)/2) + 1769472*root(729*a^4*b^4*d^6 + 27
*a^2*b^2*d^2 + a^2 - b^2, d, k)^3*a^3*b^4*tan(c/2 + (d*x)/2) - 1769472*roo
t(729*a^4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^3*a^5*b^2*tan(c/2 +
(d*x)/2) - 5308416*root(729*a^4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k
)^4*a^3*b^5*tan(c/2 + (d*x)/2) + 5308416*root(729*a^4*b^4*d^6 + 27*a^2*b^2
*d^2 + a^2 - b^2, d, k)^4*a^5*b^3*tan(c/2 + (d*x)/2) - 1990656*root(729...
```

3.389. $\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.390 $\int \frac{1}{a+b\sin^3(c+dx)}\,dx$

3.390.1 Optimal result

Integrand size = 14, antiderivative size = 245

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx = \frac{2\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}d} + \frac{2\arctan\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}d} - \frac{2\arctan\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}d}$$

output
```
2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a
^(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*ta
n(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(2/3)/d/(a^(2/3)+(
-1)^(1/3)*b^(2/3))^(1/2)-2/3*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(2/3)*a^(1/3)
*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(2/3)/d/(a^(2/3
)-(-1)^(2/3)*b^(2/3))^(1/2)
```

3.390.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx = \$\text{Aborted}$$

input
```
Integrate[(a + b*Sin[c + d*x]^3)^(-1),x]
```

output
```
$Aborted
```

3.390.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin(c+dx)^3}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)}-\frac{1}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)-\sqrt[3]{a}\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\sin(c+\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\arctan\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}-\frac{2\arctan\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

input
```
Int[(a + b*Sin[c + d*x]^3)^(-1),x]
```

output
```
(2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(
3*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*ArcTan[((-1)^(2/3)*b^(1/3) + a^(
1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*a^(2/3)*Sqr
t[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)
^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*
a^(2/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*d)
```

3.390.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.390.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.46 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.34

method	result
derivativedivides	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}}{3d}$
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}}{3d}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+\left(729a^6d^6-729a^4b^2d^6\right)\_Z^6+243a^4d^4\_Z^4+27a^2d^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{i(dx+c)}+\left(-\frac{486d^5a^6}{b}+48\right.\right.$

input
```
int(1/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1
/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

3.390.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.25 (sec) , antiderivative size = 25429, normalized size of antiderivative = 103.79

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.390.6 Sympy [F]

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx = \int \frac{1}{a+b\sin^3{(c+dx)}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c)**3),x)
```

output
```
Integral(1/(a + b*sin(c + d*x)**3), x)
```

3.390.7 Maxima [F]

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx = \int \frac{1}{b\sin{(dx+c)}^3+a}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate(1/(b*sin(d*x + c)^3 + a), x)
```

3.390.8 Giac [F]

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx=\int \frac{1}{b\sin\left(dx+c\right)^3+a}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(1/(b*sin(d*x + c)^3 + a), x)
```

3.390.9 Mupad [B] (verification not implemented)

Time = 14.89 (sec) , antiderivative size = 609, normalized size of antiderivative = 2.49

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx = \frac{\sum_{k=1}^{6}\ln\left(-\frac{8192\,a\,b^3\,\left(-729\,a^5+243\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^4\,b-324\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^4\,\mathrm{root}\left(d^6+27\,a^2\,d^4+243\,a^4\,d^2+729\,a^4\,\left(a^2-b^2\right),d,k\right)+972\,a^3\,b^2+a\right.}{}\right.}{}$$

input
```
int(1/(a + b*sin(c + d*x)^3),x)
```

output
```
symsum(log(-(8192*a*b^3*(972*a^3*b^2 - 729*a^5 - 9*a*root(d^6 + 27*a^2*d^4
 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^4 - 162*a^3*root(d^6 + 27*a^2*
d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^2 - 4*tan(c/2 + (d*x)/2)*ro
ot(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5 + 243*a^4
*b*tan(c/2 + (d*x)/2) - 324*tan(c/2 + (d*x)/2)*a^4*root(d^6 + 27*a^2*d^4 +
 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k) + 24*b*tan(c/2 + (d*x)/2)*root(d
^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^4 - 72*a^2*tan(
c/2 + (d*x)/2)*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2),
d, k)^3 + 36*a*b*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2)
, d, k)^3 + 243*b*a^3*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 -
 b^2), d, k) + 648*tan(c/2 + (d*x)/2)*a^2*b^2*root(d^6 + 27*a^2*d^4 + 243*
a^4*d^2 + 729*a^4*(a^2 - b^2), d, k) + 216*a^2*b*tan(c/2 + (d*x)/2)*root(d
^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^2))/root(d^6 +
27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5)*root(729*a^4*b^2*
d^6 - 729*a^6*d^6 - 243*a^4*d^4 - 27*a^2*d^2 - 1, d, k), k, 1, 6)/d
```

3.391 $\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.391.1 Optimal result

Integrand size = 23, antiderivative size = 299

$$\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{2(-1)^{2/3}b^{2/3}\arctan\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\left(a^{2/3}-(-1)^{2/3}b^{2/3}\right)^{3/2}d} - \frac{2b^{2/3}\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\left(a^{2/3}-b^{2/3}\right)^{3/2}d} + \frac{2\sqrt[3]{-1}b^{2/3}\arctan\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}\left(a^{2/3}+\sqrt[3]{-1}b^{2/3}\right)^{3/2}d} + \frac{\sec(c+dx)(b-a\sin(c+dx))}{(-a^2+b^2)\,d}$$

output
```
2/3*(-1)^(2/3)*b^(2/3)*arctan(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*
c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(2/3)/(a^(2/3)-(-1)^(2/3)*b^(2/3
))^(3/2)/d-2/3*b^(2/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3
)-b^(2/3))^(1/2))/a^(2/3)/(a^(2/3)-b^(2/3))^(3/2)/d+2/3*(-1)^(1/3)*b^(2/3)
*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3
)*b^(2/3))^(1/2))/a^(2/3)/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(3/2)/d+sec(d*x+c)*
(b-a*sin(d*x+c))/(-a^2+b^2)/d
```

3.391.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.52 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.44

$$\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$= \frac{-6b+6b\cos(c+dx)-ib\cos(c+dx)\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&,\frac{2b\arctan\left(\frac{\sin}{\cos(}\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

output
```
(-6*b + 6*b*Cos[c + d*x] - I*b*Cos[c + d*x]*RootSum[(-I)*b + (3*I)*b*#1^2
+ 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & , (2*b*ArcTan[Sin[c + d*x]/(Cos[c +
 d*x] - #1)] - I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (4*I)*a*ArcTan[Sin[
c + d*x]/(Cos[c + d*x] - #1)]*#1 + 2*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#
1 - 12*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (6*I)*b*Log[1 - 2
*Cos[c + d*x]*#1 + #1^2]*#1^2 - (4*I)*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
- #1)]*#1^3 - 2*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 + 2*b*ArcTan[Sin[
c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]
*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ] + 6*a*Sin[c + d*x])/(
6*(a - b)*(a + b)*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]))
```

3.391.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos(c+dx)^2\left(a+b\sin(c+dx)^3\right)}dx$$

$$\Big\downarrow \text{ 3707}$$

$$\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}dx$$

input
```
Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

output
```
$Aborted
```

3.391.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3707
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Cos[e + f*x])^m*(a +
 b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.391.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.23 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.54

method	result
derivativedivides	$\frac{-\frac{2}{(2a-2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(-\_R^4 b+2\_R^3 a-6\_R^2 b+2\_R a-b\right.}{\_R^5 a+2\_R^3 a+4\_R^2}\right.}{3(a-b)(a+b)}}{d}$
default	$\frac{-\frac{2}{(2a-2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(-\_R^4 b+2\_R^3 a-6\_R^2 b+2\_R a-b\right.}{\_R^5 a+2\_R^3 a+4\_R^2}\right.}{3(a-b)(a+b)}}{d}$
risch	Expression too large to display

input
```
int(sec(d*x+c)^2/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/(2*a-2*b)/(tan(1/2*d*x+1/2*c)+1)+1/3*b/(a-b)/(a+b)*sum((-_R^4*b+2*
_R^3*a-6*_R^2*b+2*_R*a-b)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1
/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-2/(2*a+2*b)/(tan
(1/2*d*x+1/2*c)-1))
```

3.391.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.89 (sec) , antiderivative size = 59362, normalized size of antiderivative = 198.54

$$\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.391.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sin(d*x+c)**3),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*sin(c + d*x)**3), x)
```

3.391.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sec(dx+c)^2}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
-(2*b*cos(2*d*x + 2*c)*cos(d*x + c) + 2*b*cos(d*x + c) - ((a^2 - b^2)*d*co
s(2*d*x + 2*c)^2 + (a^2 - b^2)*d*sin(2*d*x + 2*c)^2 + 2*(a^2 - b^2)*d*cos(
2*d*x + 2*c) + (a^2 - b^2)*d)*integrate(2*(6*a*b^2*cos(4*d*x + 4*c)^2 - 48
*a*b^2*cos(3*d*x + 3*c)^2 + 6*a*b^2*cos(2*d*x + 2*c)^2 + 6*a*b^2*sin(4*d*x
 + 4*c)^2 - 48*a*b^2*sin(3*d*x + 3*c)^2 - 3*b^3*cos(d*x + c)*sin(2*d*x + 2
*c) + 6*a*b^2*sin(2*d*x + 2*c)^2 - b^3*sin(d*x + c) - (2*a*b^2*cos(4*d*x +
 4*c) - 2*a*b^2*cos(2*d*x + 2*c) - b^3*sin(5*d*x + 5*c) + 6*b^3*sin(3*d*x
+ 3*c) - b^3*sin(d*x + c))*cos(6*d*x + 6*c) + (8*a*b^2*cos(3*d*x + 3*c) +
3*b^3*sin(4*d*x + 4*c) - 3*b^3*sin(2*d*x + 2*c))*cos(5*d*x + 5*c) - (12*a*
b^2*cos(2*d*x + 2*c) + 3*b^3*sin(d*x + c) - 2*a*b^2 + 2*(8*a^2*b - 9*b^3)*
sin(3*d*x + 3*c))*cos(4*d*x + 4*c) + 2*(4*a*b^2*cos(d*x + c) - (8*a^2*b -
9*b^3)*sin(2*d*x + 2*c))*cos(3*d*x + 3*c) + (3*b^3*sin(d*x + c) - 2*a*b^2)
*cos(2*d*x + 2*c) - (b^3*cos(5*d*x + 5*c) - 6*b^3*cos(3*d*x + 3*c) + b^3*c
os(d*x + c) + 2*a*b^2*sin(4*d*x + 4*c) - 2*a*b^2*sin(2*d*x + 2*c))*sin(6*d
*x + 6*c) - (3*b^3*cos(4*d*x + 4*c) - 3*b^3*cos(2*d*x + 2*c) - 8*a*b^2*sin
(3*d*x + 3*c) + b^3)*sin(5*d*x + 5*c) + (3*b^3*cos(d*x + c) - 12*a*b^2*sin
(2*d*x + 2*c) + 2*(8*a^2*b - 9*b^3)*cos(3*d*x + 3*c))*sin(4*d*x + 4*c) + 2
*(4*a*b^2*sin(d*x + c) + 3*b^3 + (8*a^2*b - 9*b^3)*cos(2*d*x + 2*c))*sin(3
*d*x + 3*c))/(a^2*b^2 - b^4 + (a^2*b^2 - b^4)*cos(6*d*x + 6*c)^2 + 9*(a^2*
b^2 - b^4)*cos(4*d*x + 4*c)^2 + 64*(a^4 - a^2*b^2)*cos(3*d*x + 3*c)^2 +...
```

3.391.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx=\int \frac{\sec\left(dx+c\right)^2}{b\sin\left(dx+c\right)^3+a}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.391.9 Mupad [B] (verification not implemented)

Time = 17.20 (sec) , antiderivative size = 19737, normalized size of antiderivative = 66.01

$$\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x)^3)),x)
```

output
```
(a^2*symsum(log((8192*(a^2*b^20*cos(c/2 + (d*x)/2) - 12*a*b^21*sin(c/2 + (
d*x)/2) - 7*a^4*b^18*cos(c/2 + (d*x)/2) + 21*a^6*b^16*cos(c/2 + (d*x)/2) -
 35*a^8*b^14*cos(c/2 + (d*x)/2) + 35*a^10*b^12*cos(c/2 + (d*x)/2) - 21*a^1
2*b^10*cos(c/2 + (d*x)/2) + 7*a^14*b^8*cos(c/2 + (d*x)/2) - a^16*b^6*cos(c
/2 + (d*x)/2) + 84*a^3*b^19*sin(c/2 + (d*x)/2) - 252*a^5*b^17*sin(c/2 + (d
*x)/2) + 420*a^7*b^15*sin(c/2 + (d*x)/2) - 420*a^9*b^13*sin(c/2 + (d*x)/2)
 + 252*a^11*b^11*sin(c/2 + (d*x)/2) - 84*a^13*b^9*sin(c/2 + (d*x)/2) + 12*
a^15*b^7*sin(c/2 + (d*x)/2) - 198*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6
 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 8
1*a^2*b^4*d^2 - b^4, d, k)*a^3*b^20*sin(c/2 + (d*x)/2) + 714*root(2187*a^8
*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^
4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^5*b^18*sin(c/2 + (
d*x)/2) - 1470*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6
- 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4
, d, k)*a^7*b^16*sin(c/2 + (d*x)/2) + 1890*root(2187*a^8*b^2*d^6 - 2187*a^
6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^
2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^9*b^14*sin(c/2 + (d*x)/2) - 1554*roo
t(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1
458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^11*b^12*
sin(c/2 + (d*x)/2) + 798*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729...
```

3.392 $\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$

3.392.1 Optimal result

Integrand size = 23, antiderivative size = 1093

$$\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = -\frac{2(-1)^{2/3}a^{2/3}b^{8/3}\arctan\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,(a^2-b^2)^2 d}$$
$$-\frac{2b^2(2a^2+b^2)\arctan\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,(a^2-b^2)^2 d}$$
$$+\frac{2a^{2/3}b^{8/3}\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{\sqrt{a^{2/3}-b^{2/3}}\,(a^2-b^2)^2 d}$$
$$+\frac{2b^2(2a^2+b^2)\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}\,(a^2-b^2)^2 d}$$
$$+\frac{2b^{4/3}(a^2+2b^2)\arctan\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,(a^2-b^2)^2 d}$$
$$-\frac{2\sqrt[3]{-1}a^{2/3}b^{8/3}\arctan\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}\,(a^2-b^2)^2 d}$$
$$+\frac{2b^2(2a^2+b^2)\arctan\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}\,(a^2-b^2)^2 d}$$
$$-\frac{2b^{4/3}(a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}\,(a^2-b^2)^2 d}$$
$$-\frac{2b^{4/3}(a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}\,(a^2-b^2)^2 d}$$
$$+\frac{\cos(c+dx)}{12(a+b)d(1-\sin(c+dx))^2}+\frac{\cos(c+dx)}{12(a+b)d(1-\sin(c+dx))}$$
$$+\frac{(a+4b)\cos(c+dx)}{4(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{12(a-b)d(1+\sin(c+dx))^2}$$
$$-\frac{(a-4b)\cos(c+dx)}{4(a-b)^2d(1+\sin(c+dx))}-\frac{\cos(c+dx)}{12(a-b)d(1+\sin(c+dx))}$$

output
```
1/12*cos(d*x+c)/(a+b)/d/(1-sin(d*x+c))^2+1/12*cos(d*x+c)/(a+b)/d/(1-sin(d*
x+c))+1/4*(a+4*b)*cos(d*x+c)/(a+b)^2/d/(1-sin(d*x+c))-1/12*cos(d*x+c)/(a-b
)/d/(1+sin(d*x+c))^2-1/4*(a-4*b)*cos(d*x+c)/(a-b)^2/d/(1+sin(d*x+c))-1/12*
cos(d*x+c)/(a-b)/d/(1+sin(d*x+c))+2*a^(2/3)*b^(8/3)*arctan((b^(1/3)+a^(1/3
)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/(a^2-b^2)^2/d/(a^(2/3)-b^(2
/3))^(1/2)+2/3*b^2*(2*a^2+b^2)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))
/(a^(2/3)-b^(2/3))^(1/2))/a^(2/3)/(a^2-b^2)^2/d/(a^(2/3)-b^(2/3))^(1/2)+2/
3*b^(4/3)*(a^2+2*b^2)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)
-b^(2/3))^(1/2))/(a^2-b^2)^2/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*b^(4/3)*(a^2+2*
b^2)*arctanh((b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/((-1)^(1/3)*a
^(2/3)+b^(2/3))^(1/2))/(a^2-b^2)^2/d/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2)-2/
3*b^(4/3)*(a^2+2*b^2)*arctanh((b^(1/3)-(-1)^(1/3)*a^(1/3)*tan(1/2*d*x+1/2*
c))/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2))/(a^2-b^2)^2/d/(-(-1)^(2/3)*a^(2/3
)+b^(2/3))^(1/2)-2*(-1)^(1/3)*a^(2/3)*b^(8/3)*arctan(((-1)^(2/3)*b^(1/3)+a
^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/(a^2-b^2)^2
/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)+2/3*b^2*(2*a^2+b^2)*arctan(((-1)^(2/
3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))
/a^(2/3)/(a^2-b^2)^2/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)-2*(-1)^(2/3)*a^(
2/3)*b^(8/3)*arctan(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/
3)-(-1)^(2/3)*b^(2/3))^(1/2))/(a^2-b^2)^2/d/(a^(2/3)-(-1)^(2/3)*b^(2/3)...
```

3.392.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.06 (sec) , antiderivative size = 679, normalized size of antiderivative = 0.62

$$\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$= \frac{4ib^2\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&, \frac{2a^2\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+4b^2\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

output
```
((4*I)*b^2*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#
1^6 & , (2*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 4*b^2*ArcTan[Sin
[c + d*x]/(Cos[c + d*x] - #1)] - I*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] -
 (2*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (12*I)*a*b*ArcTan[Sin[c + d
*x]/(Cos[c + d*x] - #1)]*#1 + 6*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 -
 20*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 16*b^2*ArcTan[Sin[
c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (10*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1
+ #1^2]*#1^2 + (8*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (12*I)*a
*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - 6*a*b*Log[1 - 2*Cos[c +
 d*x]*#1 + #1^2]*#1^3 + 2*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^
4 + 4*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*a^2*Log[1 - 2*
Cos[c + d*x]*#1 + #1^2]*#1^4 - (2*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]
*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ] + Sec[c + d*x]^3*(4*a
^2*b + 32*b^3 - 3*b*(5*a^2 + 13*b^2)*Cos[c + d*x] + 12*b*(a^2 + 2*b^2)*Cos
[2*(c + d*x)] - 5*a^2*b*Cos[3*(c + d*x)] - 13*b^3*Cos[3*(c + d*x)] + 12*a^
3*Sin[c + d*x] - 30*a*b^2*Sin[c + d*x] + 4*a^3*Sin[3*(c + d*x)] - 22*a*b^2
*Sin[3*(c + d*x)]))/(24*(a - b)^2*(a + b)^2*d)
```

3.392.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\cos(c+dx)^4\,(a+b\sin(c+dx)^3)}dx$$
$$\Big\downarrow \text{3707}$$
$$\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}dx$$

input
```
Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

output
```
$Aborted
```

3.392.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3707
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Cos[e + f*x])^m*(a +
 b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.392.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.09 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.27

method	result
derivativedivides	$\frac{\frac{b^2\left(\sum_{\_R=\operatorname{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left((2a^2+b^2)\_R^4-6ab\_R^3+2(4a^2+5b^2)\_R^2-6a\_Rb+2a^2+b^2\right)\ln}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right.}{3(a-b)^2(a+b)^2}}{}$
default	$\frac{\frac{b^2\left(\sum_{\_R=\operatorname{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left((2a^2+b^2)\_R^4-6ab\_R^3+2(4a^2+5b^2)\_R^2-6a\_Rb+2a^2+b^2\right)\ln}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right.}{3(a-b)^2(a+b)^2}}{}$
risch	Expression too large to display

input
```
int(sec(d*x+c)^4/(a+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*b^2/(a-b)^2/(a+b)^2*sum(((2*a^2+b^2)*_R^4-6*a*b*_R^3+2*(4*a^2+5*b
^2)*_R^2-6*a*_R*b+2*a^2+b^2)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*
x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-2/3/(tan(1/2*
d*x+1/2*c)-1)^3/(2*a+2*b)-1/(2*a+2*b)/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(2*a+5*
b)/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)-2/3/(tan(1/2*d*x+1/2*c)+1)^3/(2*a-2*b)+1
/(2*a-2*b)/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(2*a-5*b)/(a-b)^2/(tan(1/2*d*x+1/2
*c)+1))
```

3.392.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.20 (sec) , antiderivative size = 85064, normalized size of antiderivative = 77.83

$$\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.392.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sin(d*x+c)**3),x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*sin(c + d*x)**3), x)
```

3.392.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.392.8 Giac [F]

$$\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sec(dx+c)^4}{b\sin(dx+c)^3+a}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.392.9 Mupad [B] (verification not implemented)

Time = 24.62 (sec) , antiderivative size = 323390, normalized size of antiderivative = 295.87

$$\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x)^3)),x)
```

output
```
(14*b^3*cos(c/2 + (d*x)/2)^6)/(3*(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c
/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6
- 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*
a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/
2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2
)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(
c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*s
in(c/2 + (d*x)/2)^2)) - (4*a^3*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^3)/
(3*(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c/2 + (d*x)/2)^6 - a^4*d*sin(c/
2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/
2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin
(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*
b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b^4*d*cos(c/2 + (d*x)/
2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*
x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2)) + (6*b^3
*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4)/(a^4*d*cos(c/2 + (d*x)/2)^6 +
b^4*d*cos(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 +
(d*x)/2)^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x
)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c
/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin...
```

3.393 $\int \frac{\cos^7(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

3.393.1 Optimal result

Integrand size = 23, antiderivative size = 288

$$\int \frac{\cos^7(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = -\frac{2(2a^2+3a^{4/3}b^{2/3}+b^2)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{7/3}d} + \frac{2(2a^2-3a^{4/3}b^{2/3}+b^2)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{7/3}d} - \frac{(2a^2-3a^{4/3}b^{2/3}+b^2)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}b^{7/3}d} - \frac{\sin(c+dx)}{b^2d} - \frac{\sin(c+dx)\,(a^2-b^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx))}{3ab^2d\,(a+b\sin^3(c+dx))}$$

output
```
2/9*(2*a^2-3*a^(4/3)*b^(2/3)+b^2)*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(5/3)/b
^(7/3)/d-1/9*(2*a^2-3*a^(4/3)*b^(2/3)+b^2)*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(
d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(5/3)/b^(7/3)/d-sin(d*x+c)/b^2/d-1/3*sin(d*
x+c)*(a^2-b^2+3*a*b*sin(d*x+c)+3*b^2*sin(d*x+c)^2)/a/b^2/d/(a+b*sin(d*x+c)
^3)-2/9*(2*a^2+3*a^(4/3)*b^(2/3)+b^2)*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*
x+c))/a^(1/3)*3^(1/2))/a^(5/3)/b^(7/3)/d*3^(1/2)
```

3.393.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.13 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.52

$$\int \frac{\cos^7(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$$
$$= \frac{\frac{4\sqrt{3}(a^2-b^2)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}b^{7/3}}+\frac{6\sqrt[3]{-1}\left(2\sqrt[3]{-1}a^{2/3}+3b^{2/3}\right)\log\left(-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)}{\sqrt[3]{a}b^{7/3}}+\frac{6(2a^{2/3}-3b^{2/3})\text{lo}}{}}{}$$

input
```
Integrate[Cos[c + d*x]^7/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
((4*Sqrt[3]*(a^2 - b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]
*a^(1/3))])/(a^(5/3)*b^(7/3)) + (6*(-1)^(1/3)*(2*(-1)^(1/3)*a^(2/3) + 3*b^
(2/3))*Log[-((-1)^(2/3)*a^(1/3)) - b^(1/3)*Sin[c + d*x]])/(a^(1/3)*b^(7/3)
) + (6*(2*a^(2/3) - 3*b^(2/3))*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(a^(1/
3)*b^(7/3)) - (4*(a^2 - b^2)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(a^(5/3)
*b^(7/3)) - (6*(-1)^(1/3)*(2*a^(2/3) + 3*(-1)^(1/3)*b^(2/3))*Log[a^(1/3) +
 (-1)^(2/3)*b^(1/3)*Sin[c + d*x]])/(a^(1/3)*b^(7/3)) + (2*(a^2 - b^2)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(a^(5/3)
*b^(7/3)) - (18*Sin[c + d*x])/b^2 - (27*Hypergeometric2F1[2/3, 2, 5/3, -((
b*Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(a*b) + 18/(b*(a + b*Sin[c + d*x]^3)
) + (6*(1 - a^2/b^2)*Sin[c + d*x])/(a*(a + b*Sin[c + d*x]^3)))/(18*d)
```

3.393.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3702, 2397, 25, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^7(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^7}{(a+b\sin(c+dx)^3)^2}dx$$

$\Big\downarrow$ 3702

$$\frac{\int \frac{(1-\sin^2(c+dx))^3}{(b\sin^3(c+dx)+a)^2}d\sin(c+dx)}{d}$$

$\Big\downarrow$ 2397

$$\frac{-\frac{\int -\frac{-3ab\sin^3(c+dx)+6ab\sin(c+dx)+a^2+2b^2}{b\sin^3(c+dx)+a}d\sin(c+dx)}{3ab^2}-\frac{\sin(c+dx)(a^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)-b^2)}{3ab^2(a+b\sin^3(c+dx))}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{-3ab\sin^3(c+dx)+6ab\sin(c+dx)+a^2+2b^2}{b\sin^3(c+dx)+a}d\sin(c+dx)}{3ab^2}-\frac{\sin(c+dx)(a^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)-b^2)}{3ab^2(a+b\sin^3(c+dx))}}{d}$$

$\Big\downarrow$ 2426

$$\frac{\frac{\int \left(\frac{2(2a^2+3b\sin(c+dx)a+b^2)}{b\sin^3(c+dx)+a}-3a\right)d\sin(c+dx)}{3ab^2}-\frac{\sin(c+dx)(a^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)-b^2)}{3ab^2(a+b\sin^3(c+dx))}}{d}$$

$\Big\downarrow$ 2009

$$\frac{\frac{-\frac{2(3a^{4/3}b^{2/3}+2a^2+b^2)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}}-\frac{(-3a^{4/3}b^{2/3}+2a^2+b^2)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{2(-3a^{4/3}b^{2/3}+2a^2+\cdots}{\cdots}}{3ab^2}\cdots}{d}$$

input
```
Int[Cos[c + d*x]^7/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
(((-2*(2*a^2 + 3*a^(4/3)*b^(2/3) + b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c
+ d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(1/3)) + (2*(2*a^2 - 3*a^(4
/3)*b^(2/3) + b^2)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(3*a^(2/3)*b^(1/3)
) - ((2*a^2 - 3*a^(4/3)*b^(2/3) + b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c
 + d*x] + b^(2/3)*Sin[c + d*x]^2])/(3*a^(2/3)*b^(1/3)) - 3*a*Sin[c + d*x])
/(3*a*b^2) - (Sin[c + d*x]*(a^2 - b^2 + 3*a*b*Sin[c + d*x] + 3*b^2*Sin[c +
 d*x]^2))/(3*a*b^2*(a + b*Sin[c + d*x]^3)))/d
```

3.393. $\int \frac{\cos^7(c+dx)}{(a+b\sin^3(c+dx))^2}dx$

3.393.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.393. $\int \frac{\cos^7(c+dx)}{(a+b\sin^3(c+dx))^2} \, dx$

3.393.4 Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-\frac{\frac{\sin(dx+c)}{b^2}-\frac{-\left(\sin^2(dx+c)\right)b-\frac{\left(a^2-b^2\right)\sin(dx+c)}{3a}+b}{a+b\left(\sin^3(dx+c)\right)}+\frac{2\left(2a^2+b^2\right)\left(\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c\right.}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{3}}{}$
default	$-\frac{\frac{\sin(dx+c)}{b^2}-\frac{-\left(\sin^2(dx+c)\right)b-\frac{\left(a^2-b^2\right)\sin(dx+c)}{3a}+b}{a+b\left(\sin^3(dx+c)\right)}+\frac{2\left(2a^2+b^2\right)\left(\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c\right.}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{3}}{}$
risch	$\frac{i e^{i(dx+c)}}{2b^2d}-\frac{i e^{-i(dx+c)}}{2b^2d}-\frac{2i\left(3ab\,e^{5i(dx+c)}+6ab\,e^{3i(dx+c)}+2ia^2e^{4i(dx+c)}-2ib^2e^{4i(dx+c)}+3ab\,e^{i(dx+c)}-2ia^2e^{2i(dx+c}\right.}{3a\,b^2d\left(b\,e^{6i(dx+c)}-3b\,e^{4i(dx+c)}+3b\,e^{2i(dx+c)}-8ia\,e^{3i(dx+c)}-b\right)}$

input
```
int(cos(d*x+c)^7/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

3.393. $\int \frac{\cos^7(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

output
```
-1/d*(sin(d*x+c)/b^2-1/b^2*((-b*sin(d*x+c)^2-1/3*(a^2-b^2)/a*sin(d*x+c)+b)
/(a+b*sin(d*x+c)^3)+2/3/a*((2*a^2+b^2)*(1/3/b/(1/b*a)^(2/3)*ln(sin(d*x+c)+
(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*ln(sin(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c
)+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)
^(1/3)*sin(d*x+c)-1)))+3*a*b*(-1/3/b/(1/b*a)^(1/3)*ln(sin(d*x+c)+(1/b*a)^(
1/3))+1/6/b/(1/b*a)^(1/3)*ln(sin(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)
^(2/3))+1/3*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*si
n(d*x+c)-1))))))
```

3.393.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 3624, normalized size of antiderivative = 12.58

$$\int \frac{\cos^7(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^7/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
-1/36*(36*a*b*cos(d*x + c)^4 - 108*a*b*cos(d*x + c)^2 + 2*(a^2*b^2*d - (a*
b^3*d*cos(d*x + c)^2 - a*b^3*d)*sin(d*x + c))*(4^(1/3)*(I*sqrt(3) + 1)*((8
*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 +
 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(
3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3)
 + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3)))*log(3/4*(
4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d
^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2
/3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*b^2 + 6*a
^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*
b^7*d^3))^(1/3)))^2*a^5*b^5*d^2 + 144*a^5*b + 72*a^3*b^3 - (4*a^6*b^2 + 4*
a^4*b^4 + a^2*b^6)*(4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b
^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*
d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^
6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*
a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3)))*d + 4*(8*a^6 + 39*a^4*b^2 + 6*a^2*b^
4 + b^6)*sin(d*x + c)) + 36*a*b - ((a^2*b^2*d - (a*b^3*d*cos(d*x + c)^2 -
a*b^3*d)*sin(d*x + c))*(4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a
^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*
b^7*d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2...
```

3.393.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^7(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**7/(a+b*sin(d*x+c)**3)**2,x)
```

output
```
Timed out
```

3.393.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.91

$$\int \frac{\cos^7(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx=$$

$$-\frac{\frac{3\left(3\,ab\sin(dx+c)^2-3\,ab+(a^2-b^2)\sin(dx+c)\right)}{ab^3\sin(dx+c)^3+a^2b^2}+\frac{9\sin(dx+c)}{b^2}-\frac{2\sqrt{3}\left(3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2+b^2\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(3\cdots}{\cdots}}{9\,d}$$

input
```
integrate(cos(d*x+c)^7/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/9*(3*(3*a*b*sin(d*x + c)^2 - 3*a*b + (a^2 - b^2)*sin(d*x + c))/(a*b^3*s
in(d*x + c)^3 + a^2*b^2) + 9*sin(d*x + c)/b^2 - 2*sqrt(3)*(3*a*b*(a/b)^(1/
3) + 2*a^2 + b^2)*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)
^(1/3))/(a*b^3*(a/b)^(2/3)) - (3*a*b*(a/b)^(1/3) - 2*a^2 - b^2)*log(sin(d*
x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 2
*(3*a*b*(a/b)^(1/3) - 2*a^2 - b^2)*log((a/b)^(1/3) + sin(d*x + c))/(a*b^3*
(a/b)^(2/3)))/d
```

3.393.8 Giac [F]

$$\int \frac{\cos^7(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx=\int \frac{\cos\left(dx+c\right)^7}{\left(b\sin\left(dx+c\right)^3+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^7/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.393.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.33

$$\int \frac{\cos^7(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$
$$=\frac{\sum_{k=1}^{3}\ln\left(\frac{8\,a^2+4\,b^2+\mathrm{root}\left(729\,a^5\,b^7\,d^3+648\,a^5\,b^3\,d+324\,a^3\,b^5\,d+120\,a^4\,b^2-48\,a^2\,b^4-8\,b^6-64\,a^6,d,k\right)^2\,a^2\,b^4\,27+12\,a\,b\,\sin(c+d\,x)+\mathrm{root}(729\,a}{}\right.}{}$$
$$-\frac{\sin\left(c+d\,x\right)}{b^2\,d}-\frac{b\sin\left(c+d\,x\right)^2-b+\frac{\sin(c+d\,x)\,\left(a^2-b^2\right)}{3\,a}}{d\,\left(b^3\sin\left(c+d\,x\right)^3+a\,b^2\right)}$$

input
```
int(cos(c + d*x)^7/(a + b*sin(c + d*x)^3)^2,x)
```

output
```
symsum(log((8*a^2 + 4*b^2 + 27*root(729*a^5*b^7*d^3 + 648*a^5*b^3*d + 324*
a^3*b^5*d + 120*a^4*b^2 - 48*a^2*b^4 - 8*b^6 - 64*a^6, d, k)^2*a^2*b^4 + 1
2*a*b*sin(c + d*x) + 6*root(729*a^5*b^7*d^3 + 648*a^5*b^3*d + 324*a^3*b^5*
d + 120*a^4*b^2 - 48*a^2*b^4 - 8*b^6 - 64*a^6, d, k)*b^4*sin(c + d*x) + 12
*root(729*a^5*b^7*d^3 + 648*a^5*b^3*d + 324*a^3*b^5*d + 120*a^4*b^2 - 48*a
^2*b^4 - 8*b^6 - 64*a^6, d, k)*a^2*b^2*sin(c + d*x))/(3*a*b^2))*root(729*a
^5*b^7*d^3 + 648*a^5*b^3*d + 324*a^3*b^5*d + 120*a^4*b^2 - 48*a^2*b^4 - 8*
b^6 - 64*a^6, d, k), k, 1, 3)/d - sin(c + d*x)/(b^2*d) - (b*sin(c + d*x)^2
 - b + (sin(c + d*x)*(a^2 - b^2))/(3*a))/(d*(a*b^2 + b^3*sin(c + d*x)^3))
```

3.394 $\int \frac{\cos^5(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

3.394.1 Optimal result

Integrand size = 23, antiderivative size = 238

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx &= -\frac{2\left(a^{4/3}+b^{4/3}\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{5/3}d}-\frac{2\left(a^{4/3}-b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{5/3}d}\\&\quad+\frac{\left(a^{4/3}-b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}b^{5/3}d}\\&\quad+\frac{\sin(c+dx)\left(b-a\sin(c+dx)-2b\sin^2(c+dx)\right)}{3abd\left(a+b\sin^3(c+dx)\right)}\end{aligned}$$

output
```
-2/9*(a^(4/3)-b^(4/3))*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(5/3)/b^(5/3)/d+1/
9*(a^(4/3)-b^(4/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+
c)^2)/a^(5/3)/b^(5/3)/d+1/3*sin(d*x+c)*(b-a*sin(d*x+c)-2*b*sin(d*x+c)^2)/a
/b/d/(a+b*sin(d*x+c)^3)-2/9*(a^(4/3)+b^(4/3))*arctan(1/3*(a^(1/3)-2*b^(1/3
)*sin(d*x+c))/a^(1/3)*3^(1/2))/a^(5/3)/b^(5/3)/d*3^(1/2)
```

3.394.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.96 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.08

$$\int \frac{\cos^5(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \frac{-\frac{4\sqrt{3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}\sqrt[3]{b}}+\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{a^{5/3}\sqrt[3]{b}}-\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{a^{5/3}\sqrt[3]{b}}+\frac{9\,\text{Hyperg}}{}}{}$$

input
```
Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
((-4*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])
/(a^(5/3)*b^(1/3)) + (4*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(a^(5/3)*b^(1
/3)) - (2*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x
]^2])/(a^(5/3)*b^(1/3)) + (9*Hypergeometric2F1[2/3, 1, 5/3, -((b*Sin[c + d
*x]^3)/a)]*Sin[c + d*x]^2)/(a*b) - (9*Hypergeometric2F1[2/3, 2, 5/3, -((b*
Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(a*b) + 12/(b*(a + b*Sin[c + d*x]^3))
+ (6*Sin[c + d*x])/(a*(a + b*Sin[c + d*x]^3)))/(18*d)
```

3.394.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3042, 3702, 2397, 27, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^5}{\left(a+b\sin(c+dx)^3\right)^2}dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int \frac{\left(1-\sin^2(c+dx)\right)^2}{\left(b\sin^3(c+dx)+a\right)^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2397}$$

$$\frac{\frac{\sin(c+dx)\left(-a\sin(c+dx)-2b\sin^2(c+dx)+b\right)}{3ab\left(a+b\sin^3(c+dx)\right)}-\frac{\int -\frac{2b(b+a\sin(c+dx))}{b\sin^3(c+dx)+a}d\sin(c+dx)}{3ab^2}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\int \frac{b+a\sin(c+dx)}{b\sin^3(c+dx)+a}d\sin(c+dx)}{3ab}+\frac{\sin(c+dx)\left(-a\sin(c+dx)-2b\sin^2(c+dx)+b\right)}{3ab\left(a+b\sin^3(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{2399}$$

$$\frac{\frac{2\left(\frac{\int \frac{\sqrt[3]{a}\left(a^{4/3}+2b^{4/3}\right)+\sqrt[3]{b}\left(a^{4/3}-b^{4/3}\right)\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\frac{1}{3}\left(\frac{a^{2/3}}{\sqrt[3]{b}}-\frac{b}{a^{2/3}}\right)\int \frac{1}{\sqrt[3]{b}\sin(c+dx)+\sqrt[3]{a}}d\sin(c+dx)\right)}{3ab}+\frac{\sin(c+dx)\left(-a\sin(c+dx)\right.}{3ab\left(a+b\sin^3\right.}}{d}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{2\left(\frac{\int \frac{\sqrt[3]{a}\left(a^{4/3}+2b^{4/3}\right)+\sqrt[3]{b}\left(a^{4/3}-b^{4/3}\right)\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\frac{a^{2/3}}{\sqrt[3]{b}}-\frac{b}{a^{2/3}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3\sqrt[3]{b}}\right)}{3ab}+\frac{\sin(c+dx)\left(-a\sin(c+dx)-2b\sin^2(c\right.}{3ab\left(a+b\sin^3(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(a^{4/3}+b^{4/3}\right)\int \frac{1}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)+\frac{1}{2}\left(\frac{a^{4/3}}{\sqrt[3]{b}}-b\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)\right)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\right.}{3ab}}{d}$$

$$\Big\downarrow \text{25}$$

3.394. $\displaystyle\int \frac{\cos^5(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(a^{4/3}+b^{4/3}\right)\int \frac{1}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)-\frac{1}{2}\left(\frac{a^{4/3}}{\sqrt[3]{b}}-b\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)\right)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\cdots}{3ab}\cdots}{d}$$

$\big\downarrow$ 27

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(a^{4/3}+b^{4/3}\right)\int \frac{1}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)-\frac{1}{2}\sqrt[3]{b}\left(\frac{a^{4/3}}{\sqrt[3]{b}}-b\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)\cdots}{3a^{2/3}\sqrt[3]{b}}\cdots}{3ab}\cdots}{d}$$

$\big\downarrow$ 1082

$$\frac{2\left(\frac{\frac{3\left(a^{4/3}+b^{4/3}\right)\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\sqrt[3]{b}\left(\frac{a^{4/3}}{\sqrt[3]{b}}-b\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\cdots}{3ab}\cdots}{d}$$

$\big\downarrow$ 217

$$\frac{2\left(\frac{-\frac{1}{2}\sqrt[3]{b}\left(\frac{a^{4/3}}{\sqrt[3]{b}}-b\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)-\frac{\sqrt{3}\left(a^{4/3}+b^{4/3}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\frac{a^{2/3}}{\sqrt[3]{b}}-\frac{b}{a^{2/3}}\right)\log\left(\sqrt[3]{a}\cdots\right.}{3\sqrt[3]{b}\cdots}\cdots}{3ab}\cdots}{d}$$

$\big\downarrow$ 1103

3.394. $\displaystyle\int \frac{\cos^5(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

$$\frac{\frac{2\left(\frac{\frac{1}{2}\left(\frac{a^{4/3}}{\sqrt[3]{b}}-b\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)-\frac{\sqrt{3}\left(a^{4/3}+b^{4/3}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\frac{a^{2/3}}{\sqrt[3]{b}}-\frac{b}{a^{2/3}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+\cdots\right.}{3\sqrt[3]{b}}\cdots\right)}{3ab}\cdots}{d}$$

input
```
Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
((2*(-1/3*((a^(2/3)/b^(1/3) - b/a^(2/3))*Log[a^(1/3) + b^(1/3)*Sin[c + d*x
]])/b^(1/3) + (-((Sqrt[3]*(a^(4/3) + b^(4/3))*ArcTan[(1 - (2*b^(1/3)*Sin[c
 + d*x])/a^(1/3))/Sqrt[3]])/b^(1/3)) + ((a^(4/3)/b^(1/3) - b)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/2)/(3*a^(2/3)*b^
(1/3))))/(3*a*b) + (Sin[c + d*x]*(b - a*Sin[c + d*x] - 2*b*Sin[c + d*x]^2)
)/(3*a*b*(a + b*Sin[c + d*x]^3)))/d
```

3.394.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.394. $\int \frac{\cos^5(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

3.394.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{\frac{-\frac{\sin^2(dx+c)}{3b}+\frac{\sin(dx+c)}{3a}+\frac{2}{3b}}{a+b\left(\sin^3(dx+c)\right)}+\frac{2b\left(\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin}{(}\right.}{}\right.}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{3}}{}$
default	$\frac{\frac{-\frac{\sin^2(dx+c)}{3b}+\frac{\sin(dx+c)}{3a}+\frac{2}{3b}}{a+b\left(\sin^3(dx+c)\right)}+\frac{2b\left(\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin}{(}\right.}{}\right.}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{3}}{}$
risch	$-\frac{2i\left(a\,e^{5i(dx+c)}+6a\,e^{3i(dx+c)}-2ib\,e^{4i(dx+c)}+a\,e^{i(dx+c)}+2ib\,e^{2i(dx+c)}\right)}{3abd\left(b\,e^{6i(dx+c)}-3b\,e^{4i(dx+c)}+3b\,e^{2i(dx+c)}-8ia\,e^{3i(dx+c)}-b\right)}+\left(\sum_{\_R=\text{RootOf}\left(729a^5b^5d^3\_Z^3+108a^3b^3a\right.}\right.$

input
```
int(cos(d*x+c)^5/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*((-1/3*sin(d*x+c)^2/b+1/3/a*sin(d*x+c)+2/3/b)/(a+b*sin(d*x+c)^3)+2/3/a
/b*(b*(1/3/b/(1/b*a)^(2/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3
)*ln(sin(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))+a*(-1/3/b/
(1/b*a)^(1/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))+1/6/b/(1/b*a)^(1/3)*ln(sin(d*x+
c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3*3^(1/2)/b/(1/b*a)^(1/3)*a
rctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))))
```

3.394.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 2255, normalized size of antiderivative = 9.47

$$\int \frac{\cos^5(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
1/36*(12*a*cos(d*x + c)^2 - 2*(a^2*b*d - (a*b^2*d*cos(d*x + c)^2 - a*b^2*d
)*sin(d*x + c))*(4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4
 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4
 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))*log(1/4*(4^(1/3
)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^
(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) -
 (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))^2*a^5*b^3*d^2 - (4^(1/3)*(I*sqrt(3) +
1)*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)
*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a
^5*b^5*d^3))^(1/3)))*a^2*b^4*d + 8*a^3*b + 4*(a^4 + b^4)*sin(d*x + c)) + (
(a^2*b*d - (a*b^2*d*cos(d*x + c)^2 - a*b^2*d)*sin(d*x + c))*(4^(1/3)*(I*sq
rt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) -
 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 -
 b^4)/(a^5*b^5*d^3))^(1/3))) + 3*sqrt(1/3)*(a^2*b*d - (a*b^2*d*cos(d*x + c
)^2 - a*b^2*d)*sin(d*x + c))*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/
(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1
)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/
3)))^2*a^2*b^2*d^2 + 64)/(a^2*b^2*d^2)))*log(1/4*(4^(1/3)*(I*sqrt(3) + 1)*
((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-
I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a...
```

3.394.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5/(a+b*sin(d*x+c)**3)**2,x)
```

output
```
Timed out
```

3.394.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.89

$$\int \frac{\cos^5(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx =$$

$$-\frac{\frac{3\left(a\sin(dx+c)^2-b\sin(dx+c)-2a\right)}{ab^2\sin(dx+c)^3+a^2b}-\frac{2\sqrt{3}\left(a\left(\frac{a}{b}\right)^{\frac{1}{3}}+b\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(a\left(\frac{a}{b}\right)^{\frac{1}{3}}-b\right)\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin\cdots\right.}{ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}\cdots}{9d}$$

input
```
integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/9*(3*(a*sin(d*x + c)^2 - b*sin(d*x + c) - 2*a)/(a*b^2*sin(d*x + c)^3 +
a^2*b) - 2*sqrt(3)*(a*(a/b)^(1/3) + b)*arctan(-1/3*sqrt(3)*((a/b)^(1/3) -
2*sin(d*x + c))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - (a*(a/b)^(1/3) - b)*log
(sin(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/
3)) + 2*(a*(a/b)^(1/3) - b)*log((a/b)^(1/3) + sin(d*x + c))/(a*b^2*(a/b)^(
2/3)))/d
```

3.394.8 Giac [F]

$$\int \frac{\cos^5(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \int \frac{\cos\left(dx+c\right)^5}{\left(b\sin\left(dx+c\right)^3+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.394.9 Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.85

$$\int \frac{\cos^5(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$$
$$= \frac{\sum_{k=1}^{3} \ln\left(\frac{4\,b+4\,a\,\sin(c+d\,x)+\mathrm{root}\left(729\,a^5\,b^5\,d^3+108\,a^3\,b^3\,d-8\,b^4+8\,a^4,d,k\right)^2\,a^2\,b^3\,81+\mathrm{root}\left(729\,a^5\,b^5\,d^3+108\,a^3\,b^3\,d-8\,b^4+8\,a^4,d,k\right)\,b^3\,\sin(c}{a\,b\,9}\right.}{d}$$
$$+ \frac{\frac{\sin(c+d\,x)}{3\,a}+\frac{2}{3\,b}-\frac{\sin(c+d\,x)^2}{3\,b}}{d\,\left(b\sin\left(c+d\,x\right)^3+a\right)}$$

input

```
int(cos(c + d*x)^5/(a + b*sin(c + d*x)^3)^2,x)
```

output

```
symsum(log((4*b + 4*a*sin(c + d*x) + 81*root(729*a^5*b^5*d^3 + 108*a^3*b^3
*d - 8*b^4 + 8*a^4, d, k)^2*a^2*b^3 + 18*root(729*a^5*b^5*d^3 + 108*a^3*b^
3*d - 8*b^4 + 8*a^4, d, k)*b^3*sin(c + d*x))/(9*a*b))*root(729*a^5*b^5*d^3
 + 108*a^3*b^3*d - 8*b^4 + 8*a^4, d, k), k, 1, 3)/d + (sin(c + d*x)/(3*a)
+ 2/(3*b) - sin(c + d*x)^2/(3*b))/(d*(a + b*sin(c + d*x)^3))
```

3.395 $\int \frac{\cos^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

3.395.1 Optimal result

Integrand size = 23, antiderivative size = 183

$$\int \frac{\cos^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = -\frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}\sqrt[3]{b}d}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d} + \frac{a+b\sin(c+dx)}{3abd\,(a+b\sin^3(c+dx))}$$

output
```
2/9*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(5/3)/b^(1/3)/d-1/9*ln(a^(2/3)-a^(1/3
)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(5/3)/b^(1/3)/d+1/3*(a+b*sin(
d*x+c))/a/b/d/(a+b*sin(d*x+c)^3)-2/9*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x
+c))/a^(1/3)*3^(1/2))/a^(5/3)/b^(1/3)/d*3^(1/2)
```

3.395.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.01

$$\int \frac{\cos^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \frac{-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}\sqrt[3]{b}}+\frac{3\sin(c+dx)}{a(a+b\sin^3(c+dx))}+\frac{\frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{a^{5/3}}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{a^{5/3}}+\cdots}{b}}{9d}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
((-2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])
/(a^(5/3)*b^(1/3)) + (3*Sin[c + d*x])/(a*(a + b*Sin[c + d*x]^3)) + ((2*b^(
2/3)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/a^(5/3) - (b^(2/3)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/a^(5/3) + 3/(a +
b*Sin[c + d*x]^3))/b)/(9*d)
```

3.395.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 3702, 2393, 27, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^3}{(a+b\sin(c+dx)^3)^2}dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int \frac{1-\sin^2(c+dx)}{(b\sin^3(c+dx)+a)^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2393}$$

$$\frac{\frac{a+b\sin(c+dx)}{3ab(a+b\sin^3(c+dx))}-\frac{\int -\frac{2}{b\sin^3(c+dx)+a}d\sin(c+dx)}{3a}}{d}$$

$\downarrow$ 27

$$\frac{\frac{2\int \frac{1}{b\sin^3(c+dx)+a}d\sin(c+dx)}{3a}+\frac{a+b\sin(c+dx)}{3ab(a+b\sin^3(c+dx))}}{d}$$

$\downarrow$ 750

$$\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}\sin(c+dx)+\sqrt[3]{a}}d\sin(c+dx)}{3a^{2/3}}\right)}{3a}+\frac{a+b\sin(c+dx)}{3ab(a+b\sin^3(c+dx))}}{d}$$

$\downarrow$ 16

$$\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{a+b\sin(c+dx)}{3ab(a+b\sin^3(c+dx))}}{d}$$

$\downarrow$ 1142

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)\right)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right.}{3a}$$
$$\overline{\qquad d \qquad}$$

$\downarrow$ 25

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)\right)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right.}{3a}$$
$$\overline{\qquad d \qquad}$$

3.395. $\int \frac{\cos^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

$\Big\downarrow$ 27

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right.}{3a}}{d}$$

$\Big\downarrow$ 1082

$$\frac{\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+}{d}$$

$\Big\downarrow$ 217

$$\frac{\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{a+b\sin(c+dx)}{3ab\left(a+b\sin^3(c+dx)\right)}}{d}$$

$\Big\downarrow$ 1103

3.395. $\displaystyle\int \frac{\cos^3(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

$$\frac{\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{a+b\sin(c+dx)}{3ab(a+b\sin^3(c+dx))}}{d}$$

input
```
Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
((2*(Log[a^(1/3) + b^(1/3)*Sin[c + d*x]]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]
*ArcTan[(1 - (2*b^(1/3)*Sin[c + d*x])/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^
(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]/(2*b^(1/3))
)/(3*a^(2/3))))/(3*a) + (a + b*Sin[c + d*x])/(3*a*b*(a + b*Sin[c + d*x]^3)
))/d
```

3.395.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.395. $\int \frac{\cos^3(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.395. $\int \frac{\cos^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

3.395.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.02 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{4\left(b\,e^{4i(dx+c)}-b\,e^{2i(dx+c)}+2ia\,e^{3i(dx+c)}\right)}{3abd\left(b\,e^{6i(dx+c)}-3b\,e^{4i(dx+c)}+3b\,e^{2i(dx+c)}-8ia\,e^{3i(dx+c)}-b\right)}+\left(\sum_{\_R=\operatorname{RootOf}\left(729a^5 b\, d^3\_Z^3-8\right)}\_R\ln\left(e^{2i(\cdots\right.\right.$
derivativedivides	$\frac{\frac{\sin(dx+c)}{3a\left(a+b\left(\sin^3(dx+c)\right)\right)}+\frac{\frac{2\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\cdots\right.}{3}\cdots\right.}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\cdots}{a}}{d}$
default	$\frac{\frac{\sin(dx+c)}{3a\left(a+b\left(\sin^3(dx+c)\right)\right)}+\frac{\frac{2\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\cdots\right.}{3}\cdots\right.}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\cdots}{a}}{d}$

input
```
int(cos(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
-4/3/a/b/d/(exp(6*I*(d*x+c))*b-3*b*exp(4*I*(d*x+c))+3*b*exp(2*I*(d*x+c))-8
*I*a*exp(3*I*(d*x+c))-b)*(b*exp(4*I*(d*x+c))-b*exp(2*I*(d*x+c))+2*I*a*exp(
3*I*(d*x+c)))+sum(_R*ln(exp(2*I*(d*x+c))+9*I*a^2*d*_R*exp(I*(d*x+c))-1),_R
=RootOf(729*_Z^3*a^5*b*d^3-8))
```

3.395.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 665, normalized size of antiderivative = 3.63

$$\int \frac{\cos^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$$

$$= \left[\frac{3\,a^2b\sin(dx+c)+3\,a^3+3\,\sqrt{\frac{1}{3}}\left(a^2b-\left(ab^2\cos(dx+c)^2-ab^2\right)\sin(dx+c)\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(-\frac{3\,(a^2b)^{\frac{1}{3}}a\sin}{}\right.}{} \right.$$

input
```
integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
[1/9*(3*a^2*b*sin(d*x + c) + 3*a^3 + 3*sqrt(1/3)*(a^2*b - (a*b^2*cos(d*x +
 c)^2 - a*b^2)*sin(d*x + c))*sqrt(-(a^2*b)^(1/3)/b)*log(-(3*(a^2*b)^(1/3)*
a*sin(d*x + c) + a^2 + 3*sqrt(1/3)*(2*a*b*cos(d*x + c)^2 - 2*a*b - (a^2*b)
^(2/3)*sin(d*x + c) + (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) + 2*(a*b*cos
(d*x + c)^2 - a*b)*sin(d*x + c))/((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)
) + (a^2*b)^(2/3)*((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(-a*b*cos(d
*x + c)^2 + a*b - (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)^(1/3)*a) - 2*(a^2*b
)^(2/3)*((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(a*b*sin(d*x + c) + (
a^2*b)^(2/3)))/(a^4*b*d - (a^3*b^2*d*cos(d*x + c)^2 - a^3*b^2*d)*sin(d*x +
 c)), 1/9*(3*a^2*b*sin(d*x + c) + 3*a^3 + 6*sqrt(1/3)*(a^2*b - (a*b^2*cos(
d*x + c)^2 - a*b^2)*sin(d*x + c))*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(
2*(a^2*b)^(2/3)*sin(d*x + c) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2)
 + (a^2*b)^(2/3)*((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(-a*b*cos(d*
x + c)^2 + a*b - (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)^(1/3)*a) - 2*(a^2*b)
^(2/3)*((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(a*b*sin(d*x + c) + (a
^2*b)^(2/3)))/(a^4*b*d - (a^3*b^2*d*cos(d*x + c)^2 - a^3*b^2*d)*sin(d*x +
c))]
```

3.395.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+b*sin(d*x+c)**3)**2,x)
```

output
```
Timed out
```

3.395.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.89

$$\int \frac{\cos^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \frac{\frac{3\,(b\sin(dx+c)+a)}{ab^2\sin(dx+c)^3+a^2b} + \frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right)}{ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{9\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/9*(3*(b*sin(d*x + c) + a)/(a*b^2*sin(d*x + c)^3 + a^2*b) + 2*sqrt(3)*arc
tan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1/3))/(a*b*(a/b)^(2
/3)) - log(sin(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(a*b*(
a/b)^(2/3)) + 2*log((a/b)^(1/3) + sin(d*x + c))/(a*b*(a/b)^(2/3)))/d
```

3.395.8 Giac [F]

$$\int \frac{\cos^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \int \frac{\cos(dx+c)^3}{\left(b\sin(dx+c)^3+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.395.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.94

$$\int \frac{\cos^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \frac{\frac{\sin(c+d\,x)}{3\,a}+\frac{1}{3\,b}}{d\,\left(b\sin\left(c+d\,x\right)^3+a\right)} + \frac{2\,\ln\left(\frac{2\,b^{5/3}}{a^{2/3}}+\frac{2\,b^2\,\sin(c+d\,x)}{a}\right)}{9\,a^{5/3}\,b^{1/3}\,d} + \frac{\ln\left(\frac{2\,b^2\,\sin(c+d\,x)}{a}+\frac{b^{5/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{5/3}\,b^{1/3}\,d} - \frac{\ln\left(\frac{2\,b^2\,\sin(c+d\,x)}{a}-\frac{b^{5/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{5/3}\,b^{1/3}\,d}$$

input

```
int(cos(c + d*x)^3/(a + b*sin(c + d*x)^3)^2,x)
```

output

```
(sin(c + d*x)/(3*a) + 1/(3*b))/(d*(a + b*sin(c + d*x)^3)) + (2*log((2*b^(5
/3))/a^(2/3) + (2*b^2*sin(c + d*x))/a))/(9*a^(5/3)*b^(1/3)*d) + (log((2*b^
2*sin(c + d*x))/a + (b^(5/3)*(3^(1/2)*1i - 1))/a^(2/3))*(3^(1/2)*1i - 1))/
(9*a^(5/3)*b^(1/3)*d) - (log((2*b^2*sin(c + d*x))/a - (b^(5/3)*(3^(1/2)*1i
 + 1))/a^(2/3))*(3^(1/2)*1i + 1))/(9*a^(5/3)*b^(1/3)*d)
```

3.396 $\int \frac{\cos(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

3.396.1 Optimal result

Integrand size = 21, antiderivative size = 176

$$\int \frac{\cos(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = -\frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}\sqrt[3]{b}d} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d} + \frac{\sin(c+dx)}{3ad\,(a+b\sin^3(c+dx))}$$

output
```
2/9*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(5/3)/b^(1/3)/d-1/9*ln(a^(2/3)-a^(1/3
)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(5/3)/b^(1/3)/d+1/3*sin(d*x+c
)/a/d/(a+b*sin(d*x+c)^3)-2/9*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(
1/3)*3^(1/2))/a^(5/3)/b^(1/3)/d*3^(1/2)
```

3.396.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.86

$$\int \frac{\cos(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \frac{-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{\sqrt[3]{b}}+\frac{3a^{2/3}\sin(c+dx)}{a+b\sin^3(c+dx)}}{9a^{5/3}d}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
((-2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])
/b^(1/3) + (2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] - Log[a^(2/3) - a^(1/3)*
b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/b^(1/3) + (3*a^(2/3)*Sin[c
 + d*x])/(a + b*Sin[c + d*x]^3))/(9*a^(5/3)*d)
```

3.396.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 3702, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(c+dx)}{(a+b\sin(c+dx)^3)^2}dx$$

$$\Big\downarrow \; 3702$$

$$\frac{\int \frac{1}{(b\sin^3(c+dx)+a)^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \; 749$$

$$\frac{\frac{2\int \frac{1}{b\sin^3(c+dx)+a}d\sin(c+dx)}{3a}+\frac{\sin(c+dx)}{3a\left(a+b\sin^3(c+dx)\right)}}{d}$$

$\Big\downarrow$ 750

$$\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}\sin(c+dx)+\sqrt[3]{a}}d\sin(c+dx)}{3a^{2/3}}\right)}{3a}+\frac{\sin(c+dx)}{3a\left(a+b\sin^3(c+dx)\right)}}{d}$$

$\Big\downarrow$ 16

$$\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{\sin(c+dx)}{3a\left(a+b\sin^3(c+dx)\right)}}{d}$$

$\Big\downarrow$ 1142

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)\right)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)\right)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}}{d}$$

$\Big\downarrow$ 27

3.396. $\int \frac{\cos(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right.}{3a}}{d}$$

$\downarrow$ 1082

$$\frac{\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+}{d}$$

$\downarrow$ 217

$$\frac{\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{b^{2/3}\sin^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+a^{2/3}}d\sin(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{\sin(c+dx)}{3a\left(a+b\sin^3(c+dx)\right)}}{d}$$

$\downarrow$ 1103

$$\frac{\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{\sin(c+dx)}{3a\left(a+b\sin^3(c+dx)\right)}}{d}$$

3.396. $\displaystyle\int\frac{\cos(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

input
```
Int[Cos[c + d*x]/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
((2*(Log[a^(1/3) + b^(1/3)*Sin[c + d*x]]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]
*ArcTan[(1 - (2*b^(1/3)*Sin[c + d*x])/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^
(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]/(2*b^(1/3))
)/(3*a^(2/3))))/(3*a) + Sin[c + d*x]/(3*a*(a + b*Sin[c + d*x]^3)))/d
```

3.396.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.396.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.90 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{4\left(e^{4i(dx+c)}-e^{2i(dx+c)}\right)}{3ad\left(b\,e^{6i(dx+c)}-3b\,e^{4i(dx+c)}+3b\,e^{2i(dx+c)}-8ia\,e^{3i(dx+c)}-b\right)}+\left(\sum_{\_R=\text{RootOf}\left(729a^5b\,d^3\_Z^3-8\right)}\_R\ln\left(e^{2i(d\right.\right.$
derivativedivides	$\frac{\frac{\sin(dx+c)}{3a\left(a+b\left(\sin^3(dx+c)\right)\right)}+\frac{\frac{2\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right.}{3}\right.}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}}{d}$
default	$\frac{\frac{\sin(dx+c)}{3a\left(a+b\left(\sin^3(dx+c)\right)\right)}+\frac{\frac{2\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right.}{3}\right.}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}}{d}$

input
```
int(cos(d*x+c)/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
-4/3/a/d/(exp(6*I*(d*x+c))*b-3*b*exp(4*I*(d*x+c))+3*b*exp(2*I*(d*x+c))-8*I
*a*exp(3*I*(d*x+c))-b)*(exp(4*I*(d*x+c))-exp(2*I*(d*x+c)))+sum(_R*ln(exp(2
*I*(d*x+c))+9*I*a^2*d*_R*exp(I*(d*x+c))-1),_R=RootOf(729*_Z^3*a^5*b*d^3-8)
)
```

3.396.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(137) = 274$.

Time = 0.36 (sec) , antiderivative size = 655, normalized size of antiderivative = 3.72

$$\int \frac{\cos(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

$$= \left[\frac{3\,a^2b\sin\left(dx+c\right)+3\,\sqrt{\frac{1}{3}}\left(a^2b-\left(ab^2\cos\left(dx+c\right)^2-ab^2\right)\sin\left(dx+c\right)\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(-\frac{3\left(a^2b\right)^{\frac{1}{3}}a\sin(dx+c)}{}\right.}{}\right.$$

input
```
integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
[1/9*(3*a^2*b*sin(d*x + c) + 3*sqrt(1/3)*(a^2*b - (a*b^2*cos(d*x + c)^2 -
a*b^2)*sin(d*x + c))*sqrt(-(a^2*b)^(1/3)/b)*log(-(3*(a^2*b)^(1/3)*a*sin(d*
x + c) + a^2 + 3*sqrt(1/3)*(2*a*b*cos(d*x + c)^2 - 2*a*b - (a^2*b)^(2/3)*s
in(d*x + c) + (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) + 2*(a*b*cos(d*x + c
)^2 - a*b)*sin(d*x + c))/((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)) + (a^2
*b)^(2/3)*((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(-a*b*cos(d*x + c)^
2 + a*b - (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)^(1/3)*a) - 2*(a^2*b)^(2/3)*
((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(a*b*sin(d*x + c) + (a^2*b)^(
2/3)))/(a^4*b*d - (a^3*b^2*d*cos(d*x + c)^2 - a^3*b^2*d)*sin(d*x + c)), 1/
9*(3*a^2*b*sin(d*x + c) + 6*sqrt(1/3)*(a^2*b - (a*b^2*cos(d*x + c)^2 - a*b
^2)*sin(d*x + c))*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*
sin(d*x + c) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + (a^2*b)^(2/3)
*((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(-a*b*cos(d*x + c)^2 + a*b -
 (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)^(1/3)*a) - 2*(a^2*b)^(2/3)*((b*cos(d
*x + c)^2 - b)*sin(d*x + c) - a)*log(a*b*sin(d*x + c) + (a^2*b)^(2/3)))/(a
^4*b*d - (a^3*b^2*d*cos(d*x + c)^2 - a^3*b^2*d)*sin(d*x + c))]
```

3.396.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(162) = 324$.

Time = 92.90 (sec) , antiderivative size = 617, normalized size of antiderivative = 3.51

$$\int \frac{\cos(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$$

$$= \begin{cases} \frac{\tilde{\infty} x\cos(c)}{\sin^6(c)} \\ \frac{\sin(c+dx)}{a^2 d} \\ -\frac{1}{5b^2 d\sin^5(c+dx)} \\ \frac{\tilde{\infty}\sin(c+dx)}{d} \\ \frac{x\cos(c)}{(a+b\sin^3(c))^2} \\ -\frac{2a\sqrt[3]{-\frac{a}{b}}\log\left(-\sqrt[3]{-\frac{a}{b}}+\sin(c+dx)\right)}{9a^3d+9a^2bd\sin^3(c+dx)} + \frac{a\sqrt[3]{-\frac{a}{b}}\log\left(4\left(-\frac{a}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-\frac{a}{b}}\sin(c+dx)+4\sin^2(c+dx)\right)}{9a^3d+9a^2bd\sin^3(c+dx)} + \frac{2\sqrt{3}a\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\right.}{9a^3d+9a^2bd\sin^3(} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+b*sin(d*x+c)**3)**2,x)
```

output
```
Piecewise((zoo*x*cos(c)/sin(c)**6, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (sin(c
 + d*x)/(a**2*d), Eq(b, 0)), (-1/(5*b**2*d*sin(c + d*x)**5), Eq(a, 0)), (z
oo*sin(c + d*x)/d, Eq(b, -a/sin(c + d*x)**3)), (x*cos(c)/(a + b*sin(c)**3)
**2, Eq(d, 0)), (-2*a*(-a/b)**(1/3)*log(-(-a/b)**(1/3) + sin(c + d*x))/(9*
a**3*d + 9*a**2*b*d*sin(c + d*x)**3) + a*(-a/b)**(1/3)*log(4*(-a/b)**(2/3)
 + 4*(-a/b)**(1/3)*sin(c + d*x) + 4*sin(c + d*x)**2)/(9*a**3*d + 9*a**2*b*
d*sin(c + d*x)**3) + 2*sqrt(3)*a*(-a/b)**(1/3)*atan(sqrt(3)/3 + 2*sqrt(3)*
sin(c + d*x)/(3*(-a/b)**(1/3)))/(9*a**3*d + 9*a**2*b*d*sin(c + d*x)**3) -
2*a*(-a/b)**(1/3)*log(2)/(9*a**3*d + 9*a**2*b*d*sin(c + d*x)**3) + 3*a*sin
(c + d*x)/(9*a**3*d + 9*a**2*b*d*sin(c + d*x)**3) - 2*b*(-a/b)**(1/3)*log(
-(-a/b)**(1/3) + sin(c + d*x))*sin(c + d*x)**3/(9*a**3*d + 9*a**2*b*d*sin(
c + d*x)**3) + b*(-a/b)**(1/3)*log(4*(-a/b)**(2/3) + 4*(-a/b)**(1/3)*sin(c
 + d*x) + 4*sin(c + d*x)**2)*sin(c + d*x)**3/(9*a**3*d + 9*a**2*b*d*sin(c
+ d*x)**3) + 2*sqrt(3)*b*(-a/b)**(1/3)*sin(c + d*x)**3*atan(sqrt(3)/3 + 2*
sqrt(3)*sin(c + d*x)/(3*(-a/b)**(1/3)))/(9*a**3*d + 9*a**2*b*d*sin(c + d*x
)**3) - 2*b*(-a/b)**(1/3)*log(2)*sin(c + d*x)**3/(9*a**3*d + 9*a**2*b*d*si
n(c + d*x)**3), True))
```

3.396.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.88

$$\int \frac{\cos(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$$
$$= \frac{\frac{3\sin(dx+c)}{ab\sin(dx+c)^3+a^2} + \frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right)}{ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{9d}$$

input
```
integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/9*(3*sin(d*x + c)/(a*b*sin(d*x + c)^3 + a^2) + 2*sqrt(3)*arctan(-1/3*sqr
t(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) - log(s
in(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(a*b*(a/b)^(2/3))
+ 2*log((a/b)^(1/3) + sin(d*x + c))/(a*b*(a/b)^(2/3)))/d
```

3.396.8 Giac [F]

$$\int \frac{\cos(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx=\int \frac{\cos(dx+c)}{\left(b\sin(dx+c)^3+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.396.9 Mupad [B] (verification not implemented)

Time = 14.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.94

$$\int \frac{\cos(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx=\frac{\sin(c+d\,x)}{3\,a\,d\,\left(b\sin(c+d\,x)^3+a\right)}+\frac{2\,\ln\left(\frac{2\,b^{5/3}}{a^{2/3}}+\frac{2\,b^2\,\sin(c+d\,x)}{a}\right)}{9\,a^{5/3}\,b^{1/3}\,d}+\frac{\ln\left(\frac{2\,b^2\,\sin(c+d\,x)}{a}+\frac{b^{5/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{5/3}\,b^{1/3}\,d}-\frac{\ln\left(\frac{2\,b^2\,\sin(c+d\,x)}{a}-\frac{b^{5/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{5/3}\,b^{1/3}\,d}$$

input
```
int(cos(c + d*x)/(a + b*sin(c + d*x)^3)^2,x)
```

output
```
sin(c + d*x)/(3*a*d*(a + b*sin(c + d*x)^3)) + (2*log((2*b^(5/3))/a^(2/3) +
 (2*b^2*sin(c + d*x))/a))/(9*a^(5/3)*b^(1/3)*d) + (log((2*b^2*sin(c + d*x)
)/a + (b^(5/3)*(3^(1/2)*1i - 1))/a^(2/3))*(3^(1/2)*1i - 1))/(9*a^(5/3)*b^(
1/3)*d) - (log((2*b^2*sin(c + d*x))/a - (b^(5/3)*(3^(1/2)*1i + 1))/a^(2/3)
)*(3^(1/2)*1i + 1))/(9*a^(5/3)*b^(1/3)*d)
```

3.397 $\int \frac{\sec(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

3.397.1 Optimal result

Integrand size = 21, antiderivative size = 587

$$\begin{aligned}
\int \frac{\sec(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx
&= -\frac{\sqrt[3]{b}\left(a^{4/3}-2b^{4/3}\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}\left(a^2-b^2\right)d} \\
&\quad -\frac{\sqrt[3]{b}\left(a^2-2a^{2/3}b^{4/3}+b^2\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^2-b^2\right)^2 d}-\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} \\
&\quad +\frac{\log(1+\sin(c+dx))}{2(a-b)^2d}-\frac{\sqrt[3]{b}\left(a^{4/3}+2b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\left(a^2-b^2\right)d} \\
&\quad -\frac{\sqrt[3]{b}\left(a^2+2a^{2/3}b^{4/3}+b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3\sqrt[3]{a}\left(a^2-b^2\right)^2 d} \\
&\quad +\frac{\sqrt[3]{b}\left(a^{4/3}+2b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{18a^{5/3}\left(a^2-b^2\right)d} \\
&\quad +\frac{\sqrt[3]{b}\left(a^2+2a^{2/3}b^{4/3}+b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6\sqrt[3]{a}\left(a^2-b^2\right)^2 d} \\
&\quad -\frac{2ab\log\left(a+b\sin^3(c+dx)\right)}{3\left(a^2-b^2\right)^2 d}+\frac{b(a-\sin(c+dx)(b-a\sin(c+dx)))}{3a\left(a^2-b^2\right)d\left(a+b\sin^3(c+dx)\right)}
\end{aligned}$$

output
```
-1/2*ln(1-sin(d*x+c))/(a+b)^2/d+1/2*ln(1+sin(d*x+c))/(a-b)^2/d-1/9*b^(1/3)
*(a^(4/3)+2*b^(4/3))*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(5/3)/(a^2-b^2)/d-1/
3*b^(1/3)*(a^2+2*a^(2/3)*b^(4/3)+b^2)*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(1/
3)/(a^2-b^2)^2/d+1/18*b^(1/3)*(a^(4/3)+2*b^(4/3))*ln(a^(2/3)-a^(1/3)*b^(1/
3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(5/3)/(a^2-b^2)/d+1/6*b^(1/3)*(a^2+2
*a^(2/3)*b^(4/3)+b^2)*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*
x+c)^2)/a^(1/3)/(a^2-b^2)^2/d-2/3*a*b*ln(a+b*sin(d*x+c)^3)/(a^2-b^2)^2/d+1
/3*b*(a-sin(d*x+c)*(b-a*sin(d*x+c)))/a/(a^2-b^2)/d/(a+b*sin(d*x+c)^3)-1/9*
b^(1/3)*(a^(4/3)-2*b^(4/3))*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1
/3)*3^(1/2))/a^(5/3)/(a^2-b^2)/d*3^(1/2)-1/3*b^(1/3)*(a^2-2*a^(2/3)*b^(4/3
)+b^2)*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2))/a^(1/3)/
(a^2-b^2)^2/d*3^(1/2)
```

3.397.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.38 (sec) , antiderivative size = 564, normalized size of antiderivative = 0.96

$$\int \frac{\sec(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$$
$$= \frac{\frac{12\sqrt{3}\sqrt[3]{a}b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{(a^2-b^2)^2}+\frac{4\sqrt{3}b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}(a^2-b^2)}-\frac{9\log(1-\sin(c+dx))}{(a+b)^2}+\frac{9\log(1+\sin(c+dx))}{(a-b)^2}-\cdots}{}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
((12*Sqrt[3]*a^(1/3)*b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sq
rt[3]*a^(1/3))])/(a^2 - b^2)^2 + (4*Sqrt[3]*b^(5/3)*ArcTan[(a^(1/3) - 2*b^
(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/(a^(5/3)*(a^2 - b^2)) - (9*Log[1 -
 Sin[c + d*x]])/(a + b)^2 + (9*Log[1 + Sin[c + d*x]])/(a - b)^2 - (12*a^(1
/3)*b^(5/3)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(a^2 - b^2)^2 - (4*b^(5/3
)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(a^(5/3)*(a^2 - b^2)) + (6*a^(1/3)*
b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^
2])/(a^2 - b^2)^2 + (2*b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x]
+ b^(2/3)*Sin[c + d*x]^2])/(a^(5/3)*(a^2 - b^2)) - (12*a*b*Log[a + b*Sin[c
 + d*x]^3])/(a^2 - b^2)^2 + (9*b*(a^2 + b^2)*Hypergeometric2F1[2/3, 1, 5/3
, -((b*Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(a*(a^2 - b^2)^2) + (9*b*Hyperg
eometric2F1[2/3, 2, 5/3, -((b*Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(a^3 - a
*b^2) + (6*b)/((a^2 - b^2)*(a + b*Sin[c + d*x]^3)) - (6*b^2*Sin[c + d*x])/
(a*(a^2 - b^2)*(a + b*Sin[c + d*x]^3)))/(18*d)
```

3.397.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 561, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3702, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\sin^3(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)\left(a+b\sin(c+dx)^3\right)^2}dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int \frac{1}{(1-\sin^2(c+dx))\left(b\sin^3(c+dx)+a\right)^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{7276}$$

$$\frac{\int \left(\frac{b(b\sin^2(c+dx)-a\sin(c+dx)+b)}{(b^2-a^2)\left(b\sin^3(c+dx)+a\right)^2}-\frac{1}{2(a+b)^2(\sin(c+dx)-1)}+\frac{1}{2(a-b)^2(\sin(c+dx)+1)}+\frac{b(-2ab\sin^2(c+dx)+(a^2+b^2)\sin(c+dx)-2ab)}{(a^2-b^2)^2\left(b\sin^3(c+dx)+a\right)}\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b(a-\sin(c+dx)(b-a\sin(c+dx)))}{3a(a^2-b^2)(a+b\sin^3(c+dx))}-\frac{2ab\log(a+b\sin^3(c+dx))}{3(a^2-b^2)^2}-\frac{\sqrt[3]{b}(-2a^{2/3}b^{4/3}+a^2+b^2)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}(a^2-b^2)^2}-\frac{\sqrt[3]{b}(a^{4/3}-2b^{4/3}}{}$$

input
```
Int[Sec[c + d*x]/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
(-1/3*(b^(1/3)*(a^(4/3) - 2*b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d
*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*(a^2 - b^2)) - (b^(1/3)*(a^2 - 2
*a^(2/3)*b^(4/3) + b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(1/3)*(a^2 - b^2)^2) - Log[1 - Sin[c + d*x]]/(2*(a
+ b)^2) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2) - (b^(1/3)*(a^(4/3) + 2*b^(4
/3))*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*(a^2 - b^2)) - (b^(1/
3)*(a^2 + 2*a^(2/3)*b^(4/3) + b^2)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(3
*a^(1/3)*(a^2 - b^2)^2) + (b^(1/3)*(a^(4/3) + 2*b^(4/3))*Log[a^(2/3) - a^(
1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(18*a^(5/3)*(a^2 - b^
2)) + (b^(1/3)*(a^2 + 2*a^(2/3)*b^(4/3) + b^2)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(6*a^(1/3)*(a^2 - b^2)^2) - (2*
a*b*Log[a + b*Sin[c + d*x]^3])/(3*(a^2 - b^2)^2) + (b*(a - Sin[c + d*x]*(b
 - a*Sin[c + d*x])))/(3*a*(a^2 - b^2)*(a + b*Sin[c + d*x]^3)))/d
```

3.397.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.397. $\int \frac{\sec(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.397.4 Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.66

method	result
derivativedivides	$$-\frac{\ln(\sin(dx+c)-1)}{2(a+b)^2}+\frac{\ln(1+\sin(dx+c))}{2(a-b)^2}+b\left(\frac{\left(\frac{a^2}{3}-\frac{b^2}{3}\right)\left(\sin^2(dx+c)\right)-\frac{b\left(a^2-b^2\right)\sin(dx+c)}{3a}+\frac{a^2}{3}-\frac{b^2}{3}}{a+b\left(\sin^3(dx+c)\right)}+\frac{2\left(-4a^2b+b^3\right)\left(\frac{\ln\left(\sin(dx+\cdots\right.}{3b\left(\cdots\right.}\cdots\right.}{\cdots}\cdots\right.$$
default	$$-\frac{\ln(\sin(dx+c)-1)}{2(a+b)^2}+\frac{\ln(1+\sin(dx+c))}{2(a-b)^2}+b\left(\frac{\left(\frac{a^2}{3}-\frac{b^2}{3}\right)\left(\sin^2(dx+c)\right)-\frac{b\left(a^2-b^2\right)\sin(dx+c)}{3a}+\frac{a^2}{3}-\frac{b^2}{3}}{a+b\left(\sin^3(dx+c)\right)}+\frac{2\left(-4a^2b+b^3\right)\left(\frac{\ln\left(\sin(dx+\cdots\right.}{3b\left(\cdots\right.}\cdots\right.}{\cdots}\cdots\right.$$
risch	$$-\frac{ix}{a^2-2ab+b^2}-\frac{ic}{d(a^2-2ab+b^2)}+\frac{ix}{a^2+2ab+b^2}+\frac{ic}{d(a^2+2ab+b^2)}+\frac{4ia^6b\,d^3x}{a^9d^3-2a^7b^2d^3+a^5b^4d^3}+\frac{4ia^6b\,d^2c}{a^9d^3-2a^7b^2d^3+\cdots}\cdots$$

input
```
int(sec(d*x+c)/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

3.397. $\int \frac{\sec(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

output
```
1/d*(-1/2/(a+b)^2*ln(sin(d*x+c)-1)+1/2/(a-b)^2*ln(1+sin(d*x+c))+b/(a-b)^2/
(a+b)^2*(((1/3*a^2-1/3*b^2)*sin(d*x+c)^2-1/3*b*(a^2-b^2)/a*sin(d*x+c)+1/3*
a^2-1/3*b^2)/(a+b*sin(d*x+c)^3)+2/3/a*((-4*a^2*b+b^3)*(1/3/b/(1/b*a)^(2/3)
*ln(sin(d*x+c)+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*ln(sin(d*x+c)^2-(1/b*a)^
(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))+(2*a^3+a*b^2)*(-1/3/b/(1/b*a)^(1/3)*
ln(sin(d*x+c)+(1/b*a)^(1/3))+1/6/b/(1/b*a)^(1/3)*ln(sin(d*x+c)^2-(1/b*a)^(
1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1
/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))-a^2*ln(a+b*sin(d*x+c)^3))))
```

3.397.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.60 (sec) , antiderivative size = 10855, normalized size of antiderivative = 18.49

$$\int \frac{\sec(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.397.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)/(a+b*sin(d*x+c)**3)**2,x)
```

output
```
Timed out
```

3.397.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.82

$$\int \frac{\sec(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$$
$$= \frac{\frac{4\sqrt{3}\left(2a^3\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)-2a^2b\left(2\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{a}{b}\right)+ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}-2a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2\left(2a^2b\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\right)-2a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}-\cdots\right.}{a^5\left(\frac{a}{b}\cdots\right.}}{}$$

input
```
integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/18*(4*sqrt(3)*(2*a^3*((a/b)^(2/3) + 1) - 2*a^2*b*(2*(a/b)^(1/3) + a/b) +
 a*b^2*(a/b)^(2/3) + b^3*(a/b)^(1/3))*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2
*sin(d*x + c))/(a/b)^(1/3))/((a^5*(a/b)^(2/3) - 2*a^3*b^2*(a/b)^(2/3) + a*
b^4*(a/b)^(2/3))*(a/b)^(1/3)) - 2*(2*a^2*b*(3*(a/b)^(2/3) - 2) - 2*a^3*(a/
b)^(1/3) - a*b^2*(a/b)^(1/3) + b^3)*log(sin(d*x + c)^2 - (a/b)^(1/3)*sin(d
*x + c) + (a/b)^(2/3))/(a^5*(a/b)^(2/3) - 2*a^3*b^2*(a/b)^(2/3) + a*b^4*(a
/b)^(2/3)) - 4*(a^2*b*(3*(a/b)^(2/3) + 4) + 2*a^3*(a/b)^(1/3) + a*b^2*(a/b
)^(1/3) - b^3)*log((a/b)^(1/3) + sin(d*x + c))/(a^5*(a/b)^(2/3) - 2*a^3*b^
2*(a/b)^(2/3) + a*b^4*(a/b)^(2/3)) + 6*(a*b*sin(d*x + c)^2 - b^2*sin(d*x +
 c) + a*b)/(a^4 - a^2*b^2 + (a^3*b - a*b^3)*sin(d*x + c)^3) + 9*log(sin(d*
x + c) + 1)/(a^2 - 2*a*b + b^2) - 9*log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b
^2))/d
```

3.397.8 Giac [F]

$$\int \frac{\sec(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \int \frac{\sec(dx+c)}{(b\sin(dx+c)^3+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.397.9 Mupad [B] (verification not implemented)

Time = 14.82 (sec) , antiderivative size = 980, normalized size of antiderivative = 1.67

$$\int \frac{\sec(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

$$= \frac{\sum_{k=1}^{3}\ln\left(\frac{\frac{8\,b^6}{27}-\frac{16\,a^2\,b^4}{27}}{a^7-2\,a^5\,b^2+a^3\,b^4}+\mathrm{root}(1458\,a^7\,b^2\,z^3-729\,a^5\,b^4\,z^3-729\,a^9\,z^3-1458\,a^6\,b\,z^2-108\,a^3\,b^2\,z-64\,a^2\right.}{}$$

$$-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)}{d\,\left(2\,a^2+4\,a\,b+2\,b^2\right)}+\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{d\,\left(2\,a^2-4\,a\,b+2\,b^2\right)}+\frac{\frac{b}{3\,(a^2-b^2)}+\frac{b\sin(c+d\,x)^2}{3\,(a^2-b^2)}-\frac{b^2\,\sin(c+d\,x)}{3\,a\,(a^2-b^2)}}{d\,\left(b\sin\left(c+d\,x\right)^3+a\right)}$$

input
```
int(1/(cos(c + d*x)*(a + b*sin(c + d*x)^3)^2),x)
```

output
```
symsum(log(((8*b^6)/27 - (16*a^2*b^4)/27)/(a^7 + a^3*b^4 - 2*a^5*b^2) + ro
ot(1458*a^7*b^2*z^3 - 729*a^5*b^4*z^3 - 729*a^9*z^3 - 1458*a^6*b*z^2 - 108
*a^3*b^2*z - 64*a^2*b + 8*b^3, z, k)*(((32*a*b^7)/27 + (128*a^3*b^5)/27)/(
a^7 + a^3*b^4 - 2*a^5*b^2) - root(1458*a^7*b^2*z^3 - 729*a^5*b^4*z^3 - 729
*a^9*z^3 - 1458*a^6*b*z^2 - 108*a^3*b^2*z - 64*a^2*b + 8*b^3, z, k)*(root(
1458*a^7*b^2*z^3 - 729*a^5*b^4*z^3 - 729*a^9*z^3 - 1458*a^6*b*z^2 - 108*a^
3*b^2*z - 64*a^2*b + 8*b^3, z, k)*((16*a^3*b^9 - 77*a^5*b^7 + 34*a^7*b^5 +
 27*a^9*b^3)/(a^7 + a^3*b^4 - 2*a^5*b^2) + root(1458*a^7*b^2*z^3 - 729*a^5
*b^4*z^3 - 729*a^9*z^3 - 1458*a^6*b*z^2 - 108*a^3*b^2*z - 64*a^2*b + 8*b^3
, z, k)*((36*a^4*b^10 + 108*a^6*b^8 - 324*a^8*b^6 + 180*a^10*b^4)/(a^7 + a
^3*b^4 - 2*a^5*b^2) + (sin(c + d*x)*(4374*a^5*b^9 - 7290*a^7*b^7 + 1458*a^
9*b^5 + 1458*a^11*b^3))/(27*(a^7 + a^3*b^4 - 2*a^5*b^2))) + (sin(c + d*x)*
(216*a^2*b^10 - 864*a^4*b^8 - 1836*a^6*b^6 + 2484*a^8*b^4))/(27*(a^7 + a^3
*b^4 - 2*a^5*b^2))) + ((64*a^2*b^8)/9 - (353*a^4*b^6)/9 + (388*a^6*b^4)/9)
/(a^7 + a^3*b^4 - 2*a^5*b^2) + (sin(c + d*x)*(96*a*b^9 - 408*a^3*b^7 + 447
*a^5*b^5))/(27*(a^7 + a^3*b^4 - 2*a^5*b^2))) + (sin(c + d*x)*(16*b^8 + 134
*a^2*b^6 - 236*a^4*b^4))/(27*(a^7 + a^3*b^4 - 2*a^5*b^2))) + (8*a*b^5*sin(
c + d*x))/(9*(a^7 + a^3*b^4 - 2*a^5*b^2)))*root(1458*a^7*b^2*z^3 - 729*a^5
*b^4*z^3 - 729*a^9*z^3 - 1458*a^6*b*z^2 - 108*a^3*b^2*z - 64*a^2*b + 8*b^3
, z, k), k, 1, 3)/d - log(sin(c + d*x) - 1)/(d*(4*a*b + 2*a^2 + 2*b^2))...
```

3.398 $\int \frac{\sec^3(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

3.398.1 Optimal result

Integrand size = 23, antiderivative size = 747

$$\int \frac{\sec^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = -\frac{b^{5/3}\left(4a^2-3a^{4/3}b^{2/3}+2b^2\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}\left(a^2-b^2\right)^2 d}$$
$$-\frac{b^{5/3}\left(4a^{8/3}-9a^2b^{2/3}+8a^{2/3}b^2-3b^{8/3}\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^2-b^2\right)^3 d}$$
$$-\frac{(a+7b)\log(1-\sin(c+dx))}{4(a+b)^3d}+\frac{(a-7b)\log(1+\sin(c+dx))}{4(a-b)^3d}$$
$$+\frac{b^{5/3}\left(4a^2+3a^{4/3}b^{2/3}+2b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\left(a^2-b^2\right)^2 d}$$
$$+\frac{b^{5/3}\left(3b^{2/3}(3a^2+b^2)+4a^{2/3}(a^2+2b^2)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3\sqrt[3]{a}\left(a^2-b^2\right)^3 d}$$
$$-\frac{b^{5/3}\left(4a^2+3a^{4/3}b^{2/3}+2b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{18a^{5/3}\left(a^2-b^2\right)^2 d}$$
$$-\frac{b^{5/3}\left(3b^{2/3}(3a^2+b^2)+4a^{2/3}(a^2+2b^2)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6\sqrt[3]{a}\left(a^2-b^2\right)^3 d}$$
$$+\frac{2ab(a^2+5b^2)\log\left(a+b\sin^3(c+dx)\right)}{3\left(a^2-b^2\right)^3 d}$$
$$+\frac{1}{4(a+b)^2d(1-\sin(c+dx))}-\frac{1}{4(a-b)^2d(1+\sin(c+dx))}$$
$$-\frac{b\left(a(a^2+2b^2)-b\sin(c+dx)\left(2a^2+b^2-3ab\sin(c+dx)\right)\right)}{3a\left(a^2-b^2\right)^2 d\left(a+b\sin^3(c+dx)\right)}$$

output
```
-1/4*(a+7*b)*ln(1-sin(d*x+c))/(a+b)^3/d+1/4*(a-7*b)*ln(1+sin(d*x+c))/(a-b)
^3/d+1/9*b^(5/3)*(4*a^2+3*a^(4/3)*b^(2/3)+2*b^2)*ln(a^(1/3)+b^(1/3)*sin(d*
x+c))/a^(5/3)/(a^2-b^2)^2/d+1/3*b^(5/3)*(3*b^(2/3)*(3*a^2+b^2)+4*a^(2/3)*(
a^2+2*b^2))*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(1/3)/(a^2-b^2)^3/d-1/18*b^(5
/3)*(4*a^2+3*a^(4/3)*b^(2/3)+2*b^2)*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+
b^(2/3)*sin(d*x+c)^2)/a^(5/3)/(a^2-b^2)^2/d-1/6*b^(5/3)*(3*b^(2/3)*(3*a^2+
b^2)+4*a^(2/3)*(a^2+2*b^2))*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/3)*
sin(d*x+c)^2)/a^(1/3)/(a^2-b^2)^3/d+2/3*a*b*(a^2+5*b^2)*ln(a+b*sin(d*x+c)^
3)/(a^2-b^2)^3/d+1/4/(a+b)^2/d/(1-sin(d*x+c))-1/4/(a-b)^2/d/(1+sin(d*x+c))
-1/3*b*(a*(a^2+2*b^2)-b*sin(d*x+c)*(2*a^2+b^2-3*a*b*sin(d*x+c)))/a/(a^2-b^
2)^2/d/(a+b*sin(d*x+c)^3)-1/9*b^(5/3)*(4*a^2-3*a^(4/3)*b^(2/3)+2*b^2)*arct
an(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2))/a^(5/3)/(a^2-b^2)^2
/d*3^(1/2)-1/3*b^(5/3)*(4*a^(8/3)-9*a^2*b^(2/3)+8*a^(2/3)*b^2-3*b^(8/3))*a
rctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2))/a^(1/3)/(a^2-b^2
)^3/d*3^(1/2)
```

3.398.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.43 (sec) , antiderivative size = 724, normalized size of antiderivative = 0.97

$$\int \frac{\sec^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$$

$$= \frac{-\frac{2b^{5/3}(2a^2+b^2)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(a^2-b^2)^2} - \frac{4\sqrt[3]{a}b^{5/3}(a^2+2b^2)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}(a^2-b^2)^3} - \frac{(a+7b)\log(1-\sin(c+dx))}{4(a+b)^3} + \frac{(}{}}{}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
((-2*b^(5/3)*(2*a^2 + b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt
[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(a^2 - b^2)^2) - (4*a^(1/3)*b^(5/3)*(a^2
 + 2*b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/(S
qrt[3]*(a^2 - b^2)^3) - ((a + 7*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^3) +
((a - 7*b)*Log[1 + Sin[c + d*x]])/(4*(a - b)^3) + (2*b^(5/3)*(2*a^2 + b^2)
*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*(a^2 - b^2)^2) + (4*a^(1/
3)*b^(5/3)*(a^2 + 2*b^2)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(3*(a^2 - b^
2)^3) - (b^(5/3)*(2*a^2 + b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x]
+ b^(2/3)*Sin[c + d*x]^2])/(9*a^(5/3)*(a^2 - b^2)^2) - (2*a^(1/3)*b^(5/3)*
(a^2 + 2*b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c +
 d*x]^2])/(3*(a^2 - b^2)^3) + (2*a*b*(a^2 + 5*b^2)*Log[a + b*Sin[c + d*x]^
3])/(3*(a^2 - b^2)^3) + 1/(4*(a + b)^2*(1 - Sin[c + d*x])) - (3*b^3*(3*a^2
 + b^2)*Hypergeometric2F1[2/3, 1, 5/3, -((b*Sin[c + d*x]^3)/a)]*Sin[c + d*
x]^2)/(2*a*(a^2 - b^2)^3) - (3*b^3*Hypergeometric2F1[2/3, 2, 5/3, -((b*Sin
[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(2*a*(a^2 - b^2)^2) - 1/(4*(a - b)^2*(1 +
 Sin[c + d*x])) - (b*(a^2 + 2*b^2))/(3*(a^2 - b^2)^2*(a + b*Sin[c + d*x]^3
)) + (a*b^2*(2 + b^2/a^2)*Sin[c + d*x])/(3*(a^2 - b^2)^2*(a + b*Sin[c + d*
x]^3)))/d
```

3.398.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 715, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3702, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^3\left(a+b\sin(c+dx)^3\right)^2}dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int \frac{1}{(1-\sin^2(c+dx))^2(b\sin^3(c+dx)+a)^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{7293}$$

$$\frac{\int \left(\frac{(2a(a^2+5b^2)\sin^2(c+dx)-3b(3a^2+b^2)\sin(c+dx)+4a(a^2+2b^2))b^2}{(a^2-b^2)^3(b\sin^3(c+dx)+a)} + \frac{(2a^2-3b\sin(c+dx)a+b^2+(a^2+2b^2)\sin^2(c+dx))b^2}{(a^2-b^2)^2(b\sin^3(c+dx)+a)^2} + \frac{-a-7b}{4(a+b)^3(\sin(c+\dots}\right.}{d}$$

$\downarrow$ 2009

$$\frac{-\frac{b(a(a^2+2b^2)-b\sin(c+dx)(2a^2-3ab\sin(c+dx)+b^2))}{3a(a^2-b^2)^2(a+b\sin^3(c+dx))} + \frac{2ab(a^2+5b^2)\log(a+b\sin^3(c+dx))}{3(a^2-b^2)^3} - \frac{b^{5/3}(-3a^{4/3}b^{2/3}+4a^2+2b^2)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{\dots}}{\sqrt{3}\dots}\right.}{3\sqrt{3}a^{5/3}(a^2-b^2)^2}}{\dots}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
(-1/3*(b^(5/3)*(4*a^2 - 3*a^(4/3)*b^(2/3) + 2*b^2)*ArcTan[(a^(1/3) - 2*b^(
1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*(a^2 - b^2)^2) - (
b^(5/3)*(4*a^(8/3) - 9*a^2*b^(2/3) + 8*a^(2/3)*b^2 - 3*b^(8/3))*ArcTan[(a^
(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*(a^2
- b^2)^3) - ((a + 7*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^3) + ((a - 7*b)*L
og[1 + Sin[c + d*x]])/(4*(a - b)^3) + (b^(5/3)*(4*a^2 + 3*a^(4/3)*b^(2/3)
+ 2*b^2)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*(a^2 - b^2)^2) +
(b^(5/3)*(3*b^(2/3)*(3*a^2 + b^2) + 4*a^(2/3)*(a^2 + 2*b^2))*Log[a^(1/3) +
 b^(1/3)*Sin[c + d*x]])/(3*a^(1/3)*(a^2 - b^2)^3) - (b^(5/3)*(4*a^2 + 3*a^
(4/3)*b^(2/3) + 2*b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3
)*Sin[c + d*x]^2])/(18*a^(5/3)*(a^2 - b^2)^2) - (b^(5/3)*(3*b^(2/3)*(3*a^2
 + b^2) + 4*a^(2/3)*(a^2 + 2*b^2))*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d
*x] + b^(2/3)*Sin[c + d*x]^2])/(6*a^(1/3)*(a^2 - b^2)^3) + (2*a*b*(a^2 + 5
*b^2)*Log[a + b*Sin[c + d*x]^3])/(3*(a^2 - b^2)^3) + 1/(4*(a + b)^2*(1 - S
in[c + d*x])) - 1/(4*(a - b)^2*(1 + Sin[c + d*x])) - (b*(a*(a^2 + 2*b^2) -
 b*Sin[c + d*x]*(2*a^2 + b^2 - 3*a*b*Sin[c + d*x])))/(3*a*(a^2 - b^2)^2*(a
 + b*Sin[c + d*x]^3)))/d
```

3.398.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.398. $\int \frac{\sec^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.398.4 Maple [A] (verified)

Time = 4.78 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.65

method	result
derivativedivides	$-\frac{1}{4(a+b)^2(\sin(dx+c)-1)}+\frac{(-a-7b)\ln(\sin(dx+c)-1)}{4(a+b)^3}-\frac{1}{4(a-b)^2(1+\sin(dx+c))}+\frac{(a-7b)\ln(1+\sin(dx+c))}{4(a-b)^3}+\frac{b^2\left(\frac{(-a^2b+b^3)(\sin^2(d\ldots}{\ldots}\right.}{\ldots}$
default	$-\frac{1}{4(a+b)^2(\sin(dx+c)-1)}+\frac{(-a-7b)\ln(\sin(dx+c)-1)}{4(a+b)^3}-\frac{1}{4(a-b)^2(1+\sin(dx+c))}+\frac{(a-7b)\ln(1+\sin(dx+c))}{4(a-b)^3}+\frac{b^2\left(\frac{(-a^2b+b^3)(\sin^2(d\ldots}{\ldots}\right.}{\ldots}$
risch	Expression too large to display

input
```
int(sec(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

3.398. $\int \frac{\sec^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

output
```
1/d*(-1/4/(a+b)^2/(sin(d*x+c)-1)+1/4/(a+b)^3*(-a-7*b)*ln(sin(d*x+c)-1)-1/4
/(a-b)^2/(1+sin(d*x+c))+1/4*(a-7*b)/(a-b)^3*ln(1+sin(d*x+c))+b^2/(a-b)^3/(
a+b)^3*(((-a^2*b+b^3)*sin(d*x+c)^2+1/3*(2*a^4-a^2*b^2-b^4)/a*sin(d*x+c)-1/
3*(a^4+a^2*b^2-2*b^4)/b)/(a+b*sin(d*x+c)^3)+2/3/a*((8*a^4+11*a^2*b^2-b^4)*
(1/3/b/(1/b*a)^(2/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*ln(s
in(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))+(-15*a^3*b-3*a*b
^3)*(-1/3/b/(1/b*a)^(1/3)*ln(sin(d*x+c)+(1/b*a)^(1/3))+1/6/b/(1/b*a)^(1/3)
*ln(sin(d*x+c)^2-(1/b*a)^(1/3)*sin(d*x+c)+(1/b*a)^(2/3))+1/3*3^(1/2)/b/(1/
b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*sin(d*x+c)-1)))+1/3*(3*a^4+
15*a^2*b^2)/b*ln(a+b*sin(d*x+c)^3))))
```

3.398.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.04 (sec) , antiderivative size = 15989, normalized size of antiderivative = 21.40

$$\int \frac{\sec^3(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.398.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**3/(a+b*sin(d*x+c)**3)**2,x)
```

output
```
Timed out
```

3.398.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 788, normalized size of antiderivative = 1.05

$$\int \frac{\sec^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/36*(8*sqrt(3)*(5*a^3*b^2*(3*(a/b)^(2/3) + 2) - a^2*b^3*(11*(a/b)^(1/3)
+ 10*a/b) - 2*a^4*b*(4*(a/b)^(1/3) + a/b) + 3*a*b^4*(a/b)^(2/3) + b^5*(a/b
)^(1/3) + 2*a^5)*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^
(1/3))/((a^7*(a/b)^(2/3) - 3*a^5*b^2*(a/b)^(2/3) + 3*a^3*b^4*(a/b)^(2/3) -
 a*b^6*(a/b)^(2/3))*(a/b)^(1/3)) - 4*(a^2*b^3*(30*(a/b)^(2/3) - 11) + 2*a^
4*b*(3*(a/b)^(2/3) - 4) - 15*a^3*b^2*(a/b)^(1/3) - 3*a*b^4*(a/b)^(1/3) + b
^5)*log(sin(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(a^7*(a/b
)^(2/3) - 3*a^5*b^2*(a/b)^(2/3) + 3*a^3*b^4*(a/b)^(2/3) - a*b^6*(a/b)^(2/3
)) - 8*(a^2*b^3*(15*(a/b)^(2/3) + 11) + a^4*b*(3*(a/b)^(2/3) + 8) + 15*a^3
*b^2*(a/b)^(1/3) + 3*a*b^4*(a/b)^(1/3) - b^5)*log((a/b)^(1/3) + sin(d*x +
c))/(a^7*(a/b)^(2/3) - 3*a^5*b^2*(a/b)^(2/3) + 3*a^3*b^4*(a/b)^(2/3) - a*b
^6*(a/b)^(2/3)) - 9*(a - 7*b)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) + 9*(a + 7*b)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b
^3) - 6*(3*(a^3*b + 3*a*b^3)*sin(d*x + c)^4 - 8*a^3*b - 4*a*b^3 - 2*(5*a^2
*b^2 + b^4)*sin(d*x + c)^3 + 2*(a^3*b - a*b^3)*sin(d*x + c)^2 + (3*a^4 + 7
*a^2*b^2 + 2*b^4)*sin(d*x + c))/(a^6 - 2*a^4*b^2 + a^2*b^4 - (a^5*b - 2*a^
3*b^3 + a*b^5)*sin(d*x + c)^5 + (a^5*b - 2*a^3*b^3 + a*b^5)*sin(d*x + c)^3
 - (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(d*x + c)^2))/d
```

3.398.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \int \frac{\sec\left(dx+c\right)^3}{\left(b\sin\left(dx+c\right)^3+a\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.398.9 Mupad [B] (verification not implemented)

Time = 14.27 (sec) , antiderivative size = 1605, normalized size of antiderivative = 2.15

$$\int \frac{\sec^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x)^3)^2),x)
```

output
```
symsum(log(root(2187*a^9*b^2*z^3 - 2187*a^7*b^4*z^3 + 729*a^5*b^6*z^3 - 72
9*a^11*z^3 + 7290*a^6*b^3*z^2 + 1458*a^8*b*z^2 - 972*a^5*b^2*z + 324*a^3*b
^4*z + 216*a^2*b^3 - 8*b^5, z, k)*(((32*a*b^11)/27 + (2173*a^3*b^9)/27 + (
847*a^5*b^7)/3 - 20*a^7*b^5)/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a
^9*b^2) - root(2187*a^9*b^2*z^3 - 2187*a^7*b^4*z^3 + 729*a^5*b^6*z^3 - 729
*a^11*z^3 + 7290*a^6*b^3*z^2 + 1458*a^8*b*z^2 - 972*a^5*b^2*z + 324*a^3*b^
4*z + 216*a^2*b^3 - 8*b^5, z, k)*(((32*a^2*b^12)/3 - (1017*a^4*b^10)/4 + 3
25*a^6*b^8 + (4153*a^8*b^6)/12 - (63*a^10*b^4)/2)/(a^11 + a^3*b^8 - 4*a^5*
b^6 + 6*a^7*b^4 - 4*a^9*b^2) + root(2187*a^9*b^2*z^3 - 2187*a^7*b^4*z^3 +
729*a^5*b^6*z^3 - 729*a^11*z^3 + 7290*a^6*b^3*z^2 + 1458*a^8*b*z^2 - 972*a
^5*b^2*z + 324*a^3*b^4*z + 216*a^2*b^3 - 8*b^5, z, k)*((16*a^3*b^13 - (563
*a^5*b^11)/2 + 303*a^7*b^9 + 188*a^9*b^7 - 239*a^11*b^5 + (27*a^13*b^3)/2)
/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2) + root(2187*a^9*b^2*
z^3 - 2187*a^7*b^4*z^3 + 729*a^5*b^6*z^3 - 729*a^11*z^3 + 7290*a^6*b^3*z^2
 + 1458*a^8*b*z^2 - 972*a^5*b^2*z + 324*a^3*b^4*z + 216*a^2*b^3 - 8*b^5, z
, k)*((36*a^4*b^14 + 36*a^6*b^12 - 504*a^8*b^10 + 936*a^10*b^8 - 684*a^12*
b^6 + 180*a^14*b^4)/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2) +
 (sin(c + d*x)*(17496*a^5*b^13 - 64152*a^7*b^11 + 81648*a^9*b^9 - 34992*a^
11*b^7 - 5832*a^13*b^5 + 5832*a^15*b^3))/(108*(a^11 + a^3*b^8 - 4*a^5*b^6
+ 6*a^7*b^4 - 4*a^9*b^2))) - (sin(c + d*x)*(13824*a^4*b^12 - 864*a^2*b^...
```

3.399 $\int \frac{\cos^4(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

3.399.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\cos^4(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Int}\left(\frac{\cos^4(c+dx)}{(a+b\sin^3(c+dx))^2},x\right)$$

output
```
Unintegrable(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x)
```

3.399.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 394, normalized size of antiderivative = 17.13

$$\int \frac{\cos^4(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \frac{-i\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&, \frac{2b\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-ib\log\left(1-2\cos(c+dx)\#1+\#1^2\right.}{}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
((-I)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 &
 , (2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*b*Log[1 - 2*Cos[c + d
*x]*#1 + #1^2] + (4*I)*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + 2*a
*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 + 12*b*ArcTan[Sin[c + d*x]/(Cos[c +
d*x] - #1)]*#1^2 - (6*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (4*I)*
a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - 2*a*Log[1 - 2*Cos[c + d*
x]*#1 + #1^2]*#1^3 + 2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I
*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3
 + b*#1^5) & ] + (24*Cos[c + d*x]*(a + b*Sin[c + d*x]))/(4*a + 3*b*Sin[c +
 d*x] - b*Sin[3*(c + d*x)]))/(18*a*b*d)
```

3.399.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3707}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^4}{(a+b\sin(c+dx)^3)^2}dx$$

$$\Big\downarrow \text{3707}$$

$$\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}dx$$

input
```
Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
$Aborted
```

3.399.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3707
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Cos[e + f*x])^m*(a +
 b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.399.4 Maple [N/A] (verified)

Time = 1.74 (sec) , antiderivative size = 241, normalized size of antiderivative = 10.48

method	result
derivativedivides	$\frac{\frac{-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}+\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3b}+\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}+\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3b}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3a}+\frac{2}{3b}}{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+a}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\right.}\right.}{}}{d}$
default	$\frac{\frac{-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}+\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3b}+\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}+\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3b}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3a}+\frac{2}{3b}}{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+a}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\right.}\right.}{}}{d}$
risch	$-\frac{2\,\mathrm{e}^{i(dx+c)}\left(2ia\,\mathrm{e}^{3i(dx+c)}+b\,\mathrm{e}^{4i(dx+c)}+2ia\,\mathrm{e}^{i(dx+c)}-b\right)}{3abd\left(b\,\mathrm{e}^{6i(dx+c)}-3b\,\mathrm{e}^{4i(dx+c)}+3b\,\mathrm{e}^{2i(dx+c)}-8ia\,\mathrm{e}^{3i(dx+c)}-b\right)}+\left(\sum_{\_R=\mathrm{RootOf}\left(531441a^{10}b^8d^6\_Z^6+59049a^8b^6d\right.}\right.$

input
```
int(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/3/a*tan(1/2*d*x+1/2*c)^5+1/3/b*tan(1/2*d*x+1/2*c)^4+4/3/a*tan(1
/2*d*x+1/2*c)^3+2/3/b*tan(1/2*d*x+1/2*c)^2+1/3/a*tan(1/2*d*x+1/2*c)+1/3/b)
/(a*tan(1/2*d*x+1/2*c)^6+3*tan(1/2*d*x+1/2*c)^4*a+8*tan(1/2*d*x+1/2*c)^3*b
+3*tan(1/2*d*x+1/2*c)^2*a+a)+2/9/a/b*sum((_R^4*b+_R^3*a+_R*a+b)/(_R^5*a+2*
_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+
8*_Z^3*b+3*_Z^2*a+a)))
```

3.399.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 9.40 (sec) , antiderivative size = 9984, normalized size of antiderivative = 434.09

$$\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.399.6 Sympy [N/A]

Not integrable

Time = 148.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \int \frac{\cos^4\left(c+dx\right)}{\left(a+b\sin^3\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)**4/(a+b*sin(d*x+c)**3)**2,x)
```

output
```
Integral(cos(c + d*x)**4/(a + b*sin(c + d*x)**3)**2, x)
```

3.399.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.399.8 Giac [N/A]

Not integrable

Time = 6.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{\cos^4(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \int \frac{\cos\left(dx+c\right)^4}{\left(b\sin\left(dx+c\right)^3+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.399.9 Mupad [B] (verification not implemented)

Time = 15.05 (sec) , antiderivative size = 2431, normalized size of antiderivative = 105.70

$$\int \frac{\cos^4(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^4/(a + b*sin(c + d*x)^3)^2,x)
```

output
```
2/(3*d*(a*b + 8*b^2*tan(c/2 + (d*x)/2)^3 + 3*a*b*tan(c/2 + (d*x)/2)^2 + 3*
a*b*tan(c/2 + (d*x)/2)^4 + a*b*tan(c/2 + (d*x)/2)^6)) + symsum(log((638976
*a^2*b^4 - 655360*b^6 - 8192*a^6 + 24576*a^4*b^2 - 2949120*root(531441*a^1
0*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2
 + a^6 - 64*b^6, d, k)*a^3*b^5 + 2138112*root(531441*a^10*b^8*d^6 + 59049*
a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d
, k)*a^5*b^3 - 9437184*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187
*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)*b^8*tan(c/2 +
 (d*x)/2) - 786432*a*b^5*tan(c/2 + (d*x)/2) + 98304*a^5*b*tan(c/2 + (d*x)/
2) - 21233664*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*
d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^2*a^2*b^8 + 18579456*r
oot(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^
4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^2*a^4*b^6 + 2654208*root(531441*a^10*
b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 +
 a^6 - 64*b^6, d, k)^2*a^6*b^4 - 167215104*root(531441*a^10*b^8*d^6 + 5904
9*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6,
 d, k)^3*a^5*b^7 + 113467392*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4
+ 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^3*a^7*b
^5 - 107495424*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4
*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^4*a^6*b^8 + 107495...
```

3.399. $\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

3.400 $\int \frac{\cos^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

3.400.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\cos^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx=\text{Int}\left(\frac{\cos^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2},x\right)$$

output
```
Unintegrable(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x)
```

3.400.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 273, normalized size of antiderivative = 11.87

$$\int \frac{\cos^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$
$$=\frac{-i\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&,\frac{2\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-i\log\left(1-2\cos(c+dx)\#1+\#1^2\right)\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
((-I)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 &
 , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x]*
#1 + #1^2] + 12*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (6*I)*Log[
1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
 #1)]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^
2 - 2*b*#1^3 + b*#1^5) & ] + (12*Sin[2*(c + d*x)])/(4*a + 3*b*Sin[c + d*x]
 - b*Sin[3*(c + d*x)]))/(18*a*d)
```

3.400.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3707}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\cos(c+dx)^2}{(a+b\sin(c+dx)^3)^2}dx$$
$$\Big\downarrow \ 3707$$
$$\int \frac{\cos^2(c+dx)}{(a+b\sin^3(c+dx))^2}dx$$

input
```
Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
$Aborted
```

3.400.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3707
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Cos[e + f*x])^m*(a +
 b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.400.4 Maple [N/A] (verified)

Time = 2.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 7.61

method	result
derivativedivides	$\frac{\frac{-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3a}}{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+a}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\right.}\right.}{}}{d}$
default	$\frac{\frac{-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3a}}{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+a}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\right.}\right.}{}}{d}$
risch	Expression too large to display

input
```
int(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/3/a*tan(1/2*d*x+1/2*c)^5+1/3/a*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d
*x+1/2*c)^6+3*tan(1/2*d*x+1/2*c)^4*a+8*tan(1/2*d*x+1/2*c)^3*b+3*tan(1/2*d*
x+1/2*c)^2*a+a)+2/9/a*sum((_R^4+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(
1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a)))
```

3.400.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 3.24 (sec) , antiderivative size = 36403, normalized size of antiderivative = 1582.74

$$\int \frac{\cos^2(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.400.6 Sympy [N/A]

Not integrable

Time = 125.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\cos^2(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*sin(d*x+c)**3)**2,x)
```

output
```
Integral(cos(c + d*x)**2/(a + b*sin(c + d*x)**3)**2, x)
```

3.400.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.400.8 Giac [N/A]

Not integrable

Time = 4.41 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{\cos^2(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \int \frac{\cos(dx+c)^2}{\left(b\sin(dx+c)^3+a\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.400.9 Mupad [B] (verification not implemented)

Time = 15.24 (sec) , antiderivative size = 1648, normalized size of antiderivative = 71.65

$$\int \frac{\cos^2(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^2/(a + b*sin(c + d*x)^3)^2,x)
```

output
```
symsum(log(-((131072*b^2)/243 - (16384*a^2)/243 + (8192*root(531441*a^12*b
^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^
2*b^2 + a^4 + 64*b^4, d, k)*a^4*tan(c/2 + (d*x)/2))/27 + (1048576*root(531
441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d
^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)*b^4*tan(c/2 + (d*x)/2))/27 + (262144
*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*
a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^2*a^2*b^4)/3 - (131072*root
(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b
^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^2*a^4*b^2)/3 - 98304*root(531441
*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2
- 16*a^2*b^2 + a^4 + 64*b^4, d, k)^3*a^5*b^3 + 442368*root(531441*a^12*b^4
*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*
b^2 + a^4 + 64*b^4, d, k)^4*a^6*b^4 + 221184*root(531441*a^12*b^4*d^6 - 53
1441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4
 + 64*b^4, d, k)^4*a^8*b^2 + 7962624*root(531441*a^12*b^4*d^6 - 531441*a^1
0*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^
4, d, k)^5*a^7*b^5 - 5971968*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^
6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)
^5*a^9*b^3 + (131072*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 1968
3*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)*a*b^...
```

3.400. $\int \frac{\cos^2(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

3.401 $\int \frac{1}{(a+b\sin^3(c+dx))^2}\,dx$

3.401.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{(a+b\sin^3(c+dx))^2}\,dx = \text{Int}\left(\frac{1}{(a+b\sin^3(c+dx))^2},x\right)$$

output
```
Unintegrable(1/(a+b*sin(d*x+c)^3)^2,x)
```

3.401.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 502, normalized size of antiderivative = 35.86

$$\int \frac{1}{(a+b\sin^3(c+dx))^2}\,dx = \frac{\frac{i\text{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&,\frac{2b^2\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-ib^2\log\left(1-2\cos(c+dx)\#1+\#1^2\right)+4iab\arctan\left(\frac{\sin(}{\cos(c+}\right.}{}\right.}{}}{}$$

input
```
Integrate[(a + b*Sin[c + d*x]^3)^(-2),x]
```

output
```
((I*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & ,
 (2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*b^2*Log[1 - 2*Cos[c +
 d*x]*#1 + #1^2] + (4*I)*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 +
 2*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 - 24*a^2*ArcTan[Sin[c + d*x]/(
Cos[c + d*x] - #1)]*#1^2 + 12*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^2 + (12*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (6*I)*b^2*Log[
1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (4*I)*a*b*ArcTan[Sin[c + d*x]/(Cos[c
+ d*x] - #1)]*#1^3 - 2*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 + 2*b^2*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*b^2*Log[1 - 2*Cos[c + d*
x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ])/(a^2 -
b^2) - (12*b*Cos[c + d*x]*(-3*a + a*Cos[2*(c + d*x)] + 2*b*Sin[c + d*x]))/
((a - b)*(a + b)*(4*a + 3*b*Sin[c + d*x] - b*Sin[3*(c + d*x)])))/(18*a*d)
```

3.401.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3693}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sin^3(c+dx)\right)^2}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(a+b\sin(c+dx)^3)^2}dx$$
$$\Big\downarrow \text{ 3693}$$
$$\int \frac{1}{\left(a+b\sin^3(c+dx)\right)^2}dx$$

input
```
Int[(a + b*Sin[c + d*x]^3)^(-2),x]
```

output
```
$Aborted
```

3.401.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3693
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Unintegrable[(a + b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, e, f, n, p
}, x]
```

3.401.4 Maple [N/A] (verified)

Time = 2.88 (sec) , antiderivative size = 349, normalized size of antiderivative = 24.93

method	result
derivativedivides	$\frac{\frac{2b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a\left(a^2-b^2\right)}-\frac{2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^2-b^2\right)}+\frac{8b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a\left(a^2-b^2\right)}+\frac{8b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^2-b^2\right)}-\frac{2b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3a\left(a^2-b^2\right)}+\frac{2b}{3a^2-3b^2}}{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+a}+\frac{\_R=\text{RootOf}(}{}$
default	$\frac{\frac{2b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a\left(a^2-b^2\right)}-\frac{2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^2-b^2\right)}+\frac{8b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a\left(a^2-b^2\right)}+\frac{8b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^2-b^2\right)}-\frac{2b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3a\left(a^2-b^2\right)}+\frac{2b}{3a^2-3b^2}}{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+a}+\frac{\_R=\text{RootOf}(}{}$
risch	Expression too large to display

input
```
int(1/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(1/3*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)^5-1/3/(a^2-b^2)*b*tan(1/2*d
*x+1/2*c)^4+4/3*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3+4/3/(a^2-b^2)*b*tan(1
/2*d*x+1/2*c)^2-1/3*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)+1/3/(a^2-b^2)*b)/(a
*tan(1/2*d*x+1/2*c)^6+3*tan(1/2*d*x+1/2*c)^4*a+8*tan(1/2*d*x+1/2*c)^3*b+3*
tan(1/2*d*x+1/2*c)^2*a+a)+1/9/a/(a^2-b^2)*sum(((3*a^2-2*b^2)*_R^4-2*a*b*_R
^3+6*a^2*_R^2-2*a*_R*b+3*a^2-2*b^2)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan
(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

3.401.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 8.47 (sec) , antiderivative size = 70185, normalized size of antiderivative = 5013.21

$$\int \frac{1}{(a+b\sin^3(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.401.6 Sympy [N/A]

Not integrable

Time = 99.86 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+b\sin^3(c+dx))^2}\,dx = \int \frac{1}{(a+b\sin^3{(c+dx)})^2}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c)**3)**2,x)
```

output
```
Integral((a + b*sin(c + d*x)**3)**(-2), x)
```

3.401.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\sin^3(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.401.8 Giac [N/A]

Not integrable

Time = 4.34 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.21

$$\int \frac{1}{(a+b\sin^3(c+dx))^2}\,dx = \int \frac{1}{\left(b\sin\left(dx+c\right)^3+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.401.9 Mupad [B] (verification not implemented)

Time = 17.02 (sec) , antiderivative size = 1567, normalized size of antiderivative = 111.93

$$\int \frac{1}{(a+b\sin^3(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*sin(c + d*x)^3)^2,x)
```

output
```
symsum(log(- (8192*(80*b^6 - 270*a^2*b^4))/(243*(a^7 + a^3*b^4 - 2*a^5*b^2
)) - root(1594323*a^14*b^2*d^6 - 1594323*a^12*b^4*d^6 + 531441*a^10*b^6*d^
6 - 531441*a^16*d^6 - 59049*a^10*b^2*d^4 + 59049*a^8*b^4*d^4 - 177147*a^12
*d^4 + 8019*a^6*b^2*d^2 - 19683*a^8*d^2 + 432*a^2*b^2 - 64*b^4 - 729*a^4,
d, k)*((8192*(144*a*b^7 + 648*a^3*b^5 - 2187*a^5*b^3))/(243*(a^7 + a^3*b^4
 - 2*a^5*b^2)) - root(1594323*a^14*b^2*d^6 - 1594323*a^12*b^4*d^6 + 531441
*a^10*b^6*d^6 - 531441*a^16*d^6 - 59049*a^10*b^2*d^4 + 59049*a^8*b^4*d^4 -
 177147*a^12*d^4 + 8019*a^6*b^2*d^2 - 19683*a^8*d^2 + 432*a^2*b^2 - 64*b^4
 - 729*a^4, d, k)*(root(1594323*a^14*b^2*d^6 - 1594323*a^12*b^4*d^6 + 5314
41*a^10*b^6*d^6 - 531441*a^16*d^6 - 59049*a^10*b^2*d^4 + 59049*a^8*b^4*d^4
 - 177147*a^12*d^4 + 8019*a^6*b^2*d^2 - 19683*a^8*d^2 + 432*a^2*b^2 - 64*b
^4 - 729*a^4, d, k)*((8192*(26973*a^7*b^5 - 20412*a^5*b^7 + 39366*a^9*b^3)
)/(243*(a^7 + a^3*b^4 - 2*a^5*b^2)) - root(1594323*a^14*b^2*d^6 - 1594323*
a^12*b^4*d^6 + 531441*a^10*b^6*d^6 - 531441*a^16*d^6 - 59049*a^10*b^2*d^4
+ 59049*a^8*b^4*d^4 - 177147*a^12*d^4 + 8019*a^6*b^2*d^2 - 19683*a^8*d^2 +
 432*a^2*b^2 - 64*b^4 - 729*a^4, d, k)*(root(1594323*a^14*b^2*d^6 - 159432
3*a^12*b^4*d^6 + 531441*a^10*b^6*d^6 - 531441*a^16*d^6 - 59049*a^10*b^2*d^
4 + 59049*a^8*b^4*d^4 - 177147*a^12*d^4 + 8019*a^6*b^2*d^2 - 19683*a^8*d^2
 + 432*a^2*b^2 - 64*b^4 - 729*a^4, d, k)*((8192*(236196*a^7*b^9 - 649539*a
^9*b^7 + 590490*a^11*b^5 - 177147*a^13*b^3))/(243*(a^7 + a^3*b^4 - 2*a^...
```

3.401. $\int \frac{1}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

3.402 $\int \frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

3.402.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx=\mathrm{Int}\left(\frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2},x\right)$$

output
```
Unintegrable(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x)
```

3.402.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 845, normalized size of antiderivative = 36.74

$$\int \frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \frac{-\frac{ib\,\mathrm{RootSum}\left[-ib+3ib\#1^2+8a\#1^3-3ib\#1^4+ib\#1^6\&,\frac{16a^2b\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+2b^3\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-8ia^2b\log\left(1-2\cos(c\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
(((-I)*b*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^
6 & , (16*a^2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 2*b^3*ArcTan[Si
n[c + d*x]/(Cos[c + d*x] - #1)] - (8*I)*a^2*b*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2] - I*b^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (20*I)*a^3*ArcTan[Sin[c
+ d*x]/(Cos[c + d*x] - #1)]*#1 + (16*I)*a*b^2*ArcTan[Sin[c + d*x]/(Cos[c +
 d*x] - #1)]*#1 + 10*a^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 + 8*a*b^2*Lo
g[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 - 120*a^2*b*ArcTan[Sin[c + d*x]/(Cos[c
+ d*x] - #1)]*#1^2 + 12*b^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2
+ (60*I)*a^2*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (6*I)*b^3*Log[1 -
2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (20*I)*a^3*ArcTan[Sin[c + d*x]/(Cos[c + d
*x] - #1)]*#1^3 - (16*I)*a*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1
^3 - 10*a^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 - 8*a*b^2*Log[1 - 2*Cos
[c + d*x]*#1 + #1^2]*#1^3 + 16*a^2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #
1)]*#1^4 + 2*b^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (8*I)*a^2
*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - I*b^3*Log[1 - 2*Cos[c + d*x]*#
1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ])/(a*(a^2 - b
^2)^2) + (18*Sin[(c + d*x)/2])/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])) + (18*Sin[(c + d*x)/2])/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])) + (12*b*Cos[c + d*x]*(-2*a^3 - 7*a*b^2 + 3*a*b^2*Cos[2*(c + d*x)] +
 2*b*(2*a^2 + b^2)*Sin[c + d*x]))/(a*(a - b)^2*(a + b)^2*(4*a + 3*b*Sin...
```

3.402.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3707}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\cos(c+dx)^2\,(a+b\sin(c+dx)^3)^2}dx$$

$$\Big\downarrow \; 3707$$

$$\int \frac{\sec^2(c+dx)}{(a+b\sin^3(c+dx))^2}dx$$

input
```
Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
$Aborted
```

3.402.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3707
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Cos[e + f*x])^m*(a +
 b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.402.4 Maple [N/A] (verified)

Time = 6.54 (sec) , antiderivative size = 398, normalized size of antiderivative = 17.30

method	result
derivativedivides	$2b\left(\frac{-\frac{\left(2a^2+b^2\right)b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}+\left(-\frac{a^2}{3}+\frac{4b^2}{3}\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4b\left(a^2+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}+\left(-\frac{2a^2}{3}-\frac{10b^2}{3}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+a}\right.$
default	$2b\left(\frac{-\frac{\left(2a^2+b^2\right)b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}+\left(-\frac{a^2}{3}+\frac{4b^2}{3}\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4b\left(a^2+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}+\left(-\frac{2a^2}{3}-\frac{10b^2}{3}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+a}\right.$
risch	Expression too large to display

input
```
int(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b/(a-b)^2/(a+b)^2*((-1/3*(2*a^2+b^2)*b/a*tan(1/2*d*x+1/2*c)^5+(-1/3
*a^2+4/3*b^2)*tan(1/2*d*x+1/2*c)^4-4/3*b*(a^2+2*b^2)/a*tan(1/2*d*x+1/2*c)^
3+(-2/3*a^2-10/3*b^2)*tan(1/2*d*x+1/2*c)^2+1/3*(2*a^2+b^2)*b/a*tan(1/2*d*x
+1/2*c)-1/3*a^2-2/3*b^2)/(a*tan(1/2*d*x+1/2*c)^6+3*tan(1/2*d*x+1/2*c)^4*a+
8*tan(1/2*d*x+1/2*c)^3*b+3*tan(1/2*d*x+1/2*c)^2*a+a)+1/18/a*sum((b*(-11*a^
2+2*b^2)*_R^4+2*a*(5*a^2+4*b^2)*_R^3-54*a^2*b*_R^2+2*a*(5*a^2+4*b^2)*_R-11
*a^2*b+2*b^3)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R
=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-1/(a+b)^2/(tan(1/2*d*x+1/2*
c)-1)-1/(a-b)^2/(tan(1/2*d*x+1/2*c)+1))
```

3.402.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 43.88 (sec) , antiderivative size = 102913, normalized size of antiderivative = 4474.48

$$\int \frac{\sec^2(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.402.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**2/(a+b*sin(d*x+c)**3)**2,x)
```

output
```
Timed out
```

3.402.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.402.8 Giac [N/A]

Not integrable

Time = 4.34 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{\sec^2(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \int \frac{\sec(dx+c)^2}{\left(b\sin(dx+c)^3+a\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.402.9 Mupad [B] (verification not implemented)

Time = 19.15 (sec) , antiderivative size = 3148, normalized size of antiderivative = 136.87

$$\int \frac{\sec^2(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x)^3)^2),x)
```

output

```
symsum(log(5479612416*a^8*b^36 - 180486144*a^6*b^38 - root(5314410*a^16*b^
4*d^6 - 5314410*a^14*b^6*d^6 - 2657205*a^18*b^2*d^6 + 2657205*a^12*b^8*d^6
 - 531441*a^10*b^10*d^6 + 531441*a^20*d^6 + 11514555*a^12*b^4*d^4 + 206671
5*a^14*b^2*d^4 + 1062882*a^10*b^6*d^4 - 295245*a^8*b^8*d^4 + 984150*a^8*b^
4*d^2 - 98415*a^6*b^6*d^2 + 15625*a^4*b^4 - 2000*a^2*b^6 + 64*b^8, d, k)*(
tan(c/2 + (d*x)/2)*(764411904*a^6*b^40 - 27805483008*a^8*b^38 + 4372973568
00*a^10*b^36 - 3672461721600*a^12*b^34 + 19250011791360*a^14*b^32 - 691506
35753472*a^16*b^30 + 180165872001024*a^18*b^28 - 352655758540800*a^20*b^26
 + 529923028377600*a^22*b^24 - 618699706859520*a^24*b^22 + 563713761042432
*a^26*b^20 - 399760062234624*a^28*b^18 + 218398602240000*a^30*b^16 - 90108
039168000*a^32*b^14 + 27130620764160*a^34*b^12 - 5617221156864*a^36*b^10 +
 713536708608*a^38*b^8 - 41803776000*a^40*b^6) - root(5314410*a^16*b^4*d^6
 - 5314410*a^14*b^6*d^6 - 2657205*a^18*b^2*d^6 + 2657205*a^12*b^8*d^6 - 53
1441*a^10*b^10*d^6 + 531441*a^20*d^6 + 11514555*a^12*b^4*d^4 + 2066715*a^1
4*b^2*d^4 + 1062882*a^10*b^6*d^4 - 295245*a^8*b^8*d^4 + 984150*a^8*b^4*d^2
 - 98415*a^6*b^6*d^2 + 15625*a^4*b^4 - 2000*a^2*b^6 + 64*b^8, d, k)*(root(
5314410*a^16*b^4*d^6 - 5314410*a^14*b^6*d^6 - 2657205*a^18*b^2*d^6 + 26572
05*a^12*b^8*d^6 - 531441*a^10*b^10*d^6 + 531441*a^20*d^6 + 11514555*a^12*b
^4*d^4 + 2066715*a^14*b^2*d^4 + 1062882*a^10*b^6*d^4 - 295245*a^8*b^8*d^4
+ 984150*a^8*b^4*d^2 - 98415*a^6*b^6*d^2 + 15625*a^4*b^4 - 2000*a^2*b^6...
```

3.402. $\int \frac{\sec^2(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

3.403 $\int \frac{\sec^4(c+dx)}{(a+b\sin^3(c+dx))^2} dx$

3.403.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sec^4(c+dx)}{(a+b\sin^3(c+dx))^2} dx = \text{Int}\left(\frac{\sec^4(c+dx)}{(a+b\sin^3(c+dx))^2}, x\right)$$

output
```
Unintegrable(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x)
```

3.403.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.25 (sec) , antiderivative size = 1158, normalized size of antiderivative = 50.35

$$\int \frac{\sec^4(c+dx)}{(a+b\sin^3(c+dx))^2} dx = \text{Too large to display}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
((4*I)*b^2*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#
1^6 & , (14*a^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 74*a^2*b^2*ArcT
an[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 2*b^4*ArcTan[Sin[c + d*x]/(Cos[c +
d*x] - #1)] - (7*I)*a^4*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - (37*I)*a^2*b^2
*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - I*b^4*Log[1 - 2*Cos[c + d*x]*#1 + #1^
2] + (144*I)*a^3*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + (36*I)*a*
b^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + 72*a^3*b*Log[1 - 2*Cos[c
 + d*x]*#1 + #1^2]*#1 + 18*a*b^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 - 18
0*a^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 372*a^2*b^2*ArcTan[S
in[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 12*b^4*ArcTan[Sin[c + d*x]/(Cos[c
+ d*x] - #1)]*#1^2 + (90*I)*a^4*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (
186*I)*a^2*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (6*I)*b^4*Log[1 -
2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (144*I)*a^3*b*ArcTan[Sin[c + d*x]/(Cos[c
+ d*x] - #1)]*#1^3 - (36*I)*a*b^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^3 - 72*a^3*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 - 18*a*b^3*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^3 + 14*a^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
 - #1)]*#1^4 + 74*a^2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 +
2*b^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (7*I)*a^4*Log[1 - 2*
Cos[c + d*x]*#1 + #1^2]*#1^4 - (37*I)*a^2*b^2*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2]*#1^4 - I*b^4*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I...
```

3.403.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3707}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\cos(c+dx)^4\left(a+b\sin(c+dx)^3\right)^2}dx$$
$$\Big\downarrow \; 3707$$
$$\int \frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}dx$$

3.403. $\int \frac{\sec^4(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

input
```
Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2,x]
```

output
```
$Aborted
```

3.403.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3707
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Cos[e + f*x])^m*(a +
 b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.403.4 Maple [N/A] (verified)

Time = 10.45 (sec) , antiderivative size = 525, normalized size of antiderivative = 22.83

method	result
derivativedivides	$2b^2\left(\frac{\frac{(a^4+7a^2b^2+b^4)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}-3b^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4b^2(2a^2+b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\left(6a^2b+6b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{(a^4+}{}}{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+a}\right.$
default	$2b^2\left(\frac{\frac{(a^4+7a^2b^2+b^4)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a}-3b^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4b^2(2a^2+b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\left(6a^2b+6b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{(a^4+}{}}{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+a}\right.$
risch	Expression too large to display

input
```
int(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b^2/(a-b)^3/(a+b)^3*((1/3*(a^4+7*a^2*b^2+b^4)/a*tan(1/2*d*x+1/2*c)^
5-3*b^3*tan(1/2*d*x+1/2*c)^4+4*b^2*(2*a^2+b^2)/a*tan(1/2*d*x+1/2*c)^3+(6*a
^2*b+6*b^3)*tan(1/2*d*x+1/2*c)^2-1/3*(a^4+7*a^2*b^2+b^4)/a*tan(1/2*d*x+1/2
*c)+2*a^2*b+b^3)/(a*tan(1/2*d*x+1/2*c)^6+3*tan(1/2*d*x+1/2*c)^4*a+8*tan(1/
2*d*x+1/2*c)^3*b+3*tan(1/2*d*x+1/2*c)^2*a+a)+1/18/a*sum(((19*a^4+28*a^2*b^
2-2*b^4)*_R^4+18*a*b*(-4*a^2-b^2)*_R^3+6*a^2*(11*a^2+34*b^2)*_R^2+18*a*b*(
-4*a^2-b^2)*_R+19*a^4+28*a^2*b^2-2*b^4)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln
(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a)))-1
/3/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^3+1/2/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^2-(
a-4*b)/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)-1/3/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^3
-1/2/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^2-(a+4*b)/(a+b)^3/(tan(1/2*d*x+1/2*c)-
1))
```

3.403.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 140.12 (sec) , antiderivative size = 133123, normalized size of antiderivative = 5787.96

$$\int \frac{\sec^4(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.403.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**4/(a+b*sin(d*x+c)**3)**2,x)
```

output
```
Timed out
```

3.403.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.403.8 Giac [N/A]

Not integrable

Time = 4.53 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{\sec^4(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \int \frac{\sec{(dx+c)}^4}{\left(b\sin{(dx+c)}^3+a\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.403.9 Mupad [B] (verification not implemented)

Time = 23.83 (sec) , antiderivative size = 4657, normalized size of antiderivative = 202.48

$$\int \frac{\sec^4(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x)^3)^2),x)
```

output
```
symsum(log(26838024192*a^8*b^54 - tan(c/2 + (d*x)/2)*(7962624000*a^7*b^55
- 508612608000*a^9*b^53 + 8841498624000*a^11*b^51 - 82283765760000*a^13*b^
49 + 501714984960000*a^15*b^47 - 2205295497216000*a^17*b^45 + 737918163763
2000*a^19*b^43 - 19451488075776000*a^21*b^41 + 41318016122880000*a^23*b^39
 - 71811432161280000*a^25*b^37 + 103155513237504000*a^27*b^35 - 1232249069
07648000*a^29*b^33 + 122756816093184000*a^31*b^31 - 101967282708480000*a^3
3*b^29 + 70396872007680000*a^35*b^27 - 40129785593856000*a^37*b^25 + 18687
625592832000*a^39*b^23 - 6994754113536000*a^41*b^21 + 2053854351360000*a^4
3*b^19 - 455730831360000*a^45*b^17 + 71860690944000*a^47*b^15 - 7177310208
000*a^49*b^13 + 341397504000*a^51*b^11) - 392822784*a^6*b^56 - root(186004
35*a^18*b^6*d^6 - 18600435*a^16*b^8*d^6 - 11160261*a^20*b^4*d^6 + 11160261
*a^14*b^10*d^6 + 3720087*a^22*b^2*d^6 - 3720087*a^12*b^12*d^6 + 531441*a^1
0*b^14*d^6 - 531441*a^24*d^6 - 173879622*a^14*b^6*d^4 - 155830311*a^12*b^8
*d^4 - 23225940*a^16*b^4*d^4 - 6475707*a^10*b^10*d^4 + 688905*a^8*b^12*d^4
 - 11565585*a^8*b^8*d^2 + 3750705*a^10*b^6*d^2 + 433755*a^6*b^10*d^2 - 117
649*a^4*b^8 + 5488*a^2*b^10 - 64*b^12, d, k)*(tan(c/2 + (d*x)/2)*(76441190
4*a^6*b^58 - 61439606784*a^8*b^56 + 2110475575296*a^10*b^54 - 33643637121024*a^12*b^52 + 319697763065856*a^14*b^50 - 2067381036048384*a^16*b^48 + 98
10082122817536*a^18*b^46 - 35797302942326784*a^20*b^44 + 10361376601303449
6*a^22*b^42 - 243004699498881024*a^24*b^40 + 468678655511248896*a^26*b^...
```

3.403. $\int \frac{\sec^4(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

3.404 $\int \frac{\cos^7(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.404.1 Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{\cos^7(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\left(\sqrt{a}+\sqrt{b}\right)^3\arctan\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d} - \frac{\left(\sqrt{a}-\sqrt{b}\right)^3\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d} - \frac{3\sin(c+dx)}{bd} + \frac{\sin^3(c+dx)}{3bd}$$

output
```
-3*sin(d*x+c)/b/d+1/3*sin(d*x+c)^3/b/d-1/2*arctanh(b^(1/4)*sin(d*x+c)/a^(1
/4))*(a^(1/2)-b^(1/2))^3/a^(3/4)/b^(7/4)/d+1/2*arctan(b^(1/4)*sin(d*x+c)/a
^(1/4))*(a^(1/2)+b^(1/2))^3/a^(3/4)/b^(7/4)/d
```

3.404.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.58

$$\int \frac{\cos^7(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{3\left(\sqrt{a}-\sqrt{b}\right)^3\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)+3i\left(\sqrt{a}+\sqrt{b}\right)^3\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c+dx)\right)-3i\left(\sqrt{a}+\sqrt{b}\right)}{}$$

input
```
Integrate[Cos[c + d*x]^7/(a - b*Sin[c + d*x]^4),x]
```

output
```
(3*(Sqrt[a] - Sqrt[b])^3*Log[a^(1/4) - b^(1/4)*Sin[c + d*x]] + (3*I)*(Sqrt
[a] + Sqrt[b])^3*Log[a^(1/4) - I*b^(1/4)*Sin[c + d*x]] - (3*I)*(Sqrt[a] +
Sqrt[b])^3*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]] - 3*(Sqrt[a] - Sqrt[b])^3
*Log[a^(1/4) + b^(1/4)*Sin[c + d*x]] - 36*a^(3/4)*b^(3/4)*Sin[c + d*x] + 4
*a^(3/4)*b^(3/4)*Sin[c + d*x]^3)/(12*a^(3/4)*b^(7/4)*d)
```

3.404.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3702, 1485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^7(c+dx)}{a-b\sin^4(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(c+dx)^7}{a-b\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{ 3702}$$

$$\frac{\int \frac{(1-\sin^2(c+dx))^3}{a-b\sin^4(c+dx)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 1485}$$

$$\frac{\int \left(\frac{\sin^2(c+dx)}{b}-\frac{3}{b}+\frac{-((a+3b)\sin^2(c+dx))+3a+b}{b(a-b\sin^4(c+dx))}\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\left(\sqrt{a}+\sqrt{b}\right)^3\arctan\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^3\text{arctanh}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}}+\frac{\sin^3(c+dx)}{3b}-\frac{3\sin(c+dx)}{b}}{d}$$

input
```
Int[Cos[c + d*x]^7/(a - b*Sin[c + d*x]^4),x]
```

output
```
(((Sqrt[a] + Sqrt[b])^3*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)
*b^(7/4)) - ((Sqrt[a] - Sqrt[b])^3*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)]
)/(2*a^(3/4)*b^(7/4)) - (3*Sin[c + d*x])/b + Sin[c + d*x]^3/(3*b))/d
```

3.404.3.1 Defintions of rubi rules used

rule 1485
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.404.4 Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.36

method	result
derivativedivides	$-\frac{-\frac{\frac{\left(\sin^{3}(dx+c)\right)}{3}-3\sin(dx+c)}{b}+\frac{\frac{(-3a-b)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}-\frac{(a+3b)\left(2\arctan\left(\frac{\sin(dx+}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{4}}{b}}{d}$
default	$-\frac{-\frac{\frac{\left(\sin^{3}(dx+c)\right)}{3}-3\sin(dx+c)}{b}+\frac{\frac{(-3a-b)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}-\frac{(a+3b)\left(2\arctan\left(\frac{\sin(dx+}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{4}}{b}}{d}$
risch	$\frac{11i\,e^{i(dx+c)}}{8bd}-\frac{11i\,e^{-i(dx+c)}}{8bd}+\left(\sum_{\_R=\text{RootOf}\left(256a^{3}b^{7}d^{4}\_Z^{4}+\left(192a^{4}b^{4}d^{2}+640a^{3}b^{5}d^{2}+192a^{2}b^{6}d^{2}\right)\_Z^{2}-a^{6}+6a^{5}b\right.}\right.$

input
```
int(cos(d*x+c)^7/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-1/b*(1/3*sin(d*x+c)^3-3*sin(d*x+c))+1/b*(1/4*(-3*a-b)*(1/b*a)^(1/4)
/a*(ln((sin(d*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b*a)^(1/4)))+2*arctan(sin
(d*x+c)/(1/b*a)^(1/4)))-1/4*(a+3*b)/b/(1/b*a)^(1/4)*(2*arctan(sin(d*x+c)/(
1/b*a)^(1/4))-ln((sin(d*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b*a)^(1/4))))))
```

3.404.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1429 vs. $2(101) = 202$.

Time = 0.60 (sec) , antiderivative size = 1429, normalized size of antiderivative = 10.91

$$\int \frac{\cos^7(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/12*(3*b*d*sqrt(-(a*b^3*d^2*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*
b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) + 6*a^2 + 20*a*b + 6*b^
2)/(a*b^3*d^2))*log(1/2*(a^6 + 12*a^5*b - 27*a^4*b^2 + 27*a^2*b^4 - 12*a*b
^5 - b^6)*sin(d*x + c) + 1/2*((a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((a^6 + 30*a^5
*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^
4)) - (3*a^5*b^2 + 46*a^4*b^3 + 60*a^3*b^4 + 18*a^2*b^5 + a*b^6)*d)*sqrt(-
(a*b^3*d^2*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4
+ 30*a*b^5 + b^6)/(a^3*b^7*d^4)) + 6*a^2 + 20*a*b + 6*b^2)/(a*b^3*d^2))) -
 3*b*d*sqrt((a*b^3*d^2*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 +
255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) - 6*a^2 - 20*a*b - 6*b^2)/(a*
b^3*d^2))*log(1/2*(a^6 + 12*a^5*b - 27*a^4*b^2 + 27*a^2*b^4 - 12*a*b^5 - b
^6)*sin(d*x + c) + 1/2*((a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((a^6 + 30*a^5*b + 2
55*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) +
(3*a^5*b^2 + 46*a^4*b^3 + 60*a^3*b^4 + 18*a^2*b^5 + a*b^6)*d)*sqrt((a*b^3*
d^2*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*
b^5 + b^6)/(a^3*b^7*d^4)) - 6*a^2 - 20*a*b - 6*b^2)/(a*b^3*d^2))) - 3*b*d*
sqrt(-(a*b^3*d^2*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^
2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) + 6*a^2 + 20*a*b + 6*b^2)/(a*b^3*d^
2))*log(-1/2*(a^6 + 12*a^5*b - 27*a^4*b^2 + 27*a^2*b^4 - 12*a*b^5 - b^6)*s
in(d*x + c) + 1/2*((a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((a^6 + 30*a^5*b + 255...
```

3.404.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^7(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**7/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.404.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.35

$$\int \frac{\cos^7(c+dx)}{a-b\sin^4(c+dx)}\,dx$$
$$= \frac{\frac{4\left(\sin(dx+c)^3-9\,\sin(dx+c)\right)}{b}+\frac{3\left(\frac{2\left(b\left(3\,\sqrt{a}+\sqrt{b}\right)+a^{\frac{3}{2}}+3\,a\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\sin(dx+c)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(b\left(3\,\sqrt{a}-\sqrt{b}\right)+a^{\frac{3}{2}}-3\,a\sqrt{b}\right)\log\left(\frac{\sqrt{b}\sin(dx+c)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin(dx+c)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\right)}{b}}{12\,d}$$

input
```
integrate(cos(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/12*(4*(sin(d*x + c)^3 - 9*sin(d*x + c))/b + 3*(2*(b*(3*sqrt(a) + sqrt(b)
) + a^(3/2) + 3*a*sqrt(b))*arctan(sqrt(b)*sin(d*x + c)/sqrt(sqrt(a)*sqrt(b
)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (b*(3*sqrt(a) - sqrt(b)) + a
^(3/2) - 3*a*sqrt(b))*log((sqrt(b)*sin(d*x + c) - sqrt(sqrt(a)*sqrt(b)))/(
sqrt(b)*sin(d*x + c) + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
b))*sqrt(b)))/b)/d
```

3.404.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(101) = 202$.

Time = 0.72 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.75

$$\int \frac{\cos^7(c+dx)}{a-b\sin^4(c+dx)}\,dx$$
$$= \frac{\frac{8\left(b^2\sin(dx+c)^3-9\,b^2\sin(dx+c)\right)}{b^3}-\frac{6\sqrt{2}\left(\left(-ab^3\right)^{\frac{3}{4}}(a+3\,b)-\left(-ab^3\right)^{\frac{1}{4}}\left(3\,ab^2+b^3\right)\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^4}-\frac{6\sqrt{2}\left(\left(-ab^3\right)\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
1/24*(8*(b^2*sin(d*x + c)^3 - 9*b^2*sin(d*x + c))/b^3 - 6*sqrt(2)*((-a*b^3
)^(3/4)*(a + 3*b) - (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*arctan(1/2*sqrt(2)*(sq
rt(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b^4) - 6*sqrt(2)*((-
a*b^3)^(3/4)*(a + 3*b) - (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*arctan(-1/2*sqrt(
2)*(sqrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b^4) + 3*sqrt(
2)*((-a*b^3)^(3/4)*(a + 3*b) + (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*log(sin(d*x
 + c)^2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^4) - 3*sqrt
(2)*((-a*b^3)^(3/4)*(a + 3*b) + (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*log(sin(d*
x + c)^2 - sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^4))/d
```

3.404.9 Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 1931, normalized size of antiderivative = 14.74

$$\int \frac{\cos^7(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^7/(a - b*sin(c + d*x)^4),x)
```

output
```
(atan((a^3*sin(c + d*x)*(- (a^3*b^7)^(1/2)/(16*b^7) - (3*a)/(8*b^3) - 5/(4
*b^2) - 3/(8*a*b) - (15*(a^3*b^7)^(1/2))/(16*a*b^6) - (15*(a^3*b^7)^(1/2))
/(16*a^2*b^5) - (a^3*b^7)^(1/2)/(16*a^3*b^4))^(1/2)*8i)/(92*a*b + (120*(a^
3*b^7)^(1/2))/b^3 + 120*a^2 + 6*b^2 + (36*a^3)/b + (2*a^4)/b^2 + (36*(a^3*
b^7)^(1/2))/(a*b^2) + (2*(a^3*b^7)^(1/2))/(a^2*b) + (6*a^2*(a^3*b^7)^(1/2)
)/b^5 + (92*a*(a^3*b^7)^(1/2))/b^4) + (b^3*sin(c + d*x)*(- (a^3*b^7)^(1/2)
/(16*b^7) - (3*a)/(8*b^3) - 5/(4*b^2) - 3/(8*a*b) - (15*(a^3*b^7)^(1/2))/(
16*a*b^6) - (15*(a^3*b^7)^(1/2))/(16*a^2*b^5) - (a^3*b^7)^(1/2)/(16*a^3*b^
4))^(1/2)*8i)/(92*a*b + (120*(a^3*b^7)^(1/2))/b^3 + 120*a^2 + 6*b^2 + (36*
a^3)/b + (2*a^4)/b^2 + (36*(a^3*b^7)^(1/2))/(a*b^2) + (2*(a^3*b^7)^(1/2))/
(a^2*b) + (6*a^2*(a^3*b^7)^(1/2))/b^5 + (92*a*(a^3*b^7)^(1/2))/b^4) + (a*b
^2*sin(c + d*x)*(- (a^3*b^7)^(1/2)/(16*b^7) - (3*a)/(8*b^3) - 5/(4*b^2) -
3/(8*a*b) - (15*(a^3*b^7)^(1/2))/(16*a*b^6) - (15*(a^3*b^7)^(1/2))/(16*a^2
*b^5) - (a^3*b^7)^(1/2)/(16*a^3*b^4))^(1/2)*120i)/(92*a*b + (120*(a^3*b^7)
^(1/2))/b^3 + 120*a^2 + 6*b^2 + (36*a^3)/b + (2*a^4)/b^2 + (36*(a^3*b^7)^(
1/2))/(a*b^2) + (2*(a^3*b^7)^(1/2))/(a^2*b) + (6*a^2*(a^3*b^7)^(1/2))/b^5
+ (92*a*(a^3*b^7)^(1/2))/b^4) + (a^2*b*sin(c + d*x)*(- (a^3*b^7)^(1/2)/(16
*b^7) - (3*a)/(8*b^3) - 5/(4*b^2) - 3/(8*a*b) - (15*(a^3*b^7)^(1/2))/(16*a
*b^6) - (15*(a^3*b^7)^(1/2))/(16*a^2*b^5) - (a^3*b^7)^(1/2)/(16*a^3*b^4))^
(1/2)*120i)/(92*a*b + (120*(a^3*b^7)^(1/2))/b^3 + 120*a^2 + 6*b^2 + (36...
```

3.405 $\int \frac{\cos^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.405.1 Optimal result

Integrand size = 24, antiderivative size = 113

$$\int \frac{\cos^5(c+dx)}{a-b\sin^4(c+dx)}\,dx=\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\arctan\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}+\frac{\left(a-2\sqrt{a}\sqrt{b}+b\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}-\frac{\sin(c+dx)}{bd}$$

output
```
-sin(d*x+c)/b/d+1/2*arctan(b^(1/4)*sin(d*x+c)/a^(1/4))*(a^(1/2)+b^(1/2))^2
/a^(3/4)/b^(5/4)/d+1/2*arctanh(b^(1/4)*sin(d*x+c)/a^(1/4))*(a+b-2*a^(1/2)*
b^(1/2))/a^(3/4)/b^(5/4)/d
```

3.405.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.67

$$\int \frac{\cos^5(c+dx)}{a-b\sin^4(c+dx)}\,dx=\frac{-\left(\sqrt{a}-\sqrt{b}\right)^2\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)+i\left(\left(\sqrt{a}+\sqrt{b}\right)^2\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c+dx)\right)-\left(\sqrt{a}+\sqrt{b}\right)^2\right.}{4a^{3/4}b^{5/4}}$$

input
```
Integrate[Cos[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

output
```
(-((Sqrt[a] - Sqrt[b])^2*Log[a^(1/4) - b^(1/4)*Sin[c + d*x]]) + I*((Sqrt[a
] + Sqrt[b])^2*Log[a^(1/4) - I*b^(1/4)*Sin[c + d*x]] - (Sqrt[a] + Sqrt[b])
^2*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]] - I*(Sqrt[a] - Sqrt[b])^2*Log[a^(
1/4) + b^(1/4)*Sin[c + d*x]]) - 4*a^(3/4)*b^(1/4)*Sin[c + d*x])/(4*a^(3/4)
*b^(5/4)*d)
```

3.405.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3702, 1485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos(c+dx)^5}{a-b\sin(c+dx)^4}dx$$

$$\Big\downarrow \ 3702$$

$$\frac{\int \frac{\left(1-\sin^2(c+dx)\right)^2}{a-b\sin^4(c+dx)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \ 1485$$

$$\frac{\int \left(\frac{-2b\sin^2(c+dx)+a+b}{b\left(a-b\sin^4(c+dx)\right)}-\frac{1}{b}\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\arctan\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}+\frac{\left(-2\sqrt{a}\sqrt{b}+a+b\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}}-\frac{\sin(c+dx)}{b}}{d}$$

input
```
Int[Cos[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

output
```
(((Sqrt[a] + Sqrt[b])^2*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)
*b^(5/4)) + ((a - 2*Sqrt[a]*Sqrt[b] + b)*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^
(1/4)])/(2*a^(3/4)*b^(5/4)) - Sin[c + d*x]/b)/d
```

3.405.3.1 Defintions of rubi rules used

rule 1485
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.405.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{-\frac{\sin(dx+c)}{b}+\frac{\frac{(a+b)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{b}}{d}$
default	$\frac{-\frac{\sin(dx+c)}{b}+\frac{\frac{(a+b)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{b}}{d}$
risch	$\frac{ie^{i(dx+c)}}{2bd}-\frac{ie^{-i(dx+c)}}{2bd}+\left(\sum_{\_R=\mathrm{RootOf}\left(256a^3b^5d^4\_Z^4+(128a^3b^3d^2+128a^2b^4d^2)\_Z^2-a^4+4a^3b-6a^2b^2+4ab^3-b\right.}\right.$

input
```
int(cos(d*x+c)^5/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-sin(d*x+c)/b+1/b*(1/4*(a+b)*(1/b*a)^(1/4)/a*(ln((sin(d*x+c)+(1/b*a)^
(1/4))/(sin(d*x+c)-(1/b*a)^(1/4)))+2*arctan(sin(d*x+c)/(1/b*a)^(1/4)))+1/2
/(1/b*a)^(1/4)*(2*arctan(sin(d*x+c)/(1/b*a)^(1/4))-ln((sin(d*x+c)+(1/b*a)^
(1/4))/(sin(d*x+c)-(1/b*a)^(1/4)))))
```

3.405.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. $2(85) = 170$.

Time = 0.44 (sec) , antiderivative size = 1041, normalized size of antiderivative = 9.21

$$\int \frac{\cos^5(c+dx)}{a-b\sin^4(c+dx)}\,dx =$$

$$-\frac{bd\sqrt{-\frac{ab^2d^2\sqrt{\frac{a^4+12a^3b+38a^2b^2+12ab^3+b^4}{a^3b^5d^4}}+4a+4b}{ab^2d^2}}\log\left(\frac{1}{2}\left(a^4+4a^3b-10a^2b^2+4ab^3+b^4\right)\sin(dx+c)+\frac{1}{2}\left(2a\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/4*(b*d*sqrt(-(a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 +
b^4)/(a^3*b^5*d^4)) + 4*a + 4*b)/(a*b^2*d^2))*log(1/2*(a^4 + 4*a^3*b - 10*
a^2*b^2 + 4*a*b^3 + b^4)*sin(d*x + c) + 1/2*(2*a^3*b^4*d^3*sqrt((a^4 + 12*
a^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) - (a^4*b + 7*a^3*b^2 +
 7*a^2*b^3 + a*b^4)*d)*sqrt(-(a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2
+ 12*a*b^3 + b^4)/(a^3*b^5*d^4)) + 4*a + 4*b)/(a*b^2*d^2))) - b*d*sqrt((a*
b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4))
 - 4*a - 4*b)/(a*b^2*d^2))*log(1/2*(a^4 + 4*a^3*b - 10*a^2*b^2 + 4*a*b^3 +
 b^4)*sin(d*x + c) + 1/2*(2*a^3*b^4*d^3*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2
+ 12*a*b^3 + b^4)/(a^3*b^5*d^4)) + (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)
*d)*sqrt((a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a
^3*b^5*d^4)) - 4*a - 4*b)/(a*b^2*d^2))) - b*d*sqrt(-(a*b^2*d^2*sqrt((a^4 +
 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) + 4*a + 4*b)/(a*b^
2*d^2))*log(-1/2*(a^4 + 4*a^3*b - 10*a^2*b^2 + 4*a*b^3 + b^4)*sin(d*x + c)
 + 1/2*(2*a^3*b^4*d^3*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/
(a^3*b^5*d^4)) - (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt(-(a*b^2*d
^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) + 4*
a + 4*b)/(a*b^2*d^2))) + b*d*sqrt((a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2
*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) - 4*a - 4*b)/(a*b^2*d^2))*log(-1/2*(
a^4 + 4*a^3*b - 10*a^2*b^2 + 4*a*b^3 + b^4)*sin(d*x + c) + 1/2*(2*a^3*b...
```

3.405.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.405.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.40

$$\int \frac{\cos^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\frac{\frac{2\left(b\left(2\sqrt{a}+\sqrt{b}\right)+a\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\sin(dx+c)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(b\left(2\sqrt{a}-\sqrt{b}\right)-a\sqrt{b}\right)\log\left(\frac{\sqrt{b}\sin(dx+c)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin(dx+c)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{b}-\frac{4\sin(dx+c)}{b}}{4d}$$

input
```
integrate(cos(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/4*((2*(b*(2*sqrt(a) + sqrt(b)) + a*sqrt(b))*arctan(sqrt(b)*sin(d*x + c)/
sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (b*(2*sqr
t(a) - sqrt(b)) - a*sqrt(b))*log((sqrt(b)*sin(d*x + c) - sqrt(sqrt(a)*sqrt
(b)))/(sqrt(b)*sin(d*x + c) + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a
)*sqrt(b))*sqrt(b)))/b - 4*sin(d*x + c)/b)/d
```

3.405.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(85) = 170.

Time = 0.64 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.75

$$\int \frac{\cos^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{8\sin(dx+c)}{b}-\frac{2\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}\left(ab+b^2\right)-2\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^3}-\frac{2\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}\left(ab+b^2\right)-2\left(-ab^3\right)^{\frac{3}{4}}\right)\text{arc}}{ab^3}$$

input
```
integrate(cos(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
-1/8*(8*sin(d*x + c)/b - 2*sqrt(2)*((-a*b^3)^(1/4)*(a*b + b^2) - 2*(-a*b^3
)^(3/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)
^(1/4))/(a*b^3) - 2*sqrt(2)*((-a*b^3)^(1/4)*(a*b + b^2) - 2*(-a*b^3)^(3/4)
)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4)
)/(a*b^3) - sqrt(2)*((-a*b^3)^(1/4)*(a*b + b^2) + 2*(-a*b^3)^(3/4))*log(si
n(d*x + c)^2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^3) + s
qrt(2)*((-a*b^3)^(1/4)*(a*b + b^2) + 2*(-a*b^3)^(3/4))*log(sin(d*x + c)^2
- sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^3))/d
```

3.405.9 Mupad [B] (verification not implemented)

Time = 14.40 (sec) , antiderivative size = 1097, normalized size of antiderivative = 9.71

$$\int \frac{\cos^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$= \frac{2\,\mathrm{atanh}\left(\frac{8\,b^3\,\sin(c+d\,x)\,\sqrt{\frac{\sqrt{a^3\,b^5}}{16\,a\,b^5}-\frac{1}{4\,a\,b}-\frac{1}{4\,b^2}+\frac{3\,\sqrt{a^3\,b^5}}{8\,a^2\,b^4}+\frac{\sqrt{a^3\,b^5}}{16\,a^3\,b^3}}}{\frac{2\,\sqrt{a^3\,b^5}}{a^2}-24\,a\,b+\frac{14\,\sqrt{a^3\,b^5}}{b^2}-4\,a^2-4\,b^2+\frac{14\,\sqrt{a^3\,b^5}}{a\,b}+\frac{2\,a\,\sqrt{a^3\,b^5}}{b^3}}+\frac{48\,a\,b^2\,\sin(c+d\,x)\,\sqrt{\frac{\sqrt{a^3\,b^5}}{16\,a\,b^5}-\frac{1}{4\,a\,b}-\frac{1}{4\,b^2}+\frac{3\,\sqrt{a^3\,b^5}}{8\,a^2\,b^4}+\frac{\sqrt{a^3\,b^5}}{16\,a^3\,b^3}}}{\frac{2\,\sqrt{a^3\,b^5}}{a^2}-24\,a\,b+\frac{14\,\sqrt{a^3\,b^5}}{b^2}-4\,a^2-4\,b^2+\frac{14\,\sqrt{a^3\,b^5}}{a\,b}+\frac{2\,a\,\sqrt{a^3\,b^5}}{b^3}}+\cdots\right.}{d}$$

$$-\frac{2\,\mathrm{atanh}\left(\frac{8\,b^3\,\sin(c+d\,x)\,\sqrt{-\frac{1}{4\,b^2}-\frac{1}{4\,a\,b}-\frac{\sqrt{a^3\,b^5}}{16\,a\,b^5}-\frac{3\,\sqrt{a^3\,b^5}}{8\,a^2\,b^4}-\frac{\sqrt{a^3\,b^5}}{16\,a^3\,b^3}}}{24\,a\,b+\frac{2\,\sqrt{a^3\,b^5}}{a^2}+\frac{14\,\sqrt{a^3\,b^5}}{b^2}+4\,a^2+4\,b^2+\frac{14\,\sqrt{a^3\,b^5}}{a\,b}+\frac{2\,a\,\sqrt{a^3\,b^5}}{b^3}}+\frac{48\,a\,b^2\,\sin(c+d\,x)\,\sqrt{-\frac{1}{4\,b^2}-\frac{1}{4\,a\,b}-\frac{\sqrt{a^3\,b^5}}{16\,a\,b^5}-\frac{3\,\sqrt{a^3\,b^5}}{8\,a^2\,b^4}-\frac{\sqrt{a^3\,b^5}}{16\,a^3\,b^3}}}{24\,a\,b+\frac{2\,\sqrt{a^3\,b^5}}{a^2}+\frac{14\,\sqrt{a^3\,b^5}}{b^2}+4\,a^2+4\,b^2+\frac{14\,\sqrt{a^3\,b^5}}{a\,b}+\frac{2\,a\,\sqrt{a^3\,b^5}}{b^3}}\cdots\right.}{d}$$

$$-\frac{\sin\left(c+d\,x\right)}{b\,d}$$

input
```
int(cos(c + d*x)^5/(a - b*sin(c + d*x)^4),x)
```

output
```
(2*atanh((8*b^3*sin(c + d*x)*((a^3*b^5)^(1/2)/(16*a*b^5) - 1/(4*a*b) - 1/(
4*b^2) + (3*(a^3*b^5)^(1/2))/(8*a^2*b^4) + (a^3*b^5)^(1/2)/(16*a^3*b^3))^(
1/2))/((2*(a^3*b^5)^(1/2))/a^2 - 24*a*b + (14*(a^3*b^5)^(1/2))/b^2 - 4*a^2
 - 4*b^2 + (14*(a^3*b^5)^(1/2))/(a*b) + (2*a*(a^3*b^5)^(1/2))/b^3) + (48*a
*b^2*sin(c + d*x)*((a^3*b^5)^(1/2)/(16*a*b^5) - 1/(4*a*b) - 1/(4*b^2) + (3
*(a^3*b^5)^(1/2))/(8*a^2*b^4) + (a^3*b^5)^(1/2)/(16*a^3*b^3))^(1/2))/((2*(
a^3*b^5)^(1/2))/a^2 - 24*a*b + (14*(a^3*b^5)^(1/2))/b^2 - 4*a^2 - 4*b^2 +
(14*(a^3*b^5)^(1/2))/(a*b) + (2*a*(a^3*b^5)^(1/2))/b^3) + (8*a^2*b*sin(c +
 d*x)*((a^3*b^5)^(1/2)/(16*a*b^5) - 1/(4*a*b) - 1/(4*b^2) + (3*(a^3*b^5)^(
1/2))/(8*a^2*b^4) + (a^3*b^5)^(1/2)/(16*a^3*b^3))^(1/2))/((2*(a^3*b^5)^(1/
2))/a^2 - 24*a*b + (14*(a^3*b^5)^(1/2))/b^2 - 4*a^2 - 4*b^2 + (14*(a^3*b^5
)^(1/2))/(a*b) + (2*a*(a^3*b^5)^(1/2))/b^3))*((a^2*(a^3*b^5)^(1/2) + b^2*(
a^3*b^5)^(1/2) - 4*a^2*b^4 - 4*a^3*b^3 + 6*a*b*(a^3*b^5)^(1/2))/(16*a^3*b^
5))^(1/2))/d - (2*atanh((8*b^3*sin(c + d*x)*(- 1/(4*b^2) - 1/(4*a*b) - (a^
3*b^5)^(1/2)/(16*a*b^5) - (3*(a^3*b^5)^(1/2))/(8*a^2*b^4) - (a^3*b^5)^(1/2
)/(16*a^3*b^3))^(1/2))/(24*a*b + (2*(a^3*b^5)^(1/2))/a^2 + (14*(a^3*b^5)^(
1/2))/b^2 + 4*a^2 + 4*b^2 + (14*(a^3*b^5)^(1/2))/(a*b) + (2*a*(a^3*b^5)^(1
/2))/b^3) + (48*a*b^2*sin(c + d*x)*(- 1/(4*b^2) - 1/(4*a*b) - (a^3*b^5)^(1
/2)/(16*a*b^5) - (3*(a^3*b^5)^(1/2))/(8*a^2*b^4) - (a^3*b^5)^(1/2)/(16*a^3
*b^3))^(1/2))/(24*a*b + (2*(a^3*b^5)^(1/2))/a^2 + (14*(a^3*b^5)^(1/2))/...
```

3.405. $\int \frac{\cos^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.406 $\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.406.1 Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d} - \frac{\left(\sqrt{a}-\sqrt{b}\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d}$$

output
```
-1/2*arctanh(b^(1/4)*sin(d*x+c)/a^(1/4))*(a^(1/2)-b^(1/2))/a^(3/4)/b^(3/4)
/d+1/2*arctan(b^(1/4)*sin(d*x+c)/a^(1/4))*(a^(1/2)+b^(1/2))/a^(3/4)/b^(3/4
)/d
```

3.406.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.68

$$\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)+i\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c+dx)\right)-i\left(\sqrt{a}+\sqrt{b}\right)\log\left(\right.}{4a^{3/4}b^{3/4}d}$$

input
```
Integrate[Cos[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

output
```
((Sqrt[a] - Sqrt[b])*Log[a^(1/4) - b^(1/4)*Sin[c + d*x]] + I*(Sqrt[a] + Sq
rt[b])*Log[a^(1/4) - I*b^(1/4)*Sin[c + d*x]] - I*(Sqrt[a] + Sqrt[b])*Log[a
^(1/4) + I*b^(1/4)*Sin[c + d*x]] - (Sqrt[a] - Sqrt[b])*Log[a^(1/4) + b^(1/
4)*Sin[c + d*x]])/(4*a^(3/4)*b^(3/4)*d)
```

3.406.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3042, 3702, 1481, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(c+dx)^3}{a-b\sin(c+dx)^4}dx$$
$$\Big\downarrow \text{3702}$$
$$\frac{\int \frac{1-\sin^2(c+dx)}{a-b\sin^4(c+dx)}d\sin(c+dx)}{d}$$
$$\Big\downarrow \text{1481}$$
$$\frac{-\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\int \frac{1}{-b\sin^2(c+dx)-\sqrt{a}\sqrt{b}}d\sin(c+dx)-\frac{1}{2}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{\sqrt{a}\sqrt{b}-b\sin^2(c+dx)}d\sin(c+dx)}{d}$$
$$\Big\downarrow \text{218}$$
$$\frac{\frac{\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\arctan\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{3/4}}-\frac{1}{2}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{\sqrt{a}\sqrt{b}-b\sin^2(c+dx)}d\sin(c+dx)}{d}$$
$$\Big\downarrow \text{221}$$
$$\frac{\frac{\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\arctan\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{3/4}}-\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{arctanh}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{3/4}}}{d}$$

input
```
Int[Cos[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

output
```
(((1 + Sqrt[b]/Sqrt[a])*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(1/4)
*b^(3/4)) - ((1 - Sqrt[b]/Sqrt[a])*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)]
)/(2*a^(1/4)*b^(3/4)))/d
```

3.406.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1481
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Simp[(e/2 + c*(d/(2*q)))   Int[1/(-q + c*x^2), x], x] + Simp[(
e/2 - c*(d/(2*q)))   Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] &
& NeQ[c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.406. $\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.406.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(67) = 134$.

Time = 0.72 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{d}$
default	$\frac{\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{d}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(256a^3b^3d^4\_Z^4+64a^2b^2d^2\_Z^2-a^2+2ab-b^2\right)}\_R\ln\left(\mathrm{e}^{2i(dx+c)}+\left(-\frac{128ia^3b^2d^3\_R^3}{a^2-b^2}+\left(-\frac{24}{a}\right.\right.\right.$

input
```
int(cos(d*x+c)^3/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*(1/b*a)^(1/4)/a*(ln((sin(d*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b*a
)^(1/4)))+2*arctan(sin(d*x+c)/(1/b*a)^(1/4)))+1/4/b/(1/b*a)^(1/4)*(2*arcta
n(sin(d*x+c)/(1/b*a)^(1/4))-ln((sin(d*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b
*a)^(1/4)))))
```

3.406.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(67) = 134$.

Time = 0.37 (sec) , antiderivative size = 631, normalized size of antiderivative = 6.64

$$\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$$
$$= \frac{1}{4}\sqrt{-\frac{abd^2\sqrt{\frac{a^2+2\,ab+b^2}{a^3b^3d^4}}+2}{abd^2}}\log\left(\frac{1}{2}\left(a^2-b^2\right)\sin\left(dx+c\right)\right.$$
$$\left.+\frac{1}{2}\left(a^3b^2d^3\sqrt{\frac{a^2+2\,ab+b^2}{a^3b^3d^4}}-\left(a^2b+ab^2\right)d\right)\sqrt{-\frac{abd^2\sqrt{\frac{a^2+2\,ab+b^2}{a^3b^3d^4}}+2}{abd^2}}\right)$$
$$-\frac{1}{4}\sqrt{\frac{abd^2\sqrt{\frac{a^2+2\,ab+b^2}{a^3b^3d^4}}-2}{abd^2}}\log\left(\frac{1}{2}\left(a^2-b^2\right)\sin\left(dx+c\right)\right.$$
$$\left.+\frac{1}{2}\left(a^3b^2d^3\sqrt{\frac{a^2+2\,ab+b^2}{a^3b^3d^4}}+\left(a^2b+ab^2\right)d\right)\sqrt{\frac{abd^2\sqrt{\frac{a^2+2\,ab+b^2}{a^3b^3d^4}}-2}{abd^2}}\right)$$
$$-\frac{1}{4}\sqrt{-\frac{abd^2\sqrt{\frac{a^2+2\,ab+b^2}{a^3b^3d^4}}+2}{abd^2}}\log\left(-\frac{1}{2}\left(a^2-b^2\right)\sin\left(dx+c\right)\right.$$
$$\left.+\frac{1}{2}\left(a^3b^2d^3\sqrt{\frac{a^2+2\,ab+b^2}{a^3b^3d^4}}-\left(a^2b+ab^2\right)d\right)\sqrt{-\frac{abd^2\sqrt{\frac{a^2+2\,ab+b^2}{a^3b^3d^4}}+2}{abd^2}}\right)$$
$$+\frac{1}{4}\sqrt{\frac{abd^2\sqrt{\frac{a^2+2\,ab+b^2}{a^3b^3d^4}}-2}{abd^2}}\log\left(-\frac{1}{2}\left(a^2-b^2\right)\sin\left(dx+c\right)\right.$$
$$\left.+\frac{1}{2}\left(a^3b^2d^3\sqrt{\frac{a^2+2\,ab+b^2}{a^3b^3d^4}}+\left(a^2b+ab^2\right)d\right)\sqrt{\frac{abd^2\sqrt{\frac{a^2+2\,ab+b^2}{a^3b^3d^4}}-2}{abd^2}}\right)$$

input
```
integrate(cos(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

3.406. $\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

output
```
1/4*sqrt(-(a*b*d^2*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) + 2)/(a*b*d^2))
*log(1/2*(a^2 - b^2)*sin(d*x + c) + 1/2*(a^3*b^2*d^3*sqrt((a^2 + 2*a*b + b
^2)/(a^3*b^3*d^4)) - (a^2*b + a*b^2)*d)*sqrt(-(a*b*d^2*sqrt((a^2 + 2*a*b +
 b^2)/(a^3*b^3*d^4)) + 2)/(a*b*d^2))) - 1/4*sqrt((a*b*d^2*sqrt((a^2 + 2*a*
b + b^2)/(a^3*b^3*d^4)) - 2)/(a*b*d^2))*log(1/2*(a^2 - b^2)*sin(d*x + c) +
 1/2*(a^3*b^2*d^3*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) + (a^2*b + a*b^2
)*d)*sqrt((a*b*d^2*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) - 2)/(a*b*d^2))
) - 1/4*sqrt(-(a*b*d^2*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) + 2)/(a*b*d
^2))*log(-1/2*(a^2 - b^2)*sin(d*x + c) + 1/2*(a^3*b^2*d^3*sqrt((a^2 + 2*a*
b + b^2)/(a^3*b^3*d^4)) - (a^2*b + a*b^2)*d)*sqrt(-(a*b*d^2*sqrt((a^2 + 2*
a*b + b^2)/(a^3*b^3*d^4)) + 2)/(a*b*d^2))) + 1/4*sqrt((a*b*d^2*sqrt((a^2 +
 2*a*b + b^2)/(a^3*b^3*d^4)) - 2)/(a*b*d^2))*log(-1/2*(a^2 - b^2)*sin(d*x
+ c) + 1/2*(a^3*b^2*d^3*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) + (a^2*b +
 a*b^2)*d)*sqrt((a*b*d^2*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) - 2)/(a*b
*d^2)))
```

3.406.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.406.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.27

$$\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\frac{2\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\sin(dx+c)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\frac{\sqrt{b}\sin(dx+c)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin(dx+c)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{4\,d}$$

input
```
integrate(cos(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/4*(2*(sqrt(a) + sqrt(b))*arctan(sqrt(b)*sin(d*x + c)/sqrt(sqrt(a)*sqrt(b
)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (sqrt(a) - sqrt(b))*log((sqr
t(b)*sin(d*x + c) - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*sin(d*x + c) + sqrt(sq
rt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/d
```

3.406.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(67) = 134$.

Time = 0.86 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.95

$$\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$$
$$= \frac{2\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2-\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^3} + \frac{2\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2-\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{ab^3}$$

input
```
integrate(cos(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
1/8*(2*sqrt(2)*((-a*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(s
qrt(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b^3) + 2*sqrt(2)*((
-a*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1
/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b^3) + sqrt(2)*((-a*b^3)^(1/4)*b^2
+ (-a*b^3)^(3/4))*log(sin(d*x + c)^2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) +
 sqrt(-a/b))/(a*b^3) - sqrt(2)*((-a*b^3)^(1/4)*b^2 + (-a*b^3)^(3/4))*log(s
in(d*x + c)^2 - sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^3))/d
```

3.406.9 Mupad [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 489, normalized size of antiderivative = 5.15

$$\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)}\,dx =$$
$$-\frac{2\,\mathrm{atanh}\left(\frac{8\,b^3\,\sin(c+d\,x)\,\sqrt{-\frac{1}{8\,a\,b}-\frac{\sqrt{a^3\,b^3}}{16\,a^2\,b^3}-\frac{\sqrt{a^3\,b^3}}{16\,a^3\,b^2}}}{2\,a\,b+\frac{2\,\sqrt{a^3\,b^3}}{a}+2\,b^2+\frac{2\,b\,\sqrt{a^3\,b^3}}{a^2}}+\frac{8\,a\,b^2\,\sin(c+d\,x)\,\sqrt{-\frac{1}{8\,a\,b}-\frac{\sqrt{a^3\,b^3}}{16\,a^2\,b^3}-\frac{\sqrt{a^3\,b^3}}{16\,a^3\,b^2}}}{2\,a\,b+\frac{2\,\sqrt{a^3\,b^3}}{a}+2\,b^2+\frac{2\,b\,\sqrt{a^3\,b^3}}{a^2}}\right)\sqrt{-\frac{a\,\sqrt{a^3\,b^3}+b\,\sqrt{a^3\,b^3}+2\,a^2}{16\,a^3\,b^3}}}{d}$$
$$-\frac{2\,\mathrm{atanh}\left(\frac{8\,b^3\,\sin(c+d\,x)\,\sqrt{\frac{\sqrt{a^3\,b^3}}{16\,a^2\,b^3}-\frac{1}{8\,a\,b}+\frac{\sqrt{a^3\,b^3}}{16\,a^3\,b^2}}}{2\,a\,b-\frac{2\,\sqrt{a^3\,b^3}}{a}+2\,b^2-\frac{2\,b\,\sqrt{a^3\,b^3}}{a^2}}+\frac{8\,a\,b^2\,\sin(c+d\,x)\,\sqrt{\frac{\sqrt{a^3\,b^3}}{16\,a^2\,b^3}-\frac{1}{8\,a\,b}+\frac{\sqrt{a^3\,b^3}}{16\,a^3\,b^2}}}{2\,a\,b-\frac{2\,\sqrt{a^3\,b^3}}{a}+2\,b^2-\frac{2\,b\,\sqrt{a^3\,b^3}}{a^2}}\right)\sqrt{\frac{a\,\sqrt{a^3\,b^3}+b\,\sqrt{a^3\,b^3}-2\,a^2\,b^2}{16\,a^3\,b^3}}}{d}$$

input
```
int(cos(c + d*x)^3/(a - b*sin(c + d*x)^4),x)
```

output
```
- (2*atanh((8*b^3*sin(c + d*x)*(- 1/(8*a*b) - (a^3*b^3)^(1/2)/(16*a^2*b^3)
 - (a^3*b^3)^(1/2)/(16*a^3*b^2))^(1/2))/(2*a*b + (2*(a^3*b^3)^(1/2))/a + 2
*b^2 + (2*b*(a^3*b^3)^(1/2))/a^2) + (8*a*b^2*sin(c + d*x)*(- 1/(8*a*b) - (
a^3*b^3)^(1/2)/(16*a^2*b^3) - (a^3*b^3)^(1/2)/(16*a^3*b^2))^(1/2))/(2*a*b
+ (2*(a^3*b^3)^(1/2))/a + 2*b^2 + (2*b*(a^3*b^3)^(1/2))/a^2))*(-(a*(a^3*b^
3)^(1/2) + b*(a^3*b^3)^(1/2) + 2*a^2*b^2)/(16*a^3*b^3))^(1/2))/d - (2*atan
h((8*b^3*sin(c + d*x)*((a^3*b^3)^(1/2)/(16*a^2*b^3) - 1/(8*a*b) + (a^3*b^3
)^(1/2)/(16*a^3*b^2))^(1/2))/(2*a*b - (2*(a^3*b^3)^(1/2))/a + 2*b^2 - (2*b
*(a^3*b^3)^(1/2))/a^2) + (8*a*b^2*sin(c + d*x)*((a^3*b^3)^(1/2)/(16*a^2*b^
3) - 1/(8*a*b) + (a^3*b^3)^(1/2)/(16*a^3*b^2))^(1/2))/(2*a*b - (2*(a^3*b^3
)^(1/2))/a + 2*b^2 - (2*b*(a^3*b^3)^(1/2))/a^2))*((a*(a^3*b^3)^(1/2) + b*(
a^3*b^3)^(1/2) - 2*a^2*b^2)/(16*a^3*b^3))^(1/2))/d
```

3.406. $\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.407 $\int \frac{\cos(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.407.1 Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{\cos(c+dx)}{a-b\sin^4(c+dx)}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}d}$$

output
```
1/2*arctan(b^(1/4)*sin(d*x+c)/a^(1/4))/a^(3/4)/b^(1/4)/d+1/2*arctanh(b^(1/
4)*sin(d*x+c)/a^(1/4))/a^(3/4)/b^(1/4)/d
```

3.407.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int \frac{\cos(c+dx)}{a-b\sin^4(c+dx)}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)+\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}d}$$

input
```
Integrate[Cos[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

output
```
(ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)] + ArcTanh[(b^(1/4)*Sin[c + d*x])/a
^(1/4)])/(2*a^(3/4)*b^(1/4)*d)
```

3.407.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 3702, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(c+dx)}{a-b\sin(c+dx)^4}dx$$

$$\Big\downarrow \; 3702$$

$$\frac{\int \frac{1}{a-b\sin^4(c+dx)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \; 756$$

$$\frac{\frac{\int \frac{1}{\sqrt{a}-\sqrt{b}\sin^2(c+dx)}d\sin(c+dx)}{2\sqrt{a}}+\frac{\int \frac{1}{\sqrt{b}\sin^2(c+dx)+\sqrt{a}}d\sin(c+dx)}{2\sqrt{a}}}{d}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\int \frac{1}{\sqrt{a}-\sqrt{b}\sin^2(c+dx)}d\sin(c+dx)}{2\sqrt{a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}}{d}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}}{d}$$

input
```
Int[Cos[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

output
```
(ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)]/(2*a^(3/4)*b^(1/4)) + ArcTanh[(b^(
1/4)*Sin[c + d*x])/a^(1/4)]/(2*a^(3/4)*b^(1/4)))/d
```

3.407.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.407.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.59 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.68

method	result	size
risch	$\sum_{\_R=\mathrm{RootOf}\left(256a^{3}b\,d^{4}\_Z^{4}-1\right)}\_R\ln\left(\mathrm{e}^{2i(dx+c)}+8iad\_R\,\mathrm{e}^{i(dx+c)}-1\right)$	48
derivativedivides	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4da}$	68
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4da}$	68

input
```
int(cos(d*x+c)/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(exp(2*I*(d*x+c))+8*I*a*d*_R*exp(I*(d*x+c))-1),_R=RootOf(256*_Z^4
*a^3*b*d^4-1))
```

3.407.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.27

$$\int \frac{\cos(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{1}{4}\left(\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\frac{1}{2}\,ad\left(\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+\frac{1}{2}\sin(dx+c)\right) - \frac{1}{4}\left(\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\frac{1}{2}\,ad\left(\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}-\frac{1}{2}\sin(dx+c)\right) + \frac{1}{4}i\left(\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\frac{1}{2}i\,ad\left(\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+\frac{1}{2}\sin(dx+c)\right) - \frac{1}{4}i\left(\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\frac{1}{2}i\,ad\left(\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}-\frac{1}{2}\sin(dx+c)\right)$$

input
```
integrate(cos(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/4*(1/(a^3*b*d^4))^(1/4)*log(1/2*a*d*(1/(a^3*b*d^4))^(1/4) + 1/2*sin(d*x
+ c)) - 1/4*(1/(a^3*b*d^4))^(1/4)*log(1/2*a*d*(1/(a^3*b*d^4))^(1/4) - 1/2*
sin(d*x + c)) + 1/4*I*(1/(a^3*b*d^4))^(1/4)*log(1/2*I*a*d*(1/(a^3*b*d^4))^
(1/4) + 1/2*sin(d*x + c)) - 1/4*I*(1/(a^3*b*d^4))^(1/4)*log(1/2*I*a*d*(1/(
a^3*b*d^4))^(1/4) - 1/2*sin(d*x + c))
```

3.407.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(63) = 126$.

Time = 2.57 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.82

$$\int \frac{\cos(c+dx)}{a-b\sin^4(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty}x\cos(c)}{\sin^4(c)} & \text{for } a=0\wedge b=0\wedge d=0 \\ \frac{1}{3bd\sin^3(c+dx)} & \text{for } a=0 \\ \frac{\sin(c+dx)}{ad} & \text{for } b=0 \\ \frac{x\cos(c)}{a-b\sin^4(c)} & \text{for } d=0 \\ -\frac{\sqrt[4]{\frac{a}{b}}\log\left(-\sqrt[4]{\frac{a}{b}}+\sin(c+dx)\right)}{4ad}+\frac{\sqrt[4]{\frac{a}{b}}\log\left(\sqrt[4]{\frac{a}{b}}+\sin(c+dx)\right)}{4ad}+\frac{\sqrt[4]{\frac{a}{b}}\operatorname{atan}\left(\frac{\sin(c+dx)}{\sqrt[4]{\frac{a}{b}}}\right)}{2ad} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a-b*sin(d*x+c)**4),x)
```

output
```
Piecewise((zoo*x*cos(c)/sin(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (1/(3*
b*d*sin(c + d*x)**3), Eq(a, 0)), (sin(c + d*x)/(a*d), Eq(b, 0)), (x*cos(c)
/(a - b*sin(c)**4), Eq(d, 0)), (-(a/b)**(1/4)*log(-(a/b)**(1/4) + sin(c +
d*x))/(4*a*d) + (a/b)**(1/4)*log((a/b)**(1/4) + sin(c + d*x))/(4*a*d) + (a
/b)**(1/4)*atan(sin(c + d*x)/(a/b)**(1/4))/(2*a*d), True))
```

3.407.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int \frac{\cos(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\frac{2\arctan\left(\frac{\sqrt{b}\sin(dx+c)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}-\frac{\log\left(\frac{\sqrt{b}\sin(dx+c)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin(dx+c)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{4\,d}$$

input
```
integrate(cos(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/4*(2*arctan(sqrt(b)*sin(d*x + c)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sq
rt(a)*sqrt(b))) - log((sqrt(b)*sin(d*x + c) - sqrt(sqrt(a)*sqrt(b)))/(sqrt
(b)*sin(d*x + c) + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
)/d
```

3.407.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(51) = 102$.

Time = 0.85 (sec) , antiderivative size = 224, normalized size of antiderivative = 3.15

$$\int \frac{\cos(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\frac{2\sqrt{2}(-ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab} + \frac{2\sqrt{2}(-ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab} + \frac{\sqrt{2}(-ab^3)^{\frac{1}{4}}\log\left(\sin(dx+\right.}{}}{8\,d}$$

input
```
integrate(cos(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
1/8*(2*sqrt(2)*(-a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) + 2
*sin(d*x + c))/(-a/b)^(1/4))/(a*b) + 2*sqrt(2)*(-a*b^3)^(1/4)*arctan(-1/2*
sqrt(2)*(sqrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b) + sqrt
(2)*(-a*b^3)^(1/4)*log(sin(d*x + c)^2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c)
+ sqrt(-a/b))/(a*b) - sqrt(2)*(-a*b^3)^(1/4)*log(sin(d*x + c)^2 - sqrt(2)*
(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b))/d
```

3.407.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{\cos(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\mathrm{atan}\left(\frac{b^{1/4}\sin(c+d\,x)}{a^{1/4}}\right)+\mathrm{atanh}\left(\frac{b^{1/4}\sin(c+d\,x)}{a^{1/4}}\right)}{2\,a^{3/4}\,b^{1/4}\,d}$$

input
```
int(cos(c + d*x)/(a - b*sin(c + d*x)^4),x)
```

output
```
(atan((b^(1/4)*sin(c + d*x))/a^(1/4)) + atanh((b^(1/4)*sin(c + d*x))/a^(1/
4)))/(2*a^(3/4)*b^(1/4)*d)
```

3.408 $\int \frac{\sec(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.408.1 Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \frac{\sec(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)d} + \frac{\operatorname{arctanh}(\sin(c+dx))}{(a-b)d} - \frac{\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)d}$$

output
```
arctanh(sin(d*x+c))/(a-b)/d-1/2*b^(1/4)*arctanh(b^(1/4)*sin(d*x+c)/a^(1/4)
)/a^(3/4)/d/(a^(1/2)-b^(1/2))+1/2*b^(1/4)*arctan(b^(1/4)*sin(d*x+c)/a^(1/4
))/a^(3/4)/d/(a^(1/2)+b^(1/2))
```

3.408.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.57

$$\int \frac{\sec(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{4a^{3/4}\operatorname{arctanh}(\sin(c+dx)) + \sqrt[4]{b}\left(\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right) + i\left(\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt[4]{a}-i\sqrt[4]{\cdots}\right.\right.\right.}{4a^{3/4}(\cdots}$$

input
```
Integrate[Sec[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

output
```
(4*a^(3/4)*ArcTanh[Sin[c + d*x]] + b^(1/4)*((Sqrt[a] + Sqrt[b])*Log[a^(1/4
) - b^(1/4)*Sin[c + d*x]] + I*((Sqrt[a] - Sqrt[b])*Log[a^(1/4) - I*b^(1/4)
*Sin[c + d*x]] + (-Sqrt[a] + Sqrt[b])*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]
] + I*(Sqrt[a] + Sqrt[b])*Log[a^(1/4) + b^(1/4)*Sin[c + d*x]])))/(4*a^(3/4
)*(a - b)*d)
```

3.408.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3702, 1485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)\left(a-b\sin(c+dx)^4\right)}dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int \frac{1}{\left(1-\sin^2(c+dx)\right)\left(a-b\sin^4(c+dx)\right)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{1485}$$

$$\frac{\int \left(-\frac{b\left(\sin^2(c+dx)+1\right)}{(a-b)\left(a-b\sin^4(c+dx)\right)}-\frac{1}{(a-b)\left(\sin^2(c+dx)-1\right)}\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)}-\frac{\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)}+\frac{\text{arctanh}(\sin(c+dx))}{a-b}}{d}$$

input
```
Int[Sec[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

output
```
((b^(1/4)*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sq
rt[b])) + ArcTanh[Sin[c + d*x]]/(a - b) - (b^(1/4)*ArcTanh[(b^(1/4)*Sin[c
+ d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] - Sqrt[b])))/d
```

3.408.3.1 Defintions of rubi rules used

rule 1485
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.408.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(89) = 178$.

Time = 1.22 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.58

method	result
derivativedivides	$\frac{-\frac{\ln(\sin(dx+c)-1)}{2a-2b}+\frac{b\left(-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)}{\sin(dx+c)-\left(\frac{a}{b}\right)}\right.}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{a-b}}{d}$
default	$\frac{-\frac{\ln(\sin(dx+c)-1)}{2a-2b}+\frac{b\left(-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)}{\sin(dx+c)-\left(\frac{a}{b}\right)}\right.}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{a-b}}{d}$
risch	$\frac{\ln(e^{i(dx+c)}+i)}{d(a-b)}-\frac{\ln(e^{i(dx+c)}-i)}{d(a-b)}+2\left(\sum_{\_R=\mathrm{RootOf}\left((4096a^5d^4-8192a^4b\,d^4+4096a^3b^2d^4)\_Z^4-256a^2b\,d^2\_Z^2-b\right)}\right.$

input
```
int(sec(d*x+c)/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(2*a-2*b)*ln(sin(d*x+c)-1)+b/(a-b)*(-1/4*(1/b*a)^(1/4)/a*(ln((sin(
d*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b*a)^(1/4)))+2*arctan(sin(d*x+c)/(1/b
*a)^(1/4)))+1/4/b/(1/b*a)^(1/4)*(2*arctan(sin(d*x+c)/(1/b*a)^(1/4))-ln((si
n(d*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b*a)^(1/4)))))+1/(2*a-2*b)*ln(1+sin
(d*x+c)))
```

3.408.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1329 vs. $2(89) = 178$.

Time = 0.42 (sec) , antiderivative size = 1329, normalized size of antiderivative = 11.36

$$\int \frac{\sec(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/4*((a - b)*d*sqrt(((a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 +
b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3
 - 2*a^2*b + a*b^2)*d^2))*log(1/2*(a*b + b^2)*sin(d*x + c) + 1/2*((a^5 - 2
*a^4*b + a^3*b^2)*d^3*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5
*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) - (a^2*b + a*b^2)*d)*sqrt(((a^3 - 2*a^2*
b + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 -
4*a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 - 2*a^2*b + a*b^2)*d^2))) - (a - b
)*d*sqrt(-((a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7
- 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) - 2*b)/((a^3 - 2*a^2*b
+ a*b^2)*d^2))*log(1/2*(a*b + b^2)*sin(d*x + c) + 1/2*((a^5 - 2*a^4*b + a^
3*b^2)*d^3*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^
4*b^3 + a^3*b^4)*d^4)) + (a^2*b + a*b^2)*d)*sqrt(-((a^3 - 2*a^2*b + a*b^2)
*d^2*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3
+ a^3*b^4)*d^4)) - 2*b)/((a^3 - 2*a^2*b + a*b^2)*d^2))) - (a - b)*d*sqrt((
(a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b +
 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 - 2*a^2*b + a*b^2)*d^
2))*log(-1/2*(a*b + b^2)*sin(d*x + c) + 1/2*((a^5 - 2*a^4*b + a^3*b^2)*d^3
*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^
3*b^4)*d^4)) - (a^2*b + a*b^2)*d)*sqrt(((a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((
a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^...
```

3.408.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int \frac{\sec\left(c+dx\right)}{a-b\sin^4\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)/(a-b*sin(d*x+c)**4),x)
```

output
```
Integral(sec(c + d*x)/(a - b*sin(c + d*x)**4), x)
```

3.408.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.43

$$\int \frac{\sec(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\frac{b\left(\frac{2\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\sin(dx+c)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\log\left(\frac{\sqrt{b}\sin(dx+c)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin(dx+c)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\right)}{a-b}+\frac{2\log(\sin(dx+c)+1)}{a-b}-\frac{2\log(\sin(dx+c)-1)}{a-b}}{4d}$$

input
```
integrate(sec(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/4*(b*(2*(sqrt(a) - sqrt(b))*arctan(sqrt(b)*sin(d*x + c)/sqrt(sqrt(a)*sqr
t(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (sqrt(a) + sqrt(b))*log((
sqrt(b)*sin(d*x + c) - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*sin(d*x + c) + sqrt
(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a - b) + 2*l
og(sin(d*x + c) + 1)/(a - b) - 2*log(sin(d*x + c) - 1)/(a - b))/d
```

3.408.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(89) = 178.

Time = 0.85 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.16

$$\int \frac{\sec(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{4\left(\left(-ab^3\right)^{\frac{1}{4}}b^2+\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^2b^2-\sqrt{2}ab^3}+\frac{4\left(\left(-ab^3\right)^{\frac{1}{4}}b^2+\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^2b^2-\sqrt{2}ab^3}$$

input
```
integrate(sec(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
-1/8*(4*((-a*b^3)^(1/4)*b^2 + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(sqrt(2)*
(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)*a^2*b^2 - sqrt(2)*a*
b^3) + 4*((-a*b^3)^(1/4)*b^2 + (-a*b^3)^(3/4))*arctan(-1/2*sqrt(2)*(sqrt(2
)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)*a^2*b^2 - sqrt(2)*
a*b^3) + (sqrt(2)*(-a*b^3)^(1/4)*b^2 - sqrt(2)*(-a*b^3)^(3/4))*log(sin(d*x
 + c)^2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a^2*b^2 - a*b^3
) - (sqrt(2)*(-a*b^3)^(1/4)*b^2 - sqrt(2)*(-a*b^3)^(3/4))*log(sin(d*x + c)
^2 - sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a^2*b^2 - a*b^3) - 4
*log(abs(sin(d*x + c) + 1))/(a - b) + 4*log(abs(sin(d*x + c) - 1))/(a - b)
)/d
```

3.408.9 Mupad [B] (verification not implemented)

Time = 16.54 (sec) , antiderivative size = 3891, normalized size of antiderivative = 33.26

$$\int \frac{\sec(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)*(a - b*sin(c + d*x)^4)),x)
```

output
```
(atan(((b^5*sin(c + d*x)*3i + ((((32*a*b^7 + 64*a^2*b^6 - 224*a^3*b^5 + 12
8*a^4*b^4 - (sin(c + d*x)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a
^5*b^4))/(4*(a - b)))/(2*(a - b)) + (sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240
*a^2*b^5))/2)/(2*(a - b)) - 10*a*b^5 + 2*b^6)*1i)/(2*(a - b)))/(a - b) + (
b^5*sin(c + d*x)*3i - ((((32*a*b^7 + 64*a^2*b^6 - 224*a^3*b^5 + 128*a^4*b^
4 + (sin(c + d*x)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4))
/(4*(a - b)))/(2*(a - b)) - (sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240*a^2*b^5
))/2)/(2*(a - b)) - 10*a*b^5 + 2*b^6)*1i)/(2*(a - b)))/(a - b))/((3*b^5*si
n(c + d*x) + (((32*a*b^7 + 64*a^2*b^6 - 224*a^3*b^5 + 128*a^4*b^4 - (sin(c
 + d*x)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4))/(4*(a - b
)))/(2*(a - b)) + (sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240*a^2*b^5))/2)/(2*(
a - b)) - 10*a*b^5 + 2*b^6)/(2*(a - b)))/(a - b) - (3*b^5*sin(c + d*x) - (
((32*a*b^7 + 64*a^2*b^6 - 224*a^3*b^5 + 128*a^4*b^4 + (sin(c + d*x)*(512*a
^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4))/(4*(a - b)))/(2*(a - b)
) - (sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240*a^2*b^5))/2)/(2*(a - b)) - 10*a
*b^5 + 2*b^6)/(2*(a - b)))/(a - b)))*1i)/(d*(a - b)) - (atan((((((2*a^2*b
 + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2
)*(64*a*b^7 + 128*a^2*b^6 - 448*a^3*b^5 + 256*a^4*b^4 + sin(c + d*x)*((2*a
^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^
(1/2)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4)) - sin(c ...
```

3.409 $\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.409.1 Optimal result

Integrand size = 24, antiderivative size = 175

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx &= \frac{b^{3/4}\arctan\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^2 d}+\frac{(a-5b)\operatorname{arctanh}(\sin(c+dx))}{2(a-b)^2 d}\\ &\quad+\frac{b^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^2 d}+\frac{1}{4(a-b)d(1-\sin(c+dx))}\\ &\quad-\frac{1}{4(a-b)d(1+\sin(c+dx))}\end{aligned}$$

output
```
1/2*(a-5*b)*arctanh(sin(d*x+c))/(a-b)^2/d+1/4/(a-b)/d/(1-sin(d*x+c))-1/4/(
a-b)/d/(1+sin(d*x+c))+1/2*b^(3/4)*arctanh(b^(1/4)*sin(d*x+c)/a^(1/4))/a^(3
/4)/d/(a^(1/2)-b^(1/2))^2+1/2*b^(3/4)*arctan(b^(1/4)*sin(d*x+c)/a^(1/4))/a
^(3/4)/d/(a^(1/2)+b^(1/2))^2
```

3.409.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.46

$$\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx =$$
$$-\frac{-\frac{2(a-5b)\text{arctanh}(\sin(c+dx))}{(a-b)^2}+\frac{b^{3/4}\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^2}-\frac{ib^{3/4}\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^2}+\frac{ib^{3/4}\log\left(\sqrt[4]{a}+i\sqrt[4]{b}\sin(\right.}{a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^2}}{4d}$$

input
```
Integrate[Sec[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

output
```
-1/4*((-2*(a - 5*b)*ArcTanh[Sin[c + d*x]])/(a - b)^2 + (b^(3/4)*Log[a^(1/4
) - b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] - Sqrt[b])^2) - (I*b^(3/4)*Lo
g[a^(1/4) - I*b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] + Sqrt[b])^2) + (I*
b^(3/4)*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] + Sqrt[b]
)^2) - (b^(3/4)*Log[a^(1/4) + b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] - S
qrt[b])^2) + 1/((a - b)*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x
])))/d
```

3.409.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3702, 1485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\cos(c+dx)^3\left(a-b\sin(c+dx)^4\right)}dx$$
$$\Big\downarrow \ 3702$$
$$\frac{\int \frac{1}{\left(1-\sin^2(c+dx)\right)^2\left(a-b\sin^4(c+dx)\right)}d\sin(c+dx)}{d}$$

$$\downarrow 1485$$

$$\frac{\int \left(\frac{5b-a}{2(a-b)^2(\sin^2(c+dx)-1)} + \frac{b(2b\sin^2(c+dx)+a+b)}{(a-b)^2(a-b\sin^4(c+dx))} + \frac{1}{4(a-b)(\sin(c+dx)-1)^2} + \frac{1}{4(a-b)(\sin(c+dx)+1)^2}\right) d\sin(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{b^{3/4}\arctan\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^2} + \frac{b^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^2} + \frac{(a-5b)\text{arctanh}(\sin(c+dx))}{2(a-b)^2} + \frac{1}{4(a-b)(1-\sin(c+dx))} - \frac{1}{4(a-b)(\sin(c+d}}{d}$$

input
```
Int[Sec[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

output
```
((b^(3/4)*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sq
rt[b])^2) + ((a - 5*b)*ArcTanh[Sin[c + d*x]])/(2*(a - b)^2) + (b^(3/4)*Arc
Tanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] - Sqrt[b])^2) +
1/(4*(a - b)*(1 - Sin[c + d*x])) - 1/(4*(a - b)*(1 + Sin[c + d*x])))/d
```

3.409.3.1 Defintions of rubi rules used

rule 1485
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.409. $\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.409.4 Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{-\frac{1}{(4a-4b)(\sin(dx+c)-1)}+\frac{(-a+5b)\ln(\sin(dx+c)-1)}{4(a-b)^2}-\frac{b\left(\frac{(-a-b)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{2\,\mathrm{arc}}{}\right.}{(a-b)^2}}{d}$
default	$\frac{-\frac{1}{(4a-4b)(\sin(dx+c)-1)}+\frac{(-a+5b)\ln(\sin(dx+c)-1)}{4(a-b)^2}-\frac{b\left(\frac{(-a-b)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{2\,\mathrm{arc}}{}\right.}{(a-b)^2}}{d}$
risch	$-\frac{i\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{d(a-b)\left(e^{2i(dx+c)}+1\right)^2}-\frac{\ln\left(e^{i(dx+c)}-i\right)a}{2d(a^2-2ab+b^2)}+\frac{5\ln\left(e^{i(dx+c)}-i\right)b}{2d(a^2-2ab+b^2)}+\frac{\ln\left(e^{i(dx+c)}+i\right)a}{2(a^2-2ab+b^2)d}-\frac{5\ln\left(e^{i(dx+c)}+i\right)b}{2(a^2-2ab+b^2)d}+8\Big($

input
```
int(sec(d*x+c)^3/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(4*a-4*b)/(sin(d*x+c)-1)+1/4/(a-b)^2*(-a+5*b)*ln(sin(d*x+c)-1)-b/(
a-b)^2*(1/4*(-a-b)*(1/b*a)^(1/4)/a*(ln((sin(d*x+c)+(1/b*a)^(1/4))/(sin(d*x
+c)-(1/b*a)^(1/4)))+2*arctan(sin(d*x+c)/(1/b*a)^(1/4)))+1/2/(1/b*a)^(1/4)*
(2*arctan(sin(d*x+c)/(1/b*a)^(1/4))-ln((sin(d*x+c)+(1/b*a)^(1/4))/(sin(d*x
+c)-(1/b*a)^(1/4)))))-1/(4*a-4*b)/(1+sin(d*x+c))+1/4*(a-5*b)/(a-b)^2*ln(1+
sin(d*x+c)))
```

3.409.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2529 vs. 2(139) = 278.

Time = 0.83 (sec) , antiderivative size = 2529, normalized size of antiderivative = 14.45

$$\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/4*((a^2 - 2*a*b + b^2)*d*sqrt(((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 +
 a*b^4)*d^2*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^
11 - 8*a^10*b + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5
*b^6 - 8*a^4*b^7 + a^3*b^8)*d^4)) + 4*a*b^2 + 4*b^3)/((a^5 - 4*a^4*b + 6*a
^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2))*cos(d*x + c)^2*log(1/2*(a^2*b^2 + 6*a*b^
3 + b^4)*sin(d*x + c) + 1/2*(2*(a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^
3*b^4)*d^3*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^1
1 - 8*a^10*b + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*
b^6 - 8*a^4*b^7 + a^3*b^8)*d^4)) - (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)
*d)*sqrt(((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2*sqrt((a^4*b^
3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 28*a^9*b
^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 + a^3*b
^8)*d^4)) + 4*a*b^2 + 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b
^4)*d^2))) - (a^2 - 2*a*b + b^2)*d*sqrt(-((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a
^2*b^3 + a*b^4)*d^2*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b
^7)/((a^11 - 8*a^10*b + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5
+ 28*a^5*b^6 - 8*a^4*b^7 + a^3*b^8)*d^4)) - 4*a*b^2 - 4*b^3)/((a^5 - 4*a^4
*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2))*cos(d*x + c)^2*log(1/2*(a^2*b^2
+ 6*a*b^3 + b^4)*sin(d*x + c) + 1/2*(2*(a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*
b^3 + a^3*b^4)*d^3*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 +...
```

3.409.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx=\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a-b*sin(d*x+c)**4),x)
```

output
```
Integral(sec(c + d*x)**3/(a - b*sin(c + d*x)**4), x)
```

3.409.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.39

$$\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\frac{b\left(\frac{2\left(b\left(2\sqrt{a}-\sqrt{b}\right)-a\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\sin(dx+c)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(b\left(2\sqrt{a}+\sqrt{b}\right)+a\sqrt{b}\right)\log\left(\frac{\sqrt{b}\sin(dx+c)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin(dx+c)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\right)}{a^2-2\,ab+b^2}-\frac{(a-5\,b)\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}+\frac{(a-5\,b)\,\mathrm{l}}{a^2}}{4\,d}$$

input
```
integrate(sec(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
-1/4*(b*(2*(b*(2*sqrt(a) - sqrt(b)) - a*sqrt(b))*arctan(sqrt(b)*sin(d*x +
c)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (b*(2*
sqrt(a) + sqrt(b)) + a*sqrt(b))*log((sqrt(b)*sin(d*x + c) - sqrt(sqrt(a)*s
qrt(b)))/(sqrt(b)*sin(d*x + c) + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))*sqrt(b)))/(a^2 - 2*a*b + b^2) - (a - 5*b)*log(sin(d*x + c) +
 1)/(a^2 - 2*a*b + b^2) + (a - 5*b)*log(sin(d*x + c) - 1)/(a^2 - 2*a*b + b
^2) + 2*sin(d*x + c)/((a - b)*sin(d*x + c)^2 - a + b))/d
```

3.409.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. 2(139) = 278.

Time = 0.82 (sec) , antiderivative size = 475, normalized size of antiderivative = 2.71

$$\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\frac{4\left(\left(-ab^3\right)^{\frac{1}{4}}\left(ab+b^2\right)+2\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^3b-2\sqrt{2}a^2b^2+\sqrt{2}ab^3}+\frac{4\left(\left(-ab^3\right)^{\frac{1}{4}}\left(ab+b^2\right)+2\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\,\mathrm{s}\right.}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{\sqrt{2}a^3b-2\sqrt{2}a^2b^2+\sqrt{2}ab^3}}{}$$

input
```
integrate(sec(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
1/8*(4*((-a*b^3)^(1/4)*(a*b + b^2) + 2*(-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*
(sqrt(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)*a^3*b - 2*s
qrt(2)*a^2*b^2 + sqrt(2)*a*b^3) + 4*((-a*b^3)^(1/4)*(a*b + b^2) + 2*(-a*b^
3)^(3/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/
b)^(1/4))/(sqrt(2)*a^3*b - 2*sqrt(2)*a^2*b^2 + sqrt(2)*a*b^3) - (2*sqrt(2)
*(-a*b^3)^(3/4) - (-a*b^3)^(1/4)*(sqrt(2)*a*b + sqrt(2)*b^2))*log(sin(d*x
+ c)^2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a^3*b - 2*a^2*b^
2 + a*b^3) + (2*sqrt(2)*(-a*b^3)^(3/4) - (-a*b^3)^(1/4)*(sqrt(2)*a*b + sqr
t(2)*b^2))*log(sin(d*x + c)^2 - sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-
a/b))/(a^3*b - 2*a^2*b^2 + a*b^3) + 2*(a - 5*b)*log(abs(sin(d*x + c) + 1))
/(a^2 - 2*a*b + b^2) - 2*(a - 5*b)*log(abs(sin(d*x + c) - 1))/(a^2 - 2*a*b
 + b^2) - 4*sin(d*x + c)/((sin(d*x + c)^2 - 1)*(a - b)))/d
```

3.409.9 Mupad [B] (verification not implemented)

Time = 18.34 (sec) , antiderivative size = 7758, normalized size of antiderivative = 44.33

$$\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^3*(a - b*sin(c + d*x)^4)),x)
```

output
```
(atan((((((128*a*b^11 + 256*a^2*b^10 - 3456*a^3*b^9 + 8960*a^4*b^8 - 1088
0*a^5*b^7 + 6912*a^6*b^6 - 2176*a^7*b^5 + 256*a^8*b^4)/(2*(a^4 - 4*a^3*b -
 4*a*b^3 + b^4 + 6*a^2*b^2)) - (sin(c + d*x)*(-(a^2*(a^3*b^3)^(1/2) + b^2*
(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7
- 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2)*(512*a^2*b^11 - 2560*
a^3*b^10 + 4608*a^4*b^9 - 2560*a^5*b^8 - 2560*a^6*b^7 + 4608*a^7*b^6 - 256
0*a^8*b^5 + 512*a^9*b^4))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*(-(
a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*
(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(
1/2) + (sin(c + d*x)*(48*a*b^10 - 16*b^11 + 1024*a^2*b^9 - 2208*a^3*b^8 +
1264*a^4*b^7 - 144*a^5*b^6 + 32*a^6*b^5))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 +
 6*a^2*b^2))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*
a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3
+ 6*a^5*b^2)))^(1/2) - (200*a*b^9 + 480*a^2*b^8 - 784*a^3*b^7 + 96*a^4*b^6
 + 8*a^5*b^5)/(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(-(a^2*(a^3
*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3
)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) + (
sin(c + d*x)*(11*a*b^8 + 27*b^9 - 7*a^2*b^7 + a^3*b^6))/(a^4 - 4*a^3*b - 4
*a*b^3 + b^4 + 6*a^2*b^2))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) -
4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3...
```

3.409. $\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.410 $\int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.410.1 Optimal result

Integrand size = 24, antiderivative size = 249

$$\int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{b^{5/4}\arctan\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^3 d} + \frac{(3a^2-6ab+35b^2)\operatorname{arctanh}(\sin(c+dx))}{8(a-b)^3 d} - \frac{b^{5/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^3 d} + \frac{1}{16(a-b)d(1-\sin(c+dx))^2} + \frac{3a-11b}{16(a-b)^2 d(1-\sin(c+dx))} - \frac{1}{16(a-b)d(1+\sin(c+dx))^2} - \frac{3a-11b}{16(a-b)^2 d(1+\sin(c+dx))}$$

output
```
1/8*(3*a^2-6*a*b+35*b^2)*arctanh(sin(d*x+c))/(a-b)^3/d+1/16/(a-b)/d/(1-sin
(d*x+c))^2+1/16*(3*a-11*b)/(a-b)^2/d/(1-sin(d*x+c))-1/16/(a-b)/d/(1+sin(d*
x+c))^2+1/16*(-3*a+11*b)/(a-b)^2/d/(1+sin(d*x+c))-1/2*b^(5/4)*arctanh(b^(1
/4)*sin(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)-b^(1/2))^3+1/2*b^(5/4)*arctan(b
^(1/4)*sin(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)+b^(1/2))^3
```

3.410.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.87 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.27

$$\int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$$
$$= \frac{\frac{2(3a^2-6ab+35b^2)\text{arctanh}(\sin(c+dx))}{(a-b)^3} + \frac{4b^{5/4}\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^3} + \frac{4ib^{5/4}\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^3} - \frac{4ib^{5/4}\log\left(\sqrt[4]{a}+i\right.}{a^{3/4}\left(\sqrt{a}-\right.}}{}$$

input
```
Integrate[Sec[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

output
```
((2*(3*a^2 - 6*a*b + 35*b^2)*ArcTanh[Sin[c + d*x]])/(a - b)^3 + (4*b^(5/4)
*Log[a^(1/4) - b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] - Sqrt[b])^3) + ((
4*I)*b^(5/4)*Log[a^(1/4) - I*b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] + Sq
rt[b])^3) - ((4*I)*b^(5/4)*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]])/(a^(3/4)
*(Sqrt[a] + Sqrt[b])^3) - (4*b^(5/4)*Log[a^(1/4) + b^(1/4)*Sin[c + d*x]])/
(a^(3/4)*(Sqrt[a] - Sqrt[b])^3) + 1/((a - b)*(-1 + Sin[c + d*x])^2) + (-3*
a + 11*b)/((a - b)^2*(-1 + Sin[c + d*x])) - 1/((a - b)*(1 + Sin[c + d*x])^
2) + (-3*a + 11*b)/((a - b)^2*(1 + Sin[c + d*x])))/(16*d)
```

3.410.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3702, 1485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\cos(c+dx)^5\left(a-b\sin(c+dx)^4\right)}dx$$
$$\Big\downarrow \text{3702}$$

$$\frac{\int \frac{1}{\left(1-\sin^2(c+dx)\right)^3\left(a-b\sin^4(c+dx)\right)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{1485}$$

$$\frac{\int \left(\frac{\left(-\left((a+3b)\sin^2(c+dx)\right)-3a-b\right)b^2}{(a-b)^3\left(a-b\sin^4(c+dx)\right)}+\frac{-3a^2+6ba-35b^2}{8(a-b)^3\left(\sin^2(c+dx)-1\right)}+\frac{3a-11b}{16(a-b)^2(\sin(c+dx)-1)^2}+\frac{3a-11b}{16(a-b)^2(\sin(c+dx)+1)^2}-\frac{1}{8(a-b)(\sin(c+}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^{5/4}\arctan\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^3}-\frac{b^{5/4}\text{arctanh}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^3}+\frac{(3a^2-6ab+35b^2)\text{arctanh}(\sin(c+dx))}{8(a-b)^3}+\frac{3a-11b}{16(a-b)^2(1-\sin(c+dx))}-\frac{}{16(}}{d}$$

input
```
Int[Sec[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

output
```
((b^(5/4)*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sq
rt[b])^3) + ((3*a^2 - 6*a*b + 35*b^2)*ArcTanh[Sin[c + d*x]])/(8*(a - b)^3)
 - (b^(5/4)*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] -
 Sqrt[b])^3) + 1/(16*(a - b)*(1 - Sin[c + d*x])^2) + (3*a - 11*b)/(16*(a -
 b)^2*(1 - Sin[c + d*x])) - 1/(16*(a - b)*(1 + Sin[c + d*x])^2) - (3*a - 1
1*b)/(16*(a - b)^2*(1 + Sin[c + d*x])))/d
```

3.410.3.1 Defintions of rubi rules used

rule 1485
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.410. $\int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.410.4 Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.29

method	result
derivativedivides	$-\frac{1}{2(8a-8b)(1+\sin(dx+c))^2}-\frac{3a-11b}{16(a-b)^2(1+\sin(dx+c))}+\frac{\left(3a^2-6ab+35b^2\right)\ln(1+\sin(dx+c))}{16(a-b)^3}+\frac{1}{2(8a-8b)(\sin(dx+c)-1)^2}-\frac{3a}{16(a-b)^2(\text{s}}$
default	$-\frac{1}{2(8a-8b)(1+\sin(dx+c))^2}-\frac{3a-11b}{16(a-b)^2(1+\sin(dx+c))}+\frac{\left(3a^2-6ab+35b^2\right)\ln(1+\sin(dx+c))}{16(a-b)^3}+\frac{1}{2(8a-8b)(\sin(dx+c)-1)^2}-\frac{3a}{16(a-b)^2(\text{s}}$
risch	Expression too large to display

input
```
int(sec(d*x+c)^5/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/(8*a-8*b)/(1+sin(d*x+c))^2-1/16*(3*a-11*b)/(a-b)^2/(1+sin(d*x+c)
)+1/16*(3*a^2-6*a*b+35*b^2)/(a-b)^3*ln(1+sin(d*x+c))+1/2/(8*a-8*b)/(sin(d*
x+c)-1)^2-1/16*(3*a-11*b)/(a-b)^2/(sin(d*x+c)-1)+1/16/(a-b)^3*(-3*a^2+6*a*
b-35*b^2)*ln(sin(d*x+c)-1)+b^2/(a-b)^3*(1/4*(-3*a-b)*(1/b*a)^(1/4)/a*(ln((
sin(d*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b*a)^(1/4)))+2*arctan(sin(d*x+c)/
(1/b*a)^(1/4)))-1/4*(-a-3*b)/b/(1/b*a)^(1/4)*(2*arctan(sin(d*x+c)/(1/b*a)^
(1/4))-ln((sin(d*x+c)+(1/b*a)^(1/4))/(sin(d*x+c)-(1/b*a)^(1/4))))))
```

3.410.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3703 vs. $2(207) = 414$.

Time = 1.65 (sec) , antiderivative size = 3703, normalized size of antiderivative = 14.87

$$\int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/16*(4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*
b^5 + (a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 +
a*b^6)*d^2*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^
2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3
+ 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 -
220*a^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b
 + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))*cos(d*x
 + c)^4*log(1/2*(a^3*b^4 + 15*a^2*b^5 + 15*a*b^6 + b^7)*sin(d*x + c) + 1/2
*((a^10 - 3*a^9*b - 3*a^8*b^2 + 25*a^7*b^3 - 45*a^6*b^4 + 39*a^5*b^5 - 17*
a^4*b^6 + 3*a^3*b^7)*d^3*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^
3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 -
 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 +
495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)) -
(3*a^5*b^3 + 46*a^4*b^4 + 60*a^3*b^5 + 18*a^2*b^6 + a*b^7)*d)*sqrt((6*a^2*
b^3 + 20*a*b^4 + 6*b^5 + (a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3
*b^4 - 6*a^2*b^5 + a*b^6)*d^2*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 4
52*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*
b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b
^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4
)))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 ...
```

3.410.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a-b*sin(d*x+c)**4),x)
```

output
```
Integral(sec(c + d*x)**5/(a - b*sin(c + d*x)**4), x)
```

3.410.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.46

$$\int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$$
$$= \frac{\frac{4\,b^2\left(\frac{2\left(b\left(3\sqrt{a}-\sqrt{b}\right)+a^{\frac{3}{2}}-3\,a\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\sin(dx+c)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(b\left(3\sqrt{a}+\sqrt{b}\right)+a^{\frac{3}{2}}+3\,a\sqrt{b}\right)\log\left(\frac{\sqrt{b}\sin(dx+c)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin(dx+c)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{(3\,a^2-6\,ab+35\,b^2)\log(\sin(}{a^3-3\,a^2b+3\,ab^2-b}}{16\,d}$$

input
```
integrate(sec(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/16*(4*b^2*(2*(b*(3*sqrt(a) - sqrt(b)) + a^(3/2) - 3*a*sqrt(b))*arctan(sq
rt(b)*sin(d*x + c)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*s
qrt(b)) + (b*(3*sqrt(a) + sqrt(b)) + a^(3/2) + 3*a*sqrt(b))*log((sqrt(b)*s
in(d*x + c) - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*sin(d*x + c) + sqrt(sqrt(a)*
sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) + (3*a^2 - 6*a*b + 35*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b
+ 3*a*b^2 - b^3) - (3*a^2 - 6*a*b + 35*b^2)*log(sin(d*x + c) - 1)/(a^3 - 3
*a^2*b + 3*a*b^2 - b^3) - 2*((3*a - 11*b)*sin(d*x + c)^3 - (5*a - 13*b)*si
n(d*x + c))/((a^2 - 2*a*b + b^2)*sin(d*x + c)^4 - 2*(a^2 - 2*a*b + b^2)*si
n(d*x + c)^2 + a^2 - 2*a*b + b^2))/d
```

3.410.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(207) = 414$.

Time = 1.28 (sec) , antiderivative size = 630, normalized size of antiderivative = 2.53

$$\int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)}\,dx =$$
$$-\frac{\frac{8\left(\left(-ab^3\right)^{\frac{3}{4}}(a+3\,b)+\left(-ab^3\right)^{\frac{1}{4}}\left(3\,ab^2+b^3\right)\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^4b-3\,\sqrt{2}a^3b^2+3\,\sqrt{2}a^2b^3-\sqrt{2}ab^4}+\frac{8\left(\left(-ab^3\right)^{\frac{3}{4}}(a+3\,b)+\left(-ab^3\right)^{\frac{1}{4}}\left(3\,ab^2+b^3\right)\right)\arctan\left(\right.}{\sqrt{2}a^4b-3\,\sqrt{2}a^3b^2+3\,\sqrt{2}a^2b^3}}{}$$

input
```
integrate(sec(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
-1/16*(8*((-a*b^3)^(3/4)*(a + 3*b) + (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*arcta
n(1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(
2)*a^4*b - 3*sqrt(2)*a^3*b^2 + 3*sqrt(2)*a^2*b^3 - sqrt(2)*a*b^4) + 8*((-a
*b^3)^(3/4)*(a + 3*b) + (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*arctan(-1/2*sqrt(2
)*(sqrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)*a^4*b - 3
*sqrt(2)*a^3*b^2 + 3*sqrt(2)*a^2*b^3 - sqrt(2)*a*b^4) - 4*((-a*b^3)^(3/4)*
(a + 3*b) - (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*log(sin(d*x + c)^2 + sqrt(2)*(
-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(sqrt(2)*a^4*b - 3*sqrt(2)*a^3*b^2
+ 3*sqrt(2)*a^2*b^3 - sqrt(2)*a*b^4) + 4*((-a*b^3)^(3/4)*(a + 3*b) - (-a*b
^3)^(1/4)*(3*a*b^2 + b^3))*log(sin(d*x + c)^2 - sqrt(2)*(-a/b)^(1/4)*sin(d
*x + c) + sqrt(-a/b))/(sqrt(2)*a^4*b - 3*sqrt(2)*a^3*b^2 + 3*sqrt(2)*a^2*b
^3 - sqrt(2)*a*b^4) - (3*a^2 - 6*a*b + 35*b^2)*log(abs(sin(d*x + c) + 1))/
(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (3*a^2 - 6*a*b + 35*b^2)*log(abs(sin(d*x
 + c) - 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 2*(3*a*sin(d*x + c)^3 - 11*b
*sin(d*x + c)^3 - 5*a*sin(d*x + c) + 13*b*sin(d*x + c))/((a^2 - 2*a*b + b^
2)*(sin(d*x + c)^2 - 1)^2))/d
```

3.410.9 Mupad [B] (verification not implemented)

Time = 20.00 (sec) , antiderivative size = 12217, normalized size of antiderivative = 49.06

$$\int \frac{\sec^5(c + dx)}{a - b\sin^4(c + dx)}\, dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^5*(a - b*sin(c + d*x)^4)),x)
```

output
```
(atan(((((18064*a*b^13 + 256*b^14 + 119760*a^2*b^12 - 275888*a^3*b^11 + 11
6624*a^4*b^10 + 28848*a^5*b^9 - 13712*a^6*b^8 + 6768*a^7*b^7 - 720*a^8*b^6
)/(64*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^
4 - 56*a^5*b^3 + 28*a^6*b^2)) - (((4096*a*b^15 + 12288*a^2*b^14 - 251904*a
^3*b^13 + 1087488*a^4*b^12 - 2457600*a^5*b^11 + 3440640*a^6*b^10 - 3182592
*a^7*b^9 + 2002944*a^8*b^8 - 872448*a^9*b^7 + 266240*a^10*b^6 - 55296*a^11
*b^5 + 6144*a^12*b^4)/(64*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56
*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) - (sin(c + d*x)*(-(a^3*(
a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3
+ 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b
+ a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2)*(819
2*a^2*b^15 - 73728*a^3*b^14 + 286720*a^4*b^13 - 614400*a^5*b^12 + 737280*a
^6*b^11 - 344064*a^7*b^10 - 344064*a^8*b^9 + 737280*a^9*b^8 - 614400*a^10*
b^7 + 286720*a^11*b^6 - 73728*a^12*b^5 + 8192*a^13*b^4))/(16*(a^8 - 8*a^7*
b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28
*a^6*b^2)))*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*
a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))
/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a
^7*b^2)))^(1/2) - (sin(c + d*x)*(256*b^15 - 50464*a^2*b^13 + 190720*a^3*b^
12 - 280960*a^4*b^11 + 212736*a^5*b^10 - 111296*a^6*b^9 + 57088*a^7*b^8...
```

3.410. $\int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.411 $\int \frac{\cos^{10}(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.411.1 Optimal result

Integrand size = 24, antiderivative size = 252

$$\int \frac{\cos^{10}(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{17x}{16b}-\frac{4(a+b)x}{b^2}-\frac{(a+3b)x}{2b^2} -\frac{\left(\sqrt{a}-\sqrt{b}\right)^{9/2}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d} +\frac{\left(\sqrt{a}+\sqrt{b}\right)^{9/2}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d} -\frac{17\cos(c+dx)\sin(c+dx)}{16bd}-\frac{(a+3b)\cos(c+dx)\sin(c+dx)}{2b^2d} -\frac{17\cos^3(c+dx)\sin(c+dx)}{24bd}-\frac{\cos^5(c+dx)\sin(c+dx)}{6bd}$$

output
```
-17/16*x/b-4*(a+b)*x/b^2-1/2*(a+3*b)*x/b^2-17/16*cos(d*x+c)*sin(d*x+c)/b/d
-1/2*(a+3*b)*cos(d*x+c)*sin(d*x+c)/b^2/d-17/24*cos(d*x+c)^3*sin(d*x+c)/b/d
-1/6*cos(d*x+c)^5*sin(d*x+c)/b/d-1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*
x+c)/a^(1/4))*(a^(1/2)-b^(1/2))^(9/2)/a^(3/4)/b^(5/2)/d+1/2*arctan((a^(1/2
)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)+b^(1/2))^(9/2)/a^(3/4)/b^(5/
2)/d
```

3.411.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{10}(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{36b(24a+35b)(c+dx)-\frac{96\left(\sqrt{a}+\sqrt{b}\right)^5\sqrt{b}\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}}-\frac{96\left(\sqrt{a}-\sqrt{b}\right)^5\sqrt{b}\operatorname{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}}}{192b^3d}$$

input
```
Integrate[Cos[c + d*x]^10/(a - b*Sin[c + d*x]^4),x]
```

output
```
-1/192*(36*b*(24*a + 35*b)*(c + d*x) - (96*(Sqrt[a] + Sqrt[b])^5*Sqrt[b]*A
rcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt
[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) - (96*(Sqrt[a] - Sqrt[b])^5*Sqrt[b]*ArcTanh
[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*
Sqrt[-a + Sqrt[a]*Sqrt[b]]) + 3*b*(16*a + 95*b)*Sin[2*(c + d*x)] + 21*b^2*
Sin[4*(c + d*x)] + b^2*Sin[6*(c + d*x)])/(b^3*d)
```

3.411.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3703, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{10}(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(c+dx)^{10}}{a-b\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{ 3703}$$

$$\frac{\int \frac{1}{(\tan^2(c+dx)+1)^4((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 1484}$$

$$\frac{\int \left(\frac{-a-3b}{b^2(\tan^2(c+dx)+1)^2} - \frac{4(a+b)}{b^2(\tan^2(c+dx)+1)} + \frac{5a^2+10ba+b^2+4(a^2-b^2)\tan^2(c+dx)}{b^2((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)} - \frac{2}{b(\tan^2(c+dx)+1)^3} - \frac{1}{b(\tan^2(c+dx)+1)^4}\right) d\,\mathrm{t}}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{\left(\sqrt{a}-\sqrt{b}\right)^{9/2}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}} + \frac{\left(\sqrt{a}+\sqrt{b}\right)^{9/2}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}} - \frac{4(a+b)\arctan(\tan(c+dx))}{b^2} - \frac{(a+3b)\arctan(\mathrm{ta}}{2b^2}}{d}$$

input
```
Int[Cos[c + d*x]^10/(a - b*Sin[c + d*x]^4),x]
```

output
```
((-17*ArcTan[Tan[c + d*x]])/(16*b) - (4*(a + b)*ArcTan[Tan[c + d*x]])/b^2
- ((a + 3*b)*ArcTan[Tan[c + d*x]])/(2*b^2) - ((Sqrt[a] - Sqrt[b])^(9/2)*Ar
cTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(5/2))
+ ((Sqrt[a] + Sqrt[b])^(9/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])
/a^(1/4)])/(2*a^(3/4)*b^(5/2)) - Tan[c + d*x]/(6*b*(1 + Tan[c + d*x]^2)^3)
 - (17*Tan[c + d*x])/(24*b*(1 + Tan[c + d*x]^2)^2) - (17*Tan[c + d*x])/(16
*b*(1 + Tan[c + d*x]^2)) - ((a + 3*b)*Tan[c + d*x])/(2*b^2*(1 + Tan[c + d*
x]^2)))/d
```

3.411.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3703
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2
] && IntegerQ[p]
```

3.411.4 Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\frac{(a-b)\left(\frac{\left(4a\sqrt{ab}+4\sqrt{ab}\,b+a^2+6ab+b^2\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(4a\sqrt{ab}+4\sqrt{ab}\,b-a^2-6ab-b^2\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{b^2}}{d}$
default	$\frac{\frac{(a-b)\left(\frac{\left(4a\sqrt{ab}+4\sqrt{ab}\,b+a^2+6ab+b^2\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(4a\sqrt{ab}+4\sqrt{ab}\,b-a^2-6ab-b^2\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{b^2}}{d}$
risch	Expression too large to display

input
```
int(cos(d*x+c)^10/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^2*(a-b)*(1/2*(4*a*(a*b)^(1/2)+4*(a*b)^(1/2)*b+a^2+6*a*b+b^2)/(a*b
)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1
/2)-a)*(a-b))^(1/2))+1/2*(4*a*(a*b)^(1/2)+4*(a*b)^(1/2)*b-a^2-6*a*b-b^2)/(
a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(
1/2)+a)*(a-b))^(1/2)))-1/b^2*(((1/2*a+41/16*b)*tan(d*x+c)^5+(a+35/6*b)*tan
(d*x+c)^3+(1/2*a+55/16*b)*tan(d*x+c))/(1+tan(d*x+c)^2)^3+3/16*(24*a+35*b)*
arctan(tan(d*x+c))))
```

3.411.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2948 vs. 2(200) = 400.

Time = 1.63 (sec) , antiderivative size = 2948, normalized size of antiderivative = 11.70

$$\int \frac{\cos^{10}(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^10/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/48*(6*b^2*d*sqrt((a*b^5*d^2*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 2
1816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^
8)/(a^3*b^9*d^4)) - a^4 - 36*a^3*b - 126*a^2*b^2 - 84*a*b^3 - 9*b^4)/(a*b^
5*d^2))*log(9/4*a^8 + 12*a^7*b - 39*a^6*b^2 + 143/2*a^4*b^4 - 52*a^3*b^5 -
 3*a^2*b^6 + 8*a*b^7 + 1/4*b^8 - 1/4*(9*a^8 + 48*a^7*b - 156*a^6*b^2 + 286
*a^4*b^4 - 208*a^3*b^5 - 12*a^2*b^6 + 32*a*b^7 + b^8)*cos(d*x + c)^2 + 1/2
*(4*(a^4*b^7 + a^3*b^8)*d^3*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 218
16*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)
/(a^3*b^9*d^4))*cos(d*x + c)*sin(d*x + c) + (9*a^7*b^2 + 138*a^6*b^3 + 639
*a^5*b^4 + 876*a^4*b^5 + 343*a^3*b^6 + 42*a^2*b^7 + a*b^8)*d*cos(d*x + c)*
sin(d*x + c))*sqrt((a*b^5*d^2*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 2
1816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^
8)/(a^3*b^9*d^4)) - a^4 - 36*a^3*b - 126*a^2*b^2 - 84*a*b^3 - 9*b^4)/(a*b^
5*d^2)) + 1/4*(2*(a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d
^2*cos(d*x + c)^2 - (a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8
)*d^2)*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^
4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4))) - 6*
b^2*d*sqrt((a*b^5*d^2*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5
*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*
b^9*d^4)) - a^4 - 36*a^3*b - 126*a^2*b^2 - 84*a*b^3 - 9*b^4)/(a*b^5*d^2...
```

3.411.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{10}(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**10/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.411.7 Maxima [F]

$$\int \frac{\cos^{10}(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\cos\left(dx+c\right)^{10}}{b\sin\left(dx+c\right)^4-a}\,dx$$

input
```
integrate(cos(d*x+c)^10/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
-1/192*(192*b^2*d*integrate(-4*(4*(a^2*b + 10*a*b^2 + 5*b^3)*cos(6*d*x + 6
*c)^2 + 4*(72*a^3 + 53*a^2*b - 54*a*b^2 + 9*b^3)*cos(4*d*x + 4*c)^2 + 4*(a
^2*b + 10*a*b^2 + 5*b^3)*cos(2*d*x + 2*c)^2 + 4*(a^2*b + 10*a*b^2 + 5*b^3)
*sin(6*d*x + 6*c)^2 + 4*(72*a^3 + 53*a^2*b - 54*a*b^2 + 9*b^3)*sin(4*d*x +
 4*c)^2 + 2*(8*a^3 + 113*a^2*b + 50*a*b^2 - 27*b^3)*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + 4*(a^2*b + 10*a*b^2 + 5*b^3)*sin(2*d*x + 2*c)^2 - ((a^2*b +
10*a*b^2 + 5*b^3)*cos(6*d*x + 6*c) + 2*(9*a^2*b + 10*a*b^2 - 3*b^3)*cos(4*
d*x + 4*c) + (a^2*b + 10*a*b^2 + 5*b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c)
 - (a^2*b + 10*a*b^2 + 5*b^3 - 2*(8*a^3 + 113*a^2*b + 50*a*b^2 - 27*b^3)*c
os(4*d*x + 4*c) - 8*(a^2*b + 10*a*b^2 + 5*b^3)*cos(2*d*x + 2*c))*cos(6*d*x
 + 6*c) - 2*(9*a^2*b + 10*a*b^2 - 3*b^3 - (8*a^3 + 113*a^2*b + 50*a*b^2 -
27*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a^2*b + 10*a*b^2 + 5*b^3)*co
s(2*d*x + 2*c) - ((a^2*b + 10*a*b^2 + 5*b^3)*sin(6*d*x + 6*c) + 2*(9*a^2*b
 + 10*a*b^2 - 3*b^3)*sin(4*d*x + 4*c) + (a^2*b + 10*a*b^2 + 5*b^3)*sin(2*d
*x + 2*c))*sin(8*d*x + 8*c) + 2*((8*a^3 + 113*a^2*b + 50*a*b^2 - 27*b^3)*s
in(4*d*x + 4*c) + 4*(a^2*b + 10*a*b^2 + 5*b^3)*sin(2*d*x + 2*c))*sin(6*d*x
 + 6*c))/(b^4*cos(8*d*x + 8*c)^2 + 16*b^4*cos(6*d*x + 6*c)^2 + 16*b^4*cos(
2*d*x + 2*c)^2 + b^4*sin(8*d*x + 8*c)^2 + 16*b^4*sin(6*d*x + 6*c)^2 + 16*b
^4*sin(2*d*x + 2*c)^2 - 8*b^4*cos(2*d*x + 2*c) + b^4 + 4*(64*a^2*b^2 - 48*
a*b^3 + 9*b^4)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*b^4)*s...
```

3.411.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(200) = 400$.

Time = 1.01 (sec) , antiderivative size = 896, normalized size of antiderivative = 3.56

$$\int \frac{\cos^{10}(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$= \frac{\frac{24\left(15\sqrt{a^2-ab+\sqrt{ab}(a-b)}\,a^4b-62\sqrt{a^2-ab+\sqrt{ab}(a-b)}\,a^2b^3-16\sqrt{a^2-ab+\sqrt{ab}(a-b)}\,ab^4-\sqrt{a^2-ab+\sqrt{ab}(a-b)}\,b^5-3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}a \cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate(cos(d*x+c)^10/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
1/48*(24*(15*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b - 62*sqrt(a^2 - a*b
 + sqrt(a*b)*(a - b))*a^2*b^3 - 16*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b
^4 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*b^5 - 3*sqrt(a^2 - a*b + sqrt(a*b
)*(a - b))*sqrt(a*b)*a^4 - 24*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b
)*a^3*b + 46*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + 40*sq
rt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 + 5*sqrt(a^2 - a*b + sqr
t(a*b)*(a - b))*sqrt(a*b)*b^4)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(
d*x + c)/sqrt((a*b^2 + sqrt(a^2*b^4 - (a*b^2 - b^3)*a*b^2))/(a*b^2 - b^3))
))*abs(-a + b)/(3*a^5*b^3 - 12*a^4*b^4 + 14*a^3*b^5 - 4*a^2*b^6 - a*b^7) +
 24*(15*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b - 62*sqrt(a^2 - a*b - sq
rt(a*b)*(a - b))*a^2*b^3 - 16*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^4 -
sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^5 + 3*sqrt(a^2 - a*b - sqrt(a*b)*(a
- b))*sqrt(a*b)*a^4 + 24*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3
*b - 46*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 - 40*sqrt(a^
2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - 5*sqrt(a^2 - a*b - sqrt(a*b
)*(a - b))*sqrt(a*b)*b^4)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x +
 c)/sqrt((a*b^2 - sqrt(a^2*b^4 - (a*b^2 - b^3)*a*b^2))/(a*b^2 - b^3))))*ab
s(-a + b)/(3*a^5*b^3 - 12*a^4*b^4 + 14*a^3*b^5 - 4*a^2*b^6 - a*b^7) - 9*(d
*x + c)*(24*a + 35*b)/b^2 - (24*a*tan(d*x + c)^5 + 123*b*tan(d*x + c)^5 +
48*a*tan(d*x + c)^3 + 280*b*tan(d*x + c)^3 + 24*a*tan(d*x + c) + 165*b*...
```

3.411.9 Mupad [B] (verification not implemented)

Time = 18.63 (sec) , antiderivative size = 10319, normalized size of antiderivative = 40.95

$$\int \frac{\cos^{10}(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^10/(a - b*sin(c + d*x)^4),x)
```

output
```
(atan(((((tan(c + d*x)*(123962*a*b^10 - 3776*a^10*b - 128*a^11 + 11153*b^1
1 - 387826*a^2*b^9 + 2370*a^3*b^8 + 780960*a^4*b^7 - 444642*a^5*b^6 - 3875
34*a^6*b^5 + 261366*a^7*b^4 + 118095*a^8*b^3 - 74000*a^9*b^2))/(64*b^6) +
(3*(((9873*a*b^13)/16 + 8*b^14 + (198963*a^2*b^12)/16 - (13467*a^3*b^11)/8
 - (240165*a^4*b^10)/8 + (68805*a^5*b^9)/16 + (307047*a^6*b^8)/16 - 1929*a
^7*b^7 - 2766*a^8*b^6 - 152*a^9*b^5)/b^8 + (3*((3*((64*a*b^15 + 1216*a^2*b
^14 - 2064*a^3*b^13 - 480*a^4*b^12 + 1968*a^5*b^11 - 704*a^6*b^10)/b^8 - (
3*tan(c + d*x)*(a*24i + b*35i)*(49152*a^2*b^13 - 49152*a^3*b^12 - 49152*a^
4*b^11 + 49152*a^5*b^10))/(2048*b^8))*(a*24i + b*35i))/(32*b^2) - (tan(c +
 d*x)*(617264*a^2*b^11 - 1024*b^13 - 10240*a*b^12 + 46512*a^3*b^10 - 91953
6*a^4*b^9 - 469488*a^5*b^8 + 498944*a^6*b^7 + 232448*a^7*b^6 + 5120*a^8*b^
5))/(64*b^6))*(a*24i + b*35i))/(32*b^2))*(a*24i + b*35i))/(32*b^2))*(a*24i
 + b*35i)*3i)/(32*b^2) + (((tan(c + d*x)*(123962*a*b^10 - 3776*a^10*b - 12
8*a^11 + 11153*b^11 - 387826*a^2*b^9 + 2370*a^3*b^8 + 780960*a^4*b^7 - 444
642*a^5*b^6 - 387534*a^6*b^5 + 261366*a^7*b^4 + 118095*a^8*b^3 - 74000*a^9
*b^2))/(64*b^6) - (3*(((9873*a*b^13)/16 + 8*b^14 + (198963*a^2*b^12)/16 -
(13467*a^3*b^11)/8 - (240165*a^4*b^10)/8 + (68805*a^5*b^9)/16 + (307047*a^
6*b^8)/16 - 1929*a^7*b^7 - 2766*a^8*b^6 - 152*a^9*b^5)/b^8 + (3*((3*((64*a
*b^15 + 1216*a^2*b^14 - 2064*a^3*b^13 - 480*a^4*b^12 + 1968*a^5*b^11 - 704
*a^6*b^10)/b^8 + (3*tan(c + d*x)*(a*24i + b*35i)*(49152*a^2*b^13 - 4915...
```

3.411. $\int \frac{\cos^{10}(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.412 $\int \frac{\cos^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.412.1 Optimal result

Integrand size = 24, antiderivative size = 186

$$\int \frac{\cos^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{11x}{8b}-\frac{(a+3b)x}{b^2}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^{7/2}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{7/2}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2d}-\frac{11\cos(c+dx)\sin(c+dx)}{8bd}-\frac{\cos^3(c+dx)\sin(c+dx)}{4bd}$$

output
```
-11/8*x/b-(a+3*b)*x/b^2-11/8*cos(d*x+c)*sin(d*x+c)/b/d-1/4*cos(d*x+c)^3*si
n(d*x+c)/b/d+1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/
2)-b^(1/2))^(7/2)/a^(3/4)/b^2/d+1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x
+c)/a^(1/4))*(a^(1/2)+b^(1/2))^(7/2)/a^(3/4)/b^2/d
```

3.412.2 Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08

$$\int \frac{\cos^8(c+dx)}{a-b\sin^4(c+dx)}\,dx =$$

$$-\frac{4(8a+35b)(c+dx)-\frac{16\left(\sqrt{a}+\sqrt{b}\right)^4\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{16\left(\sqrt{a}-\sqrt{b}\right)^4\operatorname{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}}+24b\,\mathrm{si}}{32b^2d}$$

input
```
Integrate[Cos[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

output
```
-1/32*(4*(8*a + 35*b)*(c + d*x) - (16*(Sqrt[a] + Sqrt[b])^4*ArcTan[((Sqrt[
a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a +
Sqrt[a]*Sqrt[b]]) + (16*(Sqrt[a] - Sqrt[b])^4*ArcTanh[((Sqrt[a] - Sqrt[b])
*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqr
t[b]]) + 24*b*Sin[2*(c + d*x)] + b*Sin[4*(c + d*x)])/(b^2*d)
```

3.412.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3703, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(c+dx)^8}{a-b\sin(c+dx)^4}dx$$

$$\Big\downarrow \; 3703$$

$$\frac{\int \frac{1}{\left(\tan^2(c+dx)+1\right)^3\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \; 1484$$

$$\frac{\int \left(\frac{-a-3b}{b^2(\tan^2(c+dx)+1)} + \frac{a^2+6ba+b^2+(a-b)(a+3b)\tan^2(c+dx)}{b^2((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)} - \frac{2}{b(\tan^2(c+dx)+1)^2} - \frac{1}{b(\tan^2(c+dx)+1)^3}\right) d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\left(\sqrt{a}-\sqrt{b}\right)^{7/2}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2} + \frac{\left(\sqrt{a}+\sqrt{b}\right)^{7/2}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2} - \frac{(a+3b)\arctan(\tan(c+dx))}{b^2} - \frac{11\arctan(\tan(c+d}{8b}}{d}$$

input
```
Int[Cos[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

output
```
((-11*ArcTan[Tan[c + d*x]])/(8*b) - ((a + 3*b)*ArcTan[Tan[c + d*x]])/b^2 +
 ((Sqrt[a] - Sqrt[b])^(7/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/
a^(1/4)])/(2*a^(3/4)*b^2) + ((Sqrt[a] + Sqrt[b])^(7/2)*ArcTan[(Sqrt[Sqrt[a
] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^2) - Tan[c + d*x]/(4*b*(
1 + Tan[c + d*x]^2)^2) - (11*Tan[c + d*x])/(8*b*(1 + Tan[c + d*x]^2)))/d
```

3.412.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3703
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2
] && IntegerQ[p]
```

3.412. $\int \frac{\cos^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.412.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{(a-b)\left(\frac{\left(a\sqrt{ab}+3\sqrt{ab}\,b+3ab+b^2\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(a\sqrt{ab}+3\sqrt{ab}\,b-3ab-b^2\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{b^2}-\frac{11(\mathrm{ta}}{}}{d}$
default	$\frac{\frac{(a-b)\left(\frac{\left(a\sqrt{ab}+3\sqrt{ab}\,b+3ab+b^2\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(a\sqrt{ab}+3\sqrt{ab}\,b-3ab-b^2\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{b^2}-\frac{11(\mathrm{ta}}{}}{d}$
risch	Expression too large to display

input
```
int(cos(d*x+c)^8/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^2*(a-b)*(1/2*(a*(a*b)^(1/2)+3*(a*b)^(1/2)*b+3*a*b+b^2)/(a*b)^(1/2
)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)
*(a-b))^(1/2))+1/2*(a*(a*b)^(1/2)+3*(a*b)^(1/2)*b-3*a*b-b^2)/(a*b)^(1/2)/(
((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b
))^(1/2)))-1/b^2*((11/8*tan(d*x+c)^3*b+13/8*tan(d*x+c)*b)/(1+tan(d*x+c)^2)
^2+1/8*(35*b+8*a)*arctan(tan(d*x+c))))
```

3.412.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2433 vs. 2(144) = 288.

Time = 0.97 (sec) , antiderivative size = 2433, normalized size of antiderivative = 13.08

$$\int \frac{\cos^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/8*(b^2*d*sqrt(-(a*b^4*d^2*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484
*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4)) + a^3 + 21*a^2*b +
 35*a*b^2 + 7*b^3)/(a*b^4*d^2))*log(7/4*a^6 + 7/2*a^5*b - 63/4*a^4*b^2 + 9
*a^3*b^3 + 25/4*a^2*b^4 - 9/2*a*b^5 - 1/4*b^6 - 1/4*(7*a^6 + 14*a^5*b - 63
*a^4*b^2 + 36*a^3*b^3 + 25*a^2*b^4 - 18*a*b^5 - b^6)*cos(d*x + c)^2 + 1/2*
((a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*
a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))*cos(d*x + c)*sin(d*
x + c) - (21*a^5*b^2 + 112*a^4*b^3 + 98*a^3*b^4 + 24*a^2*b^5 + a*b^6)*d*co
s(d*x + c)*sin(d*x + c))*sqrt(-(a*b^4*d^2*sqrt((49*a^6 + 490*a^5*b + 1519*
a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4)) + a^
3 + 21*a^2*b + 35*a*b^2 + 7*b^3)/(a*b^4*d^2)) - 1/4*(2*(a^5*b^3 - 3*a^4*b^
4 + 3*a^3*b^5 - a^2*b^6)*d^2*cos(d*x + c)^2 - (a^5*b^3 - 3*a^4*b^4 + 3*a^3
*b^5 - a^2*b^6)*d^2)*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^
3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))) - b^2*d*sqrt(-(a*b^4*d^2
*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42
*a*b^5 + b^6)/(a^3*b^7*d^4)) + a^3 + 21*a^2*b + 35*a*b^2 + 7*b^3)/(a*b^4*d
^2))*log(7/4*a^6 + 7/2*a^5*b - 63/4*a^4*b^2 + 9*a^3*b^3 + 25/4*a^2*b^4 - 9
/2*a*b^5 - 1/4*b^6 - 1/4*(7*a^6 + 14*a^5*b - 63*a^4*b^2 + 36*a^3*b^3 + 25*
a^2*b^4 - 18*a*b^5 - b^6)*cos(d*x + c)^2 - 1/2*((a^4*b^5 + 3*a^3*b^6)*d^3*
sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + ...
```

3.412.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**8/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.412.7 Maxima [F]

$$\int \frac{\cos^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\cos(dx+c)^8}{b\sin(dx+c)^4-a}\,dx$$

input
```
integrate(cos(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
-1/32*(32*b^2*d*integrate(-16*(4*(a*b^2 + b^3)*cos(6*d*x + 6*c)^2 + 2*(8*a
^3 + 29*a^2*b - 20*a*b^2 + 3*b^3)*cos(4*d*x + 4*c)^2 + 4*(a*b^2 + b^3)*cos
(2*d*x + 2*c)^2 + 4*(a*b^2 + b^3)*sin(6*d*x + 6*c)^2 + 2*(8*a^3 + 29*a^2*b
 - 20*a*b^2 + 3*b^3)*sin(4*d*x + 4*c)^2 + 2*(10*a^2*b + 13*a*b^2 - 5*b^3)*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(a*b^2 + b^3)*sin(2*d*x + 2*c)^2 - (
(a*b^2 + b^3)*cos(6*d*x + 6*c) + (a^2*b + 4*a*b^2 - b^3)*cos(4*d*x + 4*c)
+ (a*b^2 + b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - (a*b^2 + b^3 - 2*(10*
a^2*b + 13*a*b^2 - 5*b^3)*cos(4*d*x + 4*c) - 8*(a*b^2 + b^3)*cos(2*d*x + 2
*c))*cos(6*d*x + 6*c) - (a^2*b + 4*a*b^2 - b^3 - 2*(10*a^2*b + 13*a*b^2 -
5*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a*b^2 + b^3)*cos(2*d*x + 2*c)
 - ((a*b^2 + b^3)*sin(6*d*x + 6*c) + (a^2*b + 4*a*b^2 - b^3)*sin(4*d*x + 4
*c) + (a*b^2 + b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*((10*a^2*b + 13
*a*b^2 - 5*b^3)*sin(4*d*x + 4*c) + 4*(a*b^2 + b^3)*sin(2*d*x + 2*c))*sin(6
*d*x + 6*c))/(b^4*cos(8*d*x + 8*c)^2 + 16*b^4*cos(6*d*x + 6*c)^2 + 16*b^4*
cos(2*d*x + 2*c)^2 + b^4*sin(8*d*x + 8*c)^2 + 16*b^4*sin(6*d*x + 6*c)^2 +
16*b^4*sin(2*d*x + 2*c)^2 - 8*b^4*cos(2*d*x + 2*c) + b^4 + 4*(64*a^2*b^2 -
 48*a*b^3 + 9*b^4)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*b^4)*
sin(4*d*x + 4*c)^2 + 16*(8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) - 2*(4*b^4*cos(6*d*x + 6*c) + 4*b^4*cos(2*d*x + 2*c) - b^4 + 2*(8*a*b^3
- 3*b^4)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*b^4*cos(2*d*x + 2*c)...
```

3.412.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 836 vs. $2(144) = 288$.

Time = 0.95 (sec) , antiderivative size = 836, normalized size of antiderivative = 4.49

$$\int \frac{\cos^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$= \frac{\frac{4\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}a^4+12\sqrt{a^2-ab+\sqrt{ab}(a-b)}a^3b-34\sqrt{a^2-ab+\sqrt{ab}(a-b)}a^2b^2-12\sqrt{a^2-ab+\sqrt{ab}(a-b)}ab^3-\sqrt{a^2-ab+\sqrt{ab}(a-b)}b^4-12\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
1/8*(4*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4 + 12*sqrt(a^2 - a*b + sq
rt(a*b)*(a - b))*a^3*b - 34*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^2 -
12*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*b^4 - 12*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 + 12*s
qrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 28*sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 + 4*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
sqrt(a*b)*b^3)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a
*b^2 + sqrt(a^2*b^4 - (a*b^2 - b^3)*a*b^2))/(a*b^2 - b^3))))*abs(-a + b)/(
3*a^5*b^2 - 12*a^4*b^3 + 14*a^3*b^4 - 4*a^2*b^5 - a*b^6) + 4*(3*sqrt(a^2 -
 a*b - sqrt(a*b)*(a - b))*a^4 + 12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3
*b - 34*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^2 - 12*sqrt(a^2 - a*b -
sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^4 + 12*sq
rt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 12*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*sqrt(a*b)*a^2*b - 28*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sq
rt(a*b)*a*b^2 - 4*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(pi*f
loor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b^2 - sqrt(a^2*b^4
- (a*b^2 - b^3)*a*b^2))/(a*b^2 - b^3))))*abs(-a + b)/(3*a^5*b^2 - 12*a^4*b
^3 + 14*a^3*b^4 - 4*a^2*b^5 - a*b^6) - (d*x + c)*(8*a + 35*b)/b^2 - (11*ta
n(d*x + c)^3 + 13*tan(d*x + c))/((tan(d*x + c)^2 + 1)^2*b))/d
```

3.412.9 Mupad [B] (verification not implemented)

Time = 16.88 (sec) , antiderivative size = 8773, normalized size of antiderivative = 47.17

$$\int \frac{\cos^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^8/(a - b*sin(c + d*x)^4),x)
```

output
```
- (atan(((((373728*a^3*b^8 - 256*b^11 - 208208*a^2*b^9 - 17552*a*b^10 + 35
296*a^4*b^7 - 240464*a^5*b^6 + 29040*a^6*b^5 + 27648*a^7*b^4 + 768*a^8*b^3
)/(64*b^5) - (((4096*a*b^12 + 53248*a^2*b^11 - 129024*a^3*b^10 + 69632*a^4
*b^9 + 14336*a^5*b^8 - 12288*a^6*b^7)/(64*b^5) - (tan(c + d*x)*(-(7*a^3*(a
^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5
+ a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b
^8))^(1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^
8))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 +
 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(
a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) + (tan(c + d*x)*(256*a*b^10 + 256*b^11
 - 70832*a^2*b^9 + 61136*a^3*b^8 + 53616*a^4*b^7 - 12432*a^5*b^6 - 29696*a
^6*b^5 - 2304*a^7*b^4))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)
^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9
)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2))*(-(7*a^3*(a^3*b^9
)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*
b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(
1/2) + (tan(c + d*x)*(336*a^8*b - 1497*a*b^8 + 96*a^9 - 1257*b^9 + 21499*a
^2*b^7 - 41861*a^3*b^6 + 27109*a^4*b^5 + 3077*a^5*b^4 - 9223*a^6*b^3 + 172
1*a^7*b^2))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a
^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) +...
```

3.412. $\int \frac{\cos^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.413 $\int \frac{\cos^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.413.1 Optimal result

Integrand size = 24, antiderivative size = 155

$$\int \frac{\cos^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{5x}{2b} - \frac{\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}d} + \frac{\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd}$$

output
```
-5/2*x/b-1/2*cos(d*x+c)*sin(d*x+c)/b/d-1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*
tan(d*x+c)/a^(1/4))*(a^(1/2)-b^(1/2))^(5/2)/a^(3/4)/b^(3/2)/d+1/2*arctan((
a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)+b^(1/2))^(5/2)/a^(3/4)
/b^(3/2)/d
```

3.413.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.25

$$\int \frac{\cos^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$$
$$= \frac{-10b(c+dx)+\frac{2(\sqrt{a}+\sqrt{b})^3\sqrt{b}\arctan\left(\frac{(\sqrt{a}+\sqrt{b})\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{2(\sqrt{a}-\sqrt{b})^3\sqrt{b}\text{arctanh}\left(\frac{(\sqrt{a}-\sqrt{b})\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}}-b\sin(2(c+dx))}{4b^2d}$$

input
```
Integrate[Cos[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

output
```
(-10*b*(c + d*x) + (2*(Sqrt[a] + Sqrt[b])^3*Sqrt[b]*ArcTan[((Sqrt[a] + Sqr
t[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*
Sqrt[b]]) + (2*(Sqrt[a] - Sqrt[b])^3*Sqrt[b]*ArcTanh[((Sqrt[a] - Sqrt[b])*
Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt
[b]]) - b*Sin[2*(c + d*x)])/(4*b^2*d)
```

3.413.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3703, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\cos(c+dx)^6}{a-b\sin(c+dx)^4}dx$$
$$\Big\downarrow \; 3703$$
$$\frac{\int \frac{1}{(\tan^2(c+dx)+1)^2((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}d\tan(c+dx)}{d}$$
$$\Big\downarrow \; 1484$$

$$\frac{\int \left(\frac{2(a-b)\tan^2(c+dx)+3a+b}{b((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)} - \frac{2}{b(\tan^2(c+dx)+1)} - \frac{1}{b(\tan^2(c+dx)+1)^2}\right) d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}} + \frac{\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}} - \frac{5\arctan(\tan(c+dx))}{2b} - \frac{\tan(c+dx)}{2b(\tan^2(c+dx)+1)}}{d}$$

input
```
Int[Cos[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

output
```
((-5*ArcTan[Tan[c + d*x]])/(2*b) - ((Sqrt[a] - Sqrt[b])^(5/2)*ArcTan[(Sqrt
[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(3/2)) + ((Sqrt[a
] + Sqrt[b])^(5/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])
/(2*a^(3/4)*b^(3/2)) - Tan[c + d*x]/(2*b*(1 + Tan[c + d*x]^2)))/d
```

3.413.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3703
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2
] && IntegerQ[p]
```

3.413. $\int \frac{\cos^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.413.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{\frac{(a-b)\left(\frac{\left(a+b+2\sqrt{ab}\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(-a-b+2\sqrt{ab}\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{b}-\frac{\frac{\tan(dx+c)}{2+2\left(\tan^2(dx+c)\right)}+\frac{5\arctan}{}}{b}}{d}$
default	$\frac{\frac{(a-b)\left(\frac{\left(a+b+2\sqrt{ab}\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(-a-b+2\sqrt{ab}\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{b}-\frac{\frac{\tan(dx+c)}{2+2\left(\tan^2(dx+c)\right)}+\frac{5\arctan}{}}{b}}{d}$
risch	$-\frac{5x}{2b}+\frac{i e^{2i(dx+c)}}{8bd}-\frac{i e^{-2i(dx+c)}}{8bd}+\left(\sum_{\_R=\operatorname{RootOf}\left(256a^3b^6d^4\_Z^4+(32a^4b^3d^2+320a^3b^4d^2+160a^2b^5d^2)\_Z^2+a^5\right.}\right.$

input
```
int(cos(d*x+c)^6/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b*(a-b)*(1/2*(a+b+2*(a*b)^(1/2))/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b)
)^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2*(-a-b
+2*(a*b)^(1/2))/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan
(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2)))-1/b*(1/2*tan(d*x+c)/(1+tan(d*x+c)^
2)+5/2*arctan(tan(d*x+c))))
```

3.413.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1751 vs. 2(111) = 222.

Time = 0.74 (sec) , antiderivative size = 1751, normalized size of antiderivative = 11.30

$$\int \frac{\cos^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/8*(b*d*sqrt((a*b^3*d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3
 + b^4)/(a^3*b^5*d^4)) - a^2 - 10*a*b - 5*b^2)/(a*b^3*d^2))*log(5/4*a^4 -
7/2*a^2*b^2 + 2*a*b^3 + 1/4*b^4 - 1/4*(5*a^4 - 14*a^2*b^2 + 8*a*b^3 + b^4)
*cos(d*x + c)^2 + 1/2*(2*a^3*b^4*d^3*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^
2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))*cos(d*x + c)*sin(d*x + c) + (5*a^4*b +
15*a^3*b^2 + 11*a^2*b^3 + a*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^3*
d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4)
) - a^2 - 10*a*b - 5*b^2)/(a*b^3*d^2)) + 1/4*(2*(a^4*b^2 - 2*a^3*b^3 + a^2
*b^4)*d^2*cos(d*x + c)^2 - (a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2)*sqrt((25*a
^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))) - b*d*sqrt(
(a*b^3*d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b
^5*d^4)) - a^2 - 10*a*b - 5*b^2)/(a*b^3*d^2))*log(5/4*a^4 - 7/2*a^2*b^2 +
2*a*b^3 + 1/4*b^4 - 1/4*(5*a^4 - 14*a^2*b^2 + 8*a*b^3 + b^4)*cos(d*x + c)^
2 - 1/2*(2*a^3*b^4*d^3*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 +
 b^4)/(a^3*b^5*d^4))*cos(d*x + c)*sin(d*x + c) + (5*a^4*b + 15*a^3*b^2 + 1
1*a^2*b^3 + a*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^3*d^2*sqrt((25*a
^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4)) - a^2 - 10*a
*b - 5*b^2)/(a*b^3*d^2)) + 1/4*(2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2*cos(
d*x + c)^2 - (a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2)*sqrt((25*a^4 + 100*a^3*b
 + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))) + b*d*sqrt(-(a*b^3*d^2...
```

3.413.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.413.7 Maxima [F]

$$\int \frac{\cos^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\cos(dx+c)^6}{b\sin(dx+c)^4-a}\,dx$$

input
```
integrate(cos(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
-1/4*(4*b*d*integrate(-4*(4*(a*b + 3*b^2)*cos(6*d*x + 6*c)^2 + 4*(40*a^2 -
 23*a*b + 3*b^2)*cos(4*d*x + 4*c)^2 + 4*(a*b + 3*b^2)*cos(2*d*x + 2*c)^2 +
 4*(a*b + 3*b^2)*sin(6*d*x + 6*c)^2 + 4*(40*a^2 - 23*a*b + 3*b^2)*sin(4*d*
x + 4*c)^2 + 2*(8*a^2 + 41*a*b - 13*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 4*(a*b + 3*b^2)*sin(2*d*x + 2*c)^2 - ((a*b + 3*b^2)*cos(6*d*x + 6*c) +
2*(5*a*b - b^2)*cos(4*d*x + 4*c) + (a*b + 3*b^2)*cos(2*d*x + 2*c))*cos(8*d
*x + 8*c) - (a*b + 3*b^2 - 2*(8*a^2 + 41*a*b - 13*b^2)*cos(4*d*x + 4*c) -
8*(a*b + 3*b^2)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 2*(5*a*b - b^2 - (8*a
^2 + 41*a*b - 13*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a*b + 3*b^2)*c
os(2*d*x + 2*c) - ((a*b + 3*b^2)*sin(6*d*x + 6*c) + 2*(5*a*b - b^2)*sin(4*
d*x + 4*c) + (a*b + 3*b^2)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*((8*a^2
+ 41*a*b - 13*b^2)*sin(4*d*x + 4*c) + 4*(a*b + 3*b^2)*sin(2*d*x + 2*c))*si
n(6*d*x + 6*c))/(b^3*cos(8*d*x + 8*c)^2 + 16*b^3*cos(6*d*x + 6*c)^2 + 16*b
^3*cos(2*d*x + 2*c)^2 + b^3*sin(8*d*x + 8*c)^2 + 16*b^3*sin(6*d*x + 6*c)^2
 + 16*b^3*sin(2*d*x + 2*c)^2 - 8*b^3*cos(2*d*x + 2*c) + b^3 + 4*(64*a^2*b
- 48*a*b^2 + 9*b^3)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*s
in(4*d*x + 4*c)^2 + 16*(8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 - 2*(4*b^3*cos(6*d*x + 6*c) + 4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 -
 3*b^3)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*b^3*cos(2*d*x + 2*c) - b
^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^...
```

3.413.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 995 vs. $2(111) = 222$.

Time = 0.86 (sec) , antiderivative size = 995, normalized size of antiderivative = 6.42

$$\int \frac{\cos^6(c+dx)}{a-b\sin^4(c+dx)}\,dx =$$

$$-\frac{\frac{5(dx+c)}{b} + \frac{\left(2\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}a^2-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}ab-\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}b^2\right)b^2|-a+b|-\left(9\sqrt{a^2-ab+\sqrt{ab}(a-\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate(cos(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
-1/2*(5*(d*x + c)/b + (2*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*
a^2 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b
 + sqrt(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) - (9*sqrt(a^2 - a*b +
 sqrt(a*b)*(a - b))*a^3*b - 15*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^2
 - 9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b + sqrt(a*b
)*(a - b))*b^4)*abs(-a + b)*abs(b) + (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b)
)*sqrt(a*b)*a^3*b - 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^
2 - 7*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - sqrt(a^2 - a*b
 + sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*abs(-a + b))*(pi*floor((d*x + c)/pi +
 1/2) + arctan(tan(d*x + c)/sqrt((a*b + sqrt(a^2*b^2 - (a*b - b^2)*a*b))/(
a*b - b^2))))/((3*a^5*b^2 - 12*a^4*b^3 + 14*a^3*b^4 - 4*a^2*b^5 - a*b^6)*a
bs(b)) + (2*(3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2 - 6*sqrt(
a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b - sqrt(a*b)*
(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) - (9*sqrt(a^2 - a*b - sqrt(a*b)*(a
 - b))*a^3*b - 15*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^2 - 9*sqrt(a^2
 - a*b - sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^
4)*abs(-a + b)*abs(b) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a
^3*b - 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 - 7*sqrt(a^
2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - sqrt(a^2 - a*b - sqrt(a*b)*
(a - b))*sqrt(a*b)*b^4)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + ar...
```

3.413.9 Mupad [B] (verification not implemented)

Time = 16.55 (sec) , antiderivative size = 3088, normalized size of antiderivative = 19.92

$$\int \frac{\cos^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^6/(a - b*sin(c + d*x)^4),x)
```

output
```
(atan((a^4*b^8*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2)
 + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6)
)^(1/2)*240i - a^3*b^9*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^
7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*
a^3*b^6))^(1/2)*108i - a^5*b^7*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2
*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/
2))/(16*a^3*b^6))^(1/2)*80i - a^6*b^6*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2
) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b
^7)^(1/2))/(16*a^3*b^6))^(1/2)*120i + a^7*b^5*sin(c + d*x)*(-(5*a^2*(a^3*b
^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*
b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*60i + a^8*b^4*sin(c + d*x)*(-(5*a^2
*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3
+ 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*8i - a^3*b^11*sin(c + d*x)*(
-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a
^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(3/2)*64i + a^4*b^10*sin(c
+ d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3
*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(3/2)*128i + a^5*b^
9*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5
+ 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(3/2)*6080i
 + a^6*b^8*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) ...
```

3.413. $\int \frac{\cos^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.414 $\int \frac{\cos^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.414.1 Optimal result

Integrand size = 24, antiderivative size = 127

$$\int \frac{\cos^4(c+dx)}{a-b\sin^4(c+dx)}\,dx=-\frac{x}{b}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}bd}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}bd}$$

output
```
-x/b+1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)-b^(1/
2))^(3/2)/a^(3/4)/b/d+1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4
))*(a^(1/2)+b^(1/2))^(3/2)/a^(3/4)/b/d
```

3.414.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.35

$$\int \frac{\cos^4(c+dx)}{a-b\sin^4(c+dx)}\,dx=\frac{-2(c+dx)+\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\operatorname{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}}}{2bd}$$

input
```
Integrate[Cos[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

output
```
(-2*(c + d*x) + ((Sqrt[a] + Sqrt[b])^2*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c +
 d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) - (
(Sqrt[a] - Sqrt[b])^2*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a +
 Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]))/(2*b*d)
```

3.414.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3703, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^4}{a-b\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{3703}$$

$$\frac{\int \frac{1}{(\tan^2(c+dx)+1)((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1484}$$

$$\frac{\int \left(\frac{(a-b)\tan^2(c+dx)+a+b}{b((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}-\frac{1}{b(\tan^2(c+dx)+1)}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b}-\frac{\arctan(\tan(c+dx))}{b}}{d}$$

input
```
Int[Cos[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

output
```
(-(ArcTan[Tan[c + d*x]]/b) + ((Sqrt[a] - Sqrt[b])^(3/2)*ArcTan[(Sqrt[Sqrt[
a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b) + ((Sqrt[a] + Sqrt[b])
^(3/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*
b))/d
```

3.414.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3703
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2
] && IntegerQ[p]
```

3.414.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(dx+c))}{b}+\frac{(a-b)\left(\frac{(\sqrt{ab}-b)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{(\sqrt{ab}+a)(a-b)}}\right)}{2\sqrt{ab}\sqrt{(\sqrt{ab}+a)(a-b)}}+\frac{(\sqrt{ab}+b)\,\mathrm{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{(\sqrt{ab}-a)(a-b)}}\right)}{2\sqrt{ab}\sqrt{(\sqrt{ab}-a)(a-b)}}\right)}{b}}{d}$
default	$\frac{-\frac{\arctan(\tan(dx+c))}{b}+\frac{(a-b)\left(\frac{(\sqrt{ab}-b)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{(\sqrt{ab}+a)(a-b)}}\right)}{2\sqrt{ab}\sqrt{(\sqrt{ab}+a)(a-b)}}+\frac{(\sqrt{ab}+b)\,\mathrm{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{(\sqrt{ab}-a)(a-b)}}\right)}{2\sqrt{ab}\sqrt{(\sqrt{ab}-a)(a-b)}}\right)}{b}}{d}$
risch	$-\frac{x}{b}+\left(\sum_{\_R=\mathrm{RootOf}(256a^3b^4d^4\_Z^4+(32a^3b^2d^2+96d^2b^3a^2)\_Z^2+a^3-3a^2b+3a\,b^2-b^3)}\_R\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{1}{}\right.\right.$

input
```
int(cos(d*x+c)^4/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b*arctan(tan(d*x+c))+1/b*(a-b)*(1/2*((a*b)^(1/2)-b)/(a*b)^(1/2)/((
(a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b)
)^(1/2))+1/2*((a*b)^(1/2)+b)/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arc
tanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))))
```

3.414.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1197 vs. 2(91) = 182.

Time = 0.56 (sec) , antiderivative size = 1197, normalized size of antiderivative = 9.43

$$\int \frac{\cos^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/8*(b*sqrt((a*b^2*d^2*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) - a - 3*b
)/(a*b^2*d^2))*log(1/4*(3*a^2 - 2*a*b - b^2)*cos(d*x + c)^2 - 3/4*a^2 + 1/
2*a*b + 1/4*b^2 + 1/2*(a^3*b^2*d^3*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4
))*cos(d*x + c)*sin(d*x + c) + (3*a^2*b + a*b^2)*d*cos(d*x + c)*sin(d*x +
c))*sqrt((a*b^2*d^2*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) - a - 3*b)/(
a*b^2*d^2)) - 1/4*(2*(a^3*b - a^2*b^2)*d^2*cos(d*x + c)^2 - (a^3*b - a^2*b
^2)*d^2)*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4))) - b*sqrt((a*b^2*d^2*sq
rt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) - a - 3*b)/(a*b^2*d^2))*log(1/4*(3
*a^2 - 2*a*b - b^2)*cos(d*x + c)^2 - 3/4*a^2 + 1/2*a*b + 1/4*b^2 - 1/2*(a^
3*b^2*d^3*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4))*cos(d*x + c)*sin(d*x +
 c) + (3*a^2*b + a*b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^2*d^2*sqrt(
(9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) - a - 3*b)/(a*b^2*d^2)) - 1/4*(2*(a^3
*b - a^2*b^2)*d^2*cos(d*x + c)^2 - (a^3*b - a^2*b^2)*d^2)*sqrt((9*a^2 + 6*
a*b + b^2)/(a^3*b^3*d^4))) + b*sqrt(-(a*b^2*d^2*sqrt((9*a^2 + 6*a*b + b^2)
/(a^3*b^3*d^4)) + a + 3*b)/(a*b^2*d^2))*log(-1/4*(3*a^2 - 2*a*b - b^2)*cos
(d*x + c)^2 + 3/4*a^2 - 1/2*a*b - 1/4*b^2 + 1/2*(a^3*b^2*d^3*sqrt((9*a^2 +
 6*a*b + b^2)/(a^3*b^3*d^4))*cos(d*x + c)*sin(d*x + c) - (3*a^2*b + a*b^2)
*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a*b^2*d^2*sqrt((9*a^2 + 6*a*b + b^2)/
(a^3*b^3*d^4)) + a + 3*b)/(a*b^2*d^2)) - 1/4*(2*(a^3*b - a^2*b^2)*d^2*cos(
d*x + c)^2 - (a^3*b - a^2*b^2)*d^2)*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3...
```

3.414.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.414.7 Maxima [F]

$$\int \frac{\cos^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\cos{(dx+c)}^4}{b\sin{(dx+c)}^4-a}\,dx$$

input
```
integrate(cos(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
-(b*integrate(-8*(4*b^2*cos(6*d*x + 6*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + 4*
b^2*sin(6*d*x + 6*c)^2 + 4*b^2*sin(2*d*x + 2*c)^2 + 4*(8*a^2 - 3*a*b)*cos(
4*d*x + 4*c)^2 - b^2*cos(2*d*x + 2*c) + 4*(8*a^2 - 3*a*b)*sin(4*d*x + 4*c)
^2 + 6*(4*a*b - b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (b^2*cos(6*d*x +
6*c) + 2*a*b*cos(4*d*x + 4*c) + b^2*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + (
8*b^2*cos(2*d*x + 2*c) - b^2 + 6*(4*a*b - b^2)*cos(4*d*x + 4*c))*cos(6*d*x
 + 6*c) - 2*(a*b - 3*(4*a*b - b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (b
^2*sin(6*d*x + 6*c) + 2*a*b*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*sin(8
*d*x + 8*c) + 2*(4*b^2*sin(2*d*x + 2*c) + 3*(4*a*b - b^2)*sin(4*d*x + 4*c)
)*sin(6*d*x + 6*c))/(b^3*cos(8*d*x + 8*c)^2 + 16*b^3*cos(6*d*x + 6*c)^2 +
16*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(8*d*x + 8*c)^2 + 16*b^3*sin(6*d*x + 6*
c)^2 + 16*b^3*sin(2*d*x + 2*c)^2 - 8*b^3*cos(2*d*x + 2*c) + b^3 + 4*(64*a^
2*b - 48*a*b^2 + 9*b^3)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b - 48*a*b^2 + 9*b^
3)*sin(4*d*x + 4*c)^2 + 16*(8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) - 2*(4*b^3*cos(6*d*x + 6*c) + 4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b
^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*b^3*cos(2*d*x + 2*c)
 - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b
^2 - 3*b^3 - 4*(8*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2
*b^3*sin(6*d*x + 6*c) + 2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d
*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3...
```

3.414.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 906 vs. $2(91) = 182$.

Time = 0.80 (sec) , antiderivative size = 906, normalized size of antiderivative = 7.13

$$\int \frac{\cos^4(c+dx)}{a-b\sin^4(c+dx)}\,dx =$$

$$-\frac{\frac{2\,(dx+c)}{b} - \frac{\left(\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}a^2-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}ab-\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}b^2\right)b^2|-a+b|+\left(3\sqrt{a^2-ab+\sqrt{ab}(a-\cdots\right.\right.}{\cdots}}{\cdots}$$

input
```
integrate(cos(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
-1/2*(2*(d*x + c)/b - ((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^
2 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b +
 sqrt(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) + (3*sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*a^3*b - 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^2 -
7*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b + sqrt(a*b)*(
a - b))*b^4)*abs(-a + b)*abs(b) + (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*s
qrt(a*b)*a^2*b^2 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 -
 sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*abs(-a + b))*(pi*floor
((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b + sqrt(a^2*b^2 - (a*b
 - b^2)*a*b))/(a*b - b^2))))/((3*a^5*b^2 - 12*a^4*b^3 + 14*a^3*b^4 - 4*a^2
*b^5 - a*b^6)*abs(b)) + ((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*
a^2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b
 - sqrt(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) - (3*sqrt(a^2 - a*b -
 sqrt(a*b)*(a - b))*a^3*b - 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^2
- 7*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b - sqrt(a*b)
*(a - b))*b^4)*abs(-a + b)*abs(b) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))
*sqrt(a*b)*a^2*b^2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3
 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*abs(-a + b))*(pi*flo
or((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b - sqrt(a^2*b^2 - (a
*b - b^2)*a*b))/(a*b - b^2))))/((3*a^5*b^2 - 12*a^4*b^3 + 14*a^3*b^4 - ...
```

3.414.9 Mupad [B] (verification not implemented)

Time = 16.03 (sec) , antiderivative size = 4299, normalized size of antiderivative = 33.85

$$\int \frac{\cos^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^4/(a - b*sin(c + d*x)^4),x)
```

output
```
atan((90*a^4*tan(c + d*x))/(10*a*b^3 + 132*a^3*b - 90*a^4 - 2*b^4 - 68*a^2
*b^2 + (18*a^5)/b) - (18*a^5*tan(c + d*x))/(10*a*b^4 - 90*a^4*b + 18*a^5 -
 2*b^5 - 68*a^2*b^3 + 132*a^3*b^2) + (2*b^4*tan(c + d*x))/(10*a*b^3 + 132*
a^3*b - 90*a^4 - 2*b^4 - 68*a^2*b^2 + (18*a^5)/b) + (68*a^2*b^2*tan(c + d*
x))/(10*a*b^3 + 132*a^3*b - 90*a^4 - 2*b^4 - 68*a^2*b^2 + (18*a^5)/b) - (1
0*a*b^3*tan(c + d*x))/(10*a*b^3 + 132*a^3*b - 90*a^4 - 2*b^4 - 68*a^2*b^2
+ (18*a^5)/b) - (132*a^3*b*tan(c + d*x))/(10*a*b^3 + 132*a^3*b - 90*a^4 -
2*b^4 - 68*a^2*b^2 + (18*a^5)/b))/(b*d) + (atan(((tan(c + d*x)*(30*a*b^4 -
 30*a^4*b + 6*a^5 - 6*b^5 - 60*a^2*b^3 + 60*a^3*b^2) + (-(3*a*(a^3*b^5)^(1
/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(36*a*b
^5 - 12*a^5*b - 4*b^6 + ((-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^
2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(64*a*b^7 + 256*a^2*b^6 - 896*a^3*b^5
 + 768*a^4*b^4 - 192*a^5*b^3 + tan(c + d*x)*(-(3*a*(a^3*b^5)^(1/2) + b*(a^
3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(768*a^2*b^7 - 768
*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)) + tan(c + d*x)*(80*a*b^6 - 16*b^7 +
 224*a^2*b^5 - 480*a^3*b^4 + 48*a^4*b^3 + 144*a^5*b^2))*(-(3*a*(a^3*b^5)^(
1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2) - 72*a
^2*b^4 + 40*a^3*b^3 + 12*a^4*b^2))*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1
/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*1i + (tan(c + d*x)*(30*a*b^
4 - 30*a^4*b + 6*a^5 - 6*b^5 - 60*a^2*b^3 + 60*a^3*b^2) - (-(3*a*(a^3*b...
```

3.414. $\int \frac{\cos^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.415 $\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.415.1 Optimal result

Integrand size = 24, antiderivative size = 125

$$\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\sqrt{\sqrt{a}-\sqrt{b}}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}d} + \frac{\sqrt{\sqrt{a}+\sqrt{b}}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}d}$$

output
```
-1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)-b^(1/2))^
(1/2)/a^(3/4)/d/b^(1/2)+1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1
/4))*(a^(1/2)+b^(1/2))^(1/2)/a^(3/4)/d/b^(1/2)
```

3.415.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.26

$$\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\frac{(\sqrt{a}\sqrt{b}+b)\arctan\left(\frac{(\sqrt{a}+\sqrt{b})\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}} + \frac{(\sqrt{a}\sqrt{b}-b)\operatorname{arctanh}\left(\frac{(\sqrt{a}-\sqrt{b})\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}}{2\sqrt{a}bd}$$

input
```
Integrate[Cos[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

output
```
(((Sqrt[a]*Sqrt[b] + b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a +
 Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + ((Sqrt[a]*Sqrt[b] - b)*Arc
Tanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[
-a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*b*d)
```

3.415.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3703, 1406, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^2}{a-b\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{3703}$$

$$\frac{\int \frac{1}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1406}$$

$$\frac{\frac{(a-b)\int \frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\tan(c+dx)}{2\sqrt{a}\sqrt{b}}-\frac{(a-b)\int \frac{1}{(a-b)\tan^2(c+dx)+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\tan(c+dx)}{2\sqrt{a}\sqrt{b}}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(a-b)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(a-b)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\sqrt{\sqrt{a}-\sqrt{b}}\left(\sqrt{a}+\sqrt{b}\right)}}{d}$$

input
```
Int[Cos[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

output
```
(-1/2*((a - b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(a^
(3/4)*Sqrt[Sqrt[a] - Sqrt[b]]*(Sqrt[a] + Sqrt[b])*Sqrt[b]) + ((a - b)*ArcT
an[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] -
Sqrt[b])*Sqrt[Sqrt[a] + Sqrt[b]]*Sqrt[b]))/d
```

3.415.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3703
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2
] && IntegerQ[p]
```

3.415. $\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.415.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.72 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.58

method	result
risch	$\sum_{\_R=\text{RootOf}\left(256a^3b^2d^4\_Z^4+32a^2b\,d^2\_Z^2+a-b\right)}\_R\ln\left(e^{2i(dx+c)}+32a^2d^2\_R^2+8iad\_R+\frac{2a}{b}-1\right.$
derivativedivides	$\frac{(a-b)\left(\frac{\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}-\frac{\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{d}$
default	$\frac{(a-b)\left(\frac{\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}-\frac{\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{d}$

input
```
int(cos(d*x+c)^2/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(exp(2*I*(d*x+c))+32*a^2*d^2*_R^2+8*I*a*d*_R+2/b*a-1),_R=RootOf(2
56*_Z^4*a^3*b^2*d^4+32*_Z^2*a^2*b*d^2+a-b))
```

3.415.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(85) = 170$.

Time = 0.36 (sec) , antiderivative size = 541, normalized size of antiderivative = 4.33

$$\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{1}{8}\sqrt{-\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}+1}{abd^2}}\log\left(\frac{1}{2}\,ad\sqrt{-\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}+1}{abd^2}}\cos(dx+c)\sin(dx+c)+\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\left(2\,a^2d^2\cos(dx+c)^2-a^2d^2\right)\sqrt{\frac{1}{a^3bd^4}}-\frac{1}{4}\right)+\frac{1}{8}\sqrt{-\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}+1}{abd^2}}\log\left(-\frac{1}{2}\,ad\sqrt{-\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}+1}{abd^2}}\cos(dx+c)\sin(dx+c)+\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\left(2\,a^2d^2\cos(dx+c)^2-a^2d^2\right)\sqrt{\frac{1}{a^3bd^4}}-\frac{1}{4}\right)+\frac{1}{8}\sqrt{\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}-1}{abd^2}}\log\left(\frac{1}{2}\,ad\sqrt{\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}-1}{abd^2}}\cos(dx+c)\sin(dx+c)-\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\left(2\,a^2d^2\cos(dx+c)^2-a^2d^2\right)\sqrt{\frac{1}{a^3bd^4}}+\frac{1}{4}\right)-\frac{1}{8}\sqrt{\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}-1}{abd^2}}\log\left(-\frac{1}{2}\,ad\sqrt{\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}-1}{abd^2}}\cos(dx+c)\sin(dx+c)-\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\left(2\,a^2d^2\cos(dx+c)^2-a^2d^2\right)\sqrt{\frac{1}{a^3bd^4}}+\frac{1}{4}\right)$$

3.415. $\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

input
```
integrate(cos(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
-1/8*sqrt(-(a*b*d^2*sqrt(1/(a^3*b*d^4)) + 1)/(a*b*d^2))*log(1/2*a*d*sqrt(-
(a*b*d^2*sqrt(1/(a^3*b*d^4)) + 1)/(a*b*d^2))*cos(d*x + c)*sin(d*x + c) + 1
/4*cos(d*x + c)^2 + 1/4*(2*a^2*d^2*cos(d*x + c)^2 - a^2*d^2)*sqrt(1/(a^3*b
*d^4)) - 1/4) + 1/8*sqrt(-(a*b*d^2*sqrt(1/(a^3*b*d^4)) + 1)/(a*b*d^2))*log
(-1/2*a*d*sqrt(-(a*b*d^2*sqrt(1/(a^3*b*d^4)) + 1)/(a*b*d^2))*cos(d*x + c)*
sin(d*x + c) + 1/4*cos(d*x + c)^2 + 1/4*(2*a^2*d^2*cos(d*x + c)^2 - a^2*d^
2)*sqrt(1/(a^3*b*d^4)) - 1/4) + 1/8*sqrt((a*b*d^2*sqrt(1/(a^3*b*d^4)) - 1)
/(a*b*d^2))*log(1/2*a*d*sqrt((a*b*d^2*sqrt(1/(a^3*b*d^4)) - 1)/(a*b*d^2))*
cos(d*x + c)*sin(d*x + c) - 1/4*cos(d*x + c)^2 + 1/4*(2*a^2*d^2*cos(d*x +
c)^2 - a^2*d^2)*sqrt(1/(a^3*b*d^4)) + 1/4) - 1/8*sqrt((a*b*d^2*sqrt(1/(a^3
*b*d^4)) - 1)/(a*b*d^2))*log(-1/2*a*d*sqrt((a*b*d^2*sqrt(1/(a^3*b*d^4)) -
1)/(a*b*d^2))*cos(d*x + c)*sin(d*x + c) - 1/4*cos(d*x + c)^2 + 1/4*(2*a^2*
d^2*cos(d*x + c)^2 - a^2*d^2)*sqrt(1/(a^3*b*d^4)) + 1/4)
```

3.415.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.415.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\cos{(dx+c)}^2}{b\sin{(dx+c)}^4-a}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
-integrate(cos(d*x + c)^2/(b*sin(d*x + c)^4 - a), x)
```

3.415.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(85) = 170$.

Time = 0.80 (sec) , antiderivative size = 559, normalized size of antiderivative = 4.47

$$\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$= \frac{\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}a^2b-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}ab^2-\sqrt{a^2-ab+\sqrt{ab}(a-b)}b^3+3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}a^2-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}ab-\sqrt{}\right.}{\overline{3a^5b-12a^4b^2+14a^3b^3-4a^2b^4-ab^5}}$$

input
```
integrate(cos(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
1/2*((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b - 6*sqrt(a^2 - a*b + sqr
t(a*b)*(a - b))*a*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*b^3 + 3*sqrt(a
^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2 - 6*sqrt(a^2 - a*b + sqrt(a*b)
*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^
2)*(pi*floor((d*x + c)/pi + 1/2) + arctan(2*tan(d*x + c)/sqrt((4*a + sqrt(
-16*(a - b)*a + 16*a^2))/(a - b))))*abs(a - b)/(3*a^5*b - 12*a^4*b^2 + 14*
a^3*b^3 - 4*a^2*b^4 - a*b^5) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*
b - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^2 - sqrt(a^2 - a*b - sqrt(a*
b)*(a - b))*b^3 - 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2 + 6*
sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b + sqrt(a^2 - a*b - sqrt(
a*b)*(a - b))*sqrt(a*b)*b^2)*(pi*floor((d*x + c)/pi + 1/2) + arctan(2*tan(
d*x + c)/sqrt((4*a - sqrt(-16*(a - b)*a + 16*a^2))/(a - b))))*abs(a - b)/(
3*a^5*b - 12*a^4*b^2 + 14*a^3*b^3 - 4*a^2*b^4 - a*b^5))/d
```

3.415.9 Mupad [B] (verification not implemented)

Time = 14.75 (sec) , antiderivative size = 1409, normalized size of antiderivative = 11.27

$$\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^2/(a - b*sin(c + d*x)^4),x)
```

output
```
(atan(((tan(c + d*x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3) + (-((a^3*b^3)^
(1/2) + a^2*b)/(16*a^3*b^2))^(1/2)*(16*a*b^3 + 16*a^3*b - 32*a^2*b^2 + tan
(c + d*x)*(64*a^4*b + 64*a^2*b^3 - 128*a^3*b^2)*(-((a^3*b^3)^(1/2) + a^2*b
)/(16*a^3*b^2))^(1/2)))*(-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2)*1i
 + (tan(c + d*x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3) - (-((a^3*b^3)^(1/2
) + a^2*b)/(16*a^3*b^2))^(1/2)*(16*a*b^3 + 16*a^3*b - 32*a^2*b^2 - tan(c +
 d*x)*(64*a^4*b + 64*a^2*b^3 - 128*a^3*b^2)*(-((a^3*b^3)^(1/2) + a^2*b)/(1
6*a^3*b^2))^(1/2)))*(-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2)*1i)/((
tan(c + d*x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3) + (-((a^3*b^3)^(1/2) +
a^2*b)/(16*a^3*b^2))^(1/2)*(16*a*b^3 + 16*a^3*b - 32*a^2*b^2 + tan(c + d*x
)*(64*a^4*b + 64*a^2*b^3 - 128*a^3*b^2)*(-((a^3*b^3)^(1/2) + a^2*b)/(16*a^
3*b^2))^(1/2)))*(-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2) - (tan(c +
 d*x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3) - (-((a^3*b^3)^(1/2) + a^2*b)/
(16*a^3*b^2))^(1/2)*(16*a*b^3 + 16*a^3*b - 32*a^2*b^2 - tan(c + d*x)*(64*a
^4*b + 64*a^2*b^3 - 128*a^3*b^2)*(-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))
^(1/2)))*(-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2)))*(-((a^3*b^3)^(1
/2) + a^2*b)/(16*a^3*b^2))^(1/2)*2i)/d + (atan(((tan(c + d*x)*(12*a*b^2 -
12*a^2*b + 4*a^3 - 4*b^3) + (((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/2)
*(16*a*b^3 + 16*a^3*b - 32*a^2*b^2 + tan(c + d*x)*(64*a^4*b + 64*a^2*b^3 -
 128*a^3*b^2)*(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/2)))*(((a^3*b...
```

3.415. $\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.416 $\int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.416.1 Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} + \frac{\sqrt{b}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d} + \frac{\tan(c+dx)}{(a-b)d}$$

output

```
-1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*b^(1/2)/a^(3/4)/d/
(a^(1/2)-b^(1/2))^(3/2)+1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1
/4))*b^(1/2)/a^(3/4)/d/(a^(1/2)+b^(1/2))^(3/2)+tan(d*x+c)/(a-b)/d
```

3.416.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.23

$$\int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\frac{\left(\sqrt{a}\sqrt{b}-b\right)\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}} + \frac{\left(\sqrt{a}\sqrt{b}+b\right)\text{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}} + 2\tan(c+dx)}{2(a-b)d}$$

input
```
Integrate[Sec[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

output
```
(((Sqrt[a]*Sqrt[b] - b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a +
 Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) + ((Sqrt[a]*Sqrt[b
] + b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b
]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + 2*Tan[c + d*x])/(2*(a - b)*d)
```

3.416.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3703, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^2\left(a-b\sin(c+dx)^4\right)}dx$$

$$\Big\downarrow \text{3703}$$

$$\frac{\int \frac{\left(\tan^2(c+dx)+1\right)^2}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1484}$$

$$\frac{\int \left(\frac{1}{a-b}-\frac{b\left(2\tan^2(c+dx)+1\right)}{(a-b)\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tan(c+dx)}{a-b}}{d}$$

input
```
Int[Sec[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

output
```
(-1/2*(Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(a^
(3/4)*(Sqrt[a] - Sqrt[b])^(3/2)) + (Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]
]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt[b])^(3/2)) + Tan[c +
d*x]/(a - b))/d
```

3.416.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3703
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2
] && IntegerQ[p]
```

3.416.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)}{a-b}-b\left(\frac{\left(-a-b+2\sqrt{ab}\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(a+b+2\sqrt{ab}\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{d}$
default	$\frac{\frac{\tan(dx+c)}{a-b}-b\left(\frac{\left(-a-b+2\sqrt{ab}\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(a+b+2\sqrt{ab}\right)\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{d}$
risch	$\frac{2i}{d(a-b)\left(e^{2i(dx+c)}+1\right)}+4\left(\sum_{\_R=\operatorname{RootOf}\left(\left(65536a^6d^4-196608a^5b\,d^4+196608a^4b^2d^4-65536a^3b^3d^4\right)\_Z^4+\left(512a^3b\,d^2\right.\right.}$

input
```
int(sec(d*x+c)^2/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a-b)*tan(d*x+c)-b*(1/2*(-a-b+2*(a*b)^(1/2))/(a*b)^(1/2)/(a-b)/(((a
*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b)
)^(1/2))+1/2*(a+b+2*(a*b)^(1/2))/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))
^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))))
```

3.416.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2589 vs. $2(102) = 204$.

Time = 0.67 (sec) , antiderivative size = 2589, normalized size of antiderivative = 18.23

$$\int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/8*((a - b)*d*sqrt(((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b
^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4
 - 6*a^4*b^5 + a^3*b^6)*d^4)) - a*b - 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 -
 a*b^3)*d^2))*cos(d*x + c)*log(3/4*a*b^2 + 1/4*b^3 - 1/4*(3*a*b^2 + b^3)*c
os(d*x + c)^2 + 1/2*(2*(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^3*sqrt((9*a
^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5
*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) + (3*a^3*b + 4
*a^2*b^2 + a*b^3)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^4 - 3*a^3*b + 3*a^
2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a
^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4)) - a*b - 3*b^
2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2)) - 1/4*(2*(a^5 - 3*a^4*b + 3*
a^3*b^2 - a^2*b^3)*d^2*cos(d*x + c)^2 - (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b
^3)*d^2)*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 2
0*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))) - (a - b)*d*sqrt(((a^
4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^
9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*
d^4)) - a*b - 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2))*cos(d*x +
c)*log(3/4*a*b^2 + 1/4*b^3 - 1/4*(3*a*b^2 + b^3)*cos(d*x + c)^2 - 1/2*(2*(
a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^3*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/
((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^...
```

3.416.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a-b*sin(d*x+c)**4),x)
```

output
```
Integral(sec(c + d*x)**2/(a - b*sin(c + d*x)**4), x)
```

3.416.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\sec(dx+c)^2}{b\sin(dx+c)^4-a}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
(((a - b)*d*cos(2*d*x + 2*c)^2 + (a - b)*d*sin(2*d*x + 2*c)^2 + 2*(a - b)*
d*cos(2*d*x + 2*c) + (a - b)*d)*integrate(4*(4*b^2*cos(6*d*x + 6*c)^2 + 4*
b^2*cos(2*d*x + 2*c)^2 + 4*b^2*sin(6*d*x + 6*c)^2 + 4*b^2*sin(2*d*x + 2*c)
^2 - 12*(8*a*b - 3*b^2)*cos(4*d*x + 4*c)^2 - b^2*cos(2*d*x + 2*c) - 12*(8*
a*b - 3*b^2)*sin(4*d*x + 4*c)^2 + 2*(8*a*b - 15*b^2)*sin(4*d*x + 4*c)*sin(
2*d*x + 2*c) - (b^2*cos(6*d*x + 6*c) - 6*b^2*cos(4*d*x + 4*c) + b^2*cos(2*
d*x + 2*c))*cos(8*d*x + 8*c) + (8*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b -
15*b^2)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) + 2*(3*b^2 + (8*a*b - 15*b^2)*c
os(2*d*x + 2*c))*cos(4*d*x + 4*c) - (b^2*sin(6*d*x + 6*c) - 6*b^2*sin(4*d*
x + 4*c) + b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*(4*b^2*sin(2*d*x + 2
*c) + (8*a*b - 15*b^2)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c))/(a*b^2 - b^3 +
(a*b^2 - b^3)*cos(8*d*x + 8*c)^2 + 16*(a*b^2 - b^3)*cos(6*d*x + 6*c)^2 + 4
*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*cos(4*d*x + 4*c)^2 + 16*(a*b^2 -
b^3)*cos(2*d*x + 2*c)^2 + (a*b^2 - b^3)*sin(8*d*x + 8*c)^2 + 16*(a*b^2 - b
^3)*sin(6*d*x + 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*sin(4*d
*x + 4*c)^2 + 16*(8*a^2*b - 11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 16*(a*b^2 - b^3)*sin(2*d*x + 2*c)^2 + 2*(a*b^2 - b^3 - 4*(a*b^2 -
b^3)*cos(6*d*x + 6*c) - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(4*d*x + 4*c) -
4*(a*b^2 - b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a*b^2 - b^3 - 2*(8
*a^2*b - 11*a*b^2 + 3*b^3)*cos(4*d*x + 4*c) - 4*(a*b^2 - b^3)*cos(2*d*x...
```

3.416.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1211 vs. $2(102) = 204$.

Time = 0.84 (sec) , antiderivative size = 1211, normalized size of antiderivative = 8.53

$$\int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
-1/2*((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5 - 9*sqrt(a^2 -
a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b + 2*sqrt(a^2 - a*b - sqrt(a*b)*(a
 - b))*sqrt(a*b)*a^3*b^2 + 10*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b
)*a^2*b^3 - 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^4 - sqrt(a
^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^5 - 2*(3*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(
a*b)*a*b^2 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(a - b)^2
+ (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b - 12*sqrt(a^2 - a*b - sqrt(
a*b)*(a - b))*a^3*b^2 + 14*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^3 - 4
*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^4 - sqrt(a^2 - a*b - sqrt(a*b)*(a
 - b))*b^5)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x +
c)/sqrt((a^2 - a*b + sqrt((a^2 - a*b)^2 - (a^2 - a*b)*(a^2 - 2*a*b + b^2))
)/(a^2 - 2*a*b + b^2))))/(3*a^8 - 21*a^7*b + 59*a^6*b^2 - 85*a^5*b^3 + 65*
a^4*b^4 - 23*a^3*b^5 + a^2*b^6 + a*b^7) - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a
 - b))*sqrt(a*b)*a^5 - 9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4
*b + 2*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 + 10*sqrt(a^2
 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^3 - 5*sqrt(a^2 - a*b + sqrt(a*
b)*(a - b))*sqrt(a*b)*a*b^4 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b
)*b^5 - 2*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(
a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b + sqrt(...
```

3.416.9 Mupad [B] (verification not implemented)

Time = 15.54 (sec) , antiderivative size = 2832, normalized size of antiderivative = 19.94

$$\int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^2*(a - b*sin(c + d*x)^4)),x)
```

output
```
tan(c + d*x)/(d*(a - b)) + (atan(((((2*(8*a*b^4 - 16*a^2*b^3 + 8*a^3*b^2))
/(a - b) - (4*tan(c + d*x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3
*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*(16*a^5*
b - 16*a^2*b^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*((3*a*(a^3*b^3)^(1/2)
+ b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*
a^4*b^2)))^(1/2) - (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b^2))/(a - b))*((3
*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b -
 a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*1i - (((2*(8*a*b^4 - 16*a^2*b^3 + 8*a^
3*b^2))/(a - b) + (4*tan(c + d*x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2
) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*(
16*a^5*b - 16*a^2*b^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*((3*a*(a^3*b^3)
^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b
^3 - 3*a^4*b^2)))^(1/2) + (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b^2))/(a -
b))*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*
a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*1i)/((((2*(8*a*b^4 - 16*a^2*b^3
 + 8*a^3*b^2))/(a - b) - (4*tan(c + d*x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^
3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^
(1/2)*(16*a^5*b - 16*a^2*b^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*((3*a*(a
^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6
+ a^3*b^3 - 3*a^4*b^2)))^(1/2) - (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b...
```

3.416. $\int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.417 $\int \frac{\sec^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.417.1 Optimal result

Integrand size = 24, antiderivative size = 161

$$\int \frac{\sec^4(c+dx)}{a-b\sin^4(c+dx)}\,dx=\frac{b\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d}+\frac{b\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}d}+\frac{(a-3b)\tan(c+dx)}{(a-b)^2d}+\frac{\tan^3(c+dx)}{3(a-b)d}$$

output
```
1/2*b*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2
)-b^(1/2))^(5/2)+1/2*b*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/
a^(3/4)/d/(a^(1/2)+b^(1/2))^(5/2)+(a-3*b)*tan(d*x+c)/(a-b)^2/d+1/3*tan(d*x
+c)^3/(a-b)/d
```

3.417.2 Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.27

$$\int \frac{\sec^4(c+dx)}{a-b\sin^4(c+dx)}\,dx=\frac{\frac{3\left(\sqrt{a}-\sqrt{b}\right)^2b\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}}-\frac{3\left(\sqrt{a}+\sqrt{b}\right)^2b\operatorname{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}}+4(a-4b)\tan(c+dx)+2(a-}{6(a-b)^2d}$$

input
```
Integrate[Sec[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

output
```
((3*(Sqrt[a] - Sqrt[b])^2*b*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt
[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) - (3*(Sqrt[a]
+ Sqrt[b])^2*b*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a
]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + 4*(a - 4*b)*Tan[c + d*
x] + 2*(a - b)*Sec[c + d*x]^2*Tan[c + d*x])/(6*(a - b)^2*d)
```

3.417.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3703, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{a-b\sin^4(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^4\left(a-b\sin(c+dx)^4\right)}dx$$

$$\Big\downarrow \text{3703}$$

$$\frac{\int \frac{\left(\tan^2(c+dx)+1\right)^3}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1484}$$

$$\frac{\int \left(\frac{\tan^2(c+dx)}{a-b}+\frac{b(a+3b)\tan^2(c+dx)+b(a+b)}{(a-b)^2\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}+\frac{a-3b}{(a-b)^2}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{b\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tan^3(c+dx)}{3(a-b)}+\frac{(a-3b)\tan(c+dx)}{(a-b)^2}}{d}$$

input
```
Int[Sec[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

output
```
((b*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sq
rt[a] - Sqrt[b])^(5/2)) + (b*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])
/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt[b])^(5/2)) + ((a - 3*b)*Tan[c + d*x]
)/(a - b)^2 + Tan[c + d*x]^3/(3*(a - b)))/d
```

3.417.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3703
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2
] && IntegerQ[p]
```

3.417.4 Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tan^3(dx+c)\right)}{3}-\frac{\left(\tan^3(dx+c)\right)b}{3}+\tan(dx+c)a-3\tan(dx+c)b}{(a-b)^2}+\frac{b\left(\frac{\left(a\sqrt{ab}+3\sqrt{ab}\,b-3ab-b^2\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(a\sqrt{a\right.}}{}\right.}{a-b}}{d}$
default	$\frac{\frac{\frac{a\left(\tan^3(dx+c)\right)}{3}-\frac{\left(\tan^3(dx+c)\right)b}{3}+\tan(dx+c)a-3\tan(dx+c)b}{(a-b)^2}+\frac{b\left(\frac{\left(a\sqrt{ab}+3\sqrt{ab}\,b-3ab-b^2\right)\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\left(a\sqrt{a\right.}}{}\right.}{a-b}}{d}$
risch	$-\frac{4i\left(3b\,e^{4i(dx+c)}-3a\,e^{2i(dx+c)}+9b\,e^{2i(dx+c)}-a+4b\right)}{3d(a-b)^2\left(e^{2i(dx+c)}+1\right)^3}+16\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16777216a^8d^4-83886080a^7b\,d^4+167772\right.\right.}\right.$

input
```
int(sec(d*x+c)^4/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a-b)^2*(1/3*a*tan(d*x+c)^3-1/3*tan(d*x+c)^3*b+tan(d*x+c)*a-3*tan(d
*x+c)*b)+b/(a-b)*(1/2*(a*(a*b)^(1/2)+3*(a*b)^(1/2)*b-3*a*b-b^2)/(a*b)^(1/2
)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1
/2)-a)*(a-b))^(1/2))+1/2*(a*(a*b)^(1/2)+3*(a*b)^(1/2)*b+3*a*b+b^2)/(a*b)^(
1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(
1/2)+a)*(a-b))^(1/2))))
```

3.417.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4113 vs. 2(123) = 246.

Time = 0.96 (sec) , antiderivative size = 4113, normalized size of antiderivative = 25.55

$$\int \frac{\sec^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/24*(3*(a^2 - 2*a*b + b^2)*d*sqrt(-(a^2*b^2 + 10*a*b^3 + 5*b^4 - (a^6 - 5
*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2*sqrt((25*a^4*b^5
 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^1
1*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b
^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4)))/((a^6 - 5*a^5*b + 10*a^4*b
^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2))*cos(d*x + c)^3*log(5/4*a^2*b^4
+ 5/2*a*b^5 + 1/4*b^6 - 1/4*(5*a^2*b^4 + 10*a*b^5 + b^6)*cos(d*x + c)^2 +
1/2*((a^9 - 2*a^8*b - 5*a^7*b^2 + 20*a^6*b^3 - 25*a^5*b^4 + 14*a^4*b^5 - 3
*a^3*b^6)*d^3*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^
9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8
*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4
))*cos(d*x + c)*sin(d*x + c) + (15*a^4*b^3 + 35*a^3*b^4 + 13*a^2*b^5 + a*b
^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a^2*b^2 + 10*a*b^3 + 5*b^4 - (a^6
- 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2*sqrt((25*a^4*
b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*
a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^
6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4)))/((a^6 - 5*a^5*b + 10*a^
4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2)) - 1/4*(2*(a^7*b - 5*a^6*b^2
+ 10*a^5*b^3 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^2*cos(d*x + c)^2 - (a^7
*b - 5*a^6*b^2 + 10*a^5*b^3 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^2)*sq...
```

3.417.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int \frac{\sec^4{(c+dx)}}{a-b\sin^4{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a-b*sin(d*x+c)**4),x)
```

output
```
Integral(sec(c + d*x)**4/(a - b*sin(c + d*x)**4), x)
```

3.417.7 Maxima [F]

$$\int \frac{\sec^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\sec(dx+c)^4}{b\sin(dx+c)^4-a}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
-1/3*(36*(a - 2*b)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - 12*(b*sin(4*d*x + 4
*c) - (a - 3*b)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - 3*((a^2 - 2*a*b + b^2
)*d*cos(6*d*x + 6*c)^2 + 9*(a^2 - 2*a*b + b^2)*d*cos(4*d*x + 4*c)^2 + 9*(a
^2 - 2*a*b + b^2)*d*cos(2*d*x + 2*c)^2 + (a^2 - 2*a*b + b^2)*d*sin(6*d*x +
 6*c)^2 + 9*(a^2 - 2*a*b + b^2)*d*sin(4*d*x + 4*c)^2 + 18*(a^2 - 2*a*b + b
^2)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*(a^2 - 2*a*b + b^2)*d*sin(2*d*
x + 2*c)^2 + 6*(a^2 - 2*a*b + b^2)*d*cos(2*d*x + 2*c) + (a^2 - 2*a*b + b^2
)*d + 2*(3*(a^2 - 2*a*b + b^2)*d*cos(4*d*x + 4*c) + 3*(a^2 - 2*a*b + b^2)*
d*cos(2*d*x + 2*c) + (a^2 - 2*a*b + b^2)*d)*cos(6*d*x + 6*c) + 6*(3*(a^2 -
 2*a*b + b^2)*d*cos(2*d*x + 2*c) + (a^2 - 2*a*b + b^2)*d)*cos(4*d*x + 4*c)
 + 6*((a^2 - 2*a*b + b^2)*d*sin(4*d*x + 4*c) + (a^2 - 2*a*b + b^2)*d*sin(2
*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-8*(4*b^3*cos(6*d*x + 6*c)^2 + 4*
b^3*cos(2*d*x + 2*c)^2 + 4*b^3*sin(6*d*x + 6*c)^2 + 4*b^3*sin(2*d*x + 2*c)
^2 - b^3*cos(2*d*x + 2*c) - 4*(8*a^2*b + 13*a*b^2 - 6*b^3)*cos(4*d*x + 4*c
)^2 - 4*(8*a^2*b + 13*a*b^2 - 6*b^3)*sin(4*d*x + 4*c)^2 + 2*(4*a*b^2 - 11*
b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (b^3*cos(6*d*x + 6*c) + b^3*cos(2
*d*x + 2*c) - 2*(a*b^2 + 2*b^3)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + (8*b^
3*cos(2*d*x + 2*c) - b^3 + 2*(4*a*b^2 - 11*b^3)*cos(4*d*x + 4*c))*cos(6*d*
x + 6*c) + 2*(a*b^2 + 2*b^3 + (4*a*b^2 - 11*b^3)*cos(2*d*x + 2*c))*cos(4*d
*x + 4*c) - (b^3*sin(6*d*x + 6*c) + b^3*sin(2*d*x + 2*c) - 2*(a*b^2 + 2...
```

3.417.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2183 vs. $2(123) = 246$.

Time = 0.93 (sec) , antiderivative size = 2183, normalized size of antiderivative = 13.56

$$\int \frac{\sec^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
1/6*(2*(a^2*tan(d*x + c)^3 - 2*a*b*tan(d*x + c)^3 + b^2*tan(d*x + c)^3 + 3
*a^2*tan(d*x + c) - 12*a*b*tan(d*x + c) + 9*b^2*tan(d*x + c))/(a^3 - 3*a^2
*b + 3*a*b^2 - b^3) - 3*((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*
a^3*b + 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 - 19*sqrt(
a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - 3*sqrt(a^2 - a*b + sqrt(a
*b)*(a - b))*sqrt(a*b)*b^4)*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)^2*abs(-a + b)
- (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^7*b - 15*sqrt(a^2 - a*b + sqrt(
a*b)*(a - b))*a^6*b^2 + 23*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^5*b^3 - 3
*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b^4 - 23*sqrt(a^2 - a*b + sqrt(a*
b)*(a - b))*a^3*b^5 + 19*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^6 - 3*s
qrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^7 - sqrt(a^2 - a*b + sqrt(a*b)*(a -
 b))*b^8)*abs(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*abs(-a + b) - (9*sqrt(a^2 - a
*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b - 69*sqrt(a^2 - a*b + sqrt(a*b)*(a
 - b))*sqrt(a*b)*a^8*b^2 + 216*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*
b)*a^7*b^3 - 352*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^4 + 3
06*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^5 - 114*sqrt(a^2 -
a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^6 - 16*sqrt(a^2 - a*b + sqrt(a*b)
*(a - b))*sqrt(a*b)*a^3*b^7 + 24*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(
a*b)*a^2*b^8 - 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^9 - sqr
t(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^10)*abs(-a + b))*(pi*floor...
```

3.417.9 Mupad [B] (verification not implemented)

Time = 16.48 (sec) , antiderivative size = 4664, normalized size of antiderivative = 28.97

$$\int \frac{\sec^4(c + dx)}{a - b\sin^4(c + dx)} \, dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^4*(a - b*sin(c + d*x)^4)),x)
```

output
```
tan(c + d*x)^3/(3*d*(a - b)) - (tan(c + d*x)*((2*a)/(a - b)^2 - 3/(a - b))
)/d + (atan(((((16*a*b^6 - 32*a^2*b^5 + 32*a^4*b^3 - 16*a^5*b^2)/(3*a*b^2
- 3*a^2*b + a^3 - b^3) - (4*tan(c + d*x)*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^
3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/
(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2
)*(16*a^7*b - 16*a^2*b^6 + 80*a^3*b^5 - 160*a^4*b^4 + 160*a^5*b^3 - 80*a^6
*b^2))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3
*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(
16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)
 - (4*tan(c + d*x)*(15*a*b^5 + b^6 + 15*a^2*b^4 + a^3*b^3))/(3*a*b^2 - 3*a
^2*b + a^3 - b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b
^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a
^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*1i - (((16*a*b^6 - 3
2*a^2*b^5 + 32*a^4*b^3 - 16*a^5*b^2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (4*
tan(c + d*x)*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 1
0*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5
 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*(16*a^7*b - 16*a^2*b^6 + 8
0*a^3*b^5 - 160*a^4*b^4 + 160*a^5*b^3 - 80*a^6*b^2))/(3*a*b^2 - 3*a^2*b +
a^3 - b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10
*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b...
```

3.417. $\int \frac{\sec^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.418 $\int \frac{\sec^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.418.1 Optimal result

Integrand size = 24, antiderivative size = 204

$$\int \frac{\sec^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{b^{3/2}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{7/2}d} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{7/2}d} + \frac{(a^2-3ab+6b^2)\tan(c+dx)}{(a-b)^3d} + \frac{2(a-2b)\tan^3(c+dx)}{3(a-b)^2d} + \frac{\tan^5(c+dx)}{5(a-b)d}$$

output
```
-1/2*b^(3/2)*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(3/4)/d/
(a^(1/2)-b^(1/2))^(7/2)+1/2*b^(3/2)*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x
+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)+b^(1/2))^(7/2)+(a^2-3*a*b+6*b^2)*tan(d*x+c
)/(a-b)^3/d+2/3*(a-2*b)*tan(d*x+c)^3/(a-b)^2/d+1/5*tan(d*x+c)^5/(a-b)/d
```

3.418.2 Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.24

$$\int \frac{\sec^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$$
$$= \frac{\frac{15\left(\sqrt{a}-\sqrt{b}\right)^3 b^{3/2}\arctan\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}}+\frac{15\left(\sqrt{a}+\sqrt{b}\right)^3 b^{3/2}\text{arctanh}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}}+2(8a^2-21ab+73b^2)\tan}{30(a-b)^3d}$$

input
```
Integrate[Sec[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

output
```
((15*(Sqrt[a] - Sqrt[b])^3*b^(3/2)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x
])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) + (15*(
Sqrt[a] + Sqrt[b])^3*b^(3/2)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sq
rt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + 2*(8*a^2
 - 21*a*b + 73*b^2)*Tan[c + d*x] + 4*(2*a - 7*b)*(a - b)*Sec[c + d*x]^2*Ta
n[c + d*x] + 6*(a - b)^2*Sec[c + d*x]^4*Tan[c + d*x])/(30*(a - b)^3*d)
```

3.418.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3703, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\cos(c+dx)^6\left(a-b\sin(c+dx)^4\right)}dx$$
$$\Big\downarrow \text{ 3703}$$
$$\frac{\int \frac{\left(\tan^2(c+dx)+1\right)^4}{(a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$
$$\Big\downarrow \text{ 1484}$$

$$\frac{\int \left(\frac{\tan^4(c+dx)}{a-b}+\frac{2(a-2b)\tan^2(c+dx)}{(a-b)^2}+\frac{a^2-3ba+6b^2}{(a-b)^3}-\frac{4(a+b)\tan^2(c+dx)b^2+(3a+b)b^2}{(a-b)^3((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a)}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^{3/2}\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{7/2}}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{7/2}}+\frac{(a^2-3ab+6b^2)\tan(c+dx)}{(a-b)^3}+\frac{\tan^5(c+dx)}{5(a-b)}+\frac{2(a-2b)\tan^3(c+dx)}{3(a-b)^2}}{d}$$

input
```
Int[Sec[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

output
```
(-1/2*(b^(3/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(a^
(3/4)*(Sqrt[a] - Sqrt[b])^(7/2)) + (b^(3/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]
]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt[b])^(7/2)) + ((a^2 -
3*a*b + 6*b^2)*Tan[c + d*x])/(a - b)^3 + (2*(a - 2*b)*Tan[c + d*x]^3)/(3*(
a - b)^2) + Tan[c + d*x]^5/(5*(a - b)))/d
```

3.418.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3703
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2
] && IntegerQ[p]
```

3.418. $\int \frac{\sec^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.418.4 Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.52

method	result
derivativedivides	$\frac{\frac{\frac{a^2\left(\tan^5(dx+c)\right)}{5}-\frac{2ab\left(\tan^5(dx+c)\right)}{5}+\frac{b^2\left(\tan^5(dx+c)\right)}{5}+\frac{2a^2\left(\tan^3(dx+c)\right)}{3}-2ab\left(\tan^3(dx+c)\right)+\frac{4b^2\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)a^2-}{(a-b)^3}}{}$
default	$\frac{\frac{\frac{a^2\left(\tan^5(dx+c)\right)}{5}-\frac{2ab\left(\tan^5(dx+c)\right)}{5}+\frac{b^2\left(\tan^5(dx+c)\right)}{5}+\frac{2a^2\left(\tan^3(dx+c)\right)}{3}-2ab\left(\tan^3(dx+c)\right)+\frac{4b^2\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)a^2-}{(a-b)^3}}{}$
risch	Expression too large to display

input
```
int(sec(d*x+c)^6/(a-b*sin(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a-b)^3*(1/5*a^2*tan(d*x+c)^5-2/5*a*b*tan(d*x+c)^5+1/5*b^2*tan(d*x+
c)^5+2/3*a^2*tan(d*x+c)^3-2*a*b*tan(d*x+c)^3+4/3*b^2*tan(d*x+c)^3+tan(d*x+
c)*a^2-3*a*b*tan(d*x+c)+6*tan(d*x+c)*b^2)-b^2/(a-b)^2*(1/2*(4*a*(a*b)^(1/2
)+4*(a*b)^(1/2)*b+a^2+6*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))
^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2*(4*a*(a*
b)^(1/2)+4*(a*b)^(1/2)*b-a^2-6*a*b-b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)
*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))))
```

3.418.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5587 vs. 2(160) = 320.

Time = 1.59 (sec) , antiderivative size = 5587, normalized size of antiderivative = 27.39

$$\int \frac{\sec^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

output
```
Too large to include
```

3.418.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**6/(a-b*sin(d*x+c)**4),x)
```

output
```
Timed out
```

3.418.7 Maxima [F]

$$\int \frac{\sec^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \int -\frac{\sec{(dx+c)}^6}{b\sin{(dx+c)}^4-a}\,dx$$

input
```
integrate(sec(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/15*(300*(a*b - 5*b^2)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - 10*(48*b^2*sin
(6*d*x + 6*c) + 3*(a*b + 3*b^2)*sin(8*d*x + 8*c) + 2*(8*a^2 - 21*a*b + 49*
b^2)*sin(4*d*x + 4*c) + 8*(a^2 - 3*a*b + 8*b^2)*sin(2*d*x + 2*c))*cos(10*d
*x + 10*c) + 50*(6*(a*b - 5*b^2)*sin(6*d*x + 6*c) - 16*(a^2 - 3*a*b + 5*b^
2)*sin(4*d*x + 4*c) - (8*a^2 - 27*a*b + 55*b^2)*sin(2*d*x + 2*c))*cos(8*d*
x + 8*c) - 200*((8*a^2 - 21*a*b + 25*b^2)*sin(4*d*x + 4*c) + 4*(a^2 - 3*a*
b + 5*b^2)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 15*((a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*d*cos(10*d*x + 10*c)^2 + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*co
s(8*d*x + 8*c)^2 + 100*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(6*d*x + 6*c)^
2 + 100*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x + 4*c)^2 + 25*(a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x + 2*c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 -
b^3)*d*sin(10*d*x + 10*c)^2 + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(8*d
*x + 8*c)^2 + 100*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(6*d*x + 6*c)^2 + 1
00*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(4*d*x + 4*c)^2 + 100*(a^3 - 3*a^2
*b + 3*a*b^2 - b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*(a^3 - 3*a^2*
b + 3*a*b^2 - b^3)*d*sin(2*d*x + 2*c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d + 2*(5*(a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*d*cos(8*d*x + 8*c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 -
b^3)*d*cos(6*d*x + 6*c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x +
 4*c) + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3...
```

3.418.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3105 vs. $2(160) = 320$.

Time = 1.00 (sec) , antiderivative size = 3105, normalized size of antiderivative = 15.22

$$\int \frac{\sec^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

output
```
1/30*(2*(3*a^4*tan(d*x + c)^5 - 12*a^3*b*tan(d*x + c)^5 + 18*a^2*b^2*tan(d
*x + c)^5 - 12*a*b^3*tan(d*x + c)^5 + 3*b^4*tan(d*x + c)^5 + 10*a^4*tan(d*
x + c)^3 - 50*a^3*b*tan(d*x + c)^3 + 90*a^2*b^2*tan(d*x + c)^3 - 70*a*b^3*
tan(d*x + c)^3 + 20*b^4*tan(d*x + c)^3 + 15*a^4*tan(d*x + c) - 75*a^3*b*ta
n(d*x + c) + 195*a^2*b^2*tan(d*x + c) - 225*a*b^3*tan(d*x + c) + 90*b^4*ta
n(d*x + c))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5) - 15
*(4*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 - 3*sqrt(a^2
- a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^3 - 7*sqrt(a^2 - a*b + sqrt(a*b
)*(a - b))*sqrt(a*b)*a*b^4 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)
*b^5)*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)^2*abs(-a +
 b) + (9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^9*b^2 - 69*sqrt(a^2 - a*b +
 sqrt(a*b)*(a - b))*a^8*b^3 + 216*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^7*
b^4 - 352*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^6*b^5 + 306*sqrt(a^2 - a*b
 + sqrt(a*b)*(a - b))*a^5*b^6 - 114*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^
4*b^7 - 16*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^8 + 24*sqrt(a^2 - a*b
 + sqrt(a*b)*(a - b))*a^2*b^9 - 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^
10 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*b^11)*abs(a^5 - 5*a^4*b + 10*a^3*
b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*abs(-a + b) - (3*sqrt(a^2 - a*b + sqrt(a
*b)*(a - b))*sqrt(a*b)*a^14*b - 18*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqr
t(a*b)*a^13*b^2 - 19*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^12...
```

3.418.9 Mupad [B] (verification not implemented)

Time = 17.54 (sec) , antiderivative size = 6534, normalized size of antiderivative = 32.03

$$\int \frac{\sec^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^6*(a - b*sin(c + d*x)^4)),x)
```

output
```
(atan(((((4*(4*a*b^8 - 4*a^2*b^7 - 24*a^3*b^6 + 56*a^4*b^5 - 44*a^5*b^4 +
12*a^6*b^3))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2) - (
4*tan(c + d*x)*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 +
 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(
a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 3
5*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2)*(16*a^9*b - 16*a^2*b^8 + 112*
a^3*b^7 - 336*a^4*b^6 + 560*a^5*b^5 - 560*a^6*b^4 + 336*a^7*b^3 - 112*a^8*
b^2))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))*((7*a^3*(
a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4
 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a
^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 -
 21*a^8*b^2)))^(1/2) - (4*tan(c + d*x)*(28*a*b^7 + b^8 + 70*a^2*b^6 + 28*a
^3*b^5 + a^4*b^4))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^
2))*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5
 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1
/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 +
 35*a^7*b^3 - 21*a^8*b^2)))^(1/2)*1i - (((4*(4*a*b^8 - 4*a^2*b^7 - 24*a^3*
b^6 + 56*a^4*b^5 - 44*a^5*b^4 + 12*a^6*b^3))/(5*a*b^4 - 5*a^4*b + a^5 - b^
5 - 10*a^2*b^3 + 10*a^3*b^2) + (4*tan(c + d*x)*((7*a^3*(a^3*b^7)^(1/2) + b
^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21...
```

3.418. $\int \frac{\sec^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

3.419 $\int \cos^m(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx$

3.419.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \cos^m(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx = \text{Int}\left(\cos^m(e+fx) \left(a+b\sin^4(e+fx)\right)^p, x\right)$$

output
```
Unintegrable(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x)
```

3.419.2 Mathematica [N/A]

Not integrable

Time = 6.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^m(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx = \int \cos^m(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx$$

input
```
Integrate[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
Integrate[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^4)^p, x]
```

3.419.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3707}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^m(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^m \left(a+b\sin(e+fx)^4\right)^p dx$$

$$\Big\downarrow \text{3707}$$

$$\int \cos^m(e+fx) \left(a+b\sin^4(e+fx)\right)^p dx$$

input
```
Int[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
$Aborted
```

3.419.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3707
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Cos[e + f*x])^m*(a +
 b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.419.4 Maple [N/A] (verified)

Not integrable

Time = 1.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(\cos^{m}\left(fx+e\right)\right)\left(a+b\left(\sin^{4}\left(fx+e\right)\right)\right)^{p}dx$$

input
```
int(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x)
```

output
```
int(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x)
```

3.419.5 Fricas [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \cos^{m}(e+fx)\left(a+b\sin^{4}(e+fx)\right)^{p}\,dx=\int \left(b\sin\left(fx+e\right)^{4}+a\right)^{p}\cos\left(fx+e\right)^{m}\,dx$$

input
```
integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

output
```
integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^m,
 x)
```

3.419.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{m}(e+fx)\left(a+b\sin^{4}(e+fx)\right)^{p}\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**m*(a+b*sin(f*x+e)**4)**p,x)
```

output
```
Timed out
```

3.419.7 Maxima [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^m\,dx$$

input
```
integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^m, x)
```

3.419.8 Giac [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^m\,dx$$

input
```
integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^m, x)
```

3.419.9 Mupad [N/A]

Not integrable

Time = 18.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^m\,\left(b\sin\left(e+f\,x\right)^4+a\right)^p\,dx$$

input
```
int(cos(e + f*x)^m*(a + b*sin(e + f*x)^4)^p,x)
```

output
```
int(cos(e + f*x)^m*(a + b*sin(e + f*x)^4)^p, x)
```

3.420 $\int \cos^5(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx$

3.420.1 Optimal result

Integrand size = 23, antiderivative size = 197

$$\int \cos^5(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx = \frac{\sin(e+fx) \left(a+b\sin^4(e+fx)\right)^{1+p}}{bf(5+4p)} - \frac{(a-b(5+4p)) \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{b\sin^4(e+fx)}{a}\right) \sin(e+fx) \left(a+b\sin^4(e+fx)\right)^p \left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}}{bf(5+4p)} - \frac{2 \text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{b\sin^4(e+fx)}{a}\right) \sin^3(e+fx) \left(a+b\sin^4(e+fx)\right)^p \left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}}{3f}$$

output
```
sin(f*x+e)*(a+b*sin(f*x+e)^4)^(p+1)/b/f/(5+4*p)-(a-b*(5+4*p))*hypergeom([1
/4, -p],[5/4],-b*sin(f*x+e)^4/a)*sin(f*x+e)*(a+b*sin(f*x+e)^4)^p/b/f/(5+4*
p)/((1+b*sin(f*x+e)^4/a)^p)-2/3*hypergeom([3/4, -p],[7/4],-b*sin(f*x+e)^4/
a)*sin(f*x+e)^3*(a+b*sin(f*x+e)^4)^p/f/((1+b*sin(f*x+e)^4/a)^p)
```

3.420.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.72

$$\int \cos^5(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx = \frac{\sin(e+fx) \left(a+b\sin^4(e+fx)\right)^p \left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p} \left(15 \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{b\sin^4(e+fx)}{a}\right) - 1\right.}{}$$

input
```
Integrate[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
(Sin[e + f*x]*(a + b*Sin[e + f*x]^4)^p*(15*Hypergeometric2F1[1/4, -p, 5/4,
 -((b*Sin[e + f*x]^4)/a)] - 10*Hypergeometric2F1[3/4, -p, 7/4, -((b*Sin[e
+ f*x]^4)/a)]*Sin[e + f*x]^2 + 3*Hypergeometric2F1[5/4, -p, 9/4, -((b*Sin[
e + f*x]^4)/a)]*Sin[e + f*x]^4))/(15*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

3.420.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3702, 1519, 25, 1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^5 \left(a+b\sin(e+fx)^4\right)^p dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)^2 \left(b\sin^4(e+fx)+a\right)^p d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{1519}$$

$$\frac{\frac{\int -\left(\left(2b(4p+5)\sin^2(e+fx)+a-b(4p+5)\right)\left(b\sin^4(e+fx)+a\right)^p\right)d\sin(e+fx)}{b(4p+5)}+\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{p+1}}{b(4p+5)}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{p+1}}{b(4p+5)}-\frac{\int \left(2b(4p+5)\sin^2(e+fx)+a-5b-4bp\right)\left(b\sin^4(e+fx)+a\right)^p d\sin(e+fx)}{b(4p+5)}}{f}$$

$$\Big\downarrow \text{1516}$$

$$\frac{\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{p+1}}{b(4p+5)}-\frac{\int \left(2b(4p+5)\sin^2(e+fx)\left(b\sin^4(e+fx)+a\right)^p+a\left(1-\frac{b(4p+5)}{a}\right)\left(b\sin^4(e+fx)+a\right)^p\right)d\sin(e+fx)}{b(4p+5)}}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sin(e+fx)(a+b\sin^4(e+fx))^{p+1}}{b(4p+5)}-\frac{(a-4bp-5b)\sin(e+fx)(a+b\sin^4(e+fx))^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{b\sin^4(e+fx)}{a}\right)}{b(4}}{f}$$

input
```
Int[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
((Sin[e + f*x]*(a + b*Sin[e + f*x]^4)^(1 + p))/(b*(5 + 4*p)) - (((a - 5*b
- 4*b*p)*Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e +
f*x]*(a + b*Sin[e + f*x]^4)^p)/(1 + (b*Sin[e + f*x]^4)/a)^p + (2*b*(5 + 4*
p)*Hypergeometric2F1[3/4, -p, 7/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^3
*(a + b*Sin[e + f*x]^4)^p)/(3*(1 + (b*Sin[e + f*x]^4)/a)^p))/(b*(5 + 4*p))
)/f
```

3.420.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.420.4 Maple [F]

$$\int \left(\cos^{5}\left(fx+e\right)\right)\left(a+b\left(\sin^{4}\left(fx+e\right)\right)\right)^{p}dx$$

input
```
int(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x)
```

output
```
int(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x)
```

3.420.5 Fricas [F]

$$\int \cos^5(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

output
```
integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^5,
 x)
```

3.420.6 Sympy [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5*(a+b*sin(f*x+e)**4)**p,x)
```

output
```
Timed out
```

3.420.7 Maxima [F]

$$\int \cos^5(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^5, x)
```

3.420.8 Giac [F]

$$\int \cos^5(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^5, x)
```

3.420.9 Mupad [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^5\,\left(b\sin\left(e+f\,x\right)^4+a\right)^p\,dx$$

input
```
int(cos(e + f*x)^5*(a + b*sin(e + f*x)^4)^p,x)
```

output
```
int(cos(e + f*x)^5*(a + b*sin(e + f*x)^4)^p, x)
```

3.421 $\int \cos^3(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx$

3.421.1 Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \cos^3(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}}{f} - \frac{\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{b\sin^4(e+fx)}{a}\right)\sin^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}}{3f}$$

output
```
hypergeom([1/4, -p],[5/4],-b*sin(f*x+e)^4/a)*sin(f*x+e)*(a+b*sin(f*x+e)^4)
^p/f/((1+b*sin(f*x+e)^4/a)^p)-1/3*hypergeom([3/4, -p],[7/4],-b*sin(f*x+e)^
4/a)*sin(f*x+e)^3*(a+b*sin(f*x+e)^4)^p/f/((1+b*sin(f*x+e)^4/a)^p)
```

3.421.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.76

$$\int \cos^3(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx = -\frac{\sin(e+fx)\left(-3\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{b\sin^4(e+fx)}{a}\right)+\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{b\sin^4(e\cdots}{a}\right.\right.}{3f}$$

input
```
Integrate[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
-1/3*(Sin[e + f*x]*(-3*Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4
)/a)] + Hypergeometric2F1[3/4, -p, 7/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f
*x]^2)*(a + b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

3.421.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3702, 1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^3 \left(a+b\sin(e+fx)^4\right)^p dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int \left(1-\sin^2(e+fx)\right) \left(b\sin^4(e+fx)+a\right)^p d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{1516}$$

$$\frac{\int \left(\left(b\sin^4(e+fx)+a\right)^p - \sin^2(e+fx)\left(b\sin^4(e+fx)+a\right)^p\right) d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p \left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{b\sin^4(e+fx)}{a}\right) - \frac{1}{3}\sin^3(e+f}{f}$$

input
```
Int[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
((Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]*(a
 + b*Sin[e + f*x]^4)^p)/(1 + (b*Sin[e + f*x]^4)/a)^p - (Hypergeometric2F1[
3/4, -p, 7/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^3*(a + b*Sin[e + f*x]^
4)^p)/(3*(1 + (b*Sin[e + f*x]^4)/a)^p))/f
```

3.421.3.1 Defintions of rubi rules used

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.421.4 Maple [F]

$$\int \left(\cos^{3}\left(fx+e\right)\right)\left(a+b\left(\sin^{4}\left(fx+e\right)\right)\right)^{p}dx$$

input
```
int(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x)
```

output
```
int(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x)
```

3.421.5 Fricas [F]

$$\int \cos^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

output
```
integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^3,
 x)
```

3.421.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**4)**p,x)
```

output
```
Timed out
```

3.421.7 Maxima [F]

$$\int \cos^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^3, x)
```

3.421.8 Giac [F]

$$\int \cos^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^3, x)
```

3.421.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^3\,\left(b\sin\left(e+f\,x\right)^4+a\right)^p\,dx$$

input
```
int(cos(e + f*x)^3*(a + b*sin(e + f*x)^4)^p,x)
```

output
```
int(cos(e + f*x)^3*(a + b*sin(e + f*x)^4)^p, x)
```

3.422 $\int \cos(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

3.422.1 Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \cos(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}}{f}$$

output
```
hypergeom([1/4, -p],[5/4],-b*sin(f*x+e)^4/a)*sin(f*x+e)*(a+b*sin(f*x+e)^4)
^p/f/((1+b*sin(f*x+e)^4/a)^p)
```

3.422.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \cos(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}}{f}$$

input
```
Integrate[Cos[e + f*x]*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
(Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]*(a
+ b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

3.422.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3702, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)\left(a+b\sin(e+fx)^4\right)^p\,dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int \left(b\sin^4(e+fx)+a\right)^p d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{779}$$

$$\frac{\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}\int \left(\frac{b\sin^4(e+fx)}{a}+1\right)^p d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{778}$$

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{b\sin^4(e+fx)}{a}\right)}{f}$$

input
```
Int[Cos[e + f*x]*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
(Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]*(a
+ b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

3.422.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.422.4 Maple [F]

$$\int \cos\left(fx+e\right)\left(a+b\left(\sin^{4}\left(fx+e\right)\right)\right)^{p}dx$$

input
```
int(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x)
```

output
```
int(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x)
```

3.422.5 Fricas [F]

$$\int \cos(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^4+a\right)^p \cos\left(fx+e\right) \, dx$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

output
```
integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e), x
)
```

3.422.6 Sympy [F(-1)]

Timed out.

$$\int \cos(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)**4)**p,x)
```

output
```
Timed out
```

3.422.7 Maxima [F]

$$\int \cos(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^4+a\right)^p \cos\left(fx+e\right) \, dx$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e), x)
```

3.422.8 Giac [F]

$$\int \cos(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e), x)
```

3.422.9 Mupad [B] (verification not implemented)

Time = 13.77 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \cos(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$
$$=\frac{\sin\left(e+f\,x\right)\,\left(b\sin\left(e+f\,x\right)^4+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{4},-p;\ \frac{5}{4};\ -\frac{b\sin(e+f\,x)^4}{a}\right)}{f\,\left(\frac{b\sin(e+f\,x)^4}{a}+1\right)^p}$$

input
```
int(cos(e + f*x)*(a + b*sin(e + f*x)^4)^p,x)
```

output
```
(sin(e + f*x)*(a + b*sin(e + f*x)^4)^p*hypergeom([1/4, -p], 5/4, -(b*sin(e
 + f*x)^4)/a))/(f*((b*sin(e + f*x)^4)/a + 1)^p)
```

3.423 $\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

3.423.1 Optimal result

Integrand size = 21, antiderivative size = 158

$$\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$
$$=\frac{\operatorname{AppellF1}\left(\frac{1}{4},1,-p,\frac{5}{4},\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}}{f}$$
$$+\frac{\operatorname{AppellF1}\left(\frac{3}{4},1,-p,\frac{7}{4},\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)\sin^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}}{3f}$$

output
```
AppellF1(1/4,1,-p,5/4,sin(f*x+e)^4,-b*sin(f*x+e)^4/a)*sin(f*x+e)*(a+b*sin(
f*x+e)^4)^p/f/((1+b*sin(f*x+e)^4/a)^p)+1/3*AppellF1(3/4,1,-p,7/4,sin(f*x+e
)^4,-b*sin(f*x+e)^4/a)*sin(f*x+e)^3*(a+b*sin(f*x+e)^4)^p/f/((1+b*sin(f*x+e
)^4/a)^p)
```

3.423.2 Mathematica [F]

$$\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

input
```
Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^4)^p, x]
```

3.423.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3702, 1569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a+b\sin(e+fx)^4\right)^p}{\cos(e+fx)}dx$$

$$\Big\downarrow \ 3702$$

$$\frac{\int \frac{\left(b\sin^4(e+fx)+a\right)^p}{1-\sin^2(e+fx)}d\sin(e+fx)}{f}$$

$$\Big\downarrow \ 1569$$

$$\frac{\int \left(\frac{\left(b\sin^4(e+fx)+a\right)^p}{1-\sin^4(e+fx)}-\frac{\sin^2(e+fx)\left(b\sin^4(e+fx)+a\right)^p}{\sin^4(e+fx)-1}\right)d\sin(e+fx)}{f}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{4},1,-p,\frac{5}{4},\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)+\frac{1}{3}\sin^3(e}{f}$$

input
```
Int[Sec[e + f*x]*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
((AppellF1[1/4, 1, -p, 5/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e
 + f*x]*(a + b*Sin[e + f*x]^4)^p)/(1 + (b*Sin[e + f*x]^4)/a)^p + (AppellF1
[3/4, 1, -p, 7/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^3*
(a + b*Sin[e + f*x]^4)^p)/(3*(1 + (b*Sin[e + f*x]^4)/a)^p))/f
```

3.423.3.1 Defintions of rubi rules used

rule 1569
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !
IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.423.4 Maple [F]

$$\int \sec(fx+e)\left(a+b\left(\sin^4(fx+e)\right)\right)^p dx$$

input
```
int(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x)
```

output
```
int(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x)
```

3.423.5 Fricas [F]

$$\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \int \left(b\sin(fx+e)^4+a\right)^p \sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

output
```
integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e), x
)
```

3.423.6 Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)**4)**p,x)
```

output
```
Timed out
```

3.423.7 Maxima [F]

$$\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \int \left(b\sin\left(fx+e\right)^4+a\right)^p \sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e), x)
```

3.423.8 Giac [F]

$$\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \int \left(b\sin\left(fx+e\right)^4+a\right)^p \sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e), x)
```

3.423.9 Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx = \int \frac{\left(b\sin\left(e+f\,x\right)^4+a\right)^p}{\cos\left(e+f\,x\right)} \, dx$$

input
```
int((a + b*sin(e + f*x)^4)^p/cos(e + f*x),x)
```

output
```
int((a + b*sin(e + f*x)^4)^p/cos(e + f*x), x)
```

3.424 $\int \sec^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

3.424.1 Optimal result

Integrand size = 23, antiderivative size = 239

$$\int \sec^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$
$$=\frac{\text{AppellF1}\left(\frac{1}{4},2,-p,\frac{5}{4},\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}}{f}$$
$$+\frac{2\,\text{AppellF1}\left(\frac{3}{4},2,-p,\frac{7}{4},\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)\sin^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}}{3f}$$
$$+\frac{\text{AppellF1}\left(\frac{5}{4},2,-p,\frac{9}{4},\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)\sin^5(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}}{5f}$$

output
```
AppellF1(1/4,2,-p,5/4,sin(f*x+e)^4,-b*sin(f*x+e)^4/a)*sin(f*x+e)*(a+b*sin(
f*x+e)^4)^p/f/((1+b*sin(f*x+e)^4/a)^p)+2/3*AppellF1(3/4,2,-p,7/4,sin(f*x+e
)^4,-b*sin(f*x+e)^4/a)*sin(f*x+e)^3*(a+b*sin(f*x+e)^4)^p/f/((1+b*sin(f*x+e
)^4/a)^p)+1/5*AppellF1(5/4,2,-p,9/4,sin(f*x+e)^4,-b*sin(f*x+e)^4/a)*sin(f*
x+e)^5*(a+b*sin(f*x+e)^4)^p/f/((1+b*sin(f*x+e)^4/a)^p)
```

3.424.2 Mathematica [F]

$$\int \sec^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \sec^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

input
```
Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p, x]
```

3.424.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3702, 1569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\sin(e+fx)^4\right)^p}{\cos(e+fx)^3}dx$$

$$\Big\downarrow \; 3702$$

$$\frac{\int \frac{\left(b\sin^4(e+fx)+a\right)^p}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 1569$$

$$\frac{\int \left(\frac{\sin^4(e+fx)\left(b\sin^4(e+fx)+a\right)^p}{\left(\sin^4(e+fx)-1\right)^2}+\frac{2\sin^2(e+fx)\left(b\sin^4(e+fx)+a\right)^p}{\left(\sin^4(e+fx)-1\right)^2}+\frac{\left(b\sin^4(e+fx)+a\right)^p}{\left(\sin^4(e+fx)-1\right)^2}\right)d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 2009$$

$$\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{4},2,-p,\frac{5}{4},\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)+\frac{1}{5}\sin^5(e$$

input
```
Int[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
((AppellF1[1/4, 2, -p, 5/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e
 + f*x]*(a + b*Sin[e + f*x]^4)^p)/(1 + (b*Sin[e + f*x]^4)/a)^p + (2*Appell
F1[3/4, 2, -p, 7/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^
3*(a + b*Sin[e + f*x]^4)^p)/(3*(1 + (b*Sin[e + f*x]^4)/a)^p) + (AppellF1[5
/4, 2, -p, 9/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^5*(a
 + b*Sin[e + f*x]^4)^p)/(5*(1 + (b*Sin[e + f*x]^4)/a)^p))/f
```

3.424.3.1 Defintions of rubi rules used

rule 1569
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.424.4 Maple [F]

$$\int \left(\sec^3(fx+e)\right)\left(a+b\left(\sin^4(fx+e)\right)\right)^p dx$$

input
```
int(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x)
```

output
```
int(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x)
```

3.424.5 Fricas [F]

$$\int \sec^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^4+a\right)^p\sec\left(fx+e\right)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

output
```
integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^3,
 x)
```

3.424.6 Sympy [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**4)**p,x)
```

output
```
Timed out
```

3.424.7 Maxima [F]

$$\int \sec^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^4+a\right)^p\sec\left(fx+e\right)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^3, x)
```

3.424.8 Giac [F]

$$\int \sec^3(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx = \int \left(b \sin\left(fx+e\right)^4 + a\right)^p \sec\left(fx+e\right)^3 \, dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^3, x)
```

3.424.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx = \int \frac{\left(b \sin\left(e+f\,x\right)^4+a\right)^p}{\cos\left(e+f\,x\right)^3} \, dx$$

input
```
int((a + b*sin(e + f*x)^4)^p/cos(e + f*x)^3,x)
```

output
```
int((a + b*sin(e + f*x)^4)^p/cos(e + f*x)^3, x)
```

3.425 $\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

3.425.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\operatorname{Int}\left(\cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

output
```
Unintegrable(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

3.425.2 Mathematica [N/A]

Not integrable

Time = 3.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

input
```
Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p, x]
```

3.425.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3707}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \cos(e+fx)^4 \left(a+b\sin(e+fx)^4\right)^p dx$$

$$\Big\downarrow \text{ 3707}$$

$$\int \cos^4(e+fx) \left(a+b\sin^4(e+fx)\right)^p dx$$

input
```
Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
$Aborted
```

3.425.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3707
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Cos[e + f*x])^m*(a +
 b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.425.4 Maple [N/A] (verified)

Not integrable

Time = 1.94 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(\cos^{4}\left(fx+e\right)\right)\left(a+b\left(\sin^{4}\left(fx+e\right)\right)\right)^{p}dx$$

input
```
int(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

output
```
int(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

3.425.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

output
```
integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^4,
 x)
```

3.425.6 Sympy [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**4)**p,x)
```

output
```
Timed out
```

3.425.7 Maxima [N/A]

Not integrable

Time = 3.54 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^4, x)
```

3.425.8 Giac [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^4, x)
```

3.425.9 Mupad [N/A]

Not integrable

Time = 15.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^4\,\left(b\sin\left(e+f\,x\right)^4+a\right)^p\,dx$$

input
```
int(cos(e + f*x)^4*(a + b*sin(e + f*x)^4)^p,x)
```

output
```
int(cos(e + f*x)^4*(a + b*sin(e + f*x)^4)^p, x)
```

3.426 $\int \cos^2(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx$

3.426.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \cos^2(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx = \text{Int}\left(\cos^2(e+fx) \left(a+b\sin^4(e+fx)\right)^p, x\right)$$

output
```
Unintegrable(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

3.426.2 Mathematica [N/A]

Not integrable

Time = 4.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^2(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx = \int \cos^2(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx$$

input
```
Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p, x]
```

3.426.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3707}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(e+fx)^2\left(a+b\sin(e+fx)^4\right)^p dx$$

$$\Big\downarrow \; 3707$$

$$\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p dx$$

input
```
Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
$Aborted
```

3.426.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3707
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Cos[e + f*x])^m*(a +
 b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.426.4 Maple [N/A] (verified)

Not integrable

Time = 1.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^4\left(fx+e\right)\right)\right)^p dx$$

input
```
int(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

output
```
int(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

3.426.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

output
```
integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^2,
 x)
```

3.426.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**4)**p,x)
```

output
```
Timed out
```

3.426.7 Maxima [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^2, x)
```

3.426.8 Giac [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^2, x)
```

3.426.9 Mupad [N/A]

Not integrable

Time = 14.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^2\left(b\sin\left(e+f\,x\right)^4+a\right)^p\,dx$$

input
```
int(cos(e + f*x)^2*(a + b*sin(e + f*x)^4)^p,x)
```

output
```
int(cos(e + f*x)^2*(a + b*sin(e + f*x)^4)^p, x)
```

3.427 $\int (a+b\sin^4(e+fx))^p \, dx$

3.427.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (a+b\sin^4(e+fx))^p \, dx = \text{Int}\left((a+b\sin^4(e+fx))^p, x\right)$$

output
```
Unintegrable((a+b*sin(f*x+e)^4)^p,x)
```

3.427.2 Mathematica [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sin^4(e+fx))^p \, dx = \int (a+b\sin^4(e+fx))^p \, dx$$

input
```
Integrate[(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
Integrate[(a + b*Sin[e + f*x]^4)^p, x]
```

3.427.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3693}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin^4(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin(e+fx)^4\right)^p\,dx$$

$$\Big\downarrow \text{3693}$$

$$\int \left(a+b\sin^4(e+fx)\right)^p\,dx$$

input
```
Int[(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
$Aborted
```

3.427.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3693
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Unintegrable[(a + b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, e, f, n, p
}, x]
```

3.427.4 Maple [N/A] (verified)

Not integrable

Time = 0.65 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a+b\left(\sin^4\left(fx+e\right)\right)\right)^p dx$$

input
```
int((a+b*sin(f*x+e)^4)^p,x)
```

output
```
int((a+b*sin(f*x+e)^4)^p,x)
```

3.427.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \left(a+b\sin^4(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^4+a\right)^p \, dx$$

input
```
integrate((a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

output
```
integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p, x)
```

3.427.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sin^4(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate((a+b*sin(f*x+e)**4)**p,x)
```

output
```
Timed out
```

3.427.7 Maxima [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^4+a\right)^p\,dx$$

input
```
integrate((a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p, x)
```

3.427.8 Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^4+a\right)^p\,dx$$

input
```
integrate((a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p, x)
```

3.427.9 Mupad [N/A]

Not integrable

Time = 14.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(b\sin\left(e+f\,x\right)^4+a\right)^p\,dx$$

input
```
int((a + b*sin(e + f*x)^4)^p,x)
```

output
```
int((a + b*sin(e + f*x)^4)^p, x)
```

3.428 $\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

3.428.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\text{Int}\left(\sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

output
```
Unintegrable(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

3.428.2 Mathematica [N/A]

Not integrable

Time = 4.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

input
```
Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p, x]
```

3.428.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3707}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\sin(e+fx)^4\right)^p}{\cos(e+fx)^2}dx$$
$$\Big\downarrow \text{3707}$$
$$\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

input
```
Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
$Aborted
```

3.428.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3707
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Cos[e + f*x])^m*(a +
 b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.428.4 Maple [N/A] (verified)

Not integrable

Time = 1.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(\sec^2(fx+e)\right)\left(a+b\left(\sin^4(fx+e)\right)\right)^p dx$$

input
```
int(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

output
```
int(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

3.428.5 Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p dx = \int \left(b\sin\left(fx+e\right)^4+a\right)^p \sec\left(fx+e\right)^2 dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

output
```
integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^2,
 x)
```

3.428.6 Sympy [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**4)**p,x)
```

output
```
Timed out
```

3.428.7 Maxima [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^4+a\right)^p\sec\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^2, x)
```

3.428.8 Giac [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^4+a\right)^p\sec\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^2, x)
```

3.428.9 Mupad [N/A]

Not integrable

Time = 16.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \frac{\left(b\sin\left(e+f\,x\right)^4+a\right)^p}{\cos\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b*sin(e + f*x)^4)^p/cos(e + f*x)^2,x)
```

output
```
int((a + b*sin(e + f*x)^4)^p/cos(e + f*x)^2, x)
```

3.429 $\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

3.429.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \text{Int}\left(\sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

output
```
Unintegrable(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

3.429.2 Mathematica [N/A]

Not integrable

Time = 7.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

input
```
Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p, x]
```

3.429.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3707}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a+b\sin(e+fx)^4\right)^p}{\cos(e+fx)^4}dx$$

$$\Big\downarrow \ 3707$$

$$\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

input
```
Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p,x]
```

output
```
$Aborted
```

3.429.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3707
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Cos[e + f*x])^m*(a +
 b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.429.4 Maple [N/A] (verified)

Not integrable

Time = 1.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(\sec^4\left(fx+e\right)\right)\left(a+b\left(\sin^4\left(fx+e\right)\right)\right)^p dx$$

input
```
int(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

output
```
int(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

3.429.5 Fricas [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \int \left(b\sin\left(fx+e\right)^4+a\right)^p\sec\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

output
```
integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^4,
 x)
```

3.429.6 Sympy [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**4)**p,x)
```

output
```
Timed out
```

3.429.7 Maxima [N/A]

Not integrable

Time = 3.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^4(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^4+a\right)^p \sec\left(fx+e\right)^4 \, dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^4, x)
```

3.429.8 Giac [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^4(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx = \int \left(b\sin\left(fx+e\right)^4+a\right)^p \sec\left(fx+e\right)^4 \, dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^4, x)
```

3.429.9 Mupad [N/A]

Not integrable

Time = 17.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^4(e+fx) \left(a+b\sin^4(e+fx)\right)^p \, dx = \int \frac{\left(b\sin\left(e+f\,x\right)^4+a\right)^p}{\cos\left(e+f\,x\right)^4} \, dx$$

input
```
int((a + b*sin(e + f*x)^4)^p/cos(e + f*x)^4,x)
```

output
```
int((a + b*sin(e + f*x)^4)^p/cos(e + f*x)^4, x)
```

3.430 $\int \cos^m(e+fx) (a+b\sin^n(e+fx))^p \, dx$

3.430.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \cos^m(e+fx) (a+b\sin^n(e+fx))^p \, dx = \text{Int}(\cos^m(e+fx) (a+b\sin^n(e+fx))^p, x)$$

output
```
Unintegrable(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x)
```

3.430.2 Mathematica [N/A]

Not integrable

Time = 5.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^m(e+fx) (a+b\sin^n(e+fx))^p \, dx = \int \cos^m(e+fx) (a+b\sin^n(e+fx))^p \, dx$$

input
```
Integrate[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
Integrate[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^n)^p, x]
```

3.430.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3707}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^m(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^m\,(a+b\sin(e+fx)^n)^p\,dx$$

$$\Big\downarrow \text{3707}$$

$$\int \cos^m(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

input
```
Int[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
$Aborted
```

3.430.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3707
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Cos[e + f*x])^m*(a +
 b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.430.4 Maple [N/A] (verified)

Not integrable

Time = 0.68 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (\cos^m (fx+e)) (a+b(\sin^n (fx+e)))^p dx$$

input
```
int(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x)
```

output
```
int(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x)
```

3.430.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^m(e+fx) (a+b\sin^n(e+fx))^p \, dx = \int (b\sin(fx+e)^n+a)^p \cos(fx+e)^m \, dx$$

input
```
integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^m, x)
```

3.430.6 Sympy [N/A]

Not integrable

Time = 129.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \cos^m(e+fx) (a+b\sin^n(e+fx))^p \, dx = \int (a+b\sin^n(e+fx))^p \cos^m(e+fx) \, dx$$

input
```
integrate(cos(f*x+e)**m*(a+b*sin(f*x+e)**n)**p,x)
```

output
```
Integral((a + b*sin(e + f*x)**n)**p*cos(e + f*x)**m, x)
```

3.430.7 Maxima [N/A]

Not integrable

Time = 3.69 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int (b\sin\left(fx+e\right)^n+a)^p\cos\left(fx+e\right)^m\,dx$$

input
```
integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^m, x)
```

3.430.8 Giac [N/A]

Not integrable

Time = 4.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int (b\sin\left(fx+e\right)^n+a)^p\cos\left(fx+e\right)^m\,dx$$

input
```
integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^m, x)
```

3.430.9 Mupad [N/A]

Not integrable

Time = 14.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^m\,{\left(a+b\,\sin\left(e+f\,x\right)^n\right)}^p\,dx$$

input
```
int(cos(e + f*x)^m*(a + b*sin(e + f*x)^n)^p,x)
```

output
```
int(cos(e + f*x)^m*(a + b*sin(e + f*x)^n)^p, x)
```

3.431 $\int \cos^5(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$

3.431.1 Optimal result

Integrand size = 23, antiderivative size = 226

$$\int \cos^5(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$
$$=\frac{\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b\sin^n(e+fx)}{a}\right)\sin(e+fx)\,(a+b\sin^n(e+fx))^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}}{f}$$
$$-\frac{2\,\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{3+n}{n},-\frac{b\sin^n(e+fx)}{a}\right)\sin^3(e+fx)\,(a+b\sin^n(e+fx))^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}}{3f}$$
$$+\frac{\text{Hypergeometric2F1}\left(\frac{5}{n},-p,\frac{5+n}{n},-\frac{b\sin^n(e+fx)}{a}\right)\sin^5(e+fx)\,(a+b\sin^n(e+fx))^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}}{5f}$$

output
```
hypergeom([-p, 1/n],[1+1/n],-b*sin(f*x+e)^n/a)*sin(f*x+e)*(a+b*sin(f*x+e)^
n)^p/f/((1+b*sin(f*x+e)^n/a)^p)-2/3*hypergeom([-p, 3/n],[(3+n)/n],-b*sin(f
*x+e)^n/a)*sin(f*x+e)^3*(a+b*sin(f*x+e)^n)^p/f/((1+b*sin(f*x+e)^n/a)^p)+1/
5*hypergeom([-p, 5/n],[(5+n)/n],-b*sin(f*x+e)^n/a)*sin(f*x+e)^5*(a+b*sin(f
*x+e)^n)^p/f/((1+b*sin(f*x+e)^n/a)^p)
```

3.431.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.69

$$\int \cos^5(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$
$$= \frac{\sin(e+fx)\left(15\,\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b\sin^n(e+fx)}{a}\right)-10\,\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{3+n}{n},-\right.\right.}{}$$

input
```
Integrate[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
(Sin[e + f*x]*(15*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f
*x]^n)/a)] - 10*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*Sin[e + f*x]^n)
/a)]*Sin[e + f*x]^2 + 3*Hypergeometric2F1[5/n, -p, (5 + n)/n, -((b*Sin[e +
 f*x]^n)/a)]*Sin[e + f*x]^4)*(a + b*Sin[e + f*x]^n)^p)/(15*f*(1 + (b*Sin[e
 + f*x]^n)/a)^p)
```

3.431.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3702, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \cos(e+fx)^5\left(a+b\sin(e+fx)^n\right)^p dx$$
$$\Big\downarrow \text{ 3702}$$
$$\frac{\int \left(1-\sin^2(e+fx)\right)^2\left(b\sin^n(e+fx)+a\right)^p d\sin(e+fx)}{f}$$
$$\Big\downarrow \text{ 2432}$$
$$\frac{\int \left(\sin^4(e+fx)\left(b\sin^n(e+fx)+a\right)^p-2\sin^2(e+fx)\left(b\sin^n(e+fx)+a\right)^p+\left(b\sin^n(e+fx)+a\right)^p\right)d\sin(e+fx)}{f}$$

$$\Big\downarrow 2009$$

$$\sin(e+fx)\,(a+b\sin^n(e+fx))^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b\sin^n(e+fx)}{a}\right)+\frac{1}{5}\sin^5(e$$

input
```
Int[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
((Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e
 + f*x]*(a + b*Sin[e + f*x]^n)^p)/(1 + (b*Sin[e + f*x]^n)/a)^p - (2*Hyperg
eometric2F1[3/n, -p, (3 + n)/n, -((b*Sin[e + f*x]^n)/a)]*Sin[e + f*x]^3*(a
 + b*Sin[e + f*x]^n)^p)/(3*(1 + (b*Sin[e + f*x]^n)/a)^p) + (Hypergeometric
2F1[5/n, -p, (5 + n)/n, -((b*Sin[e + f*x]^n)/a)]*Sin[e + f*x]^5*(a + b*Sin
[e + f*x]^n)^p)/(5*(1 + (b*Sin[e + f*x]^n)/a)^p))/f
```

3.431.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.431.4 Maple [F]

$$\int \left(\cos^{5}\left(fx+e\right)\right)\left(a+b\left(\sin^{n}\left(fx+e\right)\right)\right)^{p}dx$$

input
```
int(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x)
```

output
```
int(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x)
```

3.431.5 Fricas [F]

$$\int \cos^5(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^5, x)
```

3.431.6 Sympy [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**5*(a+b*sin(f*x+e)**n)**p,x)
```

output
```
Timed out
```

3.431.7 Maxima [F]

$$\int \cos^5(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int (b\sin(fx+e)^n+a)^p\cos(fx+e)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^5, x)
```

3.431.8 Giac [F]

$$\int \cos^5(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int (b\sin(fx+e)^n+a)^p\cos(fx+e)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^5, x)
```

3.431.9 Mupad [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \cos(e+f\,x)^5\,\left(a+b\sin(e+f\,x)^n\right)^p\,dx$$

input
```
int(cos(e + f*x)^5*(a + b*sin(e + f*x)^n)^p,x)
```

output
```
int(cos(e + f*x)^5*(a + b*sin(e + f*x)^n)^p, x)
```

3.432 $\int \cos^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

3.432.1 Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \cos^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$
$$=\frac{\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b\sin^n(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}}{f}$$
$$-\frac{\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{3+n}{n},-\frac{b\sin^n(e+fx)}{a}\right)\sin^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}}{3f}$$

output
```
hypergeom([-p, 1/n],[1+1/n],-b*sin(f*x+e)^n/a)*sin(f*x+e)*(a+b*sin(f*x+e)^
n)^p/f/((1+b*sin(f*x+e)^n/a)^p)-1/3*hypergeom([-p, 3/n],[(3+n)/n],-b*sin(f
*x+e)^n/a)*sin(f*x+e)^3*(a+b*sin(f*x+e)^n)^p/f/((1+b*sin(f*x+e)^n/a)^p)
```

3.432.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.77

$$\int \cos^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx =$$
$$-\frac{\sin(e+fx)\left(-3\,\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b\sin^n(e+fx)}{a}\right)+\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{3+n}{n},\right.\right.}{3f}$$

input
```
Integrate[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
-1/3*(Sin[e + f*x]*(-3*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[
e + f*x]^n)/a)] + Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*Sin[e + f*x]^
n)/a)]*Sin[e + f*x]^2)*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n
)/a)^p)
```

3.432.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3702, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^3\,(a+b\sin(e+fx)^n)^p\,dx$$

$$\Big\downarrow \text{3702}$$

$$\frac{\int (1-\sin^2(e+fx))\,(b\sin^n(e+fx)+a)^p\,d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2432}$$

$$\frac{\int \left((b\sin^n(e+fx)+a)^p-\sin^2(e+fx)\,(b\sin^n(e+fx)+a)^p\right)d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(e+fx)\,(a+b\sin^n(e+fx))^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b\sin^n(e+fx)}{a}\right)-\frac{1}{3}\sin^3(e}{f}$$

input
```
Int[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
((Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e
 + f*x]*(a + b*Sin[e + f*x]^n)^p)/(1 + (b*Sin[e + f*x]^n)/a)^p - (Hypergeo
metric2F1[3/n, -p, (3 + n)/n, -((b*Sin[e + f*x]^n)/a)]*Sin[e + f*x]^3*(a +
 b*Sin[e + f*x]^n)^p)/(3*(1 + (b*Sin[e + f*x]^n)/a)^p))/f
```

3.432.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.432.4 Maple [F]

$$\int (\cos^3(fx+e)) (a+b(\sin^n(fx+e)))^p dx$$

input
```
int(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
```

output
```
int(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
```

3.432.5 Fricas [F]

$$\int \cos^3(e+fx)(a+b\sin^n(e+fx))^p\,dx=\int (b\sin(fx+e)^n+a)^p\cos(fx+e)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^3, x)
```

3.432.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx)(a+b\sin^n(e+fx))^p\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**n)**p,x)
```

output
```
Timed out
```

3.432.7 Maxima [F]

$$\int \cos^3(e+fx)(a+b\sin^n(e+fx))^p\,dx=\int (b\sin(fx+e)^n+a)^p\cos(fx+e)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^3, x)
```

3.432.8 Giac [F]

$$\int \cos^3(e+fx)\,(a+b\sin^n(e+fx))^p\,dx=\int (b\sin(fx+e)^n+a)^p\cos(fx+e)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^3, x)
```

3.432.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\,(a+b\sin^n(e+fx))^p\,dx=\int \cos(e+f\,x)^3\,(a+b\sin(e+f\,x)^n)^p\,dx$$

input
```
int(cos(e + f*x)^3*(a + b*sin(e + f*x)^n)^p,x)
```

output
```
int(cos(e + f*x)^3*(a + b*sin(e + f*x)^n)^p, x)
```

3.433 $\int \cos(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

3.433.1 Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \cos(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b\sin^n(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}}{f}$$

output
```
hypergeom([-p, 1/n],[1+1/n],-b*sin(f*x+e)^n/a)*sin(f*x+e)*(a+b*sin(f*x+e)^
n)^p/f/((1+b*sin(f*x+e)^n/a)^p)
```

3.433.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \cos(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b\sin^n(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}}{f}$$

input
```
Integrate[Cos[e + f*x]*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
(Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e
+ f*x]*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a)^p)
```

3.433.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3702, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(e+fx)\left(a+b\sin(e+fx)^n\right)^p dx$$

$$\Big\downarrow \; 3702$$

$$\frac{\int (b\sin^n(e+fx)+a)^p\,d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 779$$

$$\frac{(a+b\sin^n(e+fx))^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}\int \left(\frac{b\sin^n(e+fx)}{a}+1\right)^p d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 778$$

$$\frac{\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b\sin^n(e+fx)}{a}\right)}{f}$$

input
```
Int[Cos[e + f*x]*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
(Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e
+ f*x]*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a)^p)
```

3.433.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.433.4 Maple [F]

$$\int \cos(fx+e)\,(a+b(\sin^n(fx+e)))^p dx$$

input
```
int(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
```

output
```
int(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
```

3.433.5 Fricas [F]

$$\int \cos(e+fx)\,(a+b\sin^n(e+fx))^p\,dx = \int (b\sin\left(fx+e\right)^n+a)^p\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e), x)
```

3.433.6 Sympy [F]

$$\int \cos(e+fx)\,(a+b\sin^n(e+fx))^p\,dx = \int (a+b\sin^n\left(e+fx\right))^p\cos\left(e+fx\right)dx$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)**n)**p,x)
```

output
```
Integral((a + b*sin(e + f*x)**n)**p*cos(e + f*x), x)
```

3.433.7 Maxima [F]

$$\int \cos(e+fx)\,(a+b\sin^n(e+fx))^p\,dx = \int (b\sin\left(fx+e\right)^n+a)^p\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e), x)
```

3.433.8 Giac [F]

$$\int \cos(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e), x)
```

3.433.9 Mupad [B] (verification not implemented)

Time = 14.55 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \cos(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\frac{\sin\left(e+f\,x\right)\,\left(a+b\sin\left(e+f\,x\right)^n\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{n},-p;\ \frac{1}{n}+1;\ -\frac{b\sin(e+f\,x)^n}{a}\right)}{f\left(\frac{b\sin(e+f\,x)^n}{a}+1\right)^p}$$

input
```
int(cos(e + f*x)*(a + b*sin(e + f*x)^n)^p,x)
```

output
```
(sin(e + f*x)*(a + b*sin(e + f*x)^n)^p*hypergeom([1/n, -p], 1/n + 1, -(b*s
in(e + f*x)^n)/a))/(f*((b*sin(e + f*x)^n)/a + 1)^p)
```

3.434 $\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

3.434.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx = \text{Int}\left(\sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

output
```
Unintegrable(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
```

3.434.2 Mathematica [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx = \int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

input
```
Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^n)^p, x]
```

3.434.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3707}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\sin(e+fx)^n)^p}{\cos(e+fx)}dx$$

$$\Big\downarrow \text{3707}$$

$$\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

input
```
Int[Sec[e + f*x]*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
$Aborted
```

3.434.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3707
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Cos[e + f*x])^m*(a +
 b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.434.4 Maple [N/A] (verified)

Not integrable

Time = 0.59 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sec(fx+e)(a+b(\sin^n(fx+e)))^p dx$$

input
```
int(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
```

output
```
int(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
```

3.434.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sec(e+fx)(a+b\sin^n(e+fx))^p\,dx = \int (b\sin(fx+e)^n+a)^p\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e)^n + a)^p*sec(f*x + e), x)
```

3.434.6 Sympy [N/A]

Not integrable

Time = 132.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \sec(e+fx)(a+b\sin^n(e+fx))^p\,dx = \int (a+b\sin^n(e+fx))^p\sec(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)**n)**p,x)
```

output
```
Integral((a + b*sin(e + f*x)**n)**p*sec(e + f*x), x)
```

3.434.7 Maxima [N/A]

Not integrable

Time = 2.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^n+a\right)^p\sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e), x)
```

3.434.8 Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^n+a\right)^p\sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e), x)
```

3.434.9 Mupad [N/A]

Not integrable

Time = 13.65 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \frac{\left(a+b\sin\left(e+f\,x\right)^n\right)^p}{\cos\left(e+f\,x\right)}\,dx$$

input
```
int((a + b*sin(e + f*x)^n)^p/cos(e + f*x),x)
```

output
```
int((a + b*sin(e + f*x)^n)^p/cos(e + f*x), x)
```

3.435 $\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

3.435.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\mathrm{Int}\left(\sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

output
```
Unintegrable(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
```

3.435.2 Mathematica [N/A]

Not integrable

Time = 7.65 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

input
```
Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p, x]
```

3.435.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3707}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+b\sin(e+fx)^n)^p}{\cos(e+fx)^3}dx$$

$$\Big\downarrow \; 3707$$

$$\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p dx$$

input
```
Int[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
$Aborted
```

3.435.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3707
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Cos[e + f*x])^m*(a +
 b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.435.4 Maple [N/A] (verified)

Not integrable

Time = 0.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(\sec^{3}\left(fx+e\right)\right)\left(a+b(\sin^{n}\left(fx+e\right))\right)^{p}dx$$

input
```
int(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
```

output
```
int(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
```

3.435.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\sec\left(fx+e\right)^{3}\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^3, x)
```

3.435.6 Sympy [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**n)**p,x)
```

output
```
Timed out
```

3.435.7 Maxima [N/A]

Not integrable

Time = 2.73 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int (b\sin(fx+e)^n+a)^p\sec(fx+e)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^3, x)
```

3.435.8 Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int (b\sin(fx+e)^n+a)^p\sec(fx+e)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^3, x)
```

3.435.9 Mupad [N/A]

Not integrable

Time = 13.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \frac{(a+b\sin(e+f\,x)^n)^p}{\cos(e+f\,x)^3}\,dx$$

input
```
int((a + b*sin(e + f*x)^n)^p/cos(e + f*x)^3,x)
```

output
```
int((a + b*sin(e + f*x)^n)^p/cos(e + f*x)^3, x)
```

3.436 $\int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

3.436.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\operatorname{Int}\left(\cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

output
```
Unintegrable(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

3.436.2 Mathematica [N/A]

Not integrable

Time = 10.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

input
```
Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p, x]
```

3.436.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3707}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(e+fx)^4\,(a+b\sin(e+fx)^n)^p\,dx$$

$$\Big\downarrow \; 3707$$

$$\int \cos^4(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

input
```
Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
$Aborted
```

3.436.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3707
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Cos[e + f*x])^m*(a +
 b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.436.4 Maple [N/A] (verified)

Not integrable

Time = 0.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(\cos^4\left(fx+e\right)\right)\left(a+b(\sin^n\left(fx+e\right))\right)^p dx$$

input
```
int(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

output
```
int(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

3.436.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^4, x)
```

3.436.6 Sympy [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**n)**p,x)
```

output
```
Timed out
```

3.436.7 Maxima [N/A]

Not integrable

Time = 7.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int (b\sin\left(fx+e\right)^n+a)^p\cos\left(fx+e\right)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^4, x)
```

3.436.8 Giac [N/A]

Not integrable

Time = 10.63 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int (b\sin\left(fx+e\right)^n+a)^p\cos\left(fx+e\right)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^4, x)
```

3.436.9 Mupad [N/A]

Not integrable

Time = 13.49 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \cos\left(e+f\,x\right)^4\left(a+b\sin\left(e+f\,x\right)^n\right)^p\,dx$$

input
```
int(cos(e + f*x)^4*(a + b*sin(e + f*x)^n)^p,x)
```

output
```
int(cos(e + f*x)^4*(a + b*sin(e + f*x)^n)^p, x)
```

3.437 $\int \cos^2(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$

3.437.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \cos^2(e+fx)\,(a+b\sin^n(e+fx))^p\,dx = \text{Int}\left(\cos^2(e+fx)\,(a+b\sin^n(e+fx))^p,x\right)$$

output
```
Unintegrable(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
```

3.437.2 Mathematica [N/A]

Not integrable

Time = 8.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^2(e+fx)\,(a+b\sin^n(e+fx))^p\,dx = \int \cos^2(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

input
```
Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p, x]
```

3.437.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3707}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^2\,(a+b\sin(e+fx)^n)^p\,dx$$

$$\Big\downarrow \text{3707}$$

$$\int \cos^2(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

input
```
Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
$Aborted
```

3.437.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3707
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Cos[e + f*x])^m*(a +
 b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.437.4 Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^n\left(fx+e\right)\right)\right)^p dx$$

input
```
int(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
```

output
```
int(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
```

3.437.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx = \int \left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^2, x)
```

3.437.6 Sympy [N/A]

Not integrable

Time = 43.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx = \int \left(a+b\sin^n\left(e+fx\right)\right)^p\cos^2\left(e+fx\right)dx$$

input
```
integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**n)**p,x)
```

output
```
Integral((a + b*sin(e + f*x)**n)**p*cos(e + f*x)**2, x)
```

3.437.7 Maxima [N/A]

Not integrable

Time = 4.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int (b\sin{(fx+e)}^n+a)^p\cos{(fx+e)}^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^2, x)
```

3.437.8 Giac [N/A]

Not integrable

Time = 6.96 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int (b\sin{(fx+e)}^n+a)^p\cos{(fx+e)}^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^2, x)
```

3.437.9 Mupad [N/A]

Not integrable

Time = 14.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \cos{(e+f\,x)}^2\,\left(a+b\sin{(e+f\,x)}^n\right)^p\,dx$$

input
```
int(cos(e + f*x)^2*(a + b*sin(e + f*x)^n)^p,x)
```

output
```
int(cos(e + f*x)^2*(a + b*sin(e + f*x)^n)^p, x)
```

3.438 $\int (a+b\sin^n(e+fx))^p\,dx$

3.438.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (a+b\sin^n(e+fx))^p\,dx = \operatorname{Int}\left((a+b\sin^n(e+fx))^p,x\right)$$

output
```
Unintegrable((a+b*sin(f*x+e)^n)^p,x)
```

3.438.2 Mathematica [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sin^n(e+fx))^p\,dx = \int (a+b\sin^n(e+fx))^p\,dx$$

input
```
Integrate[(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
Integrate[(a + b*Sin[e + f*x]^n)^p, x]
```

3.438.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3693}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin^n(e+fx))^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sin(e+fx)^n)^p\,dx$$

$$\Big\downarrow \text{3693}$$

$$\int (a+b\sin^n(e+fx))^p\,dx$$

input
```
Int[(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
$Aborted
```

3.438.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3693
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Unintegrable[(a + b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, e, f, n, p
}, x]
```

3.438.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b(\sin^n(fx+e)))^p dx$$

input
```
int((a+b*sin(f*x+e)^n)^p,x)
```

output
```
int((a+b*sin(f*x+e)^n)^p,x)
```

3.438.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sin^n(e+fx))^p \, dx = \int (b\sin(fx+e)^n+a)^p \, dx$$

input
```
integrate((a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e)^n + a)^p, x)
```

3.438.6 Sympy [N/A]

Not integrable

Time = 4.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sin^n(e+fx))^p \, dx = \int (a+b\sin^n(e+fx))^p \, dx$$

input
```
integrate((a+b*sin(f*x+e)**n)**p,x)
```

output
```
Integral((a + b*sin(e + f*x)**n)**p, x)
```

3.438.7 Maxima [N/A]

Not integrable

Time = 2.73 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a + b\sin^n(e + fx))^p \, dx = \int (b\sin(fx + e)^n + a)^p \, dx$$

input
```
integrate((a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p, x)
```

3.438.8 Giac [N/A]

Not integrable

Time = 3.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a + b\sin^n(e + fx))^p \, dx = \int (b\sin(fx + e)^n + a)^p \, dx$$

input
```
integrate((a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p, x)
```

3.438.9 Mupad [N/A]

Not integrable

Time = 14.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a + b\sin^n(e + fx))^p \, dx = \int (a + b\sin(e + f\,x)^n)^p \, dx$$

input
```
int((a + b*sin(e + f*x)^n)^p,x)
```

output
```
int((a + b*sin(e + f*x)^n)^p, x)
```

3.439 $\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

3.439.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\text{Int}\left(\sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

output
```
Unintegrable(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
```

3.439.2 Mathematica [N/A]

Not integrable

Time = 4.77 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

input
```
Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p, x]
```

3.439.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3707}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\sin(e+fx)^n)^p}{\cos(e+fx)^2}dx$$

$$\Big\downarrow \text{3707}$$

$$\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p dx$$

input
```
Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
$Aborted
```

3.439.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3707
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Cos[e + f*x])^m*(a +
 b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.439.4 Maple [N/A] (verified)

Not integrable

Time = 0.66 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(\sec^2(fx+e)\right)(a+b(\sin^n(fx+e)))^p dx$$

input
```
int(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
```

output
```
int(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
```

3.439.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^2(e+fx)\,(a+b\sin^n(e+fx))^p\,dx = \int (b\sin(fx+e)^n+a)^p\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^2, x)
```

3.439.6 Sympy [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\,(a+b\sin^n(e+fx))^p\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**n)**p,x)
```

output
```
Timed out
```

3.439.7 Maxima [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int (b\sin(fx+e)^n+a)^p\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^2, x)
```

3.439.8 Giac [N/A]

Not integrable

Time = 3.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int (b\sin(fx+e)^n+a)^p\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^2, x)
```

3.439.9 Mupad [N/A]

Not integrable

Time = 13.74 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \frac{(a+b\sin(e+f\,x)^n)^p}{\cos(e+f\,x)^2}\,dx$$

input
```
int((a + b*sin(e + f*x)^n)^p/cos(e + f*x)^2,x)
```

output
```
int((a + b*sin(e + f*x)^n)^p/cos(e + f*x)^2, x)
```

3.440 $\int \sec^4(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$

3.440.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \sec^4(e+fx)\,(a+b\sin^n(e+fx))^p\,dx = \text{Int}\left(\sec^4(e+fx)\,(a+b\sin^n(e+fx))^p\,,x\right)$$

output
```
Unintegrable(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

3.440.2 Mathematica [N/A]

Not integrable

Time = 7.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^4(e+fx)\,(a+b\sin^n(e+fx))^p\,dx = \int \sec^4(e+fx)\,(a+b\sin^n(e+fx))^p\,dx$$

input
```
Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p, x]
```

3.440.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3707}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\sin(e+fx)^n)^p}{\cos(e+fx)^4}dx$$

$$\Big\downarrow \text{ 3707}$$

$$\int \sec^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p dx$$

input
```
Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p,x]
```

output
```
$Aborted
```

3.440.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3707
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Cos[e + f*x])^m*(a +
 b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.440.4 Maple [N/A] (verified)

Not integrable

Time = 0.94 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(\sec^4(fx+e)\right)(a+b(\sin^n(fx+e)))^p dx$$

input
```
int(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

output
```
int(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

3.440.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^4(e+fx)(a+b\sin^n(e+fx))^p\,dx = \int (b\sin(fx+e)^n+a)^p\sec(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^4, x)
```

3.440.6 Sympy [F(-1)]

Timed out.

$$\int \sec^4(e+fx)(a+b\sin^n(e+fx))^p\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**n)**p,x)
```

output
```
Timed out
```

3.440.7 Maxima [N/A]

Not integrable

Time = 3.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int (b\sin(fx+e)^n+a)^p\sec(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^4, x)
```

3.440.8 Giac [N/A]

Not integrable

Time = 3.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int (b\sin(fx+e)^n+a)^p\sec(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^4, x)
```

3.440.9 Mupad [N/A]

Not integrable

Time = 13.75 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \frac{(a+b\sin(e+f\,x)^n)^p}{\cos(e+f\,x)^4}\,dx$$

input
```
int((a + b*sin(e + f*x)^n)^p/cos(e + f*x)^4,x)
```

output
```
int((a + b*sin(e + f*x)^n)^p/cos(e + f*x)^4, x)
```

3.441 $\int \frac{\tan^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.441.1 Optimal result

Integrand size = 23, antiderivative size = 128

$$\int \frac{\tan^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{a^3\log(\cos(c+dx))}{(a+b)^4d} - \frac{a^3\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)^4d} + \frac{(3a^2+3ab+b^2)\sec^2(c+dx)}{2(a+b)^3d} - \frac{(3a+2b)\sec^4(c+dx)}{4(a+b)^2d} + \frac{\sec^6(c+dx)}{6(a+b)d}$$

output
```
a^3*ln(cos(d*x+c))/(a+b)^4/d-1/2*a^3*ln(a+b*sin(d*x+c)^2)/(a+b)^4/d+1/2*(3
*a^2+3*a*b+b^2)*sec(d*x+c)^2/(a+b)^3/d-1/4*(3*a+2*b)*sec(d*x+c)^4/(a+b)^2/
d+1/6*sec(d*x+c)^6/(a+b)/d
```

3.441.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.88

$$\int \frac{\tan^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{12a^3\log(\cos(c+dx))}{(a+b)^4} - \frac{6a^3\log\left(a+b\sin^2(c+dx)\right)}{(a+b)^4} + \frac{6(3a^2+3ab+b^2)\sec^2(c+dx)}{(a+b)^3} - \frac{3(3a+2b)\sec^4(c+dx)}{(a+b)^2} + \frac{2\sec^6(c+dx)}{a+b}}{12d}$$

input
```
Integrate[Tan[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

output
```
((12*a^3*Log[Cos[c + d*x]])/(a + b)^4 - (6*a^3*Log[a + b*Sin[c + d*x]^2])/
(a + b)^4 + (6*(3*a^2 + 3*a*b + b^2)*Sec[c + d*x]^2)/(a + b)^3 - (3*(3*a +
 2*b)*Sec[c + d*x]^4)/(a + b)^2 + (2*Sec[c + d*x]^6)/(a + b))/(12*d)
```

3.441.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3673, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^7(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\tan(c+dx)^7}{a+b\sin(c+dx)^2}dx$$

$$\Big\downarrow \ 3673$$

$$\frac{\int \frac{\sin^6(c+dx)}{(1-\sin^2(c+dx))^4(b\sin^2(c+dx)+a)}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \ 99$$

$$\frac{\int \left(\frac{a^3}{(a+b)^4(\sin^2(c+dx)-1)}-\frac{ba^3}{(a+b)^4(b\sin^2(c+dx)+a)}+\frac{3a^2+3ba+b^2}{(a+b)^3(\sin^2(c+dx)-1)^2}+\frac{3a+2b}{(a+b)^2(\sin^2(c+dx)-1)^3}+\frac{1}{(a+b)(\sin^2(c+dx)-1)^4}\right)}{2d}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{a^3\log(1-\sin^2(c+dx))}{(a+b)^4}-\frac{a^3\log(a+b\sin^2(c+dx))}{(a+b)^4}+\frac{3a^2+3ab+b^2}{(a+b)^3(1-\sin^2(c+dx))}-\frac{3a+2b}{2(a+b)^2(1-\sin^2(c+dx))^2}+\frac{1}{3(a+b)(1-\sin^2(c+dx))^3}}{2d}$$

input
```
Int[Tan[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

output
```
((a^3*Log[1 - Sin[c + d*x]^2])/(a + b)^4 - (a^3*Log[a + b*Sin[c + d*x]^2])
/(a + b)^4 + 1/(3*(a + b)*(1 - Sin[c + d*x]^2)^3) - (3*a + 2*b)/(2*(a + b)
^2*(1 - Sin[c + d*x]^2)^2) + (3*a^2 + 3*a*b + b^2)/((a + b)^3*(1 - Sin[c +
 d*x]^2)))/(2*d)
```

3.441.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.441.4 Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{a^3 \ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2(a+b)^4}+\frac{a^3 \ln(\cos(dx+c))}{(a+b)^4}-\frac{3a+2b}{4(a+b)^2 \cos(dx+c)^4}-\frac{-3a^2-3ab-b^2}{2(a+b)^3 \cos(dx+c)^2}+\frac{1}{6(a+b)\cos(dx+c)^6}}{d}$
default	$\frac{-\frac{a^3 \ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2(a+b)^4}+\frac{a^3 \ln(\cos(dx+c))}{(a+b)^4}-\frac{3a+2b}{4(a+b)^2 \cos(dx+c)^4}-\frac{-3a^2-3ab-b^2}{2(a+b)^3 \cos(dx+c)^2}+\frac{1}{6(a+b)\cos(dx+c)^6}}{d}$
risch	$\frac{6a^2 e^{10i(dx+c)}+6ab\, e^{10i(dx+c)}+2b^2 e^{10i(dx+c)}+12a^2 e^{8i(dx+c)}+4ab\, e^{8i(dx+c)}+\frac{68a^2 e^{6i(dx+c)}}{3}+\frac{52ab\, e^{6i(dx+c)}}{3}+\frac{20b^2 e^{6i(d}}{3}}{d(a+b)^3\left(e^{2i(dx+c)}+1\right)^6}$

input
```
int(tan(d*x+c)^7/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a^3/(a+b)^4*ln(a+b-b*cos(d*x+c)^2)+a^3/(a+b)^4*ln(cos(d*x+c))-1/
4*(3*a+2*b)/(a+b)^2/cos(d*x+c)^4-1/2*(-3*a^2-3*a*b-b^2)/(a+b)^3/cos(d*x+c)
^2+1/6/(a+b)/cos(d*x+c)^6)
```

3.441.5 Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.40

$$\int \frac{\tan^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{6\,a^3\cos\left(dx+c\right)^6\log\left(-b\cos\left(dx+c\right)^2+a+b\right)-12\,a^3\cos\left(dx+c\right)^6\log\left(-\cos\left(dx+c\right)\right)-6\left(3\,a^3+6\,a\right.}{12\left(a^4+4\,a^3b+6\,a^2b^2+\right.}$$

input
```
integrate(tan(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/12*(6*a^3*cos(d*x + c)^6*log(-b*cos(d*x + c)^2 + a + b) - 12*a^3*cos(d*
x + c)^6*log(-cos(d*x + c)) - 6*(3*a^3 + 6*a^2*b + 4*a*b^2 + b^3)*cos(d*x
+ c)^4 - 2*a^3 - 6*a^2*b - 6*a*b^2 - 2*b^3 + 3*(3*a^3 + 8*a^2*b + 7*a*b^2
+ 2*b^3)*cos(d*x + c)^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*co
s(d*x + c)^6)
```

3.441.6 Sympy [F]

$$\int \frac{\tan^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\tan^7\left(c+dx\right)}{a+b\sin^2\left(c+dx\right)}\,dx$$

input
```
integrate(tan(d*x+c)**7/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(tan(c + d*x)**7/(a + b*sin(c + d*x)**2), x)
```

3.441.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(120) = 240$.

Time = 0.27 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.13

$$\int \frac{\tan^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{6\,a^3\log\left(b\sin(dx+c)^2+a\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} - \frac{6\,a^3\log\left(\sin(dx+c)^2-1\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} + \frac{6\,(3\,a^2+3\,ab+b^2)\sin(dx+c)^4-3\,(9\,a^2+7\,ab+2\,b^2)\,\mathrm{s}}{(a^3+3\,a^2b+3\,ab^2+b^3)\sin(dx+c)^6-3\,(a^3+3\,a^2b+3\,ab^2+b^3)\sin(dx+c)^4-a^3-}}{12\,d}$$

input
```
integrate(tan(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/12*(6*a^3*log(b*sin(d*x + c)^2 + a)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^
3 + b^4) - 6*a^3*log(sin(d*x + c)^2 - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*
b^3 + b^4) + (6*(3*a^2 + 3*a*b + b^2)*sin(d*x + c)^4 - 3*(9*a^2 + 7*a*b +
2*b^2)*sin(d*x + c)^2 + 11*a^2 + 7*a*b + 2*b^2)/((a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*sin(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sin(d*x + c)^4 -
 a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sin(d*x
 + c)^2))/d
```

3.441.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(120) = 240$.

Time = 4.01 (sec) , antiderivative size = 603, normalized size of antiderivative = 4.71

$$\int \frac{\tan^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{30\,a^3\log\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} - \frac{60\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} + \frac{147\,a^3+\frac{1002\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+120}{}}{}$$

input
```
integrate(tan(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/60*(30*a^3*log(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 60*a^3*log(abs(-(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^
3 + b^4) + (147*a^3 + 1002*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 120
*a^2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2925*a^3*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 960*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 + 240*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4780*a^3*(cos(
d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 3600*a^2*b*(cos(d*x + c) - 1)^3/(co
s(d*x + c) + 1)^3 + 2400*a*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 +
 640*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2925*a^3*(cos(d*x + c
) - 1)^4/(cos(d*x + c) + 1)^4 + 960*a^2*b*(cos(d*x + c) - 1)^4/(cos(d*x +
c) + 1)^4 + 240*a*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1002*a^3
*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 120*a^2*b*(cos(d*x + c) - 1)^
5/(cos(d*x + c) + 1)^5 + 147*a^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6
)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*((cos(d*x + c) - 1)/(cos(d*
x + c) + 1) + 1)^6))/d
```

3.441.9 Mupad [B] (verification not implemented)

Time = 13.91 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.90

$$\int \frac{\tan^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\tan(c+d\,x)^6}{6\,d\,(a+b)} + \frac{a^2\tan(c+d\,x)^2}{2\,d\,(a+b)^3} - \frac{a^3\,\ln\left((a+b)\,\tan(c+d\,x)^2+a\right)}{d\,\left(2\,a^4+8\,a^3\,b+12\,a^2\,b^2+8\,a\,b^3+2\,b^4\right)} - \frac{a\tan(c+d\,x)^4}{4\,d\,(a+b)^2}$$

input
```
int(tan(c + d*x)^7/(a + b*sin(c + d*x)^2),x)
```

output
```
tan(c + d*x)^6/(6*d*(a + b)) + (a^2*tan(c + d*x)^2)/(2*d*(a + b)^3) - (a^3
*log(a + tan(c + d*x)^2*(a + b)))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 +
12*a^2*b^2)) - (a*tan(c + d*x)^4)/(4*d*(a + b)^2)
```

3.442 $\int \frac{\tan^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.442.1 Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{\tan^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{a^2\log(\cos(c+dx))}{(a+b)^3d} + \frac{a^2\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)^3d} - \frac{(2a+b)\sec^2(c+dx)}{2(a+b)^2d} + \frac{\sec^4(c+dx)}{4(a+b)d}$$

output
```
-a^2*ln(cos(d*x+c))/(a+b)^3/d+1/2*a^2*ln(a+b*sin(d*x+c)^2)/(a+b)^3/d-1/2*(
2*a+b)*sec(d*x+c)^2/(a+b)^2/d+1/4*sec(d*x+c)^4/(a+b)/d
```

3.442.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int \frac{\tan^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{2a^2\left(-2\log(\cos(c+dx))+\log\left(a+b\sin^2(c+dx)\right)\right)-2\left(2a^2+3ab+b^2\right)\sec^2(c+dx)+(a+b)^2\sec^4(c+d}{4(a+b)^3d}$$

input
```
Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

output
```
(2*a^2*(-2*Log[Cos[c + d*x]] + Log[a + b*Sin[c + d*x]^2]) - 2*(2*a^2 + 3*a
*b + b^2)*Sec[c + d*x]^2 + (a + b)^2*Sec[c + d*x]^4)/(4*(a + b)^3*d)
```

3.442.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3673, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^5}{a+b\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\sin^4(c+dx)}{(1-\sin^2(c+dx))^3(b\sin^2(c+dx)+a)}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(-\frac{a^2}{(a+b)^3(\sin^2(c+dx)-1)}+\frac{ba^2}{(a+b)^3(b\sin^2(c+dx)+a)}+\frac{-2a-b}{(a+b)^2(\sin^2(c+dx)-1)^2}-\frac{1}{(a+b)(\sin^2(c+dx)-1)^3}\right)d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^2\log(1-\sin^2(c+dx))}{(a+b)^3}+\frac{a^2\log(a+b\sin^2(c+dx))}{(a+b)^3}-\frac{2a+b}{(a+b)^2(1-\sin^2(c+dx))}+\frac{1}{2(a+b)(1-\sin^2(c+dx))^2}}{2d}$$

input
```
Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-((a^2*Log[1 - Sin[c + d*x]^2])/(a + b)^3) + (a^2*Log[a + b*Sin[c + d*x]^
2])/(a + b)^3 + 1/(2*(a + b)*(1 - Sin[c + d*x]^2)^2) - (2*a + b)/((a + b)^
2*(1 - Sin[c + d*x]^2)))/(2*d)
```

3.442.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.442.4 Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{a^{2} \ln\left(a+b-b\left(\cos^{2}(dx+c)\right)\right)}{2(a+b)^{3}}-\frac{2a+b}{2(a+b)^{2}\cos(dx+c)^{2}}+\frac{1}{4(a+b)\cos(dx+c)^{4}}-\frac{a^{2}\ln(\cos(dx+c))}{(a+b)^{3}}}{d}$
default	$\frac{\frac{a^{2} \ln\left(a+b-b\left(\cos^{2}(dx+c)\right)\right)}{2(a+b)^{3}}-\frac{2a+b}{2(a+b)^{2}\cos(dx+c)^{2}}+\frac{1}{4(a+b)\cos(dx+c)^{4}}-\frac{a^{2}\ln(\cos(dx+c))}{(a+b)^{3}}}{d}$
risch	$-\frac{2\left(2a\,\mathrm{e}^{6i(dx+c)}+b\,\mathrm{e}^{6i(dx+c)}+2a\,\mathrm{e}^{4i(dx+c)}+2a\,\mathrm{e}^{2i(dx+c)}+b\,\mathrm{e}^{2i(dx+c)}\right)}{d(a+b)^{2}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{4}}-\frac{a^{2}\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d(a^{3}+3a^{2}b+3a\,b^{2}+b^{3})}+\frac{a^{2}\ln\left(\mathrm{e}^{4i(dx+c)}\right.}{2d(a^{3}+3}$

input
```
int(tan(d*x+c)^5/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*a^2/(a+b)^3*ln(a+b-b*cos(d*x+c)^2)-1/2*(2*a+b)/(a+b)^2/cos(d*x+c)
^2+1/4/(a+b)/cos(d*x+c)^4-a^2/(a+b)^3*ln(cos(d*x+c)))
```

3.442.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.26

$$\int \frac{\tan^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{2\,a^2\cos(dx+c)^4\log\left(-b\cos(dx+c)^2+a+b\right)-4\,a^2\cos(dx+c)^4\log\left(-\cos(dx+c)\right)-2\,(2\,a^2+3\,ab+\dots}{4\,(a^3+3\,a^2b+3\,ab^2+b^3)d\cos(dx+c)^4}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/4*(2*a^2*cos(d*x + c)^4*log(-b*cos(d*x + c)^2 + a + b) - 4*a^2*cos(d*x +
 c)^4*log(-cos(d*x + c)) - 2*(2*a^2 + 3*a*b + b^2)*cos(d*x + c)^2 + a^2 +
2*a*b + b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cos(d*x + c)^4)
```

3.442.6 Sympy [F]

$$\int \frac{\tan^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\tan^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**5/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(tan(c + d*x)**5/(a + b*sin(c + d*x)**2), x)
```

3.442.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.69

$$\int \frac{\tan^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{2\,a^2\log\left(b\sin(dx+c)^2+a\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\,a^2\log\left(\sin(dx+c)^2-1\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\,(2\,a+b)\sin(dx+c)^2-3\,a-b}{(a^2+2\,ab+b^2)\sin(dx+c)^4-2\,(a^2+2\,ab+b^2)\sin(dx+c)^2+a^2+2\,ab+b^2}}{4\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/4*(2*a^2*log(b*sin(d*x + c)^2 + a)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*a
^2*log(sin(d*x + c)^2 - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (2*(2*a + b)*
sin(d*x + c)^2 - 3*a - b)/((a^2 + 2*a*b + b^2)*sin(d*x + c)^4 - 2*(a^2 + 2
*a*b + b^2)*sin(d*x + c)^2 + a^2 + 2*a*b + b^2))/d
```

3.442.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(88) = 176$.

Time = 2.26 (sec) , antiderivative size = 393, normalized size of antiderivative = 4.18

$$\int \frac{\tan^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{6\,a^2\log\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{25\,a^2+\frac{124\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{24\,ab(\cos(d\cdots}{\cos(dx+\cdots}}{\cdots}}{12\,a\cdots}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/12*(6*a^2*log(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 12*a^2*log(abs(-(cos(d*x + c) - 1)/
(cos(d*x + c) + 1) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (25*a^2 + 124*a
^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 24*a*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 246*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 144*a*
b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 48*b^2*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 + 124*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 +
 24*a*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 25*a^2*(cos(d*x + c) -
 1)^4/(cos(d*x + c) + 1)^4)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*((cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 1)^4))/d
```

3.442.9 Mupad [B] (verification not implemented)

Time = 14.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96

$$\int \frac{\tan^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{a^2\left(\frac{\ln\left((a+b)\tan(c+dx)^2+a\right)}{2}-\frac{\tan(c+dx)^2}{2}+\frac{\tan(c+dx)^4}{4}\right)+\frac{b^2\tan(c+dx)^4}{4}-a\,b\left(\frac{\tan(c+dx)^2}{2}-\frac{\tan(c+dx)^4}{2}\right)}{d\,(a+b)^3}$$

input
```
int(tan(c + d*x)^5/(a + b*sin(c + d*x)^2),x)
```

output
```
(a^2*(log(a + tan(c + d*x)^2*(a + b))/2 - tan(c + d*x)^2/2 + tan(c + d*x)^
4/4) + (b^2*tan(c + d*x)^4)/4 - a*b*(tan(c + d*x)^2/2 - tan(c + d*x)^4/2))
/(d*(a + b)^3)
```

3.443 $\int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.443.1 Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{a\log(\cos(c+dx))}{(a+b)^2d} - \frac{a\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)^2d} + \frac{\sec^2(c+dx)}{2(a+b)d}$$

output
```
a*ln(cos(d*x+c))/(a+b)^2/d-1/2*a*ln(a+b*sin(d*x+c)^2)/(a+b)^2/d+1/2*sec(d*
x+c)^2/(a+b)/d
```

3.443.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{a(2\log(\cos(c+dx)) - \log\left(a+b\sin^2(c+dx)\right)) + (a+b)\sec^2(c+dx)}{2(a+b)^2d}$$

input
```
Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

output
```
(a*(2*Log[Cos[c + d*x]] - Log[a + b*Sin[c + d*x]^2]) + (a + b)*Sec[c + d*x
]^2)/(2*(a + b)^2*d)
```

3.443.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3673, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^3}{a+b\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\sin^2(c+dx)}{\left(1-\sin^2(c+dx)\right)^2\left(b\sin^2(c+dx)+a\right)}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{a}{(a+b)^2\left(\sin^2(c+dx)-1\right)}-\frac{ba}{(a+b)^2\left(b\sin^2(c+dx)+a\right)}+\frac{1}{(a+b)\left(\sin^2(c+dx)-1\right)^2}\right)d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{(a+b)(1-\sin^2(c+dx))}+\frac{a\log(1-\sin^2(c+dx))}{(a+b)^2}-\frac{a\log\left(a+b\sin^2(c+dx)\right)}{(a+b)^2}}{2d}$$

input
```
Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

output
```
((a*Log[1 - Sin[c + d*x]^2])/(a + b)^2 - (a*Log[a + b*Sin[c + d*x]^2])/(a
+ b)^2 + 1/((a + b)*(1 - Sin[c + d*x]^2)))/(2*d)
```

3.443.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.443.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\frac{a\ln(\cos(dx+c))}{(a+b)^2}+\frac{1}{2(a+b)\cos(dx+c)^2}-\frac{a\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2(a+b)^2}}{d}$	58
default	$\frac{\frac{a\ln(\cos(dx+c))}{(a+b)^2}+\frac{1}{2(a+b)\cos(dx+c)^2}-\frac{a\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2(a+b)^2}}{d}$	58
risch	$\frac{2\,e^{2i(dx+c)}}{d(a+b)\left(e^{2i(dx+c)}+1\right)^2}+\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d(a^2+2ab+b^2)}-\frac{a\ln\left(e^{4i(dx+c)}-\frac{2(2a+b)e^{2i(dx+c)}}{b}+1\right)}{2d(a^2+2ab+b^2)}$	114

input
```
int(tan(d*x+c)^3/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a/(a+b)^2*ln(cos(d*x+c))+1/2/(a+b)/cos(d*x+c)^2-1/2*a/(a+b)^2*ln(a+b-
b*cos(d*x+c)^2))
```

3.443.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

$$\int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{a\cos(dx+c)^2\log\left(-b\cos(dx+c)^2+a+b\right)-2\,a\cos(dx+c)^2\log\left(-\cos(dx+c)\right)-a-b}{2\,(a^2+2\,ab+b^2)d\cos(dx+c)^2}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(a*cos(d*x + c)^2*log(-b*cos(d*x + c)^2 + a + b) - 2*a*cos(d*x + c)^2
*log(-cos(d*x + c)) - a - b)/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^2)
```

3.443.6 Sympy [F]

$$\int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**3/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(tan(c + d*x)**3/(a + b*sin(c + d*x)**2), x)
```

3.443.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.28

$$\int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{a\log\left(b\sin(dx+c)^2+a\right)}{a^2+2\,ab+b^2}-\frac{a\log\left(\sin(dx+c)^2-1\right)}{a^2+2\,ab+b^2}+\frac{1}{(a+b)\sin(dx+c)^2-a-b}}{2\,d}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/2*(a*log(b*sin(d*x + c)^2 + a)/(a^2 + 2*a*b + b^2) - a*log(sin(d*x + c)
^2 - 1)/(a^2 + 2*a*b + b^2) + 1/((a + b)*sin(d*x + c)^2 - a - b))/d
```

3.443.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(60) = 120$.

Time = 0.87 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.66

$$\int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx =$$

$$-\frac{\frac{a\log\left(a-\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{a^2+2ab+b^2}-\frac{2a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^2+2ab+b^2}+\frac{3a+\frac{10a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4b(\cos(dx+c)-1)}{\cos(dx+c)+1}}{(a^2+2ab+b^2)\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right.}}{2d}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/2*(a*log(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)
/(a^2 + 2*a*b + b^2) - 2*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
- 1))/(a^2 + 2*a*b + b^2) + (3*a + 10*a*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2)/((a^2 + 2*a*b + b^2)*((cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) + 1)^2))/d
```

3.443.9 Mupad [B] (verification not implemented)

Time = 13.59 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{a\left(\frac{\ln\left((a+b)\tan(c+dx)^2+a\right)}{2}-\frac{\tan(c+dx)^2}{2}\right)-\frac{b\tan(c+dx)^2}{2}}{d\,(a+b)^2}$$

input
```
int(tan(c + d*x)^3/(a + b*sin(c + d*x)^2),x)
```

output
```
-(a*(log(a + tan(c + d*x)^2*(a + b))/2 - tan(c + d*x)^2/2) - (b*tan(c + d*
x)^2)/2)/(d*(a + b)^2)
```

3.444 $\int \frac{\tan(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.444.1 Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\tan(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\log(\cos(c+dx))}{(a+b)d} + \frac{\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)d}$$

output
```
-ln(cos(d*x+c))/(a+b)/d+1/2*ln(a+b*sin(d*x+c)^2)/(a+b)/d
```

3.444.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\tan(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{-2\log(\cos(c+dx)) + \log\left(a+b-b\cos^2(c+dx)\right)}{2ad+2bd}$$

input
```
Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-2*Log[Cos[c + d*x]] + Log[a + b - b*Cos[c + d*x]^2])/(2*a*d + 2*b*d)
```

3.444.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3673, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(c+dx)}{a+b\sin(c+dx)^2}dx$$

$$\Big\downarrow \; 3673$$

$$\frac{\int \frac{1}{\left(1-\sin^2(c+dx)\right)\left(b\sin^2(c+dx)+a\right)}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \; 47$$

$$\frac{\frac{\int \frac{1}{1-\sin^2(c+dx)}d\sin^2(c+dx)}{a+b}+\frac{b\int \frac{1}{b\sin^2(c+dx)+a}d\sin^2(c+dx)}{a+b}}{2d}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{\log\left(a+b\sin^2(c+dx)\right)}{a+b}-\frac{\log\left(1-\sin^2(c+dx)\right)}{a+b}}{2d}$$

input
```
Int[Tan[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-(Log[1 - Sin[c + d*x]^2]/(a + b)) + Log[a + b*Sin[c + d*x]^2]/(a + b))/(
2*d)
```

3.444.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.444.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{-\frac{\ln(\cos(dx+c))}{a+b}+\frac{\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2a+2b}}{d}$	42
default	$\frac{-\frac{\ln(\cos(dx+c))}{a+b}+\frac{\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2a+2b}}{d}$	42
risch	$-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{d(a+b)}+\frac{\ln\left(e^{4i(dx+c)}-\frac{2(2a+b)e^{2i(dx+c)}}{b}+1\right)}{2d(a+b)}$	65

input
```
int(tan(d*x+c)/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(a+b)*ln(cos(d*x+c))+1/2/(a+b)*ln(a+b-b*cos(d*x+c)^2))
```

3.444.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\tan(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\log\left(-b\cos\left(dx+c\right)^2+a+b\right)-2\log\left(-\cos\left(dx+c\right)\right)}{2\left(a+b\right)d}$$

input
```
integrate(tan(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*(log(-b*cos(d*x + c)^2 + a + b) - 2*log(-cos(d*x + c)))/((a + b)*d)
```

3.444.6 Sympy [F]

$$\int \frac{\tan(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\tan\left(c+dx\right)}{a+b\sin^2\left(c+dx\right)}\,dx$$

input
```
integrate(tan(d*x+c)/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(tan(c + d*x)/(a + b*sin(c + d*x)**2), x)
```

3.444.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\tan(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{\log\left(b\sin(dx+c)^2+a\right)}{a+b}-\frac{\log\left(\sin(dx+c)^2-1\right)}{a+b}}{2\,d}$$

input
```
integrate(tan(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*(log(b*sin(d*x + c)^2 + a)/(a + b) - log(sin(d*x + c)^2 - 1)/(a + b))/
d
```

3.444.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(41) = 82$.

Time = 0.48 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.56

$$\int \frac{\tan(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{\log\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{a+b}-\frac{2\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a+b}}{2\,d}$$

input
```
integrate(tan(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(log(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(a
 + b) - 2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/(a + b))/d
```

3.444.9 Mupad [B] (verification not implemented)

Time = 13.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \frac{\tan(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\ln\left((a+b)\,\tan(c+d\,x)^2+a\right)}{d\,(2\,a+2\,b)}$$

input
```
int(tan(c + d*x)/(a + b*sin(c + d*x)^2),x)
```

output
```
log(a + tan(c + d*x)^2*(a + b))/(d*(2*a + 2*b))
```

3.445 $\int \frac{\cot(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.445.1 Optimal result

Integrand size = 21, antiderivative size = 38

$$\int \frac{\cot(c+dx)}{a+b\sin^2(c+dx)}\,dx=\frac{\log(\sin(c+dx))}{ad}-\frac{\log\left(a+b\sin^2(c+dx)\right)}{2ad}$$

output
```
ln(sin(d*x+c))/a/d-1/2*ln(a+b*sin(d*x+c)^2)/a/d
```

3.445.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\cot(c+dx)}{a+b\sin^2(c+dx)}\,dx=\frac{\log(\sin(c+dx))}{ad}-\frac{\log\left(a+b\sin^2(c+dx)\right)}{2ad}$$

input
```
Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

output
```
Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]^2]/(2*a*d)
```

3.445.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3673, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)\left(a+b\sin(c+dx)^2\right)}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\csc^2(c+dx)}{b\sin^2(c+dx)+a}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{\int \csc^2(c+dx)d\sin^2(c+dx)}{a}-\frac{b\int \frac{1}{b\sin^2(c+dx)+a}d\sin^2(c+dx)}{a}}{2d}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{\log\left(\sin^2(c+dx)\right)}{a}-\frac{b\int \frac{1}{b\sin^2(c+dx)+a}d\sin^2(c+dx)}{a}}{2d}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\log\left(\sin^2(c+dx)\right)}{a}-\frac{\log\left(a+b\sin^2(c+dx)\right)}{a}}{2d}$$

input
```
Int[Cot[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

output
```
(Log[Sin[c + d*x]^2]/a - Log[a + b*Sin[c + d*x]^2]/a)/(2*d)
```

3.445.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.445.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2a}+\frac{\ln(1+\cos(dx+c))}{2a}+\frac{\ln(\cos(dx+c)-1)}{2a}}{d}$	54
default	$\frac{-\frac{\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2a}+\frac{\ln(1+\cos(dx+c))}{2a}+\frac{\ln(\cos(dx+c)-1)}{2a}}{d}$	54
risch	$\frac{\ln\left(e^{2i(dx+c)}-1\right)}{da}-\frac{\ln\left(e^{4i(dx+c)}-\frac{2(2a+b)e^{2i(dx+c)}}{b}+1\right)}{2ad}$	60

input
```
int(cot(d*x+c)/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a*ln(a+b-b*cos(d*x+c)^2)+1/2/a*ln(1+cos(d*x+c))+1/2/a*ln(cos(d*x
+c)-1))
```

3.445.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{\cot(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\log\left(-b\cos\left(dx+c\right)^2+a+b\right)-2\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)}{2\,ad}$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(log(-b*cos(d*x + c)^2 + a + b) - 2*log(1/2*sin(d*x + c)))/(a*d)
```

3.445.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\cot\left(c+dx\right)}{a+b\sin^2\left(c+dx\right)}\,dx$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(cot(c + d*x)/(a + b*sin(c + d*x)**2), x)
```

3.445.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{\cot(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{\log\left(b\sin(dx+c)^2+a\right)}{a}-\frac{\log\left(\sin(dx+c)^2\right)}{a}}{2\,d}$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/2*(log(b*sin(d*x + c)^2 + a)/a - log(sin(d*x + c)^2)/a)/d
```

3.445.8 Giac [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\cot(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{\log\left(\sin(dx+c)^2\right)}{a} - \frac{\log\left(\left|b\sin(dx+c)^2+a\right|\right)}{a}}{2\,d}$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(log(sin(d*x + c)^2)/a - log(abs(b*sin(d*x + c)^2 + a))/a)/d
```

3.445.9 Mupad [B] (verification not implemented)

Time = 13.60 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{\cot(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\ln\left(a+a\tan(c+d\,x)^2+b\tan(c+d\,x)^2\right)-2\,\ln\left(\tan(c+d\,x)\right)}{2\,a\,d}$$

input
```
int(cot(c + d*x)/(a + b*sin(c + d*x)^2),x)
```

output
```
-(log(a + a*tan(c + d*x)^2 + b*tan(c + d*x)^2) - 2*log(tan(c + d*x)))/(2*a
*d)
```

3.446 $\int \frac{\cot^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.446.1 Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \frac{\cot^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\csc^2(c+dx)}{2ad} - \frac{(a+b)\log(\sin(c+dx))}{a^2d} + \frac{(a+b)\log\left(a+b\sin^2(c+dx)\right)}{2a^2d}$$

output
```
-1/2*csc(d*x+c)^2/a/d-(a+b)*ln(sin(d*x+c))/a^2/d+1/2*(a+b)*ln(a+b*sin(d*x+
c)^2)/a^2/d
```

3.446.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \frac{\cot^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{a\csc^2(c+dx)+(a+b)\left(2\log(\sin(c+dx))-\log\left(a+b\sin^2(c+dx)\right)\right)}{2a^2d}$$

input
```
Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

output
```
-1/2*(a*Csc[c + d*x]^2 + (a + b)*(2*Log[Sin[c + d*x]] - Log[a + b*Sin[c +
d*x]^2]))/(a^2*d)
```

3.446.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3673, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^3\left(a+b\sin(c+dx)^2\right)}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\csc^4(c+dx)\left(1-\sin^2(c+dx)\right)}{b\sin^2(c+dx)+a}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{\csc^4(c+dx)}{a}+\frac{(-a-b)\csc^2(c+dx)}{a^2}+\frac{b(a+b)}{a^2\left(b\sin^2(c+dx)+a\right)}\right)d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(a+b)\log\left(\sin^2(c+dx)\right)}{a^2}+\frac{(a+b)\log\left(a+b\sin^2(c+dx)\right)}{a^2}-\frac{\csc^2(c+dx)}{a}}{2d}$$

input
```
Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-(Csc[c + d*x]^2/a) - ((a + b)*Log[Sin[c + d*x]^2])/a^2 + ((a + b)*Log[a
+ b*Sin[c + d*x]^2])/a^2)/(2*d)
```

3.446.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.446.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{-\frac{1}{4a(1+\cos(dx+c))}+\frac{(-a-b)\ln(1+\cos(dx+c))}{2a^2}+\frac{(a+b)\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2a^2}+\frac{1}{4a(\cos(dx+c)-1)}+\frac{(-a-b)\ln(\cos(dx+c)-1)}{2a^2}}{d}$
default	$\frac{-\frac{1}{4a(1+\cos(dx+c))}+\frac{(-a-b)\ln(1+\cos(dx+c))}{2a^2}+\frac{(a+b)\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2a^2}+\frac{1}{4a(\cos(dx+c)-1)}+\frac{(-a-b)\ln(\cos(dx+c)-1)}{2a^2}}{d}$
risch	$\frac{2\,\mathrm{e}^{2i(dx+c)}}{da\left(\mathrm{e}^{2i(dx+c)}-1\right)^2}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{da}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)b}{a^2d}+\frac{\ln\left(\mathrm{e}^{4i(dx+c)}-\frac{2(2a+b)\mathrm{e}^{2i(dx+c)}}{b}+1\right)}{2ad}+\ln\left(\mathrm{e}^{4i(dx+c}\right.$

input
```
int(cot(d*x+c)^3/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4/a/(1+cos(d*x+c))+1/2*(-a-b)/a^2*ln(1+cos(d*x+c))+1/2*(a+b)/a^2*l
n(a+b-b*cos(d*x+c)^2)+1/4/a/(cos(d*x+c)-1)+1/2*(-a-b)/a^2*ln(cos(d*x+c)-1)
)
```

3.446.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.44

$$\int \frac{\cot^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\left((a+b)\cos\left(dx+c\right)^2-a-b\right)\log\left(-b\cos\left(dx+c\right)^2+a+b\right)-2\left((a+b)\cos\left(dx+c\right)^2-a-b\right)\log\left(\frac{1}{2}\sin\right.}{2\left(a^2d\cos\left(dx+c\right)^2-a^2d\right)}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*(((a + b)*cos(d*x + c)^2 - a - b)*log(-b*cos(d*x + c)^2 + a + b) - 2*(
(a + b)*cos(d*x + c)^2 - a - b)*log(1/2*sin(d*x + c)) + a)/(a^2*d*cos(d*x
+ c)^2 - a^2*d)
```

3.446.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\cot^3\left(c+dx\right)}{a+b\sin^2\left(c+dx\right)}\,dx$$

input
```
integrate(cot(d*x+c)**3/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(cot(c + d*x)**3/(a + b*sin(c + d*x)**2), x)
```

3.446.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{\cot^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{(a+b)\log\left(b\sin(dx+c)^2+a\right)}{a^2}-\frac{(a+b)\log\left(\sin(dx+c)^2\right)}{a^2}-\frac{1}{a\sin(dx+c)^2}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*((a + b)*log(b*sin(d*x + c)^2 + a)/a^2 - (a + b)*log(sin(d*x + c)^2)/a
^2 - 1/(a*sin(d*x + c)^2))/d
```

3.446.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.71

$$\int \frac{\cot^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}}{a}+\frac{4\,(a+b)\log\left(\left|-a\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)+2\,a+4\,b\right|\right)}{a^2}}{8\,d}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/8*(((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x
+ c) + 1))/a + 4*(a + b)*log(abs(-a*((cos(d*x + c) + 1)/(cos(d*x + c) - 1)
 + (cos(d*x + c) - 1)/(cos(d*x + c) + 1)) + 2*a + 4*b))/a^2)/d
```

3.446.9 Mupad [B] (verification not implemented)

Time = 14.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

$$\int \frac{\cot^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\ln\left(a+a\tan(c+d\,x)^2+b\tan(c+d\,x)^2\right)\,(a+b)}{2\,a^2\,d} - \frac{\cot(c+d\,x)^2}{2\,a\,d} - \frac{\ln\left(\tan(c+d\,x)\right)\,(a+b)}{a^2\,d}$$

input
```
int(cot(c + d*x)^3/(a + b*sin(c + d*x)^2),x)
```

output
```
(log(a + a*tan(c + d*x)^2 + b*tan(c + d*x)^2)*(a + b))/(2*a^2*d) - cot(c +
 d*x)^2/(2*a*d) - (log(tan(c + d*x))*(a + b))/(a^2*d)
```

3.447 $\int \frac{\cot^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.447.1 Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{\cot^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{(2a+b)\csc^2(c+dx)}{2a^2d} - \frac{\csc^4(c+dx)}{4ad} + \frac{(a+b)^2\log(\sin(c+dx))}{a^3d} - \frac{(a+b)^2\log\left(a+b\sin^2(c+dx)\right)}{2a^3d}$$

output
```
1/2*(2*a+b)*csc(d*x+c)^2/a^2/d-1/4*csc(d*x+c)^4/a/d+(a+b)^2*ln(sin(d*x+c))
/a^3/d-1/2*(a+b)^2*ln(a+b*sin(d*x+c)^2)/a^3/d
```

3.447.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int \frac{\cot^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{2a(2a+b)\csc^2(c+dx) - a^2\csc^4(c+dx) + 2(a+b)^2\left(2\log(\sin(c+dx)) - \log\left(a+b\sin^2(c+dx)\right)\right)}{4a^3d}$$

input
```
Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

output
```
(2*a*(2*a + b)*Csc[c + d*x]^2 - a^2*Csc[c + d*x]^4 + 2*(a + b)^2*(2*Log[Si
n[c + d*x]] - Log[a + b*Sin[c + d*x]^2]))/(4*a^3*d)
```

3.447.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3673, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(c+dx)}{a+b\sin^2(c+dx)} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^5 \left(a+b\sin(c+dx)^2\right)}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\csc^6(c+dx)\left(1-\sin^2(c+dx)\right)^2}{b\sin^2(c+dx)+a}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{\csc^6(c+dx)}{a}+\frac{(-2a-b)\csc^4(c+dx)}{a^2}+\frac{(a+b)^2\csc^2(c+dx)}{a^3}-\frac{b(a+b)^2}{a^3\left(b\sin^2(c+dx)+a\right)}\right)d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a+b)^2\log\left(\sin^2(c+dx)\right)}{a^3}-\frac{(a+b)^2\log\left(a+b\sin^2(c+dx)\right)}{a^3}+\frac{(2a+b)\csc^2(c+dx)}{a^2}-\frac{\csc^4(c+dx)}{2a}}{2d}$$

input
```
Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

output
```
(((2*a + b)*Csc[c + d*x]^2)/a^2 - Csc[c + d*x]^4/(2*a) + ((a + b)^2*Log[Si
n[c + d*x]^2])/a^3 - ((a + b)^2*Log[a + b*Sin[c + d*x]^2])/a^3)/(2*d)
```

3.447.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.447.4 Maple [A] (verified)

Time = 3.90 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.81

method	result
derivativedivides	$\frac{-\frac{1}{16a(1+\cos(dx+c))^2}-\frac{-7a-4b}{16a^2(1+\cos(dx+c))}+\frac{(a^2+2ab+b^2)\ln(1+\cos(dx+c))}{2a^3}-\frac{(a^2+2ab+b^2)\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2a^3}-\frac{\cdots}{16a(\cos(\cdots}}{d}$
default	$\frac{-\frac{1}{16a(1+\cos(dx+c))^2}-\frac{-7a-4b}{16a^2(1+\cos(dx+c))}+\frac{(a^2+2ab+b^2)\ln(1+\cos(dx+c))}{2a^3}-\frac{(a^2+2ab+b^2)\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2a^3}-\frac{\cdots}{16a(\cos(\cdots}}{d}$
risch	$-\frac{2\left(2a\,e^{6i(dx+c)}+b\,e^{6i(dx+c)}-2a\,e^{4i(dx+c)}-2b\,e^{4i(dx+c)}+2a\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}\right)}{d\,a^2\left(e^{2i(dx+c)}-1\right)^4}+\frac{\ln\left(e^{2i(dx+c)}-1\right)}{da}+\frac{2\ln\left(e^{2i(\cdots}}{a\cdots}$

input
```
int(cot(d*x+c)^5/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/16/a/(1+cos(d*x+c))^2-1/16*(-7*a-4*b)/a^2/(1+cos(d*x+c))+1/2*(a^2+
2*a*b+b^2)/a^3*ln(1+cos(d*x+c))-1/2*(a^2+2*a*b+b^2)/a^3*ln(a+b-b*cos(d*x+c
)^2)-1/16/a/(cos(d*x+c)-1)^2-1/16*(7*a+4*b)/a^2/(cos(d*x+c)-1)+1/2*(a^2+2*
a*b+b^2)/a^3*ln(cos(d*x+c)-1))
```

3.447.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(83) = 166$.

Time = 0.35 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.22

$$\int \frac{\cot^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{2\,(2\,a^2+ab)\cos{(dx+c)}^2-3\,a^2-2\,ab+2\,\left((a^2+2\,ab+b^2)\cos{(dx+c)}^4-2\,(a^2+2\,ab+b^2)\cos{(dx-}}{}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/4*(2*(2*a^2 + a*b)*cos(d*x + c)^2 - 3*a^2 - 2*a*b + 2*((a^2 + 2*a*b + b
^2)*cos(d*x + c)^4 - 2*(a^2 + 2*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b +
b^2)*log(-b*cos(d*x + c)^2 + a + b) - 4*((a^2 + 2*a*b + b^2)*cos(d*x + c)^
4 - 2*(a^2 + 2*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)*log(1/2*sin(
d*x + c)))/(a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)
```

3.447.6 Sympy [F]

$$\int \frac{\cot^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\cot^5{(c+dx)}}{a+b\sin^2{(c+dx)}}\,dx$$

input
```
integrate(cot(d*x+c)**5/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(cot(c + d*x)**5/(a + b*sin(c + d*x)**2), x)
```

3.447.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{\cot^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{2\,(a^2+2\,ab+b^2)\log\left(b\sin(dx+c)^2+a\right)}{a^3}-\frac{2\,(a^2+2\,ab+b^2)\log\left(\sin(dx+c)^2\right)}{a^3}-\frac{2\,(2\,a+b)\sin(dx+c)^2-a}{a^2\sin(dx+c)^4}}{4\,d}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/4*(2*(a^2 + 2*a*b + b^2)*log(b*sin(d*x + c)^2 + a)/a^3 - 2*(a^2 + 2*a*b
 + b^2)*log(sin(d*x + c)^2)/a^3 - (2*(2*a + b)*sin(d*x + c)^2 - a)/(a^2*si
n(d*x + c)^4))/d
```

3.447.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(83) = 166$.

Time = 0.43 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.30

$$\int \frac{\cot^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{a\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)^2+12\,a\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)+8\,b\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)}{a^2}+\frac{32\,(a^2+2\,ab+b^2)\log\left(\left|-a\left(\frac{\cos(dx+c}{\cos(dx+c}\right.\right.\right.}{a^3}}{64\,d}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/64*((a*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos
(d*x + c) + 1))^2 + 12*a*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x
 + c) - 1)/(cos(d*x + c) + 1)) + 8*b*((cos(d*x + c) + 1)/(cos(d*x + c) - 1
) + (cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/a^2 + 32*(a^2 + 2*a*b + b^2)*l
og(abs(-a*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos
(d*x + c) + 1)) + 2*a + 4*b))/a^3)/d
```

3.447.9 Mupad [B] (verification not implemented)

Time = 13.45 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \frac{\cot^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\ln(\tan(c+d\,x))\,(a^2+2\,a\,b+b^2)}{a^3\,d} - \frac{\ln\left(a+a\tan(c+d\,x)^2+b\tan(c+d\,x)^2\right)\,(a^2+2\,a\,b+b^2)}{2\,a^3\,d} - \frac{\frac{1}{4\,a}-\frac{\tan(c+d\,x)^2\,(a+b)}{2\,a^2}}{d\tan(c+d\,x)^4}$$

input
```
int(cot(c + d*x)^5/(a + b*sin(c + d*x)^2),x)
```

output
```
(log(tan(c + d*x))*(2*a*b + a^2 + b^2))/(a^3*d) - (log(a + a*tan(c + d*x)^
2 + b*tan(c + d*x)^2)*(2*a*b + a^2 + b^2))/(2*a^3*d) - (1/(4*a) - (tan(c +
 d*x)^2*(a + b))/(2*a^2))/(d*tan(c + d*x)^4)
```

3.447. $\int \frac{\cot^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.448 $\int \frac{\cot^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.448.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{\cot^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{(3a^2+3ab+b^2)\csc^2(c+dx)}{2a^3d}+\frac{(3a+b)\csc^4(c+dx)}{4a^2d}-\frac{\csc^6(c+dx)}{6ad}-\frac{(a+b)^3\log(\sin(c+dx))}{a^4d}+\frac{(a+b)^3\log\left(a+b\sin^2(c+dx)\right)}{2a^4d}$$

output

```
-1/2*(3*a^2+3*a*b+b^2)*csc(d*x+c)^2/a^3/d+1/4*(3*a+b)*csc(d*x+c)^4/a^2/d-1
/6*csc(d*x+c)^6/a/d-(a+b)^3*ln(sin(d*x+c))/a^4/d+1/2*(a+b)^3*ln(a+b*sin(d*
x+c)^2)/a^4/d
```

3.448.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int \frac{\cot^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{6a(3a^2+3ab+b^2)\csc^2(c+dx)-3a^2(3a+b)\csc^4(c+dx)+2a^3\csc^6(c+dx)+12(a+b)^3\log(\sin(c+}{12a^4d}$$

input

```
Integrate[Cot[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

output
```
-1/12*(6*a*(3*a^2 + 3*a*b + b^2)*Csc[c + d*x]^2 - 3*a^2*(3*a + b)*Csc[c +
d*x]^4 + 2*a^3*Csc[c + d*x]^6 + 12*(a + b)^3*Log[Sin[c + d*x]] - 6*(a + b)
^3*Log[a + b*Sin[c + d*x]^2])/(a^4*d)
```

3.448.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3673, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^7(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(c+dx)^7\left(a+b\sin(c+dx)^2\right)}dx$$

$$\Big\downarrow \text{ 3673}$$

$$\frac{\int \frac{\csc^8(c+dx)\left(1-\sin^2(c+dx)\right)^3}{b\sin^2(c+dx)+a}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\int \left(\frac{\csc^8(c+dx)}{a}+\frac{(-3a-b)\csc^6(c+dx)}{a^2}+\frac{\left(3a^2+3ba+b^2\right)\csc^4(c+dx)}{a^3}-\frac{(a+b)^3\csc^2(c+dx)}{a^4}+\frac{b(a+b)^3}{a^4\left(b\sin^2(c+dx)+a\right)}\right)d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{(a+b)^3\log\left(\sin^2(c+dx)\right)}{a^4}+\frac{(a+b)^3\log\left(a+b\sin^2(c+dx)\right)}{a^4}+\frac{(3a+b)\csc^4(c+dx)}{2a^2}-\frac{\left(3a^2+3ab+b^2\right)\csc^2(c+dx)}{a^3}-\frac{\csc^6(c+dx)}{3a}}{2d}$$

input
```
Int[Cot[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-(((3*a^2 + 3*a*b + b^2)*Csc[c + d*x]^2)/a^3) + ((3*a + b)*Csc[c + d*x]^4
)/(2*a^2) - Csc[c + d*x]^6/(3*a) - ((a + b)^3*Log[Sin[c + d*x]^2])/a^4 + (
(a + b)^3*Log[a + b*Sin[c + d*x]^2])/a^4)/(2*d)
```

3.448.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.448.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(113) = 226$.

Time = 7.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.09

method	result
derivativedivides	$\frac{\frac{(a^3+3a^2b+3a\,b^2+b^3)\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2a^4}-\frac{1}{48a(1+\cos(dx+c))^3}-\frac{-5a-2b}{32a^2(1+\cos(dx+c))^2}-\frac{19a^2+22ab+8b^2}{32a^3(1+\cos(dx+c))}+\frac{(-a^3-3a^2b-}{}}{}$
default	$\frac{\frac{(a^3+3a^2b+3a\,b^2+b^3)\ln\left(a+b-b\left(\cos^2(dx+c)\right)\right)}{2a^4}-\frac{1}{48a(1+\cos(dx+c))^3}-\frac{-5a-2b}{32a^2(1+\cos(dx+c))^2}-\frac{19a^2+22ab+8b^2}{32a^3(1+\cos(dx+c))}+\frac{(-a^3-3a^2b-}{}}{}$
risch	$\frac{6a^2e^{10i(dx+c)}+6ab\,e^{10i(dx+c)}+2b^2e^{10i(dx+c)}-12a^2e^{8i(dx+c)}-20ab\,e^{8i(dx+c)}-8b^2e^{8i(dx+c)}+\frac{68a^2e^{6i(dx+c)}}{3}+28ab\,e^{6i(d}}{d\,a^3(e^{2i(dx+c)}}$

input
```
int(cot(d*x+c)^7/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*(a^3+3*a^2*b+3*a*b^2+b^3)/a^4*ln(a+b-b*cos(d*x+c)^2)-1/48/a/(1+co
s(d*x+c))^3-1/32*(-5*a-2*b)/a^2/(1+cos(d*x+c))^2-1/32*(19*a^2+22*a*b+8*b^2
)/a^3/(1+cos(d*x+c))+1/2*(-a^3-3*a^2*b-3*a*b^2-b^3)/a^4*ln(1+cos(d*x+c))+1
/48/a/(cos(d*x+c)-1)^3-1/32*(-5*a-2*b)/a^2/(cos(d*x+c)-1)^2-1/32*(-19*a^2-
22*a*b-8*b^2)/a^3/(cos(d*x+c)-1)+1/2*(-a^3-3*a^2*b-3*a*b^2-b^3)/a^4*ln(cos
(d*x+c)-1))
```

3.448.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(113) = 226$.

Time = 0.40 (sec) , antiderivative size = 371, normalized size of antiderivative = 3.07

$$\int \frac{\cot^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{6\,(3\,a^3+3\,a^2b+ab^2)\cos{(dx+c)}^4+11\,a^3+15\,a^2b+6\,ab^2-3\,(9\,a^3+11\,a^2b+4\,ab^2)\cos{(dx+c)}^2+6\,((\cdots}{}$$

input
```
integrate(cot(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/12*(6*(3*a^3 + 3*a^2*b + a*b^2)*cos(d*x + c)^4 + 11*a^3 + 15*a^2*b + 6*a
*b^2 - 3*(9*a^3 + 11*a^2*b + 4*a*b^2)*cos(d*x + c)^2 + 6*((a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*cos(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x
 + c)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*cos(d*x + c)^2)*log(-b*cos(d*x + c)^2 + a + b) - 12*((a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*cos(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x +
c)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
os(d*x + c)^2)*log(1/2*sin(d*x + c)))/(a^4*d*cos(d*x + c)^6 - 3*a^4*d*cos(
d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)
```

3.448.6 Sympy [F]

$$\int \frac{\cot^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\cot^7{(c+dx)}}{a+b\sin^2{(c+dx)}}\,dx$$

input
```
integrate(cot(d*x+c)**7/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(cot(c + d*x)**7/(a + b*sin(c + d*x)**2), x)
```

3.448.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.13

$$\int \frac{\cot^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \frac{\frac{6\,(a^3+3\,a^2b+3\,ab^2+b^3)\log\left(b\sin(dx+c)^2+a\right)}{a^4} - \frac{6\,(a^3+3\,a^2b+3\,ab^2+b^3)\log\left(\sin(dx+c)^2\right)}{a^4} - \frac{6\,(3\,a^2+3\,ab+b^2)\sin(dx+c)^4-3\,(3\,a^2+ab)\sin(d}{a^3\sin(dx+c)^6}}{12\,d}$$

input
```
integrate(cot(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/12*(6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(b*sin(d*x + c)^2 + a)/a^4 - 6*
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(sin(d*x + c)^2)/a^4 - (6*(3*a^2 + 3*a*
b + b^2)*sin(d*x + c)^4 - 3*(3*a^2 + a*b)*sin(d*x + c)^2 + 2*a^2)/(a^3*sin
(d*x + c)^6))/d
```

3.448.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(113) = 226$.

Time = 0.45 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.92

$$\int \frac{\cot^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \frac{\frac{a^2\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)^3+12\,a^2\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)^2+6\,ab\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)^2+84\,a^2\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)+}{a^3}}{}$$

input
```
integrate(cot(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/384*((a^2*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(c
os(d*x + c) + 1))^3 + 12*a^2*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos
(d*x + c) - 1)/(cos(d*x + c) + 1))^2 + 6*a*b*((cos(d*x + c) + 1)/(cos(d*x
+ c) - 1) + (cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2 + 84*a^2*((cos(d*x +
c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x + c) + 1)) + 120*
a*b*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x +
 c) + 1)) + 48*b^2*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c)
- 1)/(cos(d*x + c) + 1)))/a^3 + 192*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(ab
s(-a*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x
+ c) + 1)) + 2*a + 4*b))/a^4)/d
```

3.448.9 Mupad [B] (verification not implemented)

Time = 13.95 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.14

$$\int \frac{\cot^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\ln\left(a+a\tan(c+d\,x)^2+b\tan(c+d\,x)^2\right)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{2\,a^4\,d} - \frac{\frac{1}{6\,a}-\frac{\tan(c+d\,x)^2\,(a+b)}{4\,a^2}+\frac{\tan(c+d\,x)^4\,(a+b)^2}{2\,a^3}}{d\tan\left(c+d\,x\right)^6} - \frac{\ln\left(\tan(c+d\,x)\right)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{a^4\,d}$$

input
```
int(cot(c + d*x)^7/(a + b*sin(c + d*x)^2),x)
```

output
```
(log(a + a*tan(c + d*x)^2 + b*tan(c + d*x)^2)*(3*a*b^2 + 3*a^2*b + a^3 + b
^3))/(2*a^4*d) - (1/(6*a) - (tan(c + d*x)^2*(a + b))/(4*a^2) + (tan(c + d*
x)^4*(a + b)^2)/(2*a^3))/(d*tan(c + d*x)^6) - (log(tan(c + d*x))*(3*a*b^2
+ 3*a^2*b + a^3 + b^3))/(a^4*d)
```

3.449 $\int \frac{\tan^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.449.1 Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{\tan^8(c+dx)}{a+b\sin^2(c+dx)}\,dx=\frac{a^{7/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{9/2}d}-\frac{a^3\tan(c+dx)}{(a+b)^4d}+\frac{a^2\tan^3(c+dx)}{3(a+b)^3d}-\frac{a\tan^5(c+dx)}{5(a+b)^2d}+\frac{\tan^7(c+dx)}{7(a+b)d}$$

output
```
a^(7/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/(a+b)^(9/2)/d-a^3*tan(d*x+c
)/(a+b)^4/d+1/3*a^2*tan(d*x+c)^3/(a+b)^3/d-1/5*a*tan(d*x+c)^5/(a+b)^2/d+1/
7*tan(d*x+c)^7/(a+b)/d
```

3.449.2 Mathematica [A] (verified)

Time = 4.55 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.22

$$\int \frac{\tan^8(c+dx)}{a+b\sin^2(c+dx)}\,dx=\frac{a^{7/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{9/2}d}+\frac{(-176a^3-122a^2b-66ab^2-15b^3+(122a^3+254a^2b+177ab^2+45b^3)\sec^2(c+dx)-3(a+b)^2(22a+}{105(a+b)^4d}$$

input
```
Integrate[Tan[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

output
```
(a^(7/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(9/2)*d) + (
(-176*a^3 - 122*a^2*b - 66*a*b^2 - 15*b^3 + (122*a^3 + 254*a^2*b + 177*a*b
^2 + 45*b^3)*Sec[c + d*x]^2 - 3*(a + b)^2*(22*a + 15*b)*Sec[c + d*x]^4 + 1
5*(a + b)^3*Sec[c + d*x]^6)*Tan[c + d*x])/(105*(a + b)^4*d)
```

3.449.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3674, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^8(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^8}{a+b\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3674}$$

$$\frac{\int \frac{\tan^8(c+dx)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{254}$$

$$\frac{\int \left(\frac{\tan^6(c+dx)}{a+b}-\frac{a\tan^4(c+dx)}{(a+b)^2}+\frac{a^2\tan^2(c+dx)}{(a+b)^3}+\frac{a^4}{(a+b)^4((a+b)\tan^2(c+dx)+a)}-\frac{a^3}{(a+b)^4}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^{7/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{9/2}}-\frac{a^3\tan(c+dx)}{(a+b)^4}+\frac{a^2\tan^3(c+dx)}{3(a+b)^3}+\frac{\tan^7(c+dx)}{7(a+b)}-\frac{a\tan^5(c+dx)}{5(a+b)^2}}{d}$$

input
```
Int[Tan[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

output
```
((a^(7/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a + b)^(9/2) - (a^3
*Tan[c + d*x])/(a + b)^4 + (a^2*Tan[c + d*x]^3)/(3*(a + b)^3) - (a*Tan[c +
 d*x]^5)/(5*(a + b)^2) + Tan[c + d*x]^7/(7*(a + b)))/d
```

3.449.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3674
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[f
f/f   Subst[Int[(d*ff*x)^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(p + 1)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ
[p]
```

3.449.4 Maple [A] (verified)

Time = 6.45 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{\frac{(a+b)\left(a^2+2ab+b^2\right)\left(\tan^7(dx+c)\right)}{7}-\frac{a\left(a^2+2ab+b^2\right)\left(\tan^5(dx+c)\right)}{5}+\frac{a^2\left(\tan^3(dx+c)\right)(a+b)}{3}-a^3\tan(dx+c)}{(a+b)^4}+\frac{a^4\arctan\left(\frac{(a+b)\tan(c}{\sqrt{a(a+}}\right.}{(a+b)^4\sqrt{a(a+b}}}{d}$
default	$\frac{\frac{\frac{(a+b)\left(a^2+2ab+b^2\right)\left(\tan^7(dx+c)\right)}{7}-\frac{a\left(a^2+2ab+b^2\right)\left(\tan^5(dx+c)\right)}{5}+\frac{a^2\left(\tan^3(dx+c)\right)(a+b)}{3}-a^3\tan(dx+c)}{(a+b)^4}+\frac{a^4\arctan\left(\frac{(a+b)\tan(c}{\sqrt{a(a+}}\right.}{(a+b)^4\sqrt{a(a+b}}}{d}$
risch	$-\frac{2i(122a^2b+66ab^2+15b^3+176a^3+1260a^3e^{10i(dx+c)}+105b^3e^{12i(dx+c)}+812a^3e^{2i(dx+c)}+2436a^3e^{4i(dx+c)}+315b^3e^{4i(d}}{}$

input
```
int(tan(d*x+c)^8/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a+b)^4*(1/7*(a+b)*(a^2+2*a*b+b^2)*tan(d*x+c)^7-1/5*a*(a^2+2*a*b+b^
2)*tan(d*x+c)^5+1/3*a^2*tan(d*x+c)^3*(a+b)-a^3*tan(d*x+c))+1/(a+b)^4*a^4/(
a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
```

3.449.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(106) = 212$.

Time = 0.37 (sec) , antiderivative size = 602, normalized size of antiderivative = 5.02

$$\int \frac{\tan^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \left[\frac{105\,a^3\sqrt{-\frac{a}{a+b}}\cos(dx+c)^7\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4-2\,(4\,a^2+5\,ab+b^2)\cos(dx+c)^2-4\left((2\,a^2+3\,ab+b^2)\cos(dx+c)^3-(a^2\cdots\right.}{b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2\,a\cdots}\right.}{}\right.$$
$$-\frac{105\,a^3\sqrt{\frac{a}{a+b}}\arctan\left(\frac{\left((2\,a+b)\cos(dx+c)^2-a-b\right)\sqrt{\frac{a}{a+b}}}{2\,a\cos(dx+c)\sin(dx+c)}\right)\cos(dx+c)^7+2\left((176\,a^3+122\,a^2b+66\,ab^2+15\,b^3)\cdots\right.}{}$$

input
```
integrate(tan(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/420*(105*a^3*sqrt(-a/(a + b))*cos(d*x + c)^7*log(((8*a^2 + 8*a*b + b^2)
*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + 3*a
*b + b^2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a +
b))*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*
cos(d*x + c)^2 + a^2 + 2*a*b + b^2)) - 4*((176*a^3 + 122*a^2*b + 66*a*b^2
+ 15*b^3)*cos(d*x + c)^6 - (122*a^3 + 254*a^2*b + 177*a*b^2 + 45*b^3)*cos(
d*x + c)^4 - 15*a^3 - 45*a^2*b - 45*a*b^2 - 15*b^3 + 3*(22*a^3 + 59*a^2*b
+ 52*a*b^2 + 15*b^3)*cos(d*x + c)^2)*sin(d*x + c))/((a^4 + 4*a^3*b + 6*a^2
*b^2 + 4*a*b^3 + b^4)*d*cos(d*x + c)^7), -1/210*(105*a^3*sqrt(a/(a + b))*a
rctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x +
c)*sin(d*x + c)))*cos(d*x + c)^7 + 2*((176*a^3 + 122*a^2*b + 66*a*b^2 + 15
*b^3)*cos(d*x + c)^6 - (122*a^3 + 254*a^2*b + 177*a*b^2 + 45*b^3)*cos(d*x
+ c)^4 - 15*a^3 - 45*a^2*b - 45*a*b^2 - 15*b^3 + 3*(22*a^3 + 59*a^2*b + 52
*a*b^2 + 15*b^3)*cos(d*x + c)^2)*sin(d*x + c))/((a^4 + 4*a^3*b + 6*a^2*b^2
 + 4*a*b^3 + b^4)*d*cos(d*x + c)^7)]
```

3.449.6 Sympy [F]

$$\int \frac{\tan^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\tan^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**8/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(tan(c + d*x)**8/(a + b*sin(c + d*x)**2), x)
```

3.449.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.50

$$\int \frac{\tan^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \frac{\frac{105\,a^4\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{(a+b)a}} + \frac{15\,(a^3+3\,a^2b+3\,ab^2+b^3)\tan(dx+c)^7-21\,(a^3+2\,a^2b+ab^2)\tan(dx+c)^5-105\,a^3\tan(dx+c)+35\,(a^3+a}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}}{105\,d}$$

input
```
integrate(tan(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/105*(105*a^4*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a^4 + 4*a^3*
b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*a)) + (15*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*tan(d*x + c)^7 - 21*(a^3 + 2*a^2*b + a*b^2)*tan(d*x + c)^5 - 1
05*a^3*tan(d*x + c) + 35*(a^3 + a^2*b)*tan(d*x + c)^3)/(a^4 + 4*a^3*b + 6*
a^2*b^2 + 4*a*b^3 + b^4))/d
```

3.449.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(106) = 212$.

Time = 5.09 (sec) , antiderivative size = 472, normalized size of antiderivative = 3.93

$$\int \frac{\tan^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \frac{\frac{105\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^4}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{a^2+ab}} + 15\,a^6\tan(dx+c)^7+90\,a^5b\tan(dx+c)^7+225\,a^4b^2\tan(dx+c)^7+300\,a^3b}{}$$

input
```
integrate(tan(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/105*(105*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*
x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^4/((a^4 + 4*a^3*b + 6*a^2*b^2
 + 4*a*b^3 + b^4)*sqrt(a^2 + a*b)) + (15*a^6*tan(d*x + c)^7 + 90*a^5*b*tan
(d*x + c)^7 + 225*a^4*b^2*tan(d*x + c)^7 + 300*a^3*b^3*tan(d*x + c)^7 + 22
5*a^2*b^4*tan(d*x + c)^7 + 90*a*b^5*tan(d*x + c)^7 + 15*b^6*tan(d*x + c)^7
 - 21*a^6*tan(d*x + c)^5 - 105*a^5*b*tan(d*x + c)^5 - 210*a^4*b^2*tan(d*x
+ c)^5 - 210*a^3*b^3*tan(d*x + c)^5 - 105*a^2*b^4*tan(d*x + c)^5 - 21*a*b^
5*tan(d*x + c)^5 + 35*a^6*tan(d*x + c)^3 + 140*a^5*b*tan(d*x + c)^3 + 210*
a^4*b^2*tan(d*x + c)^3 + 140*a^3*b^3*tan(d*x + c)^3 + 35*a^2*b^4*tan(d*x +
 c)^3 - 105*a^6*tan(d*x + c) - 315*a^5*b*tan(d*x + c) - 315*a^4*b^2*tan(d*
x + c) - 105*a^3*b^3*tan(d*x + c))/(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^
3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7))/d
```

3.449.9 Mupad [B] (verification not implemented)

Time = 14.83 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.18

$$\int \frac{\tan^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\tan(c+d\,x)^7}{7\,d\,(a+b)} + \frac{a^2\tan(c+d\,x)^3}{3\,d\,(a+b)^3} + \frac{a^{7/2}\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)\,(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4)}{2\,\sqrt{a}\,(a+b)^{9/2}}\right)}{d\,(a+b)^{9/2}} - \frac{a\tan(c+d\,x)^5}{5\,d\,(a+b)^2} - \frac{a^3\tan(c+d\,x)}{d\,(a+b)^4}$$

input
```
int(tan(c + d*x)^8/(a + b*sin(c + d*x)^2),x)
```

output
```
tan(c + d*x)^7/(7*d*(a + b)) + (a^2*tan(c + d*x)^3)/(3*d*(a + b)^3) + (a^(
7/2)*atan((tan(c + d*x)*(2*a + 2*b)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2
*b^2))/(2*a^(1/2)*(a + b)^(9/2))))/(d*(a + b)^(9/2)) - (a*tan(c + d*x)^5)/
(5*d*(a + b)^2) - (a^3*tan(c + d*x))/(d*(a + b)^4)
```

3.450 $\int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.450.1 Optimal result

Integrand size = 23, antiderivative size = 97

$$\int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{a^{5/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{7/2}d}+\frac{a^2\tan(c+dx)}{(a+b)^3d}-\frac{a\tan^3(c+dx)}{3(a+b)^2d}+\frac{\tan^5(c+dx)}{5(a+b)d}$$

output
```
-a^(5/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/(a+b)^(7/2)/d+a^2*tan(d*x+
c)/(a+b)^3/d-1/3*a*tan(d*x+c)^3/(a+b)^2/d+1/5*tan(d*x+c)^5/(a+b)/d
```

3.450.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.14

$$\int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{-15a^{5/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)+\sqrt{a+b}(23a^2+11ab+3b^2-(11a^2+17ab+6b^2)\sec^2(c+dx)+3(a+}{15(a+b)^{7/2}d}$$

input
```
Integrate[Tan[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-15*a^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a + b]*(23*
a^2 + 11*a*b + 3*b^2 - (11*a^2 + 17*a*b + 6*b^2)*Sec[c + d*x]^2 + 3*(a + b
)^2*Sec[c + d*x]^4)*Tan[c + d*x])/(15*(a + b)^(7/2)*d)
```

3.450.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3674, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\tan(c+dx)^6}{a+b\sin(c+dx)^2}dx$$

$$\Big\downarrow \ 3674$$

$$\frac{\int \frac{\tan^6(c+dx)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \ 254$$

$$\frac{\int \left(\frac{\tan^4(c+dx)}{a+b}-\frac{a\tan^2(c+dx)}{(a+b)^2}-\frac{a^3}{(a+b)^3((a+b)\tan^2(c+dx)+a)}+\frac{a^2}{(a+b)^3}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{a^{5/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{7/2}}+\frac{a^2\tan(c+dx)}{(a+b)^3}+\frac{\tan^5(c+dx)}{5(a+b)}-\frac{a\tan^3(c+dx)}{3(a+b)^2}}{d}$$

input
```
Int[Tan[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-((a^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a + b)^(7/2)) + (
a^2*Tan[c + d*x])/(a + b)^3 - (a*Tan[c + d*x]^3)/(3*(a + b)^2) + Tan[c + d
*x]^5/(5*(a + b)))/d
```

3.450. $\int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.450.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3674
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[f
f/f   Subst[Int[(d*ff*x)^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(p + 1)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ
[p]
```

3.450.4 Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\frac{\frac{a^2\left(\tan^5(dx+c)\right)}{5}+\frac{2ab\left(\tan^5(dx+c)\right)}{5}+\frac{b^2\left(\tan^5(dx+c)\right)}{5}-\frac{a^2\left(\tan^3(dx+c)\right)}{3}-\frac{ab\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)a^2}{(a+b)^3}-\frac{a^3\arctan\left(\frac{(a+b)\tan}{\sqrt{a(a}}\right.}{(a+b)^3\sqrt{a(a+}}}{d}$
default	$\frac{\frac{\frac{a^2\left(\tan^5(dx+c)\right)}{5}+\frac{2ab\left(\tan^5(dx+c)\right)}{5}+\frac{b^2\left(\tan^5(dx+c)\right)}{5}-\frac{a^2\left(\tan^3(dx+c)\right)}{3}-\frac{ab\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)a^2}{(a+b)^3}-\frac{a^3\arctan\left(\frac{(a+b)\tan}{\sqrt{a(a}}\right.}{(a+b)^3\sqrt{a(a+}}}{d}$
risch	$\frac{2i\left(45a^2e^{8i(dx+c)}+45ab\,e^{8i(dx+c)}+15b^2e^{8i(dx+c)}+90a^2e^{6i(dx+c)}+30ab\,e^{6i(dx+c)}+140a^2e^{4i(dx+c)}+80ab\,e^{4i(dx+c)}+30b\right.}{15d(a+b)^3\left(e^{2i(dx+c)}+1\right)^5}$

input
```
int(tan(d*x+c)^6/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a+b)^3*(1/5*a^2*tan(d*x+c)^5+2/5*a*tan(d*x+c)^5*b+1/5*b^2*tan(d*x+
c)^5-1/3*a^2*tan(d*x+c)^3-1/3*a*tan(d*x+c)^3*b+tan(d*x+c)*a^2)-a^3/(a+b)^3
/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
```

3.450.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(85) = 170$.

Time = 0.34 (sec) , antiderivative size = 472, normalized size of antiderivative = 4.87

$$\int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \left[\frac{15\,a^2\sqrt{-\frac{a}{a+b}}\cos(dx+c)^5\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4-2\,(4\,a^2+5\,ab+b^2)\cos(dx+c)^2+4\left((2\,a^2+3\,ab+b^2)\cos(dx+c)^3-(a^2+\right.}{b^2\cos(dx+c)^4-2\,(ab+b^2)\cos(dx+c)^2+a^2+2\,ab-}\right.}{}\right.$$

input
```
integrate(tan(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/60*(15*a^2*sqrt(-a/(a + b))*cos(d*x + c)^5*log(((8*a^2 + 8*a*b + b^2)*c
os(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b
 + b^2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b)
)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*co
s(d*x + c)^2 + a^2 + 2*a*b + b^2)) + 4*((23*a^2 + 11*a*b + 3*b^2)*cos(d*x
+ c)^4 - (11*a^2 + 17*a*b + 6*b^2)*cos(d*x + c)^2 + 3*a^2 + 6*a*b + 3*b^2)
*sin(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cos(d*x + c)^5), 1/30*(1
5*a^2*sqrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a
/(a + b))/(a*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c)^5 + 2*((23*a^2 + 11*
a*b + 3*b^2)*cos(d*x + c)^4 - (11*a^2 + 17*a*b + 6*b^2)*cos(d*x + c)^2 + 3
*a^2 + 6*a*b + 3*b^2)*sin(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cos
(d*x + c)^5)]
```

3.450.6 Sympy [F]

$$\int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**6/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(tan(c + d*x)**6/(a + b*sin(c + d*x)**2), x)
```

3.450.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.34

$$\int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{15\,a^3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{(a+b)a}} - \frac{3\,(a^2+2\,ab+b^2)\tan(dx+c)^5-5\,(a^2+ab)\tan(dx+c)^3+15\,a^2\tan(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3}}{15\,d}$$

input
```
integrate(tan(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/15*(15*a^3*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*sqrt((a + b)*a)) - (3*(a^2 + 2*a*b + b^2)*tan(d*x + c)^5
 - 5*(a^2 + a*b)*tan(d*x + c)^3 + 15*a^2*tan(d*x + c))/(a^3 + 3*a^2*b + 3*
a*b^2 + b^3))/d
```

3.450.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(85) = 170.

Time = 2.21 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.05

$$\int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}\,dx =$$

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^3}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{a^2+ab}} - \frac{3\,a^4\tan(dx+c)^5+12\,a^3b\tan(dx+c)^5+18\,a^2b^2\tan(dx+c)^5+12\,ab^3\tan\cdots}{\cdots}}{\cdots}$$

input
```
integrate(tan(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/15*(15*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x
 + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^3/((a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*sqrt(a^2 + a*b)) - (3*a^4*tan(d*x + c)^5 + 12*a^3*b*tan(d*x + c)^5 +
18*a^2*b^2*tan(d*x + c)^5 + 12*a*b^3*tan(d*x + c)^5 + 3*b^4*tan(d*x + c)^5
 - 5*a^4*tan(d*x + c)^3 - 15*a^3*b*tan(d*x + c)^3 - 15*a^2*b^2*tan(d*x + c
)^3 - 5*a*b^3*tan(d*x + c)^3 + 15*a^4*tan(d*x + c) + 30*a^3*b*tan(d*x + c)
 + 15*a^2*b^2*tan(d*x + c))/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a
*b^4 + b^5))/d
```

3.450.9 Mupad [B] (verification not implemented)

Time = 14.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.15

$$\int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\tan(c+d\,x)^5}{5\,d\,(a+b)} - \frac{a^{5/2}\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}{2\,\sqrt{a}\,(a+b)^{7/2}}\right)}{d\,(a+b)^{7/2}} - \frac{a\tan(c+d\,x)^3}{3\,d\,(a+b)^2} + \frac{a^2\tan(c+d\,x)}{d\,(a+b)^3}$$

input
```
int(tan(c + d*x)^6/(a + b*sin(c + d*x)^2),x)
```

output
```
tan(c + d*x)^5/(5*d*(a + b)) - (a^(5/2)*atan((tan(c + d*x)*(2*a + 2*b)*(3*
a*b^2 + 3*a^2*b + a^3 + b^3))/(2*a^(1/2)*(a + b)^(7/2))))/(d*(a + b)^(7/2)
) - (a*tan(c + d*x)^3)/(3*d*(a + b)^2) + (a^2*tan(c + d*x))/(d*(a + b)^3)
```

3.451 $\int \frac{\tan^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.451.1 Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \frac{\tan^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{a^{3/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{5/2}d} - \frac{a\tan(c+dx)}{(a+b)^2d} + \frac{\tan^3(c+dx)}{3(a+b)d}$$

output
```
a^(3/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/(a+b)^(5/2)/d-a*tan(d*x+c)/
(a+b)^2/d+1/3*tan(d*x+c)^3/(a+b)/d
```

3.451.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{\tan^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{3a^{3/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right) + \sqrt{a+b}(-4a-b+(a+b)\sec^2(c+dx))\tan(c+dx)}{3(a+b)^{5/2}d}$$

input
```
Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

output
```
(3*a^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a + b]*(-4*a
- b + (a + b)*Sec[c + d*x]^2)*Tan[c + d*x])/(3*(a + b)^(5/2)*d)
```

3.451.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3674, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^4}{a+b\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3674}$$

$$\frac{\int \frac{\tan^4(c+dx)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{254}$$

$$\frac{\int \left(\frac{a^2}{(a+b)^2((a+b)\tan^2(c+dx)+a)}-\frac{a}{(a+b)^2}+\frac{\tan^2(c+dx)}{a+b}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^{3/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{5/2}}+\frac{\tan^3(c+dx)}{3(a+b)}-\frac{a\tan(c+dx)}{(a+b)^2}}{d}$$

input
```
Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

output
```
((a^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a + b)^(5/2) - (a*T
an[c + d*x])/(a + b)^2 + Tan[c + d*x]^3/(3*(a + b)))/d
```

3.451.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3674
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[f
f/f   Subst[Int[(d*ff*x)^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(p + 1)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ
[p]
```

3.451.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tan^3(dx+c)\right)}{3}+\frac{\left(\tan^3(dx+c)\right)b}{3}-\tan(dx+c)a}{(a+b)^2}+\frac{a^2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{(a+b)^2\sqrt{a(a+b)}}}{d}$
default	$\frac{\frac{\frac{a\left(\tan^3(dx+c)\right)}{3}+\frac{\left(\tan^3(dx+c)\right)b}{3}-\tan(dx+c)a}{(a+b)^2}+\frac{a^2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{(a+b)^2\sqrt{a(a+b)}}}{d}$
risch	$-\frac{2i\left(6a\,e^{4i(dx+c)}+3b\,e^{4i(dx+c)}+6a\,e^{2i(dx+c)}+4a+b\right)}{3d(a+b)^2\left(e^{2i(dx+c)}+1\right)^3}+\frac{\sqrt{-a(a+b)}\,a\ln\left(e^{2i(dx+c)}+\frac{2i\sqrt{-a(a+b)}-2a-b}{b}\right)}{2(a+b)^3d}-\sqrt{-a(a}$

input
```
int(tan(d*x+c)^4/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a+b)^2*(1/3*a*tan(d*x+c)^3+1/3*tan(d*x+c)^3*b-tan(d*x+c)*a)+a^2/(a
+b)^2/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2)))
```

3.451.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 366, normalized size of antiderivative = 4.95

$$\int \frac{\tan^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \left[\frac{3\,a\sqrt{-\frac{a}{a+b}}\cos\left(dx+c\right)^3\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(dx+c)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(dx+c)^2-4\left(\left(2\,a^2+3\,ab+b^2\right)\cos(dx+c)^3-\left(a^2+2\,a\right.\right.}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2\,ab+b^2}\right.}{12\left(a^2+2\,ab+b^2\right)d\cos\left(\right.}\right.$$
$$-\frac{3\,a\sqrt{\frac{a}{a+b}}\arctan\left(\frac{\left(\left(2\,a+b\right)\cos(dx+c)^2-a-b\right)\sqrt{\frac{a}{a+b}}}{2\,a\cos(dx+c)\sin(dx+c)}\right)\cos\left(dx+c\right)^3+2\left(\left(4\,a+b\right)\cos\left(dx+c\right)^2-a-b\right)\sin\left(dx\right.}{6\left(a^2+2\,ab+b^2\right)d\cos\left(dx+c\right)^3}$$

input
```
integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/12*(3*a*sqrt(-a/(a + b))*cos(d*x + c)^3*log(((8*a^2 + 8*a*b + b^2)*cos(
d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + 3*a*b +
b^2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*s
in(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d
*x + c)^2 + a^2 + 2*a*b + b^2)) - 4*((4*a + b)*cos(d*x + c)^2 - a - b)*sin
(d*x + c))/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^3), -1/6*(3*a*sqrt(a/(a + b
))*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*
x + c)*sin(d*x + c)))*cos(d*x + c)^3 + 2*((4*a + b)*cos(d*x + c)^2 - a - b
)*sin(d*x + c))/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^3)]
```

3.451.6 Sympy [F]

$$\int \frac{\tan^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\tan^4\left(c+dx\right)}{a+b\sin^2\left(c+dx\right)}\,dx$$

input
```
integrate(tan(d*x+c)**4/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(tan(c + d*x)**4/(a + b*sin(c + d*x)**2), x)
```

3.451.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{\tan^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{3\,a^2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}(a^2+2\,ab+b^2)}+\frac{(a+b)\tan(dx+c)^3-3\,a\tan(dx+c)}{a^2+2\,ab+b^2}}{3\,d}$$

input
```
integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/3*(3*a^2*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(
a^2 + 2*a*b + b^2)) + ((a + b)*tan(d*x + c)^3 - 3*a*tan(d*x + c))/(a^2 + 2
*a*b + b^2))/d
```

3.451.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(64) = 128.

Time = 0.91 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.22

$$\int \frac{\tan^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^2}{(a^2+2\,ab+b^2)\sqrt{a^2+ab}}+\frac{a^2\tan(dx+c)^3+2\,ab\tan(dx+c)^3+b^2\tan(dx+c)^3-3\,a^2\tan(dx+c)-3\,ab\,\mathrm{t}}{a^3+3\,a^2b+3\,ab^2+b^3}}{3\,d}$$

input
```
integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x +
c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^2/((a^2 + 2*a*b + b^2)*sqrt(a^2 +
 a*b)) + (a^2*tan(d*x + c)^3 + 2*a*b*tan(d*x + c)^3 + b^2*tan(d*x + c)^3 -
 3*a^2*tan(d*x + c) - 3*a*b*tan(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))
/d
```

3.451.9 Mupad [B] (verification not implemented)

Time = 13.95 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.12

$$\int \frac{\tan^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\tan(c+d\,x)^3}{3\,d\,(a+b)} - \frac{a\tan(c+d\,x)}{d\,(a+b)^2} + \frac{a^{3/2}\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)\,(a^2+2\,a\,b+b^2)}{2\,\sqrt{a}\,(a+b)^{5/2}}\right)}{d\,(a+b)^{5/2}}$$

input
```
int(tan(c + d*x)^4/(a + b*sin(c + d*x)^2),x)
```

output
```
tan(c + d*x)^3/(3*d*(a + b)) - (a*tan(c + d*x))/(d*(a + b)^2) + (a^(3/2)*a
tan((tan(c + d*x)*(2*a + 2*b)*(2*a*b + a^2 + b^2))/(2*a^(1/2)*(a + b)^(5/2
))))/(d*(a + b)^(5/2))
```

3.452 $\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.452.1 Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx=-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}d}+\frac{\tan(c+dx)}{(a+b)d}$$

output
```
-arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))*a^(1/2)/(a+b)^(3/2)/d+tan(d*x+c)/(
a+b)/d
```

3.452.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx=-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}d}+\frac{\tan(c+dx)}{(a+b)d}$$

input
```
Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

output
```
-((Sqrt[a]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(3/2)*d))
+ Tan[c + d*x]/((a + b)*d)
```

3.452.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3674, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(c+dx)^2}{a+b\sin(c+dx)^2}dx$$

$$\Big\downarrow \; 3674$$

$$\frac{\int \frac{\tan^2(c+dx)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \; 262$$

$$\frac{\frac{\tan(c+dx)}{a+b}-\frac{a\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{a+b}}{d}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\tan(c+dx)}{a+b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}}}{d}$$

input
```
Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-((Sqrt[a]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a + b)^(3/2)) + T
an[c + d*x]/(a + b))/d
```

3.452.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3674
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[f
f/f   Subst[Int[(d*ff*x)^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(p + 1)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ
[p]
```

3.452.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)}{a+b}-\frac{a\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{(a+b)\sqrt{a(a+b)}}}{d}$
default	$\frac{\frac{\tan(dx+c)}{a+b}-\frac{a\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{(a+b)\sqrt{a(a+b)}}}{d}$
risch	$\frac{2i}{d(a+b)\left(e^{2i(dx+c)}+1\right)}-\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2i(dx+c)}+\frac{2i\sqrt{-a(a+b)}-2a-b}{b}\right)}{2(a+b)^2d}+\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2i(dx+c)}-\frac{2i\sqrt{-a(a+b)}+2}{b}\right.}{2(a+b)^2d}$

input
```
int(tan(d*x+c)^2/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(tan(d*x+c)/(a+b)-a/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(
a+b))^(1/2)))
```

3.452.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(45) = 90$.

Time = 0.32 (sec) , antiderivative size = 300, normalized size of antiderivative = 5.66

$$\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \left[\frac{\sqrt{-\frac{a}{a+b}}\cos(dx+c)\log\left(\frac{(8a^2+8ab+b^2)\cos(dx+c)^4-2(4a^2+5ab+b^2)\cos(dx+c)^2+4\left((2a^2+3ab+b^2)\cos(dx+c)^3-(a^2+2ab+b^2)\right.}{b^2\cos(dx+c)^4-2(ab+b^2)\cos(dx+c)^2+a^2+2ab+b^2}\right.}{4(a+b)d\cos(dx+c)}\right.$$

input
```
integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(-a/(a + b))*cos(d*x + c)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c
)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b + b^2)*co
s(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x
+ c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)
^2 + a^2 + 2*a*b + b^2)) + 4*sin(d*x + c))/((a + b)*d*cos(d*x + c)), 1/2*(
sqrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a +
b))/(a*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c) + 2*sin(d*x + c))/((a + b)
*d*cos(d*x + c))]
```

3.452.6 Sympy [F]

$$\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**2/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(tan(c + d*x)**2/(a + b*sin(c + d*x)**2), x)
```

3.452.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{a\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}(a+b)} - \frac{\tan(dx+c)}{a+b}}{d}$$

input
```
integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-(a*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a + b))
 - tan(d*x + c)/(a + b))/d
```

3.452.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.62

$$\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a}{\sqrt{a^2+ab}(a+b)} - \frac{\tan(dx+c)}{a+b}}{d}$$

input
```
integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) +
b*tan(d*x + c))/sqrt(a^2 + a*b)))*a/(sqrt(a^2 + a*b)*(a + b)) - tan(d*x +
c)/(a + b))/d
```

3.452.9 Mupad [B] (verification not implemented)

Time = 13.44 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\tan(c+d\,x)}{d\,(a+b)} - \frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)}{2\,\sqrt{a}\,\sqrt{a+b}}\right)}{d\,(a+b)^{3/2}}$$

input
```
int(tan(c + d*x)^2/(a + b*sin(c + d*x)^2),x)
```

output
```
tan(c + d*x)/(d*(a + b)) - (a^(1/2)*atan((tan(c + d*x)*(2*a + 2*b))/(2*a^(
1/2)*(a + b)^(1/2))))/(d*(a + b)^(3/2))
```

3.453 $\int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.453.1 Optimal result

Integrand size = 23, antiderivative size = 52

$$\int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\cot(c+dx)}{ad}$$

output
```
-cot(d*x+c)/a/d-arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))*(a+b)^(1/2)/a^(3/2)
/d
```

3.453.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{-\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right) - \sqrt{a}\cot(c+dx)}{a^{3/2}d}$$

input
```
Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-(Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]) - Sqrt[a]*Cot[c
 + d*x])/(a^(3/2)*d)
```

3.453.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3674, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\tan(c+dx)^2\,(a+b\sin(c+dx)^2)}dx$$
$$\Big\downarrow \text{3674}$$
$$\frac{\int \frac{\cot^2(c+dx)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$
$$\Big\downarrow \text{264}$$
$$\frac{-\frac{(a+b)\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{a}-\frac{\cot(c+dx)}{a}}{d}$$
$$\Big\downarrow \text{218}$$
$$\frac{-\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\cot(c+dx)}{a}}{d}$$

input
```
Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-((Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/a^(3/2)) - Cot
[c + d*x]/a)/d
```

3.453.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3674
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[f
f/f   Subst[Int[(d*ff*x)^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(p + 1)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ
[p]
```

3.453.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{-\frac{1}{a\tan(dx+c)}+\frac{(-a-b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a\sqrt{a(a+b)}}}{d}$
default	$\frac{-\frac{1}{a\tan(dx+c)}+\frac{(-a-b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a\sqrt{a(a+b)}}}{d}$
risch	$-\frac{2i}{ad\left(e^{2i(dx+c)}-1\right)}-\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2i(dx+c)}+\frac{2i\sqrt{-a(a+b)}-2a-b}{b}\right)}{2a^2d}+\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2i(dx+c)}-\frac{2i\sqrt{-a(a+b)}+2a}{b}\right.}{2a^2d}$

input
```
int(cot(d*x+c)^2/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a/tan(d*x+c)+1/a*(-a-b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a
*(a+b))^(1/2)))
```

3.453.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(44) = 88$.

Time = 0.31 (sec) , antiderivative size = 290, normalized size of antiderivative = 5.58

$$\int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \left[\frac{\sqrt{-\frac{a+b}{a}}\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(dx+c)^4-2\left(4a^2+5ab+b^2\right)\cos(dx+c)^2+4\left(\left(2a^2+ab\right)\cos(dx+c)^3-\left(a^2+ab\right)\cos(dx+c)\right)\sqrt{-\frac{a+b}{a}}\sin(dx}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2ab+b^2}\right.}{4\,ad\sin(dx+c)}\right.$$

input
```
integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^
2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + a*b)*cos(d*x + c)^3 - (a^2 +
 a*b)*cos(d*x + c))*sqrt(-(a + b)/a)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^
2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(
d*x + c) - 4*cos(d*x + c))/(a*d*sin(d*x + c)), 1/2*(sqrt((a + b)/a)*arctan
(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(d*x +
 c)*sin(d*x + c)))*sin(d*x + c) - 2*cos(d*x + c))/(a*d*sin(d*x + c))]
```

3.453.6 Sympy [F]

$$\int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**2/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(cot(c + d*x)**2/(a + b*sin(c + d*x)**2), x)
```

3.453.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{(a+b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a}+\frac{1}{a\tan(dx+c)}}{d}$$

input
```
integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-((a + b)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a)
 + 1/(a*tan(d*x + c)))/d
```

3.453.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.63

$$\int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(a+b)}{\sqrt{a^2+ab}a}+\frac{1}{a\tan(dx+c)}}{d}$$

input
```
integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) +
b*tan(d*x + c))/sqrt(a^2 + a*b)))*(a + b)/(sqrt(a^2 + a*b)*a) + 1/(a*tan(d
*x + c)))/d
```

3.453.9 Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\cot(c+d\,x)}{a\,d}-\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)\,\sqrt{a+b}}{a^{3/2}\,d}$$

input
```
int(cot(c + d*x)^2/(a + b*sin(c + d*x)^2),x)
```

output
```
- cot(c + d*x)/(a*d) - (atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2))*(a + b)
^(1/2))/(a^(3/2)*d)
```

3.454 $\int \frac{\cot^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.454.1 Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \frac{\cot^4(c+dx)}{a+b\sin^2(c+dx)}\,dx=\frac{(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3ad}$$

output
```
(a+b)^(3/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(5/2)/d+(a+b)*cot(d*x
+c)/a^2/d-1/3*cot(d*x+c)^3/a/d
```

3.454.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int \frac{\cot^4(c+dx)}{a+b\sin^2(c+dx)}\,dx=\frac{3(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)+\sqrt{a}\cot(c+dx)\left(4a+3b-a\csc^2(c+dx)\right)}{3a^{5/2}d}$$

input
```
Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

output
```
(3*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*Cot[
c + d*x]*(4*a + 3*b - a*Csc[c + d*x]^2))/(3*a^(5/2)*d)
```

3.454.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3674, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\tan(c+dx)^4\left(a+b\sin(c+dx)^2\right)}dx$$

$$\Big\downarrow \ 3674$$

$$\frac{\int \frac{\cot^4(c+dx)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \ 264$$

$$\frac{-\frac{(a+b)\int \frac{\cot^2(c+dx)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{a}-\frac{\cot^3(c+dx)}{3a}}{d}$$

$$\Big\downarrow \ 264$$

$$\frac{-\frac{(a+b)\left(-\frac{(a+b)\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{a}-\frac{\cot(c+dx)}{a}\right)}{a}-\frac{\cot^3(c+dx)}{3a}}{d}$$

$$\Big\downarrow \ 218$$

$$\frac{-\frac{(a+b)\left(-\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\cot(c+dx)}{a}\right)}{a}-\frac{\cot^3(c+dx)}{3a}}{d}$$

input
```
Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-1/3*Cot[c + d*x]^3/a - ((a + b)*(-((Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[
c + d*x])/Sqrt[a]])/a^(3/2)) - Cot[c + d*x]/a))/a)/d
```

3.454.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3674
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[f
f/f   Subst[Int[(d*ff*x)^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(p + 1)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ
[p]
```

3.454.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{\left(a^{2}+2ab+b^{2}\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a^{2}\sqrt{a(a+b)}}-\frac{1}{3a\tan(dx+c)^{3}}-\frac{-a-b}{a^{2}\tan(dx+c)}}{d}$
default	$\frac{\frac{\left(a^{2}+2ab+b^{2}\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a^{2}\sqrt{a(a+b)}}-\frac{1}{3a\tan(dx+c)^{3}}-\frac{-a-b}{a^{2}\tan(dx+c)}}{d}$
risch	$\frac{2i\left(6a\,e^{4i(dx+c)}+3b\,e^{4i(dx+c)}-6a\,e^{2i(dx+c)}-6b\,e^{2i(dx+c)}+4a+3b\right)}{3d\,a^{2}\left(e^{2i(dx+c)}-1\right)^{3}}+\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2i(dx+c)}+\frac{2i\sqrt{-a(a+b)}-2a-b}{b}\right)}{2a^{2}d}$

input
```
int(cot(d*x+c)^4/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*((a^2+2*a*b+b^2)/a^2/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))
^(1/2))-1/3/a/tan(d*x+c)^3-1/a^2*(-a-b)/tan(d*x+c))
```

3.454.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(61) = 122$.

Time = 0.31 (sec) , antiderivative size = 402, normalized size of antiderivative = 5.66

$$\int \frac{\cot^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \left[\frac{4\,(4\,a+3\,b)\cos{(dx+c)}^3+3\,\left((a+b)\cos{(dx+c)}^2-a-b\right)\sqrt{-\frac{a+b}{a}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(dx+c)^4-2\,(4\,a^2+5\,a}{}\right.}{12\,\left(a^2d\cos{(dx}\right.}\right.$$

input
```
integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/12*(4*(4*a + 3*b)*cos(d*x + c)^3 + 3*((a + b)*cos(d*x + c)^2 - a - b)*s
qrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a
*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + a*b)*cos(d*x + c)^3 - (a^2 + a*b)*c
os(d*x + c))*sqrt(-(a + b)/a)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d
*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x + c
) - 12*(a + b)*cos(d*x + c))/((a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
, 1/6*(2*(4*a + 3*b)*cos(d*x + c)^3 - 3*((a + b)*cos(d*x + c)^2 - a - b)*s
qrt((a + b)/a)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt((a + b)/
a)/((a + b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) - 6*(a + b)*cos(d*x +
 c))/((a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))]
```

3.454.6 Sympy [F]

$$\int \frac{\cot^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\cot^4\left(c+dx\right)}{a+b\sin^2\left(c+dx\right)}\,dx$$

input
```
integrate(cot(d*x+c)**4/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(cot(c + d*x)**4/(a + b*sin(c + d*x)**2), x)
```

3.454.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{\cot^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{3\,(a^2+2\,ab+b^2)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a^2}+\frac{3\,(a+b)\tan(dx+c)^2-a}{a^2\tan(dx+c)^3}}{3\,d}$$

input
```
integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/3*(3*(a^2 + 2*a*b + b^2)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(s
qrt((a + b)*a)*a^2) + (3*(a + b)*tan(d*x + c)^2 - a)/(a^2*tan(d*x + c)^3))
/d
```

3.454.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.69

$$\int \frac{\cot^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(a^2+2\,ab+b^2)}{\sqrt{a^2+ab}a^2}+\frac{3\,a\tan(dx+c)^2+3\,b\tan(dx+c)^2-a}{a^2\tan(dx+c)^3}}{3\,d}$$

input
```
integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x +
c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*(a^2 + 2*a*b + b^2)/(sqrt(a^2 + a*b
)*a^2) + (3*a*tan(d*x + c)^2 + 3*b*tan(d*x + c)^2 - a)/(a^2*tan(d*x + c)^3
))/d
```

3.454.9 Mupad [B] (verification not implemented)

Time = 13.45 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \frac{\cot^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)\,(a+b)^{3/2}}{a^{5/2}\,d} - \frac{\frac{1}{3\,a}-\frac{\tan(c+d\,x)^2\,(a+b)}{a^2}}{d\tan\left(c+d\,x\right)^3}$$

input
```
int(cot(c + d*x)^4/(a + b*sin(c + d*x)^2),x)
```

output
```
(atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2))*(a + b)^(3/2))/(a^(5/2)*d) - (
1/(3*a) - (tan(c + d*x)^2*(a + b))/a^2)/(d*tan(c + d*x)^3)
```

3.455 $\int \frac{\cot^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.455.1 Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{\cot^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{(a+b)^{5/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d} - \frac{(a+b)^2\cot(c+dx)}{a^3d} + \frac{(a+b)\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{5ad}$$

output
```
-(a+b)^(5/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(7/2)/d-(a+b)^2*cot(
d*x+c)/a^3/d+1/3*(a+b)*cot(d*x+c)^3/a^2/d-1/5*cot(d*x+c)^5/a/d
```

3.455.2 Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.05

$$\int \frac{\cot^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{-15(a+b)^{5/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right) - \sqrt{a}\cot(c+dx)\left(23a^2+35ab+15b^2-a(11a+5b)\csc^2(c+dx)+\right.}{15a^{7/2}d}$$

input
```
Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-15*(a + b)^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]] - Sqrt[a]*Co
t[c + d*x]*(23*a^2 + 35*a*b + 15*b^2 - a*(11*a + 5*b)*Csc[c + d*x]^2 + 3*a
^2*Csc[c + d*x]^4))/(15*a^(7/2)*d)
```

3.455.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3674, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(c+dx)}{a+b\sin^2(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\tan(c+dx)^6\left(a+b\sin(c+dx)^2\right)}dx$$
$$\Big\downarrow \text{3674}$$
$$\frac{\int \frac{\cot^6(c+dx)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$
$$\Big\downarrow \text{264}$$
$$\frac{-\frac{(a+b)\int \frac{\cot^4(c+dx)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{a}-\frac{\cot^5(c+dx)}{5a}}{d}$$
$$\Big\downarrow \text{264}$$
$$\frac{-\frac{(a+b)\left(-\frac{(a+b)\int \frac{\cot^2(c+dx)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{a}-\frac{\cot^3(c+dx)}{3a}\right)}{a}-\frac{\cot^5(c+dx)}{5a}}{d}$$
$$\Big\downarrow \text{264}$$
$$\frac{-\frac{(a+b)\left(-\frac{(a+b)\left(-\frac{(a+b)\int \frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{a}-\frac{\cot(c+dx)}{a}\right)}{a}-\frac{\cot^3(c+dx)}{3a}\right)}{a}-\frac{\cot^5(c+dx)}{5a}}{d}$$
$$\Big\downarrow \text{218}$$
$$\frac{-\frac{(a+b)\left(-\frac{(a+b)\left(-\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\cot(c+dx)}{a}\right)}{a}-\frac{\cot^3(c+dx)}{3a}\right)}{a}-\frac{\cot^5(c+dx)}{5a}}{d}$$

input
```
Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-1/5*Cot[c + d*x]^5/a - ((a + b)*(-1/3*Cot[c + d*x]^3/a - ((a + b)*(-((Sq
rt[a + b]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/a^(3/2)) - Cot[c + d
*x]/a))/a))/a)/d
```

3.455.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3674
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[f
f/f   Subst[Int[(d*ff*x)^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(p + 1)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ
[p]
```

3.455.4 Maple [A] (verified)

Time = 5.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{\left(-a^{3}-3a^{2}b-3a\,b^{2}-b^{3}\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a^{3}\sqrt{a(a+b)}}-\frac{1}{5a\tan(dx+c)^{5}}-\frac{-a-b}{3a^{2}\tan(dx+c)^{3}}-\frac{a^{2}+2ab+b^{2}}{a^{3}\tan(dx+c)}}{d}$
default	$\frac{\frac{\left(-a^{3}-3a^{2}b-3a\,b^{2}-b^{3}\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{a^{3}\sqrt{a(a+b)}}-\frac{1}{5a\tan(dx+c)^{5}}-\frac{-a-b}{3a^{2}\tan(dx+c)^{3}}-\frac{a^{2}+2ab+b^{2}}{a^{3}\tan(dx+c)}}{d}$
risch	$-\frac{2i\left(45a^{2}e^{8i(dx+c)}+45ab\,e^{8i(dx+c)}+15b^{2}e^{8i(dx+c)}-90a^{2}e^{6i(dx+c)}-150ab\,e^{6i(dx+c)}-60b^{2}e^{6i(dx+c)}+140a^{2}e^{4i(dx+c)}+\right.}{15d\,a^{3}\left(e^{2i(dx+c)}-\right.}$

input
```
int(cot(d*x+c)^6/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^3*(-a^3-3*a^2*b-3*a*b^2-b^3)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x
+c)/(a*(a+b))^(1/2))-1/5/a/tan(d*x+c)^5-1/3*(-a-b)/a^2/tan(d*x+c)^3-1/a^3*
(a^2+2*a*b+b^2)/tan(d*x+c))
```

3.455.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(84) = 168$.

Time = 0.35 (sec) , antiderivative size = 576, normalized size of antiderivative = 6.00

$$\int \frac{\cot^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

$$= \left[-\frac{4\left(23\,a^{2}+35\,ab+15\,b^{2}\right)\cos\left(dx+c\right)^{5}-20\left(7\,a^{2}+13\,ab+6\,b^{2}\right)\cos\left(dx+c\right)^{3}-15\left(\left(a^{2}+2\,ab+b^{2}\right)\cos\right.}{}\right.$$

$$-\frac{2\left(23\,a^{2}+35\,ab+15\,b^{2}\right)\cos\left(dx+c\right)^{5}-10\left(7\,a^{2}+13\,ab+6\,b^{2}\right)\cos\left(dx+c\right)^{3}-15\left(\left(a^{2}+2\,ab+b^{2}\right)\cos\right.}{30\left(a^{3}d\right.}$$

input
```
integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/60*(4*(23*a^2 + 35*a*b + 15*b^2)*cos(d*x + c)^5 - 20*(7*a^2 + 13*a*b +
 6*b^2)*cos(d*x + c)^3 - 15*((a^2 + 2*a*b + b^2)*cos(d*x + c)^4 - 2*(a^2 +
 2*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)*sqrt(-(a + b)/a)*log(((8
*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^
2 + 4*((2*a^2 + a*b)*cos(d*x + c)^3 - (a^2 + a*b)*cos(d*x + c))*sqrt(-(a +
 b)/a)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^
2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x + c) + 60*(a^2 + 2*a*b + b
^2)*cos(d*x + c))/((a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)
*sin(d*x + c)), -1/30*(2*(23*a^2 + 35*a*b + 15*b^2)*cos(d*x + c)^5 - 10*(7
*a^2 + 13*a*b + 6*b^2)*cos(d*x + c)^3 - 15*((a^2 + 2*a*b + b^2)*cos(d*x +
c)^4 - 2*(a^2 + 2*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)*sqrt((a +
 b)/a)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt((a + b)/a)/((a +
 b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) + 30*(a^2 + 2*a*b + b^2)*cos(
d*x + c))/((a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x
 + c))]
```

3.455.6 Sympy [F]

$$\int \frac{\cot^6(c+dx)}{a+b\sin^2(c+dx)}\,dx=\int \frac{\cot^6{(c+dx)}}{a+b\sin^2{(c+dx)}}\,dx$$

input
```
integrate(cot(d*x+c)**6/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(cot(c + d*x)**6/(a + b*sin(c + d*x)**2), x)
```

3.455.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.16

$$\int \frac{\cot^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{15\,(a^3+3\,a^2b+3\,ab^2+b^3)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a^3}+\frac{15\,(a^2+2\,ab+b^2)\tan(dx+c)^4-5\,(a^2+ab)\tan(dx+c)^2+3\,a^2}{a^3\tan(dx+c)^5}}{15\,d}$$

input
```
integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan((a + b)*tan(d*x + c)/sqrt
((a + b)*a))/(sqrt((a + b)*a)*a^3) + (15*(a^2 + 2*a*b + b^2)*tan(d*x + c)^
4 - 5*(a^2 + a*b)*tan(d*x + c)^2 + 3*a^2)/(a^3*tan(d*x + c)^5))/d
```

3.455.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(84) = 168$.

Time = 0.54 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.78

$$\int \frac{\cot^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{15\left(a^3+3a^2b+3ab^2+b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\sqrt{a^2+ab}a^3}+\frac{15a^2\tan(dx+c)^4+30ab\tan(dx+c)^4+15b^2\tan(}{a^3\tan}}{15d}$$

input
```
integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(pi*floor((d*x + c)/pi + 1/2)*sg
n(2*a + 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/
(sqrt(a^2 + a*b)*a^3) + (15*a^2*tan(d*x + c)^4 + 30*a*b*tan(d*x + c)^4 + 1
5*b^2*tan(d*x + c)^4 - 5*a^2*tan(d*x + c)^2 - 5*a*b*tan(d*x + c)^2 + 3*a^2
)/(a^3*tan(d*x + c)^5))/d
```

3.455.9 Mupad [B] (verification not implemented)

Time = 13.96 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{\cot^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\frac{1}{5a}-\frac{\tan(c+dx)^2(a+b)}{3a^2}+\frac{\tan(c+dx)^4(a+b)^2}{a^3}}{d\tan(c+dx)^5} - \frac{\operatorname{atan}\left(\frac{\tan(c+dx)\sqrt{a+b}}{\sqrt{a}}\right)(a+b)^{5/2}}{a^{7/2}d}$$

input
```
int(cot(c + d*x)^6/(a + b*sin(c + d*x)^2),x)
```

output
```
- (1/(5*a) - (tan(c + d*x)^2*(a + b))/(3*a^2) + (tan(c + d*x)^4*(a + b)^2)
/a^3)/(d*tan(c + d*x)^5) - (atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2))*(a
+ b)^(5/2))/(a^(7/2)*d)
```

3.456 $\int \frac{\cot^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

3.456.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{\cot^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{(a+b)^{7/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d}+\frac{(a+b)^3\cot(c+dx)}{a^4d} - \frac{(a+b)^2\cot^3(c+dx)}{3a^3d}+\frac{(a+b)\cot^5(c+dx)}{5a^2d}-\frac{\cot^7(c+dx)}{7ad}$$

output
```
(a+b)^(7/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(9/2)/d+(a+b)^3*cot(d
*x+c)/a^4/d-1/3*(a+b)^2*cot(d*x+c)^3/a^3/d+1/5*(a+b)*cot(d*x+c)^5/a^2/d-1/
7*cot(d*x+c)^7/a/d
```

3.456.2 Mathematica [A] (verified)

Time = 3.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.15

$$\int \frac{\cot^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{(a+b)^{7/2}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d} + \frac{\cot(c+dx)\left(176a^3+406a^2b+350ab^2+105b^3-a(122a^2+112ab+35b^2)\csc^2(c+dx)+3a^2(22a+7b\right.}{105a^4d}$$

input
```
Integrate[Cot[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

output
```
((a + b)^(7/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(9/2)*d) + (
Cot[c + d*x]*(176*a^3 + 406*a^2*b + 350*a*b^2 + 105*b^3 - a*(122*a^2 + 112
*a*b + 35*b^2)*Csc[c + d*x]^2 + 3*a^2*(22*a + 7*b)*Csc[c + d*x]^4 - 15*a^3
*Csc[c + d*x]^6))/(105*a^4*d)
```

3.456.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 3674, 264, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^8(c+dx)}{a+b\sin^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^8\left(a+b\sin(c+dx)^2\right)}dx$$

$$\Big\downarrow \text{3674}$$

$$\frac{\int \frac{\cot^8(c+dx)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{264}$$

$$\frac{-\frac{(a+b)\int \frac{\cot^6(c+dx)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{a}-\frac{\cot^7(c+dx)}{7a}}{d}$$

$$\Big\downarrow \text{264}$$

$$\frac{-\frac{(a+b)\left(-\frac{(a+b)\int \frac{\cot^4(c+dx)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{a}-\frac{\cot^5(c+dx)}{5a}\right)}{a}-\frac{\cot^7(c+dx)}{7a}}{d}$$

$$\Big\downarrow \text{264}$$

$$\frac{-\frac{(a+b)\left(-\frac{(a+b)\left(-\frac{(a+b)\int \frac{\cot^2(c+dx)}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{a}-\frac{\cot^3(c+dx)}{3a}\right)}{a}-\frac{\cot^5(c+dx)}{5a}\right)}{a}-\frac{\cot^7(c+dx)}{7a}}{d}$$

$$\downarrow \text{264}$$

$$\frac{-\frac{(a+b)\left(-\frac{(a+b)\left(-\frac{(a+b)\left(-\frac{(a+b)\int\frac{1}{(a+b)\tan^2(c+dx)+a}d\tan(c+dx)}{a}-\frac{\cot(c+dx)}{a}\right)}{a}-\frac{\cot^3(c+dx)}{3a}\right)}{a}-\frac{\cot^5(c+dx)}{5a}\right)}{a}-\frac{\cot^7(c+dx)}{7a}}{d}$$

$$\downarrow \text{218}$$

$$\frac{-\frac{(a+b)\left(-\frac{(a+b)\left(-\frac{(a+b)\left(-\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\cot(c+dx)}{a}\right)}{a}-\frac{\cot^3(c+dx)}{3a}\right)}{a}-\frac{\cot^5(c+dx)}{5a}\right)}{a}-\frac{\cot^7(c+dx)}{7a}}{d}$$

input
```
Int[Cot[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

output
```
(-1/7*Cot[c + d*x]^7/a - ((a + b)*(-1/5*Cot[c + d*x]^5/a - ((a + b)*(-1/3*
Cot[c + d*x]^3/a - ((a + b)*(-((Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[c + d*
x])/Sqrt[a]])/a^(3/2)) - Cot[c + d*x]/a))/a))/a))/a)/d
```

3.456.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

3.456. $\int \frac{\cot^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3674
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[f
f/f   Subst[Int[(d*ff*x)^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(p + 1)
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ
[p]
```

3.456.4 Maple [A] (verified)

Time = 10.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{-\frac{-a-b}{5a^2\tan(dx+c)^5}-\frac{-a^3-3a^2b-3a\,b^2-b^3}{a^4\tan(dx+c)}-\frac{1}{7a\tan(dx+c)^7}-\frac{a^2+2ab+b^2}{3a^3\tan(dx+c)^3}+\frac{\left(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4\right)\arctan\left(\frac{(a+b)\tan(dx}{\sqrt{a(a+b)}}\right.}{a^4\sqrt{a(a+b)}}}{d}$
default	$\frac{-\frac{-a-b}{5a^2\tan(dx+c)^5}-\frac{-a^3-3a^2b-3a\,b^2-b^3}{a^4\tan(dx+c)}-\frac{1}{7a\tan(dx+c)^7}-\frac{a^2+2ab+b^2}{3a^3\tan(dx+c)^3}+\frac{\left(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4\right)\arctan\left(\frac{(a+b)\tan(dx}{\sqrt{a(a+b)}}\right.}{a^4\sqrt{a(a+b)}}}{d}$
risch	$2i(406a^2b+350a\,b^2+105b^3+176a^3+420a\,b^2e^{12i(dx+c)}+5040a\,b^2e^{4i(dx+c)}-2212a^2b\,e^{2i(dx+c)}+630a^2b\,e^{12i(dx+c)}-2030$

input
```
int(cot(d*x+c)^8/(a+b*sin(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/5*(-a-b)/a^2/tan(d*x+c)^5-1/a^4*(-a^3-3*a^2*b-3*a*b^2-b^3)/tan(d*x
+c)-1/7/a/tan(d*x+c)^7-1/3/a^3*(a^2+2*a*b+b^2)/tan(d*x+c)^3+(a^4+4*a^3*b+6
*a^2*b^2+4*a*b^3+b^4)/a^4/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b)
)^(1/2)))
```

3.456.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(103) = 206$.

Time = 0.34 (sec) , antiderivative size = 834, normalized size of antiderivative = 7.13

$$\int \frac{\cot^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$$
$$= \left[\frac{4\,(176\,a^3+406\,a^2b+350\,ab^2+105\,b^3)\cos(dx+c)^7-28\,(58\,a^3+158\,a^2b+145\,ab^2+45\,b^3)\cos(dx+c}{}\right.$$

input
```
integrate(cot(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/420*(4*(176*a^3 + 406*a^2*b + 350*a*b^2 + 105*b^3)*cos(d*x + c)^7 - 28*
(58*a^3 + 158*a^2*b + 145*a*b^2 + 45*b^3)*cos(d*x + c)^5 + 140*(10*a^3 + 2
9*a^2*b + 28*a*b^2 + 9*b^3)*cos(d*x + c)^3 + 105*((a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cos(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c)^4
- a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*
x + c)^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(
4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + a*b)*cos(d*x + c)^3 - (a
^2 + a*b)*cos(d*x + c))*sqrt(-(a + b)/a)*sin(d*x + c) + a^2 + 2*a*b + b^2)
/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*
sin(d*x + c) - 420*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c))/((a^4*d*c
os(d*x + c)^6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*s
in(d*x + c)), 1/210*(2*(176*a^3 + 406*a^2*b + 350*a*b^2 + 105*b^3)*cos(d*x
 + c)^7 - 14*(58*a^3 + 158*a^2*b + 145*a*b^2 + 45*b^3)*cos(d*x + c)^5 + 70
*(10*a^3 + 29*a^2*b + 28*a*b^2 + 9*b^3)*cos(d*x + c)^3 - 105*((a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cos(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
(d*x + c)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*cos(d*x + c)^2)*sqrt((a + b)/a)*arctan(1/2*((2*a + b)*cos(d*x + c)^2
 - a - b)*sqrt((a + b)/a)/((a + b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c
) - 210*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c))/((a^4*d*cos(d*x + c)
^6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x +...
```

3.456.6 Sympy [F]

$$\int \frac{\cot^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \int \frac{\cot^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**8/(a+b*sin(d*x+c)**2),x)
```

output
```
Integral(cot(c + d*x)**8/(a + b*sin(c + d*x)**2), x)
```

3.456.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.32

$$\int \frac{\cot^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{105\,(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a^4} + \frac{105\,(a^3+3\,a^2b+3\,ab^2+b^3)\tan(dx+c)^6-35\,(a^3+2\,a^2b+ab^2)\tan(dx+c)^4-15\,a^3+}{a^4\tan(dx+c)^7}}{105\,d}$$

input
```
integrate(cot(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/105*(105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*arctan((a + b)*tan(
d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a^4) + (105*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*tan(d*x + c)^6 - 35*(a^3 + 2*a^2*b + a*b^2)*tan(d*x + c)^4 -
15*a^3 + 21*(a^3 + a^2*b)*tan(d*x + c)^2)/(a^4*tan(d*x + c)^7))/d
```

3.456.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(103) = 206.

Time = 0.59 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.03

$$\int \frac{\cot^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\frac{105\,(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\sqrt{a^2+ab}a^4} + \frac{105\,a^3\tan(dx+c)^6+315\,a^2b\tan(dx+c)^6+}{}}{10}$$

input
```
integrate(cot(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

output
```
1/105*(105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(pi*floor((d*x + c)
/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(
a^2 + a*b)))/(sqrt(a^2 + a*b)*a^4) + (105*a^3*tan(d*x + c)^6 + 315*a^2*b*t
an(d*x + c)^6 + 315*a*b^2*tan(d*x + c)^6 + 105*b^3*tan(d*x + c)^6 - 35*a^3
*tan(d*x + c)^4 - 70*a^2*b*tan(d*x + c)^4 - 35*a*b^2*tan(d*x + c)^4 + 21*a
^3*tan(d*x + c)^2 + 21*a^2*b*tan(d*x + c)^2 - 15*a^3)/(a^4*tan(d*x + c)^7)
)/d
```

3.456.9 Mupad [B] (verification not implemented)

Time = 16.78 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

$$\int \frac{\cot^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)\,(a+b)^{7/2}}{a^{9/2}\,d} - \frac{\frac{1}{7\,a}-\frac{\tan(c+d\,x)^2\,(a+b)}{5\,a^2}+\frac{\tan(c+d\,x)^4\,(a+b)^2}{3\,a^3}-\frac{\tan(c+d\,x)^6\,(a+b)^3}{a^4}}{d\tan\left(c+d\,x\right)^7}$$

input
```
int(cot(c + d*x)^8/(a + b*sin(c + d*x)^2),x)
```

output
```
(atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2))*(a + b)^(7/2))/(a^(9/2)*d) - (
1/(7*a) - (tan(c + d*x)^2*(a + b))/(5*a^2) + (tan(c + d*x)^4*(a + b)^2)/(3
*a^3) - (tan(c + d*x)^6*(a + b)^3)/a^4)/(d*tan(c + d*x)^7)
```

3.457 $\int \sqrt{a-a\sin^2(e+fx)}\tan^5(e+fx)\,dx$

3.457.1 Optimal result

Integrand size = 26, antiderivative size = 64

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^5(e+fx)\,dx = \frac{a^2}{3f\left(a\cos^2(e+fx)\right)^{3/2}} - \frac{2a}{f\sqrt{a\cos^2(e+fx)}} - \frac{\sqrt{a\cos^2(e+fx)}}{f}$$

output
```
1/3*a^2/f/(a*cos(f*x+e)^2)^(3/2)-2*a/f/(a*cos(f*x+e)^2)^(1/2)-(a*cos(f*x+e
)^2)^(1/2)/f
```

3.457.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.66

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^5(e+fx)\,dx = \frac{a(-6-3\cos^2(e+fx)+\sec^2(e+fx))}{3f\sqrt{a\cos^2(e+fx)}}$$

input
```
Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^5,x]
```

output
```
(a*(-6 - 3*Cos[e + f*x]^2 + Sec[e + f*x]^2))/(3*f*Sqrt[a*Cos[e + f*x]^2])
```

3.457.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3655, 3042, 25, 3684, 8, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^5\sqrt{a-a\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \tan^5(e+fx)\sqrt{a\cos^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}}{\tan\left(e+fx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sqrt{a\sin\left(\frac{1}{2}(2e+\pi)+fx\right)^2}}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)^5}dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \sqrt{a\cos^2(e+fx)}\left(1-\cos^2(e+fx)\right)^2\sec^6(e+fx)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{8}$$

$$-\frac{a^3\int \frac{\left(1-\cos^2(e+fx)\right)^2}{\left(a\cos^2(e+fx)\right)^{5/2}}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{53}$$

$$-\frac{a^3\int \left(\frac{1}{a^2\sqrt{a\cos^2(e+fx)}}-\frac{2}{a\left(a\cos^2(e+fx)\right)^{3/2}}+\frac{1}{\left(a\cos^2(e+fx)\right)^{5/2}}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\left(\frac{2\sqrt{a\cos^2(e+fx)}}{a^3}+\frac{4}{a^2\sqrt{a\cos^2(e+fx)}}-\frac{2}{3a(a\cos^2(e+fx))^{3/2}}\right)}{2f}$$

input
```
Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^5,x]
```

output
```
-1/2*(a^3*(-2/(3*a*(a*Cos[e + f*x]^2)^(3/2)) + 4/(a^2*Sqrt[a*Cos[e + f*x]^
2]) + (2*Sqrt[a*Cos[e + f*x]^2])/a^3))/f
```

3.457.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.457.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{a\left(3\left(\cos^{4}\left(fx+e\right)\right)+6\left(\cos^{2}\left(fx+e\right)\right)-1\right)}{3\cos\left(fx+e\right)^{2}\sqrt{a\left(\cos^{2}\left(fx+e\right)\right)}\, f}$	49
risch	$-\frac{\sqrt{\left(e^{2i(fx+e)}+1\right)^{2} a\, e^{-2i(fx+e)}}\,\left(3\,e^{8i(fx+e)}+36\,e^{6i(fx+e)}+50\,e^{4i(fx+e)}+36\,e^{2i(fx+e)}+3\right)}{6f\left(e^{2i(fx+e)}+1\right)^{4}}$	91

input
```
int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/3/cos(f*x+e)^2*a*(3*cos(f*x+e)^4+6*cos(f*x+e)^2-1)/(a*cos(f*x+e)^2)^(1/
2)/f
```

3.457.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^5(e+fx)\,dx = -\frac{\left(3\,\cos\left(fx+e\right)^{4}+6\,\cos\left(fx+e\right)^{2}-1\right)\sqrt{a\cos\left(fx+e\right)^{2}}}{3\,f\,\cos\left(fx+e\right)^{4}}$$

input
```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
-1/3*(3*cos(f*x + e)^4 + 6*cos(f*x + e)^2 - 1)*sqrt(a*cos(f*x + e)^2)/(f*c
os(f*x + e)^4)
```

3.457.6 Sympy [F]

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^5(e+fx)\,dx = \int \sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\tan^{5}\left(e+fx\right)dx$$

input
```
integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**5,x)
```

output
```
Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x)**5, x
)
```

3.457.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^5(e+fx)\,dx=-\frac{3\sqrt{-a\sin(fx+e)^2+a}\,a^3-\frac{6\left(a\sin(fx+e)^2-a\right)a^4+a^5}{\left(-a\sin(fx+e)^2+a\right)^{\frac{3}{2}}}}{3\,a^3f}$$

input
```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(-a*sin(f*x + e)^2 + a)*a^3 - (6*(a*sin(f*x + e)^2 - a)*a^4 +
a^5)/(-a*sin(f*x + e)^2 + a)^(3/2))/(a^3*f)
```

3.457.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(56) = 112.

Time = 2.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.00

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^5(e+fx)\,dx$$
$$=\frac{2\sqrt{a}\left(\frac{3\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-1\right)}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1}-\frac{3\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-12\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+5\,\mathrm{sgn}\left(\tan\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3}\right.}{3\,f}$$

input
```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
2/3*sqrt(a)*(3*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)/(tan(1/2*f*x + 1/2*e)^2 + 1
) - (3*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^4 - 12*sgn(tan
(1/2*f*x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^2 + 5*sgn(tan(1/2*f*x + 1/2*
e)^4 - 1))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)/f
```

3.457.9 Mupad [B] (verification not implemented)

Time = 17.39 (sec) , antiderivative size = 326, normalized size of antiderivative = 5.09

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^5(e+fx)\,dx = -\frac{\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{f} - \frac{8\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{f\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)} + \frac{16\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{3\,f\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)} - \frac{16\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{3\,f\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^3\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}$$

input
```
int(tan(e + f*x)^5*(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
(16*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f
*x*1i)*1i)/2)^2)^(1/2))/(3*f*(exp(e*2i + f*x*2i) + 1)^2*(exp(e*1i + f*x*1i
) + exp(e*3i + f*x*3i))) - (8*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x
*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(f*(exp(e*2i + f*x*2i) +
 1)*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) - (a - a*((exp(- e*1i - f*x
*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)/f - (16*exp(e*3i + f*x*3i
)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2
))/(3*f*(exp(e*2i + f*x*2i) + 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i
)))
```

3.458 $\int \sqrt{a-a\sin^2(e+fx)}\tan^3(e+fx)\,dx$

3.458.1 Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^3(e+fx)\,dx=\frac{a}{f\sqrt{a\cos^2(e+fx)}}+\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

output
```
a/f/(a*cos(f*x+e)^2)^(1/2)+(a*cos(f*x+e)^2)^(1/2)/f
```

3.458.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^3(e+fx)\,dx=\frac{a(1+\cos^2(e+fx))}{f\sqrt{a\cos^2(e+fx)}}$$

input
```
Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^3,x]
```

output
```
(a*(1 + Cos[e + f*x]^2))/(f*Sqrt[a*Cos[e + f*x]^2])
```

3.458.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3655, 3042, 25, 3684, 8, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^3\sqrt{a-a\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \tan^3(e+fx)\sqrt{a\cos^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}}{\tan\left(e+fx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sqrt{a\sin\left(\frac{1}{2}(2e+\pi)+fx\right)^2}}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3}dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \sqrt{a\cos^2(e+fx)}(1-\cos^2(e+fx))\sec^4(e+fx)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{8}$$

$$-\frac{a^2\int \frac{1-\cos^2(e+fx)}{(a\cos^2(e+fx))^{3/2}}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{53}$$

$$-\frac{a^2\int \left(\frac{1}{(a\cos^2(e+fx))^{3/2}}-\frac{1}{a\sqrt{a\cos^2(e+fx)}}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\left(-\frac{2\sqrt{a\cos^2(e+fx)}}{a^2}-\frac{2}{a\sqrt{a\cos^2(e+fx)}}\right)}{2f}$$

input
```
Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^3,x]
```

output
```
-1/2*(a^2*(-2/(a*Sqrt[a*Cos[e + f*x]^2]) - (2*Sqrt[a*Cos[e + f*x]^2])/a^2)
)/f
```

3.458.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.458. $\int \sqrt{a-a\sin^2(e+fx)}\tan^3(e+fx)\,dx$

3.458.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{a(\cos^2(fx+e)+1)}{\sqrt{a(\cos^2(fx+e))}\,f}$	28
risch	$\frac{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,\left(e^{4i(fx+e)}+6\,e^{2i(fx+e)}+1\right)}{2f\left(e^{2i(fx+e)}+1\right)^2}$	67

input
```
int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(cos(f*x+e)^2+1)/(a*cos(f*x+e)^2)^(1/2)/f
```

3.458.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^3(e+fx)\,dx = \frac{\sqrt{a\cos\left(fx+e\right)^2}\left(\cos\left(fx+e\right)^2+1\right)}{f\cos\left(fx+e\right)^2}$$

input
```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
sqrt(a*cos(f*x + e)^2)*(cos(f*x + e)^2 + 1)/(f*cos(f*x + e)^2)
```

3.458.6 Sympy [F]

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^3(e+fx)\,dx = \int \sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\tan^3\left(e+fx\right)dx$$

input
```
integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**3,x)
```

output
```
Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x)**3, x
)
```

3.458.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^3(e+fx)\,dx=\frac{\sqrt{-a\sin\left(fx+e\right)^2+a}a^2+\frac{a^3}{\sqrt{-a\sin(fx+e)^2+a}}}{a^2f}$$

input
```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
(sqrt(-a*sin(f*x + e)^2 + a)*a^2 + a^3/sqrt(-a*sin(f*x + e)^2 + a))/(a^2*f
)
```

3.458.8 Giac [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^3(e+fx)\,dx=\frac{4\sqrt{a}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)f}$$

input
```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
4*sqrt(a)*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)/((tan(1/2*f*x + 1/2*e)^4 - 1)*f)
```

3.458.9 Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.82

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^3(e+fx)\,dx$$
$$=\frac{\sqrt{2}\sqrt{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}\left(8\,\cos\left(2\,e+2\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)+7\right)}{2\,f\,\left(4\,\cos\left(2\,e+2\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)+3\right)}$$

input
```
int(tan(e + f*x)^3*(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
(2^(1/2)*(a*(cos(2*e + 2*f*x) + 1))^(1/2)*(8*cos(2*e + 2*f*x) + cos(4*e +
4*f*x) + 7))/(2*f*(4*cos(2*e + 2*f*x) + cos(4*e + 4*f*x) + 3))
```

3.459 $\int \sqrt{a-a\sin^2(e+fx)}\tan(e+fx)\,dx$

3.459.1 Optimal result

Integrand size = 24, antiderivative size = 19

$$\int \sqrt{a-a\sin^2(e+fx)}\tan(e+fx)\,dx=-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

output
```
-(a*cos(f*x+e)^2)^(1/2)/f
```

3.459.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sqrt{a-a\sin^2(e+fx)}\tan(e+fx)\,dx=-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

input
```
Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x],x]
```

output
```
-(Sqrt[a*Cos[e + f*x]^2]/f)
```

3.459.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 3655, 3042, 25, 3684, 8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)\sqrt{a-a\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \tan(e+fx)\sqrt{a\cos^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}}{\tan\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sqrt{a\sin\left(\frac{1}{2}(2e+\pi)+fx\right)^2}}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \sqrt{a\cos^2(e+fx)}\sec^2(e+fx)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{8}$$

$$-\frac{a\int \frac{1}{\sqrt{a\cos^2(e+fx)}}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{17}$$

$$-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

input
```
Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x],x]
```

output
```
-(Sqrt[a*Cos[e + f*x]^2]/f)
```

3.459.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.459.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$-\frac{\sqrt{a-a\left(\sin^2(fx+e)\right)}}{f}$	21
default	$-\frac{\sqrt{a-a\left(\sin^2(fx+e)\right)}}{f}$	21
risch	$-\frac{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\, e^{-2i(fx+e)}}\, e^{2i(fx+e)}}{2f\left(e^{2i(fx+e)}+1\right)}-\frac{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\, e^{-2i(fx+e)}}}{2f\left(e^{2i(fx+e)}+1\right)}$	99

input
```
int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
-1/f*(a-a*sin(f*x+e)^2)^(1/2)
```

3.459.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \sqrt{a-a\sin^2(e+fx)}\tan(e+fx)\,dx=-\frac{\sqrt{a\cos\left(fx+e\right)^2}}{f}$$

input
```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

output
```
-sqrt(a*cos(f*x + e)^2)/f
```

3.459.6 Sympy [F]

$$\int \sqrt{a-a\sin^2(e+fx)}\tan(e+fx)\,dx$$
$$=\int \sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\tan\left(e+fx\right)dx$$

input
```
integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e),x)
```

output
```
Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x), x)
```

3.459.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \sqrt{a-a\sin^2(e+fx)}\tan(e+fx)\,dx = -\frac{\sqrt{-a\sin\left(fx+e\right)^2+a}}{f}$$

input
```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

output
```
-sqrt(-a*sin(f*x + e)^2 + a)/f
```

3.459.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.37 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \sqrt{a-a\sin^2(e+fx)}\tan(e+fx)\,dx = \frac{2\sqrt{a}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)f}$$

input
```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")
```

output
```
2*sqrt(a)*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)/((tan(1/2*f*x + 1/2*e)^2 + 1)*f)
```

3.459.9 Mupad [B] (verification not implemented)

Time = 14.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \sqrt{a-a\sin^2(e+fx)}\tan(e+fx)\,dx = -\frac{\sqrt{a-a\sin\left(e+f\,x\right)^2}}{f}$$

input
```
int(tan(e + f*x)*(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
-(a - a*sin(e + f*x)^2)^(1/2)/f
```

3.460 $\int \cot(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

3.460.1 Optimal result

Integrand size = 24, antiderivative size = 50

$$\int \cot(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = -\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

output
```
-arctanh((a*cos(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+(a*cos(f*x+e)^2)^(1/2)/
f
```

3.460.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \cot(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \frac{-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a\cos^2(e+fx)}}{f}$$

input
```
Integrate[Cot[e + f*x]*Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
(-(Sqrt[a]*ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]) + Sqrt[a*Cos[e + f*x]^
2])/f
```

3.460.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3655, 3042, 25, 3684, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a-a\sin(e+fx)^2}}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \cot(e+fx)\sqrt{a\cos^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(e+fx+\frac{\pi}{2}\right)\left(-\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sqrt{a\sin\left(\frac{1}{2}(2e+\pi)+fx\right)^2}\tan\left(\frac{1}{2}(2e+\pi)+fx\right)dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \frac{\sqrt{a\cos^2(e+fx)}}{1-\cos^2(e+fx)}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{60}$$

$$-\frac{a\int \frac{1}{\sqrt{a\cos^2(e+fx)}(1-\cos^2(e+fx))}d\cos^2(e+fx)-2\sqrt{a\cos^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2\int \frac{1}{1-\frac{\cos^4(e+fx)}{a}}d\sqrt{a\cos^2(e+fx)}-2\sqrt{a\cos^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)-2\sqrt{a\cos^2(e+fx)}}{2f}$$

input
```
Int[Cot[e + f*x]*Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
-1/2*(2*Sqrt[a]*ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]] - 2*Sqrt[a*Cos[e +
 f*x]^2])/f
```

3.460.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.460.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

method	result
default	$\frac{a\cos(fx+e)(2\cos(fx+e)+\ln(\cos(fx+e)-1)-\ln(1+\cos(fx+e)))}{2\sqrt{a(\cos^2(fx+e))}\,f}$
risch	$\frac{\sqrt{(\mathrm{e}^{2i(fx+e)}+1)^2 a\,\mathrm{e}^{-2i(fx+e)}}\,\mathrm{e}^{2i(fx+e)}}{2f(\mathrm{e}^{2i(fx+e)}+1)}+\frac{\sqrt{(\mathrm{e}^{2i(fx+e)}+1)^2 a\,\mathrm{e}^{-2i(fx+e)}}}{2f(\mathrm{e}^{2i(fx+e)}+1)}-\frac{\ln(\mathrm{e}^{ifx}+\mathrm{e}^{-ie})\sqrt{(\mathrm{e}^{2i(fx+e)}+1)^2 a\,\mathrm{e}^{-2i(fx+e)}}\,\mathrm{e}^{i(fx}}{f(\mathrm{e}^{2i(fx+e)}+1)}$

input
```
int(cot(f*x+e)*(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*cos(f*x+e)*(2*cos(f*x+e)+ln(cos(f*x+e)-1)-ln(1+cos(f*x+e)))/(a*cos(f
*x+e)^2)^(1/2)/f
```

3.460.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int \cot(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \frac{\sqrt{a\cos{(fx+e)}^2}\left(2\,\cos{(fx+e)}-\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}\right)\right)}{2\,f\cos{(fx+e)}}$$

input
```
integrate(cot(f*x+e)*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(a*cos(f*x + e)^2)*(2*cos(f*x + e) - log(-(cos(f*x + e) + 1)/(cos(
f*x + e) - 1)))/(f*cos(f*x + e))
```

3.460.6 Sympy [F]

$$\int \cot(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$
$$=\int \sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\cot\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)*(a-a*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x), x)
```

3.460.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

$$\int \cot(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$
$$=-\frac{\sqrt{a}\log\left(\frac{2\sqrt{-a\sin(fx+e)^2+a}\sqrt{a}}{|\sin(fx+e)|}+\frac{2a}{|\sin(fx+e)|}\right)-\sqrt{-a\sin\left(fx+e\right)^2+a}}{f}$$

input
```
integrate(cot(f*x+e)*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(a)*log(2*sqrt(-a*sin(f*x + e)^2 + a)*sqrt(a)/abs(sin(f*x + e)) + 2*
a/abs(sin(f*x + e))) - sqrt(-a*sin(f*x + e)^2 + a))/f
```

3.460.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. 2(42) = 84.

Time = 0.56 (sec) , antiderivative size = 524, normalized size of antiderivative = 10.48

$$\int \cot(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-(sqrt(a)*log(abs(tan(1/2*f*x)*tan(1/2*e) - 1))*sgn(tan(1/2*f*x)^4*tan(1/2
*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3*
tan(1/2*e) - 4*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*t
an(1/2*e) + 1) - sqrt(a)*log(abs(tan(1/2*f*x) + tan(1/2*e)))*sgn(tan(1/2*f
*x)^4*tan(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*ta
n(1/2*f*x)^3*tan(1/2*e) - 4*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*t
an(1/2*f*x)*tan(1/2*e) + 1) + 2*(2*sqrt(a)*sgn(tan(1/2*f*x)^4*tan(1/2*e)^4
 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3*tan(1
/2*e) - 4*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*tan(1/
2*e) + 1)*tan(1/2*f*x)*tan(1/2*e) + sqrt(a)*sgn(tan(1/2*f*x)^4*tan(1/2*e)^
4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3*tan(
1/2*e) - 4*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*tan(1
/2*e) + 1)*tan(1/2*e)^2 - sqrt(a)*sgn(tan(1/2*f*x)^4*tan(1/2*e)^4 - 4*tan(
1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e) - 4
*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*tan(1/2*e) + 1)
)/((tan(1/2*f*x)^2 + 1)*(tan(1/2*e)^2 + 1)))/f
```

3.460.9 Mupad [F(-1)]

Timed out.

$$\int \cot(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \int \cot(e+f\,x)\,\sqrt{a-a\sin\left(e+f\,x\right)^2}\,dx$$

input
```
int(cot(e + f*x)*(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)*(a - a*sin(e + f*x)^2)^(1/2), x)
```

3.461 $\int \cot^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

3.461.1 Optimal result

Integrand size = 26, antiderivative size = 87

$$\int \cot^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \frac{3\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2f} - \frac{3\sqrt{a\cos^2(e+fx)}}{2f} - \frac{\left(a\cos^2(e+fx)\right)^{3/2}\csc^2(e+fx)}{2af}$$

output
```
-1/2*(a*cos(f*x+e)^2)^(3/2)*csc(f*x+e)^2/a/f+3/2*arctanh((a*cos(f*x+e)^2)^
(1/2)/a^(1/2))*a^(1/2)/f-3/2*(a*cos(f*x+e)^2)^(1/2)/f
```

3.461.2 Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.11

$$\int \cot^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$

$$= \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right) + \sqrt{a\cos^2(e+fx)}\left(\frac{\text{arctanh}\left(\sqrt{\cos^2(e+fx)}\right)}{\sqrt{\cos^2(e+fx)}} + (-2+\cos(2(e+fx)))\csc^2(e+f\right.}{2f}$$

input
```
Integrate[Cot[e + f*x]^3*Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
(2*Sqrt[a]*ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]] + Sqrt[a*Cos[e + f*x]^2
]*(ArcTanh[Sqrt[Cos[e + f*x]^2]]/Sqrt[Cos[e + f*x]^2] + (-2 + Cos[2*(e + f
*x)])*Csc[e + f*x]^2))/(2*f)
```

3.461.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3655, 3042, 25, 3684, 8, 51, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a-a\sin(e+fx)^2}}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \; 3655$$

$$\int \cot^3(e+fx)\sqrt{a\cos^2(e+fx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan\left(e+fx+\frac{\pi}{2}\right)^3\left(-\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}\right)dx$$

$$\Big\downarrow \; 25$$

$$-\int \sqrt{a\sin\left(\frac{1}{2}(2e+\pi)+fx\right)^2}\tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3dx$$

$$\Big\downarrow \; 3684$$

$$-\frac{\int \frac{\cos^2(e+fx)\sqrt{a\cos^2(e+fx)}}{(1-\cos^2(e+fx))^2}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \; 8$$

$$-\frac{\int \frac{(a\cos^2(e+fx))^{3/2}}{(1-\cos^2(e+fx))^2}d\cos^2(e+fx)}{2af}$$

$$\Big\downarrow \; 51$$

$$-\frac{\frac{(a\cos^2(e+fx))^{3/2}}{1-\cos^2(e+fx)}-\frac{3}{2}a\int\frac{\sqrt{a\cos^2(e+fx)}}{1-\cos^2(e+fx)}d\cos^2(e+fx)}{2af}$$

$$\Big\downarrow \textbf{60}$$

$$-\frac{\frac{(a\cos^2(e+fx))^{3/2}}{1-\cos^2(e+fx)}-\frac{3}{2}a\left(a\int\frac{1}{\sqrt{a\cos^2(e+fx)}(1-\cos^2(e+fx))}d\cos^2(e+fx)-2\sqrt{a\cos^2(e+fx)}\right)}{2af}$$

$$\Big\downarrow \textbf{73}$$

$$-\frac{\frac{(a\cos^2(e+fx))^{3/2}}{1-\cos^2(e+fx)}-\frac{3}{2}a\left(2\int\frac{1}{1-\frac{\cos^4(e+fx)}{a}}d\sqrt{a\cos^2(e+fx)}-2\sqrt{a\cos^2(e+fx)}\right)}{2af}$$

$$\Big\downarrow \textbf{219}$$

$$-\frac{\frac{(a\cos^2(e+fx))^{3/2}}{1-\cos^2(e+fx)}-\frac{3}{2}a\left(2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)-2\sqrt{a\cos^2(e+fx)}\right)}{2af}$$

input
```
Int[Cot[e + f*x]^3*Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
-1/2*((a*Cos[e + f*x]^2)^(3/2)/(1 - Cos[e + f*x]^2) - (3*a*(2*Sqrt[a]*ArcT
anh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]] - 2*Sqrt[a*Cos[e + f*x]^2]))/2)/(a*f)
```

3.461.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

3.461. $\int \cot^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.461. $\int \cot^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

3.461.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

method	result
default	$\frac{a\cos(fx+e)\left(4\cos(fx+e)\left(\sin^2(fx+e)\right)+2\cos(fx+e)+\left(-3\ln(1+\cos(fx+e))+3\ln(\cos(fx+e)-1)\right)\left(\sin^2(fx+e)\right)\right)}{4\sqrt{a\left(\cos^2(fx+e)\right)}\,(\cos(fx+e)-1)(1+\cos(fx+e))f}$
risch	$-\frac{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,\left(3\ln\left(e^{ifx}-e^{-ie}\right)e^{5i(fx+e)}-3\ln\left(e^{ifx}+e^{-ie}\right)e^{5i(fx+e)}+e^{6i(fx+e)}-6\ln\left(e^{ifx}-e^{-ie}\right)e^{3i(fx+e)}+6\ln\right.}{2f\left(e^{2i(fx+e)}+1\right)\left(e^{2i(fx+e)}-\right.}$

input
```
int(cot(f*x+e)^3*(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*cos(f*x+e)*(4*cos(f*x+e)*sin(f*x+e)^2+2*cos(f*x+e)+(-3*ln(1+cos(f*x+
e))+3*ln(cos(f*x+e)-1))*sin(f*x+e)^2)/(a*cos(f*x+e)^2)^(1/2)/(cos(f*x+e)-1
)/(1+cos(f*x+e))/f
```

3.461.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \cot^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = -\frac{\sqrt{a\cos(fx+e)^2}\left(4\cos(fx+e)^3+3\left(\cos(fx+e)^2-1\right)\log\left(-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-6\cos(fx+e)\right)}{4\left(f\cos(fx+e)^3-f\cos(fx+e)\right)}$$

input
```
integrate(cot(f*x+e)^3*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(a*cos(f*x + e)^2)*(4*cos(f*x + e)^3 + 3*(cos(f*x + e)^2 - 1)*log
(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 6*cos(f*x + e))/(f*cos(f*x + e)
^3 - f*cos(f*x + e))
```

3.461.6 Sympy [F]

$$\int \cot^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$
$$=\int \sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\cot^3\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**3*(a-a*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x)**3, x
)
```

3.461.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14

$$\int \cot^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$
$$=\frac{3\sqrt{a}\log\left(\frac{2\sqrt{-a\sin(fx+e)^2+a}\sqrt{a}}{|\sin(fx+e)|}+\frac{2a}{|\sin(fx+e)|}\right)-3\sqrt{-a\sin\left(fx+e\right)^2+a}-\frac{\left(-a\sin(fx+e)^2+a\right)^{\frac{3}{2}}}{a\sin(fx+e)^2}}{2f}$$

input
```
integrate(cot(f*x+e)^3*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(3*sqrt(a)*log(2*sqrt(-a*sin(f*x + e)^2 + a)*sqrt(a)/abs(sin(f*x + e))
 + 2*a/abs(sin(f*x + e))) - 3*sqrt(-a*sin(f*x + e)^2 + a) - (-a*sin(f*x +
e)^2 + a)^(3/2)/(a*sin(f*x + e)^2))/f
```

3.461.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1996 vs. $2(71) = 142$.

Time = 1.19 (sec) , antiderivative size = 1996, normalized size of antiderivative = 22.94

$$\int \cot^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx=\text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*(12*sqrt(a)*log(abs(tan(1/2*f*x)*tan(1/2*e) - 1))*sgn(tan(1/2*f*x)^4*t
an(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f
*x)^3*tan(1/2*e) - 4*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*
f*x)*tan(1/2*e) + 1) - 12*sqrt(a)*log(abs(tan(1/2*f*x) + tan(1/2*e)))*sgn(
tan(1/2*f*x)^4*tan(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)
^4 - 4*tan(1/2*f*x)^3*tan(1/2*e) - 4*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e
)^4 - 4*tan(1/2*f*x)*tan(1/2*e) + 1) + 16*(2*sqrt(a)*sgn(tan(1/2*f*x)^4*ta
n(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*
x)^3*tan(1/2*e) - 4*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f
*x)*tan(1/2*e) + 1)*tan(1/2*f*x)*tan(1/2*e) + sqrt(a)*sgn(tan(1/2*f*x)^4*t
an(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f
*x)^3*tan(1/2*e) - 4*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*
f*x)*tan(1/2*e) + 1)*tan(1/2*e)^2 - sqrt(a)*sgn(tan(1/2*f*x)^4*tan(1/2*e)^
4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3*tan(
1/2*e) - 4*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*tan(1
/2*e) + 1))/((tan(1/2*f*x)^2 + 1)*(tan(1/2*e)^2 + 1)) + (2*sqrt(a)*sgn(tan
(1/2*f*x)^4*tan(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4
- 4*tan(1/2*f*x)^3*tan(1/2*e) - 4*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4
 - 4*tan(1/2*f*x)*tan(1/2*e) + 1)*tan(1/2*f*x)^3*tan(1/2*e)^7 + sqrt(a)*sg
n(tan(1/2*f*x)^4*tan(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2...
```

3.461.9 Mupad [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \int \cot(e+f\,x)^3\,\sqrt{a-a\sin\left(e+f\,x\right)^2}\,dx$$

input
```
int(cot(e + f*x)^3*(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^3*(a - a*sin(e + f*x)^2)^(1/2), x)
```

3.462 $\int \sqrt{a-a\sin^2(e+fx)}\tan^6(e+fx)\,dx$

3.462.1 Optimal result

Integrand size = 26, antiderivative size = 120

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^6(e+fx)\,dx = \frac{15\text{arctanh}(\sin(e+fx))\sqrt{a\cos^2(e+fx)}\sec(e+fx)}{8f}-\frac{15\sqrt{a\cos^2(e+fx)}\tan(e+fx)}{8f}-\frac{5\sqrt{a\cos^2(e+fx)}\tan^3(e+fx)}{8f}+\frac{\sqrt{a\cos^2(e+fx)}\tan^5(e+fx)}{4f}$$

output
```
15/8*arctanh(sin(f*x+e))*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/f-15/8*(a*cos(f
*x+e)^2)^(1/2)*tan(f*x+e)/f-5/8*(a*cos(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f+1/4*
(a*cos(f*x+e)^2)^(1/2)*tan(f*x+e)^5/f
```

3.462.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.62

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^6(e+fx)\,dx = \frac{\sqrt{a\cos^2(e+fx)}\sec^5(e+fx)\left(60\text{arctanh}(\sin(e+fx))\cos^4(e+fx)-5\sin(e+fx)-15\sin(3(e+fx))\right.}{32f}$$

input
```
Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^6,x]
```

output
```
(Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x]^5*(60*ArcTanh[Sin[e + f*x]]*Cos[e + f
*x]^4 - 5*Sin[e + f*x] - 15*Sin[3*(e + f*x)] - 2*Sin[5*(e + f*x)]))/(32*f)
```

3.462.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3655, 3042, 3686, 3042, 3072, 252, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^6\sqrt{a-a\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \tan^6(e+fx)\sqrt{a\cos^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}}{\tan\left(e+fx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{3686}$$

$$\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \sin(e+fx)\tan^5(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \sin(e+fx)\tan(e+fx)^5dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \frac{\sin^6(e+fx)}{\left(1-\sin^2(e+fx)\right)^3}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{252}$$

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\left(\frac{\sin^5(e+fx)}{4(1-\sin^2(e+fx))^2}-\frac{5}{4}\int\frac{\sin^4(e+fx)}{(1-\sin^2(e+fx))^2}d\sin(e+fx)\right)}{f}$$

$\downarrow$ 252

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\left(\frac{\sin^5(e+fx)}{4(1-\sin^2(e+fx))^2}-\frac{5}{4}\left(\frac{\sin^3(e+fx)}{2(1-\sin^2(e+fx))}-\frac{3}{2}\int\frac{\sin^2(e+fx)}{1-\sin^2(e+fx)}d\sin(e+fx)\right)\right)}{f}$$

$\downarrow$ 262

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\left(\frac{\sin^5(e+fx)}{4(1-\sin^2(e+fx))^2}-\frac{5}{4}\left(\frac{\sin^3(e+fx)}{2(1-\sin^2(e+fx))}-\frac{3}{2}\left(\int\frac{1}{1-\sin^2(e+fx)}d\sin(e+fx)-\sin(e+fx)\right)\right.\right.}{f}$$

$\downarrow$ 219

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\left(\frac{\sin^5(e+fx)}{4(1-\sin^2(e+fx))^2}-\frac{5}{4}\left(\frac{\sin^3(e+fx)}{2(1-\sin^2(e+fx))}-\frac{3}{2}(\text{arctanh}(\sin(e+fx))-\sin(e+fx))\right)\right)}{f}$$

input
```
Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^6,x]
```

output
```
(Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x]*(Sin[e + f*x]^5/(4*(1 - Sin[e + f*x]^
2)^2) - (5*((-3*(ArcTanh[Sin[e + f*x]] - Sin[e + f*x]))/2 + Sin[e + f*x]^3
/(2*(1 - Sin[e + f*x]^2))))/4))/f
```

3.462.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

3.462. $\int\sqrt{a-a\sin^2(e+fx)}\tan^6(e+fx)\,dx$

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.462.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.31

method	result
default	$-\frac{\sqrt{a\left(\sin^2(fx+e)\right)}\left(8\sqrt{a\left(\sin^2(fx+e)\right)}\left(\cos^4(fx+e)\right)\sqrt{a}-15\ln\left(\frac{2\sqrt{a}\sqrt{a\left(\sin^2(fx+e)\right)}+2a}{\cos(fx+e)}\right)a\left(\cos^4(fx+e)\right)+9\sqrt{a\left(\sin^2(fx+e)\right)}\left(\mathrm{co}\cdots\right.\right.}{8\cos(fx+e)^3\sqrt{a}\,\sin(fx+e)\sqrt{a\left(\cos^2(fx+e)\right)}\,f}$
risch	$\frac{i\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,e^{2i(fx+e)}}{2f\left(e^{2i(fx+e)}+1\right)}-\frac{i\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}}{2\left(e^{2i(fx+e)}+1\right)f}+\frac{i\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\left(9\,e^{8i(fx+e)}+e^{6i(\cdots}\right.}{4f\left(e^{2i(fx+e)}+1\right)^5}$

3.462. $\int\sqrt{a-a\sin^2(e+fx)}\tan^6(e+fx)\,dx$

input
```
int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/8/cos(f*x+e)^3*(a*sin(f*x+e)^2)^(1/2)*(8*(a*sin(f*x+e)^2)^(1/2)*cos(f*x
+e)^4*a^(1/2)-15*ln(2/cos(f*x+e)*(a^(1/2)*(a*sin(f*x+e)^2)^(1/2)+a))*a*cos
(f*x+e)^4+9*(a*sin(f*x+e)^2)^(1/2)*cos(f*x+e)^2*a^(1/2)-2*a^(1/2)*(a*sin(f
*x+e)^2)^(1/2))/a^(1/2)/sin(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

3.462.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^6(e+fx)\,dx =$$
$$-\frac{\left(15\cos\left(fx+e\right)^4\log\left(-\frac{\sin(fx+e)-1}{\sin(fx+e)+1}\right)+2\left(8\cos\left(fx+e\right)^4+9\cos\left(fx+e\right)^2-2\right)\sin\left(fx+e\right)\right)\sqrt{a\cos(}}{16\,f\cos\left(fx+e\right)^5}$$

input
```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="fricas")
```

output
```
-1/16*(15*cos(f*x + e)^4*log(-(sin(f*x + e) - 1)/(sin(f*x + e) + 1)) + 2*(
8*cos(f*x + e)^4 + 9*cos(f*x + e)^2 - 2)*sin(f*x + e))*sqrt(a*cos(f*x + e)
^2)/(f*cos(f*x + e)^5)
```

3.462.6 Sympy [F]

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^6(e+fx)\,dx$$
$$=\int \sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\tan^6\left(e+fx\right)dx$$

input
```
integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**6,x)
```

output
```
Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x)**6, x
)
```

3.462.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1955 vs. 2(104) = 208.

Time = 1.13 (sec) , antiderivative size = 1955, normalized size of antiderivative = 16.29

$$\int \sqrt{a - a\sin^2(e + fx)}\tan^6(e + fx)\,dx = \text{Too large to display}$$

input
```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="maxima")
```

output
```
1/16*(8*(sin(9*f*x + 9*e) + 4*sin(7*f*x + 7*e) + 6*sin(5*f*x + 5*e) + 4*si
n(3*f*x + 3*e) + sin(f*x + e))*cos(10*f*x + 10*e) - 20*(3*sin(8*f*x + 8*e)
 + sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - 3*sin(2*f*x + 2*e))*cos(9*f*x + 9
*e) + 60*(4*sin(7*f*x + 7*e) + 6*sin(5*f*x + 5*e) + 4*sin(3*f*x + 3*e) + s
in(f*x + e))*cos(8*f*x + 8*e) - 80*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) -
3*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) + 20*(6*sin(5*f*x + 5*e) + 4*sin(3*f*
x + 3*e) + sin(f*x + e))*cos(6*f*x + 6*e) + 120*(sin(4*f*x + 4*e) + 3*sin(
2*f*x + 2*e))*cos(5*f*x + 5*e) - 20*(4*sin(3*f*x + 3*e) + sin(f*x + e))*co
s(4*f*x + 4*e) + 15*(2*(4*cos(7*f*x + 7*e) + 6*cos(5*f*x + 5*e) + 4*cos(3*
f*x + 3*e) + cos(f*x + e))*cos(9*f*x + 9*e) + cos(9*f*x + 9*e)^2 + 8*(6*co
s(5*f*x + 5*e) + 4*cos(3*f*x + 3*e) + cos(f*x + e))*cos(7*f*x + 7*e) + 16*
cos(7*f*x + 7*e)^2 + 12*(4*cos(3*f*x + 3*e) + cos(f*x + e))*cos(5*f*x + 5*
e) + 36*cos(5*f*x + 5*e)^2 + 16*cos(3*f*x + 3*e)^2 + 8*cos(3*f*x + 3*e)*co
s(f*x + e) + cos(f*x + e)^2 + 2*(4*sin(7*f*x + 7*e) + 6*sin(5*f*x + 5*e) +
 4*sin(3*f*x + 3*e) + sin(f*x + e))*sin(9*f*x + 9*e) + sin(9*f*x + 9*e)^2
+ 8*(6*sin(5*f*x + 5*e) + 4*sin(3*f*x + 3*e) + sin(f*x + e))*sin(7*f*x + 7
*e) + 16*sin(7*f*x + 7*e)^2 + 12*(4*sin(3*f*x + 3*e) + sin(f*x + e))*sin(5
*f*x + 5*e) + 36*sin(5*f*x + 5*e)^2 + 16*sin(3*f*x + 3*e)^2 + 8*sin(3*f*x
+ 3*e)*sin(f*x + e) + sin(f*x + e)^2)*log(cos(f*x + e)^2 + sin(f*x + e)^2
+ 2*sin(f*x + e) + 1) - 15*(2*(4*cos(7*f*x + 7*e) + 6*cos(5*f*x + 5*e) ...
```

3.462.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(104) = 208$.

Time = 2.31 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.95

$$\int \sqrt{a - a\sin^2(e + fx)}\tan^6(e + fx)\,dx =$$

$$-\frac{\left(15\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right|\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)-15\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\right.\right.\right.}{}$$

input
```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="giac")
```

output
```
-1/16*(15*log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e) + 2))*sgn(
tan(1/2*f*x + 1/2*e)^4 - 1) - 15*log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*
f*x + 1/2*e) - 2))*sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 32*sgn(tan(1/2*f*x +
1/2*e)^4 - 1)/(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e)) - 4*(7*(1/ta
n(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^3*sgn(tan(1/2*f*x + 1/2*e)^4 -
1) - 36*(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))*sgn(tan(1/2*f*x +
1/2*e)^4 - 1))/((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^2 - 4)^2)*
sqrt(a)/f
```

3.462.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a - a\sin^2(e + fx)}\tan^6(e + fx)\,dx = \int \tan(e + f\,x)^6\,\sqrt{a - a\sin(e + f\,x)^2}\,dx$$

input
```
int(tan(e + f*x)^6*(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^6*(a - a*sin(e + f*x)^2)^(1/2), x)
```

3.463 $\int \sqrt{a-a\sin^2(e+fx)}\tan^4(e+fx)\,dx$

3.463.1 Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^4(e+fx)\,dx = -\frac{3\text{arctanh}(\sin(e+fx))\sqrt{a\cos^2(e+fx)}\sec(e+fx)}{2f} + \frac{3\sqrt{a\cos^2(e+fx)}\tan(e+fx)}{2f} + \frac{\sqrt{a\cos^2(e+fx)}\tan^3(e+fx)}{2f}$$

output
```
-3/2*arctanh(sin(f*x+e))*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/f+3/2*(a*cos(f*
x+e)^2)^(1/2)*tan(f*x+e)/f+1/2*(a*cos(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f
```

3.463.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.60

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^4(e+fx)\,dx = \frac{a(-3\text{arctanh}(\sin(e+fx))\cos(e+fx)+(2+\cos(2(e+fx)))\tan(e+fx))}{2f\sqrt{a\cos^2(e+fx)}}$$

input
```
Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^4,x]
```

output
```
(a*(-3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x] + (2 + Cos[2*(e + f*x)])*Tan[e +
 f*x]))/(2*f*Sqrt[a*Cos[e + f*x]^2])
```

3.463.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.78, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 3655, 3042, 3686, 3042, 3072, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(e+fx)^4\sqrt{a-a\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{ 3655}$$

$$\int \tan^4(e+fx)\sqrt{a\cos^2(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}}{\tan\left(e+fx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{ 3686}$$

$$\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \sin(e+fx)\tan^3(e+fx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \sin(e+fx)\tan(e+fx)^3dx$$

$$\Big\downarrow \text{ 3072}$$

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \frac{\sin^4(e+fx)}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\left(\frac{\sin^3(e+fx)}{2\left(1-\sin^2(e+fx)\right)}-\frac{3}{2}\int \frac{\sin^2(e+fx)}{1-\sin^2(e+fx)}d\sin(e+fx)\right)}{f}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\left(\frac{\sin^3(e+fx)}{2\left(1-\sin^2(e+fx)\right)}-\frac{3}{2}\left(\int \frac{1}{1-\sin^2(e+fx)}d\sin(e+fx)-\sin(e+fx)\right)\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\left(\frac{\sin^3(e+fx)}{2(1-\sin^2(e+fx))}-\frac{3}{2}(\text{arctanh}(\sin(e+fx))-\sin(e+fx))\right)}{f}$$

input
```
Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^4,x]
```

output
```
(Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x]*((-3*(ArcTanh[Sin[e + f*x]] - Sin[e +
 f*x]))/2 + Sin[e + f*x]^3/(2*(1 - Sin[e + f*x]^2))))/f
```

3.463.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.463. $\int \sqrt{a-a\sin^2(e+fx)}\tan^4(e+fx)\,dx$

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.463.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.44

method	result
default	$\frac{\sqrt{a\left(\sin^2(fx+e)\right)}\left(2\sqrt{a\left(\sin^2(fx+e)\right)}\left(\cos^2(fx+e)\right)\sqrt{a}-3\ln\left(\frac{2\sqrt{a}\sqrt{a\left(\sin^2(fx+e)\right)}+2a}{\cos(fx+e)}\right)a\left(\cos^2(fx+e)\right)+\sqrt{a}\sqrt{a\left(\sin^2(fx+e)\right)}\right)}{2\cos(fx+e)\sqrt{a}\,\sin(fx+e)\sqrt{a\left(\cos^2(fx+e)\right)}\,f}$
risch	$-\frac{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\left(i e^{6i(fx+e)}+3\ln\left(e^{ifx}+i e^{-ie}\right)e^{5i(fx+e)}-3\ln\left(e^{ifx}-i e^{-ie}\right)e^{5i(fx+e)}+3i e^{4i(fx+e)}+6\ln\left(e^{ifx}+i e^{-i}\right.\right.}{2f\left(e^{2i(fx+e)}+1\right)^3}$

input
```
int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/2/cos(f*x+e)*(a*sin(f*x+e)^2)^(1/2)*(2*(a*sin(f*x+e)^2)^(1/2)*cos(f*x+e)
^2*a^(1/2)-3*ln(2/cos(f*x+e)*(a^(1/2)*(a*sin(f*x+e)^2)^(1/2)+a))*a*cos(f*x
+e)^2+a^(1/2)*(a*sin(f*x+e)^2)^(1/2))/a^(1/2)/sin(f*x+e)/(a*cos(f*x+e)^2)^
(1/2)/f
```

3.463.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.85

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^4(e+fx)\,dx = -\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(3\cos\left(fx+e\right)^2\log\left(-\frac{\sin(fx+e)+1}{\sin(fx+e)-1}\right)-2\left(2\cos\left(fx+e\right)^2+1\right)\sin\left(fx+e\right)\right)}{4f\cos\left(fx+e\right)^3}$$

input
```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
-1/4*sqrt(a*cos(f*x + e)^2)*(3*cos(f*x + e)^2*log(-(sin(f*x + e) + 1)/(sin
(f*x + e) - 1)) - 2*(2*cos(f*x + e)^2 + 1)*sin(f*x + e))/(f*cos(f*x + e)^3
)
```

3.463.6 Sympy [F]

$$\int \sqrt{a - a\sin^2(e + fx)}\tan^4(e + fx)\,dx$$
$$= \int \sqrt{-a\left(\sin\left(e + fx\right) - 1\right)\left(\sin\left(e + fx\right) + 1\right)}\tan^4\left(e + fx\right)dx$$

input
```
integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**4,x)
```

output
```
Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x)**4, x
)
```

3.463.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(79) = 158$.

Time = 0.42 (sec) , antiderivative size = 827, normalized size of antiderivative = 9.09

$$\int \sqrt{a - a\sin^2(e + fx)}\tan^4(e + fx)\,dx = \text{Too large to display}$$

input
```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
-1/4*(2*(sin(5*f*x + 5*e) + 2*sin(3*f*x + 3*e) + sin(f*x + e))*cos(6*f*x +
 6*e) - 6*(sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*cos(5*f*x + 5*e) + 6*(2*si
n(3*f*x + 3*e) + sin(f*x + e))*cos(4*f*x + 4*e) + 3*(2*(2*cos(3*f*x + 3*e)
 + cos(f*x + e))*cos(5*f*x + 5*e) + cos(5*f*x + 5*e)^2 + 4*cos(3*f*x + 3*e
)^2 + 4*cos(3*f*x + 3*e)*cos(f*x + e) + cos(f*x + e)^2 + 2*(2*sin(3*f*x +
3*e) + sin(f*x + e))*sin(5*f*x + 5*e) + sin(5*f*x + 5*e)^2 + 4*sin(3*f*x +
 3*e)^2 + 4*sin(3*f*x + 3*e)*sin(f*x + e) + sin(f*x + e)^2)*log(cos(f*x +
e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - 3*(2*(2*cos(3*f*x + 3*e) + c
os(f*x + e))*cos(5*f*x + 5*e) + cos(5*f*x + 5*e)^2 + 4*cos(3*f*x + 3*e)^2
+ 4*cos(3*f*x + 3*e)*cos(f*x + e) + cos(f*x + e)^2 + 2*(2*sin(3*f*x + 3*e)
 + sin(f*x + e))*sin(5*f*x + 5*e) + sin(5*f*x + 5*e)^2 + 4*sin(3*f*x + 3*e
)^2 + 4*sin(3*f*x + 3*e)*sin(f*x + e) + sin(f*x + e)^2)*log(cos(f*x + e)^2
 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) - 2*(cos(5*f*x + 5*e) + 2*cos(3*f*
x + 3*e) + cos(f*x + e))*sin(6*f*x + 6*e) + 2*(3*cos(4*f*x + 4*e) - 3*cos(
2*f*x + 2*e) - 1)*sin(5*f*x + 5*e) - 6*(2*cos(3*f*x + 3*e) + cos(f*x + e))
*sin(4*f*x + 4*e) - 4*(3*cos(2*f*x + 2*e) + 1)*sin(3*f*x + 3*e) + 12*cos(3
*f*x + 3*e)*sin(2*f*x + 2*e) + 6*cos(f*x + e)*sin(2*f*x + 2*e) - 6*cos(2*f
*x + 2*e)*sin(f*x + e) - 2*sin(f*x + e))*sqrt(a)/((2*(2*cos(3*f*x + 3*e) +
 cos(f*x + e))*cos(5*f*x + 5*e) + cos(5*f*x + 5*e)^2 + 4*cos(3*f*x + 3*e)^
2 + 4*cos(3*f*x + 3*e)*cos(f*x + e) + cos(f*x + e)^2 + 2*(2*sin(3*f*x +...
```

3.463.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(79) = 158$.

Time = 1.22 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.16

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^4(e+fx)\,dx$$

$$= \frac{\left(3\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right|\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)-3\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\right.\right.\right.}{}$$

input
```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
1/4*(3*log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e) + 2))*sgn(tan
(1/2*f*x + 1/2*e)^4 - 1) - 3*log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x
+ 1/2*e) - 2))*sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 4*(3*(1/tan(1/2*f*x + 1/2
*e) + tan(1/2*f*x + 1/2*e))^2*sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 8*sgn(tan(
1/2*f*x + 1/2*e)^4 - 1))/((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^
3 - 4/tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)))*sqrt(a)/f
```

3.463.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4\,\sqrt{a-a\sin\left(e+f\,x\right)^2}\,dx$$

input
```
int(tan(e + f*x)^4*(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^4*(a - a*sin(e + f*x)^2)^(1/2), x)
```

3.464 $\int \sqrt{a-a\sin^2(e+fx)}\tan^2(e+fx)\,dx$

3.464.1 Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^2(e+fx)\,dx = \frac{\text{arctanh}(\sin(e+fx))\sqrt{a\cos^2(e+fx)}\sec(e+fx)}{f} - \frac{\sqrt{a\cos^2(e+fx)}\tan(e+fx)}{f}$$

output
```
arctanh(sin(f*x+e))*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/f-(a*cos(f*x+e)^2)^(
1/2)*tan(f*x+e)/f
```

3.464.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.70

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^2(e+fx)\,dx = \frac{\sqrt{a\cos^2(e+fx)}\sec(e+fx)(\text{arctanh}(\sin(e+fx))-\sin(e+fx))}{f}$$

input
```
Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^2,x]
```

output
```
(Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x]*(ArcTanh[Sin[e + f*x]] - Sin[e + f*x]
))/f
```

3.464.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.70, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3655, 3042, 3686, 3042, 3072, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2\sqrt{a-a\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \tan^2(e+fx)\sqrt{a\cos^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}}{\tan\left(e+fx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3686}$$

$$\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \sin(e+fx)\tan(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \sin(e+fx)\tan(e+fx)dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \frac{\sin^2(e+fx)}{1-\sin^2(e+fx)}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{262}$$

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\left(\int \frac{1}{1-\sin^2(e+fx)}d\sin(e+fx)-\sin(e+fx)\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}(\text{arctanh}(\sin(e+fx))-\sin(e+fx))}{f}$$

3.464. $\int \sqrt{a-a\sin^2(e+fx)}\tan^2(e+fx)\,dx$

input
```
Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^2,x]
```

output
```
(Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x]*(ArcTanh[Sin[e + f*x]] - Sin[e + f*x]
))/f
```

3.464.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.464.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.68

method	result
default	$-\frac{\cos(fx+e)\sqrt{a\left(\sin^2(fx+e)\right)}\left(\sqrt{a}\,\sqrt{a\left(\sin^2(fx+e)\right)}-\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\sin^2(fx+e)\right)}+2a}{\cos(fx+e)}\right)a\right)}{\sqrt{a}\,\sin(fx+e)\sqrt{a\left(\cos^2(fx+e)\right)}\,f}$
risch	$\frac{i\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^2 a\,\mathrm{e}^{-2i(fx+e)}}\,\mathrm{e}^{2i(fx+e)}}{2f\left(\mathrm{e}^{2i(fx+e)}+1\right)}-\frac{i\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^2 a\,\mathrm{e}^{-2i(fx+e)}}}{2\left(\mathrm{e}^{2i(fx+e)}+1\right)f}-\frac{\ln\left(\mathrm{e}^{ifx}-i\mathrm{e}^{-ie}\right)\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^2 a\,\mathrm{e}^{-2i(fx+e)}}\,\mathrm{e}^{i(}}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)}$

input
```
int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-cos(f*x+e)*(a*sin(f*x+e)^2)^(1/2)*(a^(1/2)*(a*sin(f*x+e)^2)^(1/2)-ln(2/co
s(f*x+e)*(a^(1/2)*(a*sin(f*x+e)^2)^(1/2)+a))*a)/a^(1/2)/sin(f*x+e)/(a*cos(
f*x+e)^2)^(1/2)/f
```

3.464.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int\sqrt{a-a\sin^2(e+fx)}\tan^2(e+fx)\,dx = -\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(\log\left(-\frac{\sin(fx+e)-1}{\sin(fx+e)+1}\right)+2\,\sin\left(fx+e\right)\right)}{2\,f\cos\left(fx+e\right)}$$

input
```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
-1/2*sqrt(a*cos(f*x + e)^2)*(log(-(sin(f*x + e) - 1)/(sin(f*x + e) + 1)) +
 2*sin(f*x + e))/(f*cos(f*x + e))
```

3.464.6 Sympy [F]

$$\int \sqrt{a - a\sin^2(e + fx)}\tan^2(e + fx)\,dx$$
$$= \int \sqrt{-a(\sin(e + fx) - 1)(\sin(e + fx) + 1)}\tan^2(e + fx)\,dx$$

input
```
integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**2,x)
```

output
```
Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x)**2, x
)
```

3.464.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int \sqrt{a - a\sin^2(e + fx)}\tan^2(e + fx)\,dx$$
$$= \frac{\sqrt{a}\left(\log\left(\cos(fx + e)^2 + \sin(fx + e)^2 + 2\sin(fx + e) + 1\right) - \log\left(\cos(fx + e)^2 + \sin(fx + e)^2 - 2\sin(\right.\right.}{2f}$$

input
```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*(log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - l
og(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) - 2*sin(f*x + e))
/f
```

3.464.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(53) = 106$.

Time = 0.58 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.23

$$\int \sqrt{a - a\sin^2(e + fx)}\tan^2(e + fx)\,dx =$$
$$-\frac{\left(\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)} + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 2\right|\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 1\right) - \log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)} + \tan\left(\frac{1}{2}\right.\right.\right.}{2f}$$

input
```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
-1/2*(log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e) + 2))*sgn(tan(
1/2*f*x + 1/2*e)^4 - 1) - log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1
/2*e) - 2))*sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 4*sgn(tan(1/2*f*x + 1/2*e)^4
 - 1)/(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e)))*sqrt(a)/f
```

3.464.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2\,\sqrt{a-a\sin\left(e+f\,x\right)^2}\,dx$$

input
```
int(tan(e + f*x)^2*(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^2*(a - a*sin(e + f*x)^2)^(1/2), x)
```

3.465 $\int \cot^2(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

3.465.1 Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \cot^2(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = -\frac{\sqrt{a\cos^2(e+fx)}\csc(e+fx)\sec(e+fx)}{f} - \frac{\sqrt{a\cos^2(e+fx)}\tan(e+fx)}{f}$$

output
```
-csc(f*x+e)*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/f-(a*cos(f*x+e)^2)^(1/2)*tan
(f*x+e)/f
```

3.465.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int \cot^2(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = -\frac{\sqrt{a\cos^2(e+fx)}(1+\csc^2(e+fx))\tan(e+fx)}{f}$$

input
```
Integrate[Cot[e + f*x]^2*Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
-((Sqrt[a*Cos[e + f*x]^2]*(1 + Csc[e + f*x]^2)*Tan[e + f*x])/f)
```

3.465.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.67, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3655, 3042, 3686, 3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a-a\sin(e+fx)^2}}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \cot^2(e+fx)\sqrt{a\cos^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(e+fx+\frac{\pi}{2}\right)^2\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3686}$$

$$\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \cos(e+fx)\cot^2(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \sin\left(e+fx+\frac{\pi}{2}\right)\tan\left(e+fx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \csc^2(e+fx)\left(1-\sin^2(e+fx)\right)d(-\sin(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \left(\csc^2(e+fx)-1\right)d(-\sin(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}(\sin(e+fx)+\csc(e+fx))}{f}$$

input
```
Int[Cot[e + f*x]^2*Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
-((Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x]*(Csc[e + f*x] + Sin[e + f*x]))/f)
```

3.465.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.465.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\cos(fx+e)a(\cos^2(fx+e)-2)}{\sin(fx+e)\sqrt{a(\cos^2(fx+e))}\,f}$	42
risch	$\frac{i\sqrt{(e^{2i(fx+e)}+1)^2 a\,e^{-2i(fx+e)}}\,(e^{4i(fx+e)}-6\,e^{2i(fx+e)}+1)}{2(e^{2i(fx+e)}-1)f(e^{2i(fx+e)}+1)}$	81

input
```
int(cot(f*x+e)^2*(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
cos(f*x+e)*a*(cos(f*x+e)^2-2)/sin(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

3.465.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \cot^2(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \frac{\sqrt{a\cos{(fx+e)}^2}(\cos{(fx+e)}^2-2)}{f\cos{(fx+e)}\sin{(fx+e)}}$$

input
```
integrate(cot(f*x+e)^2*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*cos(f*x + e)^2)*(cos(f*x + e)^2 - 2)/(f*cos(f*x + e)*sin(f*x + e))
```

3.465.6 Sympy [F]

$$\int \cot^2(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$
$$= \int \sqrt{-a\left(\sin{(e+fx)}-1\right)\left(\sin{(e+fx)}+1\right)}\cot^2{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**2*(a-a*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x)**2, x
)
```

3.465.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \cot^2(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = -\frac{2\sqrt{a}\tan{(fx+e)}^2+\sqrt{a}}{\sqrt{\tan{(fx+e)}^2+1}f\tan{(fx+e)}}$$

input
```
integrate(cot(f*x+e)^2*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(2*sqrt(a)*tan(f*x + e)^2 + sqrt(a))/(sqrt(tan(f*x + e)^2 + 1)*f*tan(f*x
+ e))
```

3.465.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1491 vs. 2(53) = 106.

Time = 0.88 (sec) , antiderivative size = 1491, normalized size of antiderivative = 26.16

$$\int \cot^2(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^2*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(sqrt(a)*sgn(tan(1/2*f*x)^4*tan(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e
)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e) - 4*tan(1/2*f*x)*tan(1/
2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*tan(1/2*e) + 1)*tan(1/2*f*x)^3*tan(
1/2*e)^6 - 3*sqrt(a)*sgn(tan(1/2*f*x)^4*tan(1/2*e)^4 - 4*tan(1/2*f*x)^3*ta
n(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e) - 4*tan(1/2*f*x)
*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*tan(1/2*e) + 1)*tan(1/2*f*x)
^3*tan(1/2*e)^4 - 6*sqrt(a)*sgn(tan(1/2*f*x)^4*tan(1/2*e)^4 - 4*tan(1/2*f*
x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e) - 4*tan(1
/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*tan(1/2*e) + 1)*tan(1
/2*f*x)^2*tan(1/2*e)^5 + sqrt(a)*sgn(tan(1/2*f*x)^4*tan(1/2*e)^4 - 4*tan(1
/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e) - 4*
tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*tan(1/2*e) + 1)*
tan(1/2*f*x)*tan(1/2*e)^6 + 3*sqrt(a)*sgn(tan(1/2*f*x)^4*tan(1/2*e)^4 - 4*
tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)
 - 4*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*tan(1/2*e)
+ 1)*tan(1/2*f*x)^3*tan(1/2*e)^2 + 12*sqrt(a)*sgn(tan(1/2*f*x)^4*tan(1/2*e
)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3*ta
n(1/2*e) - 4*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*tan
(1/2*e) + 1)*tan(1/2*f*x)^2*tan(1/2*e)^3 + 13*sqrt(a)*sgn(tan(1/2*f*x)^4*t
an(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/...
```

3.465.9 Mupad [B] (verification not implemented)

Time = 16.81 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.54

$$\int \cot^2(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \frac{\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,\left(-e^{e\,2i+f\,x\,2i}\,6i+e^{e\,4i+f\,x\,4i}\,1i+1i\right)}{f\,\left(e^{e\,4i+f\,x\,4i}-1\right)}$$

input
```
int(cot(e + f*x)^2*(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
((a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)
*(exp(e*4i + f*x*4i)*1i - exp(e*2i + f*x*2i)*6i + 1i))/(f*(exp(e*4i + f*x*
4i) - 1))
```

3.466 $\int \cot^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

3.466.1 Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \cot^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \frac{2\sqrt{a\cos^2(e+fx)}\csc(e+fx)\sec(e+fx)}{f} - \frac{\sqrt{a\cos^2(e+fx)}\csc^3(e+fx)\sec(e+fx)}{3f} + \frac{\sqrt{a\cos^2(e+fx)}\tan(e+fx)}{f}$$

output
```
2*csc(f*x+e)*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/f-1/3*csc(f*x+e)^3*sec(f*x+
e)*(a*cos(f*x+e)^2)^(1/2)/f+(a*cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

3.466.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.52

$$\int \cot^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = -\frac{\sqrt{a\cos^2(e+fx)}(-3-6\csc^2(e+fx)+\csc^4(e+fx))\tan(e+fx)}{3f}$$

input
```
Integrate[Cot[e + f*x]^4*Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
-1/3*(Sqrt[a*Cos[e + f*x]^2]*(-3 - 6*Csc[e + f*x]^2 + Csc[e + f*x]^4)*Tan[
e + f*x])/f
```

3.466.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.59, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3655, 3042, 3686, 3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a-a\sin(e+fx)^2}}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \cot^4(e+fx)\sqrt{a\cos^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(e+fx+\frac{\pi}{2}\right)^4\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3686}$$

$$\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \cos(e+fx)\cot^4(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \sin\left(e+fx+\frac{\pi}{2}\right)\tan\left(e+fx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \csc^4(e+fx)\left(1-\sin^2(e+fx)\right)^2d(-\sin(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \left(\csc^4(e+fx)-2\csc^2(e+fx)+1\right)d(-\sin(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\left(-\sin(e+fx)+\frac{1}{3}\csc^3(e+fx)-2\csc(e+fx)\right)}{f}$$

input
```
Int[Cot[e + f*x]^4*Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
-((Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x]*(-2*Csc[e + f*x] + Csc[e + f*x]^3/3
 - Sin[e + f*x]))/f)
```

3.466.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.466. $\int \cot^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

3.466.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\cos(fx+e)a\left(3\left(\sin^4(fx+e)\right)+6\left(\sin^2(fx+e)\right)-1\right)}{3(\cos(fx+e)-1)(1+\cos(fx+e))\sin(fx+e)\sqrt{a\left(\cos^2(fx+e)\right)}\,f}$	75
risch	$\frac{i\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^2 a\,\mathrm{e}^{-2i(fx+e)}}\left(-3\,\mathrm{e}^{8i(fx+e)}+36\,\mathrm{e}^{6i(fx+e)}-50\,\mathrm{e}^{4i(fx+e)}+36\,\mathrm{e}^{2i(fx+e)}-3\right)}{6\left(\mathrm{e}^{2i(fx+e)}-1\right)^3 f\left(\mathrm{e}^{2i(fx+e)}+1\right)}$	105

input
```
int(cot(f*x+e)^4*(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*cos(f*x+e)*a*(3*sin(f*x+e)^4+6*sin(f*x+e)^2-1)/(cos(f*x+e)-1)/(1+cos(
f*x+e))/sin(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

3.466.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \cot^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$
$$=-\frac{\left(3\cos\left(fx+e\right)^4-12\cos\left(fx+e\right)^2+8\right)\sqrt{a\cos\left(fx+e\right)^2}}{3\left(f\cos\left(fx+e\right)^3-f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

input
```
integrate(cot(f*x+e)^4*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(3*cos(f*x + e)^4 - 12*cos(f*x + e)^2 + 8)*sqrt(a*cos(f*x + e)^2)/((f
*cos(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))
```

3.466.6 Sympy [F]

$$\int \cot^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$
$$=\int \sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\cot^4\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**4*(a-a*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x)**4, x
)
```

3.466.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.63

$$\int \cot^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \frac{8\sqrt{a}\tan(fx+e)^4+4\sqrt{a}\tan(fx+e)^2-\sqrt{a}}{3\sqrt{\tan(fx+e)^2+1}f\tan(fx+e)^3}$$

input
```
integrate(cot(f*x+e)^4*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(8*sqrt(a)*tan(f*x + e)^4 + 4*sqrt(a)*tan(f*x + e)^2 - sqrt(a))/(sqrt(
tan(f*x + e)^2 + 1)*f*tan(f*x + e)^3)
```

3.466.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3443 vs. 2(83) = 166.

Time = 1.82 (sec) , antiderivative size = 3443, normalized size of antiderivative = 37.84

$$\int \cot^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^4*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/24*(48*(sqrt(a)*sgn(tan(1/2*f*x)^4*tan(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan(
1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e) - 4*tan(1/2*f*x)*t
an(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*tan(1/2*e) + 1)*tan(1/2*f*x)*t
an(1/2*e)^2 - sqrt(a)*sgn(tan(1/2*f*x)^4*tan(1/2*e)^4 - 4*tan(1/2*f*x)^3*t
an(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e) - 4*tan(1/2*f*x
)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*tan(1/2*e) + 1)*tan(1/2*f*x
) - 2*sqrt(a)*sgn(tan(1/2*f*x)^4*tan(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e
)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e) - 4*tan(1/2*f*x)*tan(1/
2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*tan(1/2*e) + 1)*tan(1/2*e))/((tan(1
/2*f*x)^2 + 1)*(tan(1/2*e)^2 + 1)) + (3*sqrt(a)*sgn(tan(1/2*f*x)^4*tan(1/2
*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3*
tan(1/2*e) - 4*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*t
an(1/2*e) + 1)*tan(1/2*f*x)^5*tan(1/2*e)^10 + 3*sqrt(a)*sgn(tan(1/2*f*x)^4
*tan(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2
*f*x)^3*tan(1/2*e) - 4*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/
2*f*x)*tan(1/2*e) + 1)*tan(1/2*f*x)^4*tan(1/2*e)^11 + sqrt(a)*sgn(tan(1/2*
f*x)^4*tan(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*t
an(1/2*f*x)^3*tan(1/2*e) - 4*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*
tan(1/2*f*x)*tan(1/2*e) + 1)*tan(1/2*f*x)^3*tan(1/2*e)^12 - 18*sqrt(a)*sgn
(tan(1/2*f*x)^4*tan(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*...
```

3.466.9 Mupad [B] (verification not implemented)

Time = 17.07 (sec) , antiderivative size = 364, normalized size of antiderivative = 4.00

$$\int \cot^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \frac{\left(\frac{1\mathrm{i}}{f}-\frac{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}}{f}\right)\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1} + \frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}\,8\mathrm{i}}{f\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)} + \frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}\,16\mathrm{i}}{3\,f\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)^2\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)} + \frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}\,16\mathrm{i}}{3\,f\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)^3\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}$$

input
```
int(cot(e + f*x)^4*(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
((1i/f - (exp(e*2i + f*x*2i)*1i)/f)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 -
(exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(exp(e*2i + f*x*2i) + 1) + (exp(e*3i
+ f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)
^2)^(1/2)*8i)/(f*(exp(e*2i + f*x*2i) - 1)*(exp(e*1i + f*x*1i) + exp(e*3i +
 f*x*3i))) + (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (ex
p(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*16i)/(3*f*(exp(e*2i + f*x*2i) - 1)^2*(exp
(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) + (exp(e*3i + f*x*3i)*(a - a*((exp(
- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*16i)/(3*f*(ex
p(e*2i + f*x*2i) - 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

3.466. $\int \cot^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

3.467 $\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

3.467.1 Optimal result

Integrand size = 26, antiderivative size = 124

$$\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = -\frac{3\sqrt{a\cos^2(e+fx)}\csc(e+fx)\sec(e+fx)}{f} + \frac{\sqrt{a\cos^2(e+fx)}\csc^3(e+fx)\sec(e+fx)}{f} - \frac{\sqrt{a\cos^2(e+fx)}\csc^5(e+fx)\sec(e+fx)}{5f} - \frac{\sqrt{a\cos^2(e+fx)}\tan(e+fx)}{f}$$

output
```
-3*csc(f*x+e)*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/f+csc(f*x+e)^3*sec(f*x+e)*
(a*cos(f*x+e)^2)^(1/2)/f-1/5*csc(f*x+e)^5*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2
)/f-(a*cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

3.467.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.54

$$\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \frac{\sqrt{a\cos^2(e+fx)}(-182+235\cos(2(e+fx))-90\cos(4(e+fx))+5\cos(6(e+fx)))\csc^5(e+fx)\sec(e-}{160f}$$

input
```
Integrate[Cot[e + f*x]^6*Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[a*Cos[e + f*x]^2]*(-182 + 235*Cos[2*(e + f*x)] - 90*Cos[4*(e + f*x)]
 + 5*Cos[6*(e + f*x)])*Csc[e + f*x]^5*Sec[e + f*x])/(160*f)
```

3.467.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.50, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3655, 3042, 3686, 3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a-a\sin(e+fx)^2}}{\tan(e+fx)^6}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \cot^6(e+fx)\sqrt{a\cos^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(e+fx+\frac{\pi}{2}\right)^6\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3686}$$

$$\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \cos(e+fx)\cot^6(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \sin\left(e+fx+\frac{\pi}{2}\right)\tan\left(e+fx+\frac{\pi}{2}\right)^6dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int \csc^6(e+fx)\left(1-\sin^2(e+fx)\right)^3d(-\sin(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\int\left(\csc^6(e+fx)-3\csc^4(e+fx)+3\csc^2(e+fx)-1\right)d(-\sin(e+fx))}{f}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\left(\sin(e+fx)+\frac{1}{5}\csc^5(e+fx)-\csc^3(e+fx)+3\csc(e+fx)\right)}{f}$$

input
```
Int[Cot[e + f*x]^6*Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
-((Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x]*(3*Csc[e + f*x] - Csc[e + f*x]^3 +
Csc[e + f*x]^5/5 + Sin[e + f*x]))/f)
```

3.467.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.467. $\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.467.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\cos(fx+e)a\left(-5(\sin^6(fx+e))-15(\sin^4(fx+e))+5(\sin^2(fx+e))-1\right)}{5(\cos(fx+e)-1)^2(1+\cos(fx+e))^2\sin(fx+e)\sqrt{a(\cos^2(fx+e))}\,f}$	85
risch	$\frac{i\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,\left(5\,e^{12i(fx+e)}-90\,e^{10i(fx+e)}+235\,e^{8i(fx+e)}-364\,e^{6i(fx+e)}+235\,e^{4i(fx+e)}-90\,e^{2i(fx+e)}+5\right)}{10\left(e^{2i(fx+e)}-1\right)^5 f\left(e^{2i(fx+e)}+1\right)}$	127

input
```
int(cot(f*x+e)^6*(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*cos(f*x+e)*a*(-5*sin(f*x+e)^6-15*sin(f*x+e)^4+5*sin(f*x+e)^2-1)/(cos(f
*x+e)-1)^2/(1+cos(f*x+e))^2/sin(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

3.467.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.69

$$\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$
$$=\frac{\left(5\cos\left(fx+e\right)^6-30\cos\left(fx+e\right)^4+40\cos\left(fx+e\right)^2-16\right)\sqrt{a\cos\left(fx+e\right)^2}}{5\left(f\cos\left(fx+e\right)^5-2f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

input
```
integrate(cot(f*x+e)^6*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/5*(5*cos(f*x + e)^6 - 30*cos(f*x + e)^4 + 40*cos(f*x + e)^2 - 16)*sqrt(a
*cos(f*x + e)^2)/((f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e))
*sin(f*x + e))
```

3.467.6 Sympy [F]

$$\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \int \sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\cot^6\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**6*(a-a*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x)**6, x
)
```

3.467.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.55

$$\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = -\frac{16\sqrt{a}\tan\left(fx+e\right)^6+8\sqrt{a}\tan\left(fx+e\right)^4-2\sqrt{a}\tan\left(fx+e\right)^2+\sqrt{a}}{5\sqrt{\tan\left(fx+e\right)^2+1}f\tan\left(fx+e\right)^5}$$

input
```
integrate(cot(f*x+e)^6*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/5*(16*sqrt(a)*tan(f*x + e)^6 + 8*sqrt(a)*tan(f*x + e)^4 - 2*sqrt(a)*tan
(f*x + e)^2 + sqrt(a))/(sqrt(tan(f*x + e)^2 + 1)*f*tan(f*x + e)^5)
```

3.467.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8626 vs. 2(114) = 228.

Time = 4.03 (sec) , antiderivative size = 8626, normalized size of antiderivative = 69.56

$$\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/160*(320*(sqrt(a)*sgn(tan(1/2*f*x)^4*tan(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan
(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e) - 4*tan(1/2*f*x)*
tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*tan(1/2*e) + 1)*tan(1/2*f*x)*
tan(1/2*e)^2 - sqrt(a)*sgn(tan(1/2*f*x)^4*tan(1/2*e)^4 - 4*tan(1/2*f*x)^3*
tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e) - 4*tan(1/2*f*
x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*tan(1/2*e) + 1)*tan(1/2*f*
x) - 2*sqrt(a)*sgn(tan(1/2*f*x)^4*tan(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*
e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e) - 4*tan(1/2*f*x)*tan(1
/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*tan(1/2*e) + 1)*tan(1/2*e))/((tan(
1/2*f*x)^2 + 1)*(tan(1/2*e)^2 + 1)) - (5*sqrt(a)*sgn(tan(1/2*f*x)^4*tan(1/
2*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1/2*f*x)^3
*tan(1/2*e) - 4*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(1/2*f*x)*
tan(1/2*e) + 1)*tan(1/2*f*x)^9*tan(1/2*e)^16 + 10*sqrt(a)*sgn(tan(1/2*f*x)
^4*tan(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4 - 4*tan(1
/2*f*x)^3*tan(1/2*e) - 4*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4 - 4*tan(
1/2*f*x)*tan(1/2*e) + 1)*tan(1/2*f*x)^8*tan(1/2*e)^17 + 10*sqrt(a)*sgn(tan
(1/2*f*x)^4*tan(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(1/2*f*x)^4
- 4*tan(1/2*f*x)^3*tan(1/2*e) - 4*tan(1/2*f*x)*tan(1/2*e)^3 - tan(1/2*e)^4
 - 4*tan(1/2*f*x)*tan(1/2*e) + 1)*tan(1/2*f*x)^7*tan(1/2*e)^18 + 5*sqrt(a)
*sgn(tan(1/2*f*x)^4*tan(1/2*e)^4 - 4*tan(1/2*f*x)^3*tan(1/2*e)^3 - tan(...
```

3.467. $\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

3.467.9 Mupad [B] (verification not implemented)

Time = 24.39 (sec) , antiderivative size = 555, normalized size of antiderivative = 4.48

$$\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$$
$$= -\frac{\left(\frac{1i}{f}-\frac{e^{e\,2i+f\,x\,2i}\,1i}{f}\right)\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}}{e^{e\,2i+f\,x\,2i}+1}$$
$$-\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,12i}{f\left(e^{e\,2i+f\,x\,2i}-1\right)\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}$$
$$-\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,16i}{f\left(e^{e\,2i+f\,x\,2i}-1\right)^2\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}$$
$$-\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,144i}{5\,f\left(e^{e\,2i+f\,x\,2i}-1\right)^3\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}$$
$$-\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,128i}{5\,f\left(e^{e\,2i+f\,x\,2i}-1\right)^4\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}$$
$$-\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,64i}{5\,f\left(e^{e\,2i+f\,x\,2i}-1\right)^5\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}$$

input
```
int(cot(e + f*x)^6*(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
- ((1i/f - (exp(e*2i + f*x*2i)*1i)/f)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2
- (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(exp(e*2i + f*x*2i) + 1) - (exp(e*3
i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/
2)^2)^(1/2)*12i)/(f*(exp(e*2i + f*x*2i) - 1)*(exp(e*1i + f*x*1i) + exp(e*3
i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 -
(exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*16i)/(f*(exp(e*2i + f*x*2i) - 1)^2*(ex
p(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp
(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*144i)/(5*f*(
exp(e*2i + f*x*2i) - 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) - (ex
p(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)
*1i)/2)^2)^(1/2)*128i)/(5*f*(exp(e*2i + f*x*2i) - 1)^4*(exp(e*1i + f*x*1i)
 + exp(e*3i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i
)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*64i)/(5*f*(exp(e*2i + f*x*2i
) - 1)^5*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

3.467. $\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

3.468 $\int \frac{\tan^5(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

3.468.1 Optimal result

Integrand size = 26, antiderivative size = 65

$$\int \frac{\tan^5(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{a^2}{5f\left(a\cos^2(e+fx)\right)^{5/2}} - \frac{2a}{3f\left(a\cos^2(e+fx)\right)^{3/2}} + \frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

output
```
1/5*a^2/f/(a*cos(f*x+e)^2)^(5/2)-2/3*a/f/(a*cos(f*x+e)^2)^(3/2)+1/f/(a*cos
(f*x+e)^2)^(1/2)
```

3.468.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \frac{\tan^5(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{15-10\sec^2(e+fx)+3\sec^4(e+fx)}{15f\sqrt{a\cos^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^5/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
(15 - 10*Sec[e + f*x]^2 + 3*Sec[e + f*x]^4)/(15*f*Sqrt[a*Cos[e + f*x]^2])
```

3.468.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3655, 3042, 25, 3684, 8, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^5}{\sqrt{a-a\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\tan^5(e+fx)}{\sqrt{a\cos^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)^5\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sqrt{a\sin\left(\frac{1}{2}(2e+\pi)+fx\right)^2}\tan\left(\frac{1}{2}(2e+\pi)+fx\right)^5}dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \frac{\left(1-\cos^2(e+fx)\right)^2\sec^6(e+fx)}{\sqrt{a\cos^2(e+fx)}}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{8}$$

$$-\frac{a^3\int \frac{\left(1-\cos^2(e+fx)\right)^2}{\left(a\cos^2(e+fx)\right)^{7/2}}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{53}$$

$$-\frac{a^3\int \left(\frac{1}{\left(a\cos^2(e+fx)\right)^{7/2}}-\frac{2}{\left(a\cos^2(e+fx)\right)^{5/2}a}+\frac{1}{\left(a\cos^2(e+fx)\right)^{3/2}a^2}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\left(-\frac{2}{a^3\sqrt{a\cos^2(e+fx)}}+\frac{4}{3a^2(a\cos^2(e+fx))^{3/2}}-\frac{2}{5a(a\cos^2(e+fx))^{5/2}}\right)}{2f}$$

input
```
Int[Tan[e + f*x]^5/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
-1/2*(a^3*(-2/(5*a*(a*Cos[e + f*x]^2)^(5/2)) + 4/(3*a^2*(a*Cos[e + f*x]^2)
^(3/2)) - 2/(a^3*Sqrt[a*Cos[e + f*x]^2])))/f
```

3.468.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.468.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{15\left(\cos^4(fx+e)\right)-10\left(\cos^2(fx+e)\right)+3}{15\cos(fx+e)^4\sqrt{a\left(\cos^2(fx+e)\right)}\,f}$	48
risch	$\frac{2\,\mathrm{e}^{8i(fx+e)}+\frac{8\,\mathrm{e}^{6i(fx+e)}}{3}+\frac{116\,\mathrm{e}^{4i(fx+e)}}{15}+\frac{8\,\mathrm{e}^{2i(fx+e)}}{3}+2}{\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^2 a\,\mathrm{e}^{-2i(fx+e)}}\,\left(\mathrm{e}^{2i(fx+e)}+1\right)^4 f}$	91

input
```
int(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/cos(f*x+e)^4*(15*cos(f*x+e)^4-10*cos(f*x+e)^2+3)/(a*cos(f*x+e)^2)^(1/
2)/f
```

3.468.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\tan^5(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{\left(15\,\cos\left(fx+e\right)^4-10\,\cos\left(fx+e\right)^2+3\right)\sqrt{a\cos\left(fx+e\right)^2}}{15\,af\cos\left(fx+e\right)^6}$$

input
```
integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(15*cos(f*x + e)^4 - 10*cos(f*x + e)^2 + 3)*sqrt(a*cos(f*x + e)^2)/(a
*f*cos(f*x + e)^6)
```

3.468.6 Sympy [F]

$$\int \frac{\tan^5(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\tan^5\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a-a*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**5/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x
)
```

3.468.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int \frac{\tan^5(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{15\left(a\sin\left(fx+e\right)^2-a\right)^2a^3+10\left(a\sin\left(fx+e\right)^2-a\right)a^4+3\,a^5}{15\left(-a\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}a^3f}$$

input
```
integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/15*(15*(a*sin(f*x + e)^2 - a)^2*a^3 + 10*(a*sin(f*x + e)^2 - a)*a^4 + 3*
a^5)/((-a*sin(f*x + e)^2 + a)^(5/2)*a^3*f)
```

3.468.8 Giac [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \frac{\tan^5(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{16\left(10\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-5\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)}{15\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^5\sqrt{a}f\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-1\right)}$$

input
```
integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
16/15*(10*tan(1/2*f*x + 1/2*e)^4 - 5*tan(1/2*f*x + 1/2*e)^2 + 1)/((tan(1/2
*f*x + 1/2*e)^2 - 1)^5*sqrt(a)*f*sgn(tan(1/2*f*x + 1/2*e)^4 - 1))
```

3.468.9 Mupad [B] (verification not implemented)

Time = 21.86 (sec) , antiderivative size = 486, normalized size of antiderivative = 7.48

$$\int \frac{\tan^5(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{4\,{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}^2}}{a\,f\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\right)}-\frac{32\,{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}^2}}{3\,a\,f\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^2\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\right)}+\frac{352\,{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}^2}}{15\,a\,f\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^3\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\right)}-\frac{128\,{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}^2}}{5\,a\,f\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^4\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\right)}+\frac{64\,{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}^2}}{5\,a\,f\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^5\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\right)}$$

input
```
int(tan(e + f*x)^5/(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
(4*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2)^2)^(1/2))/(a*f*(exp(e*2i + f*x*2i) + 1)*(exp(e*1i + f*x*1i) +
 exp(e*3i + f*x*3i))) - (32*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1
i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(3*a*f*(exp(e*2i + f*x*2i)
 + 1)^2*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) + (352*exp(e*3i + f*x*3
i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/
2))/(15*a*f*(exp(e*2i + f*x*2i) + 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*
x*3i))) - (128*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (e
xp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(5*a*f*(exp(e*2i + f*x*2i) + 1)^4*(exp(
e*1i + f*x*1i) + exp(e*3i + f*x*3i))) + (64*exp(e*3i + f*x*3i)*(a - a*((ex
p(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(5*a*f*(ex
p(e*2i + f*x*2i) + 1)^5*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

3.469 $\int \frac{\tan^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

3.469.1 Optimal result

Integrand size = 26, antiderivative size = 42

$$\int \frac{\tan^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{a}{3f\left(a\cos^2(e+fx)\right)^{3/2}} - \frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

output
```
1/3*a/f/(a*cos(f*x+e)^2)^(3/2)-1/f/(a*cos(f*x+e)^2)^(1/2)
```

3.469.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{\tan^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{-3+\sec^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^3/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
(-3 + Sec[e + f*x]^2)/(3*f*Sqrt[a*Cos[e + f*x]^2])
```

3.469.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3655, 3042, 25, 3684, 8, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\tan(e+fx)^3}{\sqrt{a-a\sin(e+fx)^2}}dx$$

$$\Big\downarrow \textbf{3655}$$

$$\int \frac{\tan^3(e+fx)}{\sqrt{a\cos^2(e+fx)}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int -\frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)^3\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \textbf{25}$$

$$-\int \frac{1}{\sqrt{a\sin\left(\frac{1}{2}(2e+\pi)+fx\right)^2}\tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3}dx$$

$$\Big\downarrow \textbf{3684}$$

$$-\frac{\int \frac{(1-\cos^2(e+fx))\sec^4(e+fx)}{\sqrt{a\cos^2(e+fx)}}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \textbf{8}$$

$$-\frac{a^2\int \frac{1-\cos^2(e+fx)}{(a\cos^2(e+fx))^{5/2}}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \textbf{53}$$

$$-\frac{a^2\int \left(\frac{1}{(a\cos^2(e+fx))^{5/2}}-\frac{1}{a(a\cos^2(e+fx))^{3/2}}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{a^2\left(\frac{2}{a^2\sqrt{a\cos^2(e+fx)}}-\frac{2}{3a(a\cos^2(e+fx))^{3/2}}\right)}{2f}$$

input
```
Int[Tan[e + f*x]^3/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
-1/2*(a^2*(-2/(3*a*(a*Cos[e + f*x]^2)^(3/2)) + 2/(a^2*Sqrt[a*Cos[e + f*x]^
2])))/f
```

3.469.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.469. $\int \frac{\tan^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

3.469.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{3(\cos^2(fx+e))-1}{3\cos(fx+e)^2\sqrt{a(\cos^2(fx+e))}\,f}$	38
risch	$-\frac{2\left(3\,\mathrm{e}^{4i(fx+e)}+2\,\mathrm{e}^{2i(fx+e)}+3\right)}{3\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^2 a\,\mathrm{e}^{-2i(fx+e)}}\,\left(\mathrm{e}^{2i(fx+e)}+1\right)^2 f}$	69

input
```
int(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/cos(f*x+e)^2*(3*cos(f*x+e)^2-1)/(a*cos(f*x+e)^2)^(1/2)/f
```

3.469.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{\tan^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = -\frac{\sqrt{a\cos{(fx+e)}^2}\left(3\,\cos{(fx+e)}^2-1\right)}{3\,af\cos{(fx+e)}^4}$$

input
```
integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(a*cos(f*x + e)^2)*(3*cos(f*x + e)^2 - 1)/(a*f*cos(f*x + e)^4)
```

3.469.6 Sympy [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\tan^3{(e+fx)}}{\sqrt{-a\left(\sin{(e+fx)}-1\right)\left(\sin{(e+fx)}+1\right)}}\,dx$$

input
```
integrate(tan(f*x+e)**3/(a-a*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**3/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x
)
```

3.469.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{\tan^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{3\left(a\sin\left(fx+e\right)^2-a\right)a^2+a^3}{3\left(-a\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a^2f}$$

input
```
integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(3*(a*sin(f*x + e)^2 - a)*a^2 + a^3)/((-a*sin(f*x + e)^2 + a)^(3/2)*a^
2*f)
```

3.469.8 Giac [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

$$\int \frac{\tan^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{4\left(3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)}{3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^3\sqrt{a}f\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}$$

input
```
integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
4/3*(3*tan(1/2*f*x + 1/2*e)^2 - 1)/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*sqrt(a)
*f*sgn(tan(1/2*f*x + 1/2*e)^4 - 1))
```

3.469.9 Mupad [B] (verification not implemented)

Time = 17.46 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.38

$$\int \frac{\tan^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = -\frac{4\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,\left(2\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+3\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+3\right)}{3\,a\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^4}$$

input
```
int(tan(e + f*x)^3/(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
-(4*exp(e*2i + f*x*2i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f
*x*1i)*1i)/2)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + 3*exp(e*4i + f*x*4i) + 3))/
(3*a*f*(exp(e*2i + f*x*2i) + 1)^4)
```

3.469. $\int \frac{\tan^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

3.470 $\int \frac{\tan(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

3.470.1 Optimal result

Integrand size = 24, antiderivative size = 18

$$\int \frac{\tan(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

output
```
1/f/(a*cos(f*x+e)^2)^(1/2)
```

3.470.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\tan(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
1/(f*Sqrt[a*Cos[e + f*x]^2])
```

3.470.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 3655, 3042, 25, 3684, 8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)}{\sqrt{a-a\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\tan(e+fx)}{\sqrt{a\cos^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sqrt{a\sin\left(\frac{1}{2}(2e+\pi)+fx\right)^2}\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \frac{\sec^2(e+fx)}{\sqrt{a\cos^2(e+fx)}}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{8}$$

$$-\frac{a\int \frac{1}{(a\cos^2(e+fx))^{3/2}}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

input
```
Int[Tan[e + f*x]/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
1/(f*Sqrt[a*Cos[e + f*x]^2])
```

3.470.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.470.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{1}{\sqrt{a-a\left(\sin^2(fx+e)\right)}\,f}$	20
default	$\frac{1}{\sqrt{a-a\left(\sin^2(fx+e)\right)}\,f}$	20
risch	$\frac{2}{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\, e^{-2i(fx+e)}}\,f}$	32

input
```
int(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a-a*sin(f*x+e)^2)^(1/2)/f
```

3.470.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{\tan(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{\sqrt{a\cos\left(fx+e\right)^2}}{af\cos\left(fx+e\right)^2}$$

input
```
integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*cos(f*x + e)^2)/(a*f*cos(f*x + e)^2)
```

3.470.6 Sympy [F]

$$\int \frac{\tan(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\tan\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

input
```
integrate(tan(f*x+e)/(a-a*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

3.470.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(16) = 32$.

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.61

$$\int \frac{\tan(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{\frac{\sqrt{-a\sin(fx+e)^2+a}}{a\sin(fx+e)+a} - \frac{\sqrt{-a\sin(fx+e)^2+a}}{a\sin(fx+e)-a}}{2f}$$

input
```
integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(sqrt(-a*sin(f*x + e)^2 + a)/(a*sin(f*x + e) + a) - sqrt(-a*sin(f*x +
e)^2 + a)/(a*sin(f*x + e) - a))/f
```

3.470.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(16) = 32$.

Time = 0.42 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int \frac{\tan(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{2}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\sqrt{a}f\text{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}$$

input
```
integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
2/((tan(1/2*f*x + 1/2*e)^2 - 1)*sqrt(a)*f*sgn(tan(1/2*f*x + 1/2*e)^4 - 1))
```

3.470.9 Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.39

$$\int \frac{\tan(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{2\sqrt{2}\,(\cos(2e+2fx)+1)\,\sqrt{a\,(\cos(2e+2fx)+1)}}{a\,f\,(4\cos(2e+2fx)+\cos(4e+4fx)+3)}$$

input
```
int(tan(e + f*x)/(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
(2*2^(1/2)*(cos(2*e + 2*f*x) + 1)*(a*(cos(2*e + 2*f*x) + 1))^(1/2))/(a*f*(
4*cos(2*e + 2*f*x) + cos(4*e + 4*f*x) + 3))
```

3.471 $\int \frac{\cot(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

3.471.1 Optimal result

Integrand size = 24, antiderivative size = 31

$$\int \frac{\cot(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

output
```
-arctanh((a*cos(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)
```

3.471.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\cot(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

input
```
Integrate[Cot[e + f*x]/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

3.471.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 3655, 3042, 25, 3684, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)\sqrt{a-a\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\cot(e+fx)}{\sqrt{a\cos^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}{\sqrt{a\sin\left(\frac{1}{2}(2e+\pi)+fx\right)^2}}dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \frac{1}{\sqrt{a\cos^2(e+fx)}(1-\cos^2(e+fx))}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{1}{1-\frac{\cos^4(e+fx)}{a}}d\sqrt{a\cos^2(e+fx)}}{af}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

input
```
Int[Cot[e + f*x]/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

3.471.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.471. $\int \frac{\cot(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

3.471.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\cos(fx+e)\,\mathrm{arctanh}(\cos(fx+e))}{\sqrt{a(\cos^2(fx+e))}\,f}$	31
risch	$-\frac{2\ln(\mathrm{e}^{ifx}+\mathrm{e}^{-ie})\cos(fx+e)}{f\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^2 a\,\mathrm{e}^{-2i(fx+e)}}}+\frac{2\ln(\mathrm{e}^{ifx}-\mathrm{e}^{-ie})\cos(fx+e)}{f\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^2 a\,\mathrm{e}^{-2i(fx+e)}}}$	104

input
```
int(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*cos(f*x+e)^2)^(1/2)*cos(f*x+e)*arctanh(cos(f*x+e))/f
```

3.471.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.71

$$\int \frac{\cot(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \left[-\frac{\sqrt{a\cos(fx+e)^2}\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}\right)}{2\,af\cos(fx+e)}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{a\cos(fx+e)^2}\sqrt{-a}}{a}\right)}{af}\right]$$

input
```
integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(a*cos(f*x + e)^2)*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1))/(
a*f*cos(f*x + e)), sqrt(-a)*arctan(sqrt(a*cos(f*x + e)^2)*sqrt(-a)/a)/(a*f
)]
```

3.471.6 Sympy [F]

$$\int \frac{\cot(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\cot(e+fx)}{\sqrt{-a(\sin(e+fx)-1)(\sin(e+fx)+1)}}\,dx$$

input
```
integrate(cot(f*x+e)/(a-a*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

3.471.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(25) = 50$.

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \frac{\cot(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = -\frac{\log\left(\frac{2\sqrt{-a\sin(fx+e)^2+a}\sqrt{a}}{|\sin(fx+e)|}+\frac{2a}{|\sin(fx+e)|}\right)}{\sqrt{a}f}$$

input
```
integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-log(2*sqrt(-a*sin(f*x + e)^2 + a)*sqrt(a)/abs(sin(f*x + e)) + 2*a/abs(sin
(f*x + e)))/(sqrt(a)*f)
```

3.471.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.471.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)}{\sqrt{a-a\sin\left(e+f\,x\right)^2}}\,dx$$

input
```
int(cot(e + f*x)/(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)/(a - a*sin(e + f*x)^2)^(1/2), x)
```

3.472 $\int \frac{\cot^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

3.472.1 Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \frac{\cot^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f} - \frac{\sqrt{a\cos^2(e+fx)}\csc^2(e+fx)}{2af}$$

output
```
1/2*arctanh((a*cos(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-1/2*csc(f*x+e)^2*(a*
cos(f*x+e)^2)^(1/2)/a/f
```

3.472.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int \frac{\cot^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\sqrt{\cos^2(e+fx)}\right)\sqrt{\cos^2(e+fx)} - \cot^2(e+fx)}{2f\sqrt{a\cos^2(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]^3/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
(ArcTanh[Sqrt[Cos[e + f*x]^2]]*Sqrt[Cos[e + f*x]^2] - Cot[e + f*x]^2)/(2*f
*Sqrt[a*Cos[e + f*x]^2])
```

3.472.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3042, 3655, 3042, 25, 3684, 8, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^3\sqrt{a-a\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\cot^3(e+fx)}{\sqrt{a\cos^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(e+fx+\frac{\pi}{2}\right)^3}{\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3}{\sqrt{a\sin\left(\frac{1}{2}(2e+\pi)+fx\right)^2}}dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \frac{\cos^2(e+fx)}{\sqrt{a\cos^2(e+fx)}(1-\cos^2(e+fx))^2}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{8}$$

$$-\frac{\int \frac{\sqrt{a\cos^2(e+fx)}}{(1-\cos^2(e+fx))^2}d\cos^2(e+fx)}{2af}$$

$$\Big\downarrow \text{51}$$

$$-\frac{\frac{\sqrt{a\cos^2(e+fx)}}{1-\cos^2(e+fx)}-\frac{1}{2}a\int \frac{1}{\sqrt{a\cos^2(e+fx)}(1-\cos^2(e+fx))}d\cos^2(e+fx)}{2af}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{\sqrt{a\cos^2(e+fx)}}{1-\cos^2(e+fx)}-\int \frac{1}{1-\frac{\cos^4(e+fx)}{a}}d\sqrt{a\cos^2(e+fx)}}{2af}$$

$$\Big\downarrow 219$$

$$-\frac{\frac{\sqrt{a\cos^2(e+fx)}}{1-\cos^2(e+fx)}-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2af}$$

input
```
Int[Cot[e + f*x]^3/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
-1/2*(-(Sqrt[a]*ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]) + Sqrt[a*Cos[e +
f*x]^2]/(1 - Cos[e + f*x]^2))/(a*f)
```

3.472.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.472. $\int \frac{\cot^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.472.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\cos(fx+e)\left(2\cos(fx+e)+\left(-\ln(1+\cos(fx+e))+\ln(\cos(fx+e)-1)\right)\left(\sin^2(fx+e)\right)\right)}{4\sqrt{a\left(\cos^2(fx+e)\right)}\,(\cos(fx+e)-1)(1+\cos(fx+e))f}$	83
risch	$\frac{\left(e^{2i(fx+e)}+1\right)^2}{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\, e^{-2i(fx+e)}}\, f\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{ifx}-e^{-ie}\right)\cos(fx+e)}{f\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\, e^{-2i(fx+e)}}}+\frac{\ln\left(e^{ifx}+e^{-ie}\right)\cos(fx+e)}{f\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\, e^{-2i(fx+e)}}}$	159

input
```
int(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*cos(f*x+e)*(2*cos(f*x+e)+(-ln(1+cos(f*x+e))+ln(cos(f*x+e)-1))*sin(f*x+
e)^2)/(a*cos(f*x+e)^2)^(1/2)/(cos(f*x+e)-1)/(1+cos(f*x+e))/f
```

3.472.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.20

$$\int \frac{\cot^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = -\frac{\sqrt{a\cos(fx+e)^2}\left(\left(\cos(fx+e)^2-1\right)\log\left(-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2\cos(fx+e)\right)}{4\left(af\cos(fx+e)^3-af\cos(fx+e)\right)}$$

input
```
integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(a*cos(f*x + e)^2)*((cos(f*x + e)^2 - 1)*log(-(cos(f*x + e) - 1)/
(cos(f*x + e) + 1)) - 2*cos(f*x + e))/(a*f*cos(f*x + e)^3 - a*f*cos(f*x +
e))
```

3.472.6 Sympy [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\cot^3(e+fx)}{\sqrt{-a(\sin(e+fx)-1)(\sin(e+fx)+1)}}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a-a*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**3/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x
)
```

3.472.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

$$\int \frac{\cot^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{\frac{\log\left(\frac{2\sqrt{-a\sin(fx+e)^2+a}\sqrt{a}}{|\sin(fx+e)|}+\frac{2a}{|\sin(fx+e)|}\right)}{\sqrt{a}} - \frac{\sqrt{-a\sin(fx+e)^2+a}}{a\sin(fx+e)^2}}{2f}$$

input
```
integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(log(2*sqrt(-a*sin(f*x + e)^2 + a)*sqrt(a)/abs(sin(f*x + e)) + 2*a/abs
(sin(f*x + e)))/sqrt(a) - sqrt(-a*sin(f*x + e)^2 + a)/(a*sin(f*x + e)^2))/
f
```

3.472.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

3.472.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^3}{\sqrt{a-a\sin\left(e+f\,x\right)^2}}\,dx$$

input
```
int(cot(e + f*x)^3/(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^3/(a - a*sin(e + f*x)^2)^(1/2), x)
```

3.473 $\int \frac{\tan^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

3.473.1 Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \frac{\tan^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{3\text{arctanh}(\sin(e+fx))\cos(e+fx)}{8f\sqrt{a\cos^2(e+fx)}} - \frac{3\tan(e+fx)}{8f\sqrt{a\cos^2(e+fx)}} + \frac{\tan^3(e+fx)}{4f\sqrt{a\cos^2(e+fx)}}$$

output
```
3/8*arctanh(sin(f*x+e))*cos(f*x+e)/f/(a*cos(f*x+e)^2)^(1/2)-3/8*tan(f*x+e)
/f/(a*cos(f*x+e)^2)^(1/2)+1/4*tan(f*x+e)^3/f/(a*cos(f*x+e)^2)^(1/2)
```

3.473.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \frac{\tan^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{3\text{arctanh}(\sin(e+fx))\cos(e+fx)+\tan(e+fx)\left(3-6\sec^2(e+fx)+8\tan^2(e+fx)\right)}{8f\sqrt{a\cos^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^4/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
(3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x] + Tan[e + f*x]*(3 - 6*Sec[e + f*x]^2
 + 8*Tan[e + f*x]^2))/(8*f*Sqrt[a*Cos[e + f*x]^2])
```

3.473.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3655, 3042, 3686, 3042, 3091, 3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^4}{\sqrt{a-a\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\tan^4(e+fx)}{\sqrt{a\cos^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)^4\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cos(e+fx)\int \sec(e+fx)\tan^4(e+fx)dx}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(e+fx)\int \sec(e+fx)\tan(e+fx)^4dx}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3091}$$

$$\frac{\cos(e+fx)\left(\frac{\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3}{4}\int \sec(e+fx)\tan^2(e+fx)dx\right)}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(e+fx)\left(\frac{\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3}{4}\int \sec(e+fx)\tan(e+fx)^2dx\right)}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3091}$$

$$\frac{\cos(e+fx)\left(\frac{\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3}{4}\left(\frac{\tan(e+fx)\sec(e+fx)}{2f}-\frac{1}{2}\int\sec(e+fx)dx\right)\right)}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(e+fx)\left(\frac{\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3}{4}\left(\frac{\tan(e+fx)\sec(e+fx)}{2f}-\frac{1}{2}\int\csc\left(e+fx+\frac{\pi}{2}\right)dx\right)\right)}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cos(e+fx)\left(\frac{\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3}{4}\left(\frac{\tan(e+fx)\sec(e+fx)}{2f}-\frac{\text{arctanh}(\sin(e+fx))}{2f}\right)\right)}{\sqrt{a\cos^2(e+fx)}}$$

input
```
Int[Tan[e + f*x]^4/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
(Cos[e + f*x]*((Sec[e + f*x]*Tan[e + f*x]^3)/(4*f) - (3*(-1/2*ArcTanh[Sin[
e + f*x]]/f + (Sec[e + f*x]*Tan[e + f*x])/(2*f)))/4))/Sqrt[a*Cos[e + f*x]^
2]
```

3.473.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.473. $\int \frac{\tan^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.473.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.45

method	result
default	$-\frac{\sqrt{a\left(\sin^2(fx+e)\right)}\left(-3\ln\left(\frac{2\sqrt{a}\sqrt{a\left(\sin^2(fx+e)\right)}+2a}{\cos(fx+e)}\right)a\left(\cos^4(fx+e)\right)+5\sqrt{a\left(\sin^2(fx+e)\right)}\left(\cos^2(fx+e)\right)\sqrt{a}-2\sqrt{a}\sqrt{a\left(\sin^2(fx+e)\right.}}{8\cos(fx+e)^3a^{\frac{3}{2}}\sin(fx+e)\sqrt{a\left(\cos^2(fx+e)\right)}f}$
risch	$\frac{i\left(5\,\mathrm{e}^{6i(fx+e)}-3\,\mathrm{e}^{4i(fx+e)}+3\,\mathrm{e}^{2i(fx+e)}-5\right)}{4\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^2a\,\mathrm{e}^{-2i(fx+e)}}\left(\mathrm{e}^{2i(fx+e)}+1\right)^3f}-\frac{3\ln\left(\mathrm{e}^{ifx}-i\mathrm{e}^{-ie}\right)\cos(fx+e)}{4f\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^2a\,\mathrm{e}^{-2i(fx+e)}}}+\frac{3\ln\left(\mathrm{e}^{ifx}+i\mathrm{e}^{-ie}\right)\cos(fx+e)}{4f\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^2a\,\mathrm{e}^{-2i(fx+e)}}}$

input
```
int(tan(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/cos(f*x+e)^3*(a*sin(f*x+e)^2)^(1/2)*(-3*ln(2/cos(f*x+e)*(a^(1/2)*(a*s
in(f*x+e)^2)^(1/2)+a))*a*cos(f*x+e)^4+5*(a*sin(f*x+e)^2)^(1/2)*cos(f*x+e)^
2*a^(1/2)-2*a^(1/2)*(a*sin(f*x+e)^2)^(1/2))/a^(3/2)/sin(f*x+e)/(a*cos(f*x+
e)^2)^(1/2)/f
```

3.473.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.88

$$\int \frac{\tan^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = -\frac{\left(3\,\cos\left(fx+e\right)^4\log\left(-\frac{\sin(fx+e)-1}{\sin(fx+e)+1}\right)+2\left(5\,\cos\left(fx+e\right)^2-2\right)\sin\left(fx+e\right)\right)\sqrt{a\cos\left(fx+e\right)^2}}{16\,af\cos\left(fx+e\right)^5}$$

input
```
integrate(tan(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/16*(3*cos(f*x + e)^4*log(-(sin(f*x + e) - 1)/(sin(f*x + e) + 1)) + 2*(5
*cos(f*x + e)^2 - 2)*sin(f*x + e))*sqrt(a*cos(f*x + e)^2)/(a*f*cos(f*x + e
)^5)
```

3.473.6 Sympy [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\tan^4(e+fx)}{\sqrt{-a(\sin(e+fx)-1)(\sin(e+fx)+1)}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a-a*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**4/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x
)
```

3.473.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1518 vs. 2(79) = 158.

Time = 0.55 (sec) , antiderivative size = 1518, normalized size of antiderivative = 16.68

$$\int \frac{\tan^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/16*(4*(5*sin(7*f*x + 7*e) - 3*sin(5*f*x + 5*e) + 3*sin(3*f*x + 3*e) - 5
*sin(f*x + e))*cos(8*f*x + 8*e) - 40*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4
*e) + 2*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 16*(3*sin(5*f*x + 5*e) - 3*si
n(3*f*x + 3*e) + 5*sin(f*x + e))*cos(6*f*x + 6*e) + 24*(3*sin(4*f*x + 4*e)
 + 2*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) + 24*(3*sin(3*f*x + 3*e) - 5*sin(f
*x + e))*cos(4*f*x + 4*e) - 3*(2*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e)
+ 4*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos
(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x +
6*e)^2 + 12*(4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e
)^2 + 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) +
 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f
*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 +
 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f
*x + 2*e)^2 + 8*cos(2*f*x + 2*e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2
+ 2*sin(f*x + e) + 1) + 3*(2*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*
cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f
*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)
^2 + 12*(4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2
+ 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*s
in(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*...
```

3.473.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(79) = 158$.

Time = 1.46 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.07

$$\int \frac{\tan^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx =$$

$$-\frac{\frac{3\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right|\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}-\frac{3\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2\right|\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}-\frac{4\left(3\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)\cdots}{\left(\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\cdots\right.\right.}}{16\sqrt{a}f}$$

input
```
integrate(tan(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/16*(3*log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e) + 2))/sgn(t
an(1/2*f*x + 1/2*e)^4 - 1) - 3*log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*
x + 1/2*e) - 2))/sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 4*(3*(1/tan(1/2*f*x + 1
/2*e) + tan(1/2*f*x + 1/2*e))^3 - 20/tan(1/2*f*x + 1/2*e) - 20*tan(1/2*f*x
 + 1/2*e))/(((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^2 - 4)^2*sgn(
tan(1/2*f*x + 1/2*e)^4 - 1)))/(sqrt(a)*f)
```

3.473.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^4}{\sqrt{a-a\sin\left(e+f\,x\right)^2}}\,dx$$

input
```
int(tan(e + f*x)^4/(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^4/(a - a*sin(e + f*x)^2)^(1/2), x)
```

3.474 $\int \frac{\tan^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

3.474.1 Optimal result

Integrand size = 26, antiderivative size = 62

$$\int \frac{\tan^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}(\sin(e+fx))\cos(e+fx)}{2f\sqrt{a\cos^2(e+fx)}}+\frac{\tan(e+fx)}{2f\sqrt{a\cos^2(e+fx)}}$$

output
```
-1/2*arctanh(sin(f*x+e))*cos(f*x+e)/f/(a*cos(f*x+e)^2)^(1/2)+1/2*tan(f*x+e
)/f/(a*cos(f*x+e)^2)^(1/2)
```

3.474.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.69

$$\int \frac{\tan^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{-\operatorname{arctanh}(\sin(e+fx))\cos(e+fx)+\tan(e+fx)}{2f\sqrt{a\cos^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^2/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
(-(ArcTanh[Sin[e + f*x]]*Cos[e + f*x]) + Tan[e + f*x])/(2*f*Sqrt[a*Cos[e +
 f*x]^2])
```

3.474.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3655, 3042, 3686, 3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{\sqrt{a-a\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\tan^2(e+fx)}{\sqrt{a\cos^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)^2\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cos(e+fx)\int \sec(e+fx)\tan^2(e+fx)dx}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(e+fx)\int \sec(e+fx)\tan(e+fx)^2dx}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3091}$$

$$\frac{\cos(e+fx)\left(\frac{\tan(e+fx)\sec(e+fx)}{2f}-\frac{1}{2}\int \sec(e+fx)dx\right)}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(e+fx)\left(\frac{\tan(e+fx)\sec(e+fx)}{2f}-\frac{1}{2}\int \csc\left(e+fx+\frac{\pi}{2}\right)dx\right)}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cos(e+fx)\left(\frac{\tan(e+fx)\sec(e+fx)}{2f}-\frac{\operatorname{arctanh}(\sin(e+fx))}{2f}\right)}{\sqrt{a\cos^2(e+fx)}}$$

input
```
Int[Tan[e + f*x]^2/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
(Cos[e + f*x]*(-1/2*ArcTanh[Sin[e + f*x]]/f + (Sec[e + f*x]*Tan[e + f*x])/
(2*f)))/Sqrt[a*Cos[e + f*x]^2]
```

3.474.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.474. $\int \frac{\tan^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

3.474.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.71

method	result	size
default	$\frac{\sqrt{a\left(\sin^2(fx+e)\right)}\left(-\ln\left(\frac{2\sqrt{a}\sqrt{a\left(\sin^2(fx+e)\right)}+2a}{\cos(fx+e)}\right)a\left(\cos^2(fx+e)\right)+\sqrt{a}\sqrt{a\left(\sin^2(fx+e)\right)}\right)}{2\cos(fx+e)a^{\frac{3}{2}}\sin(fx+e)\sqrt{a\left(\cos^2(fx+e)\right)}\,f}$	106
risch	$-\frac{i\left(e^{2i(fx+e)}-1\right)}{\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\left(e^{2i(fx+e)}+1\right)f}+\frac{\ln\left(e^{ifx}-ie^{-ie}\right)\cos(fx+e)}{f\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}}-\frac{\ln\left(e^{ifx}+ie^{-ie}\right)\cos(fx+e)}{f\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}}$	163

input
```
int(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/cos(f*x+e)*(a*sin(f*x+e)^2)^(1/2)*(-ln(2/cos(f*x+e)*(a^(1/2)*(a*sin(f*
x+e)^2)^(1/2)+a))*a*cos(f*x+e)^2+a^(1/2)*(a*sin(f*x+e)^2)^(1/2))/a^(3/2)/s
in(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

3.474.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \frac{\tan^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = -\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(\cos\left(fx+e\right)^2\log\left(-\frac{\sin(fx+e)+1}{\sin(fx+e)-1}\right)-2\,\sin\left(fx+e\right)\right)}{4\,af\cos\left(fx+e\right)^3}$$

input
```
integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(a*cos(f*x + e)^2)*(cos(f*x + e)^2*log(-(sin(f*x + e) + 1)/(sin(f
*x + e) - 1)) - 2*sin(f*x + e))/(a*f*cos(f*x + e)^3)
```

3.474.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\tan^2(e+fx)}{\sqrt{-a(\sin(e+fx)-1)(\sin(e+fx)+1)}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a-a*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**2/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x
)
```

3.474.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(54) = 108$.

Time = 0.51 (sec) , antiderivative size = 527, normalized size of antiderivative = 8.50

$$\int \frac{\tan^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{4(\sin(3fx+3e)-\sin(fx+e))\cos(4fx+4e)-(2(2\cos(2fx+2e)+1)\cos(4fx+4e)+\cos(4fx$$

input
```
integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(4*(sin(3*f*x + 3*e) - sin(f*x + e))*cos(4*f*x + 4*e) - (2*(2*cos(2*f*
x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2
 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x
+ 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2
*sin(f*x + e) + 1) + (2*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*
f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4
*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*log(
cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) - 4*(cos(3*f*x + 3*e
) - cos(f*x + e))*sin(4*f*x + 4*e) + 4*(2*cos(2*f*x + 2*e) + 1)*sin(3*f*x
+ 3*e) - 8*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 8*cos(f*x + e)*sin(2*f*x +
2*e) - 8*cos(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f*x + e))/((2*(2*cos(2*f*x
+ 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2 +
 sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x +
2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sqrt(a)*f)
```

3.474.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(54) = 108$.

Time = 0.87 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.55

$$\int \frac{\tan^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$$
$$= \frac{\frac{\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right|\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)} - \frac{\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2\right|\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)} - \frac{4\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.\right.}{\left(\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)^2-4\right)\operatorname{sgn}\left(\right.}}{4\sqrt{a}f}$$

input
```
integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*(log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e) + 2))/sgn(tan(1
/2*f*x + 1/2*e)^4 - 1) - log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/
2*e) - 2))/sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 4*(1/tan(1/2*f*x + 1/2*e) + t
an(1/2*f*x + 1/2*e))/(((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^2 -
 4)*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)))/(sqrt(a)*f)
```

3.474.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^2}{\sqrt{a-a\sin\left(e+f\,x\right)^2}}\,dx$$

input
```
int(tan(e + f*x)^2/(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^2/(a - a*sin(e + f*x)^2)^(1/2), x)
```

3.475 $\int \frac{\cot^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

3.475.1 Optimal result

Integrand size = 26, antiderivative size = 25

$$\int \frac{\cot^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx=-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}$$

output
```
-cot(f*x+e)/f/(a*cos(f*x+e)^2)^(1/2)
```

3.475.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\cot^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx=-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]^2/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
-(Cot[e + f*x]/(f*Sqrt[a*Cos[e + f*x]^2]))
```

3.475.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3655, 3042, 3686, 3042, 25, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^2\sqrt{a-a\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\cot^2(e+fx)}{\sqrt{a\cos^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(e+fx+\frac{\pi}{2}\right)^2}{\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cos(e+fx)\int \cot(e+fx)\csc(e+fx)dx}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(e+fx)\int -\sec\left(e+fx-\frac{\pi}{2}\right)\tan\left(e+fx-\frac{\pi}{2}\right)dx}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\cos(e+fx)\int \sec\left(\frac{1}{2}(2e-\pi)+fx\right)\tan\left(\frac{1}{2}(2e-\pi)+fx\right)dx}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\cos(e+fx)\int 1d\csc(e+fx)}{f\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}$$

input
```
Int[Cot[e + f*x]^2/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
-(Cot[e + f*x]/(f*Sqrt[a*Cos[e + f*x]^2]))
```

3.475.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.475.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

method	result	size
default	$-\frac{\cos(fx+e)}{\sin(fx+e)\sqrt{a(\cos^2(fx+e))}\,f}$	32
risch	$-\frac{2i(\mathrm{e}^{2i(fx+e)}+1)}{\sqrt{(\mathrm{e}^{2i(fx+e)}+1)^2 a\,\mathrm{e}^{-2i(fx+e)}}\,f(\mathrm{e}^{2i(fx+e)}-1)}$	57

input
```
int(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-cos(f*x+e)/sin(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

3.475.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{\cot^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = -\frac{\sqrt{a\cos(fx+e)^2}}{af\cos(fx+e)\sin(fx+e)}$$

input
```
integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(a*cos(f*x + e)^2)/(a*f*cos(f*x + e)*sin(f*x + e))
```

3.475.6 Sympy [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\cot^2(e+fx)}{\sqrt{-a(\sin(e+fx)-1)(\sin(e+fx)+1)}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a-a*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**2/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x
)
```

3.475.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(23) = 46$.

Time = 0.39 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.60

$$\int \frac{\cot^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = -\frac{2\left(\cos(fx+e)\sin(2fx+2e)-\cos(2fx+2e)\sin(fx+e)+\sin(fx+e)\right)\sqrt{a}}{\left(a\cos(2fx+2e)^2+a\sin(2fx+2e)^2-2a\cos(2fx+2e)+a\right)f}$$

input
```
integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-2*(cos(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e)*sin(f*x + e) + sin(f*
x + e))*sqrt(a)/((a*cos(2*f*x + 2*e)^2 + a*sin(2*f*x + 2*e)^2 - 2*a*cos(2*
f*x + 2*e) + a)*f)
```

3.475.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

3.475.9 Mupad [B] (verification not implemented)

Time = 13.61 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{\cot^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = -\frac{\sqrt{2a\,\left(\cos(2e+2fx)+1\right)}}{a\,f\,\sin(2e+2fx)}$$

input
```
int(cot(e + f*x)^2/(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
-(2*a*(cos(2*e + 2*f*x) + 1))^(1/2)/(a*f*sin(2*e + 2*f*x))
```

3.475. $\int \frac{\cot^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

3.476 $\int \frac{\cot^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

3.476.1 Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \frac{\cot^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}$$

output
```
cot(f*x+e)/f/(a*cos(f*x+e)^2)^(1/2)-1/3*cot(f*x+e)*csc(f*x+e)^2/f/(a*cos(f
*x+e)^2)^(1/2)
```

3.476.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.62

$$\int \frac{\cot^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = -\frac{\cot(e+fx)\left(-3+\csc^2(e+fx)\right)}{3f\sqrt{a\cos^2(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]^4/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
-1/3*(Cot[e + f*x]*(-3 + Csc[e + f*x]^2))/(f*Sqrt[a*Cos[e + f*x]^2])
```

3.476.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3655, 3042, 3686, 3042, 25, 3086, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^4\sqrt{a-a\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\cot^4(e+fx)}{\sqrt{a\cos^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(e+fx+\frac{\pi}{2}\right)^4}{\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cos(e+fx)\int \cot^3(e+fx)\csc(e+fx)dx}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(e+fx)\int -\sec\left(e+fx-\frac{\pi}{2}\right)\tan\left(e+fx-\frac{\pi}{2}\right)^3dx}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\cos(e+fx)\int \sec\left(\frac{1}{2}(2e-\pi)+fx\right)\tan\left(\frac{1}{2}(2e-\pi)+fx\right)^3dx}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\cos(e+fx)\int \left(\csc^2(e+fx)-1\right)d\csc(e+fx)}{f\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos(e+fx)\left(\frac{1}{3}\csc^3(e+fx)-\csc(e+fx)\right)}{f\sqrt{a\cos^2(e+fx)}}$$

input
```
Int[Cot[e + f*x]^4/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
-((Cos[e + f*x]*(-Csc[e + f*x] + Csc[e + f*x]^3/3))/(f*Sqrt[a*Cos[e + f*x]
^2]))
```

3.476.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.476. $\int \frac{\cot^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

3.476.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{\cos(fx+e)\left(3\left(\cos^2(fx+e)\right)-2\right)}{3(\cos(fx+e)-1)(1+\cos(fx+e))\sin(fx+e)\sqrt{a(\cos^2(fx+e))}\,f}$	64
risch	$\frac{2i\left(e^{2i(fx+e)}+1\right)\left(3\,e^{4i(fx+e)}-2\,e^{2i(fx+e)}+3\right)}{3\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,f\left(e^{2i(fx+e)}-1\right)^3}$	81

input
```
int(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*cos(f*x+e)*(3*cos(f*x+e)^2-2)/(cos(f*x+e)-1)/(1+cos(f*x+e))/sin(f*x+e)
/(a*cos(f*x+e)^2)^(1/2)/f
```

3.476.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{\cot^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{\sqrt{a\cos\left(fx+e\right)^2}\left(3\,\cos\left(fx+e\right)^2-2\right)}{3\left(af\cos\left(fx+e\right)^3-af\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

input
```
integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(a*cos(f*x + e)^2)*(3*cos(f*x + e)^2 - 2)/((a*f*cos(f*x + e)^3 - a
*f*cos(f*x + e))*sin(f*x + e))
```

3.476.6 Sympy [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\cot^4\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a-a*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**4/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x
)
```

3.476.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(54) = 108$.

Time = 0.36 (sec) , antiderivative size = 525, normalized size of antiderivative = 8.75

$$\int \frac{\cot^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx =$$
$$-\frac{2\left((3\sin(5fx+5e)-2\sin(3fx+3e)+3\sin(fx+e))\cos(6fx+6e)+9(\sin(4fx+4e)-\sin(2\right.}{3\left(a\cos(6fx\right.}$$

input
```
integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*((3*sin(5*f*x + 5*e) - 2*sin(3*f*x + 3*e) + 3*sin(f*x + e))*cos(6*f*x
 + 6*e) + 9*(sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*cos(5*f*x + 5*e) + 3*(2*
sin(3*f*x + 3*e) - 3*sin(f*x + e))*cos(4*f*x + 4*e) - (3*cos(5*f*x + 5*e)
- 2*cos(3*f*x + 3*e) + 3*cos(f*x + e))*sin(6*f*x + 6*e) - 3*(3*cos(4*f*x +
 4*e) - 3*cos(2*f*x + 2*e) + 1)*sin(5*f*x + 5*e) - 3*(2*cos(3*f*x + 3*e) -
 3*cos(f*x + e))*sin(4*f*x + 4*e) - 2*(3*cos(2*f*x + 2*e) - 1)*sin(3*f*x +
 3*e) + 6*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 9*cos(f*x + e)*sin(2*f*x + 2
*e) + 9*cos(2*f*x + 2*e)*sin(f*x + e) - 3*sin(f*x + e))*sqrt(a)/((a*cos(6*
f*x + 6*e)^2 + 9*a*cos(4*f*x + 4*e)^2 + 9*a*cos(2*f*x + 2*e)^2 + a*sin(6*f
*x + 6*e)^2 + 9*a*sin(4*f*x + 4*e)^2 - 18*a*sin(4*f*x + 4*e)*sin(2*f*x + 2
*e) + 9*a*sin(2*f*x + 2*e)^2 - 2*(3*a*cos(4*f*x + 4*e) - 3*a*cos(2*f*x + 2
*e) + a)*cos(6*f*x + 6*e) - 6*(3*a*cos(2*f*x + 2*e) - a)*cos(4*f*x + 4*e)
- 6*a*cos(2*f*x + 2*e) - 6*(a*sin(4*f*x + 4*e) - a*sin(2*f*x + 2*e))*sin(6
*f*x + 6*e) + a)*f)
```

3.476.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

3.476.9 Mupad [B] (verification not implemented)

Time = 18.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.97

$$\int \frac{\cot^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \frac{4\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}\,\left(-\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,2\mathrm{i}+\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,3\mathrm{i}+3\mathrm{i}\right)}{3\,a\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)^3\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}$$

input
```
int(cot(e + f*x)^4/(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
(4*exp(e*2i + f*x*2i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2)^2)^(1/2)*(exp(e*4i + f*x*4i)*3i - exp(e*2i + f*x*2i)*2i + 3i)
)/(3*a*f*(exp(e*2i + f*x*2i) - 1)^3*(exp(e*2i + f*x*2i) + 1))
```

3.477 $\int \frac{\cot^6(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

3.477.1 Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{\cot^6(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx=-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}+\frac{2\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^4(e+fx)}{5f\sqrt{a\cos^2(e+fx)}}$$

output
```
-cot(f*x+e)/f/(a*cos(f*x+e)^2)^(1/2)+2/3*cot(f*x+e)*csc(f*x+e)^2/f/(a*cos(
f*x+e)^2)^(1/2)-1/5*cot(f*x+e)*csc(f*x+e)^4/f/(a*cos(f*x+e)^2)^(1/2)
```

3.477.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.51

$$\int \frac{\cot^6(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx=-\frac{\cot(e+fx)\left(15-10\csc^2(e+fx)+3\csc^4(e+fx)\right)}{15f\sqrt{a\cos^2(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]^6/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
-1/15*(Cot[e + f*x]*(15 - 10*Csc[e + f*x]^2 + 3*Csc[e + f*x]^4))/(f*Sqrt[a
*Cos[e + f*x]^2])
```

3.477.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.58, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 3655, 3042, 3686, 3042, 25, 3086, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^6\sqrt{a-a\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\cot^6(e+fx)}{\sqrt{a\cos^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(e+fx+\frac{\pi}{2}\right)^6}{\sqrt{a\sin\left(e+fx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cos(e+fx)\int \cot^5(e+fx)\csc(e+fx)dx}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(e+fx)\int -\sec\left(e+fx-\frac{\pi}{2}\right)\tan\left(e+fx-\frac{\pi}{2}\right)^5dx}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\cos(e+fx)\int \sec\left(\frac{1}{2}(2e-\pi)+fx\right)\tan\left(\frac{1}{2}(2e-\pi)+fx\right)^5dx}{\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\cos(e+fx)\int \left(\csc^2(e+fx)-1\right)^2d\csc(e+fx)}{f\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{210}$$

$$-\frac{\cos(e+fx)\int\left(\csc^4(e+fx)-2\csc^2(e+fx)+1\right)d\csc(e+fx)}{f\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos(e+fx)\left(\frac{1}{5}\csc^5(e+fx)-\frac{2}{3}\csc^3(e+fx)+\csc(e+fx)\right)}{f\sqrt{a\cos^2(e+fx)}}$$

input
```
Int[Cot[e + f*x]^6/Sqrt[a - a*Sin[e + f*x]^2],x]
```

output
```
-((Cos[e + f*x]*(Csc[e + f*x] - (2*Csc[e + f*x]^3)/3 + Csc[e + f*x]^5/5))/
(f*Sqrt[a*Cos[e + f*x]^2]))
```

3.477.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.477. $\int \frac{\cot^6(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.477.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\cos(fx+e)\left(15\left(\cos^4(fx+e)\right)-20\left(\cos^2(fx+e)\right)+8\right)}{15(\cos(fx+e)-1)^2(1+\cos(fx+e))^2\sin(fx+e)\sqrt{a\left(\cos^2(fx+e)\right)}\,f}$	74
risch	$-\frac{2i\left(e^{2i(fx+e)}+1\right)\left(15\,e^{8i(fx+e)}-20\,e^{6i(fx+e)}+58\,e^{4i(fx+e)}-20\,e^{2i(fx+e)}+15\right)}{15\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,f\left(e^{2i(fx+e)}-1\right)^5}$	103

input
```
int(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*cos(f*x+e)*(15*cos(f*x+e)^4-20*cos(f*x+e)^2+8)/(cos(f*x+e)-1)^2/(1+c
os(f*x+e))^2/sin(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

3.477.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

$$\int \frac{\cot^6(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = -\frac{\left(15\,\cos\left(fx+e\right)^4-20\,\cos\left(fx+e\right)^2+8\right)\sqrt{a\cos\left(fx+e\right)^2}}{15\,\left(af\cos\left(fx+e\right)^5-2\,af\cos\left(fx+e\right)^3+af\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

input
```
integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(15*cos(f*x + e)^4 - 20*cos(f*x + e)^2 + 8)*sqrt(a*cos(f*x + e)^2)/(
(a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*cos(f*x + e))*sin(f*x + e
))
```

3.477.6 Sympy [F]

$$\int \frac{\cot^6(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx=\int \frac{\cot^6(e+fx)}{\sqrt{-a(\sin(e+fx)-1)(\sin(e+fx)+1)}}\,dx$$

input
```
integrate(cot(f*x+e)**6/(a-a*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**6/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x
)
```

3.477.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1236 vs. 2(86) = 172.

Time = 0.38 (sec) , antiderivative size = 1236, normalized size of antiderivative = 12.88

$$\int \frac{\cot^6(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx=\text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
2/15*((15*sin(9*f*x + 9*e) - 20*sin(7*f*x + 7*e) + 58*sin(5*f*x + 5*e) - 2
0*sin(3*f*x + 3*e) + 15*sin(f*x + e))*cos(10*f*x + 10*e) + 75*(sin(8*f*x +
 8*e) - 2*sin(6*f*x + 6*e) + 2*sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*cos(9*
f*x + 9*e) + 5*(20*sin(7*f*x + 7*e) - 58*sin(5*f*x + 5*e) + 20*sin(3*f*x +
 3*e) - 15*sin(f*x + e))*cos(8*f*x + 8*e) + 100*(2*sin(6*f*x + 6*e) - 2*si
n(4*f*x + 4*e) + sin(2*f*x + 2*e))*cos(7*f*x + 7*e) + 10*(58*sin(5*f*x + 5
*e) - 20*sin(3*f*x + 3*e) + 15*sin(f*x + e))*cos(6*f*x + 6*e) + 290*(2*sin
(4*f*x + 4*e) - sin(2*f*x + 2*e))*cos(5*f*x + 5*e) + 50*(4*sin(3*f*x + 3*e
) - 3*sin(f*x + e))*cos(4*f*x + 4*e) - (15*cos(9*f*x + 9*e) - 20*cos(7*f*x
 + 7*e) + 58*cos(5*f*x + 5*e) - 20*cos(3*f*x + 3*e) + 15*cos(f*x + e))*sin
(10*f*x + 10*e) - 15*(5*cos(8*f*x + 8*e) - 10*cos(6*f*x + 6*e) + 10*cos(4*
f*x + 4*e) - 5*cos(2*f*x + 2*e) + 1)*sin(9*f*x + 9*e) - 5*(20*cos(7*f*x +
7*e) - 58*cos(5*f*x + 5*e) + 20*cos(3*f*x + 3*e) - 15*cos(f*x + e))*sin(8*
f*x + 8*e) - 20*(10*cos(6*f*x + 6*e) - 10*cos(4*f*x + 4*e) + 5*cos(2*f*x +
 2*e) - 1)*sin(7*f*x + 7*e) - 10*(58*cos(5*f*x + 5*e) - 20*cos(3*f*x + 3*e
) + 15*cos(f*x + e))*sin(6*f*x + 6*e) - 58*(10*cos(4*f*x + 4*e) - 5*cos(2*
f*x + 2*e) + 1)*sin(5*f*x + 5*e) - 50*(4*cos(3*f*x + 3*e) - 3*cos(f*x + e)
)*sin(4*f*x + 4*e) - 20*(5*cos(2*f*x + 2*e) - 1)*sin(3*f*x + 3*e) + 100*co
s(3*f*x + 3*e)*sin(2*f*x + 2*e) - 75*cos(f*x + e)*sin(2*f*x + 2*e) + 75*co
s(2*f*x + 2*e)*sin(f*x + e) - 15*sin(f*x + e))*sqrt(a)/((a*cos(10*f*x +...
```

3.477.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^6(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

3.477.9 Mupad [B] (verification not implemented)

Time = 20.96 (sec) , antiderivative size = 491, normalized size of antiderivative = 5.11

$$\int \frac{\cot^6(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = -\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,4i}{a\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)} - \frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,32i}{3\,a\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)^2\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)} - \frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,352i}{15\,a\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)^3\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)} - \frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,128i}{5\,a\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)^4\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)} - \frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,64i}{5\,a\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)^5\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}$$

input
```
int(cot(e + f*x)^6/(a - a*sin(e + f*x)^2)^(1/2),x)
```

output
```
- (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2)^2)^(1/2)*4i)/(a*f*(exp(e*2i + f*x*2i) - 1)*(exp(e*1i + f*x*1i
) + exp(e*3i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1
i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*32i)/(3*a*f*(exp(e*2i + f*x
*2i) - 1)^2*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) - (exp(e*3i + f*x*3
i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/
2)*352i)/(15*a*f*(exp(e*2i + f*x*2i) - 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i
 + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (
exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*128i)/(5*a*f*(exp(e*2i + f*x*2i) - 1)^4
*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*(
(exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*64i)/(5*
a*f*(exp(e*2i + f*x*2i) - 1)^5*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

3.478 $\int \frac{\tan^5(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

3.478.1 Optimal result

Integrand size = 26, antiderivative size = 68

$$\int \frac{\tan^5(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \frac{a^2}{7f\,(a\cos^2(e+fx))^{7/2}} - \frac{2a}{5f\,(a\cos^2(e+fx))^{5/2}} + \frac{1}{3f\,(a\cos^2(e+fx))^{3/2}}$$

output
```
1/7*a^2/f/(a*cos(f*x+e)^2)^(7/2)-2/5*a/f/(a*cos(f*x+e)^2)^(5/2)+1/3/f/(a*c
os(f*x+e)^2)^(3/2)
```

3.478.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

$$\int \frac{\tan^5(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \frac{(15-42\cos^2(e+fx)+35\cos^4(e+fx))\sec^4(e+fx)}{105f\,(a\cos^2(e+fx))^{3/2}}$$

input
```
Integrate[Tan[e + f*x]^5/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
((15 - 42*Cos[e + f*x]^2 + 35*Cos[e + f*x]^4)*Sec[e + f*x]^4)/(105*f*(a*Co
s[e + f*x]^2)^(3/2))
```

3.478.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3655, 3042, 25, 3684, 8, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^5}{\left(a-a\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\tan^5(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)^5\left(a\sin\left(e+fx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\left(a\sin\left(\frac{1}{2}(2e+\pi)+fx\right)^2\right)^{3/2}\tan\left(\frac{1}{2}(2e+\pi)+fx\right)^5}dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \frac{\left(1-\cos^2(e+fx)\right)^2\sec^6(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{8}$$

$$-\frac{a^3\int \frac{\left(1-\cos^2(e+fx)\right)^2}{\left(a\cos^2(e+fx)\right)^{9/2}}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{53}$$

$$-\frac{a^3\int \left(\frac{1}{\left(a\cos^2(e+fx)\right)^{9/2}}-\frac{2}{\left(a\cos^2(e+fx)\right)^{7/2}a}+\frac{1}{\left(a\cos^2(e+fx)\right)^{5/2}a^2}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\left(-\frac{2}{3a^3(a\cos^2(e+fx))^{3/2}}+\frac{4}{5a^2(a\cos^2(e+fx))^{5/2}}-\frac{2}{7a(a\cos^2(e+fx))^{7/2}}\right)}{2f}$$

input
```
Int[Tan[e + f*x]^5/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*(a^3*(-2/(7*a*(a*Cos[e + f*x]^2)^(7/2)) + 4/(5*a^2*(a*Cos[e + f*x]^2)
^(5/2)) - 2/(3*a^3*(a*Cos[e + f*x]^2)^(3/2))))/f
```

3.478.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.478. $\int \frac{\tan^5(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

3.478.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\sqrt{a(\cos^2(fx+e))}\,(35(\cos^4(fx+e))-42(\cos^2(fx+e))+15)}{105a^2\cos(fx+e)^8 f}$	51
risch	$\frac{\frac{8\,e^{10i(fx+e)}}{3}-\frac{32\,e^{8i(fx+e)}}{15}+\frac{304\,e^{6i(fx+e)}}{35}-\frac{32\,e^{4i(fx+e)}}{15}+\frac{8\,e^{2i(fx+e)}}{3}}{f\sqrt{(e^{2i(fx+e)}+1)^2 a\,e^{-2i(fx+e)}}\,(e^{2i(fx+e)}+1)^6 a}$	104

input
```
int(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/105/a^2/cos(f*x+e)^8*(a*cos(f*x+e)^2)^(1/2)*(35*cos(f*x+e)^4-42*cos(f*x+
e)^2+15)/f
```

3.478.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \frac{\tan^5(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \frac{(35\cos(fx+e)^4-42\cos(fx+e)^2+15)\sqrt{a\cos(fx+e)^2}}{105\,a^2 f\cos(fx+e)^8}$$

input
```
integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/105*(35*cos(f*x + e)^4 - 42*cos(f*x + e)^2 + 15)*sqrt(a*cos(f*x + e)^2)/
(a^2*f*cos(f*x + e)^8)
```

3.478.6 Sympy [F]

$$\int \frac{\tan^5(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^5(e+fx)}{(-a(\sin(e+fx)-1)(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a-a*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**5/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2)
, x)
```

3.478.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{\tan^5(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{35\left(a\sin\left(fx+e\right)^2-a\right)^2a^3+42\left(a\sin\left(fx+e\right)^2-a\right)a^4+15\,a^5}{105\left(-a\sin\left(fx+e\right)^2+a\right)^{\frac{7}{2}}a^3f}$$

input
```
integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/105*(35*(a*sin(f*x + e)^2 - a)^2*a^3 + 42*(a*sin(f*x + e)^2 - a)*a^4 + 1
5*a^5)/((-a*sin(f*x + e)^2 + a)^(7/2)*a^3*f)
```

3.478.8 Giac [A] (verification not implemented)

Time = 2.97 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.37

$$\int \frac{\tan^5(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{16\left(70\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8+35\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+21\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-7}{105\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^7a^{\frac{3}{2}}f\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4\right.}$$

input
```
integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
16/105*(70*tan(1/2*f*x + 1/2*e)^8 + 35*tan(1/2*f*x + 1/2*e)^6 + 21*tan(1/2
*f*x + 1/2*e)^4 - 7*tan(1/2*f*x + 1/2*e)^2 + 1)/((tan(1/2*f*x + 1/2*e)^2 -
 1)^7*a^(3/2)*f*sgn(tan(1/2*f*x + 1/2*e)^4 - 1))
```

3.478.9 Mupad [B] (verification not implemented)

Time = 29.97 (sec) , antiderivative size = 583, normalized size of antiderivative = 8.57

$$\int \frac{\tan^5(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \frac{16\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{3\,a^2\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)} - \frac{464\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{15\,a^2\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^3\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)} + \frac{3072\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{35\,a^2\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^4\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)} - \frac{4736\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{35\,a^2\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^5\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)} + \frac{768\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{7\,a^2\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^6\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)} - \frac{256\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{7\,a^2\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^7\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}$$

input

```
int(tan(e + f*x)^5/(a - a*sin(e + f*x)^2)^(3/2),x)
```

output

```
(16*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f
*x*1i)*1i)/2)^2)^(1/2))/(3*a^2*f*(exp(e*2i + f*x*2i) + 1)^2*(exp(e*1i + f*
x*1i) + exp(e*3i + f*x*3i))) - (464*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i
 - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(15*a^2*f*(exp(e*2
i + f*x*2i) + 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) + (3072*exp(
e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1
i)/2)^2)^(1/2))/(35*a^2*f*(exp(e*2i + f*x*2i) + 1)^4*(exp(e*1i + f*x*1i) +
 exp(e*3i + f*x*3i))) - (4736*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x
*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(35*a^2*f*(exp(e*2i + f*
x*2i) + 1)^5*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) + (768*exp(e*3i +
f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2
)^(1/2))/(7*a^2*f*(exp(e*2i + f*x*2i) + 1)^6*(exp(e*1i + f*x*1i) + exp(e*3
i + f*x*3i))) - (256*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/
2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(7*a^2*f*(exp(e*2i + f*x*2i) + 1)
^7*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

3.479 $\int \frac{\tan^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

3.479.1 Optimal result

Integrand size = 26, antiderivative size = 44

$$\int \frac{\tan^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx=\frac{a}{5f\left(a\cos^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

output
```
1/5*a/f/(a*cos(f*x+e)^2)^(5/2)-1/3/f/(a*cos(f*x+e)^2)^(3/2)
```

3.479.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{\tan^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx=\frac{a(3-5\cos^2(e+fx))}{15f\left(a\cos^2(e+fx)\right)^{5/2}}$$

input
```
Integrate[Tan[e + f*x]^3/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(a*(3 - 5*Cos[e + f*x]^2))/(15*f*(a*Cos[e + f*x]^2)^(5/2))
```

3.479.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3655, 3042, 25, 3684, 8, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^3}{(a-a\sin(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\tan^3(e+fx)}{(a\cos^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)^3\left(a\sin\left(e+fx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\left(a\sin\left(\frac{1}{2}(2e+\pi)+fx\right)^2\right)^{3/2}\tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3}dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \frac{(1-\cos^2(e+fx))\sec^4(e+fx)}{(a\cos^2(e+fx))^{3/2}}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{8}$$

$$-\frac{a^2\int \frac{1-\cos^2(e+fx)}{(a\cos^2(e+fx))^{7/2}}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{53}$$

$$-\frac{a^2\int \left(\frac{1}{(a\cos^2(e+fx))^{7/2}}-\frac{1}{a(a\cos^2(e+fx))^{5/2}}\right)d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\left(\frac{2}{3a^2(a\cos^2(e+fx))^{3/2}}-\frac{2}{5a(a\cos^2(e+fx))^{5/2}}\right)}{2f}$$

input
```
Int[Tan[e + f*x]^3/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*(a^2*(-2/(5*a*(a*Cos[e + f*x]^2)^(5/2)) + 2/(3*a^2*(a*Cos[e + f*x]^2)
^(3/2))))/f
```

3.479.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.479. $\int \frac{\tan^3(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

3.479.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,\left(5\left(\cos^2\left(fx+e\right)\right)-3\right)}{15a^2\cos\left(fx+e\right)^6 f}$	41
risch	$-\frac{8\left(5\,\mathrm{e}^{6i(fx+e)}-2\,\mathrm{e}^{4i(fx+e)}+5\,\mathrm{e}^{2i(fx+e)}\right)}{15f\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^2 a\,\mathrm{e}^{-2i(fx+e)}}\,\left(\mathrm{e}^{2i(fx+e)}+1\right)^4 a}$	82

input
```
int(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/a^2/cos(f*x+e)^6*(a*cos(f*x+e)^2)^(1/2)*(5*cos(f*x+e)^2-3)/f
```

3.479.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{\tan^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = -\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(5\,\cos\left(fx+e\right)^2-3\right)}{15\,a^2 f\cos\left(fx+e\right)^6}$$

input
```
integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/15*sqrt(a*cos(f*x + e)^2)*(5*cos(f*x + e)^2 - 3)/(a^2*f*cos(f*x + e)^6)
```

3.479.6 Sympy [F]

$$\int \frac{\tan^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\tan^3\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**3/(a-a*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**3/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2)
, x)
```

3.479.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \frac{\tan^3(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \frac{5\left(a\sin\left(fx+e\right)^2-a\right)a^2+3\,a^3}{15\left(-a\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}a^2f}$$

input
```
integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/15*(5*(a*sin(f*x + e)^2 - a)*a^2 + 3*a^3)/((-a*sin(f*x + e)^2 + a)^(5/2)
*a^2*f)
```

3.479.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(36) = 72.

Time = 0.97 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.82

$$\int \frac{\tan^3(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \frac{4\left(15\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+5\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+5\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)}{15\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^5a^{\frac{3}{2}}f\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-1\right)}$$

input
```
integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
4/15*(15*tan(1/2*f*x + 1/2*e)^6 + 5*tan(1/2*f*x + 1/2*e)^4 + 5*tan(1/2*f*x
 + 1/2*e)^2 - 1)/((tan(1/2*f*x + 1/2*e)^2 - 1)^5*a^(3/2)*f*sgn(tan(1/2*f*x
 + 1/2*e)^4 - 1))
```

3.479.9 Mupad [B] (verification not implemented)

Time = 19.30 (sec) , antiderivative size = 389, normalized size of antiderivative = 8.84

$$\int \frac{\tan^3(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = -\frac{16\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{3\,a^2\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}$$
$$+\frac{272\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{15\,a^2\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^3\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}$$
$$-\frac{128\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{5\,a^2\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^4\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}$$
$$+\frac{64\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{5\,a^2\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^5\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}$$

input
```
int(tan(e + f*x)^3/(a - a*sin(e + f*x)^2)^(3/2),x)
```

output
```
(272*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
f*x*1i)*1i)/2)^2)^(1/2))/(15*a^2*f*(exp(e*2i + f*x*2i) + 1)^3*(exp(e*1i +
f*x*1i) + exp(e*3i + f*x*3i))) - (16*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1
i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(3*a^2*f*(exp(e*2
i + f*x*2i) + 1)^2*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) - (128*exp(e
*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i
)/2)^2)^(1/2))/(5*a^2*f*(exp(e*2i + f*x*2i) + 1)^4*(exp(e*1i + f*x*1i) + e
xp(e*3i + f*x*3i))) + (64*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)
*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(5*a^2*f*(exp(e*2i + f*x*2i)
 + 1)^5*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

3.480 $\int \frac{\tan(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

3.480.1 Optimal result

Integrand size = 24, antiderivative size = 21

$$\int \frac{\tan(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \frac{1}{3f\,(a\cos^2(e+fx))^{3/2}}$$

output
```
1/3/f/(a*cos(f*x+e)^2)^(3/2)
```

3.480.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\tan(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \frac{1}{3f\,(a\cos^2(e+fx))^{3/2}}$$

input
```
Integrate[Tan[e + f*x]/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
1/(3*f*(a*Cos[e + f*x]^2)^(3/2))
```

3.480.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 3655, 3042, 25, 3684, 8, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)}{\left(a-a\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\tan(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)\left(a\sin\left(e+fx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\left(a\sin\left(\frac{1}{2}(2e+\pi)+fx\right)^2\right)^{3/2}\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \frac{\sec^2(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{8}$$

$$-\frac{a\int \frac{1}{\left(a\cos^2(e+fx)\right)^{5/2}}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

input
```
Int[Tan[e + f*x]/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
1/(3*f*(a*Cos[e + f*x]^2)^(3/2))
```

3.480.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.480. $\int \frac{\tan(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

3.480.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{1}{3\left(a-a\left(\sin^{2}\left(fx+e\right)\right)\right)^{\frac{3}{2}} f}$	21
default	$\frac{1}{3\left(a-a\left(\sin^{2}\left(fx+e\right)\right)\right)^{\frac{3}{2}} f}$	21
risch	$\frac{8\,{\mathrm e}^{2 i \left(fx+e\right)}}{3 f \sqrt{\left({\mathrm e}^{2 i \left(fx+e\right)}+1\right)^{2} a\,{\mathrm e}^{-2 i \left(fx+e\right)}}\,\left({\mathrm e}^{2 i \left(fx+e\right)}+1\right)^{2} a}$	57

input
```
int(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(a-a*sin(f*x+e)^2)^(3/2)/f
```

3.480.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{\tan(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx=\frac{\sqrt{a\cos\left(fx+e\right)^2}}{3\,a^2f\cos\left(fx+e\right)^4}$$

input
```
integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(a*cos(f*x + e)^2)/(a^2*f*cos(f*x + e)^4)
```

3.480.6 Sympy [F]

$$\int \frac{\tan(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\tan\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)/(a-a*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2), x
)
```

3.480.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(17) = 34$.

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 4.52

$$\int \frac{\tan(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \frac{\frac{1}{\sqrt{-a\sin(fx+e)^2+a}a\sin(fx+e)+\sqrt{-a\sin(fx+e)^2+a}a} - \frac{1}{\sqrt{-a\sin(fx+e)^2+a}a\sin(fx+e)-\sqrt{-a\sin}}}{6f}$$

input
```
integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/6*(1/(sqrt(-a*sin(f*x + e)^2 + a)*a*sin(f*x + e) + sqrt(-a*sin(f*x + e)^
2 + a)*a) - 1/(sqrt(-a*sin(f*x + e)^2 + a)*a*sin(f*x + e) - sqrt(-a*sin(f*
x + e)^2 + a)*a))/f
```

3.480.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(17) = 34$.

Time = 0.63 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.57

$$\int \frac{\tan(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \frac{2\left(3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+1\right)}{3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^3a^{\frac{3}{2}}f\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}$$

input
```
integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
2/3*(3*tan(1/2*f*x + 1/2*e)^4 + 1)/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*a^(3/2)
*f*sgn(tan(1/2*f*x + 1/2*e)^4 - 1))
```

3.480.9 Mupad [B] (verification not implemented)

Time = 17.01 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.43

$$\int \frac{\tan(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \frac{16\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{3\,a^2\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^4}$$

input
```
int(tan(e + f*x)/(a - a*sin(e + f*x)^2)^(3/2),x)
```

output
```
(16*exp(e*4i + f*x*4i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f
*x*1i)*1i)/2)^2)^(1/2))/(3*a^2*f*(exp(e*2i + f*x*2i) + 1)^4)
```

3.480. $\int \frac{\tan(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

3.481 $\int \frac{\cot(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

3.481.1 Optimal result

Integrand size = 24, antiderivative size = 53

$$\int \frac{\cot(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{1}{af\sqrt{a\cos^2(e+fx)}}$$

output
```
-arctanh((a*cos(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/a/f/(a*cos(f*x+e)^2)^
(1/2)
```

3.481.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{\cot(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx=\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\cos^2(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
Hypergeometric2F1[-1/2, 1, 1/2, Cos[e + f*x]^2]/(a*f*Sqrt[a*Cos[e + f*x]^2
])
```

3.481.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3655, 3042, 25, 3684, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)\left(a-a\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\cot(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(e+fx+\frac{\pi}{2}\right)}{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}{\left(a\sin\left(\frac{1}{2}(2e+\pi)+fx\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \frac{1}{\left(a\cos^2(e+fx)\right)^{3/2}\left(1-\cos^2(e+fx)\right)}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{61}$$

$$-\frac{\frac{\int \frac{1}{\sqrt{a\cos^2(e+fx)}\left(1-\cos^2(e+fx)\right)}d\cos^2(e+fx)}{a}-\frac{2}{a\sqrt{a\cos^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{2\int \frac{1}{1-\frac{\cos^4(e+fx)}{a}}d\sqrt{a\cos^2(e+fx)}}{a^2}-\frac{2}{a\sqrt{a\cos^2(e+fx)}}}{2f}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{a\cos^2(e+fx)}}}{2f}$$

input
```
Int[Cot[e + f*x]/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*((2*ArcTanh[Sqrt[a*Cos[e + f*x]^2/Sqrt[a]])/a^(3/2) - 2/(a*Sqrt[a*Co
s[e + f*x]^2]))/f
```

3.481.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.481. $\int \frac{\cot(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.481.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

method	result	size
default	$-\frac{\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^2(fx+e)\right)}+2a}{\sin(fx+e)}\right)a^2\left(\cos^2(fx+e)\right)-\sqrt{a(\cos^2(fx+e))}\,a^{\frac{3}{2}}}{a^{\frac{7}{2}}\cos(fx+e)^2 f}$	76
risch	$\frac{2}{a\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,f}-\frac{2\ln\left(e^{ifx}+e^{-ie}\right)\cos(fx+e)}{f\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,a}+\frac{2\ln\left(e^{ifx}-e^{-ie}\right)\cos(fx+e)}{f\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,a}$	144

input
```
int(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^(7/2)/cos(f*x+e)^2*(ln(2/sin(f*x+e)*(a^(1/2)*(a*cos(f*x+e)^2)^(1/2)+a
))*a^2*cos(f*x+e)^2-(a*cos(f*x+e)^2)^(1/2)*a^(3/2))/f
```

3.481.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{\cot(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx=-\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(\cos\left(fx+e\right)\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}\right)-2\right)}{2\,a^2 f\cos\left(fx+e\right)^2}$$

input
```
integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(a*cos(f*x + e)^2)*(cos(f*x + e)*log(-(cos(f*x + e) + 1)/(cos(f*x
 + e) - 1)) - 2)/(a^2*f*cos(f*x + e)^2)
```

3.481.6 Sympy [F]

$$\int \frac{\cot(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx=\int \frac{\cot(e+fx)}{(-a(\sin(e+fx)-1)(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(a-a*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2), x
)
```

3.481.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \frac{\cot(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx=-\frac{\frac{\log\left(\frac{2\sqrt{-a\sin(fx+e)^2+a}\sqrt{a}}{|\sin(fx+e)|}+\frac{2a}{|\sin(fx+e)|}\right)}{a^{\frac{3}{2}}}-\frac{1}{\sqrt{-a\sin(fx+e)^2+a}a}}{f}$$

input
```
integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-(log(2*sqrt(-a*sin(f*x + e)^2 + a)*sqrt(a)/abs(sin(f*x + e)) + 2*a/abs(si
n(f*x + e)))/a^(3/2) - 1/(sqrt(-a*sin(f*x + e)^2 + a)*a))/f
```

3.481.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.481.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+f\,x)}{\left(a-a\sin\left(e+f\,x\right)^2\right)^{3/2}}\,dx$$

input
```
int(cot(e + f*x)/(a - a*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)/(a - a*sin(e + f*x)^2)^(3/2), x)
```

3.482 $\int \frac{\cot^3(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

3.482.1 Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \frac{\cot^3(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\sqrt{a\cos^2(e+fx)}\csc^2(e+fx)}{2a^2f}$$

output
```
-1/2*arctanh((a*cos(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-1/2*csc(f*x+e)^2*(a
*cos(f*x+e)^2)^(1/2)/a^2/f
```

3.482.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int \frac{\cot^3(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = -\frac{\sqrt{a\cos^2(e+fx)}\left(\frac{\operatorname{arctanh}\left(\sqrt{\cos^2(e+fx)}\right)}{\sqrt{\cos^2(e+fx)}} + \csc^2(e+fx)\right)}{2a^2f}$$

input
```
Integrate[Cot[e + f*x]^3/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*(Sqrt[a*Cos[e + f*x]^2]*(ArcTanh[Sqrt[Cos[e + f*x]^2]]/Sqrt[Cos[e + f
*x]^2] + Csc[e + f*x]^2))/(a^2*f)
```

3.482.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3042, 3655, 3042, 25, 3684, 8, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^3\left(a-a\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\cot^3(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(e+fx+\frac{\pi}{2}\right)^3}{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3}{\left(a\sin\left(\frac{1}{2}(2e+\pi)+fx\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \frac{\cos^2(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}\left(1-\cos^2(e+fx)\right)^2}d\cos^2(e+fx)}{2f}$$

$$\Big\downarrow \text{8}$$

$$-\frac{\int \frac{1}{\sqrt{a\cos^2(e+fx)}\left(1-\cos^2(e+fx)\right)^2}d\cos^2(e+fx)}{2af}$$

$$\Big\downarrow \text{52}$$

$$-\frac{\frac{1}{2}\int \frac{1}{\sqrt{a\cos^2(e+fx)}\left(1-\cos^2(e+fx)\right)}d\cos^2(e+fx)+\frac{\sqrt{a\cos^2(e+fx)}}{a\left(1-\cos^2(e+fx)\right)}}{2af}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{\int \frac{1}{1-\frac{\cos^4(e+fx)}{a}}d\sqrt{a\cos^2(e+fx)}}{a}+\frac{\sqrt{a\cos^2(e+fx)}}{a(1-\cos^2(e+fx))}}{2af}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{a\cos^2(e+fx)}}{a(1-\cos^2(e+fx))}}{2af}$$

input
```
Int[Cot[e + f*x]^3/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*(ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/Sqrt[a] + Sqrt[a*Cos[e + f*x
]^2]/(a*(1 - Cos[e + f*x]^2)))/(a*f)
```

3.482.3.1 Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.482. $\int \frac{\cot^3(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.482.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

method	result	si
default	$\frac{-\frac{\sqrt{a\left(\cos^2(fx+e)\right)}}{2a^2\sin(fx+e)^2}-\frac{\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^2(fx+e)\right)}+2a}{\sin(fx+e)}\right)}{2a^{\frac{3}{2}}}}{f}$	6
risch	$\frac{\left(e^{2i(fx+e)}+1\right)^2}{a\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,f\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{ifx}+e^{-ie}\right)\cos(fx+e)}{f\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,a}+\frac{\ln\left(e^{ifx}-e^{-ie}\right)\cos(fx+e)}{f\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,a}$	1

input
```
int(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/2/a^2/sin(f*x+e)^2*(a*cos(f*x+e)^2)^(1/2)-1/2/a^(3/2)*ln((2*a+2*a^(1/2
)*(a*cos(f*x+e)^2)^(1/2))/sin(f*x+e)))/f
```

3.482.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26

$$\int \frac{\cot^3(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = -\frac{\sqrt{a\cos(fx+e)^2}\left((\cos(fx+e)^2-1)\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}\right)-2\cos(fx+e)\right)}{4\,(a^2f\cos(fx+e)^3-a^2f\cos(fx+e))}$$

input
```
integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(a*cos(f*x + e)^2)*((cos(f*x + e)^2 - 1)*log(-(cos(f*x + e) + 1)/
(cos(f*x + e) - 1)) - 2*cos(f*x + e))/(a^2*f*cos(f*x + e)^3 - a^2*f*cos(f*
x + e))
```

3.482.6 Sympy [F]

$$\int \frac{\cot^3(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^3(e+fx)}{(-a(\sin(e+fx)-1)(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a-a*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**3/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2)
, x)
```

3.482.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.52

$$\int \frac{\cot^3(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = -\frac{\frac{\log\left(\frac{2\sqrt{-a\sin(fx+e)^2+a}\sqrt{a}}{|\sin(fx+e)|}+\frac{2a}{|\sin(fx+e)|}\right)}{a^{\frac{3}{2}}}-\frac{1}{\sqrt{-a\sin(fx+e)^2+a}a}+\frac{1}{\sqrt{-a\sin(fx+e)^2+a}a\sin(fx+e)^2}}{2f}$$

input
```
integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(log(2*sqrt(-a*sin(f*x + e)^2 + a)*sqrt(a)/abs(sin(f*x + e)) + 2*a/ab
s(sin(f*x + e)))/a^(3/2) - 1/(sqrt(-a*sin(f*x + e)^2 + a)*a) + 1/(sqrt(-a*
sin(f*x + e)^2 + a)*a*sin(f*x + e)^2))/f
```

3.482.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^3(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.482.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+f\,x)^3}{\left(a-a\sin\left(e+f\,x\right)^2\right)^{3/2}}\,dx$$

input
```
int(cot(e + f*x)^3/(a - a*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^3/(a - a*sin(e + f*x)^2)^(3/2), x)
```

3.483 $\int \frac{\tan^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

3.483.1 Optimal result

Integrand size = 26, antiderivative size = 106

$$\int \frac{\tan^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}(\sin(e+fx))\cos(e+fx)}{8af\sqrt{a\cos^2(e+fx)}} - \frac{\tan(e+fx)}{8af\sqrt{a\cos^2(e+fx)}} + \frac{\sec^2(e+fx)\tan(e+fx)}{4af\sqrt{a\cos^2(e+fx)}}$$

output
```
-1/8*arctanh(sin(f*x+e))*cos(f*x+e)/a/f/(a*cos(f*x+e)^2)^(1/2)-1/8*tan(f*x
+e)/a/f/(a*cos(f*x+e)^2)^(1/2)+1/4*sec(f*x+e)^2*tan(f*x+e)/a/f/(a*cos(f*x+
e)^2)^(1/2)
```

3.483.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.56

$$\int \frac{\tan^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \frac{-\operatorname{arctanh}(\sin(e+fx))\cos(e+fx)+(-1+2\sec^2(e+fx))\tan(e+fx)}{8af\sqrt{a\cos^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^2/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-(ArcTanh[Sin[e + f*x]]*Cos[e + f*x]) + (-1 + 2*Sec[e + f*x]^2)*Tan[e + f
*x])/(8*a*f*Sqrt[a*Cos[e + f*x]^2])
```

3.483.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3655, 3042, 3686, 3042, 3091, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{(a-a\sin(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\tan^2(e+fx)}{(a\cos^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)^2\left(a\sin\left(e+fx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cos(e+fx)\int \sec^3(e+fx)\tan^2(e+fx)dx}{a\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(e+fx)\int \sec(e+fx)^3\tan(e+fx)^2dx}{a\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3091}$$

$$\frac{\cos(e+fx)\left(\frac{\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{1}{4}\int \sec^3(e+fx)dx\right)}{a\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(e+fx)\left(\frac{\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{1}{4}\int \csc\left(e+fx+\frac{\pi}{2}\right)^3dx\right)}{a\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\cos(e+fx)\left(\frac{1}{4}\left(-\frac{1}{2}\int \sec(e+fx)dx-\frac{\tan(e+fx)\sec(e+fx)}{2f}\right)+\frac{\tan(e+fx)\sec^3(e+fx)}{4f}\right)}{a\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(e+fx)\left(\frac{1}{4}\left(-\frac{1}{2}\int \csc\left(e+fx+\frac{\pi}{2}\right)dx-\frac{\tan(e+fx)\sec(e+fx)}{2f}\right)+\frac{\tan(e+fx)\sec^3(e+fx)}{4f}\right)}{a\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cos(e+fx)\left(\frac{1}{4}\left(-\frac{\operatorname{arctanh}(\sin(e+fx))}{2f}-\frac{\tan(e+fx)\sec(e+fx)}{2f}\right)+\frac{\tan(e+fx)\sec^3(e+fx)}{4f}\right)}{a\sqrt{a\cos^2(e+fx)}}$$

input
```
Int[Tan[e + f*x]^2/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Cos[e + f*x]*((Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (-1/2*ArcTanh[Sin[e +
 f*x]]/f - (Sec[e + f*x]*Tan[e + f*x])/(2*f))/4))/(a*Sqrt[a*Cos[e + f*x]^2
])
```

3.483.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.483. $\int \frac{\tan^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.483.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98

method	result
default	$\frac{(\ln(1+\sin(fx+e))-\ln(\sin(fx+e)-1))(\cos^4(fx+e))+2(\cos^2(fx+e))\sin(fx+e)-4\sin(fx+e)}{16a(1+\sin(fx+e))(\sin(fx+e)-1)\cos(fx+e)\sqrt{a(\cos^2(fx+e))}\,f}$
risch	$\frac{i(e^{6i(fx+e)}-7e^{4i(fx+e)}+7e^{2i(fx+e)}-1)}{4a(e^{2i(fx+e)}+1)^3\sqrt{(e^{2i(fx+e)}+1)^2 a e^{-2i(fx+e)}}\,f}-\frac{\ln(e^{ifx}+ie^{-ie})\cos(fx+e)}{4f\sqrt{(e^{2i(fx+e)}+1)^2 a e^{-2i(fx+e)}}\,a}+\frac{\ln(e^{ifx}-ie^{-ie})\cos(fx+e)}{4f\sqrt{(e^{2i(fx+e)}+1)^2 a e^{-2i(fx+e)}}\,a}$

input
```
int(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/a*((ln(1+sin(f*x+e))-ln(sin(f*x+e)-1))*cos(f*x+e)^4+2*cos(f*x+e)^2*si
n(f*x+e)-4*sin(f*x+e))/(1+sin(f*x+e))/(sin(f*x+e)-1)/cos(f*x+e)/(a*cos(f*x
+e)^2)^(1/2)/f
```

3.483.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.73

$$\int \frac{\tan^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = -\frac{\left(\cos(fx+e)^4\log\left(-\frac{\sin(fx+e)+1}{\sin(fx+e)-1}\right)+2\left(\cos(fx+e)^2-2\right)\sin(fx+e)\right)\sqrt{a\cos(fx+e)^2}}{16\,a^2 f\cos(fx+e)^5}$$

input
```
integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/16*(cos(f*x + e)^4*log(-(sin(f*x + e) + 1)/(sin(f*x + e) - 1)) + 2*(cos
(f*x + e)^2 - 2)*sin(f*x + e))*sqrt(a*cos(f*x + e)^2)/(a^2*f*cos(f*x + e)^
5)
```

3.483.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^2(e+fx)}{(-a(\sin(e+fx)-1)(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a-a*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**2/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2)
, x)
```

3.483.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1532 vs. 2(94) = 188.

Time = 0.53 (sec) , antiderivative size = 1532, normalized size of antiderivative = 14.45

$$\int \frac{\tan^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/16*(4*(sin(7*f*x + 7*e) - 7*sin(5*f*x + 5*e) + 7*sin(3*f*x + 3*e) - sin
(f*x + e))*cos(8*f*x + 8*e) - 8*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) +
 2*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 16*(7*sin(5*f*x + 5*e) - 7*sin(3*f
*x + 3*e) + sin(f*x + e))*cos(6*f*x + 6*e) + 56*(3*sin(4*f*x + 4*e) + 2*si
n(2*f*x + 2*e))*cos(5*f*x + 5*e) + 24*(7*sin(3*f*x + 3*e) - sin(f*x + e))*
cos(4*f*x + 4*e) + (2*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f
*x + 2*e) + 1)*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*
e) + 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12
*(4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*co
s(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*
x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) +
 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f
*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2
 + 8*cos(2*f*x + 2*e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x
 + e) + 1) - (2*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2
*e) + 1)*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4
*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*co
s(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*
x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*
e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*...
```

3.483.8 Giac [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{\tan^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx =$$
$$-\frac{\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)^3+\frac{4}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{4\left(\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)^2-4\right)^2a^{\frac{3}{2}}f\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}$$

input
```
integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/4*((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^3 + 4/tan(1/2*f*x +
1/2*e) + 4*tan(1/2*f*x + 1/2*e))/(((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x +
 1/2*e))^2 - 4)^2*a^(3/2)*f*sgn(tan(1/2*f*x + 1/2*e)^4 - 1))
```

3.483.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^2}{\left(a-a\sin\left(e+f\,x\right)^2\right)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^2/(a - a*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^2/(a - a*sin(e + f*x)^2)^(3/2), x)
```

3.484 $\int \frac{\cot^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

3.484.1 Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{\cot^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \frac{\operatorname{arctanh}(\sin(e+fx))\cos(e+fx)}{af\sqrt{a\cos^2(e+fx)}} - \frac{\cot(e+fx)}{af\sqrt{a\cos^2(e+fx)}}$$

output
```
arctanh(sin(f*x+e))*cos(f*x+e)/a/f/(a*cos(f*x+e)^2)^(1/2)-cot(f*x+e)/a/f/(
a*cos(f*x+e)^2)^(1/2)
```

3.484.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \frac{\cot^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]^2/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-((Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[e + f*x]^2])/(a*f*Sqrt
[a*Cos[e + f*x]^2]))
```

3.484.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3042, 3655, 3042, 3686, 3042, 3101, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^2\left(a-a\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\cot^2(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(e+fx+\frac{\pi}{2}\right)^2}{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cos(e+fx)\int \csc^2(e+fx)\sec(e+fx)dx}{a\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(e+fx)\int \csc(e+fx)^2\sec(e+fx)dx}{a\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\cos(e+fx)\int -\frac{\csc^2(e+fx)}{1-\csc^2(e+fx)}d\csc(e+fx)}{af\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\cos(e+fx)\int \frac{\csc^2(e+fx)}{1-\csc^2(e+fx)}d\csc(e+fx)}{af\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\cos(e+fx)\left(\csc(e+fx)-\int\frac{1}{1-\csc^2(e+fx)}d\csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\cos(e+fx)(\csc(e+fx)-\text{arctanh}(\csc(e+fx)))}{af\sqrt{a\cos^2(e+fx)}}$$

input
```
Int[Cot[e + f*x]^2/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-((Cos[e + f*x]*(-ArcTanh[Csc[e + f*x]] + Csc[e + f*x]))/(a*f*Sqrt[a*Cos[e
 + f*x]^2]))
```

3.484.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.484. $\int \frac{\cot^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.484.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(59) = 118$.

Time = 1.01 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.95

method	result
default	$\frac{\cos(fx+e)\sqrt{a(\sin^2(fx+e))}\left(-\ln\left(\frac{2\sqrt{a}\sqrt{a(\sin^2(fx+e))}+2a}{\cos(fx+e)}\right)a(\sin^2(fx+e))+\sqrt{a}\sqrt{a(\sin^2(fx+e))}\right)}{a^{\frac{5}{2}}(1+\cos(fx+e))(\cos(fx+e)-1)\sin(fx+e)\sqrt{a(\cos^2(fx+e))}\,f}$
risch	$-\frac{2i(e^{2i(fx+e)}+1)}{a\sqrt{(e^{2i(fx+e)}+1)^2 a\,e^{-2i(fx+e)}}\,f(e^{2i(fx+e)}-1)}-\frac{2\ln(e^{ifx}-ie^{-ie})\cos(fx+e)}{f\sqrt{(e^{2i(fx+e)}+1)^2 a\,e^{-2i(fx+e)}}\,a}+\frac{2\ln(e^{ifx}+ie^{-ie})\cos(fx+e)}{f\sqrt{(e^{2i(fx+e)}+1)^2 a\,e^{-2i(fx+e)}}\,a}$

input
```
int(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^(5/2)*cos(f*x+e)*(a*sin(f*x+e)^2)^(1/2)*(-ln(2/cos(f*x+e)*(a^(1/2)*(a*
sin(f*x+e)^2)^(1/2)+a))*a*sin(f*x+e)^2+a^(1/2)*(a*sin(f*x+e)^2)^(1/2))/(1+
cos(f*x+e))/(cos(f*x+e)-1)/sin(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

3.484.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \frac{\cot^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = -\frac{\sqrt{a\cos{(fx+e)}^2}\left(\log\left(-\frac{\sin(fx+e)-1}{\sin(fx+e)+1}\right)\sin{(fx+e)}+2\right)}{2\,a^2f\cos{(fx+e)}\sin{(fx+e)}}$$

input
```
integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(a*cos(f*x + e)^2)*(log(-(sin(f*x + e) - 1)/(sin(f*x + e) + 1))*s
in(f*x + e) + 2)/(a^2*f*cos(f*x + e)*sin(f*x + e))
```

3.484.6 Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^2{(e+fx)}}{(-a(\sin{(e+fx)}-1)(\sin{(e+fx)}+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a-a*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**2/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2)
, x)
```

3.484.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(59) = 118.

Time = 0.38 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.49

$$\int \frac{\cot^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \frac{\left(\cos{(2\,fx+2\,e)}^2+\sin{(2\,fx+2\,e)}^2-2\,\cos{(2\,fx+2\,e)}+1\right)\log\left(\cos{(fx+}\right.}{}$$

input
```
integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 - 2*cos(2*f*x + 2*e) + 1)*lo
g(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - (cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 - 2*cos(2*f*x + 2*e) + 1)*log(cos(f*x + e)^2 + si
n(f*x + e)^2 - 2*sin(f*x + e) + 1) - 4*cos(f*x + e)*sin(2*f*x + 2*e) + 4*c
os(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f*x + e))/((a*cos(2*f*x + 2*e)^2 + a*
sin(2*f*x + 2*e)^2 - 2*a*cos(2*f*x + 2*e) + a)*sqrt(a)*f)
```

3.484.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.484.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+f\,x)^2}{\left(a-a\sin\left(e+f\,x\right)^2\right)^{3/2}}\,dx$$

input
```
int(cot(e + f*x)^2/(a - a*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^2/(a - a*sin(e + f*x)^2)^(3/2), x)
```

3.485 $\int \frac{\cot^4(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

3.485.1 Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{\cot^4(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = -\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}$$

output
```
-1/3*cot(f*x+e)*csc(f*x+e)^2/a/f/(a*cos(f*x+e)^2)^(1/2)
```

3.485.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{\cot^4(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = -\frac{\cot^3(e+fx)}{3f\,(a\cos^2(e+fx))^{3/2}}$$

input
```
Integrate[Cot[e + f*x]^4/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-1/3*Cot[e + f*x]^3/(f*(a*Cos[e + f*x]^2)^(3/2))
```

3.485.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3655, 3042, 3686, 3042, 25, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^4\left(a-a\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\cot^4(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(e+fx+\frac{\pi}{2}\right)^4}{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cos(e+fx)\int \cot(e+fx)\csc^3(e+fx)dx}{a\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(e+fx)\int -\sec\left(e+fx-\frac{\pi}{2}\right)^3\tan\left(e+fx-\frac{\pi}{2}\right)dx}{a\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\cos(e+fx)\int \sec\left(\frac{1}{2}(2e-\pi)+fx\right)^3\tan\left(\frac{1}{2}(2e-\pi)+fx\right)dx}{a\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\cos(e+fx)\int \csc^2(e+fx)d\csc(e+fx)}{af\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}$$

input
```
Int[Cot[e + f*x]^4/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-1/3*(Cot[e + f*x]*Csc[e + f*x]^2)/(a*f*Sqrt[a*Cos[e + f*x]^2])
```

3.485.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.485. $\int \frac{\cot^4(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

3.485.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{\cos(fx+e)}{3\sin(fx+e)^{3}a\sqrt{a(\cos^{2}(fx+e))}\,f}$	35
risch	$\frac{8i\left(\mathrm{e}^{4i(fx+e)}+\mathrm{e}^{2i(fx+e)}\right)}{3\left(\mathrm{e}^{2i(fx+e)}-1\right)^{3}f\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^{2}a\,\mathrm{e}^{-2i(fx+e)}}\,a}$	68

input
```
int(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*cos(f*x+e)/sin(f*x+e)^3/a/(a*cos(f*x+e)^2)^(1/2)/f
```

3.485.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{\cot^4(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \frac{\sqrt{a\cos\left(fx+e\right)^2}}{3\left(a^2f\cos\left(fx+e\right)^3-a^2f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

input
```
integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(a*cos(f*x + e)^2)/((a^2*f*cos(f*x + e)^3 - a^2*f*cos(f*x + e))*si
n(f*x + e))
```

3.485.6 Sympy [F]

$$\int \frac{\cot^4(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^4\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a-a*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**4/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2)
, x)
```

3.485.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(34) = 68$.

Time = 0.34 (sec) , antiderivative size = 382, normalized size of antiderivative = 10.05

$$\int \frac{\cot^4(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{}{3\left(a^2\cos\left(6\,fx+6\,e\right)^2+9\,a^2\cos\left(4\,fx+4\,e\right)^2+9\,a^2\cos\left(2\,fx+2\,e\right)^2+a^2\sin\ldots\right.}$$

input
```
integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
8/3*(cos(3*f*x + 3*e)*sin(6*f*x + 6*e) - 3*cos(3*f*x + 3*e)*sin(4*f*x + 4*
e) - (3*cos(2*f*x + 2*e) - 1)*sin(3*f*x + 3*e) - cos(6*f*x + 6*e)*sin(3*f*
x + 3*e) + 3*cos(4*f*x + 4*e)*sin(3*f*x + 3*e) + 3*cos(3*f*x + 3*e)*sin(2*
f*x + 2*e))*sqrt(a)/((a^2*cos(6*f*x + 6*e)^2 + 9*a^2*cos(4*f*x + 4*e)^2 +
9*a^2*cos(2*f*x + 2*e)^2 + a^2*sin(6*f*x + 6*e)^2 + 9*a^2*sin(4*f*x + 4*e)
^2 - 18*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*a^2*sin(2*f*x + 2*e)^2 -
 6*a^2*cos(2*f*x + 2*e) + a^2 - 2*(3*a^2*cos(4*f*x + 4*e) - 3*a^2*cos(2*f*
x + 2*e) + a^2)*cos(6*f*x + 6*e) - 6*(3*a^2*cos(2*f*x + 2*e) - a^2)*cos(4*
f*x + 4*e) - 6*(a^2*sin(4*f*x + 4*e) - a^2*sin(2*f*x + 2*e))*sin(6*f*x + 6
*e))*f)
```

3.485.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^4(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

3.485.9 Mupad [B] (verification not implemented)

Time = 17.72 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.32

$$\int \frac{\cot^4(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx=\frac{\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,16\mathrm{i}}{3\,a^2\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^3\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}$$

input
```
int(cot(e + f*x)^4/(a - a*sin(e + f*x)^2)^(3/2),x)
```

output
```
(exp(e*4i + f*x*4i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*
1i)*1i)/2)^2)^(1/2)*16i)/(3*a^2*f*(exp(e*2i + f*x*2i) - 1)^3*(exp(e*2i + f
*x*2i) + 1))
```

3.486 $\int \frac{\cot^6(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

3.486.1 Optimal result

Integrand size = 26, antiderivative size = 77

$$\int \frac{\cot^6(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}}$$

output
```
1/3*cot(f*x+e)*csc(f*x+e)^2/a/f/(a*cos(f*x+e)^2)^(1/2)-1/5*cot(f*x+e)*csc(
f*x+e)^4/a/f/(a*cos(f*x+e)^2)^(1/2)
```

3.486.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.53

$$\int \frac{\cot^6(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = -\frac{\cot^3(e+fx)\left(-5+3\csc^2(e+fx)\right)}{15f\,(a\cos^2(e+fx))^{3/2}}$$

input
```
Integrate[Cot[e + f*x]^6/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-1/15*(Cot[e + f*x]^3*(-5 + 3*Csc[e + f*x]^2))/(f*(a*Cos[e + f*x]^2)^(3/2)
)
```

3.486.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.69, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3655, 3042, 3686, 3042, 25, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^6(a-a\sin(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\cot^6(e+fx)}{(a\cos^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(e+fx+\frac{\pi}{2}\right)^6}{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cos(e+fx)\int \cot^3(e+fx)\csc^3(e+fx)dx}{a\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(e+fx)\int -\sec\left(e+fx-\frac{\pi}{2}\right)^3\tan\left(e+fx-\frac{\pi}{2}\right)^3dx}{a\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\cos(e+fx)\int \sec\left(\frac{1}{2}(2e-\pi)+fx\right)^3\tan\left(\frac{1}{2}(2e-\pi)+fx\right)^3dx}{a\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\cos(e+fx)\int -\csc^2(e+fx)\left(1-\csc^2(e+fx)\right)d\csc(e+fx)}{af\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\cos(e+fx)\int \csc^2(e+fx)\left(1-\csc^2(e+fx)\right)d\csc(e+fx)}{af\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\cos(e+fx)\int \left(\csc^2(e+fx)-\csc^4(e+fx)\right)d\csc(e+fx)}{af\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\cos(e+fx)\left(\frac{1}{5}\csc^5(e+fx)-\frac{1}{3}\csc^3(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}$$

input
```
Int[Cot[e + f*x]^6/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-((Cos[e + f*x]*(-1/3*Csc[e + f*x]^3 + Csc[e + f*x]^5/5))/(a*f*Sqrt[a*Cos[
e + f*x]^2]))
```

3.486.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.486. $\int \frac{\cot^6(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.486.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\cos(fx+e)\left(5\left(\sin^2(fx+e)\right)-3\right)}{15a\sin(fx+e)^5\sqrt{a\left(\cos^2(fx+e)\right)}\,f}$	47
risch	$-\frac{8i\left(\mathrm{e}^{2i(fx+e)}+1\right)\left(5\,\mathrm{e}^{6i(fx+e)}+2\,\mathrm{e}^{4i(fx+e)}+5\,\mathrm{e}^{2i(fx+e)}\right)}{15\left(\mathrm{e}^{2i(fx+e)}-1\right)^5 f\sqrt{\left(\mathrm{e}^{2i(fx+e)}+1\right)^2 a\,\mathrm{e}^{-2i(fx+e)}}\,a}$	94

input
```
int(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/a*cos(f*x+e)*(5*sin(f*x+e)^2-3)/sin(f*x+e)^5/(a*cos(f*x+e)^2)^(1/2)/f
```

3.486.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{\cot^6(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = -\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(5\,\cos\left(fx+e\right)^2-2\right)}{15\left(a^2f\cos\left(fx+e\right)^5-2\,a^2f\cos\left(fx+e\right)^3+a^2f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

input
```
integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/15*sqrt(a*cos(f*x + e)^2)*(5*cos(f*x + e)^2 - 2)/((a^2*f*cos(f*x + e)^5
 - 2*a^2*f*cos(f*x + e)^3 + a^2*f*cos(f*x + e))*sin(f*x + e))
```

3.486.6 Sympy [F]

$$\int \frac{\cot^6(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx=\int \frac{\cot^6(e+fx)}{(-a(\sin(e+fx)-1)(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**6/(a-a*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**6/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2)
, x)
```

3.486.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. 2(69) = 138.

Time = 0.35 (sec) , antiderivative size = 1063, normalized size of antiderivative = 13.81

$$\int \frac{\cot^6(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
8/15*((5*sin(7*f*x + 7*e) + 2*sin(5*f*x + 5*e) + 5*sin(3*f*x + 3*e))*cos(1
0*f*x + 10*e) - 5*(5*sin(7*f*x + 7*e) + 2*sin(5*f*x + 5*e) + 5*sin(3*f*x +
 3*e))*cos(8*f*x + 8*e) - 25*(2*sin(6*f*x + 6*e) - 2*sin(4*f*x + 4*e) + si
n(2*f*x + 2*e))*cos(7*f*x + 7*e) + 10*(2*sin(5*f*x + 5*e) + 5*sin(3*f*x +
3*e))*cos(6*f*x + 6*e) + 10*(2*sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*cos(5*
f*x + 5*e) - (5*cos(7*f*x + 7*e) + 2*cos(5*f*x + 5*e) + 5*cos(3*f*x + 3*e)
)*sin(10*f*x + 10*e) + 5*(5*cos(7*f*x + 7*e) + 2*cos(5*f*x + 5*e) + 5*cos(
3*f*x + 3*e))*sin(8*f*x + 8*e) + 5*(10*cos(6*f*x + 6*e) - 10*cos(4*f*x + 4
*e) + 5*cos(2*f*x + 2*e) - 1)*sin(7*f*x + 7*e) - 10*(2*cos(5*f*x + 5*e) +
5*cos(3*f*x + 3*e))*sin(6*f*x + 6*e) - 2*(10*cos(4*f*x + 4*e) - 5*cos(2*f*
x + 2*e) + 1)*sin(5*f*x + 5*e) + 50*cos(3*f*x + 3*e)*sin(4*f*x + 4*e) + 5*
(5*cos(2*f*x + 2*e) - 1)*sin(3*f*x + 3*e) - 50*cos(4*f*x + 4*e)*sin(3*f*x
+ 3*e) - 25*cos(3*f*x + 3*e)*sin(2*f*x + 2*e))*sqrt(a)/((a^2*cos(10*f*x +
10*e)^2 + 25*a^2*cos(8*f*x + 8*e)^2 + 100*a^2*cos(6*f*x + 6*e)^2 + 100*a^2
*cos(4*f*x + 4*e)^2 + 25*a^2*cos(2*f*x + 2*e)^2 + a^2*sin(10*f*x + 10*e)^2
 + 25*a^2*sin(8*f*x + 8*e)^2 + 100*a^2*sin(6*f*x + 6*e)^2 + 100*a^2*sin(4*
f*x + 4*e)^2 - 100*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 25*a^2*sin(2*f*
x + 2*e)^2 - 10*a^2*cos(2*f*x + 2*e) + a^2 - 2*(5*a^2*cos(8*f*x + 8*e) - 1
0*a^2*cos(6*f*x + 6*e) + 10*a^2*cos(4*f*x + 4*e) - 5*a^2*cos(2*f*x + 2*e)
+ a^2)*cos(10*f*x + 10*e) - 10*(10*a^2*cos(6*f*x + 6*e) - 10*a^2*cos(4*...
```

3.486.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^6(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

3.486.9 Mupad [B] (verification not implemented)

Time = 19.60 (sec) , antiderivative size = 393, normalized size of antiderivative = 5.10

$$\int \frac{\cot^6(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = -\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,16i}{3\,a^2\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)^2\,\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}$$
$$-\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,272i}{15\,a^2\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)^3\,\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}$$
$$-\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,128i}{5\,a^2\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)^4\,\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}$$
$$-\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,64i}{5\,a^2\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)^5\,\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}$$

input
```
int(cot(e + f*x)^6/(a - a*sin(e + f*x)^2)^(3/2),x)
```

output
```
- (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2)^2)^(1/2)*16i)/(3*a^2*f*(exp(e*2i + f*x*2i) - 1)^2*(exp(e*1i +
 f*x*1i) + exp(e*3i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i
- f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*272i)/(15*a^2*f*(exp
(e*2i + f*x*2i) - 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) - (exp(e
*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i
)/2)^2)^(1/2)*128i)/(5*a^2*f*(exp(e*2i + f*x*2i) - 1)^4*(exp(e*1i + f*x*1i
) + exp(e*3i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1
i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*64i)/(5*a^2*f*(exp(e*2i + f
*x*2i) - 1)^5*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

3.487 $\int \frac{\cot^8(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx$

3.487.1 Optimal result

Integrand size = 26, antiderivative size = 115

$$\int \frac{\cot^8(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = -\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}} + \frac{2\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc^6(e+fx)}{7af\sqrt{a\cos^2(e+fx)}}$$

output
```
-1/3*cot(f*x+e)*csc(f*x+e)^2/a/f/(a*cos(f*x+e)^2)^(1/2)+2/5*cot(f*x+e)*csc
(f*x+e)^4/a/f/(a*cos(f*x+e)^2)^(1/2)-1/7*cot(f*x+e)*csc(f*x+e)^6/a/f/(a*co
s(f*x+e)^2)^(1/2)
```

3.487.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

$$\int \frac{\cot^8(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = -\frac{\cot^3(e+fx)\left(35-42\csc^2(e+fx)+15\csc^4(e+fx)\right)}{105f\left(a\cos^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Cot[e + f*x]^8/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-1/105*(Cot[e + f*x]^3*(35 - 42*Csc[e + f*x]^2 + 15*Csc[e + f*x]^4))/(f*(a
*Cos[e + f*x]^2)^(3/2))
```

3.487.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.57, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 3655, 3042, 3686, 3042, 25, 3086, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^8(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^8(a-a\sin(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{\cot^8(e+fx)}{(a\cos^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(e+fx+\frac{\pi}{2}\right)^8}{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cos(e+fx)\int \cot^5(e+fx)\csc^3(e+fx)dx}{a\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(e+fx)\int -\sec\left(e+fx-\frac{\pi}{2}\right)^3\tan\left(e+fx-\frac{\pi}{2}\right)^5dx}{a\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\cos(e+fx)\int \sec\left(\frac{1}{2}(2e-\pi)+fx\right)^3\tan\left(\frac{1}{2}(2e-\pi)+fx\right)^5dx}{a\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\cos(e+fx)\int \csc^2(e+fx)\left(1-\csc^2(e+fx)\right)^2d\csc(e+fx)}{af\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\cos(e+fx)\int\left(\csc^6(e+fx)-2\csc^4(e+fx)+\csc^2(e+fx)\right)d\csc(e+fx)}{af\sqrt{a\cos^2(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos(e+fx)\left(\frac{1}{7}\csc^7(e+fx)-\frac{2}{5}\csc^5(e+fx)+\frac{1}{3}\csc^3(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}$$

input
```
Int[Cot[e + f*x]^8/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-((Cos[e + f*x]*(Csc[e + f*x]^3/3 - (2*Csc[e + f*x]^5)/5 + Csc[e + f*x]^7/
7))/(a*f*Sqrt[a*Cos[e + f*x]^2]))
```

3.487.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.487. $\int \frac{\cot^8(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.487.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

method	result	size
default	$-\frac{\cos(fx+e)\left(35\left(\cos^4(fx+e)\right)-28\left(\cos^2(fx+e)\right)+8\right)}{105a\sin(fx+e)^7\sqrt{a\left(\cos^2(fx+e)\right)}\,f}$	57
risch	$\frac{8i\left(e^{2i(fx+e)}+1\right)\left(35\,e^{10i(fx+e)}+28\,e^{8i(fx+e)}+114\,e^{6i(fx+e)}+28\,e^{4i(fx+e)}+35\,e^{2i(fx+e)}\right)}{105\left(e^{2i(fx+e)}-1\right)^7 f\sqrt{\left(e^{2i(fx+e)}+1\right)^2 a\,e^{-2i(fx+e)}}\,a}$	116

input
```
int(cot(f*x+e)^8/(a-a*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105/a*cos(f*x+e)*(35*cos(f*x+e)^4-28*cos(f*x+e)^2+8)/sin(f*x+e)^7/(a*co
s(f*x+e)^2)^(1/2)/f
```

3.487.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

$$\int \frac{\cot^8(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(35\,\cos\left(fx+e\right)^4-28\,\cos\left(fx+e\right)^2+8\right)\sqrt{a\cos\left(fx+\right.}}{105\left(a^2f\cos\left(fx+e\right)^7-3\,a^2f\cos\left(fx+e\right)^5+3\,a^2f\cos\left(fx+e\right)^3-a^2f\cos\right.}$$

input
```
integrate(cot(f*x+e)^8/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/105*(35*cos(f*x + e)^4 - 28*cos(f*x + e)^2 + 8)*sqrt(a*cos(f*x + e)^2)/(
(a^2*f*cos(f*x + e)^7 - 3*a^2*f*cos(f*x + e)^5 + 3*a^2*f*cos(f*x + e)^3 -
a^2*f*cos(f*x + e))*sin(f*x + e))
```

3.487.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^8(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**8/(a-a*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.487.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2026 vs. 2(103) = 206.

Time = 0.36 (sec) , antiderivative size = 2026, normalized size of antiderivative = 17.62

$$\int \frac{\cot^8(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^8/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-8/105*((35*sin(11*f*x + 11*e) + 28*sin(9*f*x + 9*e) + 114*sin(7*f*x + 7*e
) + 28*sin(5*f*x + 5*e) + 35*sin(3*f*x + 3*e))*cos(14*f*x + 14*e) - 7*(35*
sin(11*f*x + 11*e) + 28*sin(9*f*x + 9*e) + 114*sin(7*f*x + 7*e) + 28*sin(5
*f*x + 5*e) + 35*sin(3*f*x + 3*e))*cos(12*f*x + 12*e) - 245*(3*sin(10*f*x
+ 10*e) - 5*sin(8*f*x + 8*e) + 5*sin(6*f*x + 6*e) - 3*sin(4*f*x + 4*e) + s
in(2*f*x + 2*e))*cos(11*f*x + 11*e) + 21*(28*sin(9*f*x + 9*e) + 114*sin(7*
f*x + 7*e) + 28*sin(5*f*x + 5*e) + 35*sin(3*f*x + 3*e))*cos(10*f*x + 10*e)
 + 196*(5*sin(8*f*x + 8*e) - 5*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) - sin
(2*f*x + 2*e))*cos(9*f*x + 9*e) - 35*(114*sin(7*f*x + 7*e) + 28*sin(5*f*x
+ 5*e) + 35*sin(3*f*x + 3*e))*cos(8*f*x + 8*e) - 798*(5*sin(6*f*x + 6*e) -
 3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*cos(7*f*x + 7*e) + 245*(4*sin(5*f*
x + 5*e) + 5*sin(3*f*x + 3*e))*cos(6*f*x + 6*e) + 196*(3*sin(4*f*x + 4*e)
- sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - (35*cos(11*f*x + 11*e) + 28*cos(9*f
*x + 9*e) + 114*cos(7*f*x + 7*e) + 28*cos(5*f*x + 5*e) + 35*cos(3*f*x + 3*
e))*sin(14*f*x + 14*e) + 7*(35*cos(11*f*x + 11*e) + 28*cos(9*f*x + 9*e) +
114*cos(7*f*x + 7*e) + 28*cos(5*f*x + 5*e) + 35*cos(3*f*x + 3*e))*sin(12*f
*x + 12*e) + 35*(21*cos(10*f*x + 10*e) - 35*cos(8*f*x + 8*e) + 35*cos(6*f*
x + 6*e) - 21*cos(4*f*x + 4*e) + 7*cos(2*f*x + 2*e) - 1)*sin(11*f*x + 11*e
) - 21*(28*cos(9*f*x + 9*e) + 114*cos(7*f*x + 7*e) + 28*cos(5*f*x + 5*e) +
 35*cos(3*f*x + 3*e))*sin(10*f*x + 10*e) - 28*(35*cos(8*f*x + 8*e) - 35...
```

3.487.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^8(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^8/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

3.487.9 Mupad [B] (verification not implemented)

Time = 30.49 (sec) , antiderivative size = 589, normalized size of antiderivative = 5.12

$$\int \frac{\cot^8(e+fx)}{(a-a\sin^2(e+fx))^{3/2}}\,dx = \frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,16i}{3\,a^2\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)^2\,\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}$$
$$+\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,464i}{15\,a^2\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)^3\,\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}$$
$$+\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,3072i}{35\,a^2\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)^4\,\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}$$
$$+\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,4736i}{35\,a^2\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)^5\,\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}$$
$$+\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,768i}{7\,a^2\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)^6\,\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}$$
$$+\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,256i}{7\,a^2\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)^7\,\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}$$

input
```
int(cot(e + f*x)^8/(a - a*sin(e + f*x)^2)^(3/2),x)
```

output
```
(exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*
1i)*1i)/2)^2)^(1/2)*16i)/(3*a^2*f*(exp(e*2i + f*x*2i) - 1)^2*(exp(e*1i + f
*x*1i) + exp(e*3i + f*x*3i))) + (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i -
f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*464i)/(15*a^2*f*(exp(e
*2i + f*x*2i) - 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) + (exp(e*3
i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/
2)^2)^(1/2)*3072i)/(35*a^2*f*(exp(e*2i + f*x*2i) - 1)^4*(exp(e*1i + f*x*1i
) + exp(e*3i + f*x*3i))) + (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1
i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*4736i)/(35*a^2*f*(exp(e*2i
+ f*x*2i) - 1)^5*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) + (exp(e*3i +
f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2
)^(1/2)*768i)/(7*a^2*f*(exp(e*2i + f*x*2i) - 1)^6*(exp(e*1i + f*x*1i) + ex
p(e*3i + f*x*3i))) + (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)
/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*256i)/(7*a^2*f*(exp(e*2i + f*x*2i
) - 1)^7*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

3.487. $\int \frac{\cot^8(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

3.488 $\int \sqrt{a+b\sin^2(e+fx)}\tan^5(e+fx)\,dx$

3.488.1 Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^5(e+fx)\,dx = \frac{(8a^2+24ab+15b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}f} - \frac{(8a^2+24ab+15b^2)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)^2f} - \frac{(8a+7b)\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{8(a+b)^2f} + \frac{\sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4(a+b)f}$$

output
```
1/8*(8*a^2+24*a*b+15*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a
+b)^(3/2)/f-1/8*(8*a+7*b)*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^2/f+
1/4*sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2)/(a+b)/f-1/8*(8*a^2+24*a*b+15*b^2
)*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)^2/f
```

3.488.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.81

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^5(e+fx)\,dx$$
$$= \frac{-\left((8a+7b)\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\right)+2(a+b)\sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}+(8a^2}{8(a+b)^2f}$$

input
```
Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^5,x]
```

output
```
(-((8*a + 7*b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2)) + 2*(a + b)*Se
c[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2) + (8*a^2 + 24*a*b + 15*b^2)*(Sqr
t[a + b]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] - Sqrt[a + b*Sin[
e + f*x]^2]))/(8*(a + b)^2*f)
```

3.488.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3673, 100, 27, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \tan(e+fx)^5\sqrt{a+b\sin(e+fx)^2}dx$$
$$\Big\downarrow \ 3673$$
$$\frac{\int \frac{\sin^4(e+fx)\sqrt{b\sin^2(e+fx)+a}}{\left(1-\sin^2(e+fx)\right)^3}d\sin^2(e+fx)}{2f}$$
$$\Big\downarrow \ 100$$
$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{2(a+b)\left(1-\sin^2(e+fx)\right)^2}-\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}\left(4(a+b)\sin^2(e+fx)+4a+3b\right)}{2\left(1-\sin^2(e+fx)\right)^2}d\sin^2(e+fx)}{2(a+b)}}{2f}$$

$\downarrow$ 27

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{2(a+b)\left(1-\sin^2(e+fx)\right)^2}-\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}\left(4(a+b)\sin^2(e+fx)+4a+3b\right)}{\left(1-\sin^2(e+fx)\right)^2}d\sin^2(e+fx)}{4(a+b)}}{2f}$$

$\downarrow$ 87

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{2(a+b)\left(1-\sin^2(e+fx)\right)^2}-\frac{\frac{(8a+7b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{(a+b)\left(1-\sin^2(e+fx)\right)}-\frac{\left(8a^2+24ab+15b^2\right)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{1-\sin^2(e+fx)}d\sin^2(e+fx)}{2(a+b)}}{4(a+b)}}{2f}$$

$\downarrow$ 60

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{2(a+b)\left(1-\sin^2(e+fx)\right)^2}-\frac{\frac{(8a+7b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{(a+b)\left(1-\sin^2(e+fx)\right)}-\frac{\left(8a^2+24ab+15b^2\right)\left((a+b)\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)-2\sqrt{a+b\sin^2(e+fx)}\right)}{2(a+b)}}{4(a+b)}}{2f}$$

$\downarrow$ 73

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{2(a+b)\left(1-\sin^2(e+fx)\right)^2}-\frac{\frac{(8a+7b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{(a+b)\left(1-\sin^2(e+fx)\right)}-\frac{\left(8a^2+24ab+15b^2\right)\left(\frac{2(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\sin^4(e+fx)}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b}-2\sqrt{a+b\sin^2(e+fx)}\right)}{2(a+b)}}{4(a+b)}}{2f}$$

$\downarrow$ 221

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{2(a+b)\left(1-\sin^2(e+fx)\right)^2}-\frac{\frac{(8a+7b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{(a+b)\left(1-\sin^2(e+fx)\right)}-\frac{\left(8a^2+24ab+15b^2\right)\left(2\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)-2\sqrt{a+b\sin^2(e+fx)}\right)}{2(a+b)}}{4(a+b)}}{2f}$$

input
```
Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^5,x]
```

3.488. $\int \sqrt{a+b\sin^2(e+fx)}\tan^5(e+fx)\,dx$

output
```
((a + b*Sin[e + f*x]^2)^(3/2)/(2*(a + b)*(1 - Sin[e + f*x]^2)^2) - (((8*a
+ 7*b)*(a + b*Sin[e + f*x]^2)^(3/2))/((a + b)*(1 - Sin[e + f*x]^2)) - ((8*
a^2 + 24*a*b + 15*b^2)*(2*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/S
qrt[a + b]] - 2*Sqrt[a + b*Sin[e + f*x]^2]))/(2*(a + b)))/(4*(a + b)))/(2*
f)
```

3.488.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

3.488. $\int \sqrt{a+b\sin^2(e+fx)}\tan^5(e+fx)\,dx$

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.488.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(157) = 314$.

Time = 2.15 (sec) , antiderivative size = 721, normalized size of antiderivative = 4.07

method	result
default	$\frac{\left(-16(a+b)^{\frac{3}{2}}\sqrt{a+b-b(\cos^2(fx+e))}\,a^2-48(a+b)^{\frac{3}{2}}\sqrt{a+b-b(\cos^2(fx+e))}\,ab-30b^2\sqrt{a+b-b(\cos^2(fx+e))}\,(a+b)^{\frac{3}{2}}+8\ln\left(\frac{2\sqrt{a+b}\sqrt{a}}{}\right.\right.}{}$

input
```
int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
1/16*((-16*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a^2-48*(a+b)^(3/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)*a*b-30*b^2*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)+8*
ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a
))*a^4+40*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*si
n(f*x+e)+a))*a^3*b+71*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2
)^(1/2)-b*sin(f*x+e)+a))*a^2*b^2+54*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^3+15*ln(2/(1+sin(f*x+e))*((a+b)
^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^4+8*ln(2/(sin(f*x+e)-
1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4+40*ln(2/(s
in(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*
b+71*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x
+e)+a))*a^2*b^2+54*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(
1/2)+b*sin(f*x+e)+a))*a*b^3+15*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos
(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^4)*cos(f*x+e)^4-2*(a+b)^(3/2)*(a+b-b*c
os(f*x+e)^2)^(3/2)*(8*a+7*b)*cos(f*x+e)^2+4*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^
2)^(3/2)*a+4*b*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2))/(a+b)^(3/2)/cos(f*x
+e)^4/(a^2+2*a*b+b^2)/f
```

3.488.5 Fricas [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.00

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^5(e+fx)\,dx$$

$$= \left[\frac{(8\,a^2+24\,ab+15\,b^2)\sqrt{a+b}\cos(fx+e)^4\log\left(\frac{b\cos(fx+e)^2-2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a+b}-2\,a-2\,b}{\cos(fx+e)^2}\right)-2\left(8\left(a^2+2\right.\right.}{16\left(a^2+2\,ab+\right.}\right.$$

$$-\frac{(8\,a^2+24\,ab+15\,b^2)\sqrt{-a-b}\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a-b}}{a+b}\right)\cos(fx+e)^4+\left(8\left(a^2+2\,ab+b^2\right)\cos\right.}{8\left(a^2+2\,ab+b^2\right)f\,c}$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
[1/16*((8*a^2 + 24*a*b + 15*b^2)*sqrt(a + b)*cos(f*x + e)^4*log((b*cos(f*x
 + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f
*x + e)^2) - 2*(8*(a^2 + 2*a*b + b^2)*cos(f*x + e)^4 + (8*a^2 + 17*a*b + 9
*b^2)*cos(f*x + e)^2 - 2*a^2 - 4*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a +
 b))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4), -1/8*((8*a^2 + 24*a*b + 15*b^
2)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b
))*cos(f*x + e)^4 + (8*(a^2 + 2*a*b + b^2)*cos(f*x + e)^4 + (8*a^2 + 17*a*
b + 9*b^2)*cos(f*x + e)^2 - 2*a^2 - 4*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)^2
+ a + b))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4)]
```

3.488.6 Sympy [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^5(e+fx)\,dx=\int \sqrt{a+b\sin^2{(e+fx)}}\tan^5{(e+fx)}\,dx$$

input
```
integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**5,x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x)**5, x)
```

3.488.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.30

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^5(e+fx)\,dx=$$
$$-\frac{16\sqrt{b\sin{(fx+e)}^2+a}\,b^3+\frac{(8a^2b^3+24ab^4+15b^5)\log\left(\frac{\sqrt{b\sin(fx+e)^2+a}-\sqrt{a+b}}{\sqrt{b\sin(fx+e)^2+a}+\sqrt{a+b}}\right)}{(a+b)^{\frac{3}{2}}}-\frac{2\left((8ab^4+9b^5)\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}-(8a\right.}{\left(b\sin(fx+e)^2+a\right)^2(a+b)+a^3+3a^2b+3ab^2}}{16b^3f}$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
-1/16*(16*sqrt(b*sin(f*x + e)^2 + a)*b^3 + (8*a^2*b^3 + 24*a*b^4 + 15*b^5)
*log((sqrt(b*sin(f*x + e)^2 + a) - sqrt(a + b))/(sqrt(b*sin(f*x + e)^2 + a
) + sqrt(a + b)))/(a + b)^(3/2) - 2*((8*a*b^4 + 9*b^5)*(b*sin(f*x + e)^2 +
 a)^(3/2) - (8*a^2*b^4 + 15*a*b^5 + 7*b^6)*sqrt(b*sin(f*x + e)^2 + a))/((b
*sin(f*x + e)^2 + a)^2*(a + b) + a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 2*(b*sin(
f*x + e)^2 + a)*(a^2 + 2*a*b + b^2)))/(b^3*f)
```

3.488.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2646 vs. $2(157) = 314$.

Time = 2.13 (sec) , antiderivative size = 2646, normalized size of antiderivative = 14.95

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^5(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
-1/4*((8*a^2 + 24*a*b + 15*b^2)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1
/2*f*x + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b))/((a + b)*sqrt(-a - b)) - 1
6*((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*t
an(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b - sqrt(a)*b)/((
sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1
/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e
)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) + a + 4*b) - 2*(8*(sqrt(a)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x +
 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a^2 + 16*(sqrt(a)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a*b + 7*(sqrt(a)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a))^7*b^2 - 56*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
+ 1/2*e)^2 + a))^6*a^(5/2) - 80*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*t
an(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2
*e)^2 + a))^6*a^(3/2)*b - 17*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2...
```

3.488.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^5(e+fx)\,dx = \int \tan(e+f\,x)^5\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(tan(e + f*x)^5*(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^5*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.489 $\int \sqrt{a+b\sin^2(e+fx)}\tan^3(e+fx)\,dx$

3.489.1 Optimal result

Integrand size = 25, antiderivative size = 118

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^3(e+fx)\,dx = -\frac{(2a+3b)\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}f} + \frac{(2a+3b)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f} + \frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2(a+b)f}$$

output
```
1/2*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2)/(a+b)/f-1/2*(2*a+3*b)*arctanh((a
+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)+1/2*(2*a+3*b)*(a+b*sin(f
*x+e)^2)^(1/2)/(a+b)/f
```

3.489.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.71

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^3(e+fx)\,dx = \frac{-\frac{(2a+3b)\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + (2+\cos(2(e+fx)))\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}$$

input
```
Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^3,x]
```

output
```
(-(((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a +
b]) + (2 + Cos[2*(e + f*x)])*Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(2
*f)
```

3.489.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3673, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^3\sqrt{a+b\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\sin^2(e+fx)\sqrt{b\sin^2(e+fx)+a}}{\left(1-\sin^2(e+fx)\right)^2}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{(a+b)\left(1-\sin^2(e+fx)\right)}-\frac{(2a+3b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{1-\sin^2(e+fx)}d\sin^2(e+fx)}{2(a+b)}}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{(a+b)\left(1-\sin^2(e+fx)\right)}-\frac{(2a+3b)\left((a+b)\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)-2\sqrt{a+b\sin^2(e+fx)}\right)}{2(a+b)}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{(a+b\sin^2(e+fx))^{3/2}}{(a+b)(1-\sin^2(e+fx))}-\frac{(2a+3b)\left(\frac{2(a+b)\int\frac{1}{\frac{a+b}{b}-\frac{\sin^4(e+fx)}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b}-2\sqrt{a+b\sin^2(e+fx)}\right)}{2(a+b)}}{2f}$$

$\Big\downarrow$ 221

$$\frac{\frac{(a+b\sin^2(e+fx))^{3/2}}{(a+b)(1-\sin^2(e+fx))}-\frac{(2a+3b)\left(2\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)-2\sqrt{a+b\sin^2(e+fx)}\right)}{2(a+b)}}{2f}$$

input
```
Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^3,x]
```

output
```
((a + b*Sin[e + f*x]^2)^(3/2)/((a + b)*(1 - Sin[e + f*x]^2)) - ((2*a + 3*b
)*(2*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] - 2*Sqrt[
a + b*Sin[e + f*x]^2]))/(2*(a + b)))/(2*f)
```

3.489.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.489.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(102) = 204$.

Time = 1.59 (sec) , antiderivative size = 403, normalized size of antiderivative = 3.42

method	result
default	$-\frac{\left(2\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}-2b\sin(fx+e)+2a}{1+\sin(fx+e)}\right)a^2+5\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}-2b\sin(fx+e)+2a}{1+\sin(fx+e)}\right)ab+3\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a}\cdots\right.\right.}{\cdots}$

input
```
int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-(2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin
(f*x+e)+a))*a^2+5*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)-b*sin(f*x+e)+a))*a*b+3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x
+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^2+2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2+5*ln(2/(sin(f*x+e)-1)*((a+b)^(1
/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b+3*ln(2/(sin(f*x+e)-1)*
((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^2-4*(a+b)^(1/2)
*(a+b-b*cos(f*x+e)^2)^(1/2)*a-6*b*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(1/2))*
cos(f*x+e)^2+2*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(1/2))/(a+b)^(3/2)/cos(f*x
+e)^2/f
```

3.489.5 Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.98

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^3(e+fx)\,dx$$
$$= \left[\frac{(2a+3b)\sqrt{a+b}\cos(fx+e)^2\log\left(\frac{b\cos(fx+e)^2+2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a+b}-2a-2b}{\cos(fx+e)^2}\right)+2\left(2(a+b)\cos(fx+\right.}{4(a+b)f\cos(fx+e)^2}\right.$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
[1/4*((2*a + 3*b)*sqrt(a + b)*cos(f*x + e)^2*log((b*cos(f*x + e)^2 + 2*sqr
t(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*
(2*(a + b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a +
b)*f*cos(f*x + e)^2), 1/2*((2*a + 3*b)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(-a - b)/(a + b))*cos(f*x + e)^2 + (2*(a + b)*cos(f*x
 + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a + b)*f*cos(f*x + e)^
2)]
```

3.489.6 Sympy [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^3(e+fx)\,dx = \int \sqrt{a+b\sin^2(e+fx)}\tan^3(e+fx)\,dx$$

input
```
integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**3,x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x)**3, x)
```

3.489.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.08

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^3(e+fx)\,dx$$
$$= \frac{4\sqrt{b\sin(fx+e)^2+a}b^2 - \frac{2\sqrt{b\sin(fx+e)^2+a}b^3}{b\sin(fx+e)^2-b} + \frac{(2ab^2+3b^3)\log\left(\frac{\sqrt{b\sin(fx+e)^2+a}-\sqrt{a+b}}{\sqrt{b\sin(fx+e)^2+a}+\sqrt{a+b}}\right)}{\sqrt{a+b}}}{4b^2f}$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/4*(4*sqrt(b*sin(f*x + e)^2 + a)*b^2 - 2*sqrt(b*sin(f*x + e)^2 + a)*b^3/(
b*sin(f*x + e)^2 - b) + (2*a*b^2 + 3*b^3)*log((sqrt(b*sin(f*x + e)^2 + a)
- sqrt(a + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/sqrt(a + b))/(b
^2*f)
```

3.489.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(102) = 204$.

Time = 0.84 (sec) , antiderivative size = 959, normalized size of antiderivative = 8.13

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^3(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
((2*a + 3*b)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
 a) - sqrt(a))/sqrt(-a - b))/sqrt(-a - b) - 4*((sqrt(a)*tan(1/2*f*x + 1/2*
e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*ta
n(1/2*f*x + 1/2*e)^2 + a))*b - sqrt(a)*b)/((sqrt(a)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/
2*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)
^2 + a))*sqrt(a) + a + 4*b) - 2*(2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
1/2*e)^2 + a))^3*a + (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))
^3*b + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2) +
 5*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*t
an(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b - 2*(
sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1
/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2 - (sqrt(a)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e
)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b + 4*(sqrt(a)*tan(1/2*f*x + 1...
```

3.489.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^3(e+fx)\,dx = \int \tan(e+f\,x)^3\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(tan(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.490 $\int \sqrt{a+b\sin^2(e+fx)}\tan(e+fx)\,dx$

3.490.1 Optimal result

Integrand size = 23, antiderivative size = 58

$$\int \sqrt{a+b\sin^2(e+fx)}\tan(e+fx)\,dx = \frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f} - \frac{\sqrt{a+b\sin^2(e+fx)}}{f}$$

output
```
arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/f-(a+b*sin(f*x+e
)^2)^(1/2)/f
```

3.490.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \sqrt{a+b\sin^2(e+fx)}\tan(e+fx)\,dx = \frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b-b\cos^2(e+fx)}}{\sqrt{a+b}}\right) - \sqrt{a+b-b\cos^2(e+fx)}}{f}$$

input
```
Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x],x]
```

output
```
(Sqrt[a + b]*ArcTanh[Sqrt[a + b - b*Cos[e + f*x]^2]/Sqrt[a + b]] - Sqrt[a
+ b - b*Cos[e + f*x]^2])/f
```

3.490.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3673, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)\sqrt{a+b\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}}{1-\sin^2(e+fx)}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{(a+b)\int \frac{1}{(1-\sin^2(e+fx))\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)-2\sqrt{a+b\sin^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\sin^4(e+fx)}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b}-2\sqrt{a+b\sin^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)-2\sqrt{a+b\sin^2(e+fx)}}{2f}$$

input
```
Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x],x]
```

output
```
(2*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] - 2*Sqrt[a
+ b*Sin[e + f*x]^2])/(2*f)
```

3.490.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.490.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(50) = 100$.

Time = 1.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.22

method	result
default	$\frac{\frac{\sqrt{a+b}\,\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}-2b\sin(fx+e)+2a}{1+\sin(fx+e)}\right)}{2}+\frac{\sqrt{a+b}\,\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}+2b\sin(fx+e)+2a}{\sin(fx+e)-1}\right)}{2}-\sqrt{a+b-b(\cos^2(}}{f}$

input
```
int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
(1/2*(a+b)^(1/2)*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)-b*sin(f*x+e)+a))+1/2*(a+b)^(1/2)*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))-(a+b-b*cos(f*x+e)^2)^(1/2))/f
```

3.490.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.50

$$\int \sqrt{a+b\sin^2(e+fx)}\tan(e+fx)\,dx = \left[\frac{\sqrt{a+b}\log\left(\frac{b\cos(fx+e)^2-2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a+b}-2a-2b}{\cos(fx+e)^2}\right)-2\sqrt{-b\cos(fx+e)^2+a+b}}{2f}, -\frac{\sqrt{-a-b}\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a-b}}{a+b}\right)+\sqrt{-b\cos(fx+e)^2+a+b}}{f}\right]$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b
)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) - 2*sqrt(-b*cos(f*x + e)^2 + a
+ b))/f, -(sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b
)/(a + b)) + sqrt(-b*cos(f*x + e)^2 + a + b))/f]
```

3.490.6 Sympy [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\tan(e+fx)\,dx = \int \sqrt{a+b\sin^2(e+fx)}\tan(e+fx)\,dx$$

input
```
integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x), x)
```

3.490.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(50) = 100$.

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.10

$$\int \sqrt{a+b\sin^2(e+fx)}\tan(e+fx)\,dx =$$
$$-\frac{\sqrt{a+b}\,\mathrm{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}(\sin(fx+e)+1)}-\frac{a}{\sqrt{ab}(\sin(fx+e)+1)}\right)-\sqrt{a+b}\,\mathrm{arsinh}\left(-\frac{b\sin(fx+e)}{\sqrt{ab}(\sin(fx+e)-1)}-\frac{a}{\sqrt{ab}(\sin(fx+e)-1)}\right)}{2f}$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(a + b)*arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) -
a/(sqrt(a*b)*(sin(f*x + e) + 1))) - sqrt(a + b)*arcsinh(-b*sin(f*x + e)/(s
qrt(a*b)*(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1))) + 2*sqrt(
b*sin(f*x + e)^2 + a))/f
```

3.490.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(50) = 100$.

Time = 0.43 (sec) , antiderivative size = 313, normalized size of antiderivative = 5.40

$$\int \sqrt{a+b\sin^2(e+fx)}\tan(e+fx)\,dx =$$
$$-\frac{2\left(\frac{(a+b)\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}-\sqrt{a}}{2\sqrt{-a-b}}\right)}{\sqrt{-a-b}}-\frac{}{\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a}\right.}\right.}{}$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")
```

output
```
-2*((a + b)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
a) - sqrt(a))/sqrt(-a - b))/sqrt(-a - b) - 2*((sqrt(a)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a))*b - sqrt(a)*b)/((sqrt(a)*tan(1/2*f*x + 1/2*e)^2
- sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2
*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))*sqrt(a) + a + 4*b))/f
```

3.490.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sin^2(e+fx)}\tan(e+fx)\,dx=\int \tan(e+f\,x)\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(tan(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.491 $\int \cot(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.491.1 Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \cot(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a+b\sin^2(e+fx)}}{f}$$

output
```
-arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+(a+b*sin(f*x+e)^2)^(1
/2)/f
```

3.491.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \cot(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{-\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a+b\sin^2(e+fx)}}{f}$$

input
```
Integrate[Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-(Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2/Sqrt[a]]) + Sqrt[a + b*Sin[e
 + f*x]^2])/f
```

3.491.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3673, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{a+b\sin(e+fx)^2}}{\tan(e+fx)}dx$$

$$\Big\downarrow \ 3673$$

$$\frac{\int \csc^2(e+fx)\sqrt{b\sin^2(e+fx)+a}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \ 60$$

$$\frac{a\int \frac{\csc^2(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)+2\sqrt{a+b\sin^2(e+fx)}}{2f}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{2a\int \frac{1}{\frac{\sin^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b}+2\sqrt{a+b\sin^2(e+fx)}}{2f}$$

$$\Big\downarrow \ 221$$

$$\frac{2\sqrt{a+b\sin^2(e+fx)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2f}$$

input
```
Int[Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Sin
[e + f*x]^2])/(2*f)
```

3.491.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.491.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)}{f}$	58

input
```
int(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*sin(f*x+e)^2)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/
2))/sin(f*x+e)))/f
```

3.491.5 Fricas [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.50

$$\int \cot(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$=\left[\frac{\sqrt{a}\log\left(\frac{2\left(b\cos(fx+e)^2+2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a}-2\,a-b\right)}{\cos(fx+e)^2-1}\right)+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{2\,f},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-b\cdots}}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*
sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) + 2*sqrt(-b*cos(f*x + e)^2 + a +
b))/f, (sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/a) + sqrt
(-b*cos(f*x + e)^2 + a + b))/f]
```

3.491.6 Sympy [F]

$$\int \cot(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2\left(e+fx\right)}\cot\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x), x)
```

3.491.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \cot(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\sqrt{a}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right) - \sqrt{b\sin\left(fx+e\right)^2+a}}{f}$$

input
```
integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - sqrt(b*sin(f*x + e)^2
 + a))/f
```

3.491.8 Giac [F]

$$\int \cot(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin\left(fx+e\right)^2+a}\cot\left(fx+e\right)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*cot(f*x + e), x)
```

3.491.9 Mupad [F(-1)]

Timed out.

$$\int \cot(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \cot(e+f\,x)\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(cot(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.492 $\int \cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.492.1 Optimal result

Integrand size = 25, antiderivative size = 110

$$\int \cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{(2a-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f} - \frac{(2a-b)\sqrt{a+b\sin^2(e+fx)}}{2af} - \frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2af}$$

output
```
-1/2*csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2)/a/f+1/2*(2*a-b)*arctanh((a+b*si
n(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-1/2*(2*a-b)*(a+b*sin(f*x+e)^2)^(1/2)/
a/f
```

3.492.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.70

$$\int \cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{(2a-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)-\sqrt{a}(2+\csc^2(e+fx))\sqrt{a+b\sin^2(e+fx)}}{2\sqrt{a}f}$$

input
```
Integrate[Cot[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
((2*a - b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2/Sqrt[a]] - Sqrt[a]*(2 + Csc[
e + f*x]^2)*Sqrt[a + b*Sin[e + f*x]^2])/(2*Sqrt[a]*f)
```

3.492.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3673, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin(e+fx)^2}}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \csc^4(e+fx)\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{-\frac{(2a-b)\int \csc^2(e+fx)\sqrt{b\sin^2(e+fx)+a}d\sin^2(e+fx)}{2a}-\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{a}}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{-\frac{(2a-b)\left(a\int \frac{\csc^2(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)+2\sqrt{a+b\sin^2(e+fx)}\right)}{2a}-\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{a}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{-\frac{(2a-b)\left(\frac{2a\int \frac{1}{\frac{\sin^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b}+2\sqrt{a+b\sin^2(e+fx)}\right)}{2a}-\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{a}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{(2a-b)\left(2\sqrt{a+b\sin^2(e+fx)}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)\right)}{2a}-\frac{\csc^2(e+fx)(a+b\sin^2(e+fx))^{3/2}}{a}}{2f}$$

input
```
Int[Cot[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-((Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2))/a) - ((2*a - b)*(-2*Sqrt[
a]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Sin[e + f*x]
^2]))/(2*a))/(2*f)
```

3.492.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.492.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{-\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}-\frac{\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}}{2\sin\left(fx+e\right)^{2}}-\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)}{2\sqrt{a}}+\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)}{f}$	122

input
```
int(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(a+b*sin(f*x+e)^2)^(1/2)-1/2/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-1/2/a
^(1/2)*b*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))+a^(1/2)*l
n((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e)))/f
```

3.492.5 Fricas [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.17

$$\int \cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$= \left[-\frac{((2a-b)\cos(fx+e)^2-2a+b)\sqrt{a}\log\left(\frac{2\left(b\cos(fx+e)^2+2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a}-2a-b\right)}{\cos(fx+e)^2-1}\right)+2(2a\cos(fx}{4\left(af\cos(fx+e)^2-af\right)}\right.$$
$$-\frac{((2a-b)\cos(fx+e)^2-2a+b)\sqrt{-a}\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a}}{a}\right)+\left(2a\cos(fx+e)^2-3a\right)\sqrt{-}}{2\left(af\cos(fx+e)^2-af\right)}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(a)*log(2*(b*cos(f*x + e)^
2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 -
 1)) + 2*(2*a*cos(f*x + e)^2 - 3*a)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a*f*
cos(f*x + e)^2 - a*f), -1/2*(((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(-a)
*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/a) + (2*a*cos(f*x + e)^2
- 3*a)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a*f*cos(f*x + e)^2 - a*f)]
```

3.492.6 Sympy [F]

$$\int \cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2(e+fx)}\cot^3(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x)**3, x)
```

3.492.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.03

$$\int \cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$=\frac{2\sqrt{a}\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)-\frac{b\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{\sqrt{a}}-2\sqrt{b\sin(fx+e)^2+a}+\frac{\sqrt{b\sin(fx+e)^2+a}b}{a}-\frac{\left(b\sin(fx+e)^2\ldots}{a\sin(fx+e\ldots}}{2f}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(2*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - b*arcsinh(a/(sqr
t(a*b)*abs(sin(f*x + e))))/sqrt(a) - 2*sqrt(b*sin(f*x + e)^2 + a) + sqrt(b
*sin(f*x + e)^2 + a)*b/a - (b*sin(f*x + e)^2 + a)^(3/2)/(a*sin(f*x + e)^2)
)/f
```

3.492.8 Giac [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.492.9 Mupad [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \cot(e+f\,x)^3\,\sqrt{b\sin(e+f\,x)^2+a}\,dx$$

input
```
int(cot(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.493 $\int \cot^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.493.1 Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \cot^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{(8a^2-8ab-b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f} + \frac{(8a^2-8ab-b^2)\sqrt{a+b\sin^2(e+fx)}}{8a^2f} + \frac{(8a+b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{8a^2f} - \frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4af}$$

output
```
-1/8*(8*a^2-8*a*b-b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f
+1/8*(8*a+b)*csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2)/a^2/f-1/4*csc(f*x+e)^4*
(a+b*sin(f*x+e)^2)^(3/2)/a/f+1/8*(8*a^2-8*a*b-b^2)*(a+b*sin(f*x+e)^2)^(1/2
)/a^2/f
```

3.493.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.62

$$\int \cot^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$=\frac{(-8a^2+8ab+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}(8a+(8a-b)\csc^2(e+fx)-2a\csc^4(e+fx))\sqrt{a+b\,\mathrm{s}}}{8a^{3/2}f}$$

input
```
Integrate[Cot[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
((-8*a^2 + 8*a*b + b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] + Sqrt
[a]*(8*a + (8*a - b)*Csc[e + f*x]^2 - 2*a*Csc[e + f*x]^4)*Sqrt[a + b*Sin[e
 + f*x]^2])/(8*a^(3/2)*f)
```

3.493.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3673, 100, 27, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin(e+fx)^2}}{\tan(e+fx)^5}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \csc^6(e+fx)\left(1-\sin^2(e+fx)\right)^2\sqrt{b\sin^2(e+fx)+a}\,d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{100}$$

$$\frac{\frac{\int -\frac{1}{2}\csc^4(e+fx)\left(-4a\sin^2(e+fx)+8a+b\right)\sqrt{b\sin^2(e+fx)+a}\,d\sin^2(e+fx)}{2a}-\frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2a}}{2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \csc^4(e+fx)\left(-4a\sin^2(e+fx)+8a+b\right)\sqrt{b\sin^2(e+fx)+a}d\sin^2(e+fx)}{4a}-\frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2a}}{2f}$$

$\Big\downarrow$ 87

$$-\frac{-\frac{\left(8a^2-8ab-b^2\right)\int \csc^2(e+fx)\sqrt{b\sin^2(e+fx)+a}d\sin^2(e+fx)}{2a}-\frac{(8a+b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{a}}{4a}-\frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2a}}{2f}$$

$\Big\downarrow$ 60

$$-\frac{-\frac{\left(8a^2-8ab-b^2\right)\left(a\int \frac{\csc^2(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)+2\sqrt{a+b\sin^2(e+fx)}\right)}{2a}-\frac{(8a+b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{a}}{4a}-\frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3}}{2a}}{2f}$$

$\Big\downarrow$ 73

$$-\frac{-\frac{\left(8a^2-8ab-b^2\right)\left(\frac{2a\int \frac{1}{\frac{\sin^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b}+2\sqrt{a+b\sin^2(e+fx)}\right)}{2a}-\frac{(8a+b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{a}}{4a}-\frac{\csc^4(e+fx)\left(a+b\sin^2(e+\right.}{2a}}{2f}$$

$\Big\downarrow$ 221

$$-\frac{-\frac{\left(8a^2-8ab-b^2\right)\left(2\sqrt{a+b\sin^2(e+fx)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)\right)}{2a}-\frac{(8a+b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{a}}{4a}-\frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx\right.}{2a}}{2f}$$

input
```
Int[Cot[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-1/2*(Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2))/a - (-(((8*a + b)*Csc[
e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2))/a) - ((8*a^2 - 8*a*b - b^2)*(-2*S
qrt[a]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Sin[e +
f*x]^2]))/(2*a))/(4*a))/(2*f)
```

3.493. $\int \cot^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.493.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.493. $\int \cot^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.493.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.28

method	result
default	$\frac{\sqrt{a+b\left(\sin^{2}(fx+e)\right)}-\frac{\sqrt{a+b\left(\sin^{2}(fx+e)\right)}}{4\sin(fx+e)^{4}}-\frac{b\sqrt{a+b\left(\sin^{2}(fx+e)\right)}}{8a\sin(fx+e)^{2}}+\frac{b^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^{2}(fx+e)\right)}}{\sin(fx+e)}\right)}{8a^{\frac{3}{2}}}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^{2}(fx+e\right.}}{\sin(fx+e)}\right.}{f}$

input
```
int(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*sin(f*x+e)^2)^(1/2)-1/4/sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2)-1/8/a*
b/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)+1/8/a^(3/2)*b^2*ln((2*a+2*a^(1/2)*
(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))-a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f
*x+e)^2)^(1/2))/sin(f*x+e))+1/a^(1/2)*b*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^
2)^(1/2))/sin(f*x+e))+1/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2))/f
```

3.493.5 Fricas [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.52

$$\int \cot^{5}(e+fx)\sqrt{a+b\sin^{2}(e+fx)}\,dx$$

$$= \left[-\frac{\left((8\,a^{2}-8\,ab-b^{2})\cos\left(fx+e\right)^{4}-2\,(8\,a^{2}-8\,ab-b^{2})\cos\left(fx+e\right)^{2}+8\,a^{2}-8\,ab-b^{2}\right)\sqrt{a}\log\left(\frac{2\left(b\cos\right.}{}\right.}{16\left(a^{2}f\cos\right.}\right.$$

input
```
integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(((8*a^2 - 8*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 - 8*a*b - b^2)*co
s(f*x + e)^2 + 8*a^2 - 8*a*b - b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 - 2*sq
rt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2
*(8*a^2*cos(f*x + e)^4 - (24*a^2 - a*b)*cos(f*x + e)^2 + 14*a^2 - a*b)*sqr
t(-b*cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)
^2 + a^2*f), 1/8*(((8*a^2 - 8*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 - 8*a*b
 - b^2)*cos(f*x + e)^2 + 8*a^2 - 8*a*b - b^2)*sqrt(-a)*arctan(sqrt(-b*cos(
f*x + e)^2 + a + b)*sqrt(-a)/a) + (8*a^2*cos(f*x + e)^4 - (24*a^2 - a*b)*c
os(f*x + e)^2 + 14*a^2 - a*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^2*f*cos(
f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f)]
```

3.493.6 Sympy [F]

$$\int \cot^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2{(e+fx)}}\cot^5{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**5*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x)**5, x)
```

3.493.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.30

$$\int \cot^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx =$$

$$-\frac{8\sqrt{a}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right) - \frac{8\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{\sqrt{a}} - \frac{b^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{a^{\frac{3}{2}}} - 8\sqrt{b\sin{(fx+e)}^2+a} + \frac{8\sqrt{}}{}}{8\,f}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*(8*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - 8*b*arcsinh(a/(
sqrt(a*b)*abs(sin(f*x + e))))/sqrt(a) - b^2*arcsinh(a/(sqrt(a*b)*abs(sin(f
*x + e))))/a^(3/2) - 8*sqrt(b*sin(f*x + e)^2 + a) + 8*sqrt(b*sin(f*x + e)^
2 + a)*b/a + sqrt(b*sin(f*x + e)^2 + a)*b^2/a^2 - 8*(b*sin(f*x + e)^2 + a)
^(3/2)/(a*sin(f*x + e)^2) - (b*sin(f*x + e)^2 + a)^(3/2)*b/(a^2*sin(f*x +
e)^2) + 2*(b*sin(f*x + e)^2 + a)^(3/2)/(a*sin(f*x + e)^4))/f
```

3.493.8 Giac [F(-1)]

Timed out.

$$\int \cot^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.493.9 Mupad [F(-1)]

Timed out.

$$\int \cot^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \cot(e+f\,x)^5\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(cot(e + f*x)^5*(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^5*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.494 $\int \sqrt{a+b\sin^2(e+fx)}\tan^4(e+fx)\,dx$

3.494.1 Optimal result

Integrand size = 25, antiderivative size = 234

$$\begin{aligned}\int \sqrt{a+b\sin^2(e+fx)}\tan^4(e+fx)\,dx &= \frac{(7a+8b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\\ &\quad -\frac{4a\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3f\sqrt{a+b\sin^2(e+fx)}}\\ &\quad -\frac{(3a+4b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)f}+\frac{\sqrt{a+b\sin^2(e+fx)}\tan^3(e+fx)}{3f}\end{aligned}$$

output
```
1/3*(7*a+8*b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)
^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-4/3*a*E
llipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin
(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(3*a+4*b)*(a+b*sin(f*x+e
)^2)^(1/2)*tan(f*x+e)/(a+b)/f+1/3*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f
```

3.494.2 Mathematica [A] (verified)

Time = 2.44 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.85

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^4(e+fx)\,dx$$
$$= \frac{2a(7a+8b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-8a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)-\frac{(8}{}}{6(a+b)f\sqrt{2a+b-b\cos(2(e+fx)}}$$

input
```
Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^4,x]
```

output
```
(2*a*(7*a + 8*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x,
 -(b/a)] - 8*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e
+ f*x, -(b/a)] - ((8*a^2 + 12*a*b + b^2 + 4*(4*a^2 + 6*a*b + b^2)*Cos[2*(e
 + f*x)] - b*(4*a + 5*b)*Cos[4*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/(2
*Sqrt[2]))/(6*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.494.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3675, 369, 440, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4\sqrt{a+b\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sin^4(e+fx)\sqrt{b\sin^2(e+fx)+a}}{\left(1-\sin^2(e+fx)\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{369}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{1}{3}\int\frac{\sin^2(e+fx)\left(4b\sin^2(e+fx)+3a\right)}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)\right)}{f}$$

$$\Big\downarrow \; 440$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(-\frac{\int-\frac{b(7a+8b)\sin^2(e+fx)+a(3a+4b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a+b}-\frac{(3a+4b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)\sqrt{1-\sin^2(e+fx)}}\right)+\frac{\sin^3(\ldots}{3\ldots}\right.}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{\int\frac{b(7a+8b)\sin^2(e+fx)+a(3a+4b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a+b}-\frac{(3a+4b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)\sqrt{1-\sin^2(e+fx)}}\right)+\frac{\sin^3(e+f\ldots}{3(1-\ldots}\right.}{f}$$

$$\Big\downarrow \; 399$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{(7a+8b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-4a(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a+b}-\frac{(3a\ldots}{}\right.\right.}{f}$$

$$\Big\downarrow \; 323$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{(7a+8b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(\ldots}{\sqrt{a+b\sin^2(e+fx)}}}{a+b}\right.\right.}{f}$$

$$\Big\downarrow \; 321$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{(7a+8b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}}{a+b}-\frac{(3a\ldots}{}\right.\right.}{f}$$

$$\Big\downarrow \; 330$$

3.494. $\int\sqrt{a+b\sin^2(e+fx)}\tan^4(e+fx)\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{\frac{(7a+8b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)\right.}{\sqrt{a+b\sin^2(e+fx)}}}{a+b}\right.\right.}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{\frac{(7a+8b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right.}{\sqrt{a+b\sin^2(e+fx)}}}{a+b}\right.\right.}{f}$$

input
```
Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^4,x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((Sin[e + f*x]^3*Sqrt[a + b*Sin[e + f*x
]^2])/(3*(1 - Sin[e + f*x]^2)^(3/2)) + (-(((3*a + 4*b)*Sin[e + f*x]*Sqrt[a
 + b*Sin[e + f*x]^2])/((a + b)*Sqrt[1 - Sin[e + f*x]^2])) + (((7*a + 8*b)*
EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/Sqrt[1
 + (b*Sin[e + f*x]^2)/a] - (4*a*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], -(
b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/Sqrt[a + b*Sin[e + f*x]^2])/(a + b))
/3))/f
```

3.494.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.494. $\int\sqrt{a+b\sin^2(e+fx)}\tan^4(e+fx)\,dx$

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.494. $\int \sqrt{a+b\sin^2(e+fx)}\tan^4(e+fx)\,dx$

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.494.4 Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.62

method	result
default	$-\frac{\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\, b(4a+5b)(\cos^4(fx+e))\sin(fx+e)-2\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\,(2a^2+5ab+}{}$

input
```
int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(4*a+5*b)*cos(f*x+e)^4*
sin(f*x+e)-2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*a^2+5*a*b+3*b^2
)*cos(f*x+e)^2*sin(f*x+e)-(cos(f*x+e)^2)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(
f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*(4*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-7*EllipticE(s
in(f*x+e),(-1/a*b)^(1/2))*a-8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(
f*x+e)^2+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*
x+e))/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(a+b)/(sin
(f*x+e)-1)/(1+sin(f*x+e))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.494.5 Fricas [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\tan{(fx+e)}^4\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral(sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^4, x)
```

3.494.6 Sympy [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{a+b\sin^2{(e+fx)}}\tan^4{(e+fx)}\,dx$$

input
```
integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**4,x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x)**4, x)
```

3.494.7 Maxima [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\tan{(fx+e)}^4\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

3.494.8 Giac [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\tan{(fx+e)}^4\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

3.494.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^4(e+fx)\,dx=\int \tan(e+f\,x)^4\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(tan(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.495 $\int \sqrt{a+b\sin^2(e+fx)}\tan^2(e+fx)\,dx$

3.495.1 Optimal result

Integrand size = 25, antiderivative size = 171

$$\begin{aligned}
&\int \sqrt{a+b\sin^2(e+fx)}\tan^2(e+fx)\,dx \\
&= -\frac{2\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} \\
&\quad+\frac{a\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}} \\
&\quad+\frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f}
\end{aligned}$$

output
```
-2*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b
*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+a*EllipticF(sin(f*x+e),(
-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/
(a+b*sin(f*x+e)^2)^(1/2)+(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

3.495.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.82

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^2(e+fx)\,dx = \frac{-2\sqrt{2}a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)+(2a+b-b}{\sqrt{2}f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^2,x]
```

output
```
(-2*Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(
b/a)] + Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x
, -(b/a)] + (2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(Sqrt[2]*f*Sqrt[2
*a + b - b*Cos[2*(e + f*x)]])
```

3.495.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3675, 369, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2\sqrt{a+b\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sin^2(e+fx)\sqrt{b\sin^2(e+fx)+a}}{\left(1-\sin^2(e+fx)\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{369}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}-\int \frac{2b\sin^2(e+fx)+a}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)\right)}{f}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-2\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)+\frac{\sin(}{}\right.}{f}$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-2\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)+\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+}{\sqrt{a+b\sin^2(e+fx)}}\right.}{f}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-2\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)+\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}+\frac{\sin(}{}\right.}{f}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{2\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\right.}{\sqrt{a+b\sin^2(e+fx)}}\right.}{f}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}-\frac{2\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\right.}{f}$$

input
```
Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^2,x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^
2])/Sqrt[1 - Sin[e + f*x]^2] - (2*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*
Sqrt[a + b*Sin[e + f*x]^2])/Sqrt[1 + (b*Sin[e + f*x]^2)/a] + (a*EllipticF[
ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/Sqrt[a + b*S
in[e + f*x]^2]))/f
```

3.495. $\int\sqrt{a+b\sin^2(e+fx)}\tan^2(e+fx)\,dx$

3.495.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a,
0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] && !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.495. $\int \sqrt{a+b\sin^2(e+fx)}\tan^2(e+fx)\,dx$

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.495.4 Maple [A] (verified)

Time = 3.76 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.30

method	result
default	$\frac{\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\left(a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)-2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{}\right.}{\sqrt{-(a+b(\sin^2(fx+e)))(\sin(fx+e)-1)(1+\sin(fx+e))}\cos}$

input
```
int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a*(cos(f*x+e)^2)^(1/2)*(-b/a*c
os(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))-2*(cos(f*x
+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*EllipticE(sin(f*x+e),(-1/
a*b)^(1/2))-cos(f*x+e)^2*sin(f*x+e)*b+a*sin(f*x+e)+b*sin(f*x+e))/(-(a+b*si
n(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e
)^2)^(1/2)/f
```

3.495.5 Fricas [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^2(e+fx)\,dx=\int \sqrt{b\sin\left(fx+e\right)^2+a}\tan\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^2, x)
```

3.495.6 Sympy [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt{a+b\sin^2{(e+fx)}}\tan^2{(e+fx)}\,dx$$

input
```
integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**2,x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x)**2, x)
```

3.495.7 Maxima [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\tan{(fx+e)}^2\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

3.495.8 Giac [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\tan{(fx+e)}^2\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

3.495.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(tan(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.496 $\int \sqrt{a+b\sin^2(e+fx)}\,dx$

3.496.1 Optimal result

Integrand size = 16, antiderivative size = 51

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx=\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

output
```
(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*si
n(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)
```

3.496.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx=\frac{a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)])/(f*S
qrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.496.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a+b\sin(e+fx)^2}dx$$

$$\Big\downarrow \; 3657$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin^2(e+fx)}{a}+1}dx}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin(e+fx)^2}{a}+1}dx}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

$$\Big\downarrow \; 3656$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

input
```
Int[Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin
[e + f*x]^2)/a])
```

3.496.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.496.4 Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

method	result	size
default	$\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,E\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)}{\cos(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}\,f}$	71

input
```
int((a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.496.5 Fricas [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-b*cos(f*x + e)^2 + a + b), x)
```

3.496.6 Sympy [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{a+b\sin^2{(e+fx)}}\,dx$$

input
```
integrate((a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2), x)
```

3.496.7 Maxima [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

3.496.8 Giac [F]

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin{(fx+e)}^2+a}\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

3.496.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sin^2(e+fx)}\,dx = \begin{cases} \frac{\sqrt{a}\,\mathrm{E}\left(e+f\,x\middle|-\frac{b}{a}\right)}{f} & \text{if } 0<a \\ \int \sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx & \text{if } \neg 0<a \end{cases}$$

input
```
int((a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
piecewise(0 < a, (a^(1/2)*ellipticE(e + f*x, -b/a))/f, ~0 < a, int((a + b*
sin(e + f*x)^2)^(1/2), x))
```

3.497 $\int \cot^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.497.1 Optimal result

Integrand size = 25, antiderivative size = 174

$$\begin{aligned}&\int \cot^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx\\&=-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}\\&\quad-\frac{2\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\\&\quad+\frac{(a+b)\sqrt{\cos^2(e+fx)}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

output
```
-cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-2*EllipticE(sin(f*x+e),(-b/a)^(1/2)
)*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+
e)^2/a)^(1/2)+(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x
+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.497.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.82

$$\int \cot^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$=\frac{-((2a+b-b\cos(2(e+fx)))\cot(e+fx))-2\sqrt{2}a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}(a+b)\sqrt{\frac{2a+b}{}}}{\sqrt{2}f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Cot[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-((2*a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x]) - 2*Sqrt[2]*a*Sqrt[(2*a +
b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + Sqrt[2]*(a + b)*Sq
rt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(Sqrt[2]*
f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.497.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 3675, 375, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a+b\sin(e+fx)^2}}{\tan(e+fx)^2}dx$$
$$\Big\downarrow \text{3675}$$
$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \csc^2(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}d\sin(e+fx)}{f}$$
$$\Big\downarrow \text{375}$$
$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(2\int -\frac{2b\sin^2(e+fx)+a-b}{2\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a}\right.}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\int \frac{2b\sin^2(e+fx)+a-b}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+\right.}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left((a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-2\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)\right.}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-2\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)+\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin}{\sqrt{a+b\sin^2(e+fx)}}\right.}{f}$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-2\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)+\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}-\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{2\sqrt{a+b\sin^2(e+fx)}\int \frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fa\right.}{\sqrt{a+b\sin^2(e+fx)}}\right.}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}-\frac{2\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right.}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.}{f}$$

input
```
Int[Cot[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

3.497. $\int \cot^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-(Csc[e + f*x]*Sqrt[1 - Sin[e + f*x]^2
]*Sqrt[a + b*Sin[e + f*x]^2]) - (2*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]
*Sqrt[a + b*Sin[e + f*x]^2])/Sqrt[1 + (b*Sin[e + f*x]^2)/a] + ((a + b)*Ell
ipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/Sqrt[
a + b*Sin[e + f*x]^2]))/f
```

3.497.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 375
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^p*((c + d*x^2)^q/(e*(m + 1)))
, x] - Simp[2/(e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^(p - 1)*(c + d*
x^2)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^2, x], x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 0] && LtQ[m, -1] && GtQ[p, 0
] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.497.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

method	result
default	$\frac{b\left(\cos^4(fx+e)\right)+(-a-b)\left(\cos^2(fx+e)\right)+\sin(fx+e)\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}\,\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\left(F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a+F\left(\sin(fx+\right.\right.}{\sin(fx+e)\cos(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}\,f}$

input
```
int(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*cos(f*x+e)^4+(-a-b)*cos(f*x+e)^2+sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)
^(1/2)*(cos(f*x+e)^2)^(1/2)*(EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a+Ellipt
icF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a)
)/sin(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.497.5 Fricas [F]

$$\int \cot^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx=\int \sqrt{b\sin{(fx+e)}^2+a}\cot{(fx+e)}^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^2, x)
```

3.497.6 Sympy [F]

$$\int \cot^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx=\int \sqrt{a+b\sin^2{(e+fx)}}\cot^2{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x)**2, x)
```

3.497.7 Maxima [F]

$$\int \cot^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx=\int \sqrt{b\sin{(fx+e)}^2+a}\cot{(fx+e)}^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

3.497.8 Giac [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.497.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \cot(e+f\,x)^2\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(cot(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.498 $\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.498.1 Optimal result

Integrand size = 25, antiderivative size = 232

$$\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{(7a-b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{4(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
1/3*(3*a-b)*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a/f-1/3*cot(f*x+e)^3*(a+b*
sin(f*x+e)^2)^(1/2)/f+1/3*(7*a-b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f
*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a/f/(1+b*sin(f*x+e)^2/
a)^(1/2)-4/3*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+
e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.498.2 Mathematica [A] (verified)

Time = 3.62 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.85

$$\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$
$$=\frac{-\frac{(-8a^2-4ab+3b^2+4(4a^2+2ab-b^2)\cos(2(e+fx))+b(-4a+b)\cos(4(e+fx)))\cot(e+fx)\csc^2(e+fx)}{2\sqrt{2}}+2a(7a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx}{a}}}{6af\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Cot[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-1/2*((-8*a^2 - 4*a*b + 3*b^2 + 4*(4*a^2 + 2*a*b - b^2)*Cos[2*(e + f*x)]
+ b*(-4*a + b)*Cos[4*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2)/Sqrt[2] + 2*
a*(7*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/
a)] - 8*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x
, -(b/a)])/(6*a*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.498.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3675, 375, 27, 442, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin(e+fx)^2}}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \csc^4(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{b\sin^2(e+fx)+a}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{375}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{2}{3}\int -\frac{\csc^2(e+fx)\sqrt{1-\sin^2(e+fx)}(4b\sin^2(e+fx)+3a-b)}{2\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\frac{1}{3}\left(1-\sin^2(e+fx)\right)^{3/}\right.}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{1}{3}\int \frac{\csc^2(e+fx)\sqrt{1-\sin^2(e+fx)}(4b\sin^2(e+fx)+3a-b)}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\frac{1}{3}\left(1-\sin^2(e+fx)\right)^{3/}\right.}{f}$$

$\downarrow$ 442

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{(3a-b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}-\frac{\int -\frac{(7a-b)b\sin^2(e+fx)+a(3a-5b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx}{a}\right.\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{\int \frac{(7a-b)b\sin^2(e+fx)+a(3a-5b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}+\frac{(3a-b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}\right.\right.}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{(7a-b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-4a(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}+\frac{(3a-}{}\right.\right.}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{(7a-b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e}{\sqrt{a+b\sin^2(e+fx)}}}{a}\right.\right.}{f}$$

$\downarrow$ 321

3.498. $\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{(7a-b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}}{a}+\frac{(3a-}{}\right.\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{\frac{(7a-b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\arcsin(\sin(e+fx)\right)}{\sqrt{a+b\sin^2(e+fx)}}}{a}\right.\right.}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{\frac{(7a-b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}}{a}\right.\right.}{f}$$

input
```
Int[Cot[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-1/3*(Csc[e + f*x]^3*(1 - Sin[e + f*x]
^2)^(3/2)*Sqrt[a + b*Sin[e + f*x]^2]) + (((3*a - b)*Csc[e + f*x]*Sqrt[1 -
Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/a + (((7*a - b)*EllipticE[ArcS
in[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/Sqrt[1 + (b*Sin[e +
f*x]^2)/a] - (4*a*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])/Sqrt[a + b*Sin[e + f*x]^2])/a)/3))/f
```

3.498. $\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

3.498.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 375
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^p*((c + d*x^2)^q/(e*(m + 1)))
, x] - Simp[2/(e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^(p - 1)*(c + d*
x^2)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^2, x], x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 0] && LtQ[m, -1] && GtQ[p, 0
] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.498. $\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.498.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.51

method	result
default	$-\frac{4\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2\left(\sin^3(fx+e)\right)+4b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F(\sin(fx}{}$

input
```
int(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(4*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*
x+e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+4*b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f
*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-7*(c
os(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a^2*sin(f*x+e)^3+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1
/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b*sin(f*x+e)^3+4*a*b*sin(f*x+e)
^6-b^2*sin(f*x+e)^6+4*a^2*sin(f*x+e)^4-6*a*b*sin(f*x+e)^4+b^2*sin(f*x+e)^4
-5*a^2*sin(f*x+e)^2+2*a*b*sin(f*x+e)^2+a^2)/a/sin(f*x+e)^3/cos(f*x+e)/(a+b
*sin(f*x+e)^2)^(1/2)/f
```

3.498.5 Fricas [F]

$$\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx=\int \sqrt{b\sin\left(fx+e\right)^2+a}\cot\left(fx+e\right)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^4, x)
```

3.498.6 Sympy [F]

$$\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx=\int \sqrt{a+b\sin^2\left(e+fx\right)}\cot^4\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x)**4, x)
```

3.498.7 Maxima [F]

$$\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \sqrt{b\sin(fx+e)^2+a}\cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

3.498.8 Giac [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.498.9 Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \int \cot(e+f\,x)^4\,\sqrt{b\sin(e+f\,x)^2+a}\,dx$$

input
```
int(cot(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.499 $\int (a+b\sin^2(e+fx))^{3/2}\tan^5(e+fx)\,dx$

3.499.1 Optimal result

Integrand size = 25, antiderivative size = 220

$$\int (a+b\sin^2(e+fx))^{3/2}\tan^5(e+fx)\,dx = \frac{(8a^2+40ab+35b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8\sqrt{a+b}f} - \frac{(8a^2+40ab+35b^2)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)f} - \frac{(8a^2+40ab+35b^2)(a+b\sin^2(e+fx))^{3/2}}{24(a+b)^2f} - \frac{(8a+9b)\sec^2(e+fx)(a+b\sin^2(e+fx))^{5/2}}{8(a+b)^2f} + \frac{\sec^4(e+fx)(a+b\sin^2(e+fx))^{5/2}}{4(a+b)f}$$

output
```
-1/24*(8*a^2+40*a*b+35*b^2)*(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^2/f-1/8*(8*a+9*
b)*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(5/2)/(a+b)^2/f+1/4*sec(f*x+e)^4*(a+b*s
in(f*x+e)^2)^(5/2)/(a+b)/f+1/8*(8*a^2+40*a*b+35*b^2)*arctanh((a+b*sin(f*x+
e)^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)-1/8*(8*a^2+40*a*b+35*b^2)*(a+b*sin(
f*x+e)^2)^(1/2)/(a+b)/f
```

3.499.2 Mathematica [A] (verified)

Time = 2.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.73

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx =$$

$$-\frac{3(8a+9b)\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}-6(a+b)\sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}+(8a^2+40}{24(a+b}$$

input
```
Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
```

output
```
-1/24*(3*(8*a + 9*b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2) - 6*(a +
b)*Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(5/2) + (8*a^2 + 40*a*b + 35*b^2)
*(-3*(a + b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] + Sqrt[
a + b*Sin[e + f*x]^2]*(4*a + 3*b + b*Sin[e + f*x]^2)))/((a + b)^2*f)
```

3.499.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3673, 100, 27, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^5\left(a+b\sin(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\sin^4(e+fx)\left(b\sin^2(e+fx)+a\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^3}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{100}$$

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{5/2}}{2(a+b)\left(1-\sin^2(e+fx)\right)^2}-\frac{\int \frac{\left(b\sin^2(e+fx)+a\right)^{3/2}\left(4(a+b)\sin^2(e+fx)+4a+5b\right)}{2\left(1-\sin^2(e+fx)\right)^2}d\sin^2(e+fx)}{2(a+b)}}{2f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{5/2}}{2(a+b)\left(1-\sin^2(e+fx)\right)^2}-\frac{\int \frac{\left(b\sin^2(e+fx)+a\right)^{3/2}\left(4(a+b)\sin^2(e+fx)+4a+5b\right)}{\left(1-\sin^2(e+fx)\right)^2}d\sin^2(e+fx)}{4(a+b)}}{2f}$$

$$\Big\downarrow \; 87$$

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{5/2}}{2(a+b)\left(1-\sin^2(e+fx)\right)^2}-\frac{\frac{(8a+9b)\left(a+b\sin^2(e+fx)\right)^{5/2}}{(a+b)\left(1-\sin^2(e+fx)\right)}-\frac{\left(8a^2+40ab+35b^2\right)\int \frac{\left(b\sin^2(e+fx)+a\right)^{3/2}}{1-\sin^2(e+fx)}d\sin^2(e+fx)}{2(a+b)}}{4(a+b)}}{2f}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{5/2}}{2(a+b)\left(1-\sin^2(e+fx)\right)^2}-\frac{\frac{(8a+9b)\left(a+b\sin^2(e+fx)\right)^{5/2}}{(a+b)\left(1-\sin^2(e+fx)\right)}-\frac{\left(8a^2+40ab+35b^2\right)\left((a+b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{1-\sin^2(e+fx)}d\sin^2(e+fx)-\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}\right)}{2(a+b)}}{4(a+b)}}{2f}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{5/2}}{2(a+b)\left(1-\sin^2(e+fx)\right)^2}-\frac{\frac{(8a+9b)\left(a+b\sin^2(e+fx)\right)^{5/2}}{(a+b)\left(1-\sin^2(e+fx)\right)}-\frac{\left(8a^2+40ab+35b^2\right)\left((a+b)\left((a+b)\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)-2\sqrt{a+b\sin \cdots}\right)\right)}{2(a+b)}}{4(a+b)}}{2f}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{5/2}}{2(a+b)\left(1-\sin^2(e+fx)\right)^2}-\frac{\frac{(8a+9b)\left(a+b\sin^2(e+fx)\right)^{5/2}}{(a+b)\left(1-\sin^2(e+fx)\right)}-\frac{\left(8a^2+40ab+35b^2\right)\left((a+b)\left(\frac{2(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\sin^4(e+fx)}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b}-2\sqrt{a+b\sin^2(e+fx \cdots}\right)\right)}{2(a+b)}}{4(a+b)}}{2f}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{5/2}}{2(a+b)\left(1-\sin^2(e+fx)\right)^2}-\frac{\frac{(8a+9b)\left(a+b\sin^2(e+fx)\right)^{5/2}}{(a+b)\left(1-\sin^2(e+fx)\right)}-\frac{\left(8a^2+40ab+35b^2\right)\left((a+b)\left(2\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)-2\sqrt{a+b\sin^2(e+fx)}\right)-\frac{2}{3}\left(\cdots\right)\right)}{2(a+b)}}{4(a+b)}}{2f}$$

3.499. $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx$

input
```
Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
```

output
```
((a + b*Sin[e + f*x]^2)^(5/2)/(2*(a + b)*(1 - Sin[e + f*x]^2)^2) - (((8*a
+ 9*b)*(a + b*Sin[e + f*x]^2)^(5/2))/((a + b)*(1 - Sin[e + f*x]^2)) - ((8*
a^2 + 40*a*b + 35*b^2)*((-2*(a + b*Sin[e + f*x]^2)^(3/2))/3 + (a + b)*(2*S
qrt[a + b]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] - 2*Sqrt[a + b*
Sin[e + f*x]^2])))/(2*(a + b)))/(4*(a + b)))/(2*f)
```

3.499.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.499.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(196) = 392$.

Time = 1.71 (sec) , antiderivative size = 711, normalized size of antiderivative = 3.23

method	result
default	$\frac{16\sqrt{a+b-b(\cos^2(fx+e))}\,(a+b)^{\frac{5}{2}}\,b(\cos^6(fx+e))+\left(-64a\sqrt{a+b-b(\cos^2(fx+e))}\,(a+b)^{\frac{5}{2}}-160b\sqrt{a+b-b(\cos^2(fx+e))}\,(a+b)^{\frac{5}{2}}+24\ln\right.}{}$

input
```
int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
1/48*(16*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2)*b*cos(f*x+e)^6+(-64*a*(a+b
-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2)-160*b*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^
(5/2)+24*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin
(f*x+e)+a))*a^4+168*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^
(1/2)-b*sin(f*x+e)+a))*a^3*b+369*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*c
os(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^2+330*ln(2/(1+sin(f*x+e))*((a+b)
^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^3+105*ln(2/(1+sin(f
*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^4+24*ln(
2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*
a^4+168*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(
f*x+e)+a))*a^3*b+369*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)
^(1/2)+b*sin(f*x+e)+a))*a^2*b^2+330*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^3+105*ln(2/(sin(f*x+e)-1)*((a+b
)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^4)*cos(f*x+e)^4-6*(a
+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2)*(8*a+13*b)*cos(f*x+e)^2+12*a*(a+b-b*c
os(f*x+e)^2)^(1/2)*(a+b)^(5/2)+12*b*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2)
)/(a+b)^(5/2)/cos(f*x+e)^4/f
```

3.499.5 Fricas [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.75

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx = \left[\frac{3\left(8a^2+40ab+35b^2\right)\sqrt{a+b}\cos\left(fx+e\right)^4\log\left(\frac{b\cos(fx+e)^2-2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a+b}-2a-2b}{\cos(fx+e)^2}\right)+2 \cdots}{\cdots}\right.$$

$$-\frac{3\left(8a^2+40ab+35b^2\right)\sqrt{-a-b}\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a-b}}{a+b}\right)\cos\left(fx+e\right)^4-\left(8\left(ab+b^2\right)\cos\left(fx+e \cdots\right.\right.}{2 \cdots}$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
[1/48*(3*(8*a^2 + 40*a*b + 35*b^2)*sqrt(a + b)*cos(f*x + e)^4*log((b*cos(f
*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos
(f*x + e)^2) + 2*(8*(a*b + b^2)*cos(f*x + e)^6 - 16*(2*a^2 + 7*a*b + 5*b^2
)*cos(f*x + e)^4 - 3*(8*a^2 + 21*a*b + 13*b^2)*cos(f*x + e)^2 + 6*a^2 + 12
*a*b + 6*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a + b)*f*cos(f*x + e)^4),
 -1/24*(3*(8*a^2 + 40*a*b + 35*b^2)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x +
e)^2 + a + b)*sqrt(-a - b)/(a + b))*cos(f*x + e)^4 - (8*(a*b + b^2)*cos(f*
x + e)^6 - 16*(2*a^2 + 7*a*b + 5*b^2)*cos(f*x + e)^4 - 3*(8*a^2 + 21*a*b +
 13*b^2)*cos(f*x + e)^2 + 6*a^2 + 12*a*b + 6*b^2)*sqrt(-b*cos(f*x + e)^2 +
 a + b))/((a + b)*f*cos(f*x + e)^4)]
```

3.499.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e)**5,x)
```

output
```
Timed out
```

3.499.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.08

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx=$$

$$-\frac{16\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}b^3+48\left(ab^3+3\,b^4\right)\sqrt{b\sin\left(fx+e\right)^2+a}+\frac{3\left(8\,a^2b^3+40\,ab^4+35\,b^5\right)\log\left(\frac{\sqrt{b\sin(fx+e)^2+a}-\sqrt{a+\cdots}}{\sqrt{b\sin(fx+e)^2+a}+\sqrt{a+\cdots}}\cdots\right)}{\sqrt{a+b}}\cdots}{48\,b^3f}$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
-1/48*(16*(b*sin(f*x + e)^2 + a)^(3/2)*b^3 + 48*(a*b^3 + 3*b^4)*sqrt(b*sin
(f*x + e)^2 + a) + 3*(8*a^2*b^3 + 40*a*b^4 + 35*b^5)*log((sqrt(b*sin(f*x +
 e)^2 + a) - sqrt(a + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/sqrt
(a + b) - 6*((8*a*b^4 + 13*b^5)*(b*sin(f*x + e)^2 + a)^(3/2) - (8*a^2*b^4
+ 19*a*b^5 + 11*b^6)*sqrt(b*sin(f*x + e)^2 + a))/((b*sin(f*x + e)^2 + a)^2
 - 2*(b*sin(f*x + e)^2 + a)*(a + b) + a^2 + 2*a*b + b^2))/(b^3*f)
```

3.499.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3585 vs. $2(196) = 392$.

Time = 3.77 (sec) , antiderivative size = 3585, normalized size of antiderivative = 16.30

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
-1/12*(3*(8*a^2 + 40*a*b + 35*b^2)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*
e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*ta
n(1/2*f*x + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b))/sqrt(-a - b) - 16*(6*(s
qrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/
2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*b + 9*(sqrt(a)*tan
(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/
2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*b^2 + 18*(sqrt(a)*tan(1/2*f*x
+ 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2)*b + 39*(sqrt(a)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b
*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(a)*b^2 + 12*(sqrt(a)*tan(1/2*f*x + 1/
2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2*b + 66*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
- sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2
*f*x + 1/2*e)^2 + a))^3*a*b^2 + 88*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
1/2*e)^2 + a))^3*b^3 - 12*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
+ a))^2*a^(5/2)*b + 6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +...
```

3.499.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx=\int \tan(e+f\,x)^5\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

input
```
int(tan(e + f*x)^5*(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^5*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.500 $\int (a+b\sin^2(e+fx))^{3/2}\tan^3(e+fx)\,dx$

3.500.1 Optimal result

Integrand size = 25, antiderivative size = 148

$$\int (a+b\sin^2(e+fx))^{3/2}\tan^3(e+fx)\,dx = -\frac{\sqrt{a+b}(2a+5b)\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f}+\frac{(2a+5b)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{(2a+5b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6(a+b)f}+\frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{2(a+b)f}$$

output
```
1/6*(2*a+5*b)*(a+b*sin(f*x+e)^2)^(3/2)/(a+b)/f+1/2*sec(f*x+e)^2*(a+b*sin(f
*x+e)^2)^(5/2)/(a+b)/f-1/2*(2*a+5*b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b
)^(1/2))*(a+b)^(1/2)/f+1/2*(2*a+5*b)*(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.500.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.78

$$\int (a+b\sin^2(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{3\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}+(2a+5b)\left(-3(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)+\sqrt{a+\ldots}\right.}{6(a+b)f}$$

input
```
Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
```

output
```
(3*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2) + (2*a + 5*b)*(-3*(a + b)^(
3/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] + Sqrt[a + b*Sin[e +
f*x]^2]*(4*a + 3*b + b*Sin[e + f*x]^2)))/(6*(a + b)*f)
```

3.500.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3673, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^3\left(a+b\sin(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\sin^2(e+fx)\left(b\sin^2(e+fx)+a\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^2}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{5/2}}{(a+b)\left(1-\sin^2(e+fx)\right)}-\frac{(2a+5b)\int \frac{\left(b\sin^2(e+fx)+a\right)^{3/2}}{1-\sin^2(e+fx)}d\sin^2(e+fx)}{2(a+b)}}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{5/2}}{(a+b)\left(1-\sin^2(e+fx)\right)}-\frac{(2a+5b)\left((a+b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{1-\sin^2(e+fx)}d\sin^2(e+fx)-\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}\right)}{2(a+b)}}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{5/2}}{(a+b)\left(1-\sin^2(e+fx)\right)}-\frac{(2a+5b)\left((a+b)\left((a+b)\int\frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)-2\sqrt{a+b\sin^2(e+fx)}\right)-\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}\right)}{2(a+b)}}{2f}$$

$\big\downarrow$ 73

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{5/2}}{(a+b)\left(1-\sin^2(e+fx)\right)}-\frac{(2a+5b)\left((a+b)\left(\frac{2(a+b)\int\frac{1}{\frac{a+b}{b}-\frac{\sin^4(e+fx)}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b}-2\sqrt{a+b\sin^2(e+fx)}\right)-\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}\right)}{2(a+b)}}{2f}$$

$\big\downarrow$ 221

$$\frac{\frac{\left(a+b\sin^2(e+fx)\right)^{5/2}}{(a+b)\left(1-\sin^2(e+fx)\right)}-\frac{(2a+5b)\left((a+b)\left(2\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)-2\sqrt{a+b\sin^2(e+fx)}\right)-\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}\right)}{2(a+b)}}{2f}$$

input
```
Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
```

output
```
((a + b*Sin[e + f*x]^2)^(5/2)/((a + b)*(1 - Sin[e + f*x]^2)) - ((2*a + 5*b
)*((-2*(a + b*Sin[e + f*x]^2)^(3/2))/3 + (a + b)*(2*Sqrt[a + b]*ArcTanh[Sq
rt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] - 2*Sqrt[a + b*Sin[e + f*x]^2])))/(2
*(a + b)))/(2*f)
```

3.500.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.500.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(128) = 256$.

Time = 1.78 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.88

method	result
default	$\frac{-\frac{\sqrt{a+b-b\left(\cos^2(fx+e)\right)}\left(b\left(\cos^2(fx+e)\right)+2a-b\right)}{3}+2b\sqrt{a+b\left(\sin^2(fx+e)\right)}+2a\sqrt{a+b\left(\sin^2(fx+e)\right)}-\frac{\left(\frac{1}{2}a^2+2ab+\frac{3}{2}b^2\right)\ln\left(\frac{2\sqrt{a+b}\sqrt{a+b-b(}}{}\right.}{\sqrt{a+b}}}{}$

input
```
int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*(a+b-b*cos(f*x+e)^2)^(1/2)*(b*cos(f*x+e)^2+2*a-b)+2*b*(a+b*sin(f*x+e
)^2)^(1/2)+2*a*(a+b*sin(f*x+e)^2)^(1/2)-(1/2*a^2+2*a*b+3/2*b^2)/(a+b)^(1/2
)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+2*a)/(1+sin(
f*x+e)))-(1/2*a^2+2*a*b+3/2*b^2)/(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(
f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(sin(f*x+e)-1))+(1/4*a^2+1/2*a*b+1/4*b
^2)*(-1/(a+b)/(sin(f*x+e)-1)*(a+b-b*cos(f*x+e)^2)^(1/2)+b/(a+b)^(3/2)*ln((
2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(sin(f*x+e)-1
)))+(-1/4*a^2-1/2*a*b-1/4*b^2)*(-1/(a+b)/(1+sin(f*x+e))*(a+b-b*cos(f*x+e)^
2)^(1/2)-b/(a+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*si
n(f*x+e)+2*a)/(1+sin(f*x+e)))))/f
```

3.500.5 Fricas [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.79

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx = \left[\frac{3\,(2\,a+5\,b)\sqrt{a+b}\cos\left(fx+e\right)^2\log\left(\frac{b\cos(fx+e)^2+2\,\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a+b}-2\,a-2\,b}{\cos(fx+e)^2}\right)-2\,(2\,b\cos\left(f\right.}{12\,f\cos\left(fx+e\right)^2}\right.$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
[1/12*(3*(2*a + 5*b)*sqrt(a + b)*cos(f*x + e)^2*log((b*cos(f*x + e)^2 + 2*
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) -
 2*(2*b*cos(f*x + e)^4 - 2*(4*a + 7*b)*cos(f*x + e)^2 - 3*a - 3*b)*sqrt(-b
*cos(f*x + e)^2 + a + b))/(f*cos(f*x + e)^2), 1/6*(3*(2*a + 5*b)*sqrt(-a -
 b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b))*cos(f*x +
 e)^2 - (2*b*cos(f*x + e)^4 - 2*(4*a + 7*b)*cos(f*x + e)^2 - 3*a - 3*b)*sq
rt(-b*cos(f*x + e)^2 + a + b))/(f*cos(f*x + e)^2)]
```

3.500.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e)**3,x)
```

output
```
Timed out
```

3.500.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.14

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx = \frac{4\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}b^2+12\left(ab^2+2b^3\right)\sqrt{b\sin\left(fx+e\right)^2+a}+\frac{3\left(2a^2b^2+7ab^3+5b^4\right)\log\left(\frac{\sqrt{b\sin(fx+e)^2}\cdots}{\sqrt{b\sin(fx+e)^2}\cdots}\right.}{\sqrt{a+b}}\cdots}{12b^2f}$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/12*(4*(b*sin(f*x + e)^2 + a)^(3/2)*b^2 + 12*(a*b^2 + 2*b^3)*sqrt(b*sin(f
*x + e)^2 + a) + 3*(2*a^2*b^2 + 7*a*b^3 + 5*b^4)*log((sqrt(b*sin(f*x + e)^
2 + a) - sqrt(a + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/sqrt(a +
 b) - 6*(a*b^3 + b^4)*sqrt(b*sin(f*x + e)^2 + a)/(b*sin(f*x + e)^2 - b))/(
b^2*f)
```

3.500.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.500.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx=\int \tan(e+f\,x)^3\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

input
```
int(tan(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.501 $\int (a+b\sin^2(e+fx))^{3/2}\tan(e+fx)\,dx$

3.501.1 Optimal result

Integrand size = 23, antiderivative size = 84

$$\int (a+b\sin^2(e+fx))^{3/2}\tan(e+fx)\,dx = \frac{(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f} - \frac{(a+b)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{(a+b\sin^2(e+fx))^{3/2}}{3f}$$

output
```
(a+b)^(3/2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f-1/3*(a+b*sin(f
*x+e)^2)^(3/2)/f-(a+b)*(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.501.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int (a+b\sin^2(e+fx))^{3/2}\tan(e+fx)\,dx = \frac{3(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b-b\cos^2(e+fx)}}{\sqrt{a+b}}\right)+\sqrt{a+b-b\cos^2(e+fx)}(-4(a+b)+b\cos^2(e+fx))}{3f}$$

input
```
Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```

output
```
(3*(a + b)^(3/2)*ArcTanh[Sqrt[a + b - b*Cos[e + f*x]^2]/Sqrt[a + b]] + Sqr
t[a + b - b*Cos[e + f*x]^2]*(-4*(a + b) + b*Cos[e + f*x]^2))/(3*f)
```

3.501.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3673, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)\left(a+b\sin(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\left(b\sin^2(e+fx)+a\right)^{3/2}}{1-\sin^2(e+fx)}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{(a+b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{1-\sin^2(e+fx)}d\sin^2(e+fx)-\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{(a+b)\left((a+b)\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)-2\sqrt{a+b\sin^2(e+fx)}\right)-\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{(a+b)\left(\frac{2(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\sin^4(e+fx)}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b}-2\sqrt{a+b\sin^2(e+fx)}\right)-\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{(a+b)\left(2\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)-2\sqrt{a+b\sin^2(e+fx)}\right)-\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}}{2f}$$

input
```
Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```

output
```
((-2*(a + b*Sin[e + f*x]^2)^(3/2))/3 + (a + b)*(2*Sqrt[a + b]*ArcTanh[Sqrt
[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] - 2*Sqrt[a + b*Sin[e + f*x]^2]))/(2*f)
```

3.501.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.501. $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx$

3.501.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(72) = 144$.

Time = 1.27 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.51

method	result
default	$\frac{\frac{\sqrt{a+b-b\left(\cos^2(fx+e)\right)}\left(b\left(\cos^2(fx+e)\right)+2a-b\right)}{3}-b\sqrt{a+b(\sin^2(fx+e))}-2a\sqrt{a+b(\sin^2(fx+e))}+\frac{\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\right.}}{1+\mathrm{s}}\right.}{\sqrt{a+b}}}{f}$

input
```
int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
(1/3*(a+b-b*cos(f*x+e)^2)^(1/2)*(b*cos(f*x+e)^2+2*a-b)-b*(a+b*sin(f*x+e)^2
)^(1/2)-2*a*(a+b*sin(f*x+e)^2)^(1/2)+(1/2*a^2+a*b+1/2*b^2)/(a+b)^(1/2)*ln(
(2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+2*a)/(1+sin(f*x+e
)))+(1/2*a^2+a*b+1/2*b^2)/(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)+2*b*sin(f*x+e)+2*a)/(sin(f*x+e)-1)))/f
```

3.501.5 Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.21

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx = \left[\frac{3\,(a+b)^{\frac{3}{2}}\log\left(\frac{b\cos(fx+e)^2-2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a+b}-2\,a-2\,b}{\cos(fx+e)^2}\right)+2\left(b\cos\left(fx+e\right)^2-4\,a-4\,b\right)\sqrt{-b}}{6\,f}\right.$$

$$-\frac{3\,(a+b)\sqrt{-a-b}\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a-b}}{a+b}\right)-\left(b\cos\left(fx+e\right)^2-4\,a-4\,b\right)\sqrt{-b\cos\left(fx+e\right)^2+a}}{3\,f}$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="fricas")
```

output
```
[1/6*(3*(a + b)^(3/2)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a
 + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*(b*cos(f*x + e)^2 - 4*a
 - 4*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/f, -1/3*(3*(a + b)*sqrt(-a - b)*a
rctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b)) - (b*cos(f*x +
 e)^2 - 4*a - 4*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/f]
```

3.501.6 Sympy [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx=\int \left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan\left(e+fx\right)dx$$

input
```
integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e),x)
```

output
```
Integral((a + b*sin(e + f*x)**2)**(3/2)*tan(e + f*x), x)
```

3.501.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(72) = 144$.

Time = 0.31 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.87

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx =$$
$$-\frac{3\,(a+b)^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}(\sin(fx+e)+1)}-\frac{a}{\sqrt{ab}(\sin(fx+e)+1)}\right)-3\,(a+b)^{\frac{3}{2}}\operatorname{arsinh}\left(-\frac{b\sin(fx+e)}{\sqrt{ab}(\sin(fx+e)-1)}-\frac{a}{\sqrt{ab}(\sin(fx+e)-}\right.}{6\,f}$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="maxima")
```

output
```
-1/6*(3*(a + b)^(3/2)*arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)
) - a/(sqrt(a*b)*(sin(f*x + e) + 1))) - 3*(a + b)^(3/2)*arcsinh(-b*sin(f*x
 + e)/(sqrt(a*b)*(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1))) +
 2*(b*sin(f*x + e)^2 + a)^(3/2) + 6*sqrt(b*sin(f*x + e)^2 + a)*a + 6*sqrt(
b*sin(f*x + e)^2 + a)*b)/f
```

3.501.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1270 vs. $2(72) = 144$.

Time = 0.71 (sec) , antiderivative size = 1270, normalized size of antiderivative = 15.12

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="giac")
```

output
```
-2/3*(3*(a^2 + 2*a*b + b^2)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f
*x + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b))/sqrt(-a - b) - 2*(6*(sqrt(a)*t
an(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x +
1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*b + 3*(sqrt(a)*tan(1/2*f*x
 + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*b^2 + 18*(sqrt(a)*tan(1/2*f*x + 1/2*e)
^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2)*b + 21*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
- sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2
*f*x + 1/2*e)^2 + a))^4*sqrt(a)*b^2 + 12*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*
f*x + 1/2*e)^2 + a))^3*a^2*b + 54*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a
*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1
/2*e)^2 + a))^3*a*b^2 + 40*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/
2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
 + a))^3*b^3 - 12*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*
a^(5/2)*b + 18*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*
e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*...
```

3.501.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx = \int \tan(e+f\,x)\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

input
```
int(tan(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.502 $\int \cot(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx$

3.502.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \cot(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx = -\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{a\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}$$

output
```
-a^(3/2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/f+1/3*(a+b*sin(f*x+e)^2
)^(3/2)/f+a*(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.502.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \cot(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \frac{-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right) + \sqrt{a+b\sin^2(e+fx)}\left(4a+b\sin^2(e+fx)\right)}{3f}$$

input
```
Integrate[Cot[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-3*a^(3/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] + Sqrt[a + b*Sin[e
 + f*x]^2]*(4*a + b*Sin[e + f*x]^2))/(3*f)
```

3.502.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3673, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^{3/2}}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \csc^2(e+fx)\left(b\sin^2(e+fx)+a\right)^{3/2}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\int \csc^2(e+fx)\sqrt{b\sin^2(e+fx)+a}d\sin^2(e+fx)+\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\left(a\int \frac{\csc^2(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)+2\sqrt{a+b\sin^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{a\left(\frac{2a\int \frac{1}{\frac{\sin^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b}+2\sqrt{a+b\sin^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{a\left(2\sqrt{a+b\sin^2(e+fx)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}}{2f}$$

input
```
Int[Cot[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
((2*(a + b*Sin[e + f*x]^2)^(3/2))/3 + a*(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin
[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Sin[e + f*x]^2]))/(2*f)
```

3.502.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.502. $\int \cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.502.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\frac{b\left(\sin^2(fx+e)\right)\sqrt{a+b\left(\sin^2(fx+e)\right)}}{3}+\frac{4a\sqrt{a+b\left(\sin^2(fx+e)\right)}}{3}-a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2(fx+e)\right)}}{\sin(fx+e)}\right)}{f}$	86

input
```
int(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(1/3*b*sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)+4/3*a*(a+b*sin(f*x+e)^2)^(1/2
)-a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e)))/f
```

3.502.5 Fricas [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.24

$$\int\cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{2\left(b\cos(fx+e)^2+2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a}-2\,a-b\right)}{\cos(fx+e)^2-1}\right)-2\left(b\cos\left(fx+e\right)^2-4\,a-b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{6\,f}\right.$$

input
```
integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*(3*a^(3/2)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b
)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*(b*cos(f*x + e)^2 - 4*a - b
)*sqrt(-b*cos(f*x + e)^2 + a + b))/f, 1/3*(3*sqrt(-a)*a*arctan(sqrt(-b*cos
(f*x + e)^2 + a + b)*sqrt(-a)/a) - (b*cos(f*x + e)^2 - 4*a - b)*sqrt(-b*co
s(f*x + e)^2 + a + b))/f]
```

3.502.6 Sympy [F]

$$\int \cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}\cot\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sin(e + f*x)**2)**(3/2)*cot(e + f*x), x)
```

3.502.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=-\frac{3\,a^{\frac{3}{2}}\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)-\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}-3\,\sqrt{b\sin\left(fx+e\right)^2+a}\,a}{3\,f}$$

input
```
integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*(3*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - (b*sin(f*x + e)
^2 + a)^(3/2) - 3*sqrt(b*sin(f*x + e)^2 + a)*a)/f
```

3.502.8 Giac [F(-1)]

Timed out.

$$\int \cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.502.9 Mupad [F(-1)]

Timed out.

$$\int \cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \cot(e+f\,x)\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

input
```
int(cot(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.503 $\int \cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.503.1 Optimal result

Integrand size = 25, antiderivative size = 140

$$\int \cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{\sqrt{a}(2a-3b)\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{(2a-3b)\sqrt{a+b\sin^2(e+fx)}}{2f}-\frac{(2a-3b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6af}-\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{2af}$$

output
```
-1/6*(2*a-3*b)*(a+b*sin(f*x+e)^2)^(3/2)/a/f-1/2*csc(f*x+e)^2*(a+b*sin(f*x+
e)^2)^(5/2)/a/f+1/2*(2*a-3*b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))*a^
(1/2)/f-1/2*(2*a-3*b)*(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.503.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.64

$$\int \cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{3\sqrt{a}(2a-3b)\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)+\left(-8a+5b+b\cos(2(e+fx))-3a\csc^2(e+fx)\right)\sqrt{}}{6f}$$

input
```
Integrate[Cot[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(3*Sqrt[a]*(2*a - 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] + (-8*a
 + 5*b + b*Cos[2*(e + f*x)] - 3*a*Csc[e + f*x]^2)*Sqrt[a + b*Sin[e + f*x]^
2])/(6*f)
```

3.503.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 3673, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^{3/2}}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \csc^4(e+fx)\left(1-\sin^2(e+fx)\right)\left(b\sin^2(e+fx)+a\right)^{3/2}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{-\frac{(2a-3b)\int \csc^2(e+fx)\left(b\sin^2(e+fx)+a\right)^{3/2}d\sin^2(e+fx)}{2a}-\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{a}}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{-\frac{(2a-3b)\left(a\int \csc^2(e+fx)\sqrt{b\sin^2(e+fx)+a}d\sin^2(e+fx)+\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}\right)}{2a}-\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{a}}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{-\frac{(2a-3b)\left(a\left(a\int \frac{\csc^2(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)+2\sqrt{a+b\sin^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}\right)}{2a}-\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{a}}{2f}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{(2a-3b)\left(a\left(\frac{2a\int \frac{1}{\frac{\sin^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b}+2\sqrt{a+b\sin^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}\right)}{2a}-\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{a}}{2f}$$

$\Big\downarrow$ 221

$$-\frac{\frac{(2a-3b)\left(a\left(2\sqrt{a+b\sin^2(e+fx)}-2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}\right)}{2a}-\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{a}}{2f}$$

input
```
Int[Cot[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-((Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2))/a) - ((2*a - 3*b)*((2*(a
+ b*Sin[e + f*x]^2)^(3/2))/3 + a*(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[e + f*
x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Sin[e + f*x]^2])))/(2*a))/(2*f)
```

3.503.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*(a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.503.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.18

method	result
default	$\frac{-\frac{b\left(\sin^{2}(fx+e)\right)\sqrt{a+b\left(\sin^{2}(fx+e)\right)}}{3}-\frac{4a\sqrt{a+b\left(\sin^{2}(fx+e)\right)}}{3}+b\sqrt{a+b\left(\sin^{2}(fx+e)\right)}-\frac{a\sqrt{a+b\left(\sin^{2}(fx+e)\right)}}{2\sin(fx+e)^{2}}-\frac{3\sqrt{a}\,b\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\left(\sin^{2}(f\right.}}{\sin(fx+e)}\right.}{2}}{f}$

input
```
int(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/3*b*sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-4/3*a*(a+b*sin(f*x+e)^2)^(1/
2)+b*(a+b*sin(f*x+e)^2)^(1/2)-1/2*a/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-
3/2*a^(1/2)*b*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))+a^(3
/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e)))/f
```

3.503.5 Fricas [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.01

$$\int \cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \left[-\frac{3\left((2a-3b)\cos\left(fx+e\right)^2-2a+3b\right)\sqrt{a}\log\left(\frac{2\left(b\cos(fx+e)^2+2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a}-2a-b\right)}{\cos(fx+e)^2-1}\right.}{12\left(f\cos\left(f\right.\right.}\right.$$

$$-\frac{3\left((2a-3b)\cos\left(fx+e\right)^2-2a+3b\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a}}{a}\right)-\left(2b\cos\left(fx+e\right)^4-2\left(4a\right.\right.}{6\left(f\cos\left(fx+e\right)^2-f\right)}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*((2*a - 3*b)*cos(f*x + e)^2 - 2*a + 3*b)*sqrt(a)*log(2*(b*cos(f*
x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x +
 e)^2 - 1)) - 2*(2*b*cos(f*x + e)^4 - 2*(4*a - b)*cos(f*x + e)^2 + 11*a -
4*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(f*cos(f*x + e)^2 - f), -1/6*(3*((2*
a - 3*b)*cos(f*x + e)^2 - 2*a + 3*b)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^
2 + a + b)*sqrt(-a)/a) - (2*b*cos(f*x + e)^4 - 2*(4*a - b)*cos(f*x + e)^2
+ 11*a - 4*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(f*cos(f*x + e)^2 - f)]
```

3.503.6 Sympy [F]

$$\int \cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}\cot^3\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sin(e + f*x)**2)**(3/2)*cot(e + f*x)**3, x)
```

3.503.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\int \cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{6\,a^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right) - 9\,\sqrt{a}b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right) - 2\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}} - 6\,\sqrt{b s}}{6\,f}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/6*(6*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - 9*sqrt(a)*b*arcs
inh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - 2*(b*sin(f*x + e)^2 + a)^(3/2) - 6*
sqrt(b*sin(f*x + e)^2 + a)*a + 9*sqrt(b*sin(f*x + e)^2 + a)*b + 3*(b*sin(f
*x + e)^2 + a)^(3/2)*b/a - 3*(b*sin(f*x + e)^2 + a)^(5/2)/(a*sin(f*x + e)^
2))/f
```

3.503.8 Giac [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.503.9 Mupad [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \int \cot(e+f\,x)^3\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

input
```
int(cot(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.504 $\int \cot^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.504.1 Optimal result

Integrand size = 25, antiderivative size = 208

$$\int \cot^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{(8a^2-24ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}f} + \frac{(8a^2-24ab+3b^2)\sqrt{a+b\sin^2(e+fx)}}{8af} + \frac{(8a^2-24ab+3b^2)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24a^2f} + \frac{(8a-b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{8a^2f} - \frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{4af}$$

output
```
1/24*(8*a^2-24*a*b+3*b^2)*(a+b*sin(f*x+e)^2)^(3/2)/a^2/f+1/8*(8*a-b)*csc(f
*x+e)^2*(a+b*sin(f*x+e)^2)^(5/2)/a^2/f-1/4*csc(f*x+e)^4*(a+b*sin(f*x+e)^2)
^(5/2)/a/f-1/8*(8*a^2-24*a*b+3*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/
2))/f/a^(1/2)+1/8*(8*a^2-24*a*b+3*b^2)*(a+b*sin(f*x+e)^2)^(1/2)/a/f
```

3.504.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.59

$$\int \cot^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{-3(8a^2-24ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\sqrt{a+b\sin^2(e+fx)}(3(8a-5b)\csc^2(e}{24\sqrt{a}f}$$

input
```
Integrate[Cot[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-3*(8*a^2 - 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] +
 Sqrt[a]*Sqrt[a + b*Sin[e + f*x]^2]*(3*(8*a - 5*b)*Csc[e + f*x]^2 - 6*a*Cs
c[e + f*x]^4 + 8*(4*a - 6*b + b*Sin[e + f*x]^2)))/(24*Sqrt[a]*f)
```

3.504.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 3673, 100, 27, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^{3/2}}{\tan(e+fx)^5}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \csc^6(e+fx)\left(1-\sin^2(e+fx)\right)^2\left(b\sin^2(e+fx)+a\right)^{3/2}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{100}$$

$$\frac{\frac{\int -\frac{1}{2}\csc^4(e+fx)\left(-4a\sin^2(e+fx)+8a-b\right)\left(b\sin^2(e+fx)+a\right)^{3/2}d\sin^2(e+fx)}{2a}-\frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{2a}}{2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \csc^4(e+fx)\left(-4a\sin^2(e+fx)+8a-b\right)\left(b\sin^2(e+fx)+a\right)^{3/2}d\sin^2(e+fx)}{4a}-\frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{2a}}{2f}$$

$\Big\downarrow$ 87

$$\frac{-\frac{-\frac{\left(8a^2-24ab+3b^2\right)\int \csc^2(e+fx)\left(b\sin^2(e+fx)+a\right)^{3/2}d\sin^2(e+fx)}{2a}-\frac{(8a-b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{a}}{4a}-\frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{2a}}{2f}$$

$\Big\downarrow$ 60

$$\frac{-\frac{-\frac{\left(8a^2-24ab+3b^2\right)\left(a\int \csc^2(e+fx)\sqrt{b\sin^2(e+fx)+a}\,d\sin^2(e+fx)+\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}\right)}{2a}-\frac{(8a-b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{a}}{4a}-\frac{\csc^4(e+fx)(a+\cdots}{\cdots}}{2f}$$

$\Big\downarrow$ 60

$$\frac{-\frac{-\frac{\left(8a^2-24ab+3b^2\right)\left(a\left(a\int \frac{\csc^2(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)+2\sqrt{a+b\sin^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}\right)}{2a}-\frac{(8a-b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{a}}{4a}-\cdots}{2f}$$

$\Big\downarrow$ 73

$$\frac{-\frac{-\frac{\left(8a^2-24ab+3b^2\right)\left(a\left(\frac{2a\int \frac{1}{\frac{\sin^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b}+2\sqrt{a+b\sin^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}\right)}{2a}-\frac{(8a-b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{a}}{4a}\cdots}{2f}$$

$\Big\downarrow$ 221

$$\frac{-\frac{-\frac{\left(8a^2-24ab+3b^2\right)\left(a\left(2\sqrt{a+b\sin^2(e+fx)}-2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+b\sin^2(e+fx)\right)^{3/2}\right)}{2a}-\frac{(8a-b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{a}}{4a}-\cdots}{2f}$$

input
```
Int[Cot[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-1/2*(Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(5/2))/a - (-(((8*a - b)*Csc[
e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2))/a) - ((8*a^2 - 24*a*b + 3*b^2)*((
2*(a + b*Sin[e + f*x]^2)^(3/2))/3 + a*(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[e
 + f*x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Sin[e + f*x]^2])))/(2*a))/(4*a))/(2*f)
```

3.504. $\int \cot^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.504.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.504. $\int \cot^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.504.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.24

method	result
default	$\frac{\frac{b\left(\sin^2(fx+e)\right)\sqrt{a+b\left(\sin^2(fx+e)\right)}}{3}+\frac{4a\sqrt{a+b\left(\sin^2(fx+e)\right)}}{3}-\frac{a\sqrt{a+b\left(\sin^2(fx+e)\right)}}{4\sin(fx+e)^4}-\frac{5b\sqrt{a+b\left(\sin^2(fx+e)\right)}}{8\sin(fx+e)^2}-\frac{3b^2\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\left(\sin^2(fx+e\right)}}{\sin(fx+e)}\right.}{8\sqrt{a}}}{f}$

input
```
int(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(1/3*b*sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)+4/3*a*(a+b*sin(f*x+e)^2)^(1/2
)-1/4*a/sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2)-5/8*b/sin(f*x+e)^2*(a+b*sin(
f*x+e)^2)^(1/2)-3/8/a^(1/2)*b^2*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)
)/sin(f*x+e))-2*b*(a+b*sin(f*x+e)^2)^(1/2)+a/sin(f*x+e)^2*(a+b*sin(f*x+e)^
2)^(1/2)+3*a^(1/2)*b*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e
))-a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e)))/f
```

3.504.5 Fricas [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.12

$$\int \cot^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\left((8a^2-24ab+3b^2)\cos(fx+e)^4-2(8a^2-24ab+3b^2)\cos(fx+e)^2+8a^2-24ab+\right.}{}\right.$$

input
```
integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(3*((8*a^2 - 24*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 - 24*a*b + 3*
b^2)*cos(f*x + e)^2 + 8*a^2 - 24*a*b + 3*b^2)*sqrt(a)*log(2*(b*cos(f*x + e
)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2
 - 1)) - 2*(8*a*b*cos(f*x + e)^6 - 8*(4*a^2 - 3*a*b)*cos(f*x + e)^4 + (88*
a^2 - 87*a*b)*cos(f*x + e)^2 - 50*a^2 + 55*a*b)*sqrt(-b*cos(f*x + e)^2 + a
 + b))/(a*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f), 1/24*(3*((8*a^2
- 24*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 - 24*a*b + 3*b^2)*cos(f*x + e)
^2 + 8*a^2 - 24*a*b + 3*b^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a +
b)*sqrt(-a)/a) - (8*a*b*cos(f*x + e)^6 - 8*(4*a^2 - 3*a*b)*cos(f*x + e)^4
+ (88*a^2 - 87*a*b)*cos(f*x + e)^2 - 50*a^2 + 55*a*b)*sqrt(-b*cos(f*x + e)
^2 + a + b))/(a*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f)]
```

3.504.6 Sympy [F(-1)]

Timed out.

$$\int \cot^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)**5*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.504.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.31

$$\int \cot^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=$$

$$-\frac{24\,a^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)-72\,\sqrt{a}b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)+\frac{9\,b^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{\sqrt{a}}-8\left(b\sin\left(fx+e\right)^2+\right.}{}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/24*(24*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - 72*sqrt(a)*b*
arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) + 9*b^2*arcsinh(a/(sqrt(a*b)*abs(
sin(f*x + e))))/sqrt(a) - 8*(b*sin(f*x + e)^2 + a)^(3/2) - 24*sqrt(b*sin(f
*x + e)^2 + a)*a + 72*sqrt(b*sin(f*x + e)^2 + a)*b + 24*(b*sin(f*x + e)^2
+ a)^(3/2)*b/a - 3*(b*sin(f*x + e)^2 + a)^(3/2)*b^2/a^2 - 9*sqrt(b*sin(f*x
 + e)^2 + a)*b^2/a - 24*(b*sin(f*x + e)^2 + a)^(5/2)/(a*sin(f*x + e)^2) +
3*(b*sin(f*x + e)^2 + a)^(5/2)*b/(a^2*sin(f*x + e)^2) + 6*(b*sin(f*x + e)^
2 + a)^(5/2)/(a*sin(f*x + e)^4))/f
```

3.504.8 Giac [F(-1)]

Timed out.

$$\int \cot^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.504.9 Mupad [F(-1)]

Timed out.

$$\int \cot^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \cot(e+f\,x)^5\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

input
```
int(cot(e + f*x)^5*(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^5*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.505 $\int (a+b\sin^2(e+fx))^{3/2}\tan^4(e+fx)\,dx$

3.505.1 Optimal result

Integrand size = 25, antiderivative size = 275

$$\int (a+b\sin^2(e+fx))^{3/2}\tan^4(e+fx)\,dx = -\frac{(3a+8b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{8(a+2b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{a(5a+8b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a+2b)\sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} + \frac{(a+b\sin^2(e+fx))^{3/2}\tan^3(e+fx)}{3f}$$

output
```
-1/3*(3*a+8*b)*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+8/3*(a+2*b
)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*
sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*a*(5*a+8*b)*EllipticF
(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^
2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)-(a+2*b)*sin(f*x+e)^2*(a+b*sin(f*x+e)
^2)^(1/2)*tan(f*x+e)/f+1/3*(a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3/f
```

3.505.2 Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.77

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx = \frac{32a(a+2b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-4a(5a+8b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\text{EllipticF}\left(e+fx\right.}{12f\sqrt{2a+b}}$$

input
```
Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```

output
```
(32*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x,
-(b/a)] - 4*a*(5*a + 8*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF
[e + f*x, -(b/a)] - ((32*a^2 + 108*a*b + 18*b^2 + (64*a^2 + 160*a*b + 17*b
^2)*Cos[2*(e + f*x)] - 2*b*(6*a + 17*b)*Cos[4*(e + f*x)] - b^2*Cos[6*(e +
f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/(4*Sqrt[2]))/(12*f*Sqrt[2*a + b - b*Co
s[2*(e + f*x)]])
```

3.505.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 3675, 369, 27, 439, 25, 444, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4\left(a+b\sin(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sin^4(e+fx)\left(b\sin^2(e+fx)+a\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{369}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{3\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{1}{3}\int\frac{3\sin^2(e+fx)\sqrt{b\sin^2(e+fx)+a}\left(2b\sin^2(e+fx)+a\right)}{\left(1-\sin^2(e+fx)\right)^{3/2}}d\sin(e+fx\right.}{f}$$

$\big\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{3\left(1-\sin^2(e+fx)\right)^{3/2}}-\int\frac{\sin^2(e+fx)\sqrt{b\sin^2(e+fx)+a}\left(2b\sin^2(e+fx)+a\right)}{\left(1-\sin^2(e+fx)\right)^{3/2}}d\sin(e+fx)\right)}{f}$$

$\big\downarrow$ 439

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\int-\frac{\sin^2(e+fx)\left(b(3a+8b)\sin^2(e+fx)+2a(a+3b)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{\sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{3\left(1-\sin^2(e+fx)\right)^{3/2}}\right.}{f}$$

$\big\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\int\frac{\sin^2(e+fx)\left(b(3a+8b)\sin^2(e+fx)+2a(a+3b)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{\sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{3\left(1-\sin^2(e+fx)\right)^{3/2}}-\right.}{f}$$

$\big\downarrow$ 444

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int\frac{b\left(8b(a+2b)\sin^2(e+fx)+a(3a+8b)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3b}-\frac{1}{3}(3a+8b)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)\sqrt{a\right.}{f}$$

$\big\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\int\frac{8b(a+2b)\sin^2(e+fx)+a(3a+8b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\frac{1}{3}(3a+8b)\sqrt{1-\sin^2(e+fx)}\sin(e+\right.}{f}$$

$\big\downarrow$ 399

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(8(a+2b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-a(5a+8b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+}}\right.\right.$$

$\big\downarrow$ 323

3.505. $\int\left(a+b\sin^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx$

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(8(a+2b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{a(5a+8b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)}}}{\sqrt{a+b\sin^2(e+fx)}}\right.\right.$$

$\downarrow$ 321

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(8(a+2b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{a(5a+8b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e\right.}{\sqrt{a+b\sin^2(e+fx)}}\right.\right.$$

$\downarrow$ 330

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{8(a+2b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(5a+8b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\text{arc}\right.}{\sqrt{a+b\sin^2(e+fx)}}\right.\right.$$

$\downarrow$ 327

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{8(a+2b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(5a+8b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(s\right.}{\sqrt{a+b\sin^2(e+fx)}}\right.\right.$$

input
```
Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-(((a + 2*b)*Sin[e + f*x]^3*Sqrt[a + b
*Sin[e + f*x]^2])/Sqrt[1 - Sin[e + f*x]^2]) - ((3*a + 8*b)*Sin[e + f*x]*Sq
rt[1 - Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/3 + (Sin[e + f*x]^3*(a
+ b*Sin[e + f*x]^2)^(3/2))/(3*(1 - Sin[e + f*x]^2)^(3/2)) + ((8*(a + 2*b)*
EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/Sqrt[1
 + (b*Sin[e + f*x]^2)/a] - (a*(5*a + 8*b)*EllipticF[ArcSin[Sin[e + f*x]],
-(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/Sqrt[a + b*Sin[e + f*x]^2])/3))/f
```

3.505. $\int\left(a+b\sin^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx$

3.505.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.505. $\int (a+b\sin^2(e+fx))^{3/2}\tan^4(e+fx)\,dx$

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.505. $\int (a+b\sin^2(e+fx))^{3/2}\tan^4(e+fx)\,dx$

3.505.4 Maple [A] (verified)

Time = 4.98 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.52

method	result
default	$-\frac{\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\, b^2 (\cos^6(fx+e)) \sin(fx+e)+\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\, b(3a+7b)(\cos^4(fx+}{}$

input
```
int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*cos(f*x+e)^6*sin(f*x+
e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(3*a+7*b)*cos(f*x+e)^4*sin
(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(4*a^2+13*a*b+9*b^2)*co
s(f*x+e)^2*sin(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(-b/a*cos
(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(5*EllipticF(sin(f*x+e),(-
1/a*b)^(1/2))*a+8*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-8*EllipticE(sin(f
*x+e),(-1/a*b)^(1/2))*a-16*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x
+e)^2+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*x+e
))/(sin(f*x+e)-1)/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2
)/(1+sin(f*x+e))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.505.5 Fricas [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^4\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f
*x + e)^4, x)
```

3.505.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e)**4,x)
```

output
```
Timed out
```

3.505.7 Maxima [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^4\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

3.505.8 Giac [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^4\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

3.505.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx=\int \tan(e+f\,x)^4\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

input
```
int(tan(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.506 $\int (a+b\sin^2(e+fx))^{3/2}\tan^2(e+fx)\,dx$

3.506.1 Optimal result

Integrand size = 25, antiderivative size = 222

$$\int (a+b\sin^2(e+fx))^{3/2}\tan^2(e+fx)\,dx=\frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{(7a+8b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}+\frac{4a(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+b\sin^2(e+fx))^{3/2}\tan(e+fx)}{f}$$

output
```
4/3*b*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-1/3*(7*a+8*b)*Ellip
ticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x
+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+4/3*a*(a+b)*EllipticF(sin(f*x+e)
,(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/
f/(a+b*sin(f*x+e)^2)^(1/2)+(a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)/f
```

3.506.2 Mathematica [A] (verified)

Time = 3.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.78

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx=\frac{-8a(7a+8b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+32a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\operatorname{EllipticF}\left(e+f\right.}{24f\sqrt{2a+b-b}}$$

input
```
Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

output
```
(-8*a*(7*a + 8*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x
, -(b/a)] + 32*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[
e + f*x, -(b/a)] + Sqrt[2]*(24*a^2 + 40*a*b + 13*b^2 - 4*b*(2*a + 3*b)*Cos
[2*(e + f*x)] - b^2*Cos[4*(e + f*x)])*Tan[e + f*x])/(24*f*Sqrt[2*a + b - b
*Cos[2*(e + f*x)]])
```

3.506.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3675, 369, 403, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(e+fx)^2\left(a+b\sin(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \ 3675$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sin^2(e+fx)\left(b\sin^2(e+fx)+a\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \ 369$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{\sqrt{1-\sin^2(e+fx)}}-\int\frac{\sqrt{b\sin^2(e+fx)+a}\left(4b\sin^2(e+fx)+a\right)}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)\right)}{f}$$

$\downarrow$ 403

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\int-\frac{b(7a+8b)\sin^2(e+fx)+a(3a+4b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{\sqrt{1-\sin^2(e+fx)}}+\frac{4}{3}b\sin(\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{1}{3}\int\frac{b(7a+8b)\sin^2(e+fx)+a(3a+4b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{\sqrt{1-\sin^2(e+fx)}}+\frac{4}{3}b\sin(\right.}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(4a(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-(7a+8b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}\right.\right.}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}-(7a+8b)\int\frac{\sqrt{b\sin^2(e+}}{\sqrt{1-\sin^2(e}}\right.\right.}{f}$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}-(7a+8b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin\right.\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}-\frac{(7a+8b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e}{a}}}{\sqrt{1-\sin^2}}}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.\right.}{f}$$

$\downarrow$ 327

3.506. $\int\left(a+b\sin^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}-\frac{(7a+8b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin\right.}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.\right.}{f}$$

input
```
Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((4*b*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x
]^2]*Sqrt[a + b*Sin[e + f*x]^2])/3 + (Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^
(3/2))/Sqrt[1 - Sin[e + f*x]^2] + (-(((7*a + 8*b)*EllipticE[ArcSin[Sin[e +
 f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/Sqrt[1 + (b*Sin[e + f*x]^2)/a]
) + (4*a*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e
 + f*x]^2)/a])/Sqrt[a + b*Sin[e + f*x]^2])/3))/f
```

3.506.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.506.4 Maple [A] (verified)

Time = 5.14 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.75

method	result
default	$\frac{\sqrt{-b\left(\cos^4(fx+e)\right)+(a+b)\left(\cos^2(fx+e)\right)}\,\left(-b^2\left(\cos^4(fx+e)\right)\sin(fx+e)+4\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}\,F\left(\sin(fx+e),\right.\right.}{}$

input
```
int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(-b^2*cos(f*x+e)^4*sin(f*x+
e)+4*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(
f*x+e),(-1/a*b)^(1/2))*a^2+4*a*b*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(
a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))-7*(cos(f*x+e)^2)^(1/2)*
(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2
-8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*
x+e),(-1/a*b)^(1/2))*a*b-2*b*cos(f*x+e)^2*sin(f*x+e)*a-2*b^2*cos(f*x+e)^2*
sin(f*x+e)+3*sin(f*x+e)*a^2+6*a*b*sin(f*x+e)+3*b^2*sin(f*x+e))/(-(a+b*sin(
f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^
2)^(1/2)/f
```

3.506.5 Fricas [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f
*x + e)^2, x)
```

3.506.6 Sympy [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx=\int \left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan^2\left(e+fx\right)dx$$

input
```
integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e)**2,x)
```

output
```
Integral((a + b*sin(e + f*x)**2)**(3/2)*tan(e + f*x)**2, x)
```

3.506.7 Maxima [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

3.506.8 Giac [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

3.506.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx=\int \tan(e+f\,x)^2\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

input
```
int(tan(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.507 $\int (a+b\sin^2(e+fx))^{3/2}\,dx$

3.507.1 Optimal result

Integrand size = 16, antiderivative size = 154

$$\int (a+b\sin^2(e+fx))^{3/2}\,dx = -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{a(a+b)\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-1/3*b*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+2/3*(2*a+b)*(cos(f
*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+
e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*a*(a+b)*(cos(f*x+e)^2)^(1/2)/
cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/f
/(a+b*sin(f*x+e)^2)^(1/2)
```

3.507.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01

$$\int (a+b\sin^2(e+fx))^{3/2}\,dx = \frac{4\sqrt{2}a(2a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-2\sqrt{2}a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,\text{EllipticF}}{6\sqrt{2}f\sqrt{2a+b-b\cos(2(e+fx)}}$$

input
```
Integrate[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(4*Sqrt[2]*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e
+ f*x, -(b/a)] - 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a
]*EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e
+ f*x)])/(6*Sqrt[2]*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.507.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {3042, 3659, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin^2(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sin(e+fx)^2)^{3/2}\,dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{3}\int \frac{2b(2a+b)\sin^2(e+fx)+a(3a+b)}{\sqrt{b\sin^2(e+fx)+a}}dx-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{2b(2a+b)\sin(e+fx)^2+a(3a+b)}{\sqrt{b\sin(e+fx)^2+a}}dx-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3651}$$

$$\frac{1}{3}\left(2(2a+b)\int \sqrt{b\sin^2(e+fx)+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}dx\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(2(2a+b)\int \sqrt{b\sin(e+fx)^2+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3657}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin^2(e+fx)}{a}+1}dx}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin(e+fx)^2}{a}+1}dx}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3656}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3662}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}dx}{\sqrt{a+b\sin^2(e+fx)}}\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

3.507. $\int \left(a+b\sin^2(e+fx)\right)^{3/2} dx$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin(e+fx)^2}{a}+1}}dx}{\sqrt{a+b\sin^2(e+fx)}}\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

$$\Big\downarrow \text{3661}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}\right)-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}$$

input
```
Int[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-1/3*(b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f + ((2*(2*a
 + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (
b*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*
Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2]))/3
```

3.507.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.507.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.73

method	result
default	$-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2}{3}-\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)b}{3}+\frac{4\sqrt{\frac{\cos(2f}{2}}}{}$

input
```
int((a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/3*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+
e),(-1/a*b)^(1/2))*a^2-1/3*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(
1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b+4/3*(cos(f*x+e)^2)^(1/2)*((a+b
*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2/3*(cos(
f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)
^(1/2))*a*b+1/3*b^2*sin(f*x+e)^5+1/3*a*b*sin(f*x+e)^3-1/3*b^2*sin(f*x+e)^3
-1/3*a*b*sin(f*x+e))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.507.5 Fricas [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^(3/2), x)
```

3.507.6 Sympy [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \int \left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sin(e + f*x)**2)**(3/2), x)
```

3.507.7 Maxima [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.507.8 Giac [F]

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.507.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int((a + b*sin(e + f*x)^2)^(3/2), x)
```

3.508 $\int \cot^2(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx$

3.508.1 Optimal result

Integrand size = 25, antiderivative size = 223

$$\int \cot^2(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{f} - \frac{(7a-b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{4a(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/f+4/3*b*cos(f*x+e)*sin(f*x+e)*(a+b*si
n(f*x+e)^2)^(1/2)/f-1/3*(7*a-b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x
+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(
1/2)+4/3*a*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)
^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.508.2 Mathematica [A] (verified)

Time = 2.62 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.78

$$\int \cot^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{2}(-24a^2-8ab+3b^2+4(2a-b)b\cos(2(e+fx))+b^2\cos(4(e+fx)))\cot(e+fx)-8a(7a\ldots}{24f\sqrt{2a+b-\ldots}}$$

input
```
Integrate[Cot[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*(-24*a^2 - 8*a*b + 3*b^2 + 4*(2*a - b)*b*Cos[2*(e + f*x)] + b^2*C
os[4*(e + f*x)])*Cot[e + f*x] - 8*a*(7*a - b)*Sqrt[(2*a + b - b*Cos[2*(e +
 f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 32*a*(a + b)*Sqrt[(2*a + b - b*Cos
[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(24*f*Sqrt[2*a + b - b*Cos[2
*(e + f*x)]])
```

3.508.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3675, 375, 27, 403, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^{3/2}}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \csc^2(e+fx)\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{375}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(2\int -\frac{\sqrt{b\sin^2(e+fx)+a}(4b\sin^2(e+fx)+a-3b)}{2\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\sqrt{1-\sin^2(e+fx)}\csc(e+fx\right.}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\sqrt{1-\sin^2(e+fx)}(-\csc(e+fx))\left(a+b\sin^2(e+fx)\right)^{3/2}-\int\frac{\sqrt{b\sin^2(e+fx)+a}(4b\sin^2(e\right.}{\sqrt{1-\sin^2(e+fx}}}{f}$$

$\downarrow$ 403

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\int -\frac{(7a-b)b\sin^2(e+fx)+a(3a-5b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{4}{3}b\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{\right.}}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{1}{3}\int\frac{(7a-b)b\sin^2(e+fx)+a(3a-5b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{4}{3}b\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{\right.}}{f}$$

$\downarrow$ 399

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(4a(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-(7a-b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\right.\right.$$

$\downarrow$ 323

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}-(7a-b)\int\frac{\sqrt{b\sin^2(e+fx}}{\sqrt{1-\sin^2(e+}}\right.\right.$$

$\downarrow$ 321

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}-(7a-b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(\right.\right.$$

$\downarrow$ 330

3.508. $\int\cot^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}-\frac{(7a-b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+}{a}}}{\sqrt{1-\sin^2(e}}}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.\right.$$

$$\Big\downarrow \text{327}$$

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}-\frac{(7a-b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(\right.}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right.\right.$$

input
```
Int[Cot[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((4*b*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x
]^2]*Sqrt[a + b*Sin[e + f*x]^2])/3 - Csc[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]
*(a + b*Sin[e + f*x]^2)^(3/2) + (-(((7*a - b)*EllipticE[ArcSin[Sin[e + f*x
]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/Sqrt[1 + (b*Sin[e + f*x]^2)/a]) +
(4*a*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f
*x]^2)/a])/Sqrt[a + b*Sin[e + f*x]^2])/3))/f
```

3.508.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 375
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^p*((c + d*x^2)^q/(e*(m + 1)))
, x] - Simp[2/(e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^(p - 1)*(c + d*
x^2)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^2, x], x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 0] && LtQ[m, -1] && GtQ[p, 0
] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.508. $\int \cot^2(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx$

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.508.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.91

method	result
default	$\frac{(\cos^6(fx+e))b^2+(2ab-2b^2)(\cos^4(fx+e))+(-3a^2-2ab+b^2)(\cos^2(fx+e))+\sin(fx+e)\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,a}{3\sin(fx+e)\cos(fx+e)\sqrt{a+b(\sin^2(}}$

input
```
int(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(cos(f*x+e)^6*b^2+(2*a*b-2*b^2)*cos(f*x+e)^4+(-3*a^2-2*a*b+b^2)*cos(f*
x+e)^2+sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a
*(4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a+4*EllipticF(sin(f*x+e),(-1/a*b)
^(1/2))*b-7*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticE(sin(f*x+e),(-
1/a*b)^(1/2))*b))/sin(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.508.5 Fricas [F]

$$\int \cot^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f
*x + e)^2, x)
```

3.508.6 Sympy [F]

$$\int \cot^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\sin^2{(e+fx)}\right)^{\frac{3}{2}}\cot^2{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sin(e + f*x)**2)**(3/2)*cot(e + f*x)**2, x)
```

3.508.7 Maxima [F]

$$\int \cot^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin{(fx+e)}^2+a\right)^{\frac{3}{2}}\cot{(fx+e)}^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

3.508.8 Giac [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.508.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \cot(e+f\,x)^2\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

input
```
int(cot(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.509 $\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.509.1 Optimal result

Integrand size = 25, antiderivative size = 276

$$\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{(a-b)\cos^2(e+fx)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{(3a-5b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{\cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}+\frac{8(a-b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}-\frac{(5a-3b)(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-1/3*cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2)/f+(a-b)*cos(f*x+e)^2*cot(f*x+e)
*(a+b*sin(f*x+e)^2)^(1/2)/f+1/3*(3*a-5*b)*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f
*x+e)^2)^(1/2)/f+8/3*(a-b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(
cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-
1/3*(5*a-3*b)*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x
+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.509.2 Mathematica [A] (verified)

Time = 5.36 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.79

$$\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{-\frac{\left(-32a^2+44ab+58b^2+\left(64a^2-32ab-79b^2\right)\cos(2(e+fx))-2(6a-11b)b\cos(4(e+fx))-b^2\cos(6(e+fx))\right)\cot(e+fx)\csc^2(e+fa}{4\sqrt{2}}}{12f\sqrt{}}$$

input
```
Integrate[Cot[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-1/4*((-32*a^2 + 44*a*b + 58*b^2 + (64*a^2 - 32*a*b - 79*b^2)*Cos[2*(e +
f*x)] - 2*(6*a - 11*b)*b*Cos[4*(e + f*x)] - b^2*Cos[6*(e + f*x)])*Cot[e +
f*x]*Csc[e + f*x]^2)/Sqrt[2] + 32*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f
*x)])/a]*EllipticE[e + f*x, -(b/a)] - 4*(5*a^2 + 2*a*b - 3*b^2)*Sqrt[(2*a
+ b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(12*f*Sqrt[2*a +
b - b*Cos[2*(e + f*x)]])
```

3.509.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 3675, 375, 27, 442, 25, 403, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(e+fx)^2\right)^{3/2}}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \csc^4(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}\left(b\sin^2(e+fx)+a\right)^{3/2}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{375}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{2}{3}\int -\frac{3}{2}\csc^2(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}(2b\sin^2(e+fx)+a-b)\right.}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\int \csc^2(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}(2b\sin^2(e+fx)+a-b)\,d\sin\right.}{f}$$

$\downarrow$ 442

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\int -\frac{\sqrt{1-\sin^2(e+fx)}(2a^2-5ba+b^2+(3a-5b)b\sin^2(e+fx))}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\frac{1}{3}(1-\sin^2(e+fx)\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\int \frac{\sqrt{1-\sin^2(e+fx)}(2a^2-5ba+b^2+(3a-5b)b\sin^2(e+fx))}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)-\frac{1}{3}(1-\sin^2(e+fx))^{3/2}\right.}{f}$$

$\downarrow$ 403

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int \frac{b\left(8(a-b)b\sin^2(e+fx)+(a-3b)(3a-b)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3b}+\frac{1}{3}(3a-5b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\right.$$

$\downarrow$ 27

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\int \frac{8(a-b)b\sin^2(e+fx)+(a-3b)(3a-b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)+\frac{1}{3}(3a-5b)\sin(e+fx)\sqrt{1-\sin^2(e+\right.$$

$\downarrow$ 399

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(8(a-b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-(5a-3b)(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+\right.\right.$$

$\downarrow$ 323

3.509. $\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(8(a-b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{(5a-3b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{}}}{\sqrt{a+b\sin^2(e+fx)}}\right.\right.$$

$\downarrow$ 321

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(8(a-b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{(5a-3b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\text{si}\right.}{\sqrt{a+b\sin^2(e+fx)}}\right.\right.$$

$\downarrow$ 330

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{8(a-b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{(5a-3b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\right.}{\sqrt{a+b\sin^2(e+fx)}}\right.\right.$$

$\downarrow$ 327

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{1}{3}\left(\frac{8(a-b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{(5a-3b)(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\text{arcs}\right.}{\sqrt{a+b\sin^2(e+fx)}}\right.\right.$$

input
```
Int[Cot[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(((3*a - 5*b)*Sin[e + f*x]*Sqrt[1 - Sin
[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/3 + (a - b)*Csc[e + f*x]*(1 - Sin
[e + f*x]^2)^(3/2)*Sqrt[a + b*Sin[e + f*x]^2] - (Csc[e + f*x]^3*(1 - Sin[e
 + f*x]^2)^(3/2)*(a + b*Sin[e + f*x]^2)^(3/2))/3 + ((8*(a - b)*EllipticE[A
rcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/Sqrt[1 + (b*Sin[e
 + f*x]^2)/a] - ((5*a - 3*b)*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a
)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/Sqrt[a + b*Sin[e + f*x]^2])/3))/f
```

3.509. $\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.509.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 375
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^p*((c + d*x^2)^q/(e*(m + 1)))
, x] - Simp[2/(e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^(p - 1)*(c + d*
x^2)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^2, x], x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 0] && LtQ[m, -1] && GtQ[p, 0
] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.509. $\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.509. $\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

3.509.4 Maple [A] (verified)

Time = 3.81 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.52

method	result
default	$-\frac{-b^{2}\left(\sin^{8}(fx+e)\right)+5\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^{2}(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^{2}\left(\sin^{3}(fx+e)\right)+2b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^{2}\right.}{}}}{}$

input
```
int(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-b^2*sin(f*x+e)^8+5*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2
)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+2*b*(cos(f*x+e)^2)
^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a
*sin(f*x+e)^3-3*b^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Elli
pticF(sin(f*x+e),(-1/a*b)^(1/2))*sin(f*x+e)^3-8*(cos(f*x+e)^2)^(1/2)*((a+b
*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*sin(f*x+e
)^3+8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+
e),(-1/a*b)^(1/2))*a*b*sin(f*x+e)^3+3*a*b*sin(f*x+e)^6-3*b^2*sin(f*x+e)^6+
4*a^2*sin(f*x+e)^4-8*a*b*sin(f*x+e)^4+4*b^2*sin(f*x+e)^4-5*a^2*sin(f*x+e)^
2+5*a*b*sin(f*x+e)^2+a^2)/sin(f*x+e)^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)
/f
```

3.509.5 Fricas [F]

$$\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f
*x + e)^4, x)
```

3.509.6 Sympy [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.509.7 Maxima [F]

$$\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

3.509.8 Giac [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.509.9 Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx) \left(a+b\sin^2(e+fx)\right)^{3/2} dx = \int \cot(e+f\,x)^4 \left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2} dx$$

input
```
int(cot(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.510 $\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.510.1 Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{(8a^2+8ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{5/2}f} - \frac{(8a+5b)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)^2f} + \frac{\sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4(a+b)f}$$

output
```
1/8*(8*a^2+8*a*b+3*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b
)^(5/2)/f-1/8*(8*a+5*b)*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)^2/f+1/
4*sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)/f
```

3.510.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.81

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{(8a^2+8ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right) + \sqrt{a+b}\sec^2(e+fx)\,(-8a-5b+2(a+b)\sec^2(e+fx))\,\sqrt{a}}{8(a+b)^{5/2}f}$$

input
```
Integrate[Tan[e + f*x]^5/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] +
 Sqrt[a + b]*Sec[e + f*x]^2*(-8*a - 5*b + 2*(a + b)*Sec[e + f*x]^2)*Sqrt[a
 + b*Sin[e + f*x]^2])/(8*(a + b)^(5/2)*f)
```

3.510.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3673, 100, 27, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(e+fx)^5}{\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{ 3673}$$

$$\frac{\int \frac{\sin^4(e+fx)}{(1-\sin^2(e+fx))^3\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 100}$$

$$\frac{\frac{\sqrt{a+b\sin^2(e+fx)}}{2(a+b)(1-\sin^2(e+fx))^2}-\frac{\int \frac{4(a+b)\sin^2(e+fx)+4a+b}{2\left(1-\sin^2(e+fx)\right)^2\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{2(a+b)}}{2f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\sqrt{a+b\sin^2(e+fx)}}{2(a+b)(1-\sin^2(e+fx))^2}-\frac{\int \frac{4(a+b)\sin^2(e+fx)+4a+b}{\left(1-\sin^2(e+fx)\right)^2\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{4(a+b)}}{2f}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{\frac{\sqrt{a+b\sin^2(e+fx)}}{2(a+b)(1-\sin^2(e+fx))^2}-\frac{\frac{(8a+5b)\sqrt{a+b\sin^2(e+fx)}}{(a+b)(1-\sin^2(e+fx))}-\frac{(8a^2+8ab+3b^2)\int\frac{1}{(1-\sin^2(e+fx))\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{2(a+b)}}{4(a+b)}}{2f}$$

$\Big\downarrow$ 73

$$\frac{\frac{\sqrt{a+b\sin^2(e+fx)}}{2(a+b)(1-\sin^2(e+fx))^2}-\frac{\frac{(8a+5b)\sqrt{a+b\sin^2(e+fx)}}{(a+b)(1-\sin^2(e+fx))}-\frac{(8a^2+8ab+3b^2)\int\frac{1}{\frac{a+b}{b}-\frac{\sin^4(e+fx)}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b(a+b)}}{4(a+b)}}{2f}$$

$\Big\downarrow$ 221

$$\frac{\frac{\sqrt{a+b\sin^2(e+fx)}}{2(a+b)(1-\sin^2(e+fx))^2}-\frac{\frac{(8a+5b)\sqrt{a+b\sin^2(e+fx)}}{(a+b)(1-\sin^2(e+fx))}-\frac{(8a^2+8ab+3b^2)\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}}{4(a+b)}}{2f}$$

input

```
Int[Tan[e + f*x]^5/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output

```
(Sqrt[a + b*Sin[e + f*x]^2]/(2*(a + b)*(1 - Sin[e + f*x]^2)^2) - (-(((8*a^
2 + 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(a + b
)^(3/2)) + ((8*a + 5*b)*Sqrt[a + b*Sin[e + f*x]^2])/((a + b)*(1 - Sin[e +
f*x]^2)))/(4*(a + b)))/(2*f)
```

3.510.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.510.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(118) = 236$.

Time = 1.58 (sec) , antiderivative size = 644, normalized size of antiderivative = 4.81

method	result
default	$\frac{\left(8\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}-2b\sin(fx+e)+2a}{1+\sin(fx+e)}\right)a^4+24\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}-2b\sin(fx+e)+2a}{1+\sin(fx+e)}\right)a^3b+27\ln\left(\frac{2\sqrt{a+b}\,\sqrt{\ldots}}{\ldots}\right.\right.}{}$

input
```
int(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*((8*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin
(f*x+e)+a))*a^4+24*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(
1/2)-b*sin(f*x+e)+a))*a^3*b+27*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos
(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^2+14*ln(2/(1+sin(f*x+e))*((a+b)^(1
/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^3+3*ln(2/(1+sin(f*x+e)
)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^4+8*ln(2/(sin
(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4+24
*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+
a))*a^3*b+27*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b
*sin(f*x+e)+a))*a^2*b^2+14*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x
+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^3+3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+
b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^4)*cos(f*x+e)^4-2*(a+b)^(5/2)*(
a+b-b*cos(f*x+e)^2)^(1/2)*(8*a+5*b)*cos(f*x+e)^2+4*a*(a+b-b*cos(f*x+e)^2)^
(1/2)*(a+b)^(5/2)+4*b*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2))/(a+b)^(5/2)/
cos(f*x+e)^4/(a^2+2*a*b+b^2)/f
```

3.510.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.45

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

$$= \left[\frac{(8a^2+8ab+3b^2)\sqrt{a+b}\cos(fx+e)^4\log\left(\frac{b\cos(fx+e)^2-2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a+b}-2a-2b}{\cos(fx+e)^2}\right)-2\left((8a^2+13\right.}{16(a^3+3a^2b+3ab^2+b^3)f\cos(fx\ldots}\right.$$

$$-\frac{(8a^2+8ab+3b^2)\sqrt{-a-b}\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a-b}}{a+b}\right)\cos(fx+e)^4+\left((8a^2+13ab+5b^2)\cos\ldots\right.}{8(a^3+3a^2b+3ab^2+b^3)f\cos(fx+e)^4}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((8*a^2 + 8*a*b + 3*b^2)*sqrt(a + b)*cos(f*x + e)^4*log((b*cos(f*x +
 e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x
 + e)^2) - 2*((8*a^2 + 13*a*b + 5*b^2)*cos(f*x + e)^2 - 2*a^2 - 4*a*b - 2*
b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*c
os(f*x + e)^4), -1/8*((8*a^2 + 8*a*b + 3*b^2)*sqrt(-a - b)*arctan(sqrt(-b*
cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b))*cos(f*x + e)^4 + ((8*a^2 + 1
3*a*b + 5*b^2)*cos(f*x + e)^2 - 2*a^2 - 4*a*b - 2*b^2)*sqrt(-b*cos(f*x + e
)^2 + a + b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4)]
```

3.510.6 Sympy [F]

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\tan^5{(e+fx)}}{\sqrt{a+b\sin^2{(e+fx)}}}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**5/sqrt(a + b*sin(e + f*x)**2), x)
```

3.510.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(118) = 236$.

Time = 0.30 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.85

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\frac{(8a^2b^3+8ab^4+3b^5)\log\left(\frac{\sqrt{b\sin(fx+e)^2+a}-\sqrt{a+b}}{\sqrt{b\sin(fx+e)^2+a}+\sqrt{a+b}}\right)}{(a^2+2ab+b^2)\sqrt{a+b}} - \frac{2\left((8ab^4+5b^5)\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}-(8a^2b^4+11ab^5+3b^6)\sqrt{b\sin(fx-\cdots}\right.}{a^4+4a^3b+6a^2b^2+4ab^3+b^4+\left(b\sin(fx+e)^2+a\right)^2(a^2+2ab+b^2)-2(a^3+3a^2b+3ab^2+\cdots}}{16b^3f}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/16*((8*a^2*b^3 + 8*a*b^4 + 3*b^5)*log((sqrt(b*sin(f*x + e)^2 + a) - sqr
t(a + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/((a^2 + 2*a*b + b^2)
*sqrt(a + b)) - 2*((8*a*b^4 + 5*b^5)*(b*sin(f*x + e)^2 + a)^(3/2) - (8*a^2
*b^4 + 11*a*b^5 + 3*b^6)*sqrt(b*sin(f*x + e)^2 + a))/(a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4 + (b*sin(f*x + e)^2 + a)^2*(a^2 + 2*a*b + b^2) - 2*(
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(b*sin(f*x + e)^2 + a)))/(b^3*f)
```

3.510.8 Giac [F]

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\tan\left(fx+e\right)^5}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.510.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^5}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(tan(e + f*x)^5/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^5/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.511 $\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.511.1 Optimal result

Integrand size = 25, antiderivative size = 81

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f}$$

output
```
-1/2*(2*a+b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/f+1
/2*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)/f
```

3.511.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\frac{(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a+b}}{2f}$$

input
```
Integrate[Tan[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-1/2*(((2*a + b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(a + b)^
(3/2) - (Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(a + b))/f
```

3.511.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3673, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^3}{\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\sin^2(e+fx)}{(1-\sin^2(e+fx))^2\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{\sqrt{a+b\sin^2(e+fx)}}{(a+b)(1-\sin^2(e+fx))}-\frac{(2a+b)\int \frac{1}{(1-\sin^2(e+fx))\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{2(a+b)}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\sqrt{a+b\sin^2(e+fx)}}{(a+b)(1-\sin^2(e+fx))}-\frac{(2a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\sin^4(e+fx)}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b(a+b)}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\sqrt{a+b\sin^2(e+fx)}}{(a+b)(1-\sin^2(e+fx))}-\frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}}{2f}$$

input
```
Int[Tan[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-(((2*a + b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(a + b)^(3/
2)) + Sqrt[a + b*Sin[e + f*x]^2]/((a + b)*(1 - Sin[e + f*x]^2)))/(2*f)
```

3.511.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.511.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(69) = 138$.

Time = 1.31 (sec) , antiderivative size = 353, normalized size of antiderivative = 4.36

method	result
default	$-\left(2\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}-2b\sin(fx+e)+2a}{1+\sin(fx+e)}\right)a^2+3\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}-2b\sin(fx+e)+2a}{1+\sin(fx+e)}\right)ab+\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+\cdots}}{\cdots}\right.\right.$

input
```
int(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-(2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin
(f*x+e)+a))*a^2+3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)-b*sin(f*x+e)+a))*a*b+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e
)^2)^(1/2)-b*sin(f*x+e)+a))*b^2+2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*
cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2+3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2
)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b+ln(2/(sin(f*x+e)-1)*((a+
b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^2)*cos(f*x+e)^2+2*(
a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2))/(a+b)^(5/2)/cos(f*x+e)^2/f
```

3.511.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.72

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$= \left[\frac{(2a+b)\sqrt{a+b}\cos(fx+e)^2\log\left(\frac{b\cos(fx+e)^2+2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a+b}-2a-2b}{\cos(fx+e)^2}\right)+2\sqrt{-b\cos(fx+e)^2+\cdots}}{4(a^2+2ab+b^2)f\cos(fx+e)^2} \right.$$

input
```
integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((2*a + b)*sqrt(a + b)*cos(f*x + e)^2*log((b*cos(f*x + e)^2 + 2*sqrt(
-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*sq
rt(-b*cos(f*x + e)^2 + a + b)*(a + b))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)
^2), 1/2*((2*a + b)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sq
rt(-a - b)/(a + b))*cos(f*x + e)^2 + sqrt(-b*cos(f*x + e)^2 + a + b)*(a +
b))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2)]
```

3.511.6 Sympy [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\tan^3{(e+fx)}}{\sqrt{a+b\sin^2{(e+fx)}}}\,dx$$

input
```
integrate(tan(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**3/sqrt(a + b*sin(e + f*x)**2), x)
```

3.511.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.53

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\frac{2\sqrt{b\sin(fx+e)^2+a}\,b^3}{\left(b\sin(fx+e)^2+a\right)(a+b)-a^2-2\,ab-b^2} - \frac{(2\,ab^2+b^3)\log\left(\frac{\sqrt{b\sin(fx+e)^2+a}-\sqrt{a+b}}{\sqrt{b\sin(fx+e)^2+a}+\sqrt{a+b}}\right)}{(a+b)^{\frac{3}{2}}}}{4\,b^2f}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(b*sin(f*x + e)^2 + a)*b^3/((b*sin(f*x + e)^2 + a)*(a + b) - a
^2 - 2*a*b - b^2) - (2*a*b^2 + b^3)*log((sqrt(b*sin(f*x + e)^2 + a) - sqrt
(a + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/(a + b)^(3/2))/(b^2*f
)
```

3.511.8 Giac [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\tan{(fx+e)}^3}{\sqrt{b\sin{(fx+e)}^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.511.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx=\int \frac{{\mathrm{tan}\left(e+f\,x\right)}^3}{\sqrt{b\,{\sin\left(e+f\,x\right)}^2+a}} \,d x$$

input
```
int(tan(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.512 $\int \frac{\tan(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.512.1 Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}f}$$

output
```
arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)
```

3.512.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b-b\cos^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}f}$$

input
```
Integrate[Tan[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b - b*Cos[e + f*x]^2]/Sqrt[a + b]]/(Sqrt[a + b]*f)
```

3.512.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3673, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{a+b}{b}-\frac{\sin^4(e+fx)}{b}}d\sqrt{b\sin^2(e+fx)+a}}{bf}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f\sqrt{a+b}}$$

input
```
Int[Tan[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]]/(Sqrt[a + b]*f)
```

3.512.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.512.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(30) = 60$.

Time = 1.37 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.86

method	result	size
default	$\frac{\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}-2b\sin(fx+e)+2a}{1+\sin(fx+e)}\right)+\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}+2b\sin(fx+e)+2a}{\sin(fx+e)-1}\right)}{2\sqrt{a+b}\,f}$	103

input
```
int(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x
+e)+a))+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(
f*x+e)+a)))/(a+b)^(1/2)/f
```

3.512.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.11

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \left[\frac{\log\left(\frac{b\cos(fx+e)^2-2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a+b}-2a-2b}{\cos(fx+e)^2}\right)}{2\sqrt{a+b}f}, -\frac{\sqrt{-a-b}\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a-b}}{a+b}\right)}{(a+b)f}\right]$$

input
```
integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)
 - 2*a - 2*b)/cos(f*x + e)^2)/(sqrt(a + b)*f), -sqrt(-a - b)*arctan(sqrt(-
b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b))/((a + b)*f)]
```

3.512.6 Sympy [F]

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\tan(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

3.512.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(30) = 60.

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.94

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\frac{\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}(\sin(fx+e)+1)}-\frac{a}{\sqrt{ab}(\sin(fx+e)+1)}\right)}{\sqrt{a+b}}-\frac{\operatorname{arsinh}\left(-\frac{b\sin(fx+e)}{\sqrt{ab}(\sin(fx+e)-1)}-\frac{a}{\sqrt{ab}(\sin(fx+e)-1)}\right)}{\sqrt{a+b}}}{2f}$$

input
```
integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a/(sqrt(a*b)
*(sin(f*x + e) + 1)))/sqrt(a + b) - arcsinh(-b*sin(f*x + e)/(sqrt(a*b)*(si
n(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1)))/sqrt(a + b))/f
```

3.512.8 Giac [F]

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\tan\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.512.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(tan(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.513 $\int \frac{\cot(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.513.1 Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

output
```
-arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)
```

3.513.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

input
```
Integrate[Cot[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

3.513.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3673, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\csc^2(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{\sin^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sin^2(e+fx)+a}}{bf}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

input
```
Int[Cot[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

3.513.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.513.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

method	result	size
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2(fx+e)\right)}}{\sin(fx+e)}\right)}{\sqrt{a}\,f}$	42

input
```
int(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))/f
```

3.513.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.03

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \left[\frac{\log\left(\frac{2\left(b\cos(fx+e)^2+2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a}-2a-b\right)}{\cos(fx+e)^2-1}\right)}{2\sqrt{a}f}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a}}{a}\right)}{af}\right]$$

input
```
integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) -
 2*a - b)/(cos(f*x + e)^2 - 1))/(sqrt(a)*f), sqrt(-a)*arctan(sqrt(-b*cos(f
*x + e)^2 + a + b)*sqrt(-a)/a)/(a*f)]
```

3.513.6 Sympy [F]

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cot(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

3.513.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{\sqrt{a}f}$$

input
```
integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/(sqrt(a)*f)
```

3.513.8 Giac [F]

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.513.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)}{\sqrt{b\sin(e+f\,x)^2+a}}\,dx$$

input
```
int(cot(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.514 $\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.514.1 Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2af}$$

output
```
1/2*(2*a+b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-1/2*csc(f*
x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/a/f
```

3.514.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\frac{(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}}{2f}$$

input
```
Integrate[Cot[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(((2*a + b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/a^(3/2) - (Csc[e
+ f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/a)/(2*f)
```

3.514.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3673, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^3\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\csc^4(e+fx)(1-\sin^2(e+fx))}{\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{-\frac{(2a+b)\int \frac{\csc^2(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{2a}-\frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{-\frac{(2a+b)\int \frac{1}{\frac{\sin^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sin^2(e+fx)+a}}{ab}-\frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}}{2f}$$

input
```
Int[Cot[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(((2*a + b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/a^(3/2) - (Csc[e
+ f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/a)/(2*f)
```

3.514.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.514.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.45

method	result	size
default	$\frac{\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2(fx+e)\right)}}{\sin(fx+e)}\right)}{\sqrt{a}}-\frac{\sqrt{a+b\left(\sin^2(fx+e)\right)}}{2a\sin(fx+e)^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2(fx+e)\right)}}{\sin(fx+e)}\right)}{2a^{\frac{3}{2}}}}{f}$	109

input
```
int(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))-1/2/a/s
in(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)+1/2*b/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*
sin(f*x+e)^2)^(1/2))/sin(f*x+e)))/f
```

3.514.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.93

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \left[\frac{\left((2\,a+b)\cos\left(fx+e\right)^2-2\,a-b\right)\sqrt{a}\log\left(\frac{2\left(b\cos(fx+e)^2-2\,\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a}-2\,a-b\right)}{\cos(fx+e)^2-1}\right)+2\,\sqrt{-b\cos\left(fx\right.}}{4\left(a^2f\cos\left(fx+e\right)^2-a^2f\right)}\right.$$

$$-\frac{\left((2\,a+b)\cos\left(fx+e\right)^2-2\,a-b\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a}}{a}\right)-\sqrt{-b\cos\left(fx+e\right)^2+a+b}a}{2\left(a^2f\cos\left(fx+e\right)^2-a^2f\right)}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(((2*a + b)*cos(f*x + e)^2 - 2*a - b)*sqrt(a)*log(2*(b*cos(f*x + e)^2
 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 -
1)) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*a)/(a^2*f*cos(f*x + e)^2 - a^2*f),
 -1/2*(((2*a + b)*cos(f*x + e)^2 - 2*a - b)*sqrt(-a)*arctan(sqrt(-b*cos(f*
x + e)^2 + a + b)*sqrt(-a)/a) - sqrt(-b*cos(f*x + e)^2 + a + b)*a)/(a^2*f*
cos(f*x + e)^2 - a^2*f)]
```

3.514.6 Sympy [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cot^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**3/sqrt(a + b*sin(e + f*x)**2), x)
```

3.514.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\frac{2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{\sqrt{a}} + \frac{b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{a^{\frac{3}{2}}} - \frac{\sqrt{b\sin(fx+e)^2+a}}{a\sin(fx+e)^2}}{2\,f}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(2*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/sqrt(a) + b*arcsinh(a/(sqr
t(a*b)*abs(sin(f*x + e))))/a^(3/2) - sqrt(b*sin(f*x + e)^2 + a)/(a*sin(f*x
 + e)^2))/f
```

3.514.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.514.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^3}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(cot(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.515 $\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.515.1 Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{(8a^2+8ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f} + \frac{(8a+3b)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8a^2f} - \frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4af}$$

output
```
-1/8*(8*a^2+8*a*b+3*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)
/f+1/8*(8*a+3*b)*csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/a^2/f-1/4*csc(f*x+e
)^4*(a+b*sin(f*x+e)^2)^(1/2)/a/f
```

3.515.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

$$= \frac{-\left((8a^2+8ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)\right)+\sqrt{a}\csc^2(e+fx)\,(8a+3b-2a\csc^2(e+fx))\,\sqrt{a+b\sin}}{8a^{5/2}f}$$

input
```
Integrate[Cot[e + f*x]^5/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]) +
Sqrt[a]*Csc[e + f*x]^2*(8*a + 3*b - 2*a*Csc[e + f*x]^2)*Sqrt[a + b*Sin[e +
 f*x]^2])/(8*a^(5/2)*f)
```

3.515.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3673, 100, 27, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^5\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\csc^6(e+fx)\left(1-\sin^2(e+fx)\right)^2}{\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{100}$$

$$\frac{\frac{\int -\frac{\csc^4(e+fx)\left(-4a\sin^2(e+fx)+8a+3b\right)}{2\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{2a}-\frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2a}}{2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\csc^4(e+fx)\left(-4a\sin^2(e+fx)+8a+3b\right)}{\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{4a}-\frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2a}}{2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{-\frac{-\frac{\left(8a^2+8ab+3b^2\right)\int \frac{\csc^2(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{2a}-\frac{(8a+3b)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}}{4a}-\frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2a}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{-\frac{-\frac{\left(8a^2+8ab+3b^2\right)\int \frac{1}{\frac{\sin^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sin^2(e+fx)+a}}{ab}-\frac{(8a+3b)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}}{4a}-\frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2a}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{\frac{\left(8a^2+8ab+3b^2\right)\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(8a+3b)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}}{4a}-\frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2a}}{2f}$$

input
```
Int[Cot[e + f*x]^5/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-1/2*(Csc[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2])/a - (((8*a^2 + 8*a*b + 3
*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/a^(3/2) - ((8*a + 3*b)*
Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/a)/(4*a))/(2*f)
```

3.515.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

3.515. $\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.515.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.63

method	result
default	$\frac{-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2(fx+e)\right)}}{\sin(fx+e)}\right)}{\sqrt{a}}-\frac{\sqrt{a+b\left(\sin^2(fx+e)\right)}}{4a\sin(fx+e)^4}+\frac{3b\sqrt{a+b\left(\sin^2(fx+e)\right)}}{8a^2\sin(fx+e)^2}-\frac{3b^2\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2(fx+e)\right)}}{\sin(fx+e)}\right)}{8a^{\frac{5}{2}}}+\frac{\sqrt{a+b\left(\sin^2(fx+e\right.}}{a\sin(fx+e)^2}}{f}$

input
```
int(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))-1/4/a/
sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2)+3/8/a^2*b/sin(f*x+e)^2*(a+b*sin(f*x+
e)^2)^(1/2)-3/8/a^(5/2)*b^2*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/si
n(f*x+e))+1/a/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-b/a^(3/2)*ln((2*a+2*a^
(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e)))/f
```

3.515.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 388, normalized size of antiderivative = 3.08

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \left[\frac{\left((8\,a^2+8\,ab+3\,b^2)\cos\left(fx+e\right)^4-2\,(8\,a^2+8\,ab+3\,b^2)\cos\left(fx+e\right)^2+8\,a^2+8\,ab+3\,b^2\right)\sqrt{a}\log\left(\frac{2\left(b\right.}{}\right.}{16\left(a^3f\cos\left(fx+e\right)^4\right.}\right.$$

input
```
integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(((8*a^2 + 8*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 8*a*b + 3*b^2)
*cos(f*x + e)^2 + 8*a^2 + 8*a*b + 3*b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 +
 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)
) - 2*((8*a^2 + 3*a*b)*cos(f*x + e)^2 - 6*a^2 - 3*a*b)*sqrt(-b*cos(f*x + e
)^2 + a + b))/(a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f), 1/8
*(((8*a^2 + 8*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 8*a*b + 3*b^2)*cos(
f*x + e)^2 + 8*a^2 + 8*a*b + 3*b^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2
 + a + b)*sqrt(-a)/a) - ((8*a^2 + 3*a*b)*cos(f*x + e)^2 - 6*a^2 - 3*a*b)*s
qrt(-b*cos(f*x + e)^2 + a + b))/(a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x +
e)^2 + a^3*f)]
```

3.515.6 Sympy [F]

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cot^5\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

input
```
integrate(cot(f*x+e)**5/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**5/sqrt(a + b*sin(e + f*x)**2), x)
```

3.515.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.25

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\frac{8\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{\sqrt{a}} + \frac{8\,b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{a^{\frac{3}{2}}} + \frac{3\,b^2\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{a^{\frac{5}{2}}} - \frac{8\,\sqrt{b\sin(fx+e)^2+a}}{a\sin(fx+e)^2} - \frac{3\,\sqrt{b\sin(fx+e)^2+a}b}{a^2\sin(fx+e)^2}}{8\,f}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*(8*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/sqrt(a) + 8*b*arcsinh(a/(
sqrt(a*b)*abs(sin(f*x + e))))/a^(3/2) + 3*b^2*arcsinh(a/(sqrt(a*b)*abs(sin
(f*x + e))))/a^(5/2) - 8*sqrt(b*sin(f*x + e)^2 + a)/(a*sin(f*x + e)^2) - 3
*sqrt(b*sin(f*x + e)^2 + a)*b/(a^2*sin(f*x + e)^2) + 2*sqrt(b*sin(f*x + e)
^2 + a)/(a*sin(f*x + e)^4))/f
```

3.515.8 Giac [F]

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cot\left(fx+e\right)^5}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.515.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^5}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(cot(e + f*x)^5/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^5/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.516 $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.516.1 Optimal result

Integrand size = 25, antiderivative size = 246

$$\begin{aligned}\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx &= \frac{2(2a+b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\\ &\quad-\frac{a\sqrt{\cos^2(e+fx)}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3(a+b)f\sqrt{a+b\sin^2(e+fx)}}\\ &\quad-\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)^2f}\\ &\quad+\frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)f}\end{aligned}$$

output
```
2/3*(2*a+b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(
1/2)*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*a*E
llipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin
(f*x+e)^2/a)^(1/2)/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)-2/3*(2*a+b)*(a+b*sin(f
*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)^2/f+1/3*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1
/2)*tan(f*x+e)/(a+b)/f
```

3.516.2 Mathematica [A] (verified)

Time = 2.56 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.76

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$= \frac{4a(2a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-2a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)-\frac{(2(\cdots}{}}{6(a+b)^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Tan[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(4*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] - 2*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e +
f*x, -(b/a)] - ((2*(4*a^2 + 3*a*b + b^2)*Cos[2*(e + f*x)] + (2*a + b)*(2*a
 - b - b*Cos[4*(e + f*x)]))*Sec[e + f*x]^2*Tan[e + f*x])/Sqrt[2])/(6*(a +
b)^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.516.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3675, 372, 402, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(e+fx)^4}{\sqrt{a+b\sin(e+fx)^2}}dx$$
$$\Big\downarrow \text{3675}$$
$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sin^4(e+fx)}{(1-\sin^2(e+fx))^{5/2}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{f}$$
$$\Big\downarrow \text{372}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\int\frac{(3a+2b)\sin^2(e+fx)+a}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3(a+b)}\right)}{f}$$

$\downarrow$ 402

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\int-\frac{2b(2a+b)\sin^2(e+fx)+a(3a+b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a+b}+\frac{2(2a+b)\sin(e+fx)\sqrt{a+b\sin^2(}}{(a+b)\sqrt{1-\sin^2(e+fx)}}}{3(a+b)}\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)\sqrt{1-\sin^2(e+fx)}}-\frac{\int\frac{2b(2a+b)\sin^2(e+fx)+a(3a+b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+}{a+b}}{3(a+b)}\right.}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)\sqrt{1-\sin^2(e+fx)}}-\frac{2(2a+b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-a(}{}}{3(a+b)}\right.}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)\sqrt{1-\sin^2(e+fx)}}-\frac{2(2a+b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{a}{}}{}}{3(a+b)}\right.}{f}$$

$\downarrow$ 321

3.516. $\int\frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)\sqrt{1-\sin^2(e+fx)}}-\frac{2(2a+b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{a}{\ldots}}{\ldots}}{3(a+b)}\ldots\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)\sqrt{1-\sin^2(e+fx)}}-\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}\ldots}}{\sqrt{1-\sin^2(e+f\ldots}}}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\ldots}{3(a+b\ldots}\right.}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a+b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)\sqrt{1-\sin^2(e+fx)}}-\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+f\ldots\right.}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\ldots}{3(a+b)}\right.}{f}$$

input
```
Int[Tan[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^
2])/(3*(a + b)*(1 - Sin[e + f*x]^2)^(3/2)) - ((2*(2*a + b)*Sin[e + f*x]*Sq
rt[a + b*Sin[e + f*x]^2])/((a + b)*Sqrt[1 - Sin[e + f*x]^2]) - ((2*(2*a +
b)*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/Sqr
t[1 + (b*Sin[e + f*x]^2)/a] - (a*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], -
(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/Sqrt[a + b*Sin[e + f*x]^2])/(a + b)
)/(3*(a + b))))/f
```

3.516. $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.516.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

3.516. $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.516.4 Maple [A] (verified)

Time = 3.80 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.53

method	result
default	$-\frac{2\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\,b(2a+b)\left(\cos^4(fx+e)\right)\sin(fx+e)-\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\,\left(4a^2+7ab+3\right.}{}$

input
```
int(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(2*a+b)*cos(f*x+e)^4*
sin(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(4*a^2+7*a*b+3*b^2)*
cos(f*x+e)^2*sin(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(-b/a*c
os(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(EllipticF(sin(f*x+e),(-
1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-4*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))*a-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)
^2+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*x+e))/
(1+sin(f*x+e))/(sin(f*x+e)-1)/(a+b)^2/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*
(1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.516.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 845, normalized size of antiderivative = 3.43

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$
$$= \frac{\left(2\left(2i\,ab^2+i\,b^3\right)\sqrt{-b}\sqrt{\frac{a^2+ab}{b^2}}\cos\left(fx+e\right)^3-\left(-4i\,a^2b-4i\,ab^2-i\,b^3\right)\sqrt{-b}\cos\left(fx+e\right)^3\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+}{b}}}{}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*((2*(2*I*a*b^2 + I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)^3
- (-4*I*a^2*b - 4*I*a*b^2 - I*b^3)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt
((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*
b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b
^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-2*I*a*b^2 - I*b^3)
*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x + e)^3 - (4*I*a^2*b + 4*I*a*b^2 +
I*b^3)*sqrt(-b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)
/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f
*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a
^2 + a*b)/b^2))/b^2) + (2*(-3*I*a^2*b - 5*I*a*b^2 - 2*I*b^3)*sqrt(-b)*sqrt
((a^2 + a*b)/b^2)*cos(f*x + e)^3 - (-6*I*a^3 - 5*I*a^2*b - I*a*b^2)*sqrt(-
b)*cos(f*x + e)^3)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_
f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*s
in(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2
))/b^2) + (2*(3*I*a^2*b + 5*I*a*b^2 + 2*I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b
^2)*cos(f*x + e)^3 - (6*I*a^3 + 5*I*a^2*b + I*a*b^2)*sqrt(-b)*cos(f*x + e)
^3)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((
2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))),
(8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (a*b^
2 + b^3 - 2*(2*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a ...
```

3.516.6 Sympy [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**4/sqrt(a + b*sin(e + f*x)**2), x)
```

3.516.7 Maxima [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)^4}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.516.8 Giac [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)^4}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.516.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^4}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(tan(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.517 $\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.517.1 Optimal result

Integrand size = 25, antiderivative size = 109

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{(a+b)f}$$

output
```
-EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*s
in(f*x+e)^2)^(1/2)/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+(a+b*sin(f*x+e)^2)^(
1/2)*tan(f*x+e)/(a+b)/f
```

3.517.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{-2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\left|-\frac{b}{a}\right.\right)+\sqrt{2}(2a+b-b\cos(2(e+fx)))\tan(e+fx)}{2(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Tan[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(-2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] +
Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(2*(a + b)*f*Sqrt[2*a
 + b - b*Cos[2*(e + f*x)]])
```

3.517.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3675, 373, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{373}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)\sqrt{1-\sin^2(e+fx)}}-\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a+b}\right)}{f}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)\sqrt{1-\sin^2(e+fx)}}-\frac{\sqrt{a+b\sin^2(e+fx)}\int \frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right)}{f}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)\sqrt{1-\sin^2(e+fx)}}-\frac{\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right)}{f}$$

input
```
Int[Tan[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^
2])/((a + b)*Sqrt[1 - Sin[e + f*x]^2]) - (EllipticE[ArcSin[Sin[e + f*x]],
-(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/((a + b)*Sqrt[1 + (b*Sin[e + f*x]^2)/a
])))/f
```

3.517.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.517.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(102) = 204$.

Time = 2.83 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.04

method	result
default	$\frac{-\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\, b(\cos^2(fx+e))\sin(fx+e)+\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\,(a+b)\sin(fx+e)-a\sqrt{}}{(a+b)\sqrt{-(a+b(\sin^2(fx+e)))(\sin(fx+e)-1)(1+\sin(fx+e))}}$

input
```
int(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*cos(f*x+e)^2*sin(f*x+e)+(-b
*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a+b)*sin(f*x+e)-a*(cos(f*x+e)^2)^
(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^
2)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)))/(a+b)/(-(a+b*sin(f*x+e)^2)*
(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.517.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 730, normalized size of antiderivative = 6.70

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

$$= \frac{2\sqrt{-b\cos(fx+e)^2+a+b}\,b^2\sin(fx+e)-\left(2i\sqrt{-b}\,b^2\sqrt{\frac{a^2+ab}{b^2}}\cos(fx+e)+(2i\,ab+i\,b^2)\sqrt{-b}\cos(fx\right.}{}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(-b*cos(f*x + e)^2 + a + b)*b^2*sin(f*x + e) - (2*I*sqrt(-b)*b^
2*sqrt((a^2 + a*b)/b^2)*cos(f*x + e) + (2*I*a*b + I*b^2)*sqrt(-b)*cos(f*x
+ e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt
((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e)))
, (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (-2
*I*sqrt(-b)*b^2*sqrt((a^2 + a*b)/b^2)*cos(f*x + e) + (-2*I*a*b - I*b^2)*sq
rt(-b)*cos(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*ellipti
c_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I
*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b
^2))/b^2) - 2*(2*(-I*a*b - I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*cos(f*x +
 e) + (2*I*a^2 + I*a*b)*sqrt(-b)*cos(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/
b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
 + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b
 + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*(2*(I*a*b + I*b^2)*sqrt(-b)*sqrt((
a^2 + a*b)/b^2)*cos(f*x + e) + (-2*I*a^2 - I*a*b)*sqrt(-b)*cos(f*x + e))*s
qrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*s
qrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^
2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/((a*b^2 + b
^3)*f*cos(f*x + e))
```

3.517.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx=\int \frac{\tan^2{(e+fx)}}{\sqrt{a+b\sin^2{(e+fx)}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

3.517.7 Maxima [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx=\int \frac{\tan{(fx+e)}^2}{\sqrt{b\sin{(fx+e)}^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.517.8 Giac [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\tan\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.517.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^2}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(tan(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.518 $\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.518.1 Optimal result

Integrand size = 16, antiderivative size = 51

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*si
n(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.518.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)}{f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(f*Sqr
t[2*a + b - b*Cos[2*(e + f*x)]])
```

3.518.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3662}$$

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}dx}{\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin(e+fx)^2}{a}+1}}dx}{\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3661}$$

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

input
```
Int[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*
Sin[e + f*x]^2])
```

3.518.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.518.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.55 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\sqrt{\frac{a+b\left(\sin^{2}(fx+e)\right)}{a}}\, \operatorname{am}^{-1}\left(fx+e\middle\vert\frac{i\sqrt{b}}{\sqrt{a}}\right)}{f\sqrt{a+b\left(\sin^{2}(fx+e)\right)}}$	52

input
```
int(1/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(a+b*sin(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*InverseJacobiAM(
f*x+e,I/a^(1/2)*b^(1/2))
```

3.518.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 305, normalized size of antiderivative = 5.98

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx =$$

$$-\frac{\left(2i\sqrt{-b}b\sqrt{\frac{a^2+ab}{b^2}}+(-2i\,a-i\,b)\sqrt{-b}\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}F(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b}{b}}(\cos{(fx+e)}+i\,\mathrm{s}\right.}{}$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-((2*I*sqrt(-b)*b*sqrt((a^2 + a*b)/b^2) + (-2*I*a - I*b)*sqrt(-b))*sqrt((2
*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a
^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*
a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (-2*I*sqrt(-b)*b
*sqrt((a^2 + a*b)/b^2) + (2*I*a + I*b)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b
)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2
*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a
*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/(b^2*f)
```

3.518.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\sin^2{(e+fx)}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sin(e + f*x)**2), x)
```

3.518.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.518.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.518.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(1/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(1/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.519 $\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.519.1 Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

output
```
-cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a/f-EllipticE(sin(f*x+e),(-b/a)^(1/2)
)*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a/f/(1+b*sin(f*
x+e)^2/a)^(1/2)
```

3.519.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.95

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\sqrt{2a+b-b\cos(2(e+fx))}\cot(e+fx)}{\sqrt{2}af} - \frac{\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\left|-\frac{b}{a}\right.\right)}{f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Cot[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
-((Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Cot[e + f*x])/(Sqrt[2]*a*f)) - (Sqrt
[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)])/(f*Sqrt[2*a
 + b - b*Cos[2*(e + f*x)]])
```

3.519.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3675, 377, 25, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^2\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\csc^2(e+fx)\sqrt{1-\sin^2(e+fx)}}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{377}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int -\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}\right)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}\right)}{f}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}\right)}{f}$$

$$\Big\downarrow 327$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}\right)}{f}$$

input
```
Int[Cot[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-((Csc[e + f*x]*Sqrt[1 - Sin[e + f*x]^
2]*Sqrt[a + b*Sin[e + f*x]^2])/a) - (EllipticE[ArcSin[Sin[e + f*x]], -(b/a
)]*Sqrt[a + b*Sin[e + f*x]^2])/(a*Sqrt[1 + (b*Sin[e + f*x]^2)/a])))/f
```

3.519.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.519.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.13

method	result	size
default	$-\frac{-b\left(\cos^{4}\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^{2}\left(fx+e\right)\right)+\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^{2}\left(fx+e\right)\right)}{a}+\frac{a+b}{a}}\,a\sin\left(fx+e\right)E\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)}{a\sin\left(fx+e\right)\cos\left(fx+e\right)\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}\,f}$	120

input
```
int(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)
^2+(a+b)/a)^(1/2)*a*sin(f*x+e)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)))/a/sin
(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.519.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 723, normalized size of antiderivative = 6.82

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx =$$

$$-\frac{2\sqrt{-b\cos(fx+e)^2+a+b}b^2\cos(fx+e)+\left(2i\sqrt{-b}b^2\sqrt{\frac{a^2+ab}{b^2}}\sin(fx+e)+(2i\,ab+i\,b^2)\sqrt{-b}\sin(f\right.}{}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(-b*cos(f*x + e)^2 + a + b)*b^2*cos(f*x + e) + (2*I*sqrt(-b)*b
^2*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + (2*I*a*b + I*b^2)*sqrt(-b)*sin(f*x
 + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqr
t((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))
), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (-
2*I*sqrt(-b)*b^2*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + (-2*I*a*b - I*b^2)*s
qrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*ellipt
ic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) -
I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/
b^2))/b^2) + 2*(2*(-I*a*b - I*b^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x
+ e) + (2*I*a^2 + I*a*b)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)
/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*
a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*
b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 2*(2*(I*a*b + I*b^2)*sqrt(-b)*sqrt(
(a^2 + a*b)/b^2)*sin(f*x + e) + (-2*I*a^2 - I*a*b)*sqrt(-b)*sin(f*x + e))*
sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*
sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a
^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/(a*b^2*f*s
in(f*x + e))
```

3.519.6 Sympy [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cot^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

3.519.7 Maxima [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)^2}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.519.8 Giac [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)^2}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.519.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^2}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(cot(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.520 $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}} dx$

3.520.1 Optimal result

Integrand size = 25, antiderivative size = 240

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}} dx = \frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} + \frac{2(2a+b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{(a+b)\sqrt{\cos^2(e+fx)}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3af\sqrt{a+b\sin^2(e+fx)}}$$

output
```
2/3*(2*a+b)*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a^2/f-1/3*cot(f*x+e)*csc(f
*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/a/f+2/3*(2*a+b)*EllipticE(sin(f*x+e),(-b/
a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^2/f/(
1+b*sin(f*x+e)^2/a)^(1/2)-1/3*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec
(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/a/f/(a+b*sin(f*x+e
)^2)^(1/2)
```

3.520.2 Mathematica [A] (verified)

Time = 4.62 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.78

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\frac{(-2(4a^2+5ab+2b^2)\cos(2(e+fx))+(2a+b)(2a+3b+b\cos(4(e+fx))))\cot(e+fx)\csc^2(e+fx)}{\sqrt{2}}+4a(2a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E(e}{6a^2f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[Cot[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output
```
(((-2*(4*a^2 + 5*a*b + 2*b^2)*Cos[2*(e + f*x)] + (2*a + b)*(2*a + 3*b + b*
Cos[4*(e + f*x)]))*Cot[e + f*x]*Csc[e + f*x]^2)/Sqrt[2] + 4*a*(2*a + b)*Sq
rt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] - 2*a*(a +
 b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(6*
a^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.520.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3675, 376, 25, 445, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(e+fx)^4\sqrt{a+b\sin(e+fx)^2}}dx$$

$$\Big\downarrow \text{ 3675}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\csc^4(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 376}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int -\frac{\csc^2(e+fx)\left(2(2a+b)-(3a+b)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3a}-\frac{\sqrt{1-\sin^2(e+fx)}\csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a}\right)}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{\int \frac{\csc^2(e+fx)\left(2(2a+b)-(3a+b)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3a}-\frac{\sqrt{1-\sin^2(e+fx)}\csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a}\right)}{f}$$

$\downarrow$ 445

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{-\frac{\int \frac{2b(2a+b)\sin^2(e+fx)+a(3a+b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{2(2a+b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}}{3a}-\sqrt{1-\right.}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{-\frac{2(2a+b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-a(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{2(2a+b)\sqrt{1-\sin\right.}{3a}}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{-\frac{2(2a+b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+f}{\sqrt{a+b\sin^2(e+fx)}}}{a}}{3a}\right.}{f}$$

$\downarrow$ 321

3.520. $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{-\frac{2(2a+b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}}{a}}{3a}-\frac{2(2a+b)\sqrt{\cdots}}{\cdots}\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{-\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx))\cdots\right.}{\sqrt{a+b\sin^2(e+fx)}}}{a}}{3a}\cdots\right.}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{-\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}}{a}}{3a}\cdots\right.}{f}$$

input

```
Int[Cot[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

output

```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(-1/3*(Csc[e + f*x]^3*Sqrt[1 - Sin[e +
f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/a - ((-2*(2*a + b)*Csc[e + f*x]*Sqrt[1
 - Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/a - ((2*(2*a + b)*EllipticE
[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/Sqrt[1 + (b*Sin
[e + f*x]^2)/a] - (a*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[
1 + (b*Sin[e + f*x]^2)/a])/Sqrt[a + b*Sin[e + f*x]^2])/a)/(3*a)))/f
```

3.520. $\int\frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

3.520.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

3.520. $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.520.4 Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.46

method	result
default	$-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2\left(\sin^3(fx+e)\right)+b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e\right.}{}$

input
```
int(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+
e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e
)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-4*(cos(f
*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a^2*sin(f*x+e)^3-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2
)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b*sin(f*x+e)^3+4*a*b*sin(f*x+e)^6
+2*b^2*sin(f*x+e)^6+4*a^2*sin(f*x+e)^4-3*a*b*sin(f*x+e)^4-2*b^2*sin(f*x+e)
^4-5*a^2*sin(f*x+e)^2-a*b*sin(f*x+e)^2+a^2)/a^2/sin(f*x+e)^3/cos(f*x+e)/(a
+b*sin(f*x+e)^2)^(1/2)/f
```

3.520.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 1045, normalized size of antiderivative = 4.35

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*((2*(-2*I*a*b^2 - I*b^3 + (2*I*a*b^2 + I*b^3)*cos(f*x + e)^2)*sqrt(-b)
*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - (4*I*a^2*b + 4*I*a*b^2 + I*b^3 + (-4
*I*a^2*b - 4*I*a*b^2 - I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt(
(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt(
(a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 +
8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(2*I*a*b^2
+ I*b^3 + (-2*I*a*b^2 - I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b
^2)*sin(f*x + e) - (-4*I*a^2*b - 4*I*a*b^2 - I*b^3 + (4*I*a^2*b + 4*I*a*b^
2 + I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*
b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) +
2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*
a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(3*I*a^2*b + 5*I*a*b^2 + 2*I*b
^3 + (-3*I*a^2*b - 5*I*a*b^2 - 2*I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2
 + a*b)/b^2)*sin(f*x + e) - (6*I*a^3 + 5*I*a^2*b + I*a*b^2 + (-6*I*a^3 - 5
*I*a^2*b - I*a*b^2)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt(
(a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b
)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^
2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-3*I*a^2*b - 5*I*a*b
^2 - 2*I*b^3 + (3*I*a^2*b + 5*I*a*b^2 + 2*I*b^3)*cos(f*x + e)^2)*sqrt(-b)*
sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - (-6*I*a^3 - 5*I*a^2*b - I*a*b^2 + ...
```

3.520.6 Sympy [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**4/sqrt(a + b*sin(e + f*x)**2), x)
```

3.520.7 Maxima [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cot\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.520.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.520.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^4}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(cot(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2), x)
```

3.521 $\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

3.521.1 Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\frac{\left(8a^2-8ab-b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f}-\frac{8a^2-8ab-b^2}{8(a+b)^3f\sqrt{a+b\sin^2(e+fx)}}-\frac{(8a+3b)\sec^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\sec^4(e+fx)}{4(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
1/8*(8*a^2-8*a*b-b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^
(7/2)/f+1/8*(-8*a^2+8*a*b+b^2)/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(1/2)-1/8*(8*a
+3*b)*sec(f*x+e)^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+1/4*sec(f*x+e)^4/(a+
b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.521.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.51 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.60

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\frac{\left(-8a^2+8ab+b^2\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\sin^2(e+fx)}{a+b}\right)-\frac{1}{2}(a+b)(4}{8(a+b)^3f\sqrt{a+b\sin^2(e+fx}}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
((-8*a^2 + 8*a*b + b^2)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[e + f*x
]^2)/(a + b)] - ((a + b)*(4*a - b + (8*a + 3*b)*Cos[2*(e + f*x)])*Sec[e +
f*x]^4)/2)/(8*(a + b)^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

3.521.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3673, 100, 27, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\tan(e+fx)^5}{\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 3673$$

$$\frac{\int \frac{\sin^4(e+fx)}{\left(1-\sin^2(e+fx)\right)^3\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \ 100$$

$$\frac{\frac{1}{2(a+b)\left(1-\sin^2(e+fx)\right)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{\int \frac{4(a+b)\sin^2(e+fx)+4a-b}{2\left(1-\sin^2(e+fx)\right)^2\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+fx)}{2(a+b)}}{2f}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{2(a+b)\left(1-\sin^2(e+fx)\right)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{\int \frac{4(a+b)\sin^2(e+fx)+4a-b}{\left(1-\sin^2(e+fx)\right)^2\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+fx)}{4(a+b)}}{2f}$$

$$\Big\downarrow \ 87$$

$$\frac{\frac{1}{2(a+b)\left(1-\sin^2(e+fx)\right)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{8a+3b}{(a+b)\left(1-\sin^2(e+fx)\right)\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(8a^2-8ab-b^2\right)\int\frac{1}{\left(1-\sin^2(e+fx)\right)\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+}{2(a+b)}}{4(a+b)}}{2f}$$

$\downarrow$ 61

$$\frac{\frac{1}{2(a+b)\left(1-\sin^2(e+fx)\right)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{8a+3b}{(a+b)\left(1-\sin^2(e+fx)\right)\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(8a^2-8ab-b^2\right)\left(\frac{\int\frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+f}{a+b}\right.}{2(a+b)}}{4(a+b)}}{2f}$$

$\downarrow$ 73

$$\frac{\frac{1}{2(a+b)\left(1-\sin^2(e+fx)\right)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{8a+3b}{(a+b)\left(1-\sin^2(e+fx)\right)\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(8a^2-8ab-b^2\right)\left(\frac{2\int\frac{1}{\frac{a+b}{b}-\frac{\sin^4(e+fx)}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b(a+b)}-\frac{}{(a+}\right.}{2(a+b)}}{4(a+b)}}{2f}$$

$\downarrow$ 221

$$\frac{\frac{1}{2(a+b)\left(1-\sin^2(e+fx)\right)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{8a+3b}{(a+b)\left(1-\sin^2(e+fx)\right)\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(8a^2-8ab-b^2\right)\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{}{(a+b)\sqrt{a+}}\right.}{2(a+b)}}{4(a+b)}}{2f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(1/(2*(a + b)*(1 - Sin[e + f*x]^2)^2*Sqrt[a + b*Sin[e + f*x]^2]) - ((8*a +
 3*b)/((a + b)*(1 - Sin[e + f*x]^2)*Sqrt[a + b*Sin[e + f*x]^2]) - ((8*a^2
- 8*a*b - b^2)*((2*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(a + b
)^(3/2) - 2/((a + b)*Sqrt[a + b*Sin[e + f*x]^2])))/(2*(a + b)))/(4*(a + b)
))/(2*f)
```

3.521. $\displaystyle\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

3.521.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.521. $\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.521.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3766 vs. 2(155) = 310.

Time = 6.50 (sec) , antiderivative size = 3767, normalized size of antiderivative = 21.28

method	result	size
default	Expression too large to display	3767

input
```
int(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/(a^4*b^2*cos(f*x+e)^4+4*a^3*b^3*cos(f*x+e)^4+6*a^2*b^4*cos(f*x+e)^4+4
*a*b^5*cos(f*x+e)^4+b^6*cos(f*x+e)^4-2*a^5*b*cos(f*x+e)^2-10*a^4*b^2*cos(f
*x+e)^2-20*a^3*b^3*cos(f*x+e)^2-20*a^2*b^4*cos(f*x+e)^2-10*a*b^5*cos(f*x+e
)^2-2*b^6*cos(f*x+e)^2+a^6+6*a^5*b+15*a^4*b^2+20*a^3*b^3+15*a^2*b^4+6*a*b^
5+b^6)/cos(f*x+e)^4/(a+b)^(3/2)*(-72*(a+b)^(3/2)*(-b*cos(f*x+e)^2+(a*b^2+b
^3)/b^2)^(1/2)*a^2*b^2*cos(f*x+e)^4-40*(a+b)^(3/2)*(-b*cos(f*x+e)^2+(a*b^2
+b^3)/b^2)^(1/2)*a*b^3*cos(f*x+e)^4-38*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3
/2)*a^2*b*cos(f*x+e)^2-20*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+
e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b^3*cos(f*x+e)^4-30*ln(2/(1+sin(f*x+e))*(
(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^4*cos(f*x+e)
^4-12*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*
x+e)+a))*a*b^5*cos(f*x+e)^4+8*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(
f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4*b^2*cos(f*x+e)^8+8*ln(2/(sin(f*x+e)-1
)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b^3*cos(f*x
+e)^8-9*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(
f*x+e)+a))*a^2*b^4*cos(f*x+e)^8-10*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b
*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^5*cos(f*x+e)^8+8*ln(2/(1+sin(f*x
+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4*b^2*cos(
f*x+e)^8+8*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*s
in(f*x+e)+a))*a^3*b^3*cos(f*x+e)^8-9*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(...
```

3.521.5 Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 593, normalized size of antiderivative = 3.35

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \left[-\frac{\left((8\,a^2b-8\,ab^2-b^3)\cos\left(fx+e\right)^6-(8\,a^3-9\,ab^2-b^3)\cos\left(fx+e\right)^4\right)\sqrt{a}}{}\right.$$

$$-\frac{\left((8\,a^2b-8\,ab^2-b^3)\cos\left(fx+e\right)^6-(8\,a^3-9\,ab^2-b^3)\cos\left(fx+e\right)^4\right)\sqrt{-a-b}\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}}{a+b}\right.}{8\left((a^4b+4\,a^3b^2+6\,a^2b^3+4\,ab^4+b^5)\right.}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(((8*a^2*b - 8*a*b^2 - b^3)*cos(f*x + e)^6 - (8*a^3 - 9*a*b^2 - b^3
)*cos(f*x + e)^4)*sqrt(a + b)*log((b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x +
e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) - 2*((8*a^3 - 9*a*b
^2 - b^3)*cos(f*x + e)^4 - 2*a^3 - 6*a^2*b - 6*a*b^2 - 2*b^3 + (8*a^3 + 19
*a^2*b + 14*a*b^2 + 3*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)
)/((a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f*cos(f*x + e)^6 - (a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*f*cos(f*x + e)^4), -
1/8*(((8*a^2*b - 8*a*b^2 - b^3)*cos(f*x + e)^6 - (8*a^3 - 9*a*b^2 - b^3)*c
os(f*x + e)^4)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a
 - b)/(a + b)) - ((8*a^3 - 9*a*b^2 - b^3)*cos(f*x + e)^4 - 2*a^3 - 6*a^2*b
 - 6*a*b^2 - 2*b^3 + (8*a^3 + 19*a^2*b + 14*a*b^2 + 3*b^3)*cos(f*x + e)^2)
*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^
4 + b^5)*f*cos(f*x + e)^6 - (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a
*b^4 + b^5)*f*cos(f*x + e)^4)]
```

3.521.6 Sympy [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx=\int \frac{\tan^5{(e+fx)}}{(a+b\sin^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**5/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.521.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(157) = 314$.

Time = 0.33 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.89

$$\int \frac{\tan^5(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx=-\frac{\frac{(8a^2b^3-8ab^4-b^5)\log\left(\frac{\sqrt{b\sin(fx+e)^2+a}-\sqrt{a+b}}{\sqrt{b\sin(fx+e)^2+a}+\sqrt{a+b}}\right)}{(a^3+3a^2b+3ab^2+b^3)\sqrt{a+b}}+\frac{2\left(8a^4b^3+16a^3b^4+8a^2b^5+(8a^2b^3-8ab^4-b^5)\left(b\sin(fx+e)^2+a\right)^2-(1\cdots\right.}{(a^3+3a^2b+3ab^2+b^3)\left(b\sin(fx+e)^2+a\right)^{\frac{5}{2}}-2(a^4+4a^3b+6a^2b^2+4ab^3+b^4)\left(b\sin(fx+e)^2\cdots\right.}}{16b^3f}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/16*((8*a^2*b^3 - 8*a*b^4 - b^5)*log((sqrt(b*sin(f*x + e)^2 + a) - sqrt(
a + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/((a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*sqrt(a + b)) + 2*(8*a^4*b^3 + 16*a^3*b^4 + 8*a^2*b^5 + (8*a^2*b
^3 - 8*a*b^4 - b^5)*(b*sin(f*x + e)^2 + a)^2 - (16*a^3*b^3 + 8*a^2*b^4 - 7
*a*b^5 + b^6)*(b*sin(f*x + e)^2 + a))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(b*
sin(f*x + e)^2 + a)^(5/2) - 2*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*
(b*sin(f*x + e)^2 + a)^(3/2) + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 +
5*a*b^4 + b^5)*sqrt(b*sin(f*x + e)^2 + a)))/(b^3*f)
```

3.521.8 Giac [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx=\int \frac{\tan(fx+e)^5}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.521.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx=\int \frac{\tan(e+f\,x)^5}{(b\sin(e+f\,x)^2+a)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^5/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^5/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.522 $\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

3.522.1 Optimal result

Integrand size = 25, antiderivative size = 118

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = -\frac{(2a-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f} + \frac{2a-b}{2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^2(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-1/2*(2*a-b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f+1
/2*(2*a-b)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+1/2*sec(f*x+e)^2/(a+b)/f/(a+
b*sin(f*x+e)^2)^(1/2)
```

3.522.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{(2a-b)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\sin^2(e+fx)}{a+b}\right)+(a+b)\sec^2(e+fx)}{2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
((2*a - b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[e + f*x]^2)/(a + b)]
 + (a + b)*Sec[e + f*x]^2)/(2*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

3.522.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3673, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^3}{\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^2\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{1}{(a+b)\left(1-\sin^2(e+fx)\right)\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a-b)\int \frac{1}{\left(1-\sin^2(e+fx)\right)\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+fx)}{2(a+b)}}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{1}{(a+b)\left(1-\sin^2(e+fx)\right)\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a-b)\left(\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{a+b}-\frac{2}{(a+b)\sqrt{a+b\sin^2(e+fx)}}\right)}{2(a+b)}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{(a+b)\left(1-\sin^2(e+fx)\right)\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a-b)\left(\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\sin^4(e+fx)}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b(a+b)}-\frac{2}{(a+b)\sqrt{a+b\sin^2(e+fx)}}\right)}{2(a+b)}}{2f}$$

$$\downarrow 221$$

$$\frac{\frac{1}{(a+b)(1-\sin^2(e+fx))\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a-b)\left(\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{2}{(a+b)\sqrt{a+b\sin^2(e+fx)}}\right)}{2(a+b)}}{2f}$$

input
```
Int[Tan[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(1/((a + b)*(1 - Sin[e + f*x]^2)*Sqrt[a + b*Sin[e + f*x]^2]) - ((2*a - b)*
((2*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(a + b)^(3/2) - 2/((a
 + b)*Sqrt[a + b*Sin[e + f*x]^2])))/(2*(a + b)))/(2*f)
```

3.522.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

3.522. $\int \frac{\tan^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.522.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2536 vs. $2(102) = 204$.

Time = 4.02 (sec) , antiderivative size = 2537, normalized size of antiderivative = 21.50

method	result	size
default	Expression too large to display	2537

input
```
int(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(cos(f*x+e)^4*a^3*b^2+3*cos(f*x+e)^4*a^2*b^3+3*cos(f*x+e)^4*a*b^4+cos(
f*x+e)^4*b^5-2*cos(f*x+e)^2*a^4*b-8*cos(f*x+e)^2*a^3*b^2-12*cos(f*x+e)^2*a
^2*b^3-8*cos(f*x+e)^2*a*b^4-2*b^5*cos(f*x+e)^2+a^5+5*a^4*b+10*a^3*b^2+10*a
^2*b^3+5*a*b^4+b^5)/(a+b)^(1/2)/cos(f*x+e)^2*(2*(-b*cos(f*x+e)^2+(a*b^2+b^
3)/b^2)^(3/2)*(a+b)^(1/2)*b^2*cos(f*x+e)^4-2*ln(2/(1+sin(f*x+e))*((a+b)^(1
/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^2*cos(f*x+e)^6-ln(2/
(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*
b^3*cos(f*x+e)^6-2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(
1/2)+b*sin(f*x+e)+a))*a^2*b^2*cos(f*x+e)^6-ln(2/(sin(f*x+e)-1)*((a+b)^(1/2
)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^3*cos(f*x+e)^6-4*(a+b)^(
1/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*a*b*cos(f*x+e)^2+2*(-b*cos(f*x+e)^2+(a*b^2
+b^3)/b^2)^(1/2)*(a+b)^(1/2)*a^2*b*cos(f*x+e)^2+10*(-b*cos(f*x+e)^2+(a*b^2
+b^3)/b^2)^(1/2)*(a+b)^(1/2)*a*b^2*cos(f*x+e)^2+6*(a+b)^(1/2)*(a+b-b*cos(f
*x+e)^2)^(1/2)*a^2*b*cos(f*x+e)^2-2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2
)*(a+b)^(1/2)*a*b^2*cos(f*x+e)^6+4*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*
a*b^2*cos(f*x+e)^6+2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*(a+b)^(1/2)*a
*b*cos(f*x+e)^4+4*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)*a^2*
b*cos(f*x+e)^4-8*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)*a*b^2
*cos(f*x+e)^4+2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*b^2*cos(f*x+e)^4-12
*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)*b^3*cos(f*x+e)^4+4...
```

3.522.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(102) = 204$.

Time = 0.45 (sec) , antiderivative size = 445, normalized size of antiderivative = 3.77

$$\int \frac{\tan^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \left[-\frac{((2\,ab-b^2)\cos(fx+e)^4-(2\,a^2+ab-b^2)\cos(fx+e)^2)\sqrt{a+b}\log\left(\frac{b}{-}\right.}{4\left((a^3b+3\,a^2b^2+3\,ab^3\right.}\right.$$

input
```
integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(((2*a*b - b^2)*cos(f*x + e)^4 - (2*a^2 + a*b - b^2)*cos(f*x + e)^2)
*sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqr
t(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*((2*a^2 + a*b - b^2)*cos(f*x + e
)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3*b + 3*a^2*
b^2 + 3*a*b^3 + b^4)*f*cos(f*x + e)^4 - (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4)*f*cos(f*x + e)^2), 1/2*(((2*a*b - b^2)*cos(f*x + e)^4 - (2*a^2 +
 a*b - b^2)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a
 + b)*sqrt(-a - b)/(a + b)) - ((2*a^2 + a*b - b^2)*cos(f*x + e)^2 + a^2 +
2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3*b + 3*a^2*b^2 + 3*a*b^
3 + b^4)*f*cos(f*x + e)^4 - (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*f*
cos(f*x + e)^2)]
```

3.522.6 Sympy [F]

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\tan^3\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**3/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.522.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.64

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\frac{(2\,ab^2-b^3)\log\left(\frac{\sqrt{b\sin(fx+e)^2+a}-\sqrt{a+b}}{\sqrt{b\sin(fx+e)^2+a}+\sqrt{a+b}}\right)}{(a^2+2\,ab+b^2)\sqrt{a+b}} - \frac{2\left(2\,a^2b^2+2\,ab^3-(2\,ab^2-b^3)\left(b\sin(fx+e)^2-\right.\right.}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}(a^2+2\,ab+b^2)-(a^3+3\,a^2b+3\,ab^2+b^3)}}{4\,b^2f}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*((2*a*b^2 - b^3)*log((sqrt(b*sin(f*x + e)^2 + a) - sqrt(a + b))/(sqrt(
b*sin(f*x + e)^2 + a) + sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) -
2*(2*a^2*b^2 + 2*a*b^3 - (2*a*b^2 - b^3)*(b*sin(f*x + e)^2 + a))/((b*sin(f
*x + e)^2 + a)^(3/2)*(a^2 + 2*a*b + b^2) - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*sqrt(b*sin(f*x + e)^2 + a)))/(b^2*f)
```

3.522.8 Giac [F]

$$\int \frac{\tan^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)^3}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.522.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^3}{(b\sin(e+f\,x)^2+a)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.523 $\int \frac{\tan(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.523.1 Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \frac{\tan(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f} - \frac{1}{(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/f-1/(a+b)/f/(a+b
*sin(f*x+e)^2)^(1/2)
```

3.523.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \frac{\tan(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\frac{b\cos^2(e+fx)}{a+b}\right)}{(a+b)f\sqrt{a+b-b\cos^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
-(Hypergeometric2F1[-1/2, 1, 1/2, 1 - (b*Cos[e + f*x]^2)/(a + b)]/((a + b)
*f*Sqrt[a + b - b*Cos[e + f*x]^2]))
```

3.523.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3673, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)}{(a+b\sin(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{a+b}-\frac{2}{(a+b)\sqrt{a+b\sin^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\sin^4(e+fx)}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b(a+b)}-\frac{2}{(a+b)\sqrt{a+b\sin^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{2}{(a+b)\sqrt{a+b\sin^2(e+fx)}}}{2f}$$

input
```
Int[Tan[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(a + b)^(3/2) - 2/((a
 + b)*Sqrt[a + b*Sin[e + f*x]^2]))/(2*f)
```

3.523.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.523.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1316 vs. $2(55) = 110$.

Time = 3.62 (sec) , antiderivative size = 1317, normalized size of antiderivative = 20.90

method	result	size
default	Expression too large to display	1317

input
```
int(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a/(cos(f*x+e)^4*a^2*b^2+2*a*b^3*cos(f*x+e)^4+b^4*cos(f*x+e)^4-2*cos(f*
x+e)^2*a^3*b-6*cos(f*x+e)^2*a^2*b^2-6*a*b^3*cos(f*x+e)^2-2*b^4*cos(f*x+e)^
2+a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*(-(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(
1/2)*b^3+(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*b^2-2*(a+b-b*cos(f*x+e)^2
)^(1/2)*a^3-(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*a^2+(-b*cos(f*x+e)^2+(
a*b^2+b^3)/b^2)^(1/2)*a^3-(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^2*a+(-
b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b*a^2-4*b*(a+b-b*cos(f*x+e)^2)^(1/2)
*a^2-2*b^2*(a+b-b*cos(f*x+e)^2)^(1/2)*a+(a+b)^(1/2)*ln(2/(sin(f*x+e)-1)*((
a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3+(a+b)^(1/2)*ln(
2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*
a^3+b^2*(a+b)^(1/2)*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^
(1/2)-b*sin(f*x+e)+a))*a+2*b*(a+b)^(1/2)*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*
(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2+b^2*(a+b)^(1/2)*ln(2/(sin(
f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a+2*b*(
a+b)^(1/2)*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*s
in(f*x+e)+a))*a^2+b^2*((a+b)^(1/2)*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b
*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a+(a+b)^(1/2)*ln(2/(1+sin(f*x+e))*((
a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a+(-b*cos(f*x+e)^2+
(a*b^2+b^3)/b^2)^(1/2)*a-(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b-2*(a+b-
b*cos(f*x+e)^2)^(1/2)*a)*cos(f*x+e)^4-b*((-b*cos(f*x+e)^2+(a*b^2+b^3)/b...
```

3.523.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(55) = 110$.

Time = 0.35 (sec) , antiderivative size = 281, normalized size of antiderivative = 4.46

$$\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \left[\frac{\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{a+b}\log\left(\frac{b\cos(fx+e)^2-2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a+b}-2a}{\cos(fx+e)^2}\right.}{2\left(\left(a^2b+2ab^2+b^3\right)f\cos\left(fx+e\right)^2-\left(a^3+3a^2\right.\right.}\right.$$

$$-\frac{\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{-a-b}\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a-b}}{a+b}\right)-\sqrt{-b\cos\left(fx+e\right)^2+a+b}(a+b)}{\left(a^2b+2ab^2+b^3\right)f\cos\left(fx+e\right)^2-\left(a^3+3a^2b+3ab^2+b^3\right)f}$$

input
```
integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((b*cos(f*x + e)^2 - a - b)*sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sqr
t(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*
sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b))/((a^2*b + 2*a*b^2 + b^3)*f*cos(f*
x + e)^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f), -((b*cos(f*x + e)^2 - a - b
)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b)
) - sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b))/((a^2*b + 2*a*b^2 + b^3)*f*co
s(f*x + e)^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)]
```

3.523.6 Sympy [F]

$$\int \frac{\tan(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+fx)}{(a+b\sin^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.523.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(55) = 110$.

Time = 0.34 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.27

$$\int \frac{\tan(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = -\frac{\frac{\operatorname{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}(\sin(fx+e)+1)}-\frac{a}{\sqrt{ab}(\sin(fx+e)+1)}\right)}{(a+b)^{\frac{3}{2}}} - \frac{\operatorname{arsinh}\left(-\frac{b\sin(fx+e)}{\sqrt{ab}(\sin(fx+e)-1)}-\frac{a}{\sqrt{ab}(\sin(fx+e)-1)}\right)}{(a+b)^{\frac{3}{2}}} + \frac{2}{\sqrt{b\sin(fx+e)^2+a}a+\sqrt{b\sin(fx+e)^2+a}b}}{2f}$$

input
```
integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a/(sqrt(a*b)
*(sin(f*x + e) + 1)))/(a + b)^(3/2) - arcsinh(-b*sin(f*x + e)/(sqrt(a*b)*(
sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1)))/(a + b)^(3/2) + 2/(
sqrt(b*sin(f*x + e)^2 + a)*a + sqrt(b*sin(f*x + e)^2 + a)*b))/f
```

3.523.8 Giac [F]

$$\int \frac{\tan(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.523.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)}{\left(b\sin(e+f\,x)^2+a\right)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.524 $\int \frac{\cot(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.524.1 Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \frac{\cot(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/a/f/(a+b*sin(f*x+e)
^2)^(1/2)
```

3.524.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{\cot(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{b\sin^2(e+fx)}{a}\right)}{af\sqrt{a+b\sin^2(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sin[e + f*x]^2)/a]/(a*f*Sqrt[a + b*
Sin[e + f*x]^2])
```

3.524.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3673, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\csc^2(e+fx)}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\int \frac{\csc^2(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{a}+\frac{2}{a\sqrt{a+b\sin^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2\int \frac{1}{\frac{\sin^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sin^2(e+fx)+a}}{ab}+\frac{2}{a\sqrt{a+b\sin^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2}{a\sqrt{a+b\sin^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}}{2f}$$

input
```
Int[Cot[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a +
b*Sin[e + f*x]^2]))/(2*f)
```

3.524.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.524.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\frac{1}{a\sqrt{a+b\left(\sin^2(fx+e)\right)}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2(fx+e)\right)}}{\sin(fx+e)}\right)}{a^{\frac{3}{2}}}}{f}$	62

input
```
int(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(1/a/(a+b*sin(f*x+e)^2)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^
2)^(1/2))/sin(f*x+e)))/f
```

3.524.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(49) = 98$.

Time = 0.38 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.95

$$\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \left[\frac{\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{a}\log\left(\frac{2\left(b\cos(fx+e)^2+2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a}-2a-b\right)}{\cos(fx+e)^2-1}\right.}{2\left(a^2bf\cos\left(fx+e\right)^2-\left(a^3+a^2b\right)f\right)}\right.$$

input
```
integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((b*cos(f*x + e)^2 - a - b)*sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(
-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*sq
rt(-b*cos(f*x + e)^2 + a + b)*a)/(a^2*b*f*cos(f*x + e)^2 - (a^3 + a^2*b)*f
), ((b*cos(f*x + e)^2 - a - b)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a
+ b)*sqrt(-a)/a) - sqrt(-b*cos(f*x + e)^2 + a + b)*a)/(a^2*b*f*cos(f*x + e
)^2 - (a^3 + a^2*b)*f)]
```

3.524.6 Sympy [F]

$$\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cot\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.524.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{\cot(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = -\frac{\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{a^{\frac{3}{2}}} - \frac{1}{\sqrt{b\sin(fx+e)^2+a}a}}{f}$$

input
```
integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-(arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(3/2) - 1/(sqrt(b*sin(f*x + e
)^2 + a)*a))/f
```

3.524.8 Giac [F]

$$\int \frac{\cot(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.524.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+f\,x)}{(b\sin(e+f\,x)^2+a)^{3/2}}\,dx$$

input
```
int(cot(e + f*x)/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.525 $\int \frac{\cot^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.525.1 Optimal result

Integrand size = 25, antiderivative size = 110

$$\int \frac{\cot^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f} - \frac{2a+3b}{2a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^2(e+fx)}{2af\sqrt{a+b\sin^2(e+fx)}}$$

output
```
1/2*(2*a+3*b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/2*(-2*
a-3*b)/a^2/f/(a+b*sin(f*x+e)^2)^(1/2)-1/2*csc(f*x+e)^2/a/f/(a+b*sin(f*x+e)
^2)^(1/2)
```

3.525.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64

$$\int \frac{\cot^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \frac{-a\csc^2(e+fx) - (2a+3b)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{b\sin^2(e+fx)}{a}\right)}{2a^2f\sqrt{a+b\sin^2(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-(a*Csc[e + f*x]^2) - (2*a + 3*b)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*
Sin[e + f*x]^2)/a])/(2*a^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

3.525.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3673, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(e+fx)^3\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3673}$$

$$\frac{\int \frac{\csc^4(e+fx)\left(1-\sin^2(e+fx)\right)}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{-\frac{(2a+3b)\int \frac{\csc^2(e+fx)}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+fx)}{2a}-\frac{\csc^2(e+fx)}{a\sqrt{a+b\sin^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{-\frac{(2a+3b)\left(\frac{\int \frac{\csc^2(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{a}+\frac{2}{a\sqrt{a+b\sin^2(e+fx)}}\right)}{2a}-\frac{\csc^2(e+fx)}{a\sqrt{a+b\sin^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{-\frac{(2a+3b)\left(\frac{2\int \frac{1}{\frac{\sin^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sin^2(e+fx)+a}}{ab}+\frac{2}{a\sqrt{a+b\sin^2(e+fx)}}\right)}{2a}-\frac{\csc^2(e+fx)}{a\sqrt{a+b\sin^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{(2a+3b)\left(\frac{2}{a\sqrt{a+b\sin^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{\csc^2(e+fx)}{a\sqrt{a+b\sin^2(e+fx)}}}{2f}$$

input
```
Int[Cot[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-(Csc[e + f*x]^2/(a*Sqrt[a + b*Sin[e + f*x]^2])) - ((2*a + 3*b)*((-2*ArcT
anh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a + b*Sin[e +
 f*x]^2])))/(2*a))/(2*f)
```

3.525.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

3.525. $\int \frac{\cot^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.525.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.35

method	result
default	$\frac{-\frac{1}{a\sqrt{a+b\left(\sin^2(fx+e)\right)}}+\frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\left(\sin^2(fx+e)\right)}}{\sin(fx+e)}\right)}{a^{\frac{3}{2}}}-\frac{1}{2a\sin(fx+e)^2\sqrt{a+b\left(\sin^2(fx+e)\right)}}-\frac{3b}{2a^2\sqrt{a+b\left(\sin^2(fx+e)\right)}}+\frac{3b\ln\left(\frac{2a+2\sqrt{a}\sqrt{a-}}{\sin(}\right.}{2a}}{f}$

input
```
int(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/a/(a+b*sin(f*x+e)^2)^(1/2)+1/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)
^2)^(1/2))/sin(f*x+e))-1/2/a/sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2)-3/2/a^2
*b/(a+b*sin(f*x+e)^2)^(1/2)+3/2/a^(5/2)*b*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e
)^2)^(1/2))/sin(f*x+e)))/f
```

3.525.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(94) = 188.

Time = 0.38 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.69

$$\int \frac{\cot^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \left[\frac{\left((2\,ab+3\,b^2)\cos\left(fx+e\right)^4-(2\,a^2+7\,ab+6\,b^2)\cos\left(fx+e\right)^2+2\,a^2+5\,a\right.}{4\,(a}\right.$$

$$-\frac{\left((2\,ab+3\,b^2)\cos\left(fx+e\right)^4-(2\,a^2+7\,ab+6\,b^2)\cos\left(fx+e\right)^2+2\,a^2+5\,ab+3\,b^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-b\cos(f}}{}\right.}{2\left(a^3bf\cos\left(fx+e\right)^4-(a^4+2\,a^3b)f\cos\left(f\right.\right.}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((2*a*b + 3*b^2)*cos(f*x + e)^4 - (2*a^2 + 7*a*b + 6*b^2)*cos(f*x +
e)^2 + 2*a^2 + 5*a*b + 3*b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 - 2*sqrt(-b*
cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) + 2*((2*a
^2 + 3*a*b)*cos(f*x + e)^2 - 3*a^2 - 3*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b
))/(a^3*b*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b)*f*cos(f*x + e)^2 + (a^4 + a^3
*b)*f), -1/2*(((2*a*b + 3*b^2)*cos(f*x + e)^4 - (2*a^2 + 7*a*b + 6*b^2)*co
s(f*x + e)^2 + 2*a^2 + 5*a*b + 3*b^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)
^2 + a + b)*sqrt(-a)/a) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 - 3*a^2 - 3*a*b)
*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^3*b*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b
)*f*cos(f*x + e)^2 + (a^4 + a^3*b)*f)]
```

3.525.6 Sympy [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^3\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**3/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.525.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.06

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\frac{2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{a^{\frac{3}{2}}}+\frac{3\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{a^{\frac{5}{2}}}-\frac{2}{\sqrt{b\sin(fx+e)^2+a}a}-\frac{3\,b}{\sqrt{b\sin(fx+e)^2+a}}}{2\,f}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(3/2) + 3*b*arcsinh(a/(s
qrt(a*b)*abs(sin(f*x + e))))/a^(5/2) - 2/(sqrt(b*sin(f*x + e)^2 + a)*a) -
3*b/(sqrt(b*sin(f*x + e)^2 + a)*a^2) - 1/(sqrt(b*sin(f*x + e)^2 + a)*a*sin
(f*x + e)^2))/f
```

3.525.8 Giac [F]

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cot\left(fx+e\right)^3}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^3/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.525.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cot(e+f\,x)^3}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(cot(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.526 $\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

3.526.1 Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = -\frac{(8a^2+24ab+15b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f} + \frac{8a^2+24ab+15b^2}{8a^3f\sqrt{a+b\sin^2(e+fx)}} + \frac{(8a+5b)\csc^2(e+fx)}{8a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^4(e+fx)}{4af\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-1/8*(8*a^2+24*a*b+15*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(7/
2)/f+1/8*(8*a^2+24*a*b+15*b^2)/a^3/f/(a+b*sin(f*x+e)^2)^(1/2)+1/8*(8*a+5*b
)*csc(f*x+e)^2/a^2/f/(a+b*sin(f*x+e)^2)^(1/2)-1/4*csc(f*x+e)^4/a/f/(a+b*si
n(f*x+e)^2)^(1/2)
```

3.526.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.56

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{a\csc^2(e+fx)\left(8a+5b-2a\csc^2(e+fx)\right)+(8a^2+24ab+15b^2)\,\text{Hypergeor}}{8a^3f\sqrt{a+b\sin^2(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(a*Csc[e + f*x]^2*(8*a + 5*b - 2*a*Csc[e + f*x]^2) + (8*a^2 + 24*a*b + 15*
b^2)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sin[e + f*x]^2)/a])/(8*a^3*f*S
qrt[a + b*Sin[e + f*x]^2])
```

3.526.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3673, 100, 27, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^5\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\csc^6(e+fx)\left(1-\sin^2(e+fx)\right)^2}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{100}$$

$$\frac{\frac{\int -\frac{\csc^4(e+fx)\left(-4a\sin^2(e+fx)+8a+5b\right)}{2\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+fx)}{2a}-\frac{\csc^4(e+fx)}{2a\sqrt{a+b\sin^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\csc^4(e+fx)\left(-4a\sin^2(e+fx)+8a+5b\right)}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+fx)}{4a}-\frac{\csc^4(e+fx)}{2a\sqrt{a+b\sin^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{-\frac{-\frac{\left(8a^2+3b(8a+5b)\right)\int \frac{\csc^2(e+fx)}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+fx)}{2a}-\frac{(8a+5b)\csc^2(e+fx)}{a\sqrt{a+b\sin^2(e+fx)}}}{4a}-\frac{\csc^4(e+fx)}{2a\sqrt{a+b\sin^2(e+fx)}}}{2f}$$

$$\Big\downarrow 61$$

$$\frac{-\frac{-\frac{\left(8a^2+3b(8a+5b)\right)\left(\frac{\int \frac{\csc^2(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{a}+\frac{2}{a\sqrt{a+b\sin^2(e+fx)}}\right)}{2a}-\frac{(8a+5b)\csc^2(e+fx)}{a\sqrt{a+b\sin^2(e+fx)}}}{4a}-\frac{\csc^4(e+fx)}{2a\sqrt{a+b\sin^2(e+fx)}}}{2f}$$

$$\Big\downarrow 73$$

$$\frac{-\frac{-\frac{\left(8a^2+3b(8a+5b)\right)\left(\frac{2\int \frac{1}{\frac{\sin^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sin^2(e+fx)+a}}{ab}+\frac{2}{a\sqrt{a+b\sin^2(e+fx)}}\right)}{2a}-\frac{(8a+5b)\csc^2(e+fx)}{a\sqrt{a+b\sin^2(e+fx)}}}{4a}-\frac{\csc^4(e+fx)}{2a\sqrt{a+b\sin^2(e+fx)}}}{2f}$$

$$\Big\downarrow 221$$

$$\frac{-\frac{-\frac{\left(8a^2+3b(8a+5b)\right)\left(\frac{2}{a\sqrt{a+b\sin^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{(8a+5b)\csc^2(e+fx)}{a\sqrt{a+b\sin^2(e+fx)}}}{4a}-\frac{\csc^4(e+fx)}{2a\sqrt{a+b\sin^2(e+fx)}}}{2f}$$

input
```
Int[Cot[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-1/2*Csc[e + f*x]^4/(a*Sqrt[a + b*Sin[e + f*x]^2]) - (-(((8*a + 5*b)*Csc[
e + f*x]^2)/(a*Sqrt[a + b*Sin[e + f*x]^2])) - ((8*a^2 + 3*b*(8*a + 5*b))*(
(-2*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a + b
*Sin[e + f*x]^2])))/(2*a))/(4*a))/(2*f)
```

3.526. $\int \frac{\cot^5(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.526.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.526. $\int \frac{\cot^5(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.526.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.59

method	result
default	$\frac{\frac{1}{a\sqrt{a+b\left(\sin^2(fx+e)\right)}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\left(\sin^2(fx+e)\right)}}{\sin(fx+e)}\right)}{a^{\frac{3}{2}}}-\frac{1}{4a\sin(fx+e)^4\sqrt{a+b\left(\sin^2(fx+e)\right)}}+\frac{5b}{8a^2\sin(fx+e)^2\sqrt{a+b\left(\sin^2(fx+e)\right)}}+\frac{}{8a^3\sqrt{a+b}}$

input
```
int(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(1/a/(a+b*sin(f*x+e)^2)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^
2)^(1/2))/sin(f*x+e))-1/4/a/sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2)+5/8/a^2*
b/sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2)+15/8/a^3*b^2/(a+b*sin(f*x+e)^2)^(1
/2)-15/8/a^(7/2)*b^2*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e
))+1/a/sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2)+3/a^2*b/(a+b*sin(f*x+e)^2)^(1
/2)-3/a^(5/2)*b*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e)))/f
```

3.526.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(147) = 294$.

Time = 0.41 (sec) , antiderivative size = 652, normalized size of antiderivative = 3.90

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \left[\frac{\left((8\,a^2b+24\,ab^2+15\,b^3)\cos\left(fx+e\right)^6-(8\,a^3+48\,a^2b+87\,ab^2+45\,b^3)\cos\right.}{}\right.$$

input
```
integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(((8*a^2*b + 24*a*b^2 + 15*b^3)*cos(f*x + e)^6 - (8*a^3 + 48*a^2*b +
 87*a*b^2 + 45*b^3)*cos(f*x + e)^4 - 8*a^3 - 32*a^2*b - 39*a*b^2 - 15*b^3
+ (16*a^3 + 72*a^2*b + 102*a*b^2 + 45*b^3)*cos(f*x + e)^2)*sqrt(a)*log(2*(
b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(c
os(f*x + e)^2 - 1)) - 2*((8*a^3 + 24*a^2*b + 15*a*b^2)*cos(f*x + e)^4 + 14
*a^3 + 29*a^2*b + 15*a*b^2 - (24*a^3 + 53*a^2*b + 30*a*b^2)*cos(f*x + e)^2
)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^4*b*f*cos(f*x + e)^6 - (a^5 + 3*a^4*
b)*f*cos(f*x + e)^4 + (2*a^5 + 3*a^4*b)*f*cos(f*x + e)^2 - (a^5 + a^4*b)*f
), 1/8*(((8*a^2*b + 24*a*b^2 + 15*b^3)*cos(f*x + e)^6 - (8*a^3 + 48*a^2*b
+ 87*a*b^2 + 45*b^3)*cos(f*x + e)^4 - 8*a^3 - 32*a^2*b - 39*a*b^2 - 15*b^3
 + (16*a^3 + 72*a^2*b + 102*a*b^2 + 45*b^3)*cos(f*x + e)^2)*sqrt(-a)*arcta
n(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/a) - ((8*a^3 + 24*a^2*b + 15*a*
b^2)*cos(f*x + e)^4 + 14*a^3 + 29*a^2*b + 15*a*b^2 - (24*a^3 + 53*a^2*b +
30*a*b^2)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^4*b*f*cos(f*
x + e)^6 - (a^5 + 3*a^4*b)*f*cos(f*x + e)^4 + (2*a^5 + 3*a^4*b)*f*cos(f*x
+ e)^2 - (a^5 + a^4*b)*f)]
```

3.526.6 Sympy [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^5{(e+fx)}}{(a+b\sin^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**5/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**5/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.526.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.31

$$\int \frac{\cot^5(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx =$$
$$-\frac{\frac{8\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{a^{\frac{3}{2}}} + \frac{24\,b\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{a^{\frac{5}{2}}} + \frac{15\,b^2\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{a^{\frac{7}{2}}} - \frac{8}{\sqrt{b\sin(fx+e)^2+a}a} - \frac{24\,b}{\sqrt{b\sin(fx+e)^2+a}a^2} - \cdots}{8\,f}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/8*(8*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(3/2) + 24*b*arcsinh(a/
(sqrt(a*b)*abs(sin(f*x + e))))/a^(5/2) + 15*b^2*arcsinh(a/(sqrt(a*b)*abs(s
in(f*x + e))))/a^(7/2) - 8/(sqrt(b*sin(f*x + e)^2 + a)*a) - 24*b/(sqrt(b*s
in(f*x + e)^2 + a)*a^2) - 15*b^2/(sqrt(b*sin(f*x + e)^2 + a)*a^3) - 8/(sqr
t(b*sin(f*x + e)^2 + a)*a*sin(f*x + e)^2) - 5*b/(sqrt(b*sin(f*x + e)^2 + a
)*a^2*sin(f*x + e)^2) + 2/(sqrt(b*sin(f*x + e)^2 + a)*a*sin(f*x + e)^4))/f
```

3.526.8 Giac [F]

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\cot\left(fx+e\right)^5}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^5/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.526.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\text{Hanged}$$

input
```
int(cot(e + f*x)^5/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
\text{Hanged}
```

3.527 $\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

3.527.1 Optimal result

Integrand size = 25, antiderivative size = 292

$$\begin{aligned}\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx &= \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\sqrt{a+b\sin^2(e+fx)}}\\ &+\frac{(7a-b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)^3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\\ &-\frac{4a\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}\\ &-\frac{4a\tan(e+fx)}{3(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

output
```
1/3*(7*a-b)*b*cos(f*x+e)*sin(f*x+e)/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3
*(7*a-b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2
)*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)^3/f/(1+b*sin(f*x+e)^2/a)^(1/2)-4/3*a*Elli
pticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*
x+e)^2/a)^(1/2)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)-4/3*a*tan(f*x+e)/(a+b)^
2/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*sec(f*x+e)^2*tan(f*x+e)/(a+b)/f/(a+b*sin(
f*x+e)^2)^(1/2)
```

3.527.2 Mathematica [A] (verified)

Time = 2.78 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.67

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{2a(7a-b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-8a(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}}{6(a+b)}$$

input
```
Integrate[Tan[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(2*a*(7*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] - 8*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e +
f*x, -(b/a)] - ((8*a^2 - 21*a*b - 5*b^2 + 4*(4*a^2 - 3*a*b + b^2)*Cos[2*(e
 + f*x)] + b*(-7*a + b)*Cos[4*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/(2*
Sqrt[2]))/(6*(a + b)^3*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.527.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 3675, 372, 27, 402, 25, 402, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(e+fx)^4}{\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 3675$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sin^4(e+fx)}{\left(1-\sin^2(e+fx)\right)^{5/2}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 372$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{a+b\sin^2(e+fx)}}-\frac{\int\frac{a\left(3\sin^2(e+fx)+1\right)}{\left(1-\sin^2(e+fx)\right)^{3/2}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3(a+b)}\right)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{a+b\sin^2(e+fx)}}-\frac{a\int\frac{3\sin^2(e+fx)+1}{\left(1-\sin^2(e+fx)\right)^{3/2}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3(a+b)}\right)}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{a+b\sin^2(e+fx)}}-\frac{a\left(\frac{\int-\frac{-4b\sin^2(e+fx)+3a-b}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{a+b}+\frac{\cdots}{(a+\cdots}\right.}{3(a+b)}\right.}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{a+b\sin^2(e+fx)}}-\frac{a\left(\frac{4\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}-\frac{\int\frac{-4b\sin^2\cdots}{\sqrt{1-\sin^2(e+fx)}\cdots}}{\cdots}\right.}{3(a+b)}\right.}{f}$$

$$\Big\downarrow \; 402$$

3.527. $\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{a+b\sin^2(e+fx)}}-\frac{a\left(\frac{4\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{b(7a-b)\sin(e+fx}{a(a+b)\sqrt{a+b}}}{}\right.}{}\right.}{f}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{a+b\sin^2(e+fx)}}-\frac{a\left(\frac{4\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}-\frac{\int\frac{(7a-b)b\sin^2(}{\sqrt{1-\sin^2(e+fx}}}{}\right.}{3}\right.}{f}$$

$\Big\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{a+b\sin^2(e+fx)}}-\frac{a\left(\frac{4\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}-\frac{(7a-b)\int\frac{\sqrt{b\sin^2(}}{\sqrt{1-\sin}}}{}\right.}{}\right.}{f}$$

$\Big\downarrow$ 323

3.527. $\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{a+b\sin^2(e+fx)}}-\frac{a\left(\frac{4\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}-\frac{(7a-b)\int\frac{\sqrt{b\sin^2(\cdots}}{\sqrt{1-\sin\cdots}}}{\cdots}\right)}{\cdots}\right)}{f\cdots}$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{a+b\sin^2(e+fx)}}-\frac{a\left(\frac{4\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}-\frac{(7a-b)\int\frac{\sqrt{b\sin^2(\cdots}}{\sqrt{1-\sin\cdots}}}{\cdots}\right)}{\cdots}\right)}{f\cdots}$$

$\downarrow$ 330

3.527. $\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{a+b\sin^2(e+fx)}}-\frac{a\left(\frac{4\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{(7a-b)\sqrt{a+b\sin^2}}{}}{}\right.}{}\right.$$

f

$\downarrow$ 327

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{a+b\sin^2(e+fx)}}-\frac{a\left(\frac{4\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{(7a-b)\sqrt{a+b\sin^2}}{}}{}\right.}{}\right.$$

f

input
```
Int[Tan[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

3.527. $\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(Sin[e + f*x]/(3*(a + b)*(1 - Sin[e + f
*x]^2)^(3/2)*Sqrt[a + b*Sin[e + f*x]^2]) - (a*((4*Sin[e + f*x])/((a + b)*S
qrt[1 - Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2]) - (((7*a - b)*b*Sin[e
+ f*x]*Sqrt[1 - Sin[e + f*x]^2])/(a*(a + b)*Sqrt[a + b*Sin[e + f*x]^2]) +
(((7*a - b)*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x
]^2])/Sqrt[1 + (b*Sin[e + f*x]^2)/a] - (4*a*(a + b)*EllipticF[ArcSin[Sin[e
 + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/Sqrt[a + b*Sin[e + f*x]^
2])/(a*(a + b)))/(a + b)))/(3*(a + b))))/f
```

3.527.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

3.527. $\int \frac{\tan^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.527. $\int \frac{\tan^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.527.4 Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.26

method	result
default	$-\frac{\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\, b(7a-b)(\cos^4(fx+e))\sin(fx+e)-4\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\, a(a+b)(\cos^2}{}$

input
```
int(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(7*a-b)*cos(f*x+e)^4*si
n(f*x+e)-4*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(a+b)*cos(f*x+e)^2
*sin(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+
(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(4*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-7*EllipticE(sin(f*x+e),(-1/
a*b)^(1/2))*a+EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2+(-b*cos
(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*x+e))/(1+sin(f*x
+e))/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)
-1)/(a+b)^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.527.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 1228, normalized size of antiderivative = 4.21

$$\int \frac{\tan^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/6*((2*((7*I*a*b^3 - I*b^4)*cos(f*x + e)^5 + (-7*I*a^2*b^2 - 6*I*a*b^3 +
I*b^4)*cos(f*x + e)^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((-14*I*a^2*b^2 -
5*I*a*b^3 + I*b^4)*cos(f*x + e)^5 + (14*I*a^3*b + 19*I*a^2*b^2 + 4*I*a*b^3
 - I*b^4)*cos(f*x + e)^3)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
+ b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(c
os(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqr
t((a^2 + a*b)/b^2))/b^2) + (2*((-7*I*a*b^3 + I*b^4)*cos(f*x + e)^5 + (7*I*
a^2*b^2 + 6*I*a*b^3 - I*b^4)*cos(f*x + e)^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2
) - ((14*I*a^2*b^2 + 5*I*a*b^3 - I*b^4)*cos(f*x + e)^5 + (-14*I*a^3*b - 19
*I*a^2*b^2 - 4*I*a*b^3 + I*b^4)*cos(f*x + e)^3)*sqrt(-b))*sqrt((2*b*sqrt((
a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)
/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2
 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*(2*((3*I*a^2*b^2 + 2*I*
a*b^3 - I*b^4)*cos(f*x + e)^5 + (-3*I*a^3*b - 5*I*a^2*b^2 - I*a*b^3 + I*b^
4)*cos(f*x + e)^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + ((-6*I*a^3*b + 7*I*a^2
*b^2 + 5*I*a*b^3)*cos(f*x + e)^5 + (6*I*a^4 - I*a^3*b - 12*I*a^2*b^2 - 5*I
*a*b^3)*cos(f*x + e)^3)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a +
b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos
(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt(
(a^2 + a*b)/b^2))/b^2) - 2*(2*((-3*I*a^2*b^2 - 2*I*a*b^3 + I*b^4)*cos(f...
```

3.527.6 Sympy [F]

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\tan^4\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**4/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.527.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.527.8 Giac [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)^4}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.527.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^4}{(b\sin(e+f\,x)^2+a)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.528 $\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

3.528.1 Optimal result

Integrand size = 25, antiderivative size = 224

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = -\frac{2b\cos(e+fx)\sin(e+fx)}{(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{2\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-2*b*cos(f*x+e)*sin(f*x+e)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)-2*EllipticE(
sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2
)^(1/2)/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+EllipticF(sin(f*x+e),(-b/a)^(
1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/(a+b)/f/(
a+b*sin(f*x+e)^2)^(1/2)+tan(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.528.2 Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.65

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{-2\sqrt{2}a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}(a+b)\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}\text{E}}{\sqrt{2}(a+b)^2f\sqrt{2a+b-b\cos}}$$

input
```
Integrate[Tan[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-2*Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(
b/a)] + Sqrt[2]*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e
 + f*x, -(b/a)] + 2*(a - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(Sqrt[2]*(a + b
)^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.528.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3675, 373, 402, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^{3/2}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{373}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}-\frac{\int\frac{a-b\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{a+b}\right)}{f}$$

$\downarrow$ 402

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{2b\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\int-\frac{a\left(2b\sin^2(e+fx)+a-b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}}{a(a+b)}}{a+b}\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{\int\frac{a\left(2b\sin^2(e+fx)+a-b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a(a+b)}+\frac{2b\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{a+b\sin}}}{a+b}\right.}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{\int\frac{2b\sin^2(e+fx)+a-b}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a+b}+\frac{2b\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{a+b\sin}}}{a+b}\right.}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{2\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e}}}{a+b}}{a+b}\right.}{f}$$

$\downarrow$ 323

3.528. $\displaystyle\int\frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{2\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{}{\sqrt{1-\sin^2(e+fx)}\cdots}}{\sqrt{a+b\cdots}}}{a+b}}{a\cdots}\right.}{f}$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{2\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\cdots}{\sqrt{a+b\sin^2(e\cdots}}}{a+b}}{a+b}\right.}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{2\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{(a+b)\sqrt{\frac{b\sin^2(\cdots}{\cdots}}}{\cdots}}{a+b}\right.}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\sqrt{a+b\sin^2(e+fx)}}-\frac{\frac{\frac{2\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}\cdots}}{\cdots}}{a+b}}{a\cdots}\right.}{f}$$

input
```
Int[Tan[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

3.528. $\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(Sin[e + f*x]/((a + b)*Sqrt[1 - Sin[e +
 f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2]) - ((2*b*Sin[e + f*x]*Sqrt[1 - Sin[e +
 f*x]^2])/((a + b)*Sqrt[a + b*Sin[e + f*x]^2]) + ((2*EllipticE[ArcSin[Sin[
e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/Sqrt[1 + (b*Sin[e + f*x]^2)
/a] - ((a + b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e +
 f*x]^2)/a])/Sqrt[a + b*Sin[e + f*x]^2])/(a + b))/(a + b)))/f
```

3.528.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

3.528. $\int \frac{\tan^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.528. $\int \frac{\tan^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.528.4 Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.24

method	result
default	$\frac{\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\left(a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)+b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{}\right.}{(a+b)^2\sqrt{-(a+b(\sin^2(fx+}}$

input
```
int(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a*(cos(f*x+e)^2)^(1/2)*(-b/a*c
os(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))+b*(cos(f*x
+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*
b)^(1/2))-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*Ellip
ticE(sin(f*x+e),(-1/a*b)^(1/2))-2*cos(f*x+e)^2*sin(f*x+e)*b+a*sin(f*x+e)+b
*sin(f*x+e))/(a+b)^2/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(
1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.528.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 1048, normalized size of antiderivative = 4.68

$$\int \frac{\tan^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
((2*(-I*b^3*cos(f*x + e)^3 + (I*a*b^2 + I*b^3)*cos(f*x + e))*sqrt(-b)*sqrt
((a^2 + a*b)/b^2) - ((2*I*a*b^2 + I*b^3)*cos(f*x + e)^3 + (-2*I*a^2*b - 3*
I*a*b^2 - I*b^3)*cos(f*x + e))*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) +
 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/
b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2
)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(I*b^3*cos(f*x + e)^3 + (-I*a*b^2 - I*b
^3)*cos(f*x + e))*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((-2*I*a*b^2 - I*b^3)*c
os(f*x + e)^3 + (2*I*a^2*b + 3*I*a*b^2 + I*b^3)*cos(f*x + e))*sqrt(-b))*sq
rt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sq
rt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2
 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((I*a*b^
2 + I*b^3)*cos(f*x + e)^3 + (-I*a^2*b - 2*I*a*b^2 - I*b^3)*cos(f*x + e))*s
qrt(-b)*sqrt((a^2 + a*b)/b^2) - ((2*I*a^2*b - I*a*b^2 - I*b^3)*cos(f*x + e
)^3 + (-2*I*a^3 - I*a^2*b + 2*I*a*b^2 + I*b^3)*cos(f*x + e))*sqrt(-b))*sqr
t((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqr
t((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2
+ 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*((-I*a*b^
2 - I*b^3)*cos(f*x + e)^3 + (I*a^2*b + 2*I*a*b^2 + I*b^3)*cos(f*x + e))*sq
rt(-b)*sqrt((a^2 + a*b)/b^2) - ((-2*I*a^2*b + I*a*b^2 + I*b^3)*cos(f*x + e
)^3 + (2*I*a^3 + I*a^2*b - 2*I*a*b^2 - I*b^3)*cos(f*x + e))*sqrt(-b))*s...
```

3.528.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\tan^2\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**2/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.528.7 Maxima [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)^2}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.528.8 Giac [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)^2}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.528.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^2}{\left(b\sin(e+f\,x)^2+a\right)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.529 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

3.529.1 Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}}+\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{a(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

output
```
b*cos(f*x+e)*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+(cos(f*x+e)^2)^
(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)^(1/
2)/a/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)
```

3.529.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\frac{2a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}b\sin(2(e+fx))}{2a(a+b)f\sqrt{2a+b-b\cos(2(e+fx))}}$$

input
```
Integrate[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

output
```
(2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + S
qrt[2]*b*Sin[2*(e + f*x)])/(2*a*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)
]])
```

3.529.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 3663, 25, 3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\int -\sqrt{b\sin^2(e+fx)+a}dx}{a(a+b)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sqrt{b\sin^2(e+fx)+a}dx}{a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sin(e+fx)^2+a}dx}{a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin^2(e+fx)}{a}+1}dx}{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin(e+fx)^2}{a}+1}dx}{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{af(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

input
```
Int[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

output
```
(b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + (
EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)*f*Sqrt[1
 + (b*Sin[e + f*x]^2)/a])
```

3.529.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.529.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^2(fx+e)\right)}{a}+\frac{a+b}{a}}\,aE\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)+\left(\cos^2(fx+e)\right)\sin(fx+e)b}{a(a+b)\cos(fx+e)\sqrt{a+b\left(\sin^2(fx+e)\right)}\,f}$	103

3.529. $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

input
```
int(1/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*EllipticE(sin(f*
x+e),(-1/a*b)^(1/2))+cos(f*x+e)^2*sin(f*x+e)*b)/a/(a+b)/cos(f*x+e)/(a+b*si
n(f*x+e)^2)^(1/2)/f
```

3.529.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 938, normalized size of antiderivative = 9.29

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{-b\cos\left(fx+e\right)^2+a+b}b^3\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(2\left(i\,b^3\cos\left(fx+e\right)^2-i\,ab^2-i\,b^3\right)\sqrt{-b}\sqrt{\frac{a^2+a}{b^2}}\right.}{}$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(-b*cos(f*x + e)^2 + a + b)*b^3*cos(f*x + e)*sin(f*x + e) - (2
*(I*b^3*cos(f*x + e)^2 - I*a*b^2 - I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) -
 (2*I*a^2*b + 3*I*a*b^2 + I*b^3 + (-2*I*a*b^2 - I*b^3)*cos(f*x + e)^2)*sqr
t(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqr
t((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))
), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (2
*(-I*b^3*cos(f*x + e)^2 + I*a*b^2 + I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)
- (-2*I*a^2*b - 3*I*a*b^2 - I*b^3 + (2*I*a*b^2 + I*b^3)*cos(f*x + e)^2)*sq
rt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sq
rt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e)
)), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 2
*(2*(-I*a^2*b - 2*I*a*b^2 - I*b^3 + (I*a*b^2 + I*b^3)*cos(f*x + e)^2)*sqrt
(-b)*sqrt((a^2 + a*b)/b^2) + (2*I*a^3 + 3*I*a^2*b + I*a*b^2 + (-2*I*a^2*b
- I*a*b^2)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
 + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(
cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sq
rt((a^2 + a*b)/b^2))/b^2) + 2*(2*(I*a^2*b + 2*I*a*b^2 + I*b^3 + (-I*a*b^2
- I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + (-2*I*a^3 - 3*I*
a^2*b - I*a*b^2 + (2*I*a^2*b + I*a*b^2)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*
b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt(...
```

3.529.6 Sympy [F]

$$\int \frac{1}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{1}{(a+b\sin^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sin(e + f*x)**2)**(-3/2), x)
```

3.529.7 Maxima [F]

$$\int \frac{1}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\sin{(fx+e)}^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

3.529.8 Giac [F]

$$\int \frac{1}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\sin{(fx+e)}^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.529.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\sin{(e+f\,x)}^2+a)^{3/2}}\,dx$$

input
```
int(1/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(1/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.530 $\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

3.530.1 Optimal result

Integrand size = 25, antiderivative size = 209

$$\begin{aligned}\int \frac{\cot^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx &= \frac{\cot(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} \\ &- \frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f} \\ &- \frac{2\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} \\ &+ \frac{\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{af\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

output
```
cot(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(1/2)-2*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1
/2)/a^2/f-2*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(
1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+EllipticF(s
in(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/
a)^(1/2)/a/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.530.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.68

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{-2(a+b-b\cos(2(e+fx)))\cot(e+fx)-2\sqrt{2}a\sqrt{\frac{2a+b-b\cos(2(e+fx))}{a}}E(e+}{\sqrt{2}a^2f\sqrt{2a+b-b\cos(2(e}$$

input
```
Integrate[Cot[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(-2*(a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x] - 2*Sqrt[2]*a*Sqrt[(2*a + b
- b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + Sqrt[2]*a*Sqrt[(2*a
+ b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(Sqrt[2]*a^2*f*Sq
rt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.530.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3675, 371, 25, 445, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^2\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\csc^2(e+fx)\sqrt{1-\sin^2(e+fx)}}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{371}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sqrt{1-\sin^2(e+fx)}\csc(e+fx)}{a\sqrt{a+b\sin^2(e+fx)}}-\frac{\int -\frac{\csc^2(e+fx)\left(2-\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}\right)}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int \frac{\csc^2(e+fx)\left(2-\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}+\frac{\sqrt{1-\sin^2(e+fx)}\csc(e+fx)}{a\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{\int \frac{2b\sin^2(e+fx)+a}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{2\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}}{a}+\frac{\sqrt{1-\sin^2(e+f}{a\sqrt{a+b\sin}} \cdots\right.}{f}$$

$$\Big\downarrow \; 399$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{2\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-a\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{2\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{}}{a} \cdots}{a}\right.}{f}$$

$$\Big\downarrow \; 323$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{2\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}}{a}-\frac{2\sqrt{1-\sin^2}}{} \cdots}{a}\right.}{f}$$

$$\Big\downarrow \; 321$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{2\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}}{a}-\frac{2\sqrt{1-\sin^2(e+fx)}\csc(e+f}{a} \cdots}{a}\right.}{f}$$

$$\Big\downarrow \; 330$$

3.530. $\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{\frac{2\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}}{a}-\frac{2\sqrt{1-\cdots}}{\cdots}}{a}\right.}{f}$$

$\Big\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{\frac{2\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}}{a}-\frac{2\sqrt{1-\sin^2(e\cdots}}{\cdots}}{a}\right.}{f}$$

input
```
Int[Cot[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((Csc[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]
)/(a*Sqrt[a + b*Sin[e + f*x]^2]) + ((-2*Csc[e + f*x]*Sqrt[1 - Sin[e + f*x]
^2]*Sqrt[a + b*Sin[e + f*x]^2])/a - ((2*EllipticE[ArcSin[Sin[e + f*x]], -(
b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/Sqrt[1 + (b*Sin[e + f*x]^2)/a] - (a*Elli
pticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/Sqrt[a
 + b*Sin[e + f*x]^2])/a)/a))/f
```

3.530.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.530. $\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 371
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a
*e*2*(p + 1))), x] + Simp[1/(a*2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p + 1)
*(c + d*x^2)^(q - 1)*Simp[c*(m + 2*(p + 1) + 1) + d*(m + 2*(p + q + 1) + 1)
*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.530. $\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.530.4 Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.67

method	result
default	$\frac{2b(\cos^4(fx+e))+(-a-2b)(\cos^2(fx+e))+\sin(fx+e)\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\,a\left(F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)-2E\left(\sin(\right.\right.}{\sin(fx+e)a^2\cos(fx+e)\sqrt{a+b(\sin^2(fx+e))}\,f}$

input
```
int(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*b*cos(f*x+e)^4+(-a-2*b)*cos(f*x+e)^2+sin(f*x+e)*(cos(f*x+e)^2)^(1/2)*(-
b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*(EllipticF(sin(f*x+e),(-1/a*b)^(1/2))-2*
EllipticE(sin(f*x+e),(-1/a*b)^(1/2))))/sin(f*x+e)/a^2/cos(f*x+e)/(a+b*sin(
f*x+e)^2)^(1/2)/f
```

3.530.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 996, normalized size of antiderivative = 4.77

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
((2*(-I*b^3*cos(f*x + e)^2 + I*a*b^2 + I*b^3)*sqrt(-b)*sqrt((a^2 + a*b)/b^
2)*sin(f*x + e) - (-2*I*a^2*b - 3*I*a*b^2 - I*b^3 + (2*I*a*b^2 + I*b^3)*co
s(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
 + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(
cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sq
rt((a^2 + a*b)/b^2))/b^2) + (2*(I*b^3*cos(f*x + e)^2 - I*a*b^2 - I*b^3)*sq
rt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - (2*I*a^2*b + 3*I*a*b^2 + I*b^3
 + (-2*I*a*b^2 - I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*s
qrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 +
 a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b
+ b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-I*a^2*b - 3*I*a
*b^2 - 2*I*b^3 + (I*a*b^2 + 2*I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 +
a*b)/b^2)*sin(f*x + e) - (-2*I*a^3 - 3*I*a^2*b - I*a*b^2 + (2*I*a^2*b + I*
a*b^2)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b
^2) + 2*a + b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
+ b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b
+ b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(I*a^2*b + 3*I*a*b^2 + 2*I*b^3 + (
-I*a*b^2 - 2*I*b^3)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x
 + e) - (2*I*a^3 + 3*I*a^2*b + I*a*b^2 + (-2*I*a^2*b - I*a*b^2)*cos(f*x +
e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)...
```

3.530.6 Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^2(e+fx)}{(a+b\sin^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**2/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.530.7 Maxima [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^2}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.530.8 Giac [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^2}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.530.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+f\,x)^2}{\left(b\sin(e+f\,x)^2+a\right)^{3/2}}\,dx$$

input
```
int(cot(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.531 $\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

3.531.1 Optimal result

Integrand size = 25, antiderivative size = 297

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}}+\frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f}-\frac{(3a+4b)\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2bf}+\frac{(7a+8b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}-\frac{4(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3a^2f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
(a+b)*cot(f*x+e)*csc(f*x+e)^2/a/b/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*(7*a+8*b)
*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a^3/f-1/3*(3*a+4*b)*cot(f*x+e)*csc(f*
x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/a^2/b/f+1/3*(7*a+8*b)*EllipticE(sin(f*x+e)
,(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^
3/f/(1+b*sin(f*x+e)^2/a)^(1/2)-4/3*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2)
)*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/a^2/f/(a+b*si
n(f*x+e)^2)^(1/2)
```

3.531.2 Mathematica [A] (verified)

Time = 4.30 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.67

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\frac{\left(8a^2+37ab+24b^2-4(4a^2+11ab+8b^2)\cos(2(e+fx))+b(7a+8b)\cos(4(e+fx))\right)\cot(e+fx)\csc^2(e+fx)}{2\sqrt{2}}+}{6a^3}$$

input
```
Integrate[Cot[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(((8*a^2 + 37*a*b + 24*b^2 - 4*(4*a^2 + 11*a*b + 8*b^2)*Cos[2*(e + f*x)] +
 b*(7*a + 8*b)*Cos[4*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2)/(2*Sqrt[2])
+ 2*a*(7*a + 8*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x
, -(b/a)] - 8*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e
 + f*x, -(b/a)])/(6*a^3*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

3.531.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3042, 3675, 370, 25, 445, 27, 445, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^4\left(a+b\sin(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\csc^4(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{370}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sqrt{1-\sin^2(e+fx)}\csc^3(e+fx)}{ab\sqrt{a+b\sin^2(e+fx)}}-\frac{\int -\frac{\csc^4(e+fx)\left(-\left((2a+3b)\sin^2(e+fx)\right)+3a+4b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{ab}\right)}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int \frac{\csc^4(e+fx)\left(-\left((2a+3b)\sin^2(e+fx)\right)+3a+4b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{ab}+\frac{(a+b)\sqrt{1-\sin^2(e+fx)}\csc^3(e+fx)}{ab\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

$\downarrow$ 445

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{\int \frac{b\csc^2(e+fx)\left(-\left((3a+4b)\sin^2(e+fx)\right)+7a+8b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3a}-\frac{(3a+4b)\sqrt{1-\sin^2(e+fx)}\csc^3(e+fx)\sqrt{a+b\sin^2(e+fx}}{3a}}{ab}\right.}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{b\int \frac{\csc^2(e+fx)\left(-\left((3a+4b)\sin^2(e+fx)\right)+7a+8b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3a}-\frac{(3a+4b)\sqrt{1-\sin^2(e+fx)}\csc^3(e+fx)\sqrt{a+b\sin^2(e+fx}}{3a}}{ab}\right.}{f}$$

$\downarrow$ 445

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{b\left(-\frac{\int \frac{b(7a+8b)\sin^2(e+fx)+a(3a+4b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{(7a+8b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}\right)}{3a}-(3a+}{ab}\right.}{f}$$

$\downarrow$ 399

3.531. $\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{-\frac{b\left(-\frac{(7a+8b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-4a(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{(7a+8b)\sqrt{1\cdots}}{\cdots}\right)}{3a}\cdots}{ab}\cdots\right)}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{-\frac{b\left(-\frac{(7a+8b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e\cdots}{\sqrt{a+b\sin^2(e+fx)}}}{a}\cdots\right)}{3a}\cdots}{a\cdots}\cdots\right)}{\cdots}$$

$\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{-\frac{b\left(-\frac{(7a+8b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}}{a}-\frac{(7a+8b\cdots}{\cdots}\right)}{3a}\cdots}{ab}\cdots\right)}{f}$$

$\downarrow$ 330

3.531. $\int\frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{b\left(-\frac{\frac{(7a+8b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+\right.}{\sqrt{a+b\sin^2(e+fx)}}}{a}\right.}{3a}\right.$$

$$\Big\downarrow \text{327}$$

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(-\frac{b\left(-\frac{\frac{(7a+8b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\right.}{\sqrt{a+b\sin^2(e+fx)}}}{a}\right.}{3a}\right.$$

input
```
Int[Cot[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(((a + b)*Csc[e + f*x]^3*Sqrt[1 - Sin[e
 + f*x]^2])/(a*b*Sqrt[a + b*Sin[e + f*x]^2]) + (-1/3*((3*a + 4*b)*Csc[e +
f*x]^3*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/a - (b*(-(((7*
a + 8*b)*Csc[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2])
/a) - (((7*a + 8*b)*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin
[e + f*x]^2])/Sqrt[1 + (b*Sin[e + f*x]^2)/a] - (4*a*(a + b)*EllipticF[ArcS
in[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/Sqrt[a + b*Sin[e
 + f*x]^2])/a))/(3*a))/(a*b)))/f
```

3.531. $\int \frac{\cot^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx$

3.531.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.531. $\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.531.4 Maple [A] (verified)

Time = 3.73 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.19

method	result
default	$-\frac{4\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2\left(\sin^3(fx+e)\right)+4b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx\right.}{}$

input
```
int(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(4*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*
x+e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+4*b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f
*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-7*(c
os(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a^2*sin(f*x+e)^3-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^
(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b*sin(f*x+e)^3+7*a*b*sin(f*x+
e)^6+8*b^2*sin(f*x+e)^6+4*a^2*sin(f*x+e)^4-3*a*b*sin(f*x+e)^4-8*b^2*sin(f*
x+e)^4-5*a^2*sin(f*x+e)^2-4*a*b*sin(f*x+e)^2+a^2)/a^3/sin(f*x+e)^3/cos(f*x
+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

3.531.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 1465, normalized size of antiderivative = 4.93

$$\int \frac{\cot^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/6*((2*((7*I*a*b^3 + 8*I*b^4)*cos(f*x + e)^4 + 7*I*a^2*b^2 + 15*I*a*b^3 +
 8*I*b^4 + (-7*I*a^2*b^2 - 22*I*a*b^3 - 16*I*b^4)*cos(f*x + e)^2)*sqrt(-b)
*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - ((-14*I*a^2*b^2 - 23*I*a*b^3 - 8*I*b
^4)*cos(f*x + e)^4 - 14*I*a^3*b - 37*I*a^2*b^2 - 31*I*a*b^3 - 8*I*b^4 + (1
4*I*a^3*b + 51*I*a^2*b^2 + 54*I*a*b^3 + 16*I*b^4)*cos(f*x + e)^2)*sqrt(-b)
*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(ar
csin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f
*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b
^2) + (2*((-7*I*a*b^3 - 8*I*b^4)*cos(f*x + e)^4 - 7*I*a^2*b^2 - 15*I*a*b^3
 - 8*I*b^4 + (7*I*a^2*b^2 + 22*I*a*b^3 + 16*I*b^4)*cos(f*x + e)^2)*sqrt(-b
)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - ((14*I*a^2*b^2 + 23*I*a*b^3 + 8*I*b
^4)*cos(f*x + e)^4 + 14*I*a^3*b + 37*I*a^2*b^2 + 31*I*a*b^3 + 8*I*b^4 + (-
14*I*a^3*b - 51*I*a^2*b^2 - 54*I*a*b^3 - 16*I*b^4)*cos(f*x + e)^2)*sqrt(-b
)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(a
rcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(
f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/
b^2) - 2*(2*((3*I*a^2*b^2 + 11*I*a*b^3 + 8*I*b^4)*cos(f*x + e)^4 + 3*I*a^3
*b + 14*I*a^2*b^2 + 19*I*a*b^3 + 8*I*b^4 + (-3*I*a^3*b - 17*I*a^2*b^2 - 30
*I*a*b^3 - 16*I*b^4)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*
x + e) + ((-6*I*a^3*b - 11*I*a^2*b^2 - 4*I*a*b^3)*cos(f*x + e)^4 - 6*I*...
```

3.531.6 Sympy [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^4(e+fx)}{(a+b\sin^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+b*sin(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**4/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.531.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.531.8 Giac [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sin^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^4}{(b\sin(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.531.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{{\cot\left(e+f\,x\right)}^4}{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

input
```
int(cot(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2), x)
```

3.532 $\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.532.1 Optimal result

Integrand size = 25, antiderivative size = 218

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{(8a^2-24ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}f} - \frac{8a^2-24ab+3b^2}{24(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(8a+b)\sec^2(e+fx)}{8(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\sec^4(e+fx)}{4(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{8a^2-24ab+3b^2}{8(a+b)^4f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
1/8*(8*a^2-24*a*b+3*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+
b)^(9/2)/f+1/24*(-8*a^2+24*a*b-3*b^2)/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(3/2)-1
/8*(8*a+b)*sec(f*x+e)^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(3/2)+1/4*sec(f*x+e)^
4/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+1/8*(-8*a^2+24*a*b-3*b^2)/(a+b)^4/f/(a+
b*sin(f*x+e)^2)^(1/2)
```

3.532.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.49

$$\int \frac{\tan^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{(-8a^2+24ab-3b^2)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\sin^2(e+fx)}{a+b}\right)-\frac{3}{2}(a+\ldots}{24(a+b)^3 f\,(a+b\sin^2(e+f\ldots}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
((-8*a^2 + 24*a*b - 3*b^2)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sin[e +
 f*x]^2)/(a + b)] - (3*(a + b)*(4*a - 3*b + (8*a + b)*Cos[2*(e + f*x)])*Se
c[e + f*x]^4)/2)/(24*(a + b)^3*f*(a + b*Sin[e + f*x]^2)^(3/2))
```

3.532.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3673, 100, 27, 87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(e+fx)^5}{(a+b\sin(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \; 3673$$

$$\frac{\int \frac{\sin^4(e+fx)}{(1-\sin^2(e+fx))^3(b\sin^2(e+fx)+a)^{5/2}}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \; 100$$

$$\frac{\frac{1}{2(a+b)(1-\sin^2(e+fx))^2(a+b\sin^2(e+fx))^{3/2}}-\frac{\int \frac{4(a+b)\sin^2(e+fx)+4a-3b}{2(1-\sin^2(e+fx))^2(b\sin^2(e+fx)+a)^{5/2}}d\sin^2(e+fx)}{2(a+b)}}{2f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{2(a+b)\left(1-\sin^2(e+fx)\right)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\int \frac{4(a+b)\sin^2(e+fx)+4a-3b}{\left(1-\sin^2(e+fx)\right)^2\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin^2(e+fx)}{4(a+b)}}{2f}$$

$$\downarrow 87$$

$$\frac{\frac{1}{2(a+b)\left(1-\sin^2(e+fx)\right)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{8a+b}{(a+b)\left(1-\sin^2(e+fx)\right)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2-24ab+3b^2\right)\int \frac{1}{\left(1-\sin^2(e+fx)\right)\left(b\sin^2(e+fx)+a\right)^{5/2}} \cdots}{2(a+b)}}{4(a+b)}}{2f}$$

$$\downarrow 61$$

$$\frac{\frac{1}{2(a+b)\left(1-\sin^2(e+fx)\right)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{8a+b}{(a+b)\left(1-\sin^2(e+fx)\right)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2-24ab+3b^2\right)\left(\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)\left(b\sin^2(e+fx)+a\right)^{3} \cdots}}{a+b} \cdots\right.}{2(a+b)}}{4(a+b)}}{2f}$$

$$\downarrow 61$$

$$\frac{\frac{1}{2(a+b)\left(1-\sin^2(e+fx)\right)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{8a+b}{(a+b)\left(1-\sin^2(e+fx)\right)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2-24ab+3b^2\right)\left(\frac{\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)\sqrt{b\sin^2(e+fx)+a}}d \cdots}{a+b} \cdots}{a \cdots}\right.}{\cdots}}{4(a+b)}}{2f}$$

$$\downarrow 73$$

$$\frac{\frac{1}{2(a+b)\left(1-\sin^2(e+fx)\right)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{8a+b}{(a+b)\left(1-\sin^2(e+fx)\right)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2-24ab+3b^2\right)\left(\frac{\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\sin^4(e+fx)}{b}}d\sqrt{b\sin^2(e+fx} \cdots}{b(a+b)} \cdots}{a+b}\right.}{2 \cdots}}{4(a+b)}}{2f}$$

$$\downarrow 221$$

3.532. $\displaystyle\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\frac{1}{2(a+b)(1-\sin^2(e+fx))^2(a+b\sin^2(e+fx))^{3/2}}-\frac{\frac{8a+b}{(a+b)(1-\sin^2(e+fx))(a+b\sin^2(e+fx))^{3/2}}-\frac{(8a^2-24ab+3b^2)\left(\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\cdots}{a+b}\cdots\right)}{2(a+\cdots}}{4(a+b)}}{2f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(1/(2*(a + b)*(1 - Sin[e + f*x]^2)^2*(a + b*Sin[e + f*x]^2)^(3/2)) - ((8*a
 + b)/((a + b)*(1 - Sin[e + f*x]^2)*(a + b*Sin[e + f*x]^2)^(3/2)) - ((8*a^
2 - 24*a*b + 3*b^2)*(-2/(3*(a + b)*(a + b*Sin[e + f*x]^2)^(3/2)) + ((2*Arc
Tanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(a + b)^(3/2) - 2/((a + b)*S
qrt[a + b*Sin[e + f*x]^2]))/(a + b)))/(2*(a + b)))/(4*(a + b)))/(2*f)
```

3.532.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.532.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1908 vs. $2(194) = 388$.

Time = 4.23 (sec) , antiderivative size = 1909, normalized size of antiderivative = 8.76

method	result	size
default	Expression too large to display	1909

input
```
int(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/16*b^4/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)/(1+sin(f*x+e))*(a+
b-b*cos(f*x+e)^2)^(1/2)+7/16*b^4/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3*a/
(a+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*
a)/(sin(f*x+e)-1))+1/16*b^4/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)/(
sin(f*x+e)-1)*(a+b-b*cos(f*x+e)^2)^(1/2)-1/12*b^2*a/(b+(-a*b)^(1/2))^3/(-b
+(-a*b)^(1/2))^3/(sin(f*x+e)+(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/
b)^(1/2)+1/16*b^2/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)/(1+sin(f*x+
e))^2*(a+b-b*cos(f*x+e)^2)^(1/2)+3/16*b^3/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1
/2))^2/(a+b)^2/(1+sin(f*x+e))*(a+b-b*cos(f*x+e)^2)^(1/2)+1/16*b^2/(b+(-a*b
)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)/(sin(f*x+e)-1)^2*(a+b-b*cos(f*x+e)^2)
^(1/2)-3/16*b^3/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)^2/(sin(f*x+e)
-1)*(a+b-b*cos(f*x+e)^2)^(1/2)+1/12*b^2*(-a*b)^(1/2)/(b+(-a*b)^(1/2))^3/(-
b+(-a*b)^(1/2))^3/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b
)^(1/2)-1/12*b^2*(-a*b)^(1/2)/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(sin(
f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12*b^2*a/(b+(
-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(sin(f*x+e)-(-a*b)^(1/2)/b)^2*(-b*cos(f
*x+e)^2+(a*b+b^2)/b)^(1/2)+1/2*b^4*a^2/(b+(-a*b)^(1/2))^4/(-b+(-a*b)^(1/2)
)^4/(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e
)+2*a)/(sin(f*x+e)-1))-b^5*a/(b+(-a*b)^(1/2))^4/(-b+(-a*b)^(1/2))^4/(a+b)^
(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/...
```

3.532.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(194) = 388$.

Time = 0.58 (sec) , antiderivative size = 995, normalized size of antiderivative = 4.56

$$\int \frac{\tan^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*((8*a^2*b^2 - 24*a*b^3 + 3*b^4)*cos(f*x + e)^8 - 2*(8*a^3*b - 16*
a^2*b^2 - 21*a*b^3 + 3*b^4)*cos(f*x + e)^6 + (8*a^4 - 8*a^3*b - 37*a^2*b^2
 - 18*a*b^3 + 3*b^4)*cos(f*x + e)^4)*sqrt(a + b)*log((b*cos(f*x + e)^2 - 2
*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2)
+ 2*(3*(8*a^3*b - 16*a^2*b^2 - 21*a*b^3 + 3*b^4)*cos(f*x + e)^6 - 4*(8*a^4
 - 8*a^3*b - 37*a^2*b^2 - 18*a*b^3 + 3*b^4)*cos(f*x + e)^4 + 6*a^4 + 24*a^
3*b + 36*a^2*b^2 + 24*a*b^3 + 6*b^4 - 3*(8*a^4 + 25*a^3*b + 27*a^2*b^2 + 1
1*a*b^3 + b^4)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^5*b^2
+ 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f*cos(f*x + e)^8 -
2*(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^
7)*f*cos(f*x + e)^6 + (a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^
4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*f*cos(f*x + e)^4), -1/24*(3*((8*a^2*b^2 -
24*a*b^3 + 3*b^4)*cos(f*x + e)^8 - 2*(8*a^3*b - 16*a^2*b^2 - 21*a*b^3 + 3*
b^4)*cos(f*x + e)^6 + (8*a^4 - 8*a^3*b - 37*a^2*b^2 - 18*a*b^3 + 3*b^4)*co
s(f*x + e)^4)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a
- b)/(a + b)) - (3*(8*a^3*b - 16*a^2*b^2 - 21*a*b^3 + 3*b^4)*cos(f*x + e)^
6 - 4*(8*a^4 - 8*a^3*b - 37*a^2*b^2 - 18*a*b^3 + 3*b^4)*cos(f*x + e)^4 + 6
*a^4 + 24*a^3*b + 36*a^2*b^2 + 24*a*b^3 + 6*b^4 - 3*(8*a^4 + 25*a^3*b + 27
*a^2*b^2 + 11*a*b^3 + b^4)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)
)/((a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f*co...
```

3.532.6 Sympy [F]

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\tan^5{(e+fx)}}{\left(a+b\sin^2{(e+fx)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**5/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.532.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(194) = 388$.

Time = 0.38 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.94

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{\frac{3\,(8\,a^2b^3-24\,ab^4+3\,b^5)\log\left(\frac{\sqrt{b\sin(fx+e)^2+a}-\sqrt{a+b}}{\sqrt{b\sin(fx+e)^2+a}+\sqrt{a+b}}\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{a+b}}+\frac{2\left(8\,a^5b^3+24\,a^4b^4+24\,a^3b^5+8\,a^2b^6+3\,(8\,a^2b^3-24\,ab^4+3\,b^5)\left(b\sin(fx+e)^2+a\right)^3-5\,(8\right.}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\left(b\sin(fx+e)^2+a\right)^{\frac{7}{2}}-2\,(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+}}{48\,b^3f}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/48*(3*(8*a^2*b^3 - 24*a*b^4 + 3*b^5)*log((sqrt(b*sin(f*x + e)^2 + a) -
sqrt(a + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/((a^4 + 4*a^3*b +
 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a + b)) + 2*(8*a^5*b^3 + 24*a^4*b^4 + 24*
a^3*b^5 + 8*a^2*b^6 + 3*(8*a^2*b^3 - 24*a*b^4 + 3*b^5)*(b*sin(f*x + e)^2 +
 a)^3 - 5*(8*a^3*b^3 - 16*a^2*b^4 - 21*a*b^5 + 3*b^6)*(b*sin(f*x + e)^2 +
a)^2 + 8*(a^4*b^3 - 4*a^3*b^4 - 11*a^2*b^5 - 6*a*b^6)*(b*sin(f*x + e)^2 +
a))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(b*sin(f*x + e)^2 + a)^(7
/2) - 2*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*(b*sin(f
*x + e)^2 + a)^(5/2) + (a^6 + 6*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b
^4 + 6*a*b^5 + b^6)*(b*sin(f*x + e)^2 + a)^(3/2)))/(b^3*f)
```

3.532.8 Giac [F]

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\tan\left(fx+e\right)^5}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.532.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Hanged}$$

input
```
int(tan(e + f*x)^5/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
\text{Hanged}
```

3.533 $\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.533.1 Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{(2a-3b)\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}f} + \frac{2a-3b}{6(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\sec^2(e+fx)}{2(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2a-3b}{2(a+b)^3f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-1/2*(2*a-3*b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(7/2)/f
+1/6*(2*a-3*b)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(3/2)+1/2*sec(f*x+e)^2/(a+b)/f
/(a+b*sin(f*x+e)^2)^(3/2)+1/2*(2*a-3*b)/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.533.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.50

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{(2a-3b)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\sin^2(e+fx)}{a+b}\right)+3(a+b)\sec^2(e+}{6(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
((2*a - 3*b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sin[e + f*x]^2)/(a +
b)] + 3*(a + b)*Sec[e + f*x]^2)/(6*(a + b)^2*f*(a + b*Sin[e + f*x]^2)^(3/2
))
```

3.533.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 3673, 87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^3}{\left(a+b\sin(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^2\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{1}{(a+b)\left(1-\sin^2(e+fx)\right)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{(2a-3b)\int \frac{1}{\left(1-\sin^2(e+fx)\right)\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin^2(e+fx)}{2(a+b)}}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{1}{(a+b)\left(1-\sin^2(e+fx)\right)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{(2a-3b)\left(\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+fx)}{a+b}-\frac{2}{3(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{2(a+b)}}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{1}{(a+b)(1-\sin^2(e+fx))(a+b\sin^2(e+fx))^{3/2}}-\frac{(2a-3b)\left(\frac{\frac{\int \frac{1}{(1-\sin^2(e+fx))\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{a+b}-\frac{2}{(a+b)\sqrt{a+b\sin^2(e+fx)}}}{a+b}-\frac{2}{3(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{2(a+b)}}{2f}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{1}{(a+b)(1-\sin^2(e+fx))(a+b\sin^2(e+fx))^{3/2}}-\frac{(2a-3b)\left(\frac{\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\sin^4(e+fx)}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b(a+b)}-\frac{2}{(a+b)\sqrt{a+b\sin^2(e+fx)}}}{a+b}-\frac{2}{3(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{2(a+b)}}{2f}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{1}{(a+b)(1-\sin^2(e+fx))(a+b\sin^2(e+fx))^{3/2}}-\frac{(2a-3b)\left(\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{2}{(a+b)\sqrt{a+b\sin^2(e+fx)}}}{a+b}-\frac{2}{3(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{2(a+b)}}{2f}$$

input
```
Int[Tan[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(1/((a + b)*(1 - Sin[e + f*x]^2)*(a + b*Sin[e + f*x]^2)^(3/2)) - ((2*a - 3
*b)*(-2/(3*(a + b)*(a + b*Sin[e + f*x]^2)^(3/2)) + ((2*ArcTanh[Sqrt[a + b*
Sin[e + f*x]^2]/Sqrt[a + b]])/(a + b)^(3/2) - 2/((a + b)*Sqrt[a + b*Sin[e
+ f*x]^2]))/(a + b)))/(2*(a + b)))/(2*f)
```

3.533. $\int \frac{\tan^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.533.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.533. $\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.533.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 833 vs. $2(133) = 266$.

Time = 3.28 (sec) , antiderivative size = 834, normalized size of antiderivative = 5.45

method	result
default	$\frac{b^2\left(-\frac{\sqrt{a+b-b\left(\cos^2(fx+e)\right)}}{(a+b)(\sin(fx+e)-1)}+\frac{b\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2(fx+e)\right)}+2b\sin(fx+e)+2a}{\sin(fx+e)-1}\right)}{(a+b)^{\frac{3}{2}}}\right)}{4(b+\sqrt{-ab})^2(-b+\sqrt{-ab})^2}-\frac{b^2\left(-\frac{\sqrt{a+b-b\left(\cos^2(fx+e)\right)}}{(a+b)(1+\sin(fx+e))}-\frac{b\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b}}{}\right)}{}\right)}{4(b+\sqrt{-ab})^2(-b+\sqrt{-}}$

input
```
int(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/4*b^2/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2*(-1/(a+b)/(sin(f*x+e)-1)*(
a+b-b*cos(f*x+e)^2)^(1/2)+b/(a+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e
)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(sin(f*x+e)-1)))-1/4*b^2/(b+(-a*b)^(1/2))^2
/(-b+(-a*b)^(1/2))^2*(-1/(a+b)/(1+sin(f*x+e))*(a+b-b*cos(f*x+e)^2)^(1/2)-b
/(a+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+2
*a)/(1+sin(f*x+e))))+1/2*b^3*(a-b)/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/
(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+2*
a)/(1+sin(f*x+e)))+1/2*b^3*(a-b)/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a
+b)^(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)
/(sin(f*x+e)-1))+1/2*b^3*(a-b)/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(-a*
b)^(1/2)/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1
/2*b^3*(a-b)/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(-a*b)^(1/2)/(sin(f*x+
e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12*b^3*(-a*b)^(1/
2)/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/a/(-(-a*b)^(1/2)*b^2*cos(f*x+e)^
2+2*a*b^2*sin(f*x+e)+(-a*b)^(3/2)+b^2*(-a*b)^(1/2))*(-b*cos(f*x+e)^2+(a*b^
2+b^3)/b^2)^(1/2)*((-a*b)^(1/2)*sin(f*x+e)+2*a)+1/12*b^3*(-a*b)^(1/2)/(b+(
-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/a/(-(-a*b)^(1/2)*b^2*cos(f*x+e)^2-2*a*b
^2*sin(f*x+e)+(-a*b)^(3/2)+b^2*(-a*b)^(1/2))*(-b*cos(f*x+e)^2+(a*b^2+b^3)/
b^2)^(1/2)*((-a*b)^(1/2)*sin(f*x+e)-2*a))/f
```

3.533.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. 2(133) = 266.

Time = 0.44 (sec) , antiderivative size = 769, normalized size of antiderivative = 5.03

$$\int \frac{\tan^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \left[-\frac{3\left((2\,ab^2-3\,b^3)\cos{(fx+e)}^6-2\,(2\,a^2b-ab^2-3\,b^3)\cos{(fx+e)}^4+(2\,\cdots\right.}{12\left((a^4b^2+4\,a^3\cdots\right.}\right.$$

input
```
integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*((2*a*b^2 - 3*b^3)*cos(f*x + e)^6 - 2*(2*a^2*b - a*b^2 - 3*b^3)*
cos(f*x + e)^4 + (2*a^3 + a^2*b - 4*a*b^2 - 3*b^3)*cos(f*x + e)^2)*sqrt(a
+ b)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)
 - 2*a - 2*b)/cos(f*x + e)^2) + 2*(3*(2*a^2*b - a*b^2 - 3*b^3)*cos(f*x + e
)^4 - 3*a^3 - 9*a^2*b - 9*a*b^2 - 3*b^3 - 4*(2*a^3 + a^2*b - 4*a*b^2 - 3*b
^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^4*b^2 + 4*a^3*b^3
 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f*cos(f*x + e)^6 - 2*(a^5*b + 5*a^4*b^2 + 10
*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*f*cos(f*x + e)^4 + (a^6 + 6*a^5*b +
 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)*f*cos(f*x + e)^2),
1/6*(3*((2*a*b^2 - 3*b^3)*cos(f*x + e)^6 - 2*(2*a^2*b - a*b^2 - 3*b^3)*cos
(f*x + e)^4 + (2*a^3 + a^2*b - 4*a*b^2 - 3*b^3)*cos(f*x + e)^2)*sqrt(-a -
b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b)) - (3*(2*a^
2*b - a*b^2 - 3*b^3)*cos(f*x + e)^4 - 3*a^3 - 9*a^2*b - 9*a*b^2 - 3*b^3 -
4*(2*a^3 + a^2*b - 4*a*b^2 - 3*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2
 + a + b))/((a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f*cos(f*x +
e)^6 - 2*(a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*f*c
os(f*x + e)^4 + (a^6 + 6*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*
a*b^5 + b^6)*f*cos(f*x + e)^2)]
```

3.533.6 Sympy [F]

$$\int \frac{\tan^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^3{(e+fx)}}{\left(a+b\sin^2{(e+fx)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**3/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**3/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.533.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.71

$$\int \frac{\tan^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{\frac{3\,(2\,ab^2-3\,b^3)\log\left(\frac{\sqrt{b\sin(fx+e)^2+a}-\sqrt{a+b}}{\sqrt{b\sin(fx+e)^2+a}+\sqrt{a+b}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{a+b}} - \frac{2\left(2\,a^3b^2+4\,a^2b^3+2\,ab^4-3\,(2\,ab^2-3\,b^3)\left(b\sin(fx+e)^2 \cdots\right.\right.}{(a^3+3\,a^2b+3\,ab^2+b^3)\left(b\sin(fx+e)^2+a\right)^{\frac{5}{2}}-(a^4+4 \cdots}}{12\,b^2f}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/12*(3*(2*a*b^2 - 3*b^3)*log((sqrt(b*sin(f*x + e)^2 + a) - sqrt(a + b))/(
sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*sqrt(a + b)) - 2*(2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 - 3*(2*a*b^2 - 3*b^3)*
(b*sin(f*x + e)^2 + a)^2 + 2*(2*a^2*b^2 - a*b^3 - 3*b^4)*(b*sin(f*x + e)^2
 + a))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(b*sin(f*x + e)^2 + a)^(5/2) - (a^
4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(b*sin(f*x + e)^2 + a)^(3/2)))/(b
^2*f)
```

3.533.8 Giac [F]

$$\int \frac{\tan^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)^3}{\left(b\sin(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.533.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+f\,x)^3}{\left(b\sin(e+f\,x)^2+a\right)^{5/2}}\,dx$$

input
```
int(tan(e + f*x)^3/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(tan(e + f*x)^3/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.533. $\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.534 $\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.534.1 Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f} - \frac{1}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{1}{(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f-1/3/(a+b)/f/(a
+b*sin(f*x+e)^2)^(3/2)-1/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.534.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\frac{b\cos^2(e+fx)}{a+b}\right)}{3(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Tan[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
-1/3*Hypergeometric2F1[-3/2, 1, -1/2, 1 - (b*Cos[e + f*x]^2)/(a + b)]/((a
+ b)*f*(a + b - b*Cos[e + f*x]^2)^(3/2))
```

3.534.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3673, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\tan(e+fx)}{(a+b\sin(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \ 3673$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \ 61$$

$$\frac{\frac{\int \frac{1}{(1-\sin^2(e+fx))\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+fx)}{a+b}-\frac{2}{3(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \ 61$$

$$\frac{\frac{\frac{\int \frac{1}{(1-\sin^2(e+fx))\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{a+b}-\frac{2}{(a+b)\sqrt{a+b\sin^2(e+fx)}}}{a+b}-\frac{2}{3(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\sin^4(e+fx)}{b}}d\sqrt{b\sin^2(e+fx)+a}}{b(a+b)}-\frac{2}{(a+b)\sqrt{a+b\sin^2(e+fx)}}}{a+b}-\frac{2}{3(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{2}{(a+b)\sqrt{a+b\sin^2(e+fx)}}}{a+b}-\frac{2}{3(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}}{2f}$$

input
```
Int[Tan[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(-2/(3*(a + b)*(a + b*Sin[e + f*x]^2)^(3/2)) + ((2*ArcTanh[Sqrt[a + b*Sin[
e + f*x]^2]/Sqrt[a + b]])/(a + b)^(3/2) - 2/((a + b)*Sqrt[a + b*Sin[e + f*
x]^2]))/(a + b))/(2*f)
```

3.534.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.534. $\int \frac{\tan(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.534.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 962 vs. 2(79) = 158.

Time = 2.34 (sec) , antiderivative size = 963, normalized size of antiderivative = 10.58

method	result	size
default	Expression too large to display	963

input
```
int(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/b^3/(a+b)^(1/2)/a^2/(cos(f*x+e)^4*a^2*b^2+2*a*b^3*cos(f*x+e)^4+b^4*cos
(f*x+e)^4-2*cos(f*x+e)^2*a^3*b-6*cos(f*x+e)^2*a^2*b^2-6*a*b^3*cos(f*x+e)^2
-2*b^4*cos(f*x+e)^2+a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*(3*ln(2/(1+sin(f*x+
e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^5*cos(f
*x+e)^4+3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*si
n(f*x+e)+a))*a^2*b^5*cos(f*x+e)^4+6*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2
)*(a+b)^(1/2)*a^2*b^4*cos(f*x+e)^2-6*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b
-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b^4*cos(f*x+e)^2-6*ln(2/(1+sin
(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^5*
cos(f*x+e)^2-6*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)
+b*sin(f*x+e)+a))*a^3*b^4*cos(f*x+e)^2-6*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*
(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^5*cos(f*x+e)^2-8*a^3*b^3
*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)-8*a^2*b^4*(-b*cos(f*x
+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)+3*a^4*b^3*ln(2/(1+sin(f*x+e))*((a
+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))+3*a^4*b^3*ln(2/(sin(
f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))+6*a^3*b
^4*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e
)+a))+6*a^3*b^4*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2
)+b*sin(f*x+e)+a))+3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)
^(1/2)-b*sin(f*x+e)+a))*a^2*b^5+3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b...
```

3.534.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(79) = 158.

Time = 0.37 (sec) , antiderivative size = 521, normalized size of antiderivative = 5.73

$$\int \frac{\tan(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \left[\frac{3\left(b^2\cos(fx+e)^4 - 2(ab+b^2)\cos(fx+e)^2 + a^2 + 2ab + b^2\right)\sqrt{a+b}\log}{6\left((a^3b^2 + 3a^2b^3 + 3ab^4 + b^5)f\cos(fx+e)^4 - 2(a\right.}\right.$$

$$-\frac{3\left(b^2\cos(fx+e)^4 - 2(ab+b^2)\cos(fx+e)^2 + a^2 + 2ab + b^2\right)\sqrt{-a-b}\arctan\left(\frac{\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{-a-b}}{a+b}\right)}{3\left((a^3b^2 + 3a^2b^3 + 3ab^4 + b^5)f\cos(fx+e)^4 - 2(a^4b + 4a^3b^2 + 6a^2b^3 + 4ab^4 + b^5)f\cos\right.}$$

input
```
integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b +
 b^2)*sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b
)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*(3*(a*b + b^2)*cos(f*x + e)
^2 - 4*a^2 - 8*a*b - 4*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3*b^2 + 3
*a^2*b^3 + 3*a*b^4 + b^5)*f*cos(f*x + e)^4 - 2*(a^4*b + 4*a^3*b^2 + 6*a^2*
b^3 + 4*a*b^4 + b^5)*f*cos(f*x + e)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a
^2*b^3 + 5*a*b^4 + b^5)*f), -1/3*(3*(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*co
s(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)
^2 + a + b)*sqrt(-a - b)/(a + b)) - (3*(a*b + b^2)*cos(f*x + e)^2 - 4*a^2
- 8*a*b - 4*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3*b^2 + 3*a^2*b^3 +
3*a*b^4 + b^5)*f*cos(f*x + e)^4 - 2*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b
^4 + b^5)*f*cos(f*x + e)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*
a*b^4 + b^5)*f)]
```

3.534.6 Sympy [F]

$$\int \frac{\tan(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+fx)}{(a+b\sin^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.534.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(79) = 158$.

Time = 0.48 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.23

$$\int \frac{\tan(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\frac{2}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}a+\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}b}+\frac{6}{\sqrt{b\sin(fx+e)^2+a}\,a^2+2\sqrt{b\sin(fx+e)^2+a}\,ab+\sqrt{b\sin(fx+e)^2+a}\,b^2}+\frac{3\,\mathrm{arsinh}\left(\frac{b\sin(fx+e)}{\sqrt{ab}(\sin(fx+e)}\right.}{(a+}}{6f}$$

input
```
integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/6*(2/((b*sin(f*x + e)^2 + a)^(3/2)*a + (b*sin(f*x + e)^2 + a)^(3/2)*b)
+ 6/(sqrt(b*sin(f*x + e)^2 + a)*a^2 + 2*sqrt(b*sin(f*x + e)^2 + a)*a*b + s
qrt(b*sin(f*x + e)^2 + a)*b^2) + 3*arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(
f*x + e) + 1)) - a/(sqrt(a*b)*(sin(f*x + e) + 1)))/(a + b)^(5/2) - 3*arcsi
nh(-b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x
+ e) - 1)))/(a + b)^(5/2))/f
```

3.534.8 Giac [F]

$$\int \frac{\tan(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)}{\left(b\sin(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.534.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\tan(e+f\,x)}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(tan(e + f*x)/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(tan(e + f*x)/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.535 $\int \frac{\cot(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.535.1 Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{\cot(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{1}{3af\,(a+b\sin^2(e+fx))^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3/a/f/(a+b*sin(f*x+
e)^2)^(3/2)+1/a^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.535.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.59

$$\int \frac{\cot(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1+\frac{b\sin^2(e+fx)}{a}\right)}{3af\,(a+b\sin^2(e+fx))^{3/2}}$$

input
```
Integrate[Cot[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Sin[e + f*x]^2)/a]/(3*a*f*(a + b*S
in[e + f*x]^2)^(3/2))
```

3.535.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3673, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)\left(a+b\sin(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\csc^2(e+fx)}{\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\int \frac{\csc^2(e+fx)}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+fx)}{a}+\frac{2}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\frac{\int \frac{\csc^2(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{a}+\frac{2}{a\sqrt{a+b\sin^2(e+fx)}}}{a}+\frac{2}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\frac{2\int \frac{1}{\frac{\sin^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sin^2(e+fx)+a}}{ab}+\frac{2}{a\sqrt{a+b\sin^2(e+fx)}}}{a}+\frac{2}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{2}{a\sqrt{a+b\sin^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{2f}$$

3.535. $\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$

input
```
Int[Cot[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(2/(3*a*(a + b*Sin[e + f*x]^2)^(3/2)) + ((-2*ArcTanh[Sqrt[a + b*Sin[e + f*
x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a + b*Sin[e + f*x]^2]))/a)/(2*f)
```

3.535.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.535.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(71) = 142$.

Time = 1.64 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.13

method	result
default	$\frac{-\frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\left(\sin^2(fx+e)\right)}}{\sin(fx+e)}\right)}{a^{\frac{5}{2}}}-\frac{7\sqrt{-b\left(\cos^2(fx+e)\right)+\frac{ab+b^2}{b}}}{12a^2\sqrt{-ab}\left(\sin(fx+e)+\frac{\sqrt{-ab}}{b}\right)}+\frac{7\sqrt{-b\left(\cos^2(fx+e)\right)+\frac{ab+b^2}{b}}}{12a^2\sqrt{-ab}\left(\sin(fx+e)-\frac{\sqrt{-ab}}{b}\right)}-\frac{\sqrt{-b\left(\cos^2(fx+e)\right)+\frac{ab+b^2}{b}}}{12a^2b\left(\sin(fx+e)-\frac{\sqrt{-ab}}{b}\right)^2}-\frac{\sqrt{-}}{12a^2}}{f}$

input
```
int(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/a^(5/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))-7/12/a
^2/(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^
(1/2)+7/12/a^2/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(
a*b+b^2)/b)^(1/2)-1/12/a^2/b/(sin(f*x+e)-(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^
2+(a*b+b^2)/b)^(1/2)-1/12/a^2/b/(sin(f*x+e)+(-a*b)^(1/2)/b)^2*(-b*cos(f*x+
e)^2+(a*b+b^2)/b)^(1/2))/f
```

3.535.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(71) = 142$.

Time = 0.35 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.60

$$\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \left[\frac{3\left(b^2\cos(fx+e)^4-2\left(ab+b^2\right)\cos(fx+e)^2+a^2+2\,ab+b^2\right)\sqrt{a}\log\left(\frac{2(}{}\right.}{6\left(a^3b^2f\cos(fx+e)^4-\right.}\right.$$

input
```
integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b +
 b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*
sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*(3*a*b*cos(f*x + e)^2 - 4*a^2
 - 3*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^3*b^2*f*cos(f*x + e)^4 - 2*(
a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^5 + 2*a^4*b + a^3*b^2)*f), 1/3*(3*(
b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqr
t(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/a) - (3*a*b*cos(f*x
+ e)^2 - 4*a^2 - 3*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^3*b^2*f*cos(f*
x + e)^4 - 2*(a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^5 + 2*a^4*b + a^3*b^2
)*f)]
```

3.535.6 Sympy [F]

$$\int \frac{\cot(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(e+fx)}{(a+b\sin^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.535.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int \frac{\cot(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = -\frac{\frac{3\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{a^{\frac{5}{2}}} - \frac{3}{\sqrt{b\sin(fx+e)^2+a}a^2} - \frac{1}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}a}}{3\,f}$$

input
```
integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(3*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(5/2) - 3/(sqrt(b*sin(f
*x + e)^2 + a)*a^2) - 1/((b*sin(f*x + e)^2 + a)^(3/2)*a))/f
```

3.535.8 Giac [F]

$$\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\cot\left(fx+e\right)}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.535.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\cot(e+f\,x)}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(cot(e + f*x)/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(cot(e + f*x)/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.536 $\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.536.1 Optimal result

Integrand size = 25, antiderivative size = 143

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{(2a+5b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f} - \frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^2(e+fx)}{2af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2a+5b}{2a^3f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
1/2*(2*a+5*b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(7/2)/f+1/6*(-2*
a-5*b)/a^2/f/(a+b*sin(f*x+e)^2)^(3/2)-1/2*csc(f*x+e)^2/a/f/(a+b*sin(f*x+e)
^2)^(3/2)+1/2*(-2*a-5*b)/a^3/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.536.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.48

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{3a\csc^2(e+fx) + (2a+5b)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1+\frac{b\sin^2(e+fx)}{a}\right)}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
-1/6*(3*a*Csc[e + f*x]^2 + (2*a + 5*b)*Hypergeometric2F1[-3/2, 1, -1/2, 1
+ (b*Sin[e + f*x]^2)/a])/(a^2*f*(a + b*Sin[e + f*x]^2)^(3/2))
```

3.536.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3673, 87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^3\left(a+b\sin(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\csc^4(e+fx)\left(1-\sin^2(e+fx)\right)}{\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{-\frac{(2a+5b)\int \frac{\csc^2(e+fx)}{\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin^2(e+fx)}{2a}-\frac{\csc^2(e+fx)}{a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{-\frac{(2a+5b)\left(\frac{\int \frac{\csc^2(e+fx)}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+fx)}{a}+\frac{2}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{2a}-\frac{\csc^2(e+fx)}{a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \text{61}$$

$$-\frac{\frac{(2a+5b)\left(\frac{\frac{\int \frac{\csc^2(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{a}+\frac{2}{a\sqrt{a+b\sin^2(e+fx)}}}{a}+\frac{2}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{2a}-\frac{\csc^2(e+fx)}{a(a+b\sin^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{(2a+5b)\left(\frac{\frac{2\int \frac{1}{\frac{\sin^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sin^2(e+fx)+a}}{ab}+\frac{2}{a\sqrt{a+b\sin^2(e+fx)}}}{a}+\frac{2}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{2a}-\frac{\csc^2(e+fx)}{a(a+b\sin^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{(2a+5b)\left(\frac{\frac{2}{a\sqrt{a+b\sin^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{2a}-\frac{\csc^2(e+fx)}{a(a+b\sin^2(e+fx))^{3/2}}}{2f}$$

input
```
Int[Cot[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(-(Csc[e + f*x]^2/(a*(a + b*Sin[e + f*x]^2)^(3/2))) - ((2*a + 5*b)*(2/(3*a
*(a + b*Sin[e + f*x]^2)^(3/2)) + ((-2*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/S
qrt[a]])/a^(3/2) + 2/(a*Sqrt[a + b*Sin[e + f*x]^2]))/a))/(2*a))/(2*f)
```

3.536.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

3.536. $\int \frac{\cot^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.536.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1037 vs. $2(123) = 246$.

Time = 2.25 (sec) , antiderivative size = 1038, normalized size of antiderivative = 7.26

method	result	size
default	Expression too large to display	1038

input
```
int(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/a^(13/2)/b^2/(cos(f*x+e)^6*b^2-2*cos(f*x+e)^4*a*b-3*cos(f*x+e)^4*b^2+c
os(f*x+e)^2*a^2+4*cos(f*x+e)^2*a*b+3*cos(f*x+e)^2*b^2-a^2-2*a*b-b^2)*(3*a^
(11/2)*b^2*(a+b-b*cos(f*x+e)^2)^(1/2)-6*a^6*b^2*ln(2/sin(f*x+e)*(a^(1/2)*(
a+b-b*cos(f*x+e)^2)^(1/2)+a))+3*(a+b-b*cos(f*x+e)^2)^(1/2)*a^(7/2)*b^4+8*a
^(11/2)*b^2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)+20*a^(9/2)*(-b*cos(f*x
+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^3+6*a^(9/2)*b^3*(a+b-b*cos(f*x+e)^2)^(1/2)+
12*a^(7/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^4-27*a^5*b^3*ln(2/sin
(f*x+e)*(a^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+a))-36*a^4*b^4*ln(2/sin(f*x+e)
*(a^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+a))-15*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)+a))*a^3*b^5+3*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f*
x+e)^2)^(1/2)+a))*a^3*b^4*(2*a+5*b)*cos(f*x+e)^6+3*cos(f*x+e)^4*b^3*(2*a^(
9/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)+(a+b-b*cos(f*x+e)^2)^(1/2)*a^
(7/2)*b+4*a^(7/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b-4*ln(2/sin(f*x
+e)*(a^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+a))*a^5-16*ln(2/sin(f*x+e)*(a^(1/2
)*(a+b-b*cos(f*x+e)^2)^(1/2)+a))*a^4*b-15*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*
cos(f*x+e)^2)^(1/2)+a))*a^3*b^2)-cos(f*x+e)^2*b^2*(8*a^(11/2)*(-b*cos(f*x+
e)^2+(a*b^2+b^3)/b^2)^(1/2)+6*(a+b-b*cos(f*x+e)^2)^(1/2)*a^(9/2)*b+26*a^(9
/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b+6*(a+b-b*cos(f*x+e)^2)^(1/2)
*a^(7/2)*b^2+24*a^(7/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^2-6*ln(2
/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+a))*a^6-39*ln(2/sin(f*x...
```

3.536.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(123) = 246$.

Time = 0.38 (sec) , antiderivative size = 666, normalized size of antiderivative = 4.66

$$\int \frac{\cot^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \left[\frac{3\left((2\,ab^2+5\,b^3)\cos(fx+e)^6-(4\,a^2b+16\,ab^2+15\,b^3)\cos(fx+e)^4-2\,a\right.}{}\right.$$

$$-\frac{3\left((2\,ab^2+5\,b^3)\cos(fx+e)^6-(4\,a^2b+16\,ab^2+15\,b^3)\cos(fx+e)^4-2\,a^3-9\,a^2b-12\,ab^2-5\,b^3+(2\,a\right.}{6\left(a^4b^2f\right.}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*((2*a*b^2 + 5*b^3)*cos(f*x + e)^6 - (4*a^2*b + 16*a*b^2 + 15*b^3)
*cos(f*x + e)^4 - 2*a^3 - 9*a^2*b - 12*a*b^2 - 5*b^3 + (2*a^3 + 13*a^2*b +
 26*a*b^2 + 15*b^3)*cos(f*x + e)^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 - 2*sq
rt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) + 2
*(3*(2*a^2*b + 5*a*b^2)*cos(f*x + e)^4 + 11*a^3 + 26*a^2*b + 15*a*b^2 - 2*
(4*a^3 + 16*a^2*b + 15*a*b^2)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a +
 b))/(a^4*b^2*f*cos(f*x + e)^6 - (2*a^5*b + 3*a^4*b^2)*f*cos(f*x + e)^4 +
(a^6 + 4*a^5*b + 3*a^4*b^2)*f*cos(f*x + e)^2 - (a^6 + 2*a^5*b + a^4*b^2)*f
), -1/6*(3*((2*a*b^2 + 5*b^3)*cos(f*x + e)^6 - (4*a^2*b + 16*a*b^2 + 15*b^
3)*cos(f*x + e)^4 - 2*a^3 - 9*a^2*b - 12*a*b^2 - 5*b^3 + (2*a^3 + 13*a^2*b
 + 26*a*b^2 + 15*b^3)*cos(f*x + e)^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)
^2 + a + b)*sqrt(-a)/a) - (3*(2*a^2*b + 5*a*b^2)*cos(f*x + e)^4 + 11*a^3 +
 26*a^2*b + 15*a*b^2 - 2*(4*a^3 + 16*a^2*b + 15*a*b^2)*cos(f*x + e)^2)*sqr
t(-b*cos(f*x + e)^2 + a + b))/(a^4*b^2*f*cos(f*x + e)^6 - (2*a^5*b + 3*a^4
*b^2)*f*cos(f*x + e)^4 + (a^6 + 4*a^5*b + 3*a^4*b^2)*f*cos(f*x + e)^2 - (a
^6 + 2*a^5*b + a^4*b^2)*f)]
```

3.536.6 Sympy [F]

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cot^3\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)**3/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.536.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.09

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\frac{6\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{a^{\frac{5}{2}}} + \frac{15\,b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{a^{\frac{7}{2}}} - \frac{6}{\sqrt{b\sin(fx+e)^2+a}\,a^2} - \frac{2}{\left(b\sin(fx+e)^2+\cdots\right.}}{6\,f}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(6*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(5/2) + 15*b*arcsinh(a/(
sqrt(a*b)*abs(sin(f*x + e))))/a^(7/2) - 6/(sqrt(b*sin(f*x + e)^2 + a)*a^2)
 - 2/((b*sin(f*x + e)^2 + a)^(3/2)*a) - 15*b/(sqrt(b*sin(f*x + e)^2 + a)*a
^3) - 5*b/((b*sin(f*x + e)^2 + a)^(3/2)*a^2) - 3/((b*sin(f*x + e)^2 + a)^(
3/2)*a*sin(f*x + e)^2))/f
```

3.536.8 Giac [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(fx+e)^3}{(b\sin(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^3/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.536.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(e+f\,x)^3}{(b\sin(e+f\,x)^2+a)^{5/2}}\,dx$$

input
```
int(cot(e + f*x)^3/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(cot(e + f*x)^3/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.537 $\int \frac{\cot^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.537.1 Optimal result

Integrand size = 25, antiderivative size = 208

$$\int \frac{\cot^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = -\frac{(8a^2+40ab+35b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f} + \frac{8a^2+40ab+35b^2}{24a^3f(a+b\sin^2(e+fx))^{3/2}} + \frac{(8a+7b)\csc^2(e+fx)}{8a^2f(a+b\sin^2(e+fx))^{3/2}} - \frac{\csc^4(e+fx)}{4af(a+b\sin^2(e+fx))^{3/2}} + \frac{8a^2+40ab+35b^2}{8a^4f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
-1/8*(8*a^2+40*a*b+35*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(9/
2)/f+1/24*(8*a^2+40*a*b+35*b^2)/a^3/f/(a+b*sin(f*x+e)^2)^(3/2)+1/8*(8*a+7*
b)*csc(f*x+e)^2/a^2/f/(a+b*sin(f*x+e)^2)^(3/2)-1/4*csc(f*x+e)^4/a/f/(a+b*s
in(f*x+e)^2)^(3/2)+1/8*(8*a^2+40*a*b+35*b^2)/a^4/f/(a+b*sin(f*x+e)^2)^(1/2
)
```

3.537.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.88 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.56

$$\int \frac{\cot^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{3a\csc^4(e+fx)\,(8a+7b-2a\csc^2(e+fx))+(8a^2+40ab+35b^2)\csc^2(e+}{24a^3f\,(b+a\csc^2(e+fx))\sqrt{a+b\mathrm{s}}}$$

input
```
Integrate[Cot[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(3*a*Csc[e + f*x]^4*(8*a + 7*b - 2*a*Csc[e + f*x]^2) + (8*a^2 + 40*a*b + 3
5*b^2)*Csc[e + f*x]^2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Sin[e + f*x]
^2)/a])/(24*a^3*f*(b + a*Csc[e + f*x]^2)*Sqrt[a + b*Sin[e + f*x]^2])
```

3.537.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.89, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3673, 100, 27, 87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^5\,(a+b\sin(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\csc^6(e+fx)(1-\sin^2(e+fx))^2}{(b\sin^2(e+fx)+a)^{5/2}}d\sin^2(e+fx)}{2f}$$

$$\Big\downarrow \text{100}$$

$$\frac{\frac{\int -\frac{\csc^4(e+fx)\left(-4a\sin^2(e+fx)+8a+7b\right)}{2\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin^2(e+fx)}{2a}-\frac{\csc^4(e+fx)}{2a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{2f}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\int \frac{\csc^4(e+fx)\left(-4a\sin^2(e+fx)+8a+7b\right)}{\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin^2(e+fx)}{4a}-\frac{\csc^4(e+fx)}{2a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{2f}$$

$\Big\downarrow$ 87

$$\frac{-\frac{-\frac{\left(8a^2+5b(8a+7b)\right)\int \frac{\csc^2(e+fx)}{\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin^2(e+fx)}{2a}-\frac{(8a+7b)\csc^2(e+fx)}{a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{4a}-\frac{\csc^4(e+fx)}{2a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{2f}$$

$\Big\downarrow$ 61

$$\frac{-\frac{-\frac{\left(8a^2+5b(8a+7b)\right)\left(\frac{\int \frac{\csc^2(e+fx)}{\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin^2(e+fx)}{a}+\frac{2}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{2a}-\frac{(8a+7b)\csc^2(e+fx)}{a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{4a}-\frac{\csc^4(e+fx)}{2a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{2f}$$

$\Big\downarrow$ 61

$$\frac{-\frac{-\frac{\left(8a^2+5b(8a+7b)\right)\left(\frac{\frac{\int \frac{\csc^2(e+fx)}{\sqrt{b\sin^2(e+fx)+a}}d\sin^2(e+fx)}{a}+\frac{2}{a\sqrt{a+b\sin^2(e+fx)}}}{a}+\frac{2}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{2a}-\frac{(8a+7b)\csc^2(e+fx)}{a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{4a}-\frac{\csc^4(e+fx)}{2a\left(a+b\sin^2(e+fx)\right)}}{2f}$$

$\Big\downarrow$ 73

$$\frac{-\frac{-\frac{\left(8a^2+5b(8a+7b)\right)\left(\frac{\frac{2\int \frac{1}{\frac{\sin^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sin^2(e+fx)+a}}{ab}+\frac{2}{a\sqrt{a+b\sin^2(e+fx)}}}{a}+\frac{2}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{2a}-\frac{(8a+7b)\csc^2(e+fx)}{a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{4a}-\frac{\csc^4(e+}{2a\left(a+b\sin^2(e\right.}}{2f}$$

$\Big\downarrow$ 221

3.537. $\displaystyle\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{-\frac{-\frac{\left(8a^2+5b(8a+7b)\right)\left(\frac{\frac{2}{a\sqrt{a+b\sin^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{2a}-\frac{(8a+7b)\csc^2(e+fx)}{a\left(a+b\sin^2(e+fx)\right)^{3/2}}}{4a}-\frac{\csc^4(e+fx)}{2a(a+b\sin^2(e+fx))^{3/2}}}{2f}$$

input
```
Int[Cot[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(-1/2*Csc[e + f*x]^4/(a*(a + b*Sin[e + f*x]^2)^(3/2)) - (-(((8*a + 7*b)*Cs
c[e + f*x]^2)/(a*(a + b*Sin[e + f*x]^2)^(3/2))) - ((8*a^2 + 5*b*(8*a + 7*b
))*(2/(3*a*(a + b*Sin[e + f*x]^2)^(3/2)) + ((-2*ArcTanh[Sqrt[a + b*Sin[e +
 f*x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a + b*Sin[e + f*x]^2]))/a))/(2*a))/
(4*a))/(2*f)
```

3.537.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.537.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 987 vs. $2(184) = 368$.

Time = 3.51 (sec) , antiderivative size = 988, normalized size of antiderivative = 4.75

method	result	size
default	Expression too large to display	988

input
```
int(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/4/a^3/sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2)+11/8/a^4*b/sin(f*x+e)^2*(a
+b*sin(f*x+e)^2)^(1/2)-35/8/a^(9/2)*b^2*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^
2)^(1/2))/sin(f*x+e))+1/a^3/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-5/a^(7/2
)*b*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))-1/a^(5/2)*ln((
2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))-1/2*(a^2+4*a*b+3*b^2)/
a^4/(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)
^(1/2)+1/2*(a^2+4*a*b+3*b^2)/a^4/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)^(1/2)/b)*
(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12/a^2/b/(sin(f*x+e)-(-a*b)^(1/2)/b)
^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/6/a^3/(sin(f*x+e)-(-a*b)^(1/2)/b)
^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12/a^4*b/(sin(f*x+e)-(-a*b)^(1/2)
/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)+1/12/a^2/(-a*b)^(1/2)/(sin(f*x+e
)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)+1/6/a^3/(-a*b)^(1/2)
*b/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)+1/12/a^
4/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(
1/2)*b^2-1/12/a^2/b/(sin(f*x+e)+(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^
2)/b)^(1/2)-1/6/a^3/(sin(f*x+e)+(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^
2)/b)^(1/2)-1/12/a^4*b/(sin(f*x+e)+(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b
+b^2)/b)^(1/2)-1/12/a^2/(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f
*x+e)^2+(a*b+b^2)/b)^(1/2)-1/6/a^3/(-a*b)^(1/2)*b/(sin(f*x+e)+(-a*b)^(1/2)
/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12/a^4/(-a*b)^(1/2)/(sin(f*x+...
```

3.537.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(184) = 368$.

Time = 0.43 (sec) , antiderivative size = 984, normalized size of antiderivative = 4.73

$$\int \frac{\cot^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*((8*a^2*b^2 + 40*a*b^3 + 35*b^4)*cos(f*x + e)^8 - 2*(8*a^3*b + 56
*a^2*b^2 + 115*a*b^3 + 70*b^4)*cos(f*x + e)^6 + (8*a^4 + 88*a^3*b + 323*a^
2*b^2 + 450*a*b^3 + 210*b^4)*cos(f*x + e)^4 + 8*a^4 + 56*a^3*b + 123*a^2*b
^2 + 110*a*b^3 + 35*b^4 - 2*(8*a^4 + 64*a^3*b + 171*a^2*b^2 + 185*a*b^3 +
70*b^4)*cos(f*x + e)^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*
x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*(3*(8*a^3*b
 + 40*a^2*b^2 + 35*a*b^3)*cos(f*x + e)^6 - (32*a^4 + 232*a^3*b + 500*a^2*b
^2 + 315*a*b^3)*cos(f*x + e)^4 - 50*a^4 - 205*a^3*b - 260*a^2*b^2 - 105*a*
b^3 + (88*a^4 + 413*a^3*b + 640*a^2*b^2 + 315*a*b^3)*cos(f*x + e)^2)*sqrt(
-b*cos(f*x + e)^2 + a + b))/(a^5*b^2*f*cos(f*x + e)^8 - 2*(a^6*b + 2*a^5*b
^2)*f*cos(f*x + e)^6 + (a^7 + 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 - 2*(a
^7 + 3*a^6*b + 2*a^5*b^2)*f*cos(f*x + e)^2 + (a^7 + 2*a^6*b + a^5*b^2)*f),
 1/24*(3*((8*a^2*b^2 + 40*a*b^3 + 35*b^4)*cos(f*x + e)^8 - 2*(8*a^3*b + 56
*a^2*b^2 + 115*a*b^3 + 70*b^4)*cos(f*x + e)^6 + (8*a^4 + 88*a^3*b + 323*a^
2*b^2 + 450*a*b^3 + 210*b^4)*cos(f*x + e)^4 + 8*a^4 + 56*a^3*b + 123*a^2*b
^2 + 110*a*b^3 + 35*b^4 - 2*(8*a^4 + 64*a^3*b + 171*a^2*b^2 + 185*a*b^3 +
70*b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sq
rt(-a)/a) - (3*(8*a^3*b + 40*a^2*b^2 + 35*a*b^3)*cos(f*x + e)^6 - (32*a^4
+ 232*a^3*b + 500*a^2*b^2 + 315*a*b^3)*cos(f*x + e)^4 - 50*a^4 - 205*a^3*b
 - 260*a^2*b^2 - 105*a*b^3 + (88*a^4 + 413*a^3*b + 640*a^2*b^2 + 315*a*...
```

3.537.6 Sympy [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^5(e+fx)}{(a+b\sin^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**5/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)**5/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.537.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.35

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx =$$

$$-\frac{\frac{24\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{a^{\frac{5}{2}}}+\frac{120\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{a^{\frac{7}{2}}}+\frac{105\,b^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|\sin(fx+e)|}\right)}{a^{\frac{9}{2}}}-\frac{24}{\sqrt{b\sin(fx+e)^2+a}a^2}-\frac{8}{\left(b\sin(fx+e)^2+a\right)}}{}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/24*(24*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(5/2) + 120*b*arcsinh
(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(7/2) + 105*b^2*arcsinh(a/(sqrt(a*b)*a
bs(sin(f*x + e))))/a^(9/2) - 24/(sqrt(b*sin(f*x + e)^2 + a)*a^2) - 8/((b*s
in(f*x + e)^2 + a)^(3/2)*a) - 120*b/(sqrt(b*sin(f*x + e)^2 + a)*a^3) - 40*
b/((b*sin(f*x + e)^2 + a)^(3/2)*a^2) - 105*b^2/(sqrt(b*sin(f*x + e)^2 + a)
*a^4) - 35*b^2/((b*sin(f*x + e)^2 + a)^(3/2)*a^3) - 24/((b*sin(f*x + e)^2
+ a)^(3/2)*a*sin(f*x + e)^2) - 21*b/((b*sin(f*x + e)^2 + a)^(3/2)*a^2*sin(
f*x + e)^2) + 6/((b*sin(f*x + e)^2 + a)^(3/2)*a*sin(f*x + e)^4))/f
```

3.537.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.537.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Hanged}$$

input
```
int(cot(e + f*x)^5/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
\text{Hanged}
```

3.538 $\int \frac{\tan^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.538.1 Optimal result

Integrand size = 25, antiderivative size = 348

$$\begin{aligned}\int \frac{\tan^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx &= \frac{(5a-3b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\,(a+b\sin^2(e+fx))^{3/2}} \\ &+ \frac{8(a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^4f\sqrt{a+b\sin^2(e+fx)}} \\ &+ \frac{8(a-b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)^4f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} \\ &- \frac{(5a-3b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} \\ &- \frac{2(2a-b)\tan(e+fx)}{3(a+b)^2f\,(a+b\sin^2(e+fx))^{3/2}} + \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\,(a+b\sin^2(e+fx))^{3/2}}\end{aligned}$$

output
```
1/3*(5*a-3*b)*b*cos(f*x+e)*sin(f*x+e)/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(3/2)+8
/3*(a-b)*b*cos(f*x+e)*sin(f*x+e)/(a+b)^4/f/(a+b*sin(f*x+e)^2)^(1/2)+8/3*(a
-b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+
b*sin(f*x+e)^2)^(1/2)/(a+b)^4/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*(5*a-3*b)*E
llipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin
(f*x+e)^2/a)^(1/2)/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(1/2)-2/3*(2*a-b)*tan(f*x+
e)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(3/2)+1/3*sec(f*x+e)^2*tan(f*x+e)/(a+b)/f/
(a+b*sin(f*x+e)^2)^(3/2)
```

3.538.2 Mathematica [A] (verified)

Time = 4.17 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.68

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{2ab\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}\left(8a(a-b)E\left(e+fx\middle|-\frac{b}{a}\right)+\left(-5a^2-2ab+3b^2\right)E\right.}{}$$

input
```
Integrate[Tan[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(2*a*b*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*(8*a*(a - b)*EllipticE[e +
 f*x, -(b/a)] + (-5*a^2 - 2*a*b + 3*b^2)*EllipticF[e + f*x, -(b/a)]) + Sqr
t[2]*b*(2*a*b*(a + b)*Sin[2*(e + f*x)] + 4*(a - b)*b*(2*a + b - b*Cos[2*(e
 + f*x)])*Sin[2*(e + f*x)] - 4*(a - b)*(2*a + b - b*Cos[2*(e + f*x)])^2*Ta
n[e + f*x] + (a + b)*(2*a + b - b*Cos[2*(e + f*x)])^2*Sec[e + f*x]^2*Tan[e
 + f*x]))/(6*b*(a + b)^4*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

3.538.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 3675, 372, 402, 27, 402, 25, 27, 402, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(e+fx)^4}{\left(a+b\sin(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 3675$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sin^4(e+fx)}{\left(1-\sin^2(e+fx)\right)^{5/2}\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 372$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\int\frac{(3a-2b)\sin^2(e+fx)+a}{\left(1-\sin^2(e+fx)\right)^{3/2}\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin(e+fx)}{3(a+b)}\right)}{f}$$

$\Big\downarrow$ 402

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\int-\frac{3\left(a(a-b)-2(2a-b)b\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin(e+fx)}{a+b}+\frac{}{(a+}}{3(a+b)}\right.}{f}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a-b)\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{3\int\frac{a(a-b}{\sqrt{1-\sin^2(e}}}{}}{3(a+b)}\right.}{f}$$

$\Big\downarrow$ 402

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a-b)\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{3\left(\frac{b(5a-3b)\sin}{3(a+b)}\right.}{}}{}\right.}{f}$$

$\Big\downarrow$ 25

3.538. $\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a-b)\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{3\left(\frac{\int \frac{a\left(a(3a\cdots}{\sqrt{1-\sin^2\cdots}}}{\cdots}\right.}{\cdots}}{\cdots}\right.}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a-b)\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{3\left(\frac{\int \frac{a(3a-\cdots}{\sqrt{1-\sin^2\cdots}}}{\cdots}\right.}{\cdots}}{\cdots}\right.}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a-b)\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{3\left(\frac{\frac{8b(a-b)\sin\cdots}{(a+b\cdots}}{\cdots}\right.}{\cdots}}{\cdots}\right.}{f}$$

$$\Big\downarrow \; 25$$

3.538. $\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a-b)\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\cdots}{\cdots}\right)$$

$$\downarrow 27$$

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a-b)\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\cdots}{\cdots}\right)$$

$$\downarrow 399$$

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a-b)\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\cdots}{\cdots}\right)$$

3.538. $\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\Big\downarrow \; 323$$

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a-b)\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\cdots}{\cdots}\right.$$

$$\Big\downarrow \; 321$$

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a-b)\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\cdots}{\cdots}\right.$$

$$\Big\downarrow \; 330$$

3.538. $\displaystyle \int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a-b)\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{3\left(\frac{8(a-b)\sqrt{a}}{\cdots}\cdots\right)}{\cdots}}{\cdots}\right)$$

$\downarrow$ 327

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{3(a+b)\left(1-\sin^2(e+fx)\right)^{3/2}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(2a-b)\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{3\left(\frac{8(a-b)\sqrt{a}}{\cdots}\cdots\right)}{\cdots}}{\cdots}\right)$$

input
```
Int[Tan[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

3.538. $\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(Sin[e + f*x]/(3*(a + b)*(1 - Sin[e + f
*x]^2)^(3/2)*(a + b*Sin[e + f*x]^2)^(3/2)) - ((2*(2*a - b)*Sin[e + f*x])/(
(a + b)*Sqrt[1 - Sin[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^(3/2)) - (3*(((5*a
 - 3*b)*b*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2])/(3*(a + b)*(a + b*Sin[e +
 f*x]^2)^(3/2)) + ((8*(a - b)*b*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2])/((a
 + b)*Sqrt[a + b*Sin[e + f*x]^2]) + ((8*(a - b)*EllipticE[ArcSin[Sin[e + f
*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/Sqrt[1 + (b*Sin[e + f*x]^2)/a] -
 ((5*a - 3*b)*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*
Sin[e + f*x]^2)/a])/Sqrt[a + b*Sin[e + f*x]^2])/(a + b))/(3*(a + b))))/(a
+ b))/(3*(a + b))))/f
```

3.538.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

3.538. $\int \frac{\tan^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.538. $\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.538.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(318) = 636$.

Time = 5.88 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.92

method	result
default	$-\frac{-8\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\,b^2(a-b)(\cos^6(fx+e))\sin(fx+e)+\sqrt{-b(\cos^4(fx+e))+(a+b)(\cos^2(fx+e))}\,b(13a^2+2a}{}$

input
```
int(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-8*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*(a-b)*cos(f*x+e)^6
*sin(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(13*a^2+2*a*b-11*
b^2)*cos(f*x+e)^4*sin(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(-
b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*b*(5*EllipticF(sin(f*
x+e),(-1/a*b)^(1/2))*a^2+2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b-3*Elli
pticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)
)*a^2+8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b)*cos(f*x+e)^4-2*(-b*cos(f
*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*a^3+3*a^2*b-b^3)*cos(f*x+e)^2*sin(f*x
+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)
^(1/2)*(cos(f*x+e)^2)^(1/2)*(5*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+7*
EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-EllipticF(sin(f*x+e),(-1/a*b)^(
1/2))*a*b^2-3*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^3-8*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))*a^3+8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2)*cos(
f*x+e)^2+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^3+3*a^2*b+3*a*b^2+b
^3)*sin(f*x+e))/(a+b*sin(f*x+e)^2)^(3/2)/(1+sin(f*x+e))/(-(a+b*sin(f*x+e)^
2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/(a+b)^4/cos(f*x+e)/
f
```

3.538.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 1730, normalized size of antiderivative = 4.97

$$\int \frac{\tan^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(4*(2*((-I*a*b^4 + I*b^5)*cos(f*x + e)^7 + 2*(I*a^2*b^3 - I*b^5)*cos(
f*x + e)^5 + (-I*a^3*b^2 - I*a^2*b^3 + I*a*b^4 + I*b^5)*cos(f*x + e)^3)*sq
rt(-b)*sqrt((a^2 + a*b)/b^2) + ((-2*I*a^2*b^3 + I*a*b^4 + I*b^5)*cos(f*x +
 e)^7 + 2*(2*I*a^3*b^2 + I*a^2*b^3 - 2*I*a*b^4 - I*b^5)*cos(f*x + e)^5 + (
-2*I*a^4*b - 3*I*a^3*b^2 + I*a^2*b^3 + 3*I*a*b^4 + I*b^5)*cos(f*x + e)^3)*
sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(
sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x +
e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) +
 4*(2*((I*a*b^4 - I*b^5)*cos(f*x + e)^7 + 2*(-I*a^2*b^3 + I*b^5)*cos(f*x +
 e)^5 + (I*a^3*b^2 + I*a^2*b^3 - I*a*b^4 - I*b^5)*cos(f*x + e)^3)*sqrt(-b)
*sqrt((a^2 + a*b)/b^2) + ((2*I*a^2*b^3 - I*a*b^4 - I*b^5)*cos(f*x + e)^7 +
 2*(-2*I*a^3*b^2 - I*a^2*b^3 + 2*I*a*b^4 + I*b^5)*cos(f*x + e)^5 + (2*I*a^
4*b + 3*I*a^3*b^2 - I*a^2*b^3 - 3*I*a*b^4 - I*b^5)*cos(f*x + e)^3)*sqrt(-b
))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2
*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (
8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - (2*((-
3*I*a^2*b^3 + 2*I*a*b^4 + 5*I*b^5)*cos(f*x + e)^7 - 2*(-3*I*a^3*b^2 - I*a^
2*b^3 + 7*I*a*b^4 + 5*I*b^5)*cos(f*x + e)^5 + (-3*I*a^4*b - 4*I*a^3*b^2 +
6*I*a^2*b^3 + 12*I*a*b^4 + 5*I*b^5)*cos(f*x + e)^3)*sqrt(-b)*sqrt((a^2 + a
*b)/b^2) - ((-6*I*a^3*b^2 + 17*I*a^2*b^3 + 4*I*a*b^4 - 3*I*b^5)*cos(f*x...
```

3.538.6 Sympy [F]

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\tan^4\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**4/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.538.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.538.8 Giac [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)^4}{(b\sin(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.538.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+f\,x)^4}{(b\sin(e+f\,x)^2+a)^{5/2}}\,dx$$

input
```
int(tan(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(tan(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.539 $\int \frac{\tan^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.539.1 Optimal result

Integrand size = 25, antiderivative size = 292

$$\begin{aligned}\int \frac{\tan^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx &= -\frac{4b\cos(e+fx)\sin(e+fx)}{3(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}\\
&-\frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^3f\sqrt{a+b\sin^2(e+fx)}}\\
&-\frac{(7a-b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a(a+b)^3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\\
&+\frac{4\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3(a+b)^2f\sqrt{a+b\sin^2(e+fx)}}\\
&+\frac{\tan(e+fx)}{(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}}\end{aligned}$$

output
```
-4/3*b*cos(f*x+e)*sin(f*x+e)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(3/2)-1/3*(7*a-b
)*b*cos(f*x+e)*sin(f*x+e)/a/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(7*a-b)
*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*s
in(f*x+e)^2)^(1/2)/a/(a+b)^3/f/(1+b*sin(f*x+e)^2/a)^(1/2)+4/3*EllipticF(si
n(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a
)^(1/2)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+tan(f*x+e)/(a+b)/f/(a+b*sin(f*x
+e)^2)^(3/2)
```

3.539.2 Mathematica [A] (verified)

Time = 3.22 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.68

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{-2a^2(7a-b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)+8a^2(a+b)\left(\frac{2a+b-b\cos(}{a}\right.}{6a(a+}$$

input
```
Integrate[Tan[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(-2*a^2*(7*a - b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f
*x, -(b/a)] + 8*a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*Ellip
ticF[e + f*x, -(b/a)] + ((24*a^3 + 4*a^2*b + 5*a*b^2 + b^3 - 4*a*b*(11*a +
 3*b)*Cos[2*(e + f*x)] + (7*a - b)*b^2*Cos[4*(e + f*x)])*Tan[e + f*x])/Sqr
t[2])/(6*a*(a + b)^3*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

3.539.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 3675, 373, 402, 25, 27, 402, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(e+fx)^2}{\left(a+b\sin(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 3675$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^{3/2}\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 373$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\int\frac{a-3b\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin(e+fx)}{a+b}\right)}{f}$$

$\downarrow$ 402

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{4b\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\int-\frac{a\left(-4b\sin^2(e+fx)+3a-\right.}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)\right.}}{3a(a+b)}}{a+b}\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\int\frac{a\left(-4b\sin^2(e+fx)+3a-b\right)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}+\frac{4b\sqrt{1-\sin^2}}{3(a+b)\left(a+\right.}}{a+b}\right.}{f}$$

$\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\int\frac{-4b\sin^2(e+fx)+3a-b}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3(a+b)}+\frac{4b\sqrt{1-\sin^2}}{3(a+b)\left(a+\right.}}{a+b}\right.}{f}$$

$\downarrow$ 402

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{b(7a-b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\int-\frac{(7a-b)b\sin^2(e+fx)+a}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2}}}{a(a+b)}}{3(a+b)}}{a+b}\right.}{f}$$

3.539. $\int\frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{\int\frac{(7a-b)b\sin^2(e+fx)+a(3a-5b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a(a+b)}+\frac{b(7a-b)\sqrt{1-\sin\cdots}}{a(a+b)\sqrt{\cdots}}}{3(a+b)}}{a+b}\right.}{f}$$

$\downarrow$ 399

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{(7a-b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-4a(a+b)\int\frac{\cdots}{\sqrt{1-\sin^2(e+\cdots}}}{a(a+b)}}{3\cdots}\right.}{f}$$

$\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{(7a-b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}\cdots}}{\cdots}}{a(a+b)}}{\cdots}\right.}{f}$$

$\downarrow$ 321

3.539. $\displaystyle\int\frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{(7a-b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}\cdots}}{\sqrt{\cdots}}}{a(a+b)}}{\cdots}\right)}{f}$$

$\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{(7a-b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{4a(a\cdots}{\cdots}}{a(a+b)}}{\cdots}\right)}{f}$$

$\downarrow$ 327

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sin(e+fx)}{(a+b)\sqrt{1-\sin^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{(7a-b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{4a(a+b)\sqrt{\cdots}}{\cdots}}{a(a+b)}}{\cdots}\right)}{f}$$

input
```
Int[Tan[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

3.539. $\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(Sin[e + f*x]/((a + b)*Sqrt[1 - Sin[e +
 f*x]^2]*(a + b*Sin[e + f*x]^2)^(3/2)) - ((4*b*Sin[e + f*x]*Sqrt[1 - Sin[e
 + f*x]^2])/(3*(a + b)*(a + b*Sin[e + f*x]^2)^(3/2)) + (((7*a - b)*b*Sin[e
 + f*x]*Sqrt[1 - Sin[e + f*x]^2])/(a*(a + b)*Sqrt[a + b*Sin[e + f*x]^2]) +
 (((7*a - b)*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*
x]^2])/Sqrt[1 + (b*Sin[e + f*x]^2)/a] - (4*a*(a + b)*EllipticF[ArcSin[Sin[
e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/Sqrt[a + b*Sin[e + f*x]
^2])/(a*(a + b)))/(3*(a + b)))/(a + b)))/f
```

3.539.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

3.539. $\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.539. $\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.539.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. 2(268) = 536.

Time = 5.92 (sec) , antiderivative size = 851, normalized size of antiderivative = 2.91

method	result	size
default	Expression too large to display	851

input
```
int(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*(7*a-b)*cos(f*x+e)^4*s
in(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(11*a^2+10*a*b-b^2)
*cos(f*x+e)^2*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^
4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*b*(4*EllipticF(sin(f*x+
e),(-1/a*b)^(1/2))*a+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-7*EllipticE(
sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f
*x+e)^2+3*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(a^2+2*a*b+b^2)*sin
(f*x+e)+4*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*
(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3
+8*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*c
os(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+4*(-
b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*
x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2-7*(-b*cos
(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^
2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-6*(-b*cos(f*x+e)
^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)
/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+(-b*cos(f*x+e)^4+(a+b
)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/
2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2)/(a+b*sin(f*x+e)^2)^(3/2)/(-
(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(a+b)^3/a/cos(f...
```

3.539.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 1632, normalized size of antiderivative = 5.59

$$\int \frac{\tan^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/6*((2*((-7*I*a*b^4 + I*b^5)*cos(f*x + e)^5 - 2*(-7*I*a^2*b^3 - 6*I*a*b^4
 + I*b^5)*cos(f*x + e)^3 + (-7*I*a^3*b^2 - 13*I*a^2*b^3 - 5*I*a*b^4 + I*b^
5)*cos(f*x + e))*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((14*I*a^2*b^3 + 5*I*a*b
^4 - I*b^5)*cos(f*x + e)^5 + 2*(-14*I*a^3*b^2 - 19*I*a^2*b^3 - 4*I*a*b^4 +
 I*b^5)*cos(f*x + e)^3 + (14*I*a^4*b + 33*I*a^3*b^2 + 23*I*a^2*b^3 + 3*I*a
*b^4 - I*b^5)*cos(f*x + e))*sqrt(-b))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*
a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*
(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*s
qrt((a^2 + a*b)/b^2))/b^2) + (2*((7*I*a*b^4 - I*b^5)*cos(f*x + e)^5 - 2*(7
*I*a^2*b^3 + 6*I*a*b^4 - I*b^5)*cos(f*x + e)^3 + (7*I*a^3*b^2 + 13*I*a^2*b
^3 + 5*I*a*b^4 - I*b^5)*cos(f*x + e))*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - ((-
14*I*a^2*b^3 - 5*I*a*b^4 + I*b^5)*cos(f*x + e)^5 + 2*(14*I*a^3*b^2 + 19*I*
a^2*b^3 + 4*I*a*b^4 - I*b^5)*cos(f*x + e)^3 + (-14*I*a^4*b - 33*I*a^3*b^2
- 23*I*a^2*b^3 - 3*I*a*b^4 + I*b^5)*cos(f*x + e))*sqrt(-b))*sqrt((2*b*sqrt
((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*
b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b
^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) - 2*(2*((-3*I*a^2*b^3 - 2
*I*a*b^4 + I*b^5)*cos(f*x + e)^5 + 2*(3*I*a^3*b^2 + 5*I*a^2*b^3 + I*a*b^4
- I*b^5)*cos(f*x + e)^3 + (-3*I*a^4*b - 8*I*a^3*b^2 - 6*I*a^2*b^3 + I*b^5)
*cos(f*x + e))*sqrt(-b)*sqrt((a^2 + a*b)/b^2) + ((6*I*a^3*b^2 - 7*I*a^2...
```

3.539.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^2{(e+fx)}}{(a+b\sin^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**2/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.539.7 Maxima [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)^2}{(b\sin(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.539.8 Giac [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)^2}{(b\sin(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.539.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+f\,x)^2}{(b\sin(e+f\,x)^2+a)^{5/2}}\,dx$$

input
```
int(tan(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(tan(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.540 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.540.1 Optimal result

Integrand size = 16, antiderivative size = 223

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{3a^2(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3a(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
1/3*b*cos(f*x+e)*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*b*(2*a+
b)*cos(f*x+e)*sin(f*x+e)/a^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+2/3*(2*a+b
)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*
sin(f*x+e)^2)^(1/2)/a^2/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*(cos(f*x+
e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^
2/a)^(1/2)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.540.2 Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{2a^2(2a+b)\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right)-a^2(a+b)\left(\frac{2a+b-b\cos(2(e-}{a}\right.}{3a^2(a+b}$$

input
```
Integrate[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

output
```
(2*a^2*(2*a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*
x, -(b/a)] - a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*Elliptic
F[e + f*x, -(b/a)] - Sqrt[2]*b*(-5*a^2 - 5*a*b - b^2 + b*(2*a + b)*Cos[2*(
e + f*x)])*Sin[2*(e + f*x)])/(3*a^2*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*
x)])^(3/2))
```

3.540.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 3663, 25, 3042, 3652, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a+b\sin(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3663}$$

$$\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\int -\frac{-b\sin^2(e+fx)+3a+2b}{\left(b\sin^2(e+fx)+a\right)^{3/2}}dx}{3a(a+b)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-b\sin^2(e+fx)+3a+2b}{\left(b\sin^2(e+fx)+a\right)^{3/2}}dx}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-b\sin(e+fx)^2+3a+2b}{(b\sin(e+fx)^2+a)^{3/2}}dx}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3652}$$

$$\frac{\frac{\int \frac{2b(2a+b)\sin^2(e+fx)+a(3a+b)}{\sqrt{b\sin^2(e+fx)+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{2b(2a+b)\sin(e+fx)^2+a(3a+b)}{\sqrt{b\sin(e+fx)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3651}$$

$$\frac{\frac{2(2a+b)\int \sqrt{b\sin^2(e+fx)+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin^2(e+fx)+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(2a+b)\int \sqrt{b\sin(e+fx)^2+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3657}$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\int \sqrt{\frac{b\sin^2(e+fx)}{a}+1}dx}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

3.540. $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\int\sqrt{\frac{b\sin(e+fx)^2}{a}+1}dx}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-a(a+b)\int\frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-a(a+b)\int\frac{1}{\sqrt{b\sin(e+fx)^2+a}}dx}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3662}$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}dx}{\sqrt{a+b\sin^2(e+fx)}}}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin(e+fx)^2}{a}+1}}dx}{\sqrt{a+b\sin^2(e+fx)}}}{a(a+b)}+\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a(a+b)}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

$$\Big\downarrow \text{3661}$$

$$\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(e+fx,-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}}{a(a+b)}}{3a(a+b)}$$

input
```
Int[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

3.540. $\int\frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

output
```
(b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
 + ((2*b*(2*a + b)*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[
e + f*x]^2]) + ((2*(2*a + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e +
 f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*
x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2]))
/(a*(a + b)))/(3*a*(a + b))
```

3.540.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.540. $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.540.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(245) = 490$.

Time = 2.11 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.45

method	result
default	$-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2b\left(\sin^2(fx+e)\right)+\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e\right.}{}$

input
```
int(1/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+
e),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e
)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2-4*(c
os(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a^2*b*sin(f*x+e)^2-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a
)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2+4*a*b^2*si
n(f*x+e)^5+2*b^3*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^
(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+a^2*(cos(f*x+e)^2)^(1/2)*((
a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-4*(cos(f
*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a^3-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(s
in(f*x+e),(-1/a*b)^(1/2))*a^2*b+5*a^2*b*sin(f*x+e)^3-a*b^2*sin(f*x+e)^3-2*
b^3*sin(f*x+e)^3-5*a^2*b*sin(f*x+e)-3*a*b^2*sin(f*x+e))/(a+b*sin(f*x+e)^2)
^(3/2)/a^2/(a+b)^2/cos(f*x+e)/f
```

3.540.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 1531, normalized size of antiderivative = 6.87

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((2*(2*I*a^3*b^2 + 5*I*a^2*b^3 + 4*I*a*b^4 + I*b^5 + (2*I*a*b^4 + I*b^
5)*cos(f*x + e)^4 - 2*(2*I*a^2*b^3 + 3*I*a*b^4 + I*b^5)*cos(f*x + e)^2)*sq
rt(-b)*sqrt((a^2 + a*b)/b^2) - (-4*I*a^4*b - 12*I*a^3*b^2 - 13*I*a^2*b^3 -
 6*I*a*b^4 - I*b^5 + (-4*I*a^2*b^3 - 4*I*a*b^4 - I*b^5)*cos(f*x + e)^4 + 2
*(4*I*a^3*b^2 + 8*I*a^2*b^3 + 5*I*a*b^4 + I*b^5)*cos(f*x + e)^2)*sqrt(-b))
*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b
*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*
a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-2*I
*a^3*b^2 - 5*I*a^2*b^3 - 4*I*a*b^4 - I*b^5 + (-2*I*a*b^4 - I*b^5)*cos(f*x
+ e)^4 - 2*(-2*I*a^2*b^3 - 3*I*a*b^4 - I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqr
t((a^2 + a*b)/b^2) - (4*I*a^4*b + 12*I*a^3*b^2 + 13*I*a^2*b^3 + 6*I*a*b^4
+ I*b^5 + (4*I*a^2*b^3 + 4*I*a*b^4 + I*b^5)*cos(f*x + e)^4 + 2*(-4*I*a^3*b
^2 - 8*I*a^2*b^3 - 5*I*a*b^4 - I*b^5)*cos(f*x + e)^2)*sqrt(-b))*sqrt((2*b*
sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2
+ a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b
 + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(-3*I*a^4*b - 11
*I*a^3*b^2 - 15*I*a^2*b^3 - 9*I*a*b^4 - 2*I*b^5 + (-3*I*a^2*b^3 - 5*I*a*b^
4 - 2*I*b^5)*cos(f*x + e)^4 - 2*(-3*I*a^3*b^2 - 8*I*a^2*b^3 - 7*I*a*b^4 -
2*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2) - (-6*I*a^5 - 17*I
*a^4*b - 17*I*a^3*b^2 - 7*I*a^2*b^3 - I*a*b^4 + (-6*I*a^3*b^2 - 5*I*a^2...
```

3.540.6 Sympy [F]

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Integral((a + b*sin(e + f*x)**2)**(-5/2), x)
```

3.540.7 Maxima [F]

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

3.540.8 Giac [F]

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.540.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(1/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.541 $\int \frac{\cot^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.541.1 Optimal result

Integrand size = 25, antiderivative size = 287

$$\begin{aligned}\int \frac{\cot^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx &= \frac{\cot(e+fx)}{3af\,(a+b\sin^2(e+fx))^{3/2}}\\
&+\frac{(3a+4b)\cot(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}}-\frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3(a+b)f}\\
&-\frac{(7a+8b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\\
&+\frac{4\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3a^2f\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

output
```
1/3*cot(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(3/2)+1/3*(3*a+4*b)*cot(f*x+e)/a^2/(
a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(7*a+8*b)*cot(f*x+e)*(a+b*sin(f*x+e)^2
)^(1/2)/a^3/(a+b)/f-1/3*(7*a+8*b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f
*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^3/(a+b)/f/(1+b*sin(f
*x+e)^2/a)^(1/2)+4/3*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*
x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/a^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

3.541.2 Mathematica [A] (verified)

Time = 3.24 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.73

$$\int \frac{\cot^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \frac{-\frac{(24a^3+68a^2b+69ab^2+24b^3-4b(11a^2+19ab+8b^2)\cos(2(e+fx))+b^2(7a+8b)\cos(4(e+fx)))\cot(e+fx)}{\sqrt{2}}}{6a}$$

input
```
Integrate[Cot[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(-(((24*a^3 + 68*a^2*b + 69*a*b^2 + 24*b^3 - 4*b*(11*a^2 + 19*a*b + 8*b^2)
*Cos[2*(e + f*x)] + b^2*(7*a + 8*b)*Cos[4*(e + f*x)])*Cot[e + f*x])/Sqrt[2
]) - 2*a^2*(7*a + 8*b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[
e + f*x, -(b/a)] + 8*a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*
EllipticF[e + f*x, -(b/a)])/(6*a^3*(a + b)*f*(2*a + b - b*Cos[2*(e + f*x)]
)^(3/2))
```

3.541.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3675, 371, 25, 441, 25, 445, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\tan(e+fx)^2\,(a+b\sin(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \; 3675$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\csc^2(e+fx)\sqrt{1-\sin^2(e+fx)}}{(b\sin^2(e+fx)+a)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \; 371$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\sqrt{1-\sin^2(e+fx)}\csc(e+fx)}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\int -\frac{\csc^2(e+fx)\left(4-3\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3a}\right)}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int \frac{\csc^2(e+fx)\left(4-3\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3a}+\frac{\sqrt{1-\sin^2(e+fx)}\csc(e+fx)}{3a\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{f}$$

$\downarrow$ 441

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{(3a+4b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{\int -\frac{\csc^2(e+fx)\left(-\left((3a+4b)\sin^2(e+fx)\right)+7a+8b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a(a+b)}}{3a}+\frac{\sqrt{1-\sin^2(e+\cdots}}{3a\left(a+b\sin^{2}\cdots\right.}\right.}{f}$$

$\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{\int \frac{\csc^2(e+fx)\left(-\left((3a+4b)\sin^2(e+fx)\right)+7a+8b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a(a+b)}+\frac{(3a+4b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)}{a(a+b)\sqrt{a+b\sin^2(e+fx)}}}{3a}+\frac{\sqrt{1-\sin^2(e+f\cdots}}{3a\left(a+b\sin^2(\cdots\right.}\right.}{f}$$

$\downarrow$ 445

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{-\frac{\int \frac{b(7a+8b)\sin^2(e+fx)+a(3a+4b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{(7a+8b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}}{a(a+b)}+\frac{(3a+4b)\sqrt{1\cdots}}{a(a+b\cdots}}{3a}\right.}{f}$$

$\downarrow$ 399

3.541. $\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{\frac{(7a+8b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-4a(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{(7a+8b)\sqrt{1-\sin^2\cdots}}{\cdots}}{a(a+b)}}{3a}\cdots\right.}{f}$$

$\Big\downarrow$ 323

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{(7a+8b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d\sin(e+fx\cdots}{\sqrt{a+b\sin^2(e+fx)}}}{a}\cdots}{a(a+b)}\cdots}{3a}\cdots\right.}{f}$$

$\Big\downarrow$ 321

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{\frac{(7a+8b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}}{a}-\frac{(7a+8b)\sqrt{1-\cdots}}{\cdots}}{a(a+b)}}{3a}\cdots\right.}{f}$$

$\Big\downarrow$ 330

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{\frac{(7a+8b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\cdots\right.}{\sqrt{a+b\sin^2(e+fx)}}}{a}\cdots}{a(a+b)}\cdots}{3a}\cdots\right.}{f}$$

$\Big\downarrow$ 327

3.541. $\displaystyle\int\frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{-\frac{-\frac{(7a+8b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{4a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}}{a} -}{a(a+b)}}{3a}\right.}{f}$$

input
```
Int[Cot[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*((Csc[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]
)/(3*a*(a + b*Sin[e + f*x]^2)^(3/2)) + (((3*a + 4*b)*Csc[e + f*x]*Sqrt[1 -
 Sin[e + f*x]^2])/(a*(a + b)*Sqrt[a + b*Sin[e + f*x]^2]) + (-(((7*a + 8*b)
*Csc[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*x]^2])/a) - ((
(7*a + 8*b)*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[a + b*Sin[e + f*x
]^2])/Sqrt[1 + (b*Sin[e + f*x]^2)/a] - (4*a*(a + b)*EllipticF[ArcSin[Sin[e
 + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/Sqrt[a + b*Sin[e + f*x]^
2])/a)/(a*(a + b)))/(3*a)))/f
```

3.541.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.541. $\int \frac{\cot^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 371
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a
*e*2*(p + 1))), x] + Simp[1/(a*2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p + 1)
*(c + d*x^2)^(q - 1)*Simp[c*(m + 2*(p + 1) + 1) + d*(m + 2*(p + q + 1) + 1)
*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.541. $\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.541.4 Maple [A] (verified)

Time = 5.29 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.43

method	result
default	$(-7a\,b^2-8b^3)(\cos^6(fx+e))+(11a^2b+26a\,b^2+16b^3)(\cos^4(fx+e))-\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\,\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,ab\Big(4F\Big(\sin(fx+e),\sqrt{\;}$

input
```
int(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-7*a*b^2-8*b^3)*cos(f*x+e)^6+(11*a^2*b+26*a*b^2+16*b^3)*cos(f*x+e)^4
-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*b*(4*EllipticF(s
in(f*x+e),(-1/a*b)^(1/2))*a+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-7*Ell
ipticE(sin(f*x+e),(-1/a*b)^(1/2))*a-8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))
*b)*cos(f*x+e)^2*sin(f*x+e)+(-3*a^3-14*a^2*b-19*a*b^2-8*b^3)*cos(f*x+e)^2+
(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(4*EllipticF(sin(
f*x+e),(-1/a*b)^(1/2))*a^2+8*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b+4*El
lipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-7*EllipticE(sin(f*x+e),(-1/a*b)^(1/
2))*a^2-15*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b-8*EllipticE(sin(f*x+e)
,(-1/a*b)^(1/2))*b^2)*sin(f*x+e))/sin(f*x+e)/a^3/(a+b)/(a+b*sin(f*x+e)^2)^
(3/2)/cos(f*x+e)/f
```

3.541.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 1595, normalized size of antiderivative = 5.56

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/6*((2*(-7*I*a^3*b^2 - 22*I*a^2*b^3 - 23*I*a*b^4 - 8*I*b^5 + (-7*I*a*b^4
- 8*I*b^5)*cos(f*x + e)^4 - 2*(-7*I*a^2*b^3 - 15*I*a*b^4 - 8*I*b^5)*cos(f*
x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) - (14*I*a^4*b + 51*I
*a^3*b^2 + 68*I*a^2*b^3 + 39*I*a*b^4 + 8*I*b^5 + (14*I*a^2*b^3 + 23*I*a*b^
4 + 8*I*b^5)*cos(f*x + e)^4 + 2*(-14*I*a^3*b^2 - 37*I*a^2*b^3 - 31*I*a*b^4
 - 8*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a
*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) +
 2*a + b)/b)*(cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2
*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*(7*I*a^3*b^2 + 22*I*a^2*b^3 +
 23*I*a*b^4 + 8*I*b^5 + (7*I*a*b^4 + 8*I*b^5)*cos(f*x + e)^4 - 2*(7*I*a^2*
b^3 + 15*I*a*b^4 + 8*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)
*sin(f*x + e) - (-14*I*a^4*b - 51*I*a^3*b^2 - 68*I*a^2*b^3 - 39*I*a*b^4 -
8*I*b^5 + (-14*I*a^2*b^3 - 23*I*a*b^4 - 8*I*b^5)*cos(f*x + e)^4 + 2*(14*I*
a^3*b^2 + 37*I*a^2*b^3 + 31*I*a*b^4 + 8*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*si
n(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsi
n(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x
+ e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2)
 - 2*(2*(-3*I*a^4*b - 17*I*a^3*b^2 - 33*I*a^2*b^3 - 27*I*a*b^4 - 8*I*b^5 +
 (-3*I*a^2*b^3 - 11*I*a*b^4 - 8*I*b^5)*cos(f*x + e)^4 + 2*(3*I*a^3*b^2 + 1
4*I*a^2*b^3 + 19*I*a*b^4 + 8*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 ...
```

3.541.6 Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx=\int \frac{\cot^2(e+fx)}{(a+b\sin^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)**2/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.541.7 Maxima [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(fx+e)^2}{(b\sin(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.541.8 Giac [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(fx+e)^2}{(b\sin(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.541.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(e+f\,x)^2}{(b\sin(e+f\,x)^2+a)^{5/2}}\,dx$$

input
```
int(cot(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(cot(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.542 $\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.542.1 Optimal result

Integrand size = 25, antiderivative size = 348

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{8(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^4f} - \frac{(3a+8b)\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3bf} + \frac{8(a+2b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\left|-\frac{b}{a}\right.\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^4f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{(5a+8b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)\sec(e+fx)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{3a^3f\sqrt{a+b\sin^2(e+fx)}}$$

output
```
1/3*(a+b)*cot(f*x+e)*csc(f*x+e)^2/a/b/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*(a+3*
b)*cot(f*x+e)*csc(f*x+e)^2/a^2/b/f/(a+b*sin(f*x+e)^2)^(1/2)+8/3*(a+2*b)*co
t(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a^4/f-1/3*(3*a+8*b)*cot(f*x+e)*csc(f*x+e
)^2*(a+b*sin(f*x+e)^2)^(1/2)/a^3/b/f+8/3*(a+2*b)*EllipticE(sin(f*x+e),(-b/
a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^4/f/(
1+b*sin(f*x+e)^2/a)^(1/2)-1/3*(5*a+8*b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))
*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/a^3/f/(a+b*sin
(f*x+e)^2)^(1/2)
```

3.542.2 Mathematica [A] (verified)

Time = 3.71 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.65

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{2a^2b\left(\frac{2a+b-b\cos(2(e+fx))}{a}\right)^{3/2}\left(8(a+2b)E\left(e+fx\middle|-\frac{b}{a}\right)-(5a+8b)\,\text{EllipticF}\left(\right.\right.}{}$$

input
```
Integrate[Cot[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(2*a^2*b*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*(8*(a + 2*b)*EllipticE[e
 + f*x, -(b/a)] - (5*a + 8*b)*EllipticF[e + f*x, -(b/a)]) + Sqrt[2]*b*(4*(
a + 2*b)*(2*a + b - b*Cos[2*(e + f*x)])^2*Cot[e + f*x] - a*(2*a + b - b*Co
s[2*(e + f*x)])^2*Cot[e + f*x]*Csc[e + f*x]^2 + 2*a*b*(a + b)*Sin[2*(e + f
*x)] + 4*b*(a + 2*b)*(2*a + b - b*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/(6*
a^4*b*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

3.542.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 3675, 370, 25, 441, 27, 445, 27, 445, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\tan(e+fx)^4\left(a+b\sin(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \ 3675$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\int \frac{\csc^4(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{\left(b\sin^2(e+fx)+a\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\Big\downarrow \ 370$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{(a+b)\sqrt{1-\sin^2(e+fx)}\csc^3(e+fx)}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\int -\frac{\csc^4(e+fx)\left(3(a+2b)-(2a+5b)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3ab}\right)}{f}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\int \frac{\csc^4(e+fx)\left(3(a+2b)-(2a+5b)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\left(b\sin^2(e+fx)+a\right)^{3/2}}d\sin(e+fx)}{3ab}+\frac{(a+b)\sqrt{1-\sin^2(e+fx)}\csc^3(e+fx)}{3ab\left(a+b\sin^2(e+fx)\right)^{3/2}}\right)}{f}$$

$\Big\downarrow$ 441

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{2(a+3b)\sqrt{1-\sin^2(e+fx)}\csc^3(e+fx)}{a\sqrt{a+b\sin^2(e+fx)}}-\frac{\int -\frac{3(a+b)\csc^4(e+fx)\left(-2(a+3b)\sin^2(e+fx)+3a+8b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a(a+b)}}{3ab}+\frac{(a+b)\sqrt{\cdots}}{3a\cdots}\right.}{f}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{3\int \frac{\csc^4(e+fx)\left(-2(a+3b)\sin^2(e+fx)+3a+8b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}+\frac{2(a+3b)\sqrt{1-\sin^2(e+fx)}\csc^3(e+fx)}{a\sqrt{a+b\sin^2(e+fx)}}}{3ab}+\frac{(a+b)\sqrt{1-\sin^2\cdots}}{3ab(a+b\,\mathrm{s}\cdots}\right.}{f}$$

$\Big\downarrow$ 445

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{3\left(-\frac{\int \frac{b\csc^2(e+fx)\left(8(a+2b)-(3a+8b)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3a}-\frac{(3a+8b)\sqrt{1-\sin^2(e+fx)}\csc^3(e+fx)\sqrt{a+b\sin^2(e+f\cdots}}{3a}\right.\cdots}{a}}{3ab}\cdots\right.}{f}$$

$\Big\downarrow$ 27

3.542. $\displaystyle\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{3\left(-\frac{b\int \frac{\csc^2(e+fx)\left(8(a+2b)-(3a+8b)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{3a}-\frac{(3a+8b)\sqrt{1-\sin^2(e+fx)}\csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a}\cdots\right)}{a}}{3ab}\cdots\right)}{f}$$

$\downarrow$ 445

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{3\left(-\frac{b\left(-\frac{\int \frac{8b(a+2b)\sin^2(e+fx)+a(3a+8b)}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{8(a+2b)\sqrt{1-\sin^2(e+fx)}\csc(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}\right)}{3a}-\cdots\right)}{a}}{3ab}\cdots\right)}{f}$$

$\downarrow$ 399

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\frac{3\left(-\frac{b\left(-\frac{8(a+2b)\int \frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-a(5a+8b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{b\sin^2(e+fx)+a}}d\sin(e+fx)}{a}-\frac{8(a+2b\cdots}{\cdots}\right)}{3a}\cdots\right)}{a}}{\cdots}\cdots\right)$$

3.542. $\displaystyle\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$\downarrow$ 323

$$\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{\left(3\left(-\frac{b\left(-\frac{8(a+2b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{a(5a+8b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}d}{\sqrt{a+b\sin^2(e+fx)}}}{a}\right.}{3a}\right.}{}\right.\right.$$

$\downarrow$ 321

3.542. $\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{3\left(-\frac{b\left(-\frac{8(a+2b)\int\frac{\sqrt{b\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{a(5a+8b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),-\frac{b}{a}\right)}{\sqrt{a+b\sin^2(e+fx)}}}{a}-\frac{8(a \cdots}{\cdots}\right)}{3a}\cdots\right)}{a}\cdots\right)}{\cdots}$$

$\downarrow$ 330

3.542. $\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{3\left(-\frac{b\left(-\frac{\frac{8(a+2b)\sqrt{a+b\sin^2(e+fx)}\int\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(5a+8b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin}{\sqrt{a+b\sin^2(e+fx)}}}{a}}{3a}}{}}{}$$

$\downarrow$ 327

3.542. $\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(\frac{3\left(-\frac{b\left(-\frac{\frac{8(a+2b)\sqrt{a+b\sin^2(e+fx)}E\left(\arcsin(\sin(e+fx))\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(5a+8b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\text{EllipticF}\left(\arcsin(\sin(e+fx\cdots\right.}{\sqrt{a+b\sin^2(e+fx)}}}{a}\cdots\right)}{3a}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

input
```
Int[Cot[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(((a + b)*Csc[e + f*x]^3*Sqrt[1 - Sin[e
 + f*x]^2])/(3*a*b*(a + b*Sin[e + f*x]^2)^(3/2)) + ((2*(a + 3*b)*Csc[e + f
*x]^3*Sqrt[1 - Sin[e + f*x]^2])/(a*Sqrt[a + b*Sin[e + f*x]^2]) + (3*(-1/3*
((3*a + 8*b)*Csc[e + f*x]^3*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a + b*Sin[e + f*
x]^2])/a - (b*((-8*(a + 2*b)*Csc[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a
+ b*Sin[e + f*x]^2])/a - ((8*(a + 2*b)*EllipticE[ArcSin[Sin[e + f*x]], -(b
/a)]*Sqrt[a + b*Sin[e + f*x]^2])/Sqrt[1 + (b*Sin[e + f*x]^2)/a] - (a*(5*a
+ 8*b)*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)
/a])/Sqrt[a + b*Sin[e + f*x]^2])/a))/(3*a)))/a)/(3*a*b)))/f
```

3.542. $\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

3.542.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.542. $\int \frac{\cot^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.542. $\int \frac{\cot^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx$

3.542.4 Maple [A] (verified)

Time = 4.47 (sec) , antiderivative size = 633, normalized size of antiderivative = 1.82

method	result
default	$-\frac{5\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2b(\sin^5(fx+e))+8\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,F\left(\sin(fx\right.}{}$

input
```
int(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(5*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*
x+e),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^5+8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f
*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^5-
8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^5-16*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)
^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^5+8*sin
(f*x+e)^8*a*b^2+16*sin(f*x+e)^8*b^3+5*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e
)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3*sin(f*x+e)^3+8*(cos
(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b
)^(1/2))*a^2*b*sin(f*x+e)^3-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^
(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3*sin(f*x+e)^3-16*(cos(f*x+e)
^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)
)*a^2*b*sin(f*x+e)^3+13*sin(f*x+e)^6*a^2*b+16*sin(f*x+e)^6*a*b^2-16*b^3*si
n(f*x+e)^6+4*sin(f*x+e)^4*a^3-7*sin(f*x+e)^4*a^2*b-24*sin(f*x+e)^4*a*b^2-5
*sin(f*x+e)^2*a^3-6*sin(f*x+e)^2*a^2*b+a^3)/sin(f*x+e)^3/a^4/(a+b*sin(f*x+
e)^2)^(3/2)/cos(f*x+e)/f
```

3.542.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1971, normalized size of antiderivative = 5.66

$$\int \frac{\cot^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(4*(2*((-I*a*b^4 - 2*I*b^5)*cos(f*x + e)^6 + I*a^3*b^2 + 4*I*a^2*b^3
+ 5*I*a*b^4 + 2*I*b^5 + (2*I*a^2*b^3 + 7*I*a*b^4 + 6*I*b^5)*cos(f*x + e)^4
 + (-I*a^3*b^2 - 6*I*a^2*b^3 - 11*I*a*b^4 - 6*I*b^5)*cos(f*x + e)^2)*sqrt(
-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + ((-2*I*a^2*b^3 - 5*I*a*b^4 - 2*I*
b^5)*cos(f*x + e)^6 + 2*I*a^4*b + 9*I*a^3*b^2 + 14*I*a^2*b^3 + 9*I*a*b^4 +
 2*I*b^5 + (4*I*a^3*b^2 + 16*I*a^2*b^3 + 19*I*a*b^4 + 6*I*b^5)*cos(f*x + e
)^4 + (-2*I*a^4*b - 13*I*a^3*b^2 - 28*I*a^2*b^3 - 23*I*a*b^4 - 6*I*b^5)*co
s(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a
 + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)*(
cos(f*x + e) + I*sin(f*x + e))), (8*a^2 + 8*a*b + b^2 - 4*(2*a*b + b^2)*sq
rt((a^2 + a*b)/b^2))/b^2) + 4*(2*((I*a*b^4 + 2*I*b^5)*cos(f*x + e)^6 - I*a
^3*b^2 - 4*I*a^2*b^3 - 5*I*a*b^4 - 2*I*b^5 + (-2*I*a^2*b^3 - 7*I*a*b^4 - 6
*I*b^5)*cos(f*x + e)^4 + (I*a^3*b^2 + 6*I*a^2*b^3 + 11*I*a*b^4 + 6*I*b^5)*
cos(f*x + e)^2)*sqrt(-b)*sqrt((a^2 + a*b)/b^2)*sin(f*x + e) + ((2*I*a^2*b^
3 + 5*I*a*b^4 + 2*I*b^5)*cos(f*x + e)^6 - 2*I*a^4*b - 9*I*a^3*b^2 - 14*I*a
^2*b^3 - 9*I*a*b^4 - 2*I*b^5 + (-4*I*a^3*b^2 - 16*I*a^2*b^3 - 19*I*a*b^4 -
 6*I*b^5)*cos(f*x + e)^4 + (2*I*a^4*b + 13*I*a^3*b^2 + 28*I*a^2*b^3 + 23*I
*a*b^4 + 6*I*b^5)*cos(f*x + e)^2)*sqrt(-b)*sin(f*x + e))*sqrt((2*b*sqrt((a
^2 + a*b)/b^2) + 2*a + b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/
b^2) + 2*a + b)/b)*(cos(f*x + e) - I*sin(f*x + e))), (8*a^2 + 8*a*b + b...
```

3.542.6 Sympy [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx=\int \frac{\cot^4(e+fx)}{(a+b\sin^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+b*sin(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)**4/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.542.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.542.8 Giac [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(fx+e)^4}{(b\sin(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.542.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\sin^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(e+f\,x)^4}{(b\sin(e+f\,x)^2+a)^{5/2}}\,dx$$

input
```
int(cot(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2),x)
```

output
```
int(cot(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2), x)
```

3.543 $\int \left(a+b\sin^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx$

3.543.1 Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \left(a+b\sin^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx$$
$$= \frac{\operatorname{AppellF1}\left(\frac{1+m}{2},\frac{1+m}{2},-p,\frac{3+m}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\cos^2(e+fx)^{\frac{1+m}{2}}\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin\cdots}{\cdots}\right.\cdots}{df(1+m)}$$

output
```
AppellF1(1/2+1/2*m,1/2+1/2*m,-p,3/2+1/2*m,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*
(cos(f*x+e)^2)^(1/2+1/2*m)*(a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^(1+m)/d/f/(
1+m)/((1+b*sin(f*x+e)^2/a)^p)
```

3.543.2 Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01

$$\int \left(a+b\sin^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx$$
$$= \frac{\operatorname{AppellF1}\left(\frac{1+m}{2},\frac{1+m}{2},-p,\frac{3+m}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\cos^2(e+fx)^{\frac{1+m}{2}}\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin\cdots}{\cdots}\right.\cdots}{f(1+m)}$$

input
```
Integrate[(a + b*Sin[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```

output
```
(AppellF1[(1 + m)/2, (1 + m)/2, -p, (3 + m)/2, Sin[e + f*x]^2, -((b*Sin[e
+ f*x]^2)/a)]*(Cos[e + f*x]^2)^((1 + m)/2)*(a + b*Sin[e + f*x]^2)^p*Tan[e
+ f*x]*(d*Tan[e + f*x])^m)/(f*(1 + m)*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

3.543.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3676, 393, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\tan(e+fx))^m \left(a+b\sin(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{3676}$$

$$\frac{\sin^{-m-1}(e+fx)\cos^2(e+fx)^{\frac{m+1}{2}}(d\tan(e+fx))^{m+1}\int \sin^m(e+fx)\left(1-\sin^2(e+fx)\right)^{\frac{1}{2}(-m-1)}\left(b\sin^2(e+fx)+\right.}{df}$$

$$\Big\downarrow \text{393}$$

$$\frac{\sin^2(e+fx)^{\frac{1-m}{2}-1}\cos^2(e+fx)^{\frac{m+1}{2}}(d\tan(e+fx))^{m+1}\int \sin^2(e+fx)^{\frac{m-1}{2}}\left(1-\sin^2(e+fx)\right)^{\frac{1}{2}(-m-1)}\left(b\sin^2(e+\right.}{2df}$$

$$\Big\downarrow \text{152}$$

$$\frac{\sin^2(e+fx)^{\frac{1-m}{2}-1}\cos^2(e+fx)^{\frac{m+1}{2}}(d\tan(e+fx))^{m+1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\int \sin^2(e+fx)^{m}}{2df}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sin^2(e+fx)^{\frac{1-m}{2}+\frac{m+1}{2}-1}\cos^2(e+fx)^{\frac{m+1}{2}}(d\tan(e+fx))^{m+1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\right.}{df(m+1)}$$

input
```
Int[(a + b*Sin[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```

output
```
(AppellF1[(1 + m)/2, (1 + m)/2, -p, (3 + m)/2, Sin[e + f*x]^2, -((b*Sin[e
+ f*x]^2)/a)]*(Cos[e + f*x]^2)^((1 + m)/2)*(Sin[e + f*x]^2)^(-1 + (1 - m)/
2 + (1 + m)/2)*(a + b*Sin[e + f*x]^2)^p*(d*Tan[e + f*x])^(1 + m))/(d*f*(1
+ m)*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

3.543.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 393
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(e*x)^m/(2*x*(x^2)^(Simplify[(m + 1)/2] - 1))   Subs
t[Int[x^(Simplify[(m + 1)/2] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x]
/; FreeQ[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simp
lify[m + 2*p]] &&  !IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3676
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[f
f*(d*Tan[e + f*x])^(m + 1)*((Cos[e + f*x]^2)^((m + 1)/2)/(d*f*Sin[e + f*x]^
(m + 1)))   Subst[Int[(ff*x)^m*((a + b*ff^2*x^2)^p/(1 - ff^2*x^2)^((m + 1)/
2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] &&  !I
ntegerQ[m]
```

3.543. $\int (a+b\sin^2(e+fx))^p (d\tan(e+fx))^m \, dx$

3.543.4 Maple [F]

$$\int \left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p \left(d\tan\left(fx+e\right)\right)^m dx$$

input
```
int((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

output
```
int((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

3.543.5 Fricas [F]

$$\int \left(a+b\sin^2(e+fx)\right)^p (d\tan(e+fx))^m\, dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p \left(d\tan\left(fx+e\right)\right)^m\, dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*(d*tan(f*x + e))^m, x)
```

3.543.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(e+fx)\right)^p (d\tan(e+fx))^m\, dx = \text{Timed out}$$

input
```
integrate((a+b*sin(f*x+e)**2)**p*(d*tan(f*x+e))**m,x)
```

output
```
Timed out
```

3.543.7 Maxima [F]

$$\int (a+b\sin^2(e+fx))^p (d\tan(e+fx))^m \,dx = \int (b\sin(fx+e)^2+a)^p (d\tan(fx+e))^m \,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

3.543.8 Giac [F]

$$\int (a+b\sin^2(e+fx))^p (d\tan(e+fx))^m \,dx = \int (b\sin(fx+e)^2+a)^p (d\tan(fx+e))^m \,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

3.543.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin^2(e+fx))^p (d\tan(e+fx))^m \,dx = \int (d\tan(e+f\,x))^m \left(b\sin(e+f\,x)^2+a\right)^p \,dx$$

input
```
int((d*tan(e + f*x))^m*(a + b*sin(e + f*x)^2)^p,x)
```

output
```
int((d*tan(e + f*x))^m*(a + b*sin(e + f*x)^2)^p, x)
```

3.544 $\int (a+b\sin^2(c+dx))^p \tan^3(c+dx)\,dx$

3.544.1 Optimal result

Integrand size = 23, antiderivative size = 102

$$\int (a+b\sin^2(c+dx))^p \tan^3(c+dx)\,dx = -\frac{(a+b+bp)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sin^2(c+dx)}{a+b}\right)(a+b\sin^2(c+dx))^{1+p}}{2(a+b)^2d(1+p)} + \frac{\sec^2(c+dx)\,(a+b\sin^2(c+dx))^{1+p}}{2(a+b)d}$$

output
```
-1/2*(b*p+a+b)*hypergeom([1, p+1],[2+p],(a+b*sin(d*x+c)^2)/(a+b))*(a+b*sin
(d*x+c)^2)^(p+1)/(a+b)^2/d/(p+1)+1/2*sec(d*x+c)^2*(a+b*sin(d*x+c)^2)^(p+1)
/(a+b)/d
```

3.544.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81

$$\int (a+b\sin^2(c+dx))^p \tan^3(c+dx)\,dx = \frac{\left(-\left((a+b+bp)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sin^2(c+dx)}{a+b}\right)\right)+(a+b)(1+p)\sec^2(c+dx)\right)(a+}{2(a+b)^2d(1+p)}$$

input
```
Integrate[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^3,x]
```

output
```
((-((a + b + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^2
)/(a + b)]) + (a + b)*(1 + p)*Sec[c + d*x]^2)*(a + b*Sin[c + d*x]^2)^(1 +
p))/(2*(a + b)^2*d*(1 + p))
```

3.544.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3673, 87, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^3\left(a+b\sin(c+dx)^2\right)^p\,dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \frac{\sin^2(c+dx)\left(b\sin^2(c+dx)+a\right)^p}{\left(1-\sin^2(c+dx)\right)^2}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{\left(a+b\sin^2(c+dx)\right)^{p+1}}{(a+b)\left(1-\sin^2(c+dx)\right)}-\frac{(a+bp+b)\int \frac{\left(b\sin^2(c+dx)+a\right)^p}{1-\sin^2(c+dx)}d\sin^2(c+dx)}{a+b}}{2d}$$

$$\Big\downarrow \text{78}$$

$$\frac{\frac{\left(a+b\sin^2(c+dx)\right)^{p+1}}{(a+b)\left(1-\sin^2(c+dx)\right)}-\frac{(a+bp+b)\left(a+b\sin^2(c+dx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sin^2(c+dx)+a}{a+b}\right)}{(p+1)(a+b)^2}}{2d}$$

input
```
Int[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^3,x]
```

output
```
(-(((a + b + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^2
)/(a + b)]*(a + b*Sin[c + d*x]^2)^(1 + p))/((a + b)^2*(1 + p))) + (a + b*S
in[c + d*x]^2)^(1 + p)/((a + b)*(1 - Sin[c + d*x]^2)))/(2*d)
```

3.544.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.544.4 Maple [F]

$$\int \left(a+\left(\sin^2\left(dx+c\right)\right)b\right)^p\left(\tan^3\left(dx+c\right)\right)dx$$

input
```
int((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^3,x)
```

output
```
int((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^3,x)
```

3.544.5 Fricas [F]

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan^3(c+dx)\,dx=\int \left(b\sin\left(dx+c\right)^2+a\right)^p \tan\left(dx+c\right)^3\,dx$$

input
```
integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((-b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^3, x)
```

3.544.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan^3(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c)**2)**p*tan(d*x+c)**3,x)
```

output
```
Timed out
```

3.544.7 Maxima [F]

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan^3(c+dx)\,dx=\int \left(b\sin\left(dx+c\right)^2+a\right)^p \tan\left(dx+c\right)^3\,dx$$

input
```
integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^3, x)
```

3.544.8 Giac [F]

$$\int (a+b\sin^2(c+dx))^p \tan^3(c+dx)\,dx = \int \left(b\sin(dx+c)^2+a\right)^p \tan(dx+c)^3\,dx$$

input
```
integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^3, x)
```

3.544.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin^2(c+dx))^p \tan^3(c+dx)\,dx = \int \tan(c+d\,x)^3 \left(b\sin(c+d\,x)^2+a\right)^p\,dx$$

input
```
int(tan(c + d*x)^3*(a + b*sin(c + d*x)^2)^p,x)
```

output
```
int(tan(c + d*x)^3*(a + b*sin(c + d*x)^2)^p, x)
```

3.545 $\int (a+b\sin^2(c+dx))^p \tan(c+dx)\,dx$

3.545.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int (a+b\sin^2(c+dx))^p \tan(c+dx)\,dx = \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sin^2(c+dx)}{a+b}\right)(a+b\sin^2(c+dx))^{1+p}}{2(a+b)d(1+p)}$$

output
```
1/2*hypergeom([1, p+1],[2+p],(a+b*sin(d*x+c)^2)/(a+b))*(a+b*sin(d*x+c)^2)^
(p+1)/(a+b)/d/(p+1)
```

3.545.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int (a+b\sin^2(c+dx))^p \tan(c+dx)\,dx = \frac{(a+b-b\cos^2(c+dx))^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{b\cos^2(c+dx)}{a+b}\right)}{2(a+b)d(1+p)}$$

input
```
Integrate[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x],x]
```

output
```
((a + b - b*Cos[c + d*x]^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 -
 (b*Cos[c + d*x]^2)/(a + b)])/(2*(a + b)*d*(1 + p))
```

3.545.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3673, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(c+dx)\left(a+b\sin(c+dx)^2\right)^p\,dx$$

$$\Big\downarrow \ 3673$$

$$\frac{\int \frac{(b\sin^2(c+dx)+a)^p}{1-\sin^2(c+dx)}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \ 78$$

$$\frac{\left(a+b\sin^2(c+dx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sin^2(c+dx)+a}{a+b}\right)}{2d(p+1)(a+b)}$$

input
```
Int[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x],x]
```

output
```
(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^2)/(a + b)]*(a + b
*Sin[c + d*x]^2)^(1 + p))/(2*(a + b)*d*(1 + p))
```

3.545.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.545.4 Maple [F]

$$\int \left(a+\left(\sin^{2}\left(dx+c\right)\right)b\right)^{p}\tan\left(dx+c\right)dx$$

input
```
int((a+b*sin(d*x+c)^2)^p*tan(d*x+c),x)
```

output
```
int((a+b*sin(d*x+c)^2)^p*tan(d*x+c),x)
```

3.545.5 Fricas [F]

$$\int \left(a+b\sin^{2}(c+dx)\right)^{p}\tan(c+dx)\,dx=\int \left(b\sin\left(dx+c\right)^{2}+a\right)^{p}\tan\left(dx+c\right)\,dx$$

input
```
integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c),x, algorithm="fricas")
```

output
```
integral((-b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c), x)
```

3.545.6 Sympy [F]

$$\int \left(a+b\sin^{2}(c+dx)\right)^{p}\tan(c+dx)\,dx=\int \left(a+b\sin^{2}\left(c+dx\right)\right)^{p}\tan\left(c+dx\right)dx$$

input
```
integrate((a+b*sin(d*x+c)**2)**p*tan(d*x+c),x)
```

output
```
Integral((a + b*sin(c + d*x)**2)**p*tan(c + d*x), x)
```

3.545.7 Maxima [F]

$$\int (a+b\sin^2(c+dx))^p\tan(c+dx)\,dx=\int \left(b\sin\left(dx+c\right)^2+a\right)^p\tan\left(dx+c\right)\,dx$$

input
```
integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c), x)
```

3.545.8 Giac [F]

$$\int (a+b\sin^2(c+dx))^p\tan(c+dx)\,dx=\int \left(b\sin\left(dx+c\right)^2+a\right)^p\tan\left(dx+c\right)\,dx$$

input
```
integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c), x)
```

3.545.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin^2(c+dx))^p\tan(c+dx)\,dx=\int \tan(c+d\,x)\,\left(b\sin\left(c+d\,x\right)^2+a\right)^p\,dx$$

input
```
int(tan(c + d*x)*(a + b*sin(c + d*x)^2)^p,x)
```

output
```
int(tan(c + d*x)*(a + b*sin(c + d*x)^2)^p, x)
```

3.546 $\int \cot(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$

3.546.1 Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \cot(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sin^2(c+dx)}{a}\right)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2ad(1+p)}$$

output
```
-1/2*hypergeom([1, p+1],[2+p],1+b*sin(d*x+c)^2/a)*(a+b*sin(d*x+c)^2)^(p+1)
/a/d/(p+1)
```

3.546.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \cot(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sin^2(c+dx)}{a}\right)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2ad(1+p)}$$

input
```
Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x]^2)^p,x]
```

output
```
-1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^2)/a]*(a + b*
Sin[c + d*x]^2)^(1 + p))/(a*d*(1 + p))
```

3.546.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3673, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx) \left(a+b\sin^2(c+dx)\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(c+dx)^2\right)^p}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{\int \csc^2(c+dx) \left(b\sin^2(c+dx)+a\right)^p d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{75}$$

$$-\frac{\left(a+b\sin^2(c+dx)\right)^{p+1} \text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sin^2(c+dx)}{a}+1\right)}{2ad(p+1)}$$

input
```
Int[Cot[c + d*x]*(a + b*Sin[c + d*x]^2)^p,x]
```

output
```
-1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^2)/a]*(a + b*
Sin[c + d*x]^2)^(1 + p))/(a*d*(1 + p))
```

3.546.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.546.4 Maple [F]

$$\int \cot\left(dx+c\right)\left(a+\left(\sin^{2}\left(dx+c\right)\right)b\right)^{p}dx$$

input
```
int(cot(d*x+c)*(a+b*sin(d*x+c)^2)^p,x)
```

output
```
int(cot(d*x+c)*(a+b*sin(d*x+c)^2)^p,x)
```

3.546.5 Fricas [F]

$$\int \cot(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx=\int \left(b\sin\left(dx+c\right)^2+a\right)^p\cot\left(dx+c\right)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c), x)
```

3.546.6 Sympy [F]

$$\int \cot(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx=\int \left(a+b\sin^2\left(c+dx\right)\right)^p\cot\left(c+dx\right)dx$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)**2)**p,x)
```

output
```
Integral((a + b*sin(c + d*x)**2)**p*cot(c + d*x), x)
```

3.546.7 Maxima [F]

$$\int \cot(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx=\int \left(b\sin\left(dx+c\right)^2+a\right)^p\cot\left(dx+c\right)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c), x)
```

3.546.8 Giac [F]

$$\int \cot(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx=\int \left(b\sin\left(dx+c\right)^2+a\right)^p\cot\left(dx+c\right)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c), x)
```

3.546.9 Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx=\int \cot(c+d\,x)\,\left(b\sin\left(c+d\,x\right)^2+a\right)^p\,dx$$

input
```
int(cot(c + d*x)*(a + b*sin(c + d*x)^2)^p,x)
```

output
```
int(cot(c + d*x)*(a + b*sin(c + d*x)^2)^p, x)
```

3.547 $\int \cot^3(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$

3.547.1 Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \cot^3(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx = -\frac{\csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2ad} + \frac{(a-bp)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sin^2(c+dx)}{a}\right)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2a^2d(1+p)}$$

output
```
-1/2*csc(d*x+c)^2*(a+b*sin(d*x+c)^2)^(p+1)/a/d+1/2*(-b*p+a)*hypergeom([1,
p+1],[2+p],1+b*sin(d*x+c)^2/a)*(a+b*sin(d*x+c)^2)^(p+1)/a^2/d/(p+1)
```

3.547.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int \cot^3(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx = -\frac{\left(a\csc^2(c+dx)+\frac{(-a+bp)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sin^2(c+dx)}{a}\right)}{1+p}\right)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2a^2d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^2)^p,x]
```

output
```
-1/2*((a*Csc[c + d*x]^2 + ((-a + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1
 + (b*Sin[c + d*x]^2)/a])/(1 + p))*(a + b*Sin[c + d*x]^2)^(1 + p))/(a^2*d)
```

3.547.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3673, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sin(c+dx)^2\right)^p}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{ 3673}$$

$$\frac{\int \csc^4(c+dx)\left(1-\sin^2(c+dx)\right)\left(b\sin^2(c+dx)+a\right)^p d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{-\frac{(a-bp)\int \csc^2(c+dx)\left(b\sin^2(c+dx)+a\right)^p d\sin^2(c+dx)}{a}-\frac{\csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{p+1}}{a}}{2d}$$

$$\Big\downarrow \text{ 75}$$

$$\frac{\frac{(a-bp)\left(a+b\sin^2(c+dx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sin^2(c+dx)}{a}+1\right)}{a^2(p+1)}-\frac{\csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{p+1}}{a}}{2d}$$

input
```
Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^2)^p,x]
```

output
```
(-((Csc[c + d*x]^2*(a + b*Sin[c + d*x]^2)^(1 + p))/a) + ((a - b*p)*Hyperge
ometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^2)/a]*(a + b*Sin[c + d*x]^
2)^(1 + p))/(a^2*(1 + p)))/(2*d)
```

3.547.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.547.4 Maple [F]

$$\int \left(\cot^{3}\left(dx+c\right)\right) \left(a+\left(\sin^{2}\left(dx+c\right)\right) b\right)^{p} dx$$

input
```
int(cot(d*x+c)^3*(a+b*sin(d*x+c)^2)^p,x)
```

output
```
int(cot(d*x+c)^3*(a+b*sin(d*x+c)^2)^p,x)
```

3.547.5 Fricas [F]

$$\int \cot^3(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx=\int \left(b\sin\left(dx+c\right)^2+a\right)^p\cot\left(dx+c\right)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^3, x)
```

3.547.6 Sympy [F(-1)]

Timed out.

$$\int \cot^3(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(cot(d*x+c)**3*(a+b*sin(d*x+c)**2)**p,x)
```

output
```
Timed out
```

3.547.7 Maxima [F]

$$\int \cot^3(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx=\int \left(b\sin\left(dx+c\right)^2+a\right)^p\cot\left(dx+c\right)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^3, x)
```

3.547.8 Giac [F]

$$\int \cot^3(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx=\int \left(b\sin\left(dx+c\right)^2+a\right)^p\cot\left(dx+c\right)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^3, x)
```

3.547.9 Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx=\int \cot(c+d\,x)^3\,\left(b\sin\left(c+d\,x\right)^2+a\right)^p\,dx$$

input
```
int(cot(c + d*x)^3*(a + b*sin(c + d*x)^2)^p,x)
```

output
```
int(cot(c + d*x)^3*(a + b*sin(c + d*x)^2)^p, x)
```

3.548 $\int (a+b\sin^2(c+dx))^p \tan^4(c+dx)\,dx$

3.548.1 Optimal result

Integrand size = 23, antiderivative size = 101

$$\int (a+b\sin^2(c+dx))^p \tan^4(c+dx)\,dx$$
$$= \frac{\operatorname{AppellF1}\left(\frac{5}{2},\frac{5}{2},-p,\frac{7}{2},\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\sin^4(c+dx)\,(a+b\sin^2(c+dx))^p\left(1+\frac{b\ldots}{}\right.}{5d}$$

output
```
1/5*AppellF1(5/2,5/2,-p,7/2,sin(d*x+c)^2,-b*sin(d*x+c)^2/a)*sin(d*x+c)^4*(
a+b*sin(d*x+c)^2)^p*(cos(d*x+c)^2)^(1/2)*tan(d*x+c)/d/((1+b*sin(d*x+c)^2/a
)^p)
```

3.548.2 Mathematica [A] (verified)

Time = 4.62 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int (a+b\sin^2(c+dx))^p \tan^4(c+dx)\,dx$$
$$= \frac{\operatorname{AppellF1}\left(\frac{5}{2},\frac{5}{2},-p,\frac{7}{2},\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\sin^4(c+dx)\,(a+b\sin^2(c+dx))^p\left(\frac{a+b\sin\ldots}{}\right.}{5d}$$

input
```
Integrate[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^4,x]
```

output
```
(AppellF1[5/2, 5/2, -p, 7/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*Sqrt
[Cos[c + d*x]^2]*Sin[c + d*x]^4*(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x])/(5*
d*((a + b*Sin[c + d*x]^2)/a)^p)
```

3.548.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3675, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)\left(a+b\sin^2(c+dx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^4\left(a+b\sin(c+dx)^2\right)^p dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cos^2(c+dx)}\sec(c+dx)\int \frac{\sin^4(c+dx)\left(b\sin^2(c+dx)+a\right)^p}{\left(1-\sin^2(c+dx)\right)^{5/2}}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{395}$$

$$\frac{\sqrt{\cos^2(c+dx)}\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}\int \frac{\sin^4(c+dx)\left(\frac{b\sin^2(c+dx)}{a}+1\right)^p}{\left(1-\sin^2(c+dx)\right)^{5/2}}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{394}$$

$$\frac{\sin^4(c+dx)\sqrt{\cos^2(c+dx)}\tan(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{5}{2},\frac{5}{2},-p,\frac{7}{2},\sin^2(c+d\right.}{5d}$$

input
```
Int[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^4,x]
```

output
```
(AppellF1[5/2, 5/2, -p, 7/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*Sqrt
[Cos[c + d*x]^2]*Sin[c + d*x]^4*(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x])/(5*
d*(1 + (b*Sin[c + d*x]^2)/a)^p)
```

3.548.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.548.4 Maple [F]

$$\int \left(a+\left(\sin^{2}\left(dx+c\right)\right) b\right)^{p} \left(\tan^{4}\left(dx+c\right)\right) dx$$

input
```
int((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^4,x)
```

output
```
int((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^4,x)
```

3.548.5 Fricas [F]

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan^4(c+dx)\,dx=\int \left(b\sin\left(dx+c\right)^2+a\right)^p \tan\left(dx+c\right)^4\,dx$$

input
```
integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
integral((-b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^4, x)
```

3.548.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan^4(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c)**2)**p*tan(d*x+c)**4,x)
```

output
```
Timed out
```

3.548.7 Maxima [F]

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan^4(c+dx)\,dx=\int \left(b\sin\left(dx+c\right)^2+a\right)^p \tan\left(dx+c\right)^4\,dx$$

input
```
integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^4, x)
```

3.548.8 Giac [F]

$$\int (a+b\sin^2(c+dx))^p \tan^4(c+dx)\,dx = \int \left(b\sin\left(dx+c\right)^2+a\right)^p \tan\left(dx+c\right)^4\,dx$$

input
```
integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^4, x)
```

3.548.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin^2(c+dx))^p \tan^4(c+dx)\,dx = \int \tan(c+d\,x)^4\,\left(b\sin\left(c+d\,x\right)^2+a\right)^p\,dx$$

input
```
int(tan(c + d*x)^4*(a + b*sin(c + d*x)^2)^p,x)
```

output
```
int(tan(c + d*x)^4*(a + b*sin(c + d*x)^2)^p, x)
```

3.549 $\int (a+b\sin^2(c+dx))^p\tan^2(c+dx)\,dx$

3.549.1 Optimal result

Integrand size = 23, antiderivative size = 101

$$\int (a+b\sin^2(c+dx))^p\tan^2(c+dx)\,dx = \frac{\operatorname{AppellF1}\left(\frac{3}{2},\frac{3}{2},-p,\frac{5}{2},\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\sin^2(c+dx)\,(a+b\sin^2(c+dx))^p\left(1+\frac{b\cdots}{}\right.}{3d}$$

output
```
1/3*AppellF1(3/2,3/2,-p,5/2,sin(d*x+c)^2,-b*sin(d*x+c)^2/a)*sin(d*x+c)^2*(
a+b*sin(d*x+c)^2)^p*(cos(d*x+c)^2)^(1/2)*tan(d*x+c)/d/((1+b*sin(d*x+c)^2/a
)^p)
```

3.549.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int (a+b\sin^2(c+dx))^p\tan^2(c+dx)\,dx = \frac{\operatorname{AppellF1}\left(\frac{3}{2},\frac{3}{2},-p,\frac{5}{2},\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\sin^2(c+dx)\,(a+b\sin^2(c+dx))^p\left(\frac{a+b\sin\cdots}{}\right.}{3d}$$

input
```
Integrate[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^2,x]
```

output
```
(AppellF1[3/2, 3/2, -p, 5/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*Sqrt
[Cos[c + d*x]^2]*Sin[c + d*x]^2*(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x])/(3*
d*((a + b*Sin[c + d*x]^2)/a)^p)
```

3.549.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3675, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)\left(a+b\sin^2(c+dx)\right)^p dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(c+dx)^2\left(a+b\sin(c+dx)^2\right)^p dx$$

$$\Big\downarrow \text{ 3675}$$

$$\frac{\sqrt{\cos^2(c+dx)}\sec(c+dx)\int \frac{\sin^2(c+dx)\left(b\sin^2(c+dx)+a\right)^p}{\left(1-\sin^2(c+dx)\right)^{3/2}}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 395}$$

$$\frac{\sqrt{\cos^2(c+dx)}\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}\int \frac{\sin^2(c+dx)\left(\frac{b\sin^2(c+dx)}{a}+1\right)^p}{\left(1-\sin^2(c+dx)\right)^{3/2}}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 394}$$

$$\frac{\sin^2(c+dx)\sqrt{\cos^2(c+dx)}\tan(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},\frac{3}{2},-p,\frac{5}{2},\sin^2(c+d$$
$$3d$$

input
```
Int[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^2,x]
```

output
```
(AppellF1[3/2, 3/2, -p, 5/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*Sqrt
[Cos[c + d*x]^2]*Sin[c + d*x]^2*(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x])/(3*
d*(1 + (b*Sin[c + d*x]^2)/a)^p)
```

3.549.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.549.4 Maple [F]

$$\int \left(a+\left(\sin^2\left(dx+c\right)\right)b\right)^p\left(\tan^2\left(dx+c\right)\right)dx$$

input
```
int((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^2,x)
```

output
```
int((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^2,x)
```

3.549.5 Fricas [F]

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan^2(c+dx)\,dx = \int \left(b\sin\left(dx+c\right)^2+a\right)^p \tan\left(dx+c\right)^2\,dx$$

input
```
integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((-b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^2, x)
```

3.549.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c)**2)**p*tan(d*x+c)**2,x)
```

output
```
Timed out
```

3.549.7 Maxima [F]

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan^2(c+dx)\,dx = \int \left(b\sin\left(dx+c\right)^2+a\right)^p \tan\left(dx+c\right)^2\,dx$$

input
```
integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^2, x)
```

3.549.8 Giac [F]

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan^2(c+dx)\,dx = \int \left(b\sin\left(dx+c\right)^2+a\right)^p \tan\left(dx+c\right)^2\,dx$$

input
```
integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^2, x)
```

3.549.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan^2(c+dx)\,dx = \int \tan(c+d\,x)^2\,\left(b\sin\left(c+d\,x\right)^2+a\right)^p\,dx$$

input
```
int(tan(c + d*x)^2*(a + b*sin(c + d*x)^2)^p,x)
```

output
```
int(tan(c + d*x)^2*(a + b*sin(c + d*x)^2)^p, x)
```

3.550 $\int \cot^2(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$

3.550.1 Optimal result

Integrand size = 23, antiderivative size = 97

$$\int \cot^2(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx = -\frac{\operatorname{AppellF1}\left(-\frac{1}{2},-\frac{1}{2},-p,\frac{1}{2},\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\csc(c+dx)\sec(c+dx)\left(a+b\sin^2(\right.}{d}$$

output
```
-AppellF1(-1/2,-1/2,-p,1/2,sin(d*x+c)^2,-b*sin(d*x+c)^2/a)*csc(d*x+c)*sec(
d*x+c)*(a+b*sin(d*x+c)^2)^p*(cos(d*x+c)^2)^(1/2)/d/((1+b*sin(d*x+c)^2/a)^p
)
```

3.550.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

$$\int \cot^2(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx = -\frac{\operatorname{AppellF1}\left(-\frac{1}{2},-\frac{1}{2},-p,\frac{1}{2},\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\csc(c+dx)\sec(c+dx)\left(a+b\sin^2(\right.}{d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^2)^p,x]
```

output
```
-((AppellF1[-1/2, -1/2, -p, 1/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*
Sqrt[Cos[c + d*x]^2]*Csc[c + d*x]*Sec[c + d*x]*(a + b*Sin[c + d*x]^2)^p)/(
d*((a + b*Sin[c + d*x]^2)/a)^p))
```

3.550.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3675, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sin(c+dx)^2\right)^p}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{ 3675}$$

$$\frac{\sqrt{\cos^2(c+dx)}\sec(c+dx)\int \csc^2(c+dx)\sqrt{1-\sin^2(c+dx)}\left(b\sin^2(c+dx)+a\right)^p d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 395}$$

$$\frac{\sqrt{\cos^2(c+dx)}\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}\int \csc^2(c+dx)\sqrt{1-\sin^2(c+dx)}\Big(\frac{b\sin^2(c+dx)}{a}}{d}$$

$$\Big\downarrow \text{ 394}$$

$$-\frac{\sqrt{\cos^2(c+dx)}\csc(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},-\frac{1}{2},-p,\frac{1}{2},\sin^2(\right.}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^2)^p,x]
```

output
```
-((AppellF1[-1/2, -1/2, -p, 1/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*
Sqrt[Cos[c + d*x]^2]*Csc[c + d*x]*Sec[c + d*x]*(a + b*Sin[c + d*x]^2)^p)/(
d*(1 + (b*Sin[c + d*x]^2)/a)^p))
```

3.550.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.550.4 Maple [F]

$$\int \left(\cot^2\left(dx+c\right)\right)\left(a+\left(\sin^2\left(dx+c\right)\right)b\right)^p dx$$

input
```
int(cot(d*x+c)^2*(a+b*sin(d*x+c)^2)^p,x)
```

output
```
int(cot(d*x+c)^2*(a+b*sin(d*x+c)^2)^p,x)
```

3.550.5 Fricas [F]

$$\int \cot^2(c+dx) \left(a+b\sin^2(c+dx)\right)^p \, dx = \int \left(b\sin\left(dx+c\right)^2+a\right)^p \cot\left(dx+c\right)^2 \, dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^2, x)
```

3.550.6 Sympy [F]

$$\int \cot^2(c+dx) \left(a+b\sin^2(c+dx)\right)^p \, dx = \int \left(a+b\sin^2\left(c+dx\right)\right)^p \cot^2\left(c+dx\right) dx$$

input
```
integrate(cot(d*x+c)**2*(a+b*sin(d*x+c)**2)**p,x)
```

output
```
Integral((a + b*sin(c + d*x)**2)**p*cot(c + d*x)**2, x)
```

3.550.7 Maxima [F]

$$\int \cot^2(c+dx) \left(a+b\sin^2(c+dx)\right)^p \, dx = \int \left(b\sin\left(dx+c\right)^2+a\right)^p \cot\left(dx+c\right)^2 \, dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^2, x)
```

3.550.8 Giac [F]

$$\int \cot^2(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx=\int \left(b\sin\left(dx+c\right)^2+a\right)^p\cot\left(dx+c\right)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^2, x)
```

3.550.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx=\int \cot(c+d\,x)^2\,\left(b\sin\left(c+d\,x\right)^2+a\right)^p\,dx$$

input
```
int(cot(c + d*x)^2*(a + b*sin(c + d*x)^2)^p,x)
```

output
```
int(cot(c + d*x)^2*(a + b*sin(c + d*x)^2)^p, x)
```

3.551 $\int \cot^4(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$

3.551.1 Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \cot^4(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx = -\frac{\operatorname{AppellF1}\left(-\frac{3}{2},-\frac{3}{2},-p,-\frac{1}{2},\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\csc^3(c+dx)\sec(c+dx)\left(a+b\sin\right.}{3d}$$

output
```
-1/3*AppellF1(-3/2,-3/2,-p,-1/2,sin(d*x+c)^2,-b*sin(d*x+c)^2/a)*csc(d*x+c)
^3*sec(d*x+c)*(a+b*sin(d*x+c)^2)^p*(cos(d*x+c)^2)^(1/2)/d/((1+b*sin(d*x+c)
^2/a)^p)
```

3.551.2 Mathematica [A] (verified)

Time = 3.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int \cot^4(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx = -\frac{\operatorname{AppellF1}\left(-\frac{3}{2},-\frac{3}{2},-p,-\frac{1}{2},\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\csc^3(c+dx)\sec(c+dx)\left(a+b\sin\right.}{3d}$$

input
```
Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^2)^p,x]
```

output
```
-1/3*(AppellF1[-3/2, -3/2, -p, -1/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/
a)]*Sqrt[Cos[c + d*x]^2]*Csc[c + d*x]^3*Sec[c + d*x]*(a + b*Sin[c + d*x]^2
)^p)/(d*((a + b*Sin[c + d*x]^2)/a)^p)
```

3.551.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3675, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)\left(a+b\sin^2(c+dx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(c+dx)^2\right)^p}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{3675}$$

$$\frac{\sqrt{\cos^2(c+dx)}\sec(c+dx)\int \csc^4(c+dx)\left(1-\sin^2(c+dx)\right)^{3/2}\left(b\sin^2(c+dx)+a\right)^p d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{395}$$

$$\frac{\sqrt{\cos^2(c+dx)}\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}\int \csc^4(c+dx)\left(1-\sin^2(c+dx)\right)^{3/2}\left(\frac{b\sin^2(c+}{a}\right.}{d}$$

$$\Big\downarrow \text{394}$$

$$-\frac{\sqrt{\cos^2(c+dx)}\csc^3(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}\text{AppellF1}\left(-\frac{3}{2},-\frac{3}{2},-p,-\frac{1}{2},\sin\right.}{3d}$$

input
```
Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^2)^p,x]
```

output
```
-1/3*(AppellF1[-3/2, -3/2, -p, -1/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/
a)]*Sqrt[Cos[c + d*x]^2]*Csc[c + d*x]^3*Sec[c + d*x]*(a + b*Sin[c + d*x]^2
)^p)/(d*(1 + (b*Sin[c + d*x]^2)/a)^p)
```

3.551.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3675
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff^(m + 1
)*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]))   Subst[Int[x^m*((a + b*ff^2*x^2)
^p/(1 - ff^2*x^2)^((m + 1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b
, e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

3.551.4 Maple [F]

$$\int \left(\cot^4(dx+c)\right)\left(a+\left(\sin^2(dx+c)\right)b\right)^p dx$$

input
```
int(cot(d*x+c)^4*(a+b*sin(d*x+c)^2)^p,x)
```

output
```
int(cot(d*x+c)^4*(a+b*sin(d*x+c)^2)^p,x)
```

3.551.5 Fricas [F]

$$\int \cot^4(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx=\int \left(b\sin\left(dx+c\right)^2+a\right)^p\cot\left(dx+c\right)^4\,dx$$

input
```
integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^2)^p,x, algorithm="fricas")
```

output
```
integral((-b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^4, x)
```

3.551.6 Sympy [F(-1)]

Timed out.

$$\int \cot^4(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(cot(d*x+c)**4*(a+b*sin(d*x+c)**2)**p,x)
```

output
```
Timed out
```

3.551.7 Maxima [F]

$$\int \cot^4(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx=\int \left(b\sin\left(dx+c\right)^2+a\right)^p\cot\left(dx+c\right)^4\,dx$$

input
```
integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^4, x)
```

3.551.8 Giac [F]

$$\int \cot^4(c+dx) \left(a+b\sin^2(c+dx)\right)^p \, dx = \int \left(b\sin\left(dx+c\right)^2+a\right)^p \cot\left(dx+c\right)^4 \, dx$$

input
```
integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^4, x)
```

3.551.9 Mupad [F(-1)]

Timed out.

$$\int \cot^4(c+dx) \left(a+b\sin^2(c+dx)\right)^p \, dx = \int \cot(c+d\,x)^4 \left(b\sin\left(c+d\,x\right)^2+a\right)^p dx$$

input
```
int(cot(c + d*x)^4*(a + b*sin(c + d*x)^2)^p,x)
```

output
```
int(cot(c + d*x)^4*(a + b*sin(c + d*x)^2)^p, x)
```

3.552 $\int \frac{\cot^3(x)}{a+b\sin^3(x)}\,dx$

3.552.1 Optimal result

Integrand size = 15, antiderivative size = 153

$$\int \frac{\cot^3(x)}{a+b\sin^3(x)}\,dx = \frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}} - \frac{\csc^2(x)}{2a} - \frac{\log(\sin(x))}{a} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)}{3a^{5/3}} + \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(x)+b^{2/3}\sin^2(x)\right)}{6a^{5/3}} + \frac{\log\left(a+b\sin^3(x)\right)}{3a}$$

output
```
-1/2*csc(x)^2/a-ln(sin(x))/a-1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*sin(x))/a^(5/3
)+1/6*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(x)+b^(2/3)*sin(x)^2)/a^(5/3)+
1/3*ln(a+b*sin(x)^3)/a+1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(x))/a
^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

3.552.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.93

$$\int \frac{\cot^3(x)}{a+b\sin^3(x)}\,dx = \frac{-3a^{2/3}\csc^2(x)-6a^{2/3}\log(\sin(x))+2\left(a^{2/3}-(-1)^{2/3}b^{2/3}\right)\log\left(-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\sin(x)\right)+2\left(a^{2/3}-b^{2/}\right.}{6a^{5/3}}$$

input
```
Integrate[Cot[x]^3/(a + b*Sin[x]^3),x]
```

output
```
(-3*a^(2/3)*Csc[x]^2 - 6*a^(2/3)*Log[Sin[x]] + 2*(a^(2/3) - (-1)^(2/3)*b^(
2/3))*Log[-((-1)^(2/3)*a^(1/3)) - b^(1/3)*Sin[x]] + 2*(a^(2/3) - b^(2/3))*
Log[a^(1/3) + b^(1/3)*Sin[x]] + 2*(a^(2/3) + (-1)^(1/3)*b^(2/3))*Log[a^(1/
3) + (-1)^(2/3)*b^(1/3)*Sin[x]])/(6*a^(5/3))
```

3.552.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3709, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(x)}{a+b\sin^3(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(x)^3\left(a+b\sin(x)^3\right)}dx$$

$$\Big\downarrow \text{3709}$$

$$\int \frac{\left(1-\sin^2(x)\right)\csc^3(x)}{a+b\sin^3(x)}d\sin(x)$$

$$\Big\downarrow \text{2373}$$

$$\int \left(\frac{b\left(\sin^2(x)-1\right)}{a\left(a+b\sin^3(x)\right)}+\frac{\csc^3(x)}{a}-\frac{\csc(x)}{a}\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(x)+b^{2/3}\sin^2(x)\right)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)}{3a^{5/3}}+\frac{\log\left(a+b\sin^3(x)\right)}{3a}-\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}$$

input
```
Int[Cot[x]^3/(a + b*Sin[x]^3),x]
```

output
```
(b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(5/3)) - Csc[x]^2/(2*a) - Log[Sin[x]]/a - (b^(2/3)*Log[a^(1/3) + b^(1/3)
*Sin[x]])/(3*a^(5/3)) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[x] + b^
(2/3)*Sin[x]^2])/(6*a^(5/3)) + Log[a + b*Sin[x]^3]/(3*a)
```

3.552.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.552.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.44 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.72

method	result
risch	$\frac{2\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}-1\right)^{2}a}-i\left(\sum_{\_R=\mathrm{RootOf}\left(27\_Z^{3}a^{5}-27ia^{4}\_Z^{2}-9\_Za^{3}+ia^{2}-ib^{2}\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(-\frac{6a^{2}\_R}{b}+\frac{2ia}{b}\right)\mathrm{e}^{ix}-1\right)\right)$
default	$-\frac{1}{2a\sin(x)^{2}}-\frac{\ln(\sin(x))}{a}+\frac{\left(-\frac{\ln\left(\sin(x)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(\sin^{2}(x)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(x)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(x)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(a+b\left(\sin\right.\right.}{3b}\right.}{a}$

input
```
int(cot(x)^3/(a+b*sin(x)^3),x,method=_RETURNVERBOSE)
```

output
```
2*exp(2*I*x)/(exp(2*I*x)-1)^2/a-I*sum(_R*ln(exp(2*I*x)+(-6/b*a^2*_R+2*I/b*
a)*exp(I*x)-1),_R=RootOf(27*_Z^3*a^5-27*I*a^4*_Z^2-9*_Z*a^3+I*a^2-I*b^2))-
1/a*ln(exp(2*I*x)-1)
```

3.552.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 874, normalized size of antiderivative = 5.71

$$\int \frac{\cot^3(x)}{a+b\sin^3(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(x)^3/(a+b*sin(x)^3),x, algorithm="fricas")
```

output
```
-1/12*(2*(a*cos(x)^2 - a)*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1
/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)*log(-1/2*(3*(I*sqrt(3) + 1)*(-1/54/a^3 +
 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)*a^2 - b*sin(x) - a) - (
(a*cos(x)^2 - a)*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2
- b^2)/a^5)^(1/3) - 2/a) + 3*sqrt(1/3)*(a*cos(x)^2 - a)*sqrt(-((3*(I*sqrt(
3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)^2*a
^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5
)^(1/3) - 2/a)*a + 4)/a^2) + 6*cos(x)^2 - 6)*log(1/2*(3*(I*sqrt(3) + 1)*(-
1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)*a^2 + 3/2*sqr
t(1/3)*a^2*sqrt(-((3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2
 - b^2)/a^5)^(1/3) - 2/a)^2*a^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b
^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)*a + 4)/a^2) - 2*b*sin(x) + a)
- ((a*cos(x)^2 - a)*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a
^2 - b^2)/a^5)^(1/3) - 2/a) - 3*sqrt(1/3)*(a*cos(x)^2 - a)*sqrt(-((3*(I*sq
rt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)^
2*a^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/
a^5)^(1/3) - 2/a)*a + 4)/a^2) + 6*cos(x)^2 - 6)*log(-1/2*(3*(I*sqrt(3) + 1
)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)*a^2 + 3/2
*sqrt(1/3)*a^2*sqrt(-((3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*
(a^2 - b^2)/a^5)^(1/3) - 2/a)^2*a^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/a^3 +...
```

3.552.6 Sympy [F]

$$\int \frac{\cot^3(x)}{a + b\sin^3(x)}\,dx = \int \frac{\cot^3(x)}{a + b\sin^3(x)}\,dx$$

input
```
integrate(cot(x)**3/(a+b*sin(x)**3),x)
```

output
```
Integral(cot(x)**3/(a + b*sin(x)**3), x)
```

3.552.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99

$$\int \frac{\cot^3(x)}{a+b\sin^3(x)}\,dx = -\frac{\sqrt{3}\left(b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{2a}{b}\right)+2a\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(x)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2} + \frac{\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)\log\left(\sin(x)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(x)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}-1\right)\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(x)\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log(\sin(x))}{a} - \frac{1}{2a\sin(x)^2}$$

input
```
integrate(cot(x)^3/(a+b*sin(x)^3),x, algorithm="maxima")
```

output
```
-1/9*sqrt(3)*(b*(3*(a/b)^(1/3) - 2*a/b) + 2*a)*arctan(-1/3*sqrt(3)*((a/b)^
(1/3) - 2*sin(x))/(a/b)^(1/3))/a^2 + 1/6*(2*(a/b)^(2/3) + 1)*log(sin(x)^2
- (a/b)^(1/3)*sin(x) + (a/b)^(2/3))/(a*(a/b)^(2/3)) + 1/3*((a/b)^(2/3) - 1
)*log((a/b)^(1/3) + sin(x))/(a*(a/b)^(2/3)) - log(sin(x))/a - 1/2/(a*sin(x
)^2)
```

3.552.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.94

$$\int \frac{\cot^3(x)}{a+b\sin^3(x)}\,dx = \frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(x)\right|\right)}{3a^2} + \frac{\log\left(\left|b\sin(x)^3+a\right|\right)}{3a} - \frac{\log(|\sin(x)|)}{a} - \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(x)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(\sin(x)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(x)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2} - \frac{1}{2a\sin(x)^2}$$

input
```
integrate(cot(x)^3/(a+b*sin(x)^3),x, algorithm="giac")
```

output
```
1/3*b*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(x)))/a^2 + 1/3*log(abs(b*si
n(x)^3 + a))/a - log(abs(sin(x)))/a - 1/3*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/
3*sqrt(3)*((-a/b)^(1/3) + 2*sin(x))/(-a/b)^(1/3))/a^2 - 1/6*(-a*b^2)^(1/3)
*log(sin(x)^2 + (-a/b)^(1/3)*sin(x) + (-a/b)^(2/3))/a^2 - 1/2/(a*sin(x)^2)
```

3.552.9 Mupad [B] (verification not implemented)

Time = 15.39 (sec) , antiderivative size = 2003, normalized size of antiderivative = 13.09

$$\int \frac{\cot^3(x)}{a+b\sin^3(x)}\,dx = \text{Too large to display}$$

input
```
int(cot(x)^3/(a + b*sin(x)^3),x)
```

output
```
symsum(log(-(256*(64*b^7*tan(x/2) + 32*a*b^6 - 44*a^3*b^4 + 15*a^5*b^2 - 1
024*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*b^8*tan(x/2)
^2 - 84*a^2*b^5*tan(x/2) + 26*a^4*b^3*tan(x/2) + 48*a*b^6*tan(x/2)^2 - 16*
root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*a^2*b^6 + 328*ro
ot(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*a^4*b^4 - 165*root
(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*a^6*b^2 - 70*a^3*b^4
*tan(x/2)^2 + 25*a^5*b^2*tan(x/2)^2 - 48*root(27*a^5*e^3 - 27*a^4*e^2 + 9*
a^3*e - a^2 + b^2, e, k)^2*a^3*b^6 - 915*root(27*a^5*e^3 - 27*a^4*e^2 + 9*
a^3*e - a^2 + b^2, e, k)^2*a^5*b^4 + 630*root(27*a^5*e^3 - 27*a^4*e^2 + 9*
a^3*e - a^2 + b^2, e, k)^2*a^7*b^2 + 873*root(27*a^5*e^3 - 27*a^4*e^2 + 9*
a^3*e - a^2 + b^2, e, k)^3*a^6*b^4 - 810*root(27*a^5*e^3 - 27*a^4*e^2 + 9*
a^3*e - a^2 + b^2, e, k)^3*a^8*b^2 + 864*root(27*a^5*e^3 - 27*a^4*e^2 + 9*
a^3*e - a^2 + b^2, e, k)^4*a^7*b^4 - 405*root(27*a^5*e^3 - 27*a^4*e^2 + 9*
a^3*e - a^2 + b^2, e, k)^4*a^9*b^2 - 1296*root(27*a^5*e^3 - 27*a^4*e^2 + 9
*a^3*e - a^2 + b^2, e, k)^5*a^8*b^4 + 1215*root(27*a^5*e^3 - 27*a^4*e^2 +
9*a^3*e - a^2 + b^2, e, k)^5*a^10*b^2 - 608*root(27*a^5*e^3 - 27*a^4*e^2 +
 9*a^3*e - a^2 + b^2, e, k)*a*b^7*tan(x/2) - 8880*root(27*a^5*e^3 - 27*a^4
*e^2 + 9*a^3*e - a^2 + b^2, e, k)^2*a^3*b^6*tan(x/2)^2 + 5067*root(27*a^5*
e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^2*a^5*b^4*tan(x/2)^2 + 1050*
root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^2*a^7*b^2*tan...
```

3.553 $\int \cot(x)\sqrt{a+b\sin^3(x)}\,dx$

3.553.1 Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \cot(x)\sqrt{a+b\sin^3(x)}\,dx = -\frac{2}{3}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)+\frac{2}{3}\sqrt{a+b\sin^3(x)}$$

output
```
-2/3*arctanh((a+b*sin(x)^3)^(1/2)/a^(1/2))*a^(1/2)+2/3*(a+b*sin(x)^3)^(1/2
)
```

3.553.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \cot(x)\sqrt{a+b\sin^3(x)}\,dx = -\frac{2}{3}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)+\frac{2}{3}\sqrt{a+b\sin^3(x)}$$

input
```
Integrate[Cot[x]*Sqrt[a + b*Sin[x]^3],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Sin[x
]^3])/3
```

3.553.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3709, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x)\sqrt{a+b\sin^3(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a+b\sin(x)^3}}{\tan(x)}dx$$

$$\Big\downarrow \text{ 3709}$$

$$\int \csc(x)\sqrt{a+b\sin^3(x)}d\sin(x)$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3}\int \csc(x)\sqrt{b\sin^3(x)+a}d\sin^3(x)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{3}\left(a\int \frac{\csc(x)}{\sqrt{b\sin^3(x)+a}}d\sin^3(x)+2\sqrt{a+b\sin^3(x)}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(\frac{2a\int \frac{1}{\frac{\sin^6(x)}{b}-\frac{a}{b}}d\sqrt{b\sin^3(x)+a}}{b}+2\sqrt{a+b\sin^3(x)}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{3}\left(2\sqrt{a+b\sin^3(x)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)\right)$$

input
```
Int[Cot[x]*Sqrt[a + b*Sin[x]^3],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[x]^3]/Sqrt[a]] + 2*Sqrt[a + b*Sin[x]^3]
)/3
```

3.553.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.553. $\int \cot(x)\sqrt{a+b\sin^3(x)}\,dx$

3.553.4 Maple [F]

$$\int \cot(x)\sqrt{a+b\left(\sin^3(x)\right)}dx$$

input
```
int(cot(x)*(a+b*sin(x)^3)^(1/2),x)
```

output
```
int(cot(x)*(a+b*sin(x)^3)^(1/2),x)
```

3.553.5 Fricas [F(-1)]

Timed out.

$$\int \cot(x)\sqrt{a+b\sin^3(x)}\,dx = \text{Timed out}$$

input
```
integrate(cot(x)*(a+b*sin(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.553.6 Sympy [F]

$$\int \cot(x)\sqrt{a+b\sin^3(x)}\,dx = \int \sqrt{a+b\sin^3(x)}\cot(x)\,dx$$

input
```
integrate(cot(x)*(a+b*sin(x)**3)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(x)**3)*cot(x), x)
```

3.553.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int \cot(x)\sqrt{a+b\sin^3(x)}\,dx = \frac{1}{3}\sqrt{a}\log\left(\frac{\sqrt{b\sin(x)^3+a}-\sqrt{a}}{\sqrt{b\sin(x)^3+a}+\sqrt{a}}\right)+\frac{2}{3}\sqrt{b\sin(x)^3+a}$$

input
```
integrate(cot(x)*(a+b*sin(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(a)*log((sqrt(b*sin(x)^3 + a) - sqrt(a))/(sqrt(b*sin(x)^3 + a) + s
qrt(a))) + 2/3*sqrt(b*sin(x)^3 + a)
```

3.553.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \cot(x)\sqrt{a+b\sin^3(x)}\,dx = \frac{2\,a\arctan\left(\frac{\sqrt{b\sin(x)^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}}+\frac{2}{3}\sqrt{b\sin(x)^3+a}$$

input
```
integrate(cot(x)*(a+b*sin(x)^3)^(1/2),x, algorithm="giac")
```

output
```
2/3*a*arctan(sqrt(b*sin(x)^3 + a)/sqrt(-a))/sqrt(-a) + 2/3*sqrt(b*sin(x)^3
 + a)
```

3.553.9 Mupad [F(-1)]

Timed out.

$$\int \cot(x)\sqrt{a+b\sin^3(x)}\,dx = \int \cot(x)\,\sqrt{b\sin(x)^3+a}\,dx$$

input
```
int(cot(x)*(a + b*sin(x)^3)^(1/2),x)
```

output
```
int(cot(x)*(a + b*sin(x)^3)^(1/2), x)
```

3.554 $\int \frac{\cot(x)}{\sqrt{a+b\sin^3(x)}}\,dx$

3.554.1 Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^3(x)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

output
```
-2/3*arctanh((a+b*sin(x)^3)^(1/2)/a^(1/2))/a^(1/2)
```

3.554.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^3(x)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Integrate[Cot[x]/Sqrt[a + b*Sin[x]^3],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Sin[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

3.554.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3709, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^3(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(x)\sqrt{a+b\sin(x)^3}}dx$$

$$\Big\downarrow \text{3709}$$

$$\int \frac{\csc(x)}{\sqrt{a+b\sin^3(x)}}d\sin(x)$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{\csc(x)}{\sqrt{b\sin^3(x)+a}}d\sin^3(x)$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{\frac{\sin^6(x)}{b}-\frac{a}{b}}d\sqrt{b\sin^3(x)+a}}{3b}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Int[Cot[x]/Sqrt[a + b*Sin[x]^3],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Sin[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

3.554.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.554.4 Maple [F]

$$\int \frac{\cot(x)}{\sqrt{a+b\left(\sin^3(x)\right)}}dx$$

input
```
int(cot(x)/(a+b*sin(x)^3)^(1/2),x)
```

output
```
int(cot(x)/(a+b*sin(x)^3)^(1/2),x)
```

3.554.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^3(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(x)/(a+b*sin(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   fail
ed of mode Union(SparseUnivariatePolynomial(Expression(Complex(Integer))),
failed) cannot be coerced to mode SparseUnivariatePolynomial(Expression(Co
mplex(Int
```

3.554.6 Sympy [F]

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^3(x)}}\,dx = \int \frac{\cot(x)}{\sqrt{a+b\sin^3(x)}}\,dx$$

input
```
integrate(cot(x)/(a+b*sin(x)**3)**(1/2),x)
```

output
```
Integral(cot(x)/sqrt(a + b*sin(x)**3), x)
```

3.554.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^3(x)}}\,dx = \frac{\log\left(\frac{\sqrt{b\sin(x)^3+a}-\sqrt{a}}{\sqrt{b\sin(x)^3+a}+\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
integrate(cot(x)/(a+b*sin(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
1/3*log((sqrt(b*sin(x)^3 + a) - sqrt(a))/(sqrt(b*sin(x)^3 + a) + sqrt(a)))
/sqrt(a)
```

3.554.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^3(x)}}\,dx = \frac{2\arctan\left(\frac{\sqrt{b\sin(x)^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}}$$

input
```
integrate(cot(x)/(a+b*sin(x)^3)^(1/2),x, algorithm="giac")
```

output
```
2/3*arctan(sqrt(b*sin(x)^3 + a)/sqrt(-a))/sqrt(-a)
```

3.554.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^3(x)}}\,dx = \int \frac{\cot(x)}{\sqrt{b\sin(x)^3+a}}\,dx$$

input
```
int(cot(x)/(a + b*sin(x)^3)^(1/2),x)
```

output
```
int(cot(x)/(a + b*sin(x)^3)^(1/2), x)
```

3.555 $\int \cot(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$

3.555.1 Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \cot(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = -\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2d}+\frac{\sqrt{a+b\sin^4(c+dx)}}{2d}$$

output
```
-1/2*arctanh((a+b*sin(d*x+c)^4)^(1/2)/a^(1/2))*a^(1/2)/d+1/2*(a+b*sin(d*x+
c)^4)^(1/2)/d
```

3.555.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \cot(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = \frac{-\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)+\sqrt{a+b\sin^4(c+dx)}}{2d}$$

input
```
Integrate[Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
(-(Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]]) + Sqrt[a + b*Sin[c
 + d*x]^4])/(2*d)
```

3.555.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3708, 243, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin(c+dx)^4}}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{3708}$$

$$\frac{\int \csc^2(c+dx)\sqrt{b\sin^4(c+dx)+a}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \csc^2(c+dx)\sqrt{b\sin^4(c+dx)+a}d\sin^4(c+dx)}{4d}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\int \frac{\csc^2(c+dx)}{\sqrt{b\sin^4(c+dx)+a}}d\sin^4(c+dx)+2\sqrt{a+b\sin^4(c+dx)}}{4d}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2a\int \frac{1}{\frac{\sqrt{b\sin^4(c+dx)+a}}{b}-\frac{a}{b}}d\sqrt{b\sin^4(c+dx)+a}}{b}+2\sqrt{a+b\sin^4(c+dx)}}{4d}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{a+b\sin^4(c+dx)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{4d}$$

input
```
Int[Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]] + 2*Sqrt[a + b*Sin
[c + d*x]^4])/(4*d)
```

3.555.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3708
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^
((m + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1
- ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.555. $\int \cot(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$

3.555.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\frac{\sqrt{a+b\left(\sin^4(dx+c)\right)}}{2}-\frac{\sqrt{a}\ \ln\left(\frac{2a+2\sqrt{a}\ \sqrt{a+b\left(\sin^4(dx+c)\right)}}{\sin(dx+c)^2}\right)}{2}}{d}$	60

input
```
int(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*(a+b*sin(d*x+c)^4)^(1/2)-1/2*a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(d
*x+c)^4)^(1/2))/sin(d*x+c)^2))
```

3.555.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 195, normalized size of antiderivative = 3.31

$$\int \cot(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$$
$$=\left[\frac{\sqrt{a}\log\left(\frac{8\left(b\cos(dx+c)^4-2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}\sqrt{a}+2\,a+b\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}\right)+2\,\sqrt{b\cos\left(dx+c\right)^4-2\,b\cos}}{4\,d}\right.$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a)*log(8*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(
d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*sqrt(a) + 2*a + b)/(cos(d*x + c)^
4 - 2*cos(d*x + c)^2 + 1)) + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2
+ a + b))/d, 1/2*(sqrt(-a)*arctan(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)
^2 + a + b)*sqrt(-a)/a) + sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a +
 b))/d]
```

3.555.6 Sympy [F]

$$\int \cot(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = \int \sqrt{a+b\sin^4{(c+dx)}}\cot{(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)**4)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(c + d*x)**4)*cot(c + d*x), x)
```

3.555.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \cot(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = \frac{\sqrt{a}\log\left(\frac{\sqrt{b\sin(dx+c)^4+a}-\sqrt{a}}{\sqrt{b\sin(dx+c)^4+a}+\sqrt{a}}\right)+2\sqrt{b\sin{(dx+c)}^4+a}}{4\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(sqrt(a)*log((sqrt(b*sin(d*x + c)^4 + a) - sqrt(a))/(sqrt(b*sin(d*x +
c)^4 + a) + sqrt(a))) + 2*sqrt(b*sin(d*x + c)^4 + a))/d
```

3.555.8 Giac [F(-1)]

Timed out.

$$\int \cot(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.555.9 Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = \int \cot(c+d\,x)\,\sqrt{b\sin\left(c+d\,x\right)^4+a}\,dx$$

input
```
int(cot(c + d*x)*(a + b*sin(c + d*x)^4)^(1/2),x)
```

output
```
int(cot(c + d*x)*(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.556 $\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.556.1 Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx=-\frac{a\operatorname{arctanh}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2(a+b)^{3/2}d}+\frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2(a+b)d}$$

output
```
-1/2*a*arctanh((a+b*sin(d*x+c)^2)/(a+b)^(1/2)/(a+b*sin(d*x+c)^4)^(1/2))/(a
+b)^(3/2)/d+1/2*sec(d*x+c)^2*(a+b*sin(d*x+c)^4)^(1/2)/(a+b)/d
```

3.556.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx=-\frac{a\operatorname{arctanh}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2(a+b)^{3/2}d}+\frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2(a+b)d}$$

input
```
Integrate[Tan[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
-1/2*(a*ArcTanh[(a + b*Sin[c + d*x]^2)/(Sqrt[a + b]*Sqrt[a + b*Sin[c + d*x
]^4])])/((a + b)^(3/2)*d) + (Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]^4])/(2
*(a + b)*d)
```

3.556.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3708, 588, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^3}{\sqrt{a+b\sin(c+dx)^4}}dx$$

$$\Big\downarrow \text{3708}$$

$$\frac{\int \frac{\sin^2(c+dx)}{\left(1-\sin^2(c+dx)\right)^2\sqrt{b\sin^4(c+dx)+a}}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{588}$$

$$\frac{\frac{\sqrt{a+b\sin^4(c+dx)}}{(a+b)\left(1-\sin^2(c+dx)\right)}-\frac{a\int \frac{1}{\left(1-\sin^2(c+dx)\right)\sqrt{b\sin^4(c+dx)+a}}d\sin^2(c+dx)}{a+b}}{2d}$$

$$\Big\downarrow \text{488}$$

$$\frac{\frac{a\int \frac{1}{-\sin^4(c+dx)+a+b}d\frac{-b\sin^2(c+dx)-a}{\sqrt{b\sin^4(c+dx)+a}}}{a+b}+\frac{\sqrt{a+b\sin^4(c+dx)}}{(a+b)\left(1-\sin^2(c+dx)\right)}}{2d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{a\text{arctanh}\left(\frac{-a-b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{(a+b)^{3/2}}+\frac{\sqrt{a+b\sin^4(c+dx)}}{(a+b)\left(1-\sin^2(c+dx)\right)}}{2d}$$

input
```
Int[Tan[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

3.556. $\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}dx$

output
```
((a*ArcTanh[(-a - b*Sin[c + d*x]^2)/(Sqrt[a + b]*Sqrt[a + b*Sin[c + d*x]^4
])])/(a + b)^(3/2) + Sqrt[a + b*Sin[c + d*x]^4]/((a + b)*(1 - Sin[c + d*x]
^2)))/(2*d)
```

3.556.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 588
```
Int[(x_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 + a*d^2)))
, x] + Simp[a*(d/(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x]
, x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[Simplify[n + 2*p + 3], 0] && Ne
Q[b*c^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3708
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^
((m + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1
- ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.556.4 Maple [F]

$$\int \frac{\tan^3(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}dx$$

input
```
int(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

output
```
int(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

3.556.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(77) = 154$.

Time = 0.41 (sec) , antiderivative size = 361, normalized size of antiderivative = 4.06

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \left[\frac{\sqrt{a+b}a\cos(dx+c)^2\log\left(\frac{(ab+2b^2)\cos(dx+c)^4-4(ab+b^2)\cos(dx+c)^2+2\sqrt{b\cos(dx+c)^4-2b\cos(dx+c)^2+a+b}\left(b\cos(dx+c)^2-a\right.}{\cos(dx+c)^4}\right.}{4(a^2+2ab+b^2)d\cos(dx+c}\right.$$
$$-\frac{a\sqrt{-a-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)^4-2b\cos(dx+c)^2+a+b}\left(b\cos(dx+c)^2-a-b\right)\sqrt{-a-b}}{(ab+b^2)\cos(dx+c)^4-2(ab+b^2)\cos(dx+c)^2+a^2+2ab+b^2}\right)\cos(dx+c)^2-\sqrt{b\cos(dx+c)}}{2(a^2+2ab+b^2)d\cos(dx+c)^2}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a + b)*a*cos(d*x + c)^2*log(((a*b + 2*b^2)*cos(d*x + c)^4 - 4*(
a*b + b^2)*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 +
 a + b)*(b*cos(d*x + c)^2 - a - b)*sqrt(a + b) + 2*a^2 + 4*a*b + 2*b^2)/co
s(d*x + c)^4) + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(a +
 b))/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^2), -1/2*(a*sqrt(-a - b)*arctan(s
qrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(b*cos(d*x + c)^2 - a -
 b)*sqrt(-a - b)/((a*b + b^2)*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^
2 + a^2 + 2*a*b + b^2))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c)^4 - 2*b*cos(d
*x + c)^2 + a + b)*(a + b))/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^2)]
```

3.556.6 Sympy [F]

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\tan^3{(c+dx)}}{\sqrt{a+b\sin^4{(c+dx)}}}\,dx$$

input
```
integrate(tan(d*x+c)**3/(a+b*sin(d*x+c)**4)**(1/2),x)
```

output
```
Integral(tan(c + d*x)**3/sqrt(a + b*sin(c + d*x)**4), x)
```

3.556.7 Maxima [F]

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\tan{(dx+c)}^3}{\sqrt{b\sin{(dx+c)}^4+a}}\,dx$$

input
```
integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(d*x + c)^3/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.556.8 Giac [F]

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\tan{(dx+c)}^3}{\sqrt{b\sin{(dx+c)}^4+a}}\,dx$$

input
```
integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.556.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^3}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

input
```
int(tan(c + d*x)^3/(a + b*sin(c + d*x)^4)^(1/2),x)
```

output
```
int(tan(c + d*x)^3/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.557 $\int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.557.1 Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2\sqrt{a+b}d}$$

output
```
1/2*arctanh((a+b*sin(d*x+c)^2)/(a+b)^(1/2)/(a+b*sin(d*x+c)^4)^(1/2))/d/(a+
b)^(1/2)
```

3.557.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{a+b-b\cos^2(c+dx)}{\sqrt{a+b}\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}\right)}{2\sqrt{a+b}d}$$

input
```
Integrate[Tan[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
ArcTanh[(a + b - b*Cos[c + d*x]^2)/(Sqrt[a + b]*Sqrt[a + b - 2*b*Cos[c + d
*x]^2 + b*Cos[c + d*x]^4])]/(2*Sqrt[a + b]*d)
```

3.557.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3708, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sin(c+dx)^4}}dx$$

$$\Big\downarrow \text{3708}$$

$$\frac{\int \frac{1}{(1-\sin^2(c+dx))\sqrt{b\sin^4(c+dx)+a}}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{488}$$

$$-\frac{\int \frac{1}{-\sin^4(c+dx)+a+b}d\frac{-b\sin^2(c+dx)-a}{\sqrt{b\sin^4(c+dx)+a}}}{2d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{-a-b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2d\sqrt{a+b}}$$

input
```
Int[Tan[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
-1/2*ArcTanh[(-a - b*Sin[c + d*x]^2)/(Sqrt[a + b]*Sqrt[a + b*Sin[c + d*x]^
4])]/(Sqrt[a + b]*d)
```

3.557.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3708
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^
((m + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1
- ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.557.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.41

method	result	size
default	$\frac{\ln\left(\frac{2a+2b-2b\left(\cos^{2}(dx+c)\right)+2\sqrt{a+b}\,\sqrt{a+b-2b\left(\cos^{2}(dx+c)\right)+b\left(\cos^{4}(dx+c)\right)}}{\cos(dx+c)^{2}}\right)}{2d\sqrt{a+b}}$	72

input
```
int(tan(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d/(a+b)^(1/2)*ln((2*a+2*b-2*b*cos(d*x+c)^2+2*(a+b)^(1/2)*(a+b-2*b*cos(
d*x+c)^2+b*cos(d*x+c)^4)^(1/2))/cos(d*x+c)^2)
```

3.557.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(43) = 86.

Time = 0.39 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.71

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \left[\frac{\log\left(\frac{(ab+2b^2)\cos(dx+c)^4-4(ab+b^2)\cos(dx+c)^2-2\sqrt{b\cos(dx+c)^4-2b\cos(dx+c)^2+a+b}\left(b\cos(dx+c)^2-a-b\right)\sqrt{a+b}+2a^2+4ab+2b^2}{\cos(dx+c)^4}\right)}{4\sqrt{a+b}d}\right.$$

input

```
integrate(tan(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output

```
[1/4*log(((a*b + 2*b^2)*cos(d*x + c)^4 - 4*(a*b + b^2)*cos(d*x + c)^2 - 2*
sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(b*cos(d*x + c)^2 - a
- b)*sqrt(a + b) + 2*a^2 + 4*a*b + 2*b^2)/cos(d*x + c)^4)/(sqrt(a + b)*d),
 1/2*sqrt(-a - b)*arctan(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a +
b)*(b*cos(d*x + c)^2 - a - b)*sqrt(-a - b)/((a*b + b^2)*cos(d*x + c)^4 - 2
*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))/((a + b)*d)]
```

3.557.6 Sympy [F]

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

input

```
integrate(tan(d*x+c)/(a+b*sin(d*x+c)**4)**(1/2),x)
```

output

```
Integral(tan(c + d*x)/sqrt(a + b*sin(c + d*x)**4), x)
```

3.557.7 Maxima [F]

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx=\int \frac{\tan\left(dx+c\right)}{\sqrt{b\sin\left(dx+c\right)^4+a}}\,dx$$

input
```
integrate(tan(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.557.8 Giac [F]

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx=\int \frac{\tan\left(dx+c\right)}{\sqrt{b\sin\left(dx+c\right)^4+a}}\,dx$$

input
```
integrate(tan(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.557.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx=\int \frac{\tan(c+d\,x)}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

input
```
int(tan(c + d*x)/(a + b*sin(c + d*x)^4)^(1/2),x)
```

output
```
int(tan(c + d*x)/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.558 $\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.558.1 Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

output
```
-1/2*arctanh((a+b*sin(d*x+c)^4)^(1/2)/a^(1/2))/d/a^(1/2)
```

3.558.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

input
```
Integrate[Cot[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
-1/2*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]]/(Sqrt[a]*d)
```

3.558.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3708, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(c+dx)\sqrt{a+b\sin(c+dx)^4}}dx$$

$$\Big\downarrow \text{ 3708}$$

$$\frac{\int \frac{\csc^2(c+dx)}{\sqrt{b\sin^4(c+dx)+a}}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\int \frac{\csc^2(c+dx)}{\sqrt{b\sin^4(c+dx)+a}}d\sin^4(c+dx)}{4d}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\int \frac{1}{\frac{\sqrt{b\sin^4(c+dx)+a}}{b}-\frac{a}{b}}d\sqrt{b\sin^4(c+dx)+a}}{2bd}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

input
```
Int[Cot[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
-1/2*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]]/(Sqrt[a]*d)
```

3.558.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3708
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^
((m + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1
- ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.558.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^{4}\left(dx+c\right)\right)}}{\sin\left(dx+c\right)^{2}}\right)}{2d\sqrt{a}}$	42

input
```
int(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(d*x+c)^4)^(1/2))/sin(d*x+c)^2)
```

3.558. $\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.558.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 140, normalized size of antiderivative = 4.00

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \left[\frac{\log\left(\frac{8\left(b\cos(dx+c)^4-2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}\sqrt{a}+2\,a+b\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}\right)}{4\,\sqrt{a}d}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{b\cos(dx+c)^4-2\,b\cos(}{a}\right.}{2\,ad}\right.$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(8*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c)^
4 - 2*b*cos(d*x + c)^2 + a + b)*sqrt(a) + 2*a + b)/(cos(d*x + c)^4 - 2*cos
(d*x + c)^2 + 1))/(sqrt(a)*d), 1/2*sqrt(-a)*arctan(sqrt(b*cos(d*x + c)^4 -
 2*b*cos(d*x + c)^2 + a + b)*sqrt(-a)/a)/(a*d)]
```

3.558.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\cot\left(c+dx\right)}{\sqrt{a+b\sin^4\left(c+dx\right)}}\,dx$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c)**4)**(1/2),x)
```

output
```
Integral(cot(c + d*x)/sqrt(a + b*sin(c + d*x)**4), x)
```

3.558.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\log\left(\frac{\sqrt{b\sin(dx+c)^4+a}-\sqrt{a}}{\sqrt{b\sin(dx+c)^4+a}+\sqrt{a}}\right)}{4\,\sqrt{a}d}$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
1/4*log((sqrt(b*sin(d*x + c)^4 + a) - sqrt(a))/(sqrt(b*sin(d*x + c)^4 + a)
 + sqrt(a)))/(sqrt(a)*d)
```

3.558.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.558.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

input
```
int(cot(c + d*x)/(a + b*sin(c + d*x)^4)^(1/2),x)
```

output
```
int(cot(c + d*x)/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.559 $\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.559.1 Optimal result

Integrand size = 25, antiderivative size = 70

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}d}-\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad}$$

output
```
1/2*arctanh((a+b*sin(d*x+c)^4)^(1/2)/a^(1/2))/d/a^(1/2)-1/2*csc(d*x+c)^2*(
a+b*sin(d*x+c)^4)^(1/2)/a/d
```

3.559.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx=\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)-\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad}$$

input
```
Integrate[Cot[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
(Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]] - Csc[c + d*x]^2*Sqrt
[a + b*Sin[c + d*x]^4])/(2*a*d)
```

3.559.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3708, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(c+dx)^3\sqrt{a+b\sin(c+dx)^4}}dx$$

$$\Big\downarrow \text{ 3708}$$

$$\frac{\int \frac{\csc^4(c+dx)(1-\sin^2(c+dx))}{\sqrt{b\sin^4(c+dx)+a}}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 534}$$

$$\frac{-\int \frac{\csc^2(c+dx)}{\sqrt{b\sin^4(c+dx)+a}}d\sin^2(c+dx)-\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{a}}{2d}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{-\frac{1}{2}\int \frac{\csc^2(c+dx)}{\sqrt{b\sin^4(c+dx)+a}}d\sin^4(c+dx)-\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{a}}{2d}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{-\frac{\int \frac{1}{\frac{\sqrt{b\sin^4(c+dx)+a}}{b}-\frac{a}{b}}d\sqrt{b\sin^4(c+dx)+a}}{b}-\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{a}}{2d}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{a}}{2d}$$

input
```
Int[Cot[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
(ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]]/Sqrt[a] - (Csc[c + d*x]^2*Sqr
t[a + b*Sin[c + d*x]^4])/a)/(2*d)
```

3.559.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3708
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^
((m + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1
- ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.559. $\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.559.4 Maple [F]

$$\int \frac{\cot^3(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}dx$$

input
```
int(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

output
```
int(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

3.559.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.53

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$
$$= \left[\frac{\left(\cos\left(dx+c\right)^2-1\right)\sqrt{a}\log\left(\frac{8\left(b\cos(dx+c)^4-2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}\sqrt{a}+2\,a+b\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}\right)+2\,\sqrt{b\,\mathrm{co}}}{4\left(ad\cos\left(dx+c\right)^2-ad\right)}\right.$$
$$-\frac{\left(\cos\left(dx+c\right)^2-1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}\sqrt{-a}}{a}\right)-\sqrt{b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+\right.}}{2\left(ad\cos\left(dx+c\right)^2-ad\right)}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((cos(d*x + c)^2 - 1)*sqrt(a)*log(8*(b*cos(d*x + c)^4 - 2*b*cos(d*x +
 c)^2 + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*sqrt(a) + 2*
a + b)/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)) + 2*sqrt(b*cos(d*x + c)^4
- 2*b*cos(d*x + c)^2 + a + b))/(a*d*cos(d*x + c)^2 - a*d), -1/2*((cos(d*x
+ c)^2 - 1)*sqrt(-a)*arctan(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a
 + b)*sqrt(-a)/a) - sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b))/(
a*d*cos(d*x + c)^2 - a*d)]
```

3.559.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

input
```
integrate(cot(d*x+c)**3/(a+b*sin(d*x+c)**4)**(1/2),x)
```

output
```
Integral(cot(c + d*x)**3/sqrt(a + b*sin(c + d*x)**4), x)
```

3.559.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\frac{\log\left(\frac{\sqrt{b\sin(dx+c)^4+a}-\sqrt{a}}{\sqrt{b\sin(dx+c)^4+a}+\sqrt{a}}\right)}{\sqrt{a}} + \frac{2\sqrt{b\sin(dx+c)^4+a}}{a\sin(dx+c)^2}}{4\,d}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(log((sqrt(b*sin(d*x + c)^4 + a) - sqrt(a))/(sqrt(b*sin(d*x + c)^4 +
a) + sqrt(a)))/sqrt(a) + 2*sqrt(b*sin(d*x + c)^4 + a)/(a*sin(d*x + c)^2))/
d
```

3.559.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.559.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^3}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

input
```
int(cot(c + d*x)^3/(a + b*sin(c + d*x)^4)^(1/2),x)
```

output
```
int(cot(c + d*x)^3/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.560 $\int \frac{\cot^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.560.1 Optimal result

Integrand size = 25, antiderivative size = 108

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{(2a-b)\text{arctanh}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d} + \frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{ad} - \frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad}$$

output
```
-1/4*(2*a-b)*arctanh((a+b*sin(d*x+c)^4)^(1/2)/a^(1/2))/a^(3/2)/d+csc(d*x+c
)^2*(a+b*sin(d*x+c)^4)^(1/2)/a/d-1/4*csc(d*x+c)^4*(a+b*sin(d*x+c)^4)^(1/2)
/a/d
```

3.560.2 Mathematica [A] (verified)

Time = 3.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.31

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right) - 4a\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)} + b\sqrt{a+b\sin^4(c+dx)}\left(\frac{a\csc^4(c+dx}{b}\right.}{4a^2d}$$

input
```
Integrate[Cot[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
-1/4*(2*a^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]] - 4*a*Csc[c +
d*x]^2*Sqrt[a + b*Sin[c + d*x]^4] + b*Sqrt[a + b*Sin[c + d*x]^4]*((a*Csc[c
 + d*x]^4)/b - ArcTanh[Sqrt[1 + (b*Sin[c + d*x]^4)/a]]/Sqrt[1 + (b*Sin[c +
 d*x]^4)/a]))/(a^2*d)
```

3.560.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3708, 540, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^5\sqrt{a+b\sin(c+dx)^4}}dx$$

$$\Big\downarrow \text{3708}$$

$$\frac{\int \frac{\csc^6(c+dx)\left(1-\sin^2(c+dx)\right)^2}{\sqrt{b\sin^4(c+dx)+a}}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{540}$$

$$\frac{-\frac{\int \frac{\csc^4(c+dx)\left(4a-(2a-b)\sin^2(c+dx)\right)}{\sqrt{b\sin^4(c+dx)+a}}d\sin^2(c+dx)}{2a}-\frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2a}}{2d}$$

$$\Big\downarrow \text{534}$$

$$\frac{-\frac{-(2a-b)\int \frac{\csc^2(c+dx)}{\sqrt{b\sin^4(c+dx)+a}}d\sin^2(c+dx)-4\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2a}-\frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2a}}{2d}$$

$$\Big\downarrow \text{243}$$

$$\frac{-\frac{-\frac{1}{2}(2a-b)\int \frac{\csc^2(c+dx)}{\sqrt{b\sin^4(c+dx)+a}}d\sin^4(c+dx)-4\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2a}-\frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2a}}{2d}$$

$$\Big\downarrow \; 73$$

$$-\frac{-\frac{-\frac{(2a-b)\int \frac{1}{\frac{\sqrt{b\sin^4(c+dx)+a}}{b}-\frac{a}{b}}d\sqrt{b\sin^4(c+dx)+a}}{b}-4\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2a}-\frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2a}}{2d}$$

$$\Big\downarrow \; 221$$

$$-\frac{-\frac{\frac{(2a-b)\text{arctanh}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}}-4\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2a}-\frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2a}}{2d}$$

input
```
Int[Cot[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
(-1/2*(Csc[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]^4])/a - (((2*a - b)*ArcTanh[
Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]])/Sqrt[a] - 4*Csc[c + d*x]^2*Sqrt[a + b
*Sin[c + d*x]^4])/(2*a))/(2*d)
```

3.560.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

3.560. $\int \frac{\cot^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3708
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^
((m + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1
- ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.560.4 Maple [F]

$$\int \frac{\cot^5(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}dx$$

input
```
int(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x)
```

output
```
int(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x)
```

3.560.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 371, normalized size of antiderivative = 3.44

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$
$$= \left[-\frac{\left((2a-b)\cos(dx+c)^4-2(2a-b)\cos(dx+c)^2+2a-b\right)\sqrt{a}\log\left(\frac{8\left(b\cos(dx+c)^4-2b\cos(dx+c)^2+2\sqrt{b\cos(\cdots}\right.}{\cos(dx+c)^4-2\,c\cdots}\right.}{8\left(a^2d\cos(dx+c)^4-\cdots\right.}\right.$$

input
```
integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(((2*a - b)*cos(d*x + c)^4 - 2*(2*a - b)*cos(d*x + c)^2 + 2*a - b)*s
qrt(a)*log(8*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c
)^4 - 2*b*cos(d*x + c)^2 + a + b)*sqrt(a) + 2*a + b)/(cos(d*x + c)^4 - 2*c
os(d*x + c)^2 + 1)) + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b
)*(4*a*cos(d*x + c)^2 - 3*a))/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)
^2 + a^2*d), 1/4*(((2*a - b)*cos(d*x + c)^4 - 2*(2*a - b)*cos(d*x + c)^2 +
 2*a - b)*sqrt(-a)*arctan(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a +
 b)*sqrt(-a)/a) - sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(4*a
*cos(d*x + c)^2 - 3*a))/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a
^2*d)]
```

3.560.6 Sympy [F]

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\cot^5\left(c+dx\right)}{\sqrt{a+b\sin^4\left(c+dx\right)}}\,dx$$

input
```
integrate(cot(d*x+c)**5/(a+b*sin(d*x+c)**4)**(1/2),x)
```

output
```
Integral(cot(c + d*x)**5/sqrt(a + b*sin(c + d*x)**4), x)
```

3.560.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.54

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\frac{2\sqrt{b\sin(dx+c)^4+a}\,b}{\left(b\sin(dx+c)^4+a\right)a-a^2} - \frac{2\log\left(\frac{\sqrt{b\sin(dx+c)^4+a}-\sqrt{a}}{\sqrt{b\sin(dx+c)^4+a}+\sqrt{a}}\right)}{\sqrt{a}} + \frac{b\log\left(\frac{\sqrt{b\sin(dx+c)^4+a}-\sqrt{a}}{\sqrt{b\sin(dx+c)^4+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{8\sqrt{b\sin(dx+c)^4+a}}{a\sin(dx+c)^2}}{8\,d}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*(2*sqrt(b*sin(d*x + c)^4 + a)*b/((b*sin(d*x + c)^4 + a)*a - a^2) - 2*
log((sqrt(b*sin(d*x + c)^4 + a) - sqrt(a))/(sqrt(b*sin(d*x + c)^4 + a) + s
qrt(a)))/sqrt(a) + b*log((sqrt(b*sin(d*x + c)^4 + a) - sqrt(a))/(sqrt(b*si
n(d*x + c)^4 + a) + sqrt(a)))/a^(3/2) - 8*sqrt(b*sin(d*x + c)^4 + a)/(a*si
n(d*x + c)^2))/d
```

3.560.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.560.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^5}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

input
```
int(cot(c + d*x)^5/(a + b*sin(c + d*x)^4)^(1/2),x)
```

output
```
int(cot(c + d*x)^5/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.561 $\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.561.1 Optimal result

Integrand size = 25, antiderivative size = 411

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

$$= \frac{\cos(c+dx)\sin(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{\sqrt{a+b}\,d\sqrt{a+b\sin^4(c+dx)}\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx)\right)}$$

$$- \frac{\sqrt[4]{a}\cos^2(c+dx)E\left(2\arctan\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx)\right)\sqrt{\frac{a+2a\tan\cdots}{(\sqrt{a}\cdots}}}{(a+b)^{3/4}d\sqrt{a+b\sin^4(c+dx)}}$$

$$+ \frac{\sqrt[4]{a}\cos^2(c+dx)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx)\right)\cdots}{2(a+b)^{3/4}d\sqrt{a+b\sin^4(c+dx)}}$$

output
```
-a^(1/4)*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1
/2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticE(sin(2*arctan((
a+b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*((a+2
*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^
(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/(a+b)^(3/4)/d/(a+b*sin(d*x+c)^4)^
(1/2)+1/2*a^(1/4)*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4
)))^2)^(1/2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2
*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/
2))*((a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+
c)^2)^2)^(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/(a+b)^(3/4)/d/(a+b*sin(d
*x+c)^4)^(1/2)+cos(d*x+c)*sin(d*x+c)*(a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^
4)/d/(a+b)^(1/2)/(a+b*sin(d*x+c)^4)^(1/2)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^
2)
```

3.561.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.95 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.71

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx =$$
$$-\frac{2i\sqrt{2}\sqrt{a}\cos^2(c+dx)\left(E\left(i\text{arcsinh}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right)|\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\right.\right.\right.}{\left(\sqrt{a}+i\sqrt{b}\right)\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}d\sqrt{8a+3b-4b\cos(2(}$$

input
```
Integrate[Tan[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
((-2*I)*Sqrt[2]*Sqrt[a]*Cos[c + d*x]^2*(EllipticE[I*ArcSinh[Sqrt[1 - (I*Sq
rt[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])
] - EllipticF[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt
[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])])*Sqrt[1 + (1 - (I*Sqrt[b])/Sqrt[a]
)*Tan[c + d*x]^2]*Sqrt[1 + (1 + (I*Sqrt[b])/Sqrt[a])*Tan[c + d*x]^2])/((Sq
rt[a] + I*Sqrt[b])*Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*d*Sqrt[8*a + 3*b - 4*b*Co
s[2*(c + d*x)] + b*Cos[4*(c + d*x)]])
```

3.561.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3711, 1459, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^2}{\sqrt{a+b\sin(c+dx)^4}}dx$$

$$\Big\downarrow \text{3711}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\int \frac{\tan^2(c+dx)}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}d\tan(c+dx)}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$\Big\downarrow \text{1459}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}d\tan(c+dx)}{\sqrt{a+b}}-\frac{\sqrt{a}\int \frac{\cdots}{\sqrt{a}\sqrt{(\cdots}}\cdots\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}d\tan(c+dx)}{\sqrt{a+b}}-\frac{\int \frac{\sqrt{\cdots}}{\sqrt{(a+b)\tan\cdots}}\cdots\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\left(\frac{\sqrt[4]{a}\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}\text{Ell}\cdots}{2(a+b)^{3/4}\sqrt{(a+b)\tan^4(c+dx)+\cdots}}\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$\Big\downarrow \text{1509}$$

$$\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\left(\frac{\sqrt[4]{a}\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}\,\mathrm{Ell}}{2(a+b)^{3/4}\sqrt{(a+b)\tan^4(c+dx)+}}\right.$$

input
```
Int[Tan[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
(Cos[c + d*x]^2*Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4]*((a^
(1/4)*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (1 - Sqrt[
a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a + 2*a*Ta
n[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]
^2)^2])/(2*(a + b)^(3/4)*Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x
]^4]) - (-((Tan[c + d*x]*Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x
]^4])/(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)) + (a^(1/4)*EllipticE[2*ArcTa
n[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqr
t[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*
Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)^2])/((a + b)^(1/4)*
Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4]))/Sqrt[a + b]))/(d*S
qrt[a + b*Sin[c + d*x]^4])
```

3.561.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.561. $\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3711
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_)*((d_.)*tan[(e_.) + (f_.)*
(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff
*(a + b*Sin[e + f*x]^4)^p*((Sec[e + f*x]^2)^(2*p)/(f*Apart[a*(1 + Tan[e + f
*x]^2)^2 + b*Tan[e + f*x]^4]^p))   Subst[Int[(d*ff*x)^m*(ExpandToSum[a*(1 +
 ff^2*x^2)^2 + b*ff^4*x^4, x]^p/(1 + ff^2*x^2)^(2*p + 1)), x], x, Tan[e + f
*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[p - 1/2]
```

3.561.4 Maple [F]

$$\int \frac{\tan^2(dx+c)}{\sqrt{a+b(\sin^4(dx+c))}}dx$$

input
```
int(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

output
```
int(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

3.561.5 Fricas [F]

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\tan(dx+c)^2}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

input
```
integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
integral(tan(d*x + c)^2/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b
), x)
```

3.561.6 Sympy [F]

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\tan^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**2/(a+b*sin(d*x+c)**4)**(1/2),x)
```

output
```
Integral(tan(c + d*x)**2/sqrt(a + b*sin(c + d*x)**4), x)
```

3.561.7 Maxima [F]

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\tan(dx+c)^2}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

input
```
integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.561.8 Giac [F]

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\tan(dx+c)^2}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

input
```
integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.561.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^2}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

input
```
int(tan(c + d*x)^2/(a + b*sin(c + d*x)^4)^(1/2),x)
```

output
```
int(tan(c + d*x)^2/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.562 $\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.562.1 Optimal result

Integrand size = 16, antiderivative size = 162

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\cos^2(c+dx)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx)\right)\sqrt{\frac{a+2a\cdots}{(\cdots}}}{2\sqrt[4]{a}\sqrt[4]{a+b}\,d\sqrt{a+b\sin^4(c+dx)}}$$

output
```
1/2*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/c
os(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan((a+b)^
(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*((a+2*a*ta
n(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^(1/2)
*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/a^(1/4)/(a+b)^(1/4)/d/(a+b*sin(d*x+c)^
4)^(1/2)
```

3.562.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{2i\cos^2(c+dx)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right),\frac{\sqrt{a}+i\sqrt{b}}{\sqrt{a}-i\sqrt{b}}\right)\sqrt{1+\left(1+\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}\sqrt{2\cdots}}{\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\,d\sqrt{8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx))}}$$

input
```
Integrate[1/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
((-2*I)*Cos[c + d*x]^2*EllipticF[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*T
an[c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])]*Sqrt[1 + (1 + (
I*Sqrt[b])/Sqrt[a])*Tan[c + d*x]^2]*Sqrt[2 + (2 - ((2*I)*Sqrt[b])/Sqrt[a])
*Tan[c + d*x]^2])/(Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*d*Sqrt[8*a + 3*b - 4*b*Co
s[2*(c + d*x)] + b*Cos[4*(c + d*x)]])
```

3.562.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 3689, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a+b\sin(c+dx)^4}}dx$$

$$\Big\downarrow \text{ 3689}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\int \frac{1}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}d\tan(c+dx)}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}d\sqrt[4]{a+b}\sqrt{a+b\sin^4(c+dx)}}$$

input
```
Int[1/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
(Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)],
(1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(
a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Ta
n[c + d*x]^2)^2])/(2*a^(1/4)*(a + b)^(1/4)*d*Sqrt[a + b*Sin[c + d*x]^4])
```

3.562.3.1 Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3689
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff*(a + b*Sin[e + f*x]^4)^p*((Sec[e + f
*x]^2)^(2*p)/(f*(a + 2*a*Tan[e + f*x]^2 + (a + b)*Tan[e + f*x]^4)^p))   Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x]
, x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[p - 1/2
]
```

3.562.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(181) = 362$.

Time = 3.05 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.44

method	result
default	$-\frac{\sqrt{\left(\left(\cos^2(2dx+2c)\right)b+b-2b\cos(2dx+2c)+4a\right)\left(\sin^2(2dx+2c)\right)}\,\sqrt{-ab}\,\sqrt{\frac{\left(-b+\sqrt{-ab}\right)\left(-1+\cos(2dx+2c)\right)}{\sqrt{-ab}\,\left(1+\cos(2dx+2c)\right)}}\,\left(1+\cos(2dx+2c)\right)^2\sqrt{\frac{-b\cos(2d}{\sqrt{-ab}\,(1}}}{\left(-b+\sqrt{-ab}\right)\sqrt{\frac{\left(-1+\cos(2dx+2c)\right)\left(1+\cos(2dx+2c)\right)\left(-b\cos(2dx+2c)+2\sqrt{-ab}+b\right)\left(b\cos(2dx+2c)+2\sqrt{-ab}-b}{b}}}$

input
```
int(1/(a+b*sin(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c)+4*a)*sin(2*d*x+2*c)^2)^(1/2)*(-
a*b)^(1/2)*((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))/(-a*b)^(1/2)/(1+cos(2*d*
x+2*c)))^(1/2)*(1+cos(2*d*x+2*c))^2*((-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)/
(-a*b)^(1/2)/(1+cos(2*d*x+2*c)))^(1/2)*((b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)-b
)/(-a*b)^(1/2)/(1+cos(2*d*x+2*c)))^(1/2)*EllipticF(((-b+(-a*b)^(1/2))*(-1+
cos(2*d*x+2*c))/(-a*b)^(1/2)/(1+cos(2*d*x+2*c)))^(1/2),((b+(-a*b)^(1/2))/(
-b+(-a*b)^(1/2)))^(1/2))/(-b+(-a*b)^(1/2))/(1/b*(-1+cos(2*d*x+2*c))*(1+cos
(2*d*x+2*c))*(-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)*(b*cos(2*d*x+2*c)+2*(-a*
b)^(1/2)-b))^(1/2)/sin(2*d*x+2*c)/(cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c)
+4*a)^(1/2)/d
```

3.562.5 Fricas [F]

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\sin\left(dx+c\right)^4+a}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b), x)
```

3.562.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+b\sin^4\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c)**4)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sin(c + d*x)**4), x)
```

3.562.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sin{(dx+c)}^4+a}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.562.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sin{(dx+c)}^4+a}}\,dx$$

input
```
integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.562.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sin{(c+d\,x)}^4+a}}\,dx$$

input
```
int(1/(a + b*sin(c + d*x)^4)^(1/2),x)
```

output
```
int(1/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.563 $\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

3.563.1 Optimal result

Integrand size = 25, antiderivative size = 477

$$
\begin{aligned}
&\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx \\
&= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} \\
&\quad+\frac{\sqrt{a+b}\cos(c+dx)\sin(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx)\right)} \\
&\quad-\frac{\sqrt[4]{a+b}\cos^2(c+dx)E\left(2\arctan\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx)\right)\sqrt{\frac{a+2\cdots}{}}}{a^{3/4}d\sqrt{a+b\sin^4(c+dx)}} \\
&\quad+\frac{\sqrt[4]{a+b}\cos^2(c+dx)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx\cdots\right.}{2a^{3/4}d\sqrt{a+b\sin^4(c+dx)}}
\end{aligned}
$$

output
```
-(a+b)^(1/4)*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2
)^(1/2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticE(sin(2*arct
an((a+b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*(
(a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)
^2)^(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/a^(3/4)/d/(a+b*sin(d*x+c)^4)^
(1/2)+1/2*(a+b)^(1/4)*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^
(1/4)))^2)^(1/2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(s
in(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))
^(1/2))*((a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(
d*x+c)^2)^2)^(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/a^(3/4)/d/(a+b*sin(d
*x+c)^4)^(1/2)-cos(d*x+c)^2*cot(d*x+c)*(a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c
)^4)/a/d/(a+b*sin(d*x+c)^4)^(1/2)+cos(d*x+c)*sin(d*x+c)*(a+b)^(1/2)*(a+2*a
*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/a/d/(a+b*sin(d*x+c)^4)^(1/2)/(a^(1/2)+(a
+b)^(1/2)*tan(d*x+c)^2)
```

3.563.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.88 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.79

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\sqrt{8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx))}\cot(c+dx)}{2\sqrt{2}ad} - \frac{\cos^4(c+dx)\left(a\sec^4(c+dx)\tan(c+dx)+b\tan^5(c+dx)+\frac{\left(ia+\sqrt{a}\sqrt{b}\right)\left(E\left(i\operatorname{arcsinh}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right)|\frac{\sqrt{a}\cdots}{\sqrt{a}\cdots}\right.\right.}{\cdots}\right.}{ad\sqrt{\cos^4(c+dx)\left(a+2a\tan^2(c\cdots\right.}}$$

input
```
Integrate[Cot[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

output
```
-1/2*(Sqrt[8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]]*Cot[c +
d*x])/(Sqrt[2]*a*d) - (Cos[c + d*x]^4*(a*Sec[c + d*x]^4*Tan[c + d*x] + b*T
an[c + d*x]^5 + ((I*a + Sqrt[a]*Sqrt[b])*(EllipticE[I*ArcSinh[Sqrt[1 - (I*
Sqrt[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b
])] - EllipticF[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]], (Sq
rt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])])*Sec[c + d*x]^2*Sqrt[1 + (1 - (I
*Sqrt[b])/Sqrt[a])*Tan[c + d*x]^2]*Sqrt[1 + (1 + (I*Sqrt[b])/Sqrt[a])*Tan[
c + d*x]^2])/Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]))/(a*d*Sqrt[Cos[c + d*x]^4*(a +
 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)])
```

3.563.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3711, 1443, 27, 1459, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(c+dx)^2\sqrt{a+b\sin(c+dx)^4}}dx$$

$$\Big\downarrow \text{ 3711}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\int \frac{\cot^2(c+dx)}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}d\tan(c+dx)}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$\Big\downarrow \text{ 1443}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\left(\frac{\int \frac{(a+b)\tan^2(c+dx)}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}d\tan(c+dx)}{a}-\frac{\cot(c+dx)\sqrt{(a\cdots}}{\cdots}\cdots\right)}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\left(\frac{(a+b)\int \frac{\tan^2(c+dx)}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}d\tan(c+dx)}{a}-\frac{\cot(c+d\cdots}{\cdots}\cdots\right)}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$\Big\downarrow \text{ 1459}$$

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\left(\frac{(a+b)\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}d\tan(c+dx)}{\sqrt{a+b}}-\frac{\sqrt{a}\int \cdots}{\cdots}\cdots\right)}{a}\cdots\right)}{d\sqrt{a+b\sin^4(c+dx)}}$$

$\downarrow$ 27

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\left(\frac{(a+b)\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}d\tan(c+dx)}{\sqrt{a+b}}-\int\frac{}{\sqrt{(a+}}\right.}{a}\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$\downarrow$ 1416

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\left(\frac{(a+b)\left(\frac{\sqrt[4]{a}\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}}{2(a+b)^{3/4}\sqrt{(a+b)\tan^4(c+}}\right.}{}\right.}{d\sqrt{}}$$

$\downarrow$ 1509

$$\frac{\cos^2(c+dx)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}\left(\frac{(a+b)\left(\frac{\sqrt[4]{a}\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}}{2(a+b)^{3/4}\sqrt{(a+b)\tan^4(c+}}\right.}{}\right.}{}$$

input
```
Int[Cot[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

3.563. $\int\frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

output
```
(Cos[c + d*x]^2*Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4]*(-((
Cot[c + d*x]*Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4])/a) + (
(a + b)*((a^(1/4)*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)]
, (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt
[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*
Tan[c + d*x]^2)^2])/(2*(a + b)^(3/4)*Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)
*Tan[c + d*x]^4]) - (-((Tan[c + d*x]*Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)
*Tan[c + d*x]^4])/(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)) + (a^(1/4)*Ellip
ticE[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (1 - Sqrt[a]/Sqrt[a +
 b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a + 2*a*Tan[c + d*x]^
2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)^2])/((a
 + b)^(1/4)*Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4]))/Sqrt[a
 + b]))/a))/(d*Sqrt[a + b*Sin[c + d*x]^4])
```

3.563.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.563. $\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3711
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_)*((d_.)*tan[(e_.) + (f_.)*
(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff
*(a + b*Sin[e + f*x]^4)^p*((Sec[e + f*x]^2)^(2*p)/(f*Apart[a*(1 + Tan[e + f
*x]^2)^2 + b*Tan[e + f*x]^4]^p))   Subst[Int[(d*ff*x)^m*(ExpandToSum[a*(1 +
 ff^2*x^2)^2 + b*ff^4*x^4, x]^p/(1 + ff^2*x^2)^(2*p + 1)), x], x, Tan[e + f
*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[p - 1/2]
```

3.563.4 Maple [F]

$$\int \frac{\cot^2(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}dx$$

input
```
int(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

output
```
int(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

3.563.5 Fricas [F]

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\cot(dx+c)^2}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

input
```
integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
integral(cot(d*x + c)^2/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b
), x)
```

3.563.6 Sympy [F]

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

input
```
integrate(cot(d*x+c)**2/(a+b*sin(d*x+c)**4)**(1/2),x)
```

output
```
Integral(cot(c + d*x)**2/sqrt(a + b*sin(c + d*x)**4), x)
```

3.563.7 Maxima [F]

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\cot(dx+c)^2}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

input
```
integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.563.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.563.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^2}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

input
```
int(cot(c + d*x)^2/(a + b*sin(c + d*x)^4)^(1/2),x)
```

output
```
int(cot(c + d*x)^2/(a + b*sin(c + d*x)^4)^(1/2), x)
```

3.564 $\int (a+b\sin^4(c+dx))^p \tan^m(c+dx)\,dx$

3.564.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sin^4(c+dx))^p \tan^m(c+dx)\,dx = \text{Int}\big((a+b\sin^4(c+dx))^p \tan^m(c+dx),x\big)$$

output
```
Unintegrable((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x)
```

3.564.2 Mathematica [N/A]

Not integrable

Time = 5.57 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^4(c+dx))^p \tan^m(c+dx)\,dx = \int (a+b\sin^4(c+dx))^p \tan^m(c+dx)\,dx$$

input
```
Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^m,x]
```

output
```
Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^m, x]
```

3.564.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3714}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^m(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan(c+dx)^m\left(a+b\sin(c+dx)^4\right)^p dx$$
$$\Big\downarrow \text{3714}$$
$$\int \tan^m(c+dx)\left(a+b\sin^4(c+dx)\right)^p dx$$

input
```
Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^m,x]
```

output
```
$Aborted
```

3.564.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3714
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sin[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.564.4 Maple [N/A] (verified)

Not integrable

Time = 1.91 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(a+b\left(\sin^4\left(dx+c\right)\right)\right)^p\left(\tan^m\left(dx+c\right)\right)dx$$

input
```
int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x)
```

output
```
int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x)
```

3.564.5 Fricas [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \left(a+b\sin^4(c+dx)\right)^p\tan^m(c+dx)\,dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p\tan\left(dx+c\right)^m\,dx$$

input
```
integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^m,
 x)
```

3.564.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sin^4(c+dx)\right)^p\tan^m(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c)**m,x)
```

output
```
Timed out
```

3.564.7 Maxima [N/A]

Not integrable

Time = 2.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^m(c+dx)\,dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \tan\left(dx+c\right)^m\,dx$$

input
```
integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^m, x)
```

3.564.8 Giac [N/A]

Not integrable

Time = 2.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^m(c+dx)\,dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \tan\left(dx+c\right)^m\,dx$$

input
```
integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^m, x)
```

3.564.9 Mupad [N/A]

Not integrable

Time = 17.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^m(c+dx)\,dx = \int \tan(c+d\,x)^m\,\left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

input
```
int(tan(c + d*x)^m*(a + b*sin(c + d*x)^4)^p,x)
```

output
```
int(tan(c + d*x)^m*(a + b*sin(c + d*x)^4)^p, x)
```

3.565 $\int (a+b\sin^4(c+dx))^p \tan^3(c+dx)\,dx$

3.565.1 Optimal result

Integrand size = 23, antiderivative size = 279

$$\int (a+b\sin^4(c+dx))^p \tan^3(c+dx)\,dx$$
$$= -\frac{(a+b+2bp)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sin^4(c+dx)}{a+b}\right)(a+b\sin^4(c+dx))^{1+p}}{4(a+b)^2d(1+p)}$$
$$+\frac{\sec^2(c+dx)\,(a+b\sin^4(c+dx))^{1+p}}{2(a+b)d}$$
$$-\frac{(a+b+2bp)\,\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\sin^4(c+dx),-\frac{b\sin^4(c+dx)}{a}\right)\sin^2(c+dx)\,(a+b\sin^4(c+dx))^p\left(1+\frac{b\sin^4(c+dx)}{a}\right)^{-p}}{2(a+b)d}$$
$$+\frac{b(1+2p)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\sin^4(c+dx)}{a}\right)\sin^2(c+dx)\,(a+b\sin^4(c+dx))^p\left(1+\frac{b\sin^4(c+dx)}{a}\right)^{-p}}{2(a+b)d}$$

output
```
-1/4*(2*b*p+a+b)*hypergeom([1, p+1],[2+p],(a+b*sin(d*x+c)^4)/(a+b))*(a+b*s
in(d*x+c)^4)^(p+1)/(a+b)^2/d/(p+1)+1/2*sec(d*x+c)^2*(a+b*sin(d*x+c)^4)^(p+
1)/(a+b)/d-1/2*(2*b*p+a+b)*AppellF1(1/2,1,-p,3/2,sin(d*x+c)^4,-b*sin(d*x+c
)^4/a)*sin(d*x+c)^2*(a+b*sin(d*x+c)^4)^p/(a+b)/d/((1+b*sin(d*x+c)^4/a)^p)+
1/2*b*(1+2*p)*hypergeom([1/2, -p],[3/2],-b*sin(d*x+c)^4/a)*sin(d*x+c)^2*(a
+b*sin(d*x+c)^4)^p/(a+b)/d/((1+b*sin(d*x+c)^4/a)^p)
```

3.565.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 810 vs. $2(279) = 558$.

Time = 14.11 (sec) , antiderivative size = 810, normalized size of antiderivative = 2.90

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^3(c+dx)\,dx$$

$$= \frac{\left(-b+\sqrt{-ab}\right)\left(b+\sqrt{-ab}\right)\cos(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(-a+\sqrt{-ab}-(a+b)\tan^2(c+dx)\right)\left(a+\sqrt{\right.}}{}$$

input
```
Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^3,x]
```

output
```
((-b + Sqrt[-(a*b)])*(b + Sqrt[-(a*b)])*Cos[c + d*x]*(a + b*Sin[c + d*x]^4
)^p*(-a + Sqrt[-(a*b)] - (a + b)*Tan[c + d*x]^2)*(a + Sqrt[-(a*b)] + (a +
b)*Tan[c + d*x]^2)*((-2*(-1 + p)*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -(((a
+ b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c + d*x]^2)/(b + S
qrt[-(a*b)])]*Sec[c + d*x]^3)/(2*b*(-1 + p)*AppellF1[1 - 2*p, -p, -p, 2 -
2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c + d*x
]^2)/(b + Sqrt[-(a*b)])] + p*((b + Sqrt[-(a*b)])*AppellF1[2 - 2*p, 1 - p,
-p, 3 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec
[c + d*x]^2)/(b + Sqrt[-(a*b)])] + (b - Sqrt[-(a*b)])*AppellF1[2 - 2*p, -p
, 1 - p, 3 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b
)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])])*Sec[c + d*x]^2) + ((1 - 2*p)^2*Appe
llF1[-2*p, -p, -p, 1 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])
), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])]*Sin[c + d*x])/(p*(b*(-1 +
2*p)*AppellF1[-2*p, -p, -p, 1 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt
[-(a*b)])), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])]*Sin[2*(c + d*x)]
+ 2*p*((b + Sqrt[-(a*b)])*AppellF1[1 - 2*p, 1 - p, -p, 2 - 2*p, -(((a + b)
*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[
-(a*b)])] + (b - Sqrt[-(a*b)])*AppellF1[1 - 2*p, -p, 1 - p, 2 - 2*p, -(((a
 + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c + d*x]^2)/(b +
Sqrt[-(a*b)])])*Tan[c + d*x]))))/(2*(a + b)^2*d*(-1 + 2*p)*(a + 2*a*Tan...
```

3.565.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 3708, 594, 719, 238, 237, 504, 334, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^3\left(a+b\sin(c+dx)^4\right)^p\,dx$$

$$\Big\downarrow \text{3708}$$

$$\frac{\int \frac{\sin^2(c+dx)\left(b\sin^4(c+dx)+a\right)^p}{\left(1-\sin^2(c+dx)\right)^2}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{594}$$

$$\frac{\frac{\left(a+b\sin^4(c+dx)\right)^{p+1}}{(a+b)\left(1-\sin^2(c+dx)\right)}-\frac{\int \frac{\left(b(2p+1)\sin^2(c+dx)+a\right)\left(b\sin^4(c+dx)+a\right)^p}{1-\sin^2(c+dx)}d\sin^2(c+dx)}{a+b}}{2d}$$

$$\Big\downarrow \text{719}$$

$$\frac{\frac{\left(a+b\sin^4(c+dx)\right)^{p+1}}{(a+b)\left(1-\sin^2(c+dx)\right)}-\frac{(a+2bp+b)\int \frac{\left(b\sin^4(c+dx)+a\right)^p}{1-\sin^2(c+dx)}d\sin^2(c+dx)-b(2p+1)\int \left(b\sin^4(c+dx)+a\right)^p d\sin^2(c+dx)}{a+b}}{2d}$$

$$\Big\downarrow \text{238}$$

$$\frac{\frac{\left(a+b\sin^4(c+dx)\right)^{p+1}}{(a+b)\left(1-\sin^2(c+dx)\right)}-\frac{(a+2bp+b)\int \frac{\left(b\sin^4(c+dx)+a\right)^p}{1-\sin^2(c+dx)}d\sin^2(c+dx)-b(2p+1)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}\int \left(\frac{b\sin^4(c+dx)}{a}+1\right)^p d\sin}{a+b}}{2d}$$

$$\Big\downarrow \text{237}$$

$$\frac{\frac{\left(a+b\sin^4(c+dx)\right)^{p+1}}{(a+b)\left(1-\sin^2(c+dx)\right)}-\frac{(a+2bp+b)\int \frac{\left(b\sin^4(c+dx)+a\right)^p}{1-\sin^2(c+dx)}d\sin^2(c+dx)-b(2p+1)\sin^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}\text{Hypergeome}}{a+b}}{2d}$$

$$\Big\downarrow \text{504}$$

$$\frac{(a+b\sin^4(c+dx))^{p+1}}{(a+b)(1-\sin^2(c+dx))}-\frac{\frac{(a+2bp+b)\left(\int\frac{\left(b\sin^4(c+dx)+a\right)^p}{1-\sin^4(c+dx)}d\sin^2(c+dx)+\int\frac{\sin^2(c+dx)\left(b\sin^4(c+dx)+a\right)^p}{1-\sin^4(c+dx)}d\sin^2(c+dx)\right)-b(2p+1)\sin^2(c+dx)(a+b}{a+b}}{2d}$$

$$\Big\downarrow \text{334}$$

$$\frac{(a+b\sin^4(c+dx))^{p+1}}{(a+b)(1-\sin^2(c+dx))}-\frac{(a+2bp+b)\left((a+b\sin^4(c+dx))^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}\int\frac{\left(\frac{b\sin^4(c+dx)}{a}+1\right)^p}{1-\sin^4(c+dx)}d\sin^2(c+dx)+\int\frac{\sin^2(c+dx)\left(b\sin^4(c+dx)+a\right)^p}{1-\sin^4(c+dx)}\right.}{2d}$$

$$\Big\downarrow \text{333}$$

$$\frac{(a+b\sin^4(c+dx))^{p+1}}{(a+b)(1-\sin^2(c+dx))}-\frac{(a+2bp+b)\left(\int\frac{\sin^2(c+dx)\left(b\sin^4(c+dx)+a\right)^p}{1-\sin^4(c+dx)}d\sin^2(c+dx)+\sin^2(c+dx)(a+b\sin^4(c+dx))^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}\text{AppellF1}\right.}{}$$

$$\Big\downarrow \text{353}$$

$$\frac{(a+b\sin^4(c+dx))^{p+1}}{(a+b)(1-\sin^2(c+dx))}-\frac{(a+2bp+b)\left(\frac{1}{2}\int\frac{\left(b\sin^4(c+dx)+a\right)^p}{1-\sin^4(c+dx)}d\sin^4(c+dx)+\sin^2(c+dx)(a+b\sin^4(c+dx))^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},1,-\right.\right.}{2d}$$

$$\Big\downarrow \text{78}$$

$$\frac{(a+b\sin^4(c+dx))^{p+1}}{(a+b)(1-\sin^2(c+dx))}-\frac{(a+2bp+b)\left(\sin^2(c+dx)(a+b\sin^4(c+dx))^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\sin^4(c+dx),-\frac{b\sin^4(c+dx)}{a}\right)+\frac{(a-}{}\right.}{}$$

input
```
Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^3,x]
```

output
```
((a + b*Sin[c + d*x]^4)^(1 + p)/((a + b)*(1 - Sin[c + d*x]^2)) - (-((b*(1
+ 2*p)*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[c + d*x]^4)/a)]*Sin[c + d*
x]^2*(a + b*Sin[c + d*x]^4)^p)/(1 + (b*Sin[c + d*x]^4)/a)^p) + (a + b + 2*
b*p)*((Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^4)/(a + b)]*
(a + b*Sin[c + d*x]^4)^(1 + p))/(2*(a + b)*(1 + p)) + (AppellF1[1/2, 1, -p
, 3/2, Sin[c + d*x]^4, -((b*Sin[c + d*x]^4)/a)]*Sin[c + d*x]^2*(a + b*Sin[
c + d*x]^4)^p)/(1 + (b*Sin[c + d*x]^4)/a)^p))/(a + b))/(2*d)
```

3.565. $\int(a+b\sin^4(c+dx))^p\tan^3(c+dx)\,dx$

3.565.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && !IntegerQ[m] && IntegerQ[n]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

3.565. $\int (a+b\sin^4(c+dx))^p\tan^3(c+dx)\,dx$

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3708
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^
((m + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1
- ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.565.4 Maple [F]

$$\int \left(a+b\left(\sin^{4}\left(dx+c\right)\right)\right)^{p}\left(\tan^{3}\left(dx+c\right)\right)dx$$

input
```
int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x)
```

output
```
int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x)
```

3.565.5 Fricas [F]

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^3(c+dx)\,dx=\int \left(b\sin\left(dx+c\right)^4+a\right)^p \tan\left(dx+c\right)^3\,dx$$

input
```
integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^3,
 x)
```

3.565.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^3(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c)**3,x)
```

output
```
Timed out
```

3.565.7 Maxima [F]

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^3(c+dx)\,dx=\int \left(b\sin\left(dx+c\right)^4+a\right)^p \tan\left(dx+c\right)^3\,dx$$

input
```
integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^3, x)
```

3.565.8 Giac [F]

$$\int (a+b\sin^4(c+dx))^p \tan^3(c+dx)\,dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \tan\left(dx+c\right)^3\,dx$$

input
```
integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^3, x)
```

3.565.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin^4(c+dx))^p \tan^3(c+dx)\,dx = \int \tan(c+d\,x)^3\,\left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

input
```
int(tan(c + d*x)^3*(a + b*sin(c + d*x)^4)^p,x)
```

output
```
int(tan(c + d*x)^3*(a + b*sin(c + d*x)^4)^p, x)
```

3.566 $\int (a+b\sin^4(c+dx))^p \tan(c+dx)\,dx$

3.566.1 Optimal result

Integrand size = 21, antiderivative size = 141

$$\int (a+b\sin^4(c+dx))^p \tan(c+dx)\,dx = \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sin^4(c+dx)}{a+b}\right)(a+b\sin^4(c+dx))^{1+p}}{4(a+b)d(1+p)} + \frac{\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\sin^4(c+dx),-\frac{b\sin^4(c+dx)}{a}\right)\sin^2(c+dx)(a+b\sin^4(c+dx))^p\left(1+\frac{b\sin^4(c+dx)}{a}\right)^{-p}}{2d}$$

output
```
1/4*hypergeom([1, p+1],[2+p],(a+b*sin(d*x+c)^4)/(a+b))*(a+b*sin(d*x+c)^4)^
(p+1)/(a+b)/d/(p+1)+1/2*AppellF1(1/2,1,-p,3/2,sin(d*x+c)^4,-b*sin(d*x+c)^4
/a)*sin(d*x+c)^2*(a+b*sin(d*x+c)^4)^p/d/((1+b*sin(d*x+c)^4/a)^p)
```

3.566.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 463 vs. 2(141) = 282.

Time = 9.51 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.28

$$\int (a+b\sin^4(c+dx))^p \tan(c+dx)\,dx = \frac{2(-b+\sqrt{-ab})(b+\sqrt{-ab})(-1+2p)\,\text{AppellF1}\left(-2p\right.}{(a+b)^2dp\left(\left(b+\sqrt{-ab}\right)p\,\text{AppellF1}\left(1-2p,1-p,-p,2-2p,-\frac{(a+b)\sec^2(c+dx)}{-b+\sqrt{-ab}},\frac{(a+b)\sec^2(c+dx)}{b+\sqrt{-ab}}\right)-\left(-b+\right.\right.}$$

input
```
Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x],x]
```

output
```
(2*(-b + Sqrt[-(a*b)])*(b + Sqrt[-(a*b)])*(-1 + 2*p)*AppellF1[-2*p, -p, -p
, 1 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c
 + d*x]^2)/(b + Sqrt[-(a*b)])]*Cos[c + d*x]^2*(a + b + (a - Sqrt[-(a*b)])*
Cot[c + d*x]^2)*(a + b + (a + Sqrt[-(a*b)])*Cot[c + d*x]^2)*Sin[c + d*x]^4
*(a + b*Sin[c + d*x]^4)^p)/((a + b)^2*d*p*((b + Sqrt[-(a*b)])*p*AppellF1[1
 - 2*p, 1 - p, -p, 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])
), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])] - (-b + Sqrt[-(a*b)])*p*Ap
pellF1[1 - 2*p, -p, 1 - p, 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[
-(a*b)])), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])] + b*(-1 + 2*p)*App
ellF1[-2*p, -p, -p, 1 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)]
)), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])]*Cos[c + d*x]^2)*(8*a + 3*
b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]))
```

3.566.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3708, 504, 334, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)\left(a+b\sin^4(c+dx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)\left(a+b\sin(c+dx)^4\right)^p dx$$

$$\Big\downarrow \text{3708}$$

$$\frac{\int \frac{\left(b\sin^4(c+dx)+a\right)^p}{1-\sin^2(c+dx)}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{504}$$

$$\frac{\int \frac{\left(b\sin^4(c+dx)+a\right)^p}{1-\sin^4(c+dx)}d\sin^2(c+dx)+\int \frac{\sin^2(c+dx)\left(b\sin^4(c+dx)+a\right)^p}{1-\sin^4(c+dx)}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{334}$$

$$\frac{(a+b\sin^4(c+dx))^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}\int\frac{\left(\frac{b\sin^4(c+dx)}{a}+1\right)^p}{1-\sin^4(c+dx)}d\sin^2(c+dx)+\int\frac{\sin^2(c+dx)(b\sin^4(c+dx)+a)^p}{1-\sin^4(c+dx)}d\sin^2(c+d}{2d}$$

$\downarrow$ 333

$$\frac{\int\frac{\sin^2(c+dx)(b\sin^4(c+dx)+a)^p}{1-\sin^4(c+dx)}d\sin^2(c+dx)+\sin^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},1,-\right.}{2d}$$

$\downarrow$ 353

$$\frac{\frac{1}{2}\int\frac{(b\sin^4(c+dx)+a)^p}{1-\sin^4(c+dx)}d\sin^4(c+dx)+\sin^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\text{s}\right.}{2d}$$

$\downarrow$ 78

$$\frac{\sin^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\sin^4(c+dx),-\frac{b\sin^4(c+dx)}{a}\right)+\frac{(a+b\sin^4}{}}{2d}$$

input
```
Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x],x]
```

output
```
((Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^4)/(a + b)]*(a +
b*Sin[c + d*x]^4)^(1 + p))/(2*(a + b)*(1 + p)) + (AppellF1[1/2, 1, -p, 3/2
, Sin[c + d*x]^4, -((b*Sin[c + d*x]^4)/a)]*Sin[c + d*x]^2*(a + b*Sin[c + d
*x]^4)^p)/(1 + (b*Sin[c + d*x]^4)/a)^p)/(2*d)
```

3.566.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3708
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^
((m + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1
- ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.566.4 Maple [F]

$$\int \left(a+b\left(\sin^4\left(dx+c\right)\right)\right)^p \tan\left(dx+c\right)dx$$

input
```
int((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x)
```

output
```
int((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x)
```

3.566.5 Fricas [F]

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan(c+dx)\,dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \tan\left(dx+c\right)\,dx$$

input
```
integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c), x
)
```

3.566.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c),x)
```

output
```
Timed out
```

3.566.7 Maxima [F]

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan(c+dx)\,dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \tan\left(dx+c\right)\,dx$$

input
```
integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c), x)
```

3.566.8 Giac [F]

$$\int (a+b\sin^4(c+dx))^p \tan(c+dx)\,dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \tan\left(dx+c\right)\,dx$$

input
```
integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c), x)
```

3.566.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin^4(c+dx))^p \tan(c+dx)\,dx = \int \tan(c+d\,x)\,\left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

input
```
int(tan(c + d*x)*(a + b*sin(c + d*x)^4)^p,x)
```

output
```
int(tan(c + d*x)*(a + b*sin(c + d*x)^4)^p, x)
```

3.567 $\int \cot(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$

3.567.1 Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \cot(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sin^4(c+dx)}{a}\right)\left(a+b\sin^4(c+dx)\right)^{1+p}}{4ad(1+p)}$$

output
```
-1/4*hypergeom([1, p+1],[2+p],1+b*sin(d*x+c)^4/a)*(a+b*sin(d*x+c)^4)^(p+1)
/a/d/(p+1)
```

3.567.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \cot(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sin^4(c+dx)}{a}\right)\left(a+b\sin^4(c+dx)\right)^{1+p}}{4ad(1+p)}$$

input
```
Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x]^4)^p,x]
```

output
```
-1/4*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^4)/a]*(a + b*
Sin[c + d*x]^4)^(1 + p))/(a*d*(1 + p))
```

3.567.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3708, 243, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(c+dx)^4\right)^p}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{3708}$$

$$\frac{\int \csc^2(c+dx)\left(b\sin^4(c+dx)+a\right)^p d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \csc^2(c+dx)\left(b\sin^4(c+dx)+a\right)^p d\sin^4(c+dx)}{4d}$$

$$\Big\downarrow \text{75}$$

$$-\frac{\left(a+b\sin^4(c+dx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sin^4(c+dx)}{a}+1\right)}{4ad(p+1)}$$

input

```
Int[Cot[c + d*x]*(a + b*Sin[c + d*x]^4)^p,x]
```

output

```
-1/4*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^4)/a]*(a + b*
Sin[c + d*x]^4)^(1 + p))/(a*d*(1 + p))
```

3.567.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3708
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^
((m + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1
- ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.567.4 Maple [F]

$$\int \cot (dx+c) \left(a+b\left(\sin^4 (dx+c)\right)\right)^p dx$$

input
```
int(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x)
```

output
```
int(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x)
```

3.567.5 Fricas [F]

$$\int \cot(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \cot\left(dx+c\right) \, dx$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c), x
)
```

3.567.6 Sympy [F(-1)]

Timed out.

$$\int \cot(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)**4)**p,x)
```

output
```
Timed out
```

3.567.7 Maxima [F]

$$\int \cot(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \cot\left(dx+c\right) \, dx$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c), x)
```

3.567.8 Giac [F]

$$\int \cot(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \cot\left(dx+c\right) \, dx$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c), x)
```

3.567.9 Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx = \int \cot(c+d\,x)\,\left(b\sin\left(c+d\,x\right)^4+a\right)^p \, dx$$

input
```
int(cot(c + d*x)*(a + b*sin(c + d*x)^4)^p,x)
```

output
```
int(cot(c + d*x)*(a + b*sin(c + d*x)^4)^p, x)
```

3.568 $\int \cot^3(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx$

3.568.1 Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \cot^3(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx = \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sin^4(c+dx)}{a}\right)\left(a+b\sin^4(c+dx)\right)^{1+p}}{4ad(1+p)} - \frac{\csc^2(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sin^4(c+dx)}{a}\right)\left(a+b\sin^4(c+dx)\right)^p\left(1+\frac{b\sin^4(c+dx)}{a}\right)^{-p}}{2d}$$

output
```
1/4*hypergeom([1, p+1],[2+p],1+b*sin(d*x+c)^4/a)*(a+b*sin(d*x+c)^4)^(p+1)/
a/d/(p+1)-1/2*csc(d*x+c)^2*hypergeom([-1/2, -p],[1/2],-b*sin(d*x+c)^4/a)*(
a+b*sin(d*x+c)^4)^p/d/((1+b*sin(d*x+c)^4/a)^p)
```

3.568.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94

$$\int \cot^3(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx = \frac{\left(a+b\sin^4(c+dx)\right)^p \left(\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sin^4(c+dx)}{a}\right)\left(a+b\sin^4(c+dx)\right)}{a(1+p)} - 2\csc^2(c+dx)\,\text{Hypergeometri}\right.}{4d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^4)^p,x]
```

output
```
((a + b*Sin[c + d*x]^4)^p*((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[
c + d*x]^4)/a]*(a + b*Sin[c + d*x]^4))/(a*(1 + p)) - (2*Csc[c + d*x]^2*Hyp
ergeometric2F1[-1/2, -p, 1/2, -((b*Sin[c + d*x]^4)/a)])/(1 + (b*Sin[c + d*
x]^4)/a)^p))/(4*d)
```

3.568.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 3708, 542, 243, 75, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(c+dx)^4\right)^p}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{3708}$$

$$\frac{\int \csc^4(c+dx)\left(1-\sin^2(c+dx)\right)\left(b\sin^4(c+dx)+a\right)^p d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{542}$$

$$\frac{\int \csc^4(c+dx)\left(b\sin^4(c+dx)+a\right)^p d\sin^2(c+dx)-\int \csc^2(c+dx)\left(b\sin^4(c+dx)+a\right)^p d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \csc^4(c+dx)\left(b\sin^4(c+dx)+a\right)^p d\sin^2(c+dx)-\frac{1}{2}\int \csc^2(c+dx)\left(b\sin^4(c+dx)+a\right)^p d\sin^4(c+dx)}{2d}$$

$$\Big\downarrow \text{75}$$

$$\frac{\int \csc^4(c+dx)\left(b\sin^4(c+dx)+a\right)^p d\sin^2(c+dx)+\frac{\left(a+b\sin^4(c+dx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sin^4(c+dx)}{a}+1\right)}{2a(p+1)}}{2d}$$

$$\Big\downarrow \text{279}$$

$$\frac{(a+b\sin^4(c+dx))^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}\int \csc^4(c+dx)\left(\frac{b\sin^4(c+dx)}{a}+1\right)^p d\sin^2(c+dx)+\frac{(a+b\sin^4(c+dx))^{p+1}\,\text{Hyper}}{}}{2d}$$

$$\Big\downarrow \text{278}$$

$$\frac{\frac{(a+b\sin^4(c+dx))^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sin^4(c+dx)}{a}+1\right)}{2a(p+1)}-\csc^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}}{2d}$$

input
```
Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^4)^p,x]
```

output
```
((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^4)/a]*(a + b*Sin[
c + d*x]^4)^(1 + p))/(2*a*(1 + p)) - (Csc[c + d*x]^2*Hypergeometric2F1[-1/
2, -p, 1/2, -((b*Sin[c + d*x]^4)/a)]*(a + b*Sin[c + d*x]^4)^p)/(1 + (b*Sin
[c + d*x]^4)/a)^p)/(2*d)
```

3.568.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 542
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[c   Int[x^m*(a + b*x^2)^p, x], x] + Simp[d   Int[x^(m + 1)*(a + b*x^2
)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3708
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^
((m + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1
- ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f,
p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.568.4 Maple [F]

$$\int \left(\cot^{3}\left(dx+c\right)\right) \left(a+b\left(\sin^{4}\left(dx+c\right)\right)\right)^{p}dx$$

input
```
int(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x)
```

output
```
int(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x)
```

3.568.5 Fricas [F]

$$\int \cot^3(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx = \int \left(b\sin\left(dx+c\right)^{4}+a\right)^{p}\cot\left(dx+c\right)^{3} \, dx$$

input
```
integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^3,
 x)
```

3.568.6 Sympy [F(-1)]

Timed out.

$$\int \cot^3(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**3*(a+b*sin(d*x+c)**4)**p,x)
```

output
```
Timed out
```

3.568.7 Maxima [F]

$$\int \cot^3(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \cot\left(dx+c\right)^3 \, dx$$

input
```
integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^3, x)
```

3.568.8 Giac [F]

$$\int \cot^3(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \cot\left(dx+c\right)^3 \, dx$$

input
```
integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^3, x)
```

3.568.9 Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx=\int \cot(c+d\,x)^3\left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

input
```
int(cot(c + d*x)^3*(a + b*sin(c + d*x)^4)^p,x)
```

output
```
int(cot(c + d*x)^3*(a + b*sin(c + d*x)^4)^p, x)
```

3.569 $\int (a+b\sin^4(c+dx))^p \tan^4(c+dx)\,dx$

3.569.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sin^4(c+dx))^p \tan^4(c+dx)\,dx = \text{Int}\left((a+b\sin^4(c+dx))^p \tan^4(c+dx),x\right)$$

output
```
Unintegrable((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x)
```

3.569.2 Mathematica [N/A]

Not integrable

Time = 21.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^4(c+dx))^p \tan^4(c+dx)\,dx = \int (a+b\sin^4(c+dx))^p \tan^4(c+dx)\,dx$$

input
```
Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^4,x]
```

output
```
Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^4, x]
```

3.569.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3714}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(c+dx)^4 \left(a+b\sin(c+dx)^4\right)^p dx$$

$$\Big\downarrow \text{ 3714}$$

$$\int \tan^4(c+dx) \left(a+b\sin^4(c+dx)\right)^p dx$$

input
```
Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^4,x]
```

output
```
$Aborted
```

3.569.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3714
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sin[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.569.4 Maple [N/A] (verified)

Not integrable

Time = 1.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(a+b\left(\sin^{4}\left(dx+c\right)\right)\right)^{p}\left(\tan^{4}\left(dx+c\right)\right) dx$$

input
```
int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x)
```

output
```
int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x)
```

3.569.5 Fricas [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \left(a+b\sin^4(c+dx)\right)^p\tan^4(c+dx)\,dx=\int \left(b\sin\left(dx+c\right)^{4}+a\right)^{p}\tan\left(dx+c\right)^{4}\,dx$$

input
```
integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^4,
 x)
```

3.569.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sin^4(c+dx)\right)^p\tan^4(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c)**4,x)
```

output
```
Timed out
```

3.569.7 Maxima [N/A]

Not integrable

Time = 14.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^4(c+dx))^p \tan^4(c+dx)\,dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \tan\left(dx+c\right)^4\,dx$$

input
```
integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^4, x)
```

3.569.8 Giac [N/A]

Not integrable

Time = 3.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^4(c+dx))^p \tan^4(c+dx)\,dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \tan\left(dx+c\right)^4\,dx$$

input
```
integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^4, x)
```

3.569.9 Mupad [N/A]

Not integrable

Time = 18.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^4(c+dx))^p \tan^4(c+dx)\,dx = \int \tan(c+d\,x)^4\left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

input
```
int(tan(c + d*x)^4*(a + b*sin(c + d*x)^4)^p,x)
```

output
```
int(tan(c + d*x)^4*(a + b*sin(c + d*x)^4)^p, x)
```

3.570 $\int (a+b\sin^4(c+dx))^p \tan^2(c+dx)\,dx$

3.570.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sin^4(c+dx))^p \tan^2(c+dx)\,dx = \text{Int}\left((a+b\sin^4(c+dx))^p \tan^2(c+dx),x\right)$$

output
```
Unintegrable((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x)
```

3.570.2 Mathematica [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^4(c+dx))^p \tan^2(c+dx)\,dx = \int (a+b\sin^4(c+dx))^p \tan^2(c+dx)\,dx$$

input
```
Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^2,x]
```

output
```
Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^2, x]
```

3.570.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3714}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(c+dx)^2\left(a+b\sin(c+dx)^4\right)^p\,dx$$

$$\Big\downarrow \ 3714$$

$$\int \tan^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

input
```
Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^2,x]
```

output
```
$Aborted
```

3.570.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3714
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sin[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.570.4 Maple [N/A] (verified)

Not integrable

Time = 1.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(a+b\left(\sin^4\left(dx+c\right)\right)\right)^p \left(\tan^2\left(dx+c\right)\right) dx$$

input
```
int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x)
```

output
```
int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x)
```

3.570.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^2(c+dx)\,dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \tan\left(dx+c\right)^2\,dx$$

input
```
integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^2,
 x)
```

3.570.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c)**2,x)
```

output
```
Timed out
```

3.570.7 Maxima [N/A]

Not integrable

Time = 5.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^2(c+dx)\,dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \tan\left(dx+c\right)^2\,dx$$

input
```
integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^2, x)
```

3.570.8 Giac [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^2(c+dx)\,dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \tan\left(dx+c\right)^2\,dx$$

input
```
integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^2, x)
```

3.570.9 Mupad [N/A]

Not integrable

Time = 17.63 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^2(c+dx)\,dx = \int \tan(c+d\,x)^2\,\left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

input
```
int(tan(c + d*x)^2*(a + b*sin(c + d*x)^4)^p,x)
```

output
```
int(tan(c + d*x)^2*(a + b*sin(c + d*x)^4)^p, x)
```

3.571 $\int (a+b\sin^4(c+dx))^p\,dx$

3.571.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (a+b\sin^4(c+dx))^p\,dx = \text{Int}\left((a+b\sin^4(c+dx))^p,x\right)$$

output
```
Unintegrable((a+b*sin(d*x+c)^4)^p,x)
```

3.571.2 Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sin^4(c+dx))^p\,dx = \int (a+b\sin^4(c+dx))^p\,dx$$

input
```
Integrate[(a + b*Sin[c + d*x]^4)^p,x]
```

output
```
Integrate[(a + b*Sin[c + d*x]^4)^p, x]
```

3.571.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3693}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sin^4(c+dx)\right)^p\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a+b\sin(c+dx)^4\right)^p\,dx$$

$$\Big\downarrow \ 3693$$

$$\int \left(a+b\sin^4(c+dx)\right)^p\,dx$$

input
```
Int[(a + b*Sin[c + d*x]^4)^p,x]
```

output
```
$Aborted
```

3.571.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3693
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Unintegrable[(a + b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, e, f, n, p
}, x]
```

3.571.4 Maple [N/A] (verified)

Not integrable

Time = 0.94 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a+b\left(\sin^4\left(dx+c\right)\right)\right)^p dx$$

input
```
int((a+b*sin(d*x+c)^4)^p,x)
```

output
```
int((a+b*sin(d*x+c)^4)^p,x)
```

3.571.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \left(a+b\sin^4(c+dx)\right)^p \, dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \, dx$$

input
```
integrate((a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p, x)
```

3.571.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sin^4(c+dx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c)**4)**p,x)
```

output
```
Timed out
```

3.571.7 Maxima [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \left(a+b\sin^4(c+dx)\right)^p\,dx=\int \left(b\sin\left(dx+c\right)^4+a\right)^p\,dx$$

input
```
integrate((a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p, x)
```

3.571.8 Giac [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \left(a+b\sin^4(c+dx)\right)^p\,dx=\int \left(b\sin\left(dx+c\right)^4+a\right)^p\,dx$$

input
```
integrate((a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p, x)
```

3.571.9 Mupad [N/A]

Not integrable

Time = 14.78 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \left(a+b\sin^4(c+dx)\right)^p\,dx=\int \left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

input
```
int((a + b*sin(c + d*x)^4)^p,x)
```

output
```
int((a + b*sin(c + d*x)^4)^p, x)
```

3.572 $\int \cot^2(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx$

3.572.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \cot^2(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx = \text{Int}\left(\cot^2(c+dx) \left(a+b\sin^4(c+dx)\right)^p ,x\right)$$

output
```
Unintegrable(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x)
```

3.572.2 Mathematica [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot^2(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx = \int \cot^2(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p,x]
```

output
```
Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p, x]
```

3.572.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3714}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\sin(c+dx)^4\right)^p}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \; 3714$$

$$\int \cot^2(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx$$

input
```
Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p,x]
```

output
```
$Aborted
```

3.572.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3714
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sin[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.572.4 Maple [N/A] (verified)

Not integrable

Time = 1.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(\cot^2(dx+c)\right)\left(a+b\left(\sin^4(dx+c)\right)\right)^p dx$$

input
```
int(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x)
```

output
```
int(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x)
```

3.572.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx=\int \left(b\sin\left(dx+c\right)^4+a\right)^p\cot\left(dx+c\right)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^2,
 x)
```

3.572.6 Sympy [F(-1)]

Timed out.

$$\int \cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(cot(d*x+c)**2*(a+b*sin(d*x+c)**4)**p,x)
```

output
```
Timed out
```

3.572.7 Maxima [N/A]

Not integrable

Time = 5.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot^2(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \cot\left(dx+c\right)^2 \, dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^2, x)
```

3.572.8 Giac [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot^2(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p \cot\left(dx+c\right)^2 \, dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^2, x)
```

3.572.9 Mupad [N/A]

Not integrable

Time = 16.69 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot^2(c+dx) \left(a+b\sin^4(c+dx)\right)^p \, dx = \int \cot(c+d\,x)^2 \left(b\sin\left(c+d\,x\right)^4+a\right)^p \,dx$$

input
```
int(cot(c + d*x)^2*(a + b*sin(c + d*x)^4)^p,x)
```

output
```
int(cot(c + d*x)^2*(a + b*sin(c + d*x)^4)^p, x)
```

3.573 $\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$

3.573.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx=\operatorname{Int}\left(\cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p,x\right)$$

output
```
Unintegrable(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x)
```

3.573.2 Mathematica [N/A]

Not integrable

Time = 14.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx=\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

input
```
Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^4)^p,x]
```

output
```
Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^4)^p, x]
```

3.573.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3714}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(c+dx)^4\right)^p}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{3714}$$

$$\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

input
```
Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^4)^p,x]
```

output
```
$Aborted
```

3.573.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3714
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sin[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.573.4 Maple [N/A] (verified)

Not integrable

Time = 1.63 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(\cot^4(dx+c)\right)\left(a+b\left(\sin^4(dx+c)\right)\right)^p dx$$

input
```
int(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x)
```

output
```
int(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x)
```

3.573.5 Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx=\int \left(b\sin\left(dx+c\right)^4+a\right)^p\cot\left(dx+c\right)^4\,dx$$

input
```
integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^4,
 x)
```

3.573.6 Sympy [F(-1)]

Timed out.

$$\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(cot(d*x+c)**4*(a+b*sin(d*x+c)**4)**p,x)
```

output
```
Timed out
```

3.573.7 Maxima [N/A]

Not integrable

Time = 14.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p\cot\left(dx+c\right)^4\,dx$$

input
```
integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^4, x)
```

3.573.8 Giac [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx = \int \left(b\sin\left(dx+c\right)^4+a\right)^p\cot\left(dx+c\right)^4\,dx$$

input
```
integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^4, x)
```

3.573.9 Mupad [N/A]

Not integrable

Time = 17.77 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx = \int \cot(c+d\,x)^4\,\left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

input
```
int(cot(c + d*x)^4*(a + b*sin(c + d*x)^4)^p,x)
```

output
```
int(cot(c + d*x)^4*(a + b*sin(c + d*x)^4)^p, x)
```

3.574 $\int (a+b\sin^n(c+dx))^3 \tan^m(c+dx)\,dx$

3.574.1 Optimal result

Integrand size = 23, antiderivative size = 306

$$\int (a+b\sin^n(c+dx))^3\tan^m(c+dx)\,dx$$
$$= \frac{a^3 \text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+m)}$$
$$+ \frac{3a^2b\cos^2(c+dx)^{\frac{1+m}{2}}\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),\sin^2(c+dx)\right)\sin^n(c+dx}{d(1+m+n)}$$
$$+ \frac{3ab^2\cos^2(c+dx)^{\frac{1+m}{2}}\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1}{2}(1+m+2n),\frac{1}{2}(3+m+2n),\sin^2(c+dx)\right)\sin^{2n}(c+}{d(1+m+2n)}$$
$$+ \frac{b^3\cos^2(c+dx)^{\frac{1+m}{2}}\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1}{2}(1+m+3n),\frac{1}{2}(3+m+3n),\sin^2(c+dx)\right)\sin^{3n}(c+d}{d(1+m+3n)}$$

output
```
a^3*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(1+m)/d
/(1+m)+3*a^2*b*(cos(d*x+c)^2)^(1/2+1/2*m)*hypergeom([1/2+1/2*m, 1/2+1/2*m+
1/2*n],[3/2+1/2*m+1/2*n],sin(d*x+c)^2)*sin(d*x+c)^n*tan(d*x+c)^(1+m)/d/(1+
m+n)+3*a*b^2*(cos(d*x+c)^2)^(1/2+1/2*m)*hypergeom([1/2+1/2*m, 1/2+1/2*m+n]
,[3/2+1/2*m+n],sin(d*x+c)^2)*sin(d*x+c)^(2*n)*tan(d*x+c)^(1+m)/d/(1+m+2*n)
+b^3*(cos(d*x+c)^2)^(1/2+1/2*m)*hypergeom([1/2+1/2*m, 1/2+1/2*m+3/2*n],[3/
2+1/2*m+3/2*n],sin(d*x+c)^2)*sin(d*x+c)^(3*n)*tan(d*x+c)^(1+m)/d/(1+m+3*n)
```

3.574.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 20.72 (sec) , antiderivative size = 3544, normalized size of antiderivative = 11.58

$$\int (a + b\sin^n(c + dx))^3 \tan^m(c + dx)\, dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sin[c + d*x]^n)^3*Tan[c + d*x]^m,x]
```

output
```
(2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*((a^3*AppellF1[(1 + m)/2, m, 1, (3
+ m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*
x)/2]^2)^n*Sin[c + d*x]^n*((3*a^2*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m
 + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n) + b*(Sec[(c
 + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3
/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + 2*n) + (b
*AppellF1[(1 + m + 3*n)/2, m, 1 + 3*n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 3*n
))))*Tan[(c + d*x)/2]*Tan[c + d*x]^m*(a^3*Tan[c + d*x]^m + 3*a^2*b*Sin[c +
 d*x]^n*Tan[c + d*x]^m + 3*a*b^2*Sin[c + d*x]^(2*n)*Tan[c + d*x]^m + b^3*S
in[c + d*x]^(3*n)*Tan[c + d*x]^m))/(d*(2*m*(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^m*Sec[c + d*x]^2*((a^3*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c +
d*x]^n*((3*a^2*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n) + b*(Sec[(c + d*x)/2]^2)^n*Sin
[c + d*x]^n*((3*a*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + 2*n) + (b*AppellF1[(1 + m +
3*n)/2, m, 1 + 3*n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 3*n))))*Tan[(c + d*x)/
2]*Tan[c + d*x]^(-1 + m) + Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*...
```

3.574.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3713, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^m(c+dx)\,(a+b\sin^n(c+dx))^3\,dx$$

$\Big\downarrow$ 3042

$$\int \tan(c+dx)^m\,(a+b\sin(c+dx)^n)^3\,dx$$

$\Big\downarrow$ 3713

$$\int \left(a^3\tan^m(c+dx)+3a^2b\tan^m(c+dx)\sin^n(c+dx)+3ab^2\tan^m(c+dx)\sin^{2n}(c+dx)+b^3\tan^m(c+dx)\sin^{3n}(\right.$$

$\Big\downarrow$ 2009

$$\frac{a^3\tan^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{d(m+1)}+$$
$$\frac{3a^2b\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^n(c+dx)\,\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+3),\sin^2(c\right.}{d(m+n+1)}$$
$$\frac{3ab^2\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^{2n}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{1}{2}(m+2n+1),\frac{1}{2}(m+2n+3),\sin\right.}{d(m+2n+1)}$$
$$\frac{b^3\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^{3n}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{1}{2}(m+3n+1),\frac{1}{2}(m+3n+3),\sin^2(\right.}{d(m+3n+1)}$$

input
```
Int[(a + b*Sin[c + d*x]^n)^3*Tan[c + d*x]^m,x]
```

output
```
(a^3*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d
*x]^(1 + m))/(d*(1 + m)) + (3*a^2*b*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeom
etric2F1[(1 + m)/2, (1 + m + n)/2, (3 + m + n)/2, Sin[c + d*x]^2]*Sin[c +
d*x]^n*Tan[c + d*x]^(1 + m))/(d*(1 + m + n)) + (3*a*b^2*(Cos[c + d*x]^2)^(
(1 + m)/2)*Hypergeometric2F1[(1 + m)/2, (1 + m + 2*n)/2, (3 + m + 2*n)/2,
Sin[c + d*x]^2]*Sin[c + d*x]^(2*n)*Tan[c + d*x]^(1 + m))/(d*(1 + m + 2*n))
 + (b^3*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeometric2F1[(1 + m)/2, (1 + m +
 3*n)/2, (3 + m + 3*n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(3*n)*Tan[c + d*x]^
(1 + m))/(d*(1 + m + 3*n))
```

3.574. $\int (a+b\sin^n(c+dx))^3\tan^m(c+dx)\,dx$

3.574.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3713
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*tan[e + f*x])^m*(a
 + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

3.574.4 Maple [F]

$$\int (a+b(\sin^n(dx+c)))^3(\tan^m(dx+c))\,dx$$

input
```
int((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x)
```

output
```
int((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x)
```

3.574.5 Fricas [F]

$$\int (a+b\sin^n(c+dx))^3\tan^m(c+dx)\,dx = \int (b\sin(dx+c)^n+a)^3\tan(dx+c)^m\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x, algorithm="fricas")
```

output
```
integral((b^3*sin(d*x + c)^(3*n) + 3*a*b^2*sin(d*x + c)^(2*n) + 3*a^2*b*si
n(d*x + c)^n + a^3)*tan(d*x + c)^m, x)
```

3.574.6 Sympy [F]

$$\int (a+b\sin^n(c+dx))^3\tan^m(c+dx)\,dx=\int (a+b\sin^n{(c+dx)})^3\tan^m{(c+dx)}\,dx$$

input
```
integrate((a+b*sin(d*x+c)**n)**3*tan(d*x+c)**m,x)
```

output
```
Integral((a + b*sin(c + d*x)**n)**3*tan(c + d*x)**m, x)
```

3.574.7 Maxima [F]

$$\int (a+b\sin^n(c+dx))^3\tan^m(c+dx)\,dx=\int (b\sin{(dx+c)}^n+a)^3\tan{(dx+c)}^m\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^n + a)^3*tan(d*x + c)^m, x)
```

3.574.8 Giac [F]

$$\int (a+b\sin^n(c+dx))^3\tan^m(c+dx)\,dx=\int (b\sin{(dx+c)}^n+a)^3\tan{(dx+c)}^m\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^n + a)^3*tan(d*x + c)^m, x)
```

3.574.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin^n(c+dx))^3\tan^m(c+dx)\,dx=\int \tan(c+d\,x)^m\,\left(a+b\sin\left(c+d\,x\right)^n\right)^3\,dx$$

input
```
int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n)^3,x)
```

output
```
int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n)^3, x)
```

3.575 $\int (a+b\sin^n(c+dx))^2\tan^m(c+dx)\,dx$

3.575.1 Optimal result

Integrand size = 23, antiderivative size = 215

$$\begin{aligned}\int (a+b\sin^n(c+dx))^2\tan^m(c+dx)\,dx &= \frac{a^2\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+m)}\\ &+\frac{2ab\cos^2(c+dx)^{\frac{1+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),\sin^2(c+dx)\right)\sin^n(c+dx)}{d(1+m+n)}\\ &+\frac{b^2\cos^2(c+dx)^{\frac{1+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1}{2}(1+m+2n),\frac{1}{2}(3+m+2n),\sin^2(c+dx)\right)\sin^{2n}(c+d}{d(1+m+2n)}\end{aligned}$$

output
```
a^2*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(1+m)/d
/(1+m)+2*a*b*(cos(d*x+c)^2)^(1/2+1/2*m)*hypergeom([1/2+1/2*m, 1/2+1/2*m+1/
2*n],[3/2+1/2*m+1/2*n],sin(d*x+c)^2)*sin(d*x+c)^n*tan(d*x+c)^(1+m)/d/(1+m+
n)+b^2*(cos(d*x+c)^2)^(1/2+1/2*m)*hypergeom([1/2+1/2*m, 1/2+1/2*m+n],[3/2+
1/2*m+n],sin(d*x+c)^2)*sin(d*x+c)^(2*n)*tan(d*x+c)^(1+m)/d/(1+m+2*n)
```

3.575.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 16.74 (sec) , antiderivative size = 2368, normalized size of antiderivative = 11.01

$$\int (a + b\sin^n(c + dx))^2 \tan^m(c + dx)\, dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sin[c + d*x]^n)^2*Tan[c + d*x]^m,x]
```

output
```
(2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*((a^2*AppellF1[(1 + m)/2, m, 1, (3
+ m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*
x)/2]^2)^n*Sin[c + d*x]^n*((2*a*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m +
 n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n) + (b*AppellF1
[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 2*n)))*Tan[(c +
d*x)/2]*Tan[c + d*x]^m*(a^2*Tan[c + d*x]^m + 2*a*b*Sin[c + d*x]^n*Tan[c +
d*x]^m + b^2*Sin[c + d*x]^(2*n)*Tan[c + d*x]^m))/(d*(2*m*(Cos[c + d*x]*Sec
[(c + d*x)/2]^2)^m*Sec[c + d*x]^2*((a^2*AppellF1[(1 + m)/2, m, 1, (3 + m)/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)/2]
^2)^n*Sin[c + d*x]^n*((2*a*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n) + (b*AppellF1[1/2
+ m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 2*n)))*Tan[(c + d*x)/
2]*Tan[c + d*x]^(-1 + m) + Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/
2]^2)^m*((a^2*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((2*a
*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])/(1 + m + n) + (b*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2
 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^...
```

3.575.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3713, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^m(c+dx)\left(a+b\sin^n(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^m\left(a+b\sin(c+dx)^n\right)^2dx$$

$$\Big\downarrow \text{3713}$$

$$\int \left(a^2\tan^m(c+dx)+2ab\tan^m(c+dx)\sin^n(c+dx)+b^2\tan^m(c+dx)\sin^{2n}(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\tan^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{d(m+1)}+\frac{2ab\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^n(c+dx)\,\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+3),\sin^2(c-\right.}{d(m+n+1)}$$
$$\frac{b^2\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^{2n}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{1}{2}(m+2n+1),\frac{1}{2}(m+2n+3),\sin^2(c\right.}{d(m+2n+1)}$$

input
```
Int[(a + b*Sin[c + d*x]^n)^2*Tan[c + d*x]^m,x]
```

output
```
(a^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d
*x]^(1 + m))/(d*(1 + m)) + (2*a*b*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeomet
ric2F1[(1 + m)/2, (1 + m + n)/2, (3 + m + n)/2, Sin[c + d*x]^2]*Sin[c + d*
x]^n*Tan[c + d*x]^(1 + m))/(d*(1 + m + n)) + (b^2*(Cos[c + d*x]^2)^((1 + m
)/2)*Hypergeometric2F1[(1 + m)/2, (1 + m + 2*n)/2, (3 + m + 2*n)/2, Sin[c
+ d*x]^2]*Sin[c + d*x]^(2*n)*Tan[c + d*x]^(1 + m))/(d*(1 + m + 2*n))
```

3.575.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3713
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*tan[e + f*x])^m*(a
 + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

3.575.4 Maple [F]

$$\int (a+b(\sin^n(dx+c)))^2(\tan^m(dx+c))\,dx$$

input
```
int((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x)
```

output
```
int((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x)
```

3.575.5 Fricas [F]

$$\int (a+b\sin^n(c+dx))^2\tan^m(c+dx)\,dx=\int (b\sin(dx+c)^n+a)^2\tan(dx+c)^m\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x, algorithm="fricas")
```

output
```
integral((b^2*sin(d*x + c)^(2*n) + 2*a*b*sin(d*x + c)^n + a^2)*tan(d*x + c
)^m, x)
```

3.575.6 Sympy [F]

$$\int (a+b\sin^n(c+dx))^2\tan^m(c+dx)\,dx=\int (a+b\sin^n(c+dx))^2\tan^m(c+dx)\,dx$$

input
```
integrate((a+b*sin(d*x+c)**n)**2*tan(d*x+c)**m,x)
```

output
```
Integral((a + b*sin(c + d*x)**n)**2*tan(c + d*x)**m, x)
```

3.575.7 Maxima [F]

$$\int (a+b\sin^n(c+dx))^2\tan^m(c+dx)\,dx=\int (b\sin{(dx+c)}^n+a)^2\tan{(dx+c)}^m\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^n + a)^2*tan(d*x + c)^m, x)
```

3.575.8 Giac [F]

$$\int (a+b\sin^n(c+dx))^2\tan^m(c+dx)\,dx=\int (b\sin{(dx+c)}^n+a)^2\tan{(dx+c)}^m\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^n + a)^2*tan(d*x + c)^m, x)
```

3.575.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin^n(c+dx))^2\tan^m(c+dx)\,dx=\int \tan(c+d\,x)^m\,(a+b\sin{(c+d\,x)}^n)^2\,dx$$

input
```
int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n)^2,x)
```

output
```
int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n)^2, x)
```

3.576 $\int (a+b\sin^n(c+dx))\tan^m(c+dx)\,dx$

3.576.1 Optimal result

Integrand size = 21, antiderivative size = 124

$$\int (a+b\sin^n(c+dx))\tan^m(c+dx)\,dx$$
$$=\frac{a\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+m)}$$
$$+\frac{b\cos^2(c+dx)^{\frac{1+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),\sin^2(c+dx)\right)\sin^n(c+dx)\,\text{ta}}{d(1+m+n)}$$

output
```
a*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(1+m)/d/(
1+m)+b*(cos(d*x+c)^2)^(1/2+1/2*m)*hypergeom([1/2+1/2*m, 1/2+1/2*m+1/2*n],[
3/2+1/2*m+1/2*n],sin(d*x+c)^2)*sin(d*x+c)^n*tan(d*x+c)^(1+m)/d/(1+m+n)
```

3.576.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 14.19 (sec) , antiderivative size = 1395, normalized size of antiderivative = 11.25

$$\int (a+b\sin^n(c+dx))\tan^m(c+dx)\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sin[c + d*x]^n)*Tan[c + d*x]^m,x]
```

output
```
(2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*(a*(1 + m + n)*AppellF1[(1 + m)/2,
m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + b*(1 + m)*Appe
llF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)*Tan[(c + d*x)/2]*Tan[c
+ d*x]^m*(a*Tan[c + d*x]^m + b*Sin[c + d*x]^n*Tan[c + d*x]^m))/(d*(1 + m)*
(1 + m + n)*((2*m*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*Sec[c + d*x]^2*(a*(1
 + m + n)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] + b*(1 + m)*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d
*x]^n)*Tan[(c + d*x)/2]*Tan[c + d*x]^(-1 + m))/((1 + m)*(1 + m + n)) + (Se
c[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*(a*(1 + m + n)*Appell
F1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] +
b*(1 + m)*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2
]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)*Tan[c + d
*x]^m)/((1 + m)*(1 + m + n)) + (2*m*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(-1
+ m)*(a*(1 + m + n)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] + b*(1 + m)*AppellF1[(1 + m + n)/2, m, 1 + n, (3 +
 m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n
*Sin[c + d*x]^n)*Tan[(c + d*x)/2]*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Co
s[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*Tan[c + d*x]^m)/((1 + m...
```

3.576.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3713, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^m(c+dx)\,(a+b\sin^n(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^m\,(a+b\sin(c+dx)^n)\,dx$$

$$\Big\downarrow \text{3713}$$

$$\int (a\tan^m(c+dx)+b\tan^m(c+dx)\sin^n(c+dx))\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\tan^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{d(m+1)}+\frac{b\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^n(c+dx)\,\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+3),\sin^2(c+a\right.}{d(m+n+1)}$$

input
```
Int[(a + b*Sin[c + d*x]^n)*Tan[c + d*x]^m,x]
```

output
```
(a*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x
]^(1 + m))/(d*(1 + m)) + (b*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeometric2F1
[(1 + m)/2, (1 + m + n)/2, (3 + m + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^n*T
an[c + d*x]^(1 + m))/(d*(1 + m + n))
```

3.576.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3713
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*tan[e + f*x])^m*(a
 + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

3.576.4 Maple [F]

$$\int (a+b(\sin^n(dx+c)))\,(\tan^m(dx+c))\,dx$$

input
```
int((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x)
```

output
```
int((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x)
```

3.576.5 Fricas [F]

$$\int (a+b\sin^n(c+dx))\tan^m(c+dx)\,dx = \int (b\sin(dx+c)^n+a)\tan(dx+c)^m\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c)^n + a)*tan(d*x + c)^m, x)
```

3.576.6 Sympy [F]

$$\int (a+b\sin^n(c+dx))\tan^m(c+dx)\,dx = \int (a+b\sin^n(c+dx))\tan^m(c+dx)\,dx$$

input
```
integrate((a+b*sin(d*x+c)**n)*tan(d*x+c)**m,x)
```

output
```
Integral((a + b*sin(c + d*x)**n)*tan(c + d*x)**m, x)
```

3.576.7 Maxima [F]

$$\int (a+b\sin^n(c+dx))\tan^m(c+dx)\,dx = \int (b\sin(dx+c)^n+a)\tan(dx+c)^m\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^n + a)*tan(d*x + c)^m, x)
```

3.576.8 Giac [F]

$$\int (a+b\sin^n(c+dx))\tan^m(c+dx)\,dx=\int (b\sin{(dx+c)}^n+a)\tan{(dx+c)}^m\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^n + a)*tan(d*x + c)^m, x)
```

3.576.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin^n(c+dx))\tan^m(c+dx)\,dx=\int \tan(c+d\,x)^m\,(a+b\sin{(c+d\,x)}^n)\,dx$$

input
```
int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n),x)
```

output
```
int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n), x)
```

3.577 $\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx$

3.577.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx = \text{Int}\left(\frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)},x\right)$$

output
```
Unintegrable(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x)
```

3.577.2 Mathematica [N/A]

Not integrable

Time = 5.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx = \int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx$$

input
```
Integrate[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n),x]
```

output
```
Integrate[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n), x]
```

3.577.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3714}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)^m}{a+b\sin(c+dx)^n}dx$$
$$\Big\downarrow \text{3714}$$
$$\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}dx$$

input
```
Int[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n),x]
```

output
```
$Aborted
```

3.577.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3714
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sin[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.577.4 Maple [N/A] (verified)

Not integrable

Time = 1.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\tan^m(dx+c)}{a+b(\sin^n(dx+c))}dx$$

input
```
int(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x)
```

output
```
int(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x)
```

3.577.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx = \int \frac{\tan(dx+c)^m}{b\sin(dx+c)^n+a}\,dx$$

input
```
integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x, algorithm="fricas")
```

output
```
integral(tan(d*x + c)^m/(b*sin(d*x + c)^n + a), x)
```

3.577.6 Sympy [N/A]

Not integrable

Time = 5.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx = \int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**m/(a+b*sin(d*x+c)**n),x)
```

output
```
Integral(tan(c + d*x)**m/(a + b*sin(c + d*x)**n), x)
```

3.577.7 Maxima [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx = \int \frac{\tan(dx+c)^m}{b\sin(dx+c)^n+a}\,dx$$

input
```
integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x, algorithm="maxima")
```

output
```
integrate(tan(d*x + c)^m/(b*sin(d*x + c)^n + a), x)
```

3.577.8 Giac [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx = \int \frac{\tan(dx+c)^m}{b\sin(dx+c)^n+a}\,dx$$

input
```
integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x, algorithm="giac")
```

output
```
integrate(tan(d*x + c)^m/(b*sin(d*x + c)^n + a), x)
```

3.577.9 Mupad [N/A]

Not integrable

Time = 13.93 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx = \int \frac{\tan(c+d\,x)^m}{a+b\sin(c+d\,x)^n}\,dx$$

input
```
int(tan(c + d*x)^m/(a + b*sin(c + d*x)^n),x)
```

output
```
int(tan(c + d*x)^m/(a + b*sin(c + d*x)^n), x)
```

3.578 $\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}\,dx$

3.578.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}\,dx=\text{Int}\left(\frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2},x\right)$$

output
```
Unintegrable(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x)
```

3.578.2 Mathematica [N/A]

Not integrable

Time = 47.94 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}\,dx=\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}\,dx$$

input
```
Integrate[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n)^2,x]
```

output
```
Integrate[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n)^2, x]
```

3.578.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3714}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)^m}{(a+b\sin(c+dx)^n)^2}dx$$
$$\Big\downarrow \text{3714}$$
$$\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}dx$$

input
```
Int[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n)^2,x]
```

output
```
$Aborted
```

3.578.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3714
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sin[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.578.4 Maple [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\tan^m (dx+c)}{(a+b(\sin^n (dx+c)))^2} dx$$

input
```
int(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x)
```

output
```
int(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x)
```

3.578.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2} dx = \int \frac{\tan(dx+c)^m}{(b\sin(dx+c)^n+a)^2} dx$$

input
```
integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x, algorithm="fricas")
```

output
```
integral(tan(d*x + c)^m/(b^2*sin(d*x + c)^(2*n) + 2*a*b*sin(d*x + c)^n + a
^2), x)
```

3.578.6 Sympy [N/A]

Not integrable

Time = 107.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2} dx = \int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2} dx$$

input
```
integrate(tan(d*x+c)**m/(a+b*sin(d*x+c)**n)**2,x)
```

output
```
Integral(tan(c + d*x)**m/(a + b*sin(c + d*x)**n)**2, x)
```

3.578.7 Maxima [N/A]

Not integrable

Time = 3.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}\,dx = \int \frac{\tan(dx+c)^m}{(b\sin(dx+c)^n+a)^2}\,dx$$

input
```
integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x, algorithm="maxima")
```

output
```
integrate(tan(d*x + c)^m/(b*sin(d*x + c)^n + a)^2, x)
```

3.578.8 Giac [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}\,dx = \int \frac{\tan(dx+c)^m}{(b\sin(dx+c)^n+a)^2}\,dx$$

input
```
integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x, algorithm="giac")
```

output
```
integrate(tan(d*x + c)^m/(b*sin(d*x + c)^n + a)^2, x)
```

3.578.9 Mupad [N/A]

Not integrable

Time = 15.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}\,dx = \int \frac{\tan(c+d\,x)^m}{(a+b\sin(c+d\,x)^n)^2}\,dx$$

input
```
int(tan(c + d*x)^m/(a + b*sin(c + d*x)^n)^2,x)
```

output
```
int(tan(c + d*x)^m/(a + b*sin(c + d*x)^n)^2, x)
```

3.579 $\int \cot(x)\sqrt{a+b\sin^n(x)}\,dx$

3.579.1 Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \cot(x)\sqrt{a+b\sin^n(x)}\,dx=-\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{n}+\frac{2\sqrt{a+b\sin^n(x)}}{n}$$

output
```
-2*arctanh((a+b*sin(x)^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(a+b*sin(x)^n)^(1/2)/
n
```

3.579.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \cot(x)\sqrt{a+b\sin^n(x)}\,dx=\frac{-2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\sin^n(x)}}{n}$$

input
```
Integrate[Cot[x]*Sqrt[a + b*Sin[x]^n],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[x]^n]/Sqrt[a]] + 2*Sqrt[a + b*Sin[x]^n]
)/n
```

3.579.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3709, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x)\sqrt{a+b\sin^n(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\sin(x)^n}}{\tan(x)}dx$$

$$\Big\downarrow \text{3709}$$

$$\int \csc(x)\sqrt{a+b\sin^n(x)}d\sin(x)$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \csc(x)\sqrt{b\sin^n(x)+a}d\sin^n(x)}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\int \frac{\csc(x)}{\sqrt{b\sin^n(x)+a}}d\sin^n(x)+2\sqrt{a+b\sin^n(x)}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2a\int \frac{1}{\frac{\sin^{2n}(x)}{b}-\frac{a}{b}}d\sqrt{b\sin^n(x)+a}}{b}+2\sqrt{a+b\sin^n(x)}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{a+b\sin^n(x)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{n}$$

input
```
Int[Cot[x]*Sqrt[a + b*Sin[x]^n],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[x]^n]/Sqrt[a]] + 2*Sqrt[a + b*Sin[x]^n]
)/n
```

3.579.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.579. $\int \cot(x)\sqrt{a+b\sin^n(x)}\,dx$

3.579.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2\sqrt{a+b(\sin^n(x))}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{a+b(\sin^n(x))}}{\sqrt{a}}\right)}{n}$	38
default	$\frac{2\sqrt{a+b(\sin^n(x))}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{a+b(\sin^n(x))}}{\sqrt{a}}\right)}{n}$	38

input
```
int(cot(x)*(a+b*sin(x)^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(2*(a+b*sin(x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*sin(x)^n)^(1/2)/a^(1/2)
))
```

3.579.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.06

$$\int \cot(x)\sqrt{a+b\sin^n(x)}\,dx$$
$$= \left[\frac{\sqrt{a}\log\left(\frac{b\sin(x)^n-2\sqrt{b\sin(x)^n+a}\sqrt{a}+2a}{\sin(x)^n}\right)+2\sqrt{b\sin(x)^n+a}}{n}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{b\sin(x)^n+a}\sqrt{-a}}{a}\right)+\sqrt{b\sin(x)\right.}}{n}\right.$$

input
```
integrate(cot(x)*(a+b*sin(x)^n)^(1/2),x, algorithm="fricas")
```

output
```
[(sqrt(a)*log((b*sin(x)^n - 2*sqrt(b*sin(x)^n + a)*sqrt(a) + 2*a)/sin(x)^n
) + 2*sqrt(b*sin(x)^n + a))/n, 2*(sqrt(-a)*arctan(sqrt(b*sin(x)^n + a)*sqr
t(-a)/a) + sqrt(b*sin(x)^n + a))/n]
```

3.579.6 Sympy [F]

$$\int \cot(x)\sqrt{a+b\sin^n(x)}\,dx = \int \sqrt{a+b\sin^n{(x)}}\cot{(x)}\,dx$$

input
```
integrate(cot(x)*(a+b*sin(x)**n)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(x)**n)*cot(x), x)
```

3.579.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \cot(x)\sqrt{a+b\sin^n(x)}\,dx = \frac{\sqrt{a}\log\left(\frac{\sqrt{b\sin(x)^n+a}-\sqrt{a}}{\sqrt{b\sin(x)^n+a}+\sqrt{a}}\right)}{n} + \frac{2\sqrt{b\sin{(x)}^n+a}}{n}$$

input
```
integrate(cot(x)*(a+b*sin(x)^n)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a)*log((sqrt(b*sin(x)^n + a) - sqrt(a))/(sqrt(b*sin(x)^n + a) + sqrt(
a)))/n + 2*sqrt(b*sin(x)^n + a)/n
```

3.579.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \cot(x)\sqrt{a+b\sin^n(x)}\,dx = \frac{2\left(\frac{ab\arctan\left(\frac{\sqrt{b\sin(x)^n+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{b\sin{(x)}^n+a}\,b\right)}{bn}$$

input
```
integrate(cot(x)*(a+b*sin(x)^n)^(1/2),x, algorithm="giac")
```

output
```
2*(a*b*arctan(sqrt(b*sin(x)^n + a)/sqrt(-a))/sqrt(-a) + sqrt(b*sin(x)^n +
a)*b)/(b*n)
```

3.579.9 Mupad [F(-1)]

Timed out.

$$\int \cot(x)\sqrt{a+b\sin^n(x)}\,dx = \int \cot(x)\,\sqrt{a+b\sin(x)^n}\,dx$$

input
```
int(cot(x)*(a + b*sin(x)^n)^(1/2),x)
```

output
```
int(cot(x)*(a + b*sin(x)^n)^(1/2), x)
```

3.580 $\int \frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}\,dx$

3.580.1 Optimal result

Integrand size = 15, antiderivative size = 29

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

output
```
-2*arctanh((a+b*sin(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

3.580.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Integrate[Cot[x]/Sqrt[a + b*Sin[x]^n],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Sin[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

3.580.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3709, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(x)\sqrt{a+b\sin(x)^n}}dx$$

$$\Big\downarrow \text{ 3709}$$

$$\int \frac{\csc(x)}{\sqrt{a+b\sin^n(x)}}d\sin(x)$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int \frac{\csc(x)}{\sqrt{b\sin^n(x)+a}}d\sin^n(x)}{n}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2\int \frac{1}{\frac{\sin^{2n}(x)}{b}-\frac{a}{b}}d\sqrt{b\sin^n(x)+a}}{bn}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Int[Cot[x]/Sqrt[a + b*Sin[x]^n],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Sin[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

3.580.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.580.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b(\sin^n(x))}}{\sqrt{a}}\right)}{n\sqrt{a}}$	24
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b(\sin^n(x))}}{\sqrt{a}}\right)}{n\sqrt{a}}$	24

input
```
int(cot(x)/(a+b*sin(x)^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+b*sin(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

3.580.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.55

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}\,dx = \left[\frac{\log\left(\frac{b\sin(x)^n-2\sqrt{b\sin(x)^n+a}\sqrt{a}+2a}{\sin(x)^n}\right)}{\sqrt{a}n}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{b\sin(x)^n+a}\sqrt{-a}}{a}\right)}{an}\right]$$

input
```
integrate(cot(x)/(a+b*sin(x)^n)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*sin(x)^n - 2*sqrt(b*sin(x)^n + a)*sqrt(a) + 2*a)/sin(x)^n)/(sqrt(a
)*n), 2*sqrt(-a)*arctan(sqrt(b*sin(x)^n + a)*sqrt(-a)/a)/(a*n)]
```

3.580.6 Sympy [F]

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}\,dx = \int \frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}\,dx$$

input
```
integrate(cot(x)/(a+b*sin(x)**n)**(1/2),x)
```

output
```
Integral(cot(x)/sqrt(a + b*sin(x)**n), x)
```

3.580.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}\,dx = \frac{\log\left(\frac{\sqrt{b\sin(x)^n+a}-\sqrt{a}}{\sqrt{b\sin(x)^n+a}+\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
integrate(cot(x)/(a+b*sin(x)^n)^(1/2),x, algorithm="maxima")
```

output
```
log((sqrt(b*sin(x)^n + a) - sqrt(a))/(sqrt(b*sin(x)^n + a) + sqrt(a)))/(sq
rt(a)*n)
```

3.580.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}\,dx = \frac{2\arctan\left(\frac{\sqrt{b\sin(x)^n+a}}{\sqrt{-a}}\right)}{\sqrt{-a}n}$$

input
```
integrate(cot(x)/(a+b*sin(x)^n)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(b*sin(x)^n + a)/sqrt(-a))/(sqrt(-a)*n)
```

3.580.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}\,dx = \int \frac{\cot(x)}{\sqrt{a+b\sin(x)^n}}\,dx$$

input
```
int(cot(x)/(a + b*sin(x)^n)^(1/2),x)
```

output
```
int(cot(x)/(a + b*sin(x)^n)^(1/2), x)
```

3.581 $\int (a+b\sin^n(c+dx))^p \tan^m(c+dx)\,dx$

3.581.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sin^n(c+dx))^p \tan^m(c+dx)\,dx = \mathrm{Int}((a+b\sin^n(c+dx))^p \tan^m(c+dx),x)$$

output
```
Unintegrable((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x)
```

3.581.2 Mathematica [N/A]

Not integrable

Time = 5.77 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p \tan^m(c+dx)\,dx = \int (a+b\sin^n(c+dx))^p \tan^m(c+dx)\,dx$$

input
```
Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^m,x]
```

output
```
Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^m, x]
```

3.581.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3714}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^m(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^m\left(a+b\sin(c+dx)^n\right)^p dx$$

$$\Big\downarrow \text{3714}$$

$$\int \tan^m(c+dx)\left(a+b\sin^n(c+dx)\right)^p dx$$

input
```
Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^m,x]
```

output
```
$Aborted
```

3.581.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3714
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sin[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.581.4 Maple [N/A] (verified)

Not integrable

Time = 1.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b(\sin^n(dx+c)))^p(\tan^m(dx+c))\,dx$$

input
```
int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x)
```

output
```
int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x)
```

3.581.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p\tan^m(c+dx)\,dx=\int (b\sin(dx+c)^n+a)^p\tan(dx+c)^m\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^m, x)
```

3.581.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\sin^n(c+dx))^p\tan^m(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c)**m,x)
```

output
```
Timed out
```

3.581.7 Maxima [N/A]

Not integrable

Time = 4.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p \tan^m(c+dx)\,dx = \int (b\sin(dx+c)^n+a)^p \tan(dx+c)^m\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^m, x)
```

3.581.8 Giac [N/A]

Not integrable

Time = 6.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p \tan^m(c+dx)\,dx = \int (b\sin(dx+c)^n+a)^p \tan(dx+c)^m\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^m, x)
```

3.581.9 Mupad [N/A]

Not integrable

Time = 15.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p \tan^m(c+dx)\,dx = \int \tan(c+d\,x)^m\,(a+b\sin(c+d\,x)^n)^p\,dx$$

input
```
int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n)^p,x)
```

output
```
int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n)^p, x)
```

3.582 $\int (a+b\sin^n(c+dx))^p \tan^3(c+dx)\,dx$

3.582.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sin^n(c+dx))^p \tan^3(c+dx)\,dx = \operatorname{Int}\left((a+b\sin^n(c+dx))^p \tan^3(c+dx),x\right)$$

output
```
Unintegrable((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x)
```

3.582.2 Mathematica [N/A]

Not integrable

Time = 25.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p \tan^3(c+dx)\,dx = \int (a+b\sin^n(c+dx))^p \tan^3(c+dx)\,dx$$

input
```
Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^3,x]
```

output
```
Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^3, x]
```

3.582.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3714}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^3\left(a+b\sin(c+dx)^n\right)^p\,dx$$

$$\Big\downarrow \text{3714}$$

$$\int \tan^3(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

input
```
Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^3,x]
```

output
```
$Aborted
```

3.582.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3714
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sin[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.582.4 Maple [N/A] (verified)

Not integrable

Time = 0.91 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a + b(\sin^n (dx + c)))^p \left(\tan^3 (dx + c)\right) dx$$

input
```
int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x)
```

output
```
int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x)
```

3.582.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a + b\sin^n(c + dx))^p \tan^3(c + dx)\, dx = \int (b\sin{(dx + c)}^n + a)^p \tan{(dx + c)}^3\, dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^3, x)
```

3.582.6 Sympy [F(-1)]

Timed out.

$$\int (a + b\sin^n(c + dx))^p \tan^3(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c)**3,x)
```

output
```
Timed out
```

3.582.7 Maxima [N/A]

Not integrable

Time = 5.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p\tan^3(c+dx)\,dx=\int (b\sin{(dx+c)}^n+a)^p\tan{(dx+c)}^3\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^3, x)
```

3.582.8 Giac [N/A]

Not integrable

Time = 8.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p\tan^3(c+dx)\,dx=\int (b\sin{(dx+c)}^n+a)^p\tan{(dx+c)}^3\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^3, x)
```

3.582.9 Mupad [N/A]

Not integrable

Time = 15.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p\tan^3(c+dx)\,dx=\int \tan(c+d\,x)^3\,(a+b\sin{(c+d\,x)}^n)^p\,dx$$

input
```
int(tan(c + d*x)^3*(a + b*sin(c + d*x)^n)^p,x)
```

output
```
int(tan(c + d*x)^3*(a + b*sin(c + d*x)^n)^p, x)
```

3.583 $\int (a+b\sin^n(c+dx))^p \tan(c+dx)\,dx$

3.583.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (a+b\sin^n(c+dx))^p \tan(c+dx)\,dx = \text{Int}((a+b\sin^n(c+dx))^p \tan(c+dx),x)$$

output
```
Unintegrable((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x)
```

3.583.2 Mathematica [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sin^n(c+dx))^p \tan(c+dx)\,dx = \int (a+b\sin^n(c+dx))^p \tan(c+dx)\,dx$$

input
```
Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x],x]
```

output
```
Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x], x]
```

3.583.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3714}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)\,(a+b\sin^n(c+dx))^p\;dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(c+dx)\,(a+b\sin(c+dx)^n)^p\,dx$$

$$\Big\downarrow \; 3714$$

$$\int \tan(c+dx)\,(a+b\sin^n(c+dx))^p\,dx$$

input
```
Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x],x]
```

output
```
$Aborted
```

3.583.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3714
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sin[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.583.4 Maple [N/A] (verified)

Not integrable

Time = 0.82 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (a+b(\sin^{n}(dx+c)))^{p}\tan(dx+c)\,dx$$

input
```
int((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x)
```

output
```
int((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x)
```

3.583.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sin^{n}(c+dx))^{p}\tan(c+dx)\,dx=\int (b\sin(dx+c)^{n}+a)^{p}\tan(dx+c)\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c), x)
```

3.583.6 Sympy [N/A]

Not integrable

Time = 109.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int (a+b\sin^{n}(c+dx))^{p}\tan(c+dx)\,dx=\int (a+b\sin^{n}(c+dx))^{p}\tan(c+dx)\,dx$$

input
```
integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c),x)
```

output
```
Integral((a + b*sin(c + d*x)**n)**p*tan(c + d*x), x)
```

3.583.7 Maxima [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sin^n(c+dx))^p\tan(c+dx)\,dx=\int (b\sin{(dx+c)}^n+a)^p\tan{(dx+c)}\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c), x)
```

3.583.8 Giac [N/A]

Not integrable

Time = 6.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sin^n(c+dx))^p\tan(c+dx)\,dx=\int (b\sin{(dx+c)}^n+a)^p\tan{(dx+c)}\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c), x)
```

3.583.9 Mupad [N/A]

Not integrable

Time = 14.70 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (a+b\sin^n(c+dx))^p\tan(c+dx)\,dx=\int \tan(c+d\,x)\,\left(a+b\,{\sin\left(c+d\,x\right)}^n\right)^p\,dx$$

input
```
int(tan(c + d*x)*(a + b*sin(c + d*x)^n)^p,x)
```

output
```
int(tan(c + d*x)*(a + b*sin(c + d*x)^n)^p, x)
```

3.584 $\int \cot(c+dx)(a+b\sin^n(c+dx))^p\,dx$

3.584.1 Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \cot(c+dx)(a+b\sin^n(c+dx))^p\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sin^n(c+dx)}{a}\right)(a+b\sin^n(c+dx))^{1+p}}{adn(1+p)}$$

output
```
-hypergeom([1, p+1],[2+p],1+b*sin(d*x+c)^n/a)*(a+b*sin(d*x+c)^n)^(p+1)/a/d
/n/(p+1)
```

3.584.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \cot(c+dx)(a+b\sin^n(c+dx))^p\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sin^n(c+dx)}{a}\right)(a+b\sin^n(c+dx))^{1+p}}{adn(1+p)}$$

input
```
Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x]^n)^p,x]
```

output
```
-((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^n)/a]*(a + b*Sin
[c + d*x]^n)^(1 + p))/(a*d*n*(1 + p)))
```

3.584.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3709, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx) (a+b\sin^n(c+dx))^p \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\sin(c+dx)^n)^p}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3709}$$

$$\frac{\int \csc(c+dx) (b\sin^n(c+dx)+a)^p d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int \csc(c+dx) (b\sin^n(c+dx)+a)^p d\sin^n(c+dx)}{dn}$$

$$\Big\downarrow \text{ 75}$$

$$-\frac{(a+b\sin^n(c+dx))^{p+1} \text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sin^n(c+dx)}{a}+1\right)}{adn(p+1)}$$

input
```
Int[Cot[c + d*x]*(a + b*Sin[c + d*x]^n)^p,x]
```

output
```
-((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^n)/a]*(a + b*Sin
[c + d*x]^n)^(1 + p))/(a*d*n*(1 + p)))
```

3.584.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.584.4 Maple [F]

$$\int \cot (dx + c) (a + b(\sin^n (dx + c)))^p dx$$

input
```
int(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x)
```

output
```
int(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x)
```

3.584.5 Fricas [F]

$$\int \cot(c + dx) (a + b \sin^n(c + dx))^p \, dx = \int (b \sin (dx + c)^n + a)^p \cot (dx + c) \, dx$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c)^n + a)^p*cot(d*x + c), x)
```

3.584.6 Sympy [F]

$$\int \cot(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\int\left(a+b\sin^n\left(c+dx\right)\right)^p\cot\left(c+dx\right)dx$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)**n)**p,x)
```

output
```
Integral((a + b*sin(c + d*x)**n)**p*cot(c + d*x), x)
```

3.584.7 Maxima [F]

$$\int \cot(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\int\left(b\sin\left(dx+c\right)^n+a\right)^p\cot\left(dx+c\right)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c), x)
```

3.584.8 Giac [F]

$$\int \cot(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\int\left(b\sin\left(dx+c\right)^n+a\right)^p\cot\left(dx+c\right)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c), x)
```

3.584.9 Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\int \cot(c+d\,x)\,\left(a+b\sin\left(c+d\,x\right)^n\right)^p\,dx$$

input
```
int(cot(c + d*x)*(a + b*sin(c + d*x)^n)^p,x)
```

output
```
int(cot(c + d*x)*(a + b*sin(c + d*x)^n)^p, x)
```

3.585 $\int \cot^3(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$

3.585.1 Optimal result

Integrand size = 23, antiderivative size = 136

$$\int \cot^3(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sin^n(c+dx)}{a}\right)\left(a+b\sin^n(c+dx)\right)^{1+p}}{adn(1+p)} - \frac{\csc^2(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{2}{n},-p,-\frac{2-n}{n},-\frac{b\sin^n(c+dx)}{a}\right)\left(a+b\sin^n(c+dx)\right)^p\left(1+\frac{b\sin^n(c+dx)}{a}\right)^{-}}{2d}$$

output
```
hypergeom([1, p+1],[2+p],1+b*sin(d*x+c)^n/a)*(a+b*sin(d*x+c)^n)^(p+1)/a/d/
n/(p+1)-1/2*csc(d*x+c)^2*hypergeom([-p, -2/n],[(-2+n)/n],-b*sin(d*x+c)^n/a
)*(a+b*sin(d*x+c)^n)^p/d/((1+b*sin(d*x+c)^n/a)^p)
```

3.585.2 Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.95

$$\int \cot^3(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx = \frac{\left(a+b\sin^n(c+dx)\right)^p\left(\frac{2\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sin^n(c+dx)}{a}\right)\left(a+b\sin^n(c+dx)\right)}{an(1+p)} - \csc^2(c+dx)\,\text{Hypergeometric}\right.}{2d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^n)^p,x]
```

output
```
((a + b*Sin[c + d*x]^n)^p*((2*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Si
n[c + d*x]^n)/a]*(a + b*Sin[c + d*x]^n))/(a*n*(1 + p)) - (Csc[c + d*x]^2*H
ypergeometric2F1[-2/n, -p, (-2 + n)/n, -((b*Sin[c + d*x]^n)/a)])/(1 + (b*S
in[c + d*x]^n)/a)^p))/(2*d)
```

3.585.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3709, 2383, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\sin(c+dx)^n\right)^p}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{3709}$$

$$\frac{\int \csc^3(c+dx)\left(1-\sin^2(c+dx)\right)\left(b\sin^n(c+dx)+a\right)^p d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2383}$$

$$\frac{\int \left(\csc^3(c+dx)\left(b\sin^n(c+dx)+a\right)^p-\csc(c+dx)\left(b\sin^n(c+dx)+a\right)^p\right) d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a+b\sin^n(c+dx))^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sin^n(c+dx)}{a}+1\right)}{an(p+1)}-\frac{1}{2}\csc^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\left(\frac{b\sin^n(c+dx)}{a}+1\right)^{-}}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^n)^p,x]
```

output
```
((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^n)/a]*(a + b*Sin[
c + d*x]^n)^(1 + p))/(a*n*(1 + p)) - (Csc[c + d*x]^2*Hypergeometric2F1[-2/
n, -p, -((2 - n)/n), -((b*Sin[c + d*x]^n)/a)]*(a + b*Sin[c + d*x]^n)^p)/(2
*(1 + (b*Sin[c + d*x]^n)/a)^p))/d
```

3.585.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2383
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.585.4 Maple [F]

$$\int \left(\cot^{3}\left(dx+c\right)\right)\left(a+b(\sin^{n}\left(dx+c\right))\right)^{p}dx$$

input
```
int(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x)
```

output
```
int(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x)
```

3.585.5 Fricas [F]

$$\int \cot^3(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\int \left(b\sin\left(dx+c\right)^n+a\right)^p\cot\left(dx+c\right)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^3, x)
```

3.585.6 Sympy [F(-1)]

Timed out.

$$\int \cot^3(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(cot(d*x+c)**3*(a+b*sin(d*x+c)**n)**p,x)
```

output
```
Timed out
```

3.585.7 Maxima [F]

$$\int \cot^3(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\int \left(b\sin\left(dx+c\right)^n+a\right)^p\cot\left(dx+c\right)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^3, x)
```

3.585.8 Giac [F]

$$\int \cot^3(c+dx)\,(a+b\sin^n(c+dx))^p\,dx=\int (b\sin{(dx+c)}^n+a)^p\cot{(dx+c)}^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^3, x)
```

3.585.9 Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+dx)\,(a+b\sin^n(c+dx))^p\,dx=\int \cot(c+d\,x)^3\,(a+b\sin{(c+d\,x)}^n)^p\,dx$$

input
```
int(cot(c + d*x)^3*(a + b*sin(c + d*x)^n)^p,x)
```

output
```
int(cot(c + d*x)^3*(a + b*sin(c + d*x)^n)^p, x)
```

3.586 $\int (a+b\sin^n(c+dx))^p \tan^4(c+dx)\,dx$

3.586.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sin^n(c+dx))^p \tan^4(c+dx)\,dx = \operatorname{Int}\left((a+b\sin^n(c+dx))^p \tan^4(c+dx),x\right)$$

output
```
Unintegrable((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x)
```

3.586.2 Mathematica [N/A]

Not integrable

Time = 25.69 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p \tan^4(c+dx)\,dx = \int (a+b\sin^n(c+dx))^p \tan^4(c+dx)\,dx$$

input
```
Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^4,x]
```

output
```
Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^4, x]
```

3.586.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3714}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^4\left(a+b\sin(c+dx)^n\right)^p dx$$

$$\Big\downarrow \text{3714}$$

$$\int \tan^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p dx$$

input
```
Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^4,x]
```

output
```
$Aborted
```

3.586.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3714
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sin[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.586.4 Maple [N/A] (verified)

Not integrable

Time = 0.90 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b(\sin^n(dx+c)))^p \left(\tan^4(dx+c)\right) dx$$

input
```
int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x)
```

output
```
int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x)
```

3.586.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p \tan^4(c+dx)\, dx = \int (b\sin(dx+c)^n + a)^p \tan(dx+c)^4\, dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^4, x)
```

3.586.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\sin^n(c+dx))^p \tan^4(c+dx)\, dx = \text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c)**4,x)
```

output
```
Timed out
```

3.586.7 Maxima [N/A]

Not integrable

Time = 6.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p\tan^4(c+dx)\,dx = \int (b\sin(dx+c)^n+a)^p\tan(dx+c)^4\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^4, x)
```

3.586.8 Giac [N/A]

Not integrable

Time = 8.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p\tan^4(c+dx)\,dx = \int (b\sin(dx+c)^n+a)^p\tan(dx+c)^4\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^4, x)
```

3.586.9 Mupad [N/A]

Not integrable

Time = 15.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p\tan^4(c+dx)\,dx = \int \tan(c+d\,x)^4\,(a+b\sin(c+d\,x)^n)^p\,dx$$

input
```
int(tan(c + d*x)^4*(a + b*sin(c + d*x)^n)^p,x)
```

output
```
int(tan(c + d*x)^4*(a + b*sin(c + d*x)^n)^p, x)
```

3.587 $\int (a+b\sin^n(c+dx))^p \tan^2(c+dx)\,dx$

3.587.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sin^n(c+dx))^p \tan^2(c+dx)\,dx = \text{Int}\left((a+b\sin^n(c+dx))^p \tan^2(c+dx),x\right)$$

output
```
Unintegrable((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x)
```

3.587.2 Mathematica [N/A]

Not integrable

Time = 3.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p \tan^2(c+dx)\,dx = \int (a+b\sin^n(c+dx))^p \tan^2(c+dx)\,dx$$

input
```
Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^2,x]
```

output
```
Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^2, x]
```

3.587.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3714}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)\,(a+b\sin^n(c+dx))^p\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan(c+dx)^2\,(a+b\sin(c+dx)^n)^p\,dx$$
$$\Big\downarrow \text{3714}$$
$$\int \tan^2(c+dx)\,(a+b\sin^n(c+dx))^p\,dx$$

input
```
Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^2,x]
```

output
```
$Aborted
```

3.587.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3714
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sin[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.587.4 Maple [N/A] (verified)

Not integrable

Time = 0.87 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b(\sin^n(dx+c)))^p (\tan^2(dx+c))\,dx$$

input
```
int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x)
```

output
```
int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x)
```

3.587.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p \tan^2(c+dx)\,dx = \int (b\sin(dx+c)^n+a)^p \tan(dx+c)^2\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^2, x)
```

3.587.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\sin^n(c+dx))^p \tan^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c)**2,x)
```

output
```
Timed out
```

3.587.7 Maxima [N/A]

Not integrable

Time = 4.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p\tan^2(c+dx)\,dx = \int (b\sin(dx+c)^n+a)^p\tan(dx+c)^2\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^2, x)
```

3.587.8 Giac [N/A]

Not integrable

Time = 6.73 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p\tan^2(c+dx)\,dx = \int (b\sin(dx+c)^n+a)^p\tan(dx+c)^2\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^2, x)
```

3.587.9 Mupad [N/A]

Not integrable

Time = 14.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin^n(c+dx))^p\tan^2(c+dx)\,dx = \int \tan(c+d\,x)^2\,(a+b\sin(c+d\,x)^n)^p\,dx$$

input
```
int(tan(c + d*x)^2*(a + b*sin(c + d*x)^n)^p,x)
```

output
```
int(tan(c + d*x)^2*(a + b*sin(c + d*x)^n)^p, x)
```

3.588 $\int (a+b\sin^n(c+dx))^p \, dx$

3.588.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (a+b\sin^n(c+dx))^p \, dx = \text{Int}((a+b\sin^n(c+dx))^p, x)$$

output
```
Unintegrable((a+b*sin(d*x+c)^n)^p,x)
```

3.588.2 Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sin^n(c+dx))^p \, dx = \int (a+b\sin^n(c+dx))^p \, dx$$

input
```
Integrate[(a + b*Sin[c + d*x]^n)^p,x]
```

output
```
Integrate[(a + b*Sin[c + d*x]^n)^p, x]
```

3.588.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3693}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin^n(c+dx))^p\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a+b\sin(c+dx)^n)^p\,dx$$
$$\Big\downarrow \text{3693}$$
$$\int (a+b\sin^n(c+dx))^p\,dx$$

input
```
Int[(a + b*Sin[c + d*x]^n)^p,x]
```

output
```
$Aborted
```

3.588.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3693
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Unintegrable[(a + b*(c*Sin[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, e, f, n, p
}, x]
```

3.588.4 Maple [N/A] (verified)

Not integrable

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b(\sin^n(dx+c)))^p dx$$

input
```
int((a+b*sin(d*x+c)^n)^p,x)
```

output
```
int((a+b*sin(d*x+c)^n)^p,x)
```

3.588.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sin^n(c+dx))^p \, dx = \int (b\sin(dx+c)^n+a)^p \, dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c)^n + a)^p, x)
```

3.588.6 Sympy [N/A]

Not integrable

Time = 4.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sin^n(c+dx))^p \, dx = \int (a+b\sin^n(c+dx))^p \, dx$$

input
```
integrate((a+b*sin(d*x+c)**n)**p,x)
```

output
```
Integral((a + b*sin(c + d*x)**n)**p, x)
```

3.588.7 Maxima [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sin^n(c+dx))^p\,dx=\int (b\sin(dx+c)^n+a)^p\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p, x)
```

3.588.8 Giac [N/A]

Not integrable

Time = 2.99 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sin^n(c+dx))^p\,dx=\int (b\sin(dx+c)^n+a)^p\,dx$$

input
```
integrate((a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p, x)
```

3.588.9 Mupad [N/A]

Not integrable

Time = 14.97 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sin^n(c+dx))^p\,dx=\int (a+b\sin(c+d\,x)^n)^p\,dx$$

input
```
int((a + b*sin(c + d*x)^n)^p,x)
```

output
```
int((a + b*sin(c + d*x)^n)^p, x)
```

3.589 $\int \cot^2(c+dx) (a+b\sin^n(c+dx))^p \, dx$

3.589.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \cot^2(c+dx) (a+b\sin^n(c+dx))^p \, dx = \mathrm{Int}\left(\cot^2(c+dx) (a+b\sin^n(c+dx))^p, x\right)$$

output
```
Unintegrable(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x)
```

3.589.2 Mathematica [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot^2(c+dx) (a+b\sin^n(c+dx))^p \, dx = \int \cot^2(c+dx) (a+b\sin^n(c+dx))^p \, dx$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^n)^p,x]
```

output
```
Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^n)^p, x]
```

3.589.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3714}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\sin(c+dx)^n\right)^p}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{ 3714}$$

$$\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

input
```
Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^n)^p,x]
```

output
```
$Aborted
```

3.589.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3714
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sin[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.589.4 Maple [N/A] (verified)

Not integrable

Time = 0.76 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(\cot^2(dx+c)\right)(a+b(\sin^n(dx+c)))^p dx$$

input
```
int(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x)
```

output
```
int(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x)
```

3.589.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot^2(c+dx)(a+b\sin^n(c+dx))^p\,dx = \int (b\sin(dx+c)^n+a)^p\cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^2, x)
```

3.589.6 Sympy [N/A]

Not integrable

Time = 113.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \cot^2(c+dx)(a+b\sin^n(c+dx))^p\,dx = \int (a+b\sin^n(c+dx))^p\cot^2(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**2*(a+b*sin(d*x+c)**n)**p,x)
```

output
```
Integral((a + b*sin(c + d*x)**n)**p*cot(c + d*x)**2, x)
```

3.589.7 Maxima [N/A]

Not integrable

Time = 4.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\int (b\sin{(dx+c)}^n+a)^p\cot{(dx+c)}^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^2, x)
```

3.589.8 Giac [N/A]

Not integrable

Time = 3.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\int (b\sin{(dx+c)}^n+a)^p\cot{(dx+c)}^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^2, x)
```

3.589.9 Mupad [N/A]

Not integrable

Time = 14.57 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\int \cot(c+d\,x)^2\,(a+b\sin{(c+d\,x)}^n)^p\,dx$$

input
```
int(cot(c + d*x)^2*(a + b*sin(c + d*x)^n)^p,x)
```

output
```
int(cot(c + d*x)^2*(a + b*sin(c + d*x)^n)^p, x)
```

3.590 $\int \cot^4(c+dx)\,(a+b\sin^n(c+dx))^p\,dx$

3.590.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \cot^4(c+dx)\,(a+b\sin^n(c+dx))^p\,dx = \text{Int}\left(\cot^4(c+dx)\,(a+b\sin^n(c+dx))^p\,,x\right)$$

output
```
Unintegrable(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x)
```

3.590.2 Mathematica [N/A]

Not integrable

Time = 32.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot^4(c+dx)\,(a+b\sin^n(c+dx))^p\,dx = \int \cot^4(c+dx)\,(a+b\sin^n(c+dx))^p\,dx$$

input
```
Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^n)^p,x]
```

output
```
Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^n)^p, x]
```

3.590.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3714}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)\,(a+b\sin^n(c+dx))^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\sin(c+dx)^n)^p}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{ 3714}$$

$$\int \cot^4(c+dx)\,(a+b\sin^n(c+dx))^p\,dx$$

input
```
Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^n)^p,x]
```

output
```
$Aborted
```

3.590.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3714
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sin[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.590.4 Maple [N/A] (verified)

Not integrable

Time = 0.86 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(\cot^{4}\left(dx+c\right)\right)\left(a+b\left(\sin^{n}\left(dx+c\right)\right)\right)^{p}dx$$

input
```
int(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x)
```

output
```
int(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x)
```

3.590.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\int \left(b\sin\left(dx+c\right)^n+a\right)^p\cot\left(dx+c\right)^4\,dx$$

input
```
integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^4, x)
```

3.590.6 Sympy [F(-1)]

Timed out.

$$\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(cot(d*x+c)**4*(a+b*sin(d*x+c)**n)**p,x)
```

output
```
Timed out
```

3.590.7 Maxima [N/A]

Not integrable

Time = 6.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\int (b\sin{(dx+c)}^n+a)^p\cot{(dx+c)}^4\,dx$$

input
```
integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^4, x)
```

3.590.8 Giac [N/A]

Not integrable

Time = 4.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\int (b\sin{(dx+c)}^n+a)^p\cot{(dx+c)}^4\,dx$$

input
```
integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^4, x)
```

3.590.9 Mupad [N/A]

Not integrable

Time = 14.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\int \cot(c+d\,x)^4\,\left(a+b\sin{(c+d\,x)}^n\right)^p\,dx$$

input
```
int(cot(c + d*x)^4*(a + b*sin(c + d*x)^n)^p,x)
```

output
```
int(cot(c + d*x)^4*(a + b*sin(c + d*x)^n)^p, x)
```

3.591 $\int \frac{a+b\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx$

3.591.1 Optimal result

Integrand size = 37, antiderivative size = 107

$$\int \frac{a+b\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx=\frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}}+\frac{(2a-b)\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{\sin(2e+2fx)}}{3fg^2\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}$$

output
```
2/3*(a+b)*(d*sin(f*x+e))^(1/2)/d/f/g/(g*cos(f*x+e))^(3/2)-1/3*(2*a-b)*(sin
(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1
/2))*sin(2*f*x+2*e)^(1/2)/f/g^2/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)
```

3.591.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.95

$$\int \frac{a+b\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx=\frac{2\cos^2(e+fx)^{3/4}\left(5a\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{4},\frac{5}{4},\sin^2(e+fx)\right)\text{s}\right.}{5fg(g\cos(e+f}$$

input
```
Integrate[(a + b*Sin[e + f*x]^2)/((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*
x]]),x]
```

output
```
(2*(Cos[e + f*x]^2)^(3/4)*(5*a*Hypergeometric2F1[1/4, 7/4, 5/4, Sin[e + f*
x]^2]*Sin[e + f*x] + b*Hypergeometric2F1[5/4, 7/4, 9/4, Sin[e + f*x]^2]*Si
n[e + f*x]^3))/(5*f*g*(g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])
```

3.591.3 Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.38, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {3042, 3681, 362, 266, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin^2(e+fx)}{\sqrt{d\sin(e+fx)}(g\cos(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(e+fx)^2}{\sqrt{d\sin(e+fx)}(g\cos(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3681}$$

$$\frac{\cos^2(e+fx)^{3/4}\int \frac{b\sin^2(e+fx)+a}{\sqrt{d\sin(e+fx)}\left(1-\sin^2(e+fx)\right)^{7/4}}d\sin(e+fx)}{fg(g\cos(e+fx))^{3/2}}$$

$$\Big\downarrow \text{362}$$

$$\frac{\cos^2(e+fx)^{3/4}\left(\frac{1}{3}(2a-b)\int \frac{1}{\sqrt{d\sin(e+fx)}\left(1-\sin^2(e+fx)\right)^{3/4}}d\sin(e+fx)+\frac{2(a+b)\sqrt{d\sin(e+fx)}}{3d\left(1-\sin^2(e+fx)\right)^{3/4}}\right)}{fg(g\cos(e+fx))^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\cos^2(e+fx)^{3/4}\left(\frac{2(2a-b)\int \frac{1}{\left(1-\sin^2(e+fx)\right)^{3/4}}d\sqrt{d\sin(e+fx)}}{3d}+\frac{2(a+b)\sqrt{d\sin(e+fx)}}{3d\left(1-\sin^2(e+fx)\right)^{3/4}}\right)}{fg(g\cos(e+fx))^{3/2}}$$

$$\Big\downarrow \text{768}$$

$$\frac{\cos^2(e+fx)^{3/4}\left(\frac{2(2a-b)(d\sin(e+fx))^{3/2}\left(1-d^2\csc^4(e+fx)\right)^{3/4}\int \frac{\csc^3(e+fx)}{\left(1-d^2\csc^4(e+fx)\right)^{3/4}}d\sqrt{d\sin(e+fx)}}{3d\left(1-\sin^2(e+fx)\right)^{3/4}}+\frac{2(a+b)\sqrt{d\sin(e+fx)}}{3d\left(1-\sin^2(e+fx)\right)^{3/4}}\right)}{fg(g\cos(e+fx))^{3/2}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\cos^2(e+fx)^{3/4}\left(\frac{2(a+b)\sqrt{d\sin(e+fx)}}{3d(1-\sin^2(e+fx))^{3/4}}-\frac{2(2a-b)(d\sin(e+fx))^{3/2}(1-d^2\csc^4(e+fx))^{3/4}\int\frac{\csc(e+fx)}{(1-d^4\sin^2(e+fx))^{3/4}}d\csc(e+fx)}{3d(1-\sin^2(e+fx))^{3/4}}\right)}{fg(g\cos(e+fx))^{3/2}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\cos^2(e+fx)^{3/4}\left(\frac{2(a+b)\sqrt{d\sin(e+fx)}}{3d(1-\sin^2(e+fx))^{3/4}}-\frac{(2a-b)(d\sin(e+fx))^{3/2}(1-d^2\csc^4(e+fx))^{3/4}\int\frac{1}{(1-d^3\sin(e+fx))^{3/4}}d(d\sin(e+fx))}{3d(1-\sin^2(e+fx))^{3/4}}\right)}{fg(g\cos(e+fx))^{3/2}}$$

$$\Big\downarrow \text{230}$$

$$\frac{\cos^2(e+fx)^{3/4}\left(\frac{2(a+b)\sqrt{d\sin(e+fx)}}{3d(1-\sin^2(e+fx))^{3/4}}-\frac{2(2a-b)(d\sin(e+fx))^{3/2}(1-d^2\csc^4(e+fx))^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin(d^2\sin(e+fx)),2\right)}{3d^2(1-\sin^2(e+fx))^{3/4}}\right)}{fg(g\cos(e+fx))^{3/2}}$$

input
```
Int[(a + b*Sin[e + f*x]^2)/((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]]),x
]
```

output
```
((Cos[e + f*x]^2)^(3/4)*((2*(a + b)*Sqrt[d*Sin[e + f*x]])/(3*d*(1 - Sin[e
+ f*x]^2)^(3/4)) - (2*(2*a - b)*(1 - d^2*Csc[e + f*x]^4)^(3/4)*EllipticF[A
rcSin[d^2*Sin[e + f*x]]/2, 2]*(d*Sin[e + f*x])^(3/2))/(3*d^2*(1 - Sin[e +
f*x]^2)^(3/4))))/(f*g*(g*Cos[e + f*x])^(3/2))
```

3.591.3.1 Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

3.591. $\int \frac{a+b\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx$

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3681
```
Int[(cos[(e_.) + (f_.)*(x_)]*(c_.))^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Sin[e + f*x], x]}, Simp[ff*c^(2*IntPart[(m - 1)/2] + 1)*((c*Co
s[e + f*x])^(2*FracPart[(m - 1)/2])/(f*(Cos[e + f*x]^2)^FracPart[(m - 1)/2]
))   Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x]
, x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m]
```

3.591. $\int \frac{a+b\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx$

3.591.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(118) = 236.

Time = 7.07 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.62

method	result
default	$\frac{\left(2\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,\sqrt{-\csc(fx+e)+\cot(fx+e)+1}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\,F\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)c\right.}{}$
parts	$\frac{a\left(2\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,\sqrt{-\csc(fx+e)+\cot(fx+e)+1}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\,F\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)\right.}{3}$

input
```
int((a+b*sin(f*x+e)^2)/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
1/3/g^2/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)*(2*(-cot(f*x+e)+csc(f*
x+e)+1)^(1/2)*(-csc(f*x+e)+cot(f*x+e)+1)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1
/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*a*cos(f*x+e)-(
-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(-csc(f*x+e)+cot(f*x+e)+1)^(1/2)*(-csc(f*x
+e)+cot(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/
2))*b*cos(f*x+e)+2*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(-csc(f*x+e)+cot(f*x+e
)+1)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e
)+1)^(1/2),1/2*2^(1/2))*a-(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(-csc(f*x+e)+co
t(f*x+e)+1)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+cs
c(f*x+e)+1)^(1/2),1/2*2^(1/2))*b+tan(f*x+e)*2^(1/2)*a+tan(f*x+e)*2^(1/2)*b
)*2^(1/2)
```

3.591.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.17

$$\int \frac{a+b\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx =$$
$$-\frac{\sqrt{i\,dg}(2\,a-b)\cos\left(fx+e\right)^{2}F(\arcsin\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\,|-1)+\sqrt{-i\,dg}(2\,a-b)\cos\left(fx+e\right)}{3\,dfg^{3}\cos\left(fx+e\right.}$$

input
```
integrate((a+b*sin(f*x+e)^2)/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x,
algorithm="fricas")
```

output
```
-1/3*(sqrt(I*d*g)*(2*a - b)*cos(f*x + e)^2*elliptic_f(arcsin(cos(f*x + e)
+ I*sin(f*x + e)), -1) + sqrt(-I*d*g)*(2*a - b)*cos(f*x + e)^2*elliptic_f(
arcsin(cos(f*x + e) - I*sin(f*x + e)), -1) - 2*sqrt(g*cos(f*x + e))*sqrt(d
*sin(f*x + e))*(a + b))/(d*f*g^3*cos(f*x + e)^2)
```

3.591.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(f*x+e)**2)/(g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(1/2),
x)
```

output
```
Timed out
```

3.591.7 Maxima [F]

$$\int \frac{a+b\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx = \int \frac{b\sin(fx+e)^2+a}{(g\cos(fx+e))^{\frac{5}{2}}\sqrt{d\sin(fx+e)}}\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x,
algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)/((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x +
e))), x)
```

3.591.8 Giac [F]

$$\int \frac{a+b\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx=\int \frac{b\sin\left(fx+e\right)^2+a}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

input
```
integrate((a+b*sin(f*x+e)^2)/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x,
algorithm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)/((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x +
e))), x)
```

3.591.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx=\int \frac{b\sin\left(e+f\,x\right)^2+a}{\left(g\,\cos\left(e+f\,x\right)\right)^{5/2}\,\sqrt{d\,\sin\left(e+f\,x\right)}}\,dx$$

input
```
int((a + b*sin(e + f*x)^2)/((g*cos(e + f*x))^(5/2)*(d*sin(e + f*x))^(1/2))
,x)
```

output
```
int((a + b*sin(e + f*x)^2)/((g*cos(e + f*x))^(5/2)*(d*sin(e + f*x))^(1/2))
, x)
```

3.592 $\int (c\cos(e+fx))^m(d\sin(e+fx))^n\left(a+b\sin^2(e+fx)\right)^{}$

3.592.1 Optimal result

Integrand size = 35, antiderivative size = 137

$$\int (c\cos(e+fx))^m(d\sin(e+fx))^n\left(a+b\sin^2(e+fx)\right)^p\,dx$$
$$=\frac{c\,\text{AppellF1}\left(\frac{1+n}{2},\frac{1-m}{2},-p,\frac{3+n}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)(c\cos(e+fx))^{-1+m}\cos^2(e+fx)^{\frac{1-m}{2}}(d\sin(e+}{df(1+n)}$$

output
```
c*AppellF1(1/2+1/2*n,-1/2*m+1/2,-p,3/2+1/2*n,sin(f*x+e)^2,-b*sin(f*x+e)^2/
a)*(c*cos(f*x+e))^(-1+m)*(cos(f*x+e)^2)^(-1/2*m+1/2)*(d*sin(f*x+e))^(1+n)*
(a+b*sin(f*x+e)^2)^p/d/f/(1+n)/((1+b*sin(f*x+e)^2/a)^p)
```

3.592.2 Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99

$$\int (c\cos(e+fx))^m(d\sin(e+fx))^n\left(a+b\sin^2(e+fx)\right)^p\,dx$$
$$=\frac{\text{AppellF1}\left(\frac{1+n}{2},\frac{1-m}{2},-p,\frac{3+n}{2},\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)(c\cos(e+fx))^m\cos^2(e+fx)^{\frac{1-m}{2}}(d\sin(e+fx)}{f(1+n)}$$

input
```
Integrate[(c*Cos[e + f*x])^m*(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x]^2)^p,x
]
```

output
```
(AppellF1[(1 + n)/2, (1 - m)/2, -p, (3 + n)/2, Sin[e + f*x]^2, -((b*Sin[e
+ f*x]^2)/a)]*(c*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m)/2)*(d*Sin[e + f
*x])^n*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + n)*(1 + (b*Sin[e + f
*x]^2)/a)^p)
```

3.592.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3042, 3681, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\cos(e+fx))^m(d\sin(e+fx))^n\left(a+b\sin^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c\cos(e+fx))^m(d\sin(e+fx))^n\left(a+b\sin(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \text{ 3681}$$

$$\frac{c\cos^2(e+fx)^{\frac{1-m}{2}}(c\cos(e+fx))^{m-1}\int(d\sin(e+fx))^n\left(1-\sin^2(e+fx)\right)^{\frac{m-1}{2}}\left(b\sin^2(e+fx)+a\right)^p d\sin(e+fx)}{f}$$

$$\Big\downarrow \text{ 395}$$

$$\frac{c\cos^2(e+fx)^{\frac{1-m}{2}}(c\cos(e+fx))^{m-1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\int(d\sin(e+fx))^n\left(1-\sin^2(e+f\right.}{f}$$

$$\Big\downarrow \text{ 394}$$

$$\frac{c\cos^2(e+fx)^{\frac{1-m}{2}}(c\cos(e+fx))^{m-1}(d\sin(e+fx))^{n+1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{n+1}{2}\right.}{df(n+1)}$$

input
```
Int[(c*Cos[e + f*x])^m*(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x]^2)^p,x]
```

output
```
(c*AppellF1[(1 + n)/2, (1 - m)/2, -p, (3 + n)/2, Sin[e + f*x]^2, -((b*Sin[
e + f*x]^2)/a)]*(c*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*(d*
Sin[e + f*x])^(1 + n)*(a + b*Sin[e + f*x]^2)^p)/(d*f*(1 + n)*(1 + (b*Sin[e
 + f*x]^2)/a)^p)
```

3.592.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3681
```
Int[(cos[(e_.) + (f_.)*(x_)]*(c_.))^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Sin[e + f*x], x]}, Simp[ff*c^(2*IntPart[(m - 1)/2] + 1)*((c*Co
s[e + f*x])^(2*FracPart[(m - 1)/2])/(f*(Cos[e + f*x]^2)^FracPart[(m - 1)/2]
))   Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x]
, x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m]
```

3.592.4 Maple [F]

$$\int (c\cos (fx+e))^m (d\sin (fx+e))^n \left(a+b\left(\sin^2 (fx+e)\right)\right)^p dx$$

input
```
int((c*cos(f*x+e))^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x)
```

output
```
int((c*cos(f*x+e))^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x)
```

3.592.5 Fricas [F]

$$\int (c\cos(e+fx))^m(d\sin(e+fx))^n \left(a+b\sin^2(e+fx)\right)^p \, dx = \int \left(b\sin (fx+e)^2+a\right)^p (c\cos (fx+e))^m (d\sin (fx+e))^n \, dx$$

input
```
integrate((c*cos(f*x+e))^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x, algori
thm="fricas")
```

output
```
integral((-b*cos(f*x + e)^2 + a + b)^p*(c*cos(f*x + e))^m*(d*sin(f*x + e))
^n, x)
```

3.592.6 Sympy [F(-1)]

Timed out.

$$\int (c\cos(e+fx))^m(d\sin(e+fx))^n \left(a+b\sin^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate((c*cos(f*x+e))**m*(d*sin(f*x+e))**n*(a+b*sin(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.592.7 Maxima [F]

$$\int (c\cos(e+fx))^m(d\sin(e+fx))^n\left(a+b\sin^2(e+fx)\right)^p\,dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p(c\cos\left(fx+e\right))^m\left(d\sin\left(fx+e\right)\right)^n\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x, algori
thm="maxima")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*(c*cos(f*x + e))^m*(d*sin(f*x + e))^n,
x)
```

3.592.8 Giac [F]

$$\int (c\cos(e+fx))^m(d\sin(e+fx))^n\left(a+b\sin^2(e+fx)\right)^p\,dx = \int \left(b\sin\left(fx+e\right)^2+a\right)^p(c\cos\left(fx+e\right))^m\left(d\sin\left(fx+e\right)\right)^n\,dx$$

input
```
integrate((c*cos(f*x+e))^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x, algori
thm="giac")
```

output
```
integrate((b*sin(f*x + e)^2 + a)^p*(c*cos(f*x + e))^m*(d*sin(f*x + e))^n,
x)
```

3.592.9 Mupad [F(-1)]

Timed out.

$$\int (c\cos(e+fx))^m(d\sin(e+fx))^n\left(a+b\sin^2(e+fx)\right)^p\,dx = \int (c\,\cos\left(e+f\,x\right))^m\,\left(d\,\sin\left(e+f\,x\right)\right)^n\,\left(b\sin\left(e+f\,x\right)^2+a\right)^p\,dx$$

input
```
int((c*cos(e + f*x))^m*(d*sin(e + f*x))^n*(a + b*sin(e + f*x)^2)^p,x)
```

output
```
int((c*cos(e + f*x))^m*(d*sin(e + f*x))^n*(a + b*sin(e + f*x)^2)^p, x)
```

3.593 $\int \sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}\,dx$

3.593.1 Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}\,dx = \frac{E\left(e+fx+\tan^{-1}(b,c)\middle|-\frac{b^2+c^2}{a}\right)\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}{f\sqrt{1+\frac{(c\cos(e+fx)+b\sin(e+fx))^2}{a}}}$$

output
```
(cos(e+f*x+arctan(b,c))^2)^(1/2)/cos(e+f*x+arctan(b,c))*EllipticE(sin(e+f*
x+arctan(b,c)),((-b^2-c^2)/a)^(1/2))*(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/
2)/f/(1+(c*cos(f*x+e)+b*sin(f*x+e))^2/a)^(1/2)
```

3.593.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 325 vs. 2(79) = 158.

Time = 2.14 (sec) , antiderivative size = 325, normalized size of antiderivative = 4.11

$$\int \sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}\,dx =$$

$$-\frac{E\left(\arcsin\left(\frac{\sqrt{\frac{\sqrt{(b^2+c^2)^2}+(b^2-c^2)\cos(2(e+fx))-2bc\sin(2(e+fx))}{\sqrt{(b^2+c^2)^2}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{(b^2+c^2)^2}}{2a+b^2+c^2+\sqrt{(b^2+c^2)^2}}\right)\sqrt{2a+b^2+c^2+(-b^2+c^2)}}{\sqrt{2}\sqrt{(b^2+c^2)^2}\,f\sqrt{\frac{2a+b^2+c^2+(-b^2+c^2)\cos(2(e+fx))+2bc\sin(2(e+f}{2a+b^2+c^2+\sqrt{(b^2+c^2)^2}}}}$$

input
```
Integrate[Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^2],x]
```

output
```
-((EllipticE[ArcSin[Sqrt[(Sqrt[(b^2 + c^2)^2] + (b^2 - c^2)*Cos[2*(e + f*x
)] - 2*b*c*Sin[2*(e + f*x)])/Sqrt[(b^2 + c^2)^2]]/Sqrt[2]], (2*Sqrt[(b^2 +
 c^2)^2])/(2*a + b^2 + c^2 + Sqrt[(b^2 + c^2)^2])]*Sqrt[2*a + b^2 + c^2 +
(-b^2 + c^2)*Cos[2*(e + f*x)] + 2*b*c*Sin[2*(e + f*x)]]*(2*b*c*Cos[2*(e +
f*x)] + (b^2 - c^2)*Sin[2*(e + f*x)]))/(Sqrt[2]*Sqrt[(b^2 + c^2)^2]*f*Sqrt
[(2*a + b^2 + c^2 + (-b^2 + c^2)*Cos[2*(e + f*x)] + 2*b*c*Sin[2*(e + f*x)]
)/(2*a + b^2 + c^2 + Sqrt[(b^2 + c^2)^2])]*Sqrt[(2*b*c*Cos[2*(e + f*x)] +
(b^2 - c^2)*Sin[2*(e + f*x)])^2/(b^2 + c^2)^2]))
```

3.593.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3720, 3042, 3719, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}dx$$

$$\Big\downarrow \; 3720$$

$$\frac{\sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}\int \sqrt{\frac{(c\cos(e+fx)+b\sin(e+fx))^2}{a}+1}dx}{\sqrt{\frac{(b\sin(e+fx)+c\cos(e+fx))^2}{a}+1}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}\int \sqrt{\frac{(c\cos(e+fx)+b\sin(e+fx))^2}{a}+1}dx}{\sqrt{\frac{(b\sin(e+fx)+c\cos(e+fx))^2}{a}+1}}$$

$$\Big\downarrow \; 3719$$

$$\frac{\sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}\int \sqrt{\frac{(b^2+c^2)\sin^2(e+fx+\tan^{-1}(b,c))}{a}+1}dx}{\sqrt{\frac{(b\sin(e+fx)+c\cos(e+fx))^2}{a}+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}\int\sqrt{\frac{(b^2+c^2)\sin(e+fx+\tan^{-1}(b,c))^2}{a}+1}dx}{\sqrt{\frac{(b\sin(e+fx)+c\cos(e+fx))^2}{a}+1}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{\sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}E\left(e+fx+\tan^{-1}(b,c)|-\frac{b^2+c^2}{a}\right)}{f\sqrt{\frac{(b\sin(e+fx)+c\cos(e+fx))^2}{a}+1}}$$

input
```
Int[Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^2],x]
```

output
```
(EllipticE[e + f*x + ArcTan[b, c], -((b^2 + c^2)/a)]*Sqrt[a + (c*Cos[e + f
*x] + b*Sin[e + f*x])^2])/(f*Sqrt[1 + (c*Cos[e + f*x] + b*Sin[e + f*x])^2/
a])
```

3.593.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3719
```
Int[((a_) + (b_.)*(cos[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)*sin[(e_.) + (f_.)*
(x_)])^2)^(p_), x_Symbol] :> Int[(a + b*(Sqrt[c^2 + d^2]*Sin[ArcTan[c, d] +
 e + f*x])^2)^p, x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[p^2, 1/4] && GtQ
[a, 0]
```

rule 3720
```
Int[((a_) + (b_.)*(cos[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)*sin[(e_.) + (f_.)*
(x_)])^2)^(p_), x_Symbol] :> Simp[(a + b*(c*Sin[e + f*x] + d*Cos[e + f*x])^
2)^p/(1 + (b*(c*Sin[e + f*x] + d*Cos[e + f*x])^2)/a)^p   Int[(1 + (b*(c*Sin
[e + f*x] + d*Cos[e + f*x])^2)/a)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[p^2, 1/4] &&  !GtQ[a, 0]
```

3.593. $\int\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}\,dx$

3.593.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.42 (sec) , antiderivative size = 315013, normalized size of antiderivative = 3987.51

method	result	size
default	Expression too large to display	315013

input
```
int((a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.593.5 Fricas [F]

$$\int \sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}\,dx = \int \sqrt{(c\cos(fx+e)+b\sin(fx+e))^2+a}\,dx$$

input
```
integrate((a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(2*b*c*cos(f*x + e)*sin(f*x + e) - (b^2 - c^2)*cos(f*x + e)^2
 + b^2 + a), x)
```

3.593.6 Sympy [F]

$$\int \sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}\,dx = \int \sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}\,dx$$

input
```
integrate((a+(c*cos(f*x+e)+b*sin(f*x+e))**2)**(1/2),x)
```

output
```
Integral(sqrt(a + (b*sin(e + f*x) + c*cos(e + f*x))**2), x)
```

3.593.7 Maxima [F]

$$\int \sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}\,dx=\int \sqrt{(c\cos(fx+e)+b\sin(fx+e))^2+a}\,dx$$

input
```
integrate((a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((c*cos(f*x + e) + b*sin(f*x + e))^2 + a), x)
```

3.593.8 Giac [F]

$$\int \sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}\,dx=\int \sqrt{(c\cos(fx+e)+b\sin(fx+e))^2+a}\,dx$$

input
```
integrate((a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((c*cos(f*x + e) + b*sin(f*x + e))^2 + a), x)
```

3.593.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}\,dx=\int \sqrt{a+(c\,\cos(e+f\,x)+b\,\sin(e+f\,x))^2}\,dx$$

input
```
int((a + (c*cos(e + f*x) + b*sin(e + f*x))^2)^(1/2),x)
```

output
```
int((a + (c*cos(e + f*x) + b*sin(e + f*x))^2)^(1/2), x)
```

3.594 $\int \frac{1}{\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}\,dx$

3.594.1 Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \frac{1}{\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}\,dx = \frac{\text{EllipticF}\left(e+fx+\tan^{-1}(b,c),-\frac{b^2+c^2}{a}\right)\sqrt{1+\frac{(c\cos(e+fx)+b\sin(e+fx))^2}{a}}}{f\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}$$

output
```
(cos(e+f*x+arctan(b,c))^2)^(1/2)/cos(e+f*x+arctan(b,c))*EllipticF(sin(e+f*
x+arctan(b,c)),((-b^2-c^2)/a)^(1/2))*(1+(c*cos(f*x+e)+b*sin(f*x+e))^2/a)^(
1/2)/f/(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2)
```

3.594.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 1.86 (sec) , antiderivative size = 529, normalized size of antiderivative = 6.70

$$\int \frac{1}{\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{2a+b^2+c^2+bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}\sin\left(2(e+fx)+\arctan\left(\frac{-b^2+c^2}{2bc}\right)\right)}{2a+b^2+c^2-bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}},\frac{2a+b^2+c^2+bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}\sin\left(2(e+fx)+\text{arcta}\right.}{2a+b^2+c^2+bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}}\right.}{}$$

input
```
Integrate[1/Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^2],x]
```

output
```
(Sqrt[2]*AppellF1[1/2, 1/2, 1/2, 3/2, (2*a + b^2 + c^2 + b*c*Sqrt[(b^2 + c
^2)^2/(b^2*c^2)]*Sin[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]])/(2*a + b
^2 + c^2 - b*c*Sqrt[(b^2 + c^2)^2/(b^2*c^2)]), (2*a + b^2 + c^2 + b*c*Sqrt
[(b^2 + c^2)^2/(b^2*c^2)]*Sin[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]])
/(2*a + b^2 + c^2 + b*c*Sqrt[(b^2 + c^2)^2/(b^2*c^2)])]*Sec[2*(e + f*x) +
ArcTan[(-b^2 + c^2)/(2*b*c)]]*Sqrt[-((b*c*Sqrt[(b^2 + c^2)^2/(b^2*c^2)]*(-
1 + Sin[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]]))/(2*a + b^2 + c^2 + b
*c*Sqrt[(b^2 + c^2)^2/(b^2*c^2)]))]*Sqrt[-((b*c*Sqrt[(b^2 + c^2)^2/(b^2*c^
2)]*(1 + Sin[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]]))/(2*a + b^2 + c^
2 - b*c*Sqrt[(b^2 + c^2)^2/(b^2*c^2)]))]*Sqrt[2*a + b^2 + c^2 + b*c*Sqrt[(
b^2 + c^2)^2/(b^2*c^2)]*Sin[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]]])/
(b*c*Sqrt[(b^2 + c^2)^2/(b^2*c^2)]*f)
```

3.594.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3720, 3042, 3719, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}}dx$$

$$\Big\downarrow \text{ 3720}$$

$$\frac{\sqrt{\frac{(b\sin(e+fx)+c\cos(e+fx))^2}{a}+1}\int \frac{1}{\sqrt{\frac{(c\cos(e+fx)+b\sin(e+fx))^2}{a}+1}}dx}{\sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\frac{(b\sin(e+fx)+c\cos(e+fx))^2}{a}+1}\int \frac{1}{\sqrt{\frac{(c\cos(e+fx)+b\sin(e+fx))^2}{a}+1}}dx}{\sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}}$$

$$\Big\downarrow \; 3719$$

$$\frac{\sqrt{\frac{(b\sin(e+fx)+c\cos(e+fx))^2}{a}+1}\int \frac{1}{\sqrt{\frac{(b^2+c^2)\sin^2\left(e+fx+\tan^{-1}(b,c)\right)}{a}+1}}dx}{\sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\frac{(b\sin(e+fx)+c\cos(e+fx))^2}{a}+1}\int \frac{1}{\sqrt{\frac{(b^2+c^2)\sin\left(e+fx+\tan^{-1}(b,c)\right)^2}{a}+1}}dx}{\sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}}$$

$$\Big\downarrow \; 3661$$

$$\frac{\sqrt{\frac{(b\sin(e+fx)+c\cos(e+fx))^2}{a}+1}\,\text{EllipticF}\left(e+fx+\tan^{-1}(b,c),-\frac{b^2+c^2}{a}\right)}{f\sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}}$$

input
```
Int[1/Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^2],x]
```

output
```
(EllipticF[e + f*x + ArcTan[b, c], -((b^2 + c^2)/a)]*Sqrt[1 + (c*Cos[e + f
*x] + b*Sin[e + f*x])^2/a])/(f*Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^
2])
```

3.594.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3719
```
Int[((a_) + (b_.)*(cos[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)*sin[(e_.) + (f_.)*
(x_)])^2)^(p_), x_Symbol] :> Int[(a + b*(Sqrt[c^2 + d^2]*Sin[ArcTan[c, d] +
 e + f*x])^2)^p, x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[p^2, 1/4] && GtQ
[a, 0]
```

3.594. $\int \frac{1}{\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}\,dx$

rule 3720
```
Int[((a_) + (b_.)*(cos[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)*sin[(e_.) + (f_.)*
(x_)])^2)^(p_), x_Symbol] :> Simp[(a + b*(c*Sin[e + f*x] + d*Cos[e + f*x])^
2)^p/(1 + (b*(c*Sin[e + f*x] + d*Cos[e + f*x])^2)/a)^p   Int[(1 + (b*(c*Sin
[e + f*x] + d*Cos[e + f*x])^2)/a)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[p^2, 1/4] &&  !GtQ[a, 0]
```

3.594.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.64 (sec) , antiderivative size = 2174, normalized size of antiderivative = 27.52

method	result	size
default	Expression too large to display	2174

input
```
int(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/f*((RootOf((c^2+a)*_Z^4-4*b*c*_Z^3+(4*b^2-2*c^2+2*a)*_Z^2+4*_Z*b*c+c^2+
a,index=1)*sin(f*x+e)+cos(f*x+e)-1)/(RootOf((c^2+a)*_Z^4-4*b*c*_Z^3+(4*b^2
-2*c^2+2*a)*_Z^2+4*_Z*b*c+c^2+a,index=2)*sin(f*x+e)+cos(f*x+e)-1)*(RootOf(
(c^2+a)*_Z^4-4*b*c*_Z^3+(4*b^2-2*c^2+2*a)*_Z^2+4*_Z*b*c+c^2+a,index=2)-Roo
tOf((c^2+a)*_Z^4-4*b*c*_Z^3+(4*b^2-2*c^2+2*a)*_Z^2+4*_Z*b*c+c^2+a,index=4)
)/(RootOf((c^2+a)*_Z^4-4*b*c*_Z^3+(4*b^2-2*c^2+2*a)*_Z^2+4*_Z*b*c+c^2+a,in
dex=1)-RootOf((c^2+a)*_Z^4-4*b*c*_Z^3+(4*b^2-2*c^2+2*a)*_Z^2+4*_Z*b*c+c^2+
a,index=4)))^(1/2)*(RootOf((c^2+a)*_Z^4-4*b*c*_Z^3+(4*b^2-2*c^2+2*a)*_Z^2+
4*_Z*b*c+c^2+a,index=2)*sin(f*x+e)+cos(f*x+e)-1)^2*(-(RootOf((c^2+a)*_Z^4-
4*b*c*_Z^3+(4*b^2-2*c^2+2*a)*_Z^2+4*_Z*b*c+c^2+a,index=2)-RootOf((c^2+a)*_
Z^4-4*b*c*_Z^3+(4*b^2-2*c^2+2*a)*_Z^2+4*_Z*b*c+c^2+a,index=1))*(RootOf((c^
2+a)*_Z^4-4*b*c*_Z^3+(4*b^2-2*c^2+2*a)*_Z^2+4*_Z*b*c+c^2+a,index=3)*sin(f*
x+e)+cos(f*x+e)-1)/(RootOf((c^2+a)*_Z^4-4*b*c*_Z^3+(4*b^2-2*c^2+2*a)*_Z^2+
4*_Z*b*c+c^2+a,index=1)-RootOf((c^2+a)*_Z^4-4*b*c*_Z^3+(4*b^2-2*c^2+2*a)*_
Z^2+4*_Z*b*c+c^2+a,index=3))/(RootOf((c^2+a)*_Z^4-4*b*c*_Z^3+(4*b^2-2*c^2+
2*a)*_Z^2+4*_Z*b*c+c^2+a,index=2)*sin(f*x+e)+cos(f*x+e)-1))^(1/2)*(-(RootO
f((c^2+a)*_Z^4-4*b*c*_Z^3+(4*b^2-2*c^2+2*a)*_Z^2+4*_Z*b*c+c^2+a,index=2)-R
ootOf((c^2+a)*_Z^4-4*b*c*_Z^3+(4*b^2-2*c^2+2*a)*_Z^2+4*_Z*b*c+c^2+a,index=
1))*(RootOf((c^2+a)*_Z^4-4*b*c*_Z^3+(4*b^2-2*c^2+2*a)*_Z^2+4*_Z*b*c+c^2+a,
index=4)*sin(f*x+e)+cos(f*x+e)-1)/(RootOf((c^2+a)*_Z^4-4*b*c*_Z^3+(4*b^...
```

3.594.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 1623, normalized size of antiderivative = 20.54

$$\int \frac{1}{\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="fricas")
```

output
```
-((b^3 + b*c^2 - I*c^3 + 2*a*b - I*(b^2 + 2*a)*c - 2*(b^3 - 3*I*b^2*c - 3*
b*c^2 + I*c^3)*sqrt((a*b^6 - 4*I*a*b*c^5 + a*c^6 + a^2*b^4 - 4*I*a^2*b*c^3
 - (5*a*b^2 - a^2)*c^4 - (5*a*b^4 + 6*a^2*b^2)*c^2 + 4*I*(a*b^5 + a^2*b^3)
*c)/(b^8 + 4*b^6*c^2 + 6*b^4*c^4 + 4*b^2*c^6 + c^8)))*sqrt((b^4 + 2*I*b*c^
3 - c^4 + 2*a*b^2 - 2*a*c^2 + 2*I*(b^3 + 2*a*b)*c + 2*(b^4 + 2*b^2*c^2 + c
^4)*sqrt((a*b^6 - 4*I*a*b*c^5 + a*c^6 + a^2*b^4 - 4*I*a^2*b*c^3 - (5*a*b^2
 - a^2)*c^4 - (5*a*b^4 + 6*a^2*b^2)*c^2 + 4*I*(a*b^5 + a^2*b^3)*c)/(b^8 +
4*b^6*c^2 + 6*b^4*c^4 + 4*b^2*c^6 + c^8)))/(b^4 + 2*b^2*c^2 + c^4))*ellipt
ic_f(arcsin(sqrt((b^4 + 2*I*b*c^3 - c^4 + 2*a*b^2 - 2*a*c^2 + 2*I*(b^3 + 2
*a*b)*c + 2*(b^4 + 2*b^2*c^2 + c^4)*sqrt((a*b^6 - 4*I*a*b*c^5 + a*c^6 + a^
2*b^4 - 4*I*a^2*b*c^3 - (5*a*b^2 - a^2)*c^4 - (5*a*b^4 + 6*a^2*b^2)*c^2 +
4*I*(a*b^5 + a^2*b^3)*c)/(b^8 + 4*b^6*c^2 + 6*b^4*c^4 + 4*b^2*c^6 + c^8)))
/(b^4 + 2*b^2*c^2 + c^4))*(cos(f*x + e) + I*sin(f*x + e))), (b^4 + c^4 + 8
*a*b^2 + 2*(b^2 + 4*a)*c^2 + 8*a^2 - 4*(b^4 - 2*I*b*c^3 - c^4 + 2*a*b^2 -
2*a*c^2 - 2*I*(b^3 + 2*a*b)*c)*sqrt((a*b^6 - 4*I*a*b*c^5 + a*c^6 + a^2*b^4
 - 4*I*a^2*b*c^3 - (5*a*b^2 - a^2)*c^4 - (5*a*b^4 + 6*a^2*b^2)*c^2 + 4*I*(
a*b^5 + a^2*b^3)*c)/(b^8 + 4*b^6*c^2 + 6*b^4*c^4 + 4*b^2*c^6 + c^8)))/(b^4
 + 2*b^2*c^2 + c^4)) + (b^3 + b*c^2 + I*c^3 + 2*a*b + I*(b^2 + 2*a)*c - 2*
(b^3 + 3*I*b^2*c - 3*b*c^2 - I*c^3)*sqrt((a*b^6 + 4*I*a*b*c^5 + a*c^6 + a^
2*b^4 + 4*I*a^2*b*c^3 - (5*a*b^2 - a^2)*c^4 - (5*a*b^4 + 6*a^2*b^2)*c^2...
```

3.594.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}\,dx = \int \frac{1}{\sqrt{a+(b\sin{(e+fx)}+c\cos{(e+fx)})^2}}\,dx$$

input
```
integrate(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + (b*sin(e + f*x) + c*cos(e + f*x))**2), x)
```

3.594.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}\,dx=\int \frac{1}{\sqrt{(c\cos{(fx+e)}+b\sin{(fx+e)})^2+a}}\,dx$$

input
```
integrate(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((c*cos(f*x + e) + b*sin(f*x + e))^2 + a), x)
```

3.594.8 Giac [F]

$$\int \frac{1}{\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}\,dx=\int \frac{1}{\sqrt{(c\cos{(fx+e)}+b\sin{(fx+e)})^2+a}}\,dx$$

input
```
integrate(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((c*cos(f*x + e) + b*sin(f*x + e))^2 + a), x)
```

3.594.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}\,dx =\int \frac{1}{\sqrt{a+{\left(c\,\cos{(e+f\,x)}+b\,\sin{(e+f\,x)}\right)}^2}}\,dx$$

input
```
int(1/(a + (c*cos(e + f*x) + b*sin(e + f*x))^2)^(1/2),x)
```

output
```
int(1/(a + (c*cos(e + f*x) + b*sin(e + f*x))^2)^(1/2), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
   print ("Enter grade_antiderivative for sagemath")
   print("Enter grade_antiderivative, result=",result)
   print("Enter grade_antiderivative, optimal=",optimal)
   print("type(anti)=",type(result))
   print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
   if result.has(I):
      if optimal.has(I): #both result and optimal complex
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
         else:
            grade = "B"
            grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
      else: #result contains complex but optimal is not
         grade = "C"
         grade_annotation ="Result contains complex when optimal does not."
   else: # result do not contain complex, this assumes optimal do not as well
      if leaf_count_result <= 2*leaf_count_optimal:
         grade = "A"
         grade_annotation =" "
      else:
         grade = "B"
         grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
   grade = "C"
   grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```